CTE3240Mi College EEM van wanden
19 okt. 2011
Hoogenbaom

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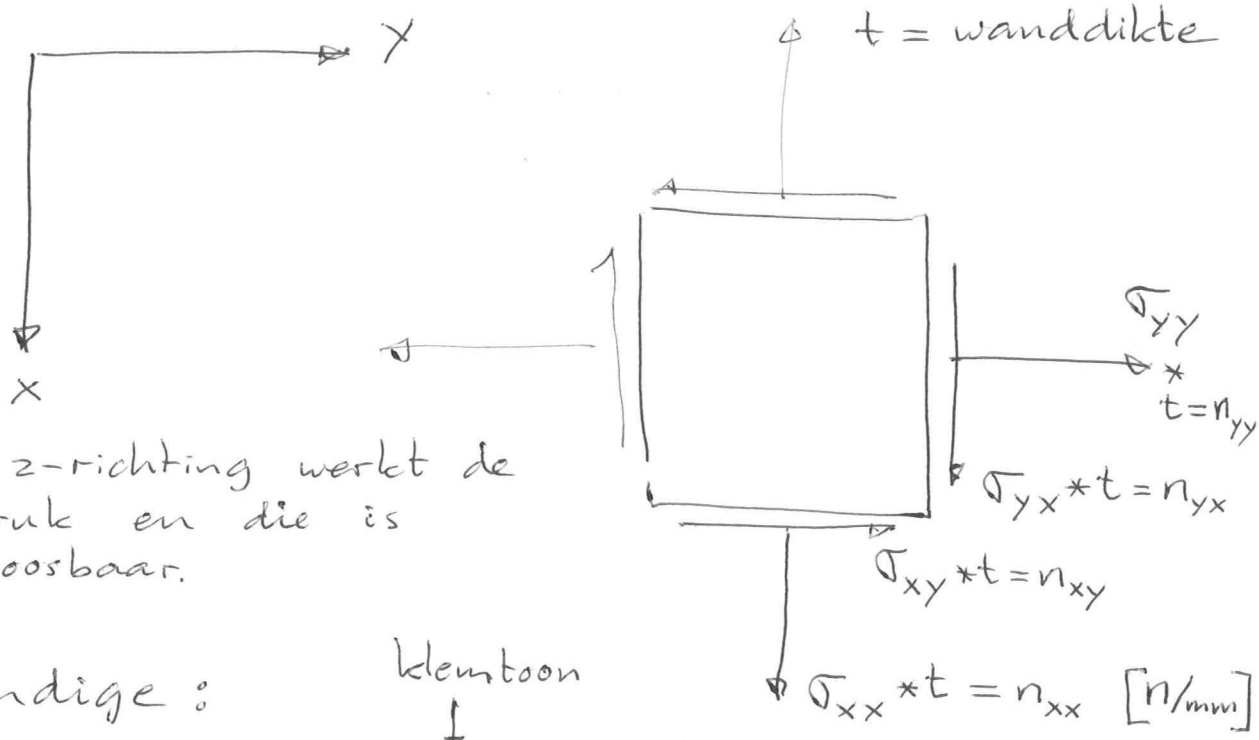
kubusje
geslepen uit een wand



In de z-richting werkt de luchtdruk en die is verwaarloosbaar.

wiskundige: klemtoon ↓
"kubusje is infinitesimaal" = oneindig klein

ingenieur:
"kubusje is niet te groot en niet te klein"

veel kleiner dan de breedte en hoogte van de wand, anders is de spanning op een vlakje niet gelijkmatig verdeeld.

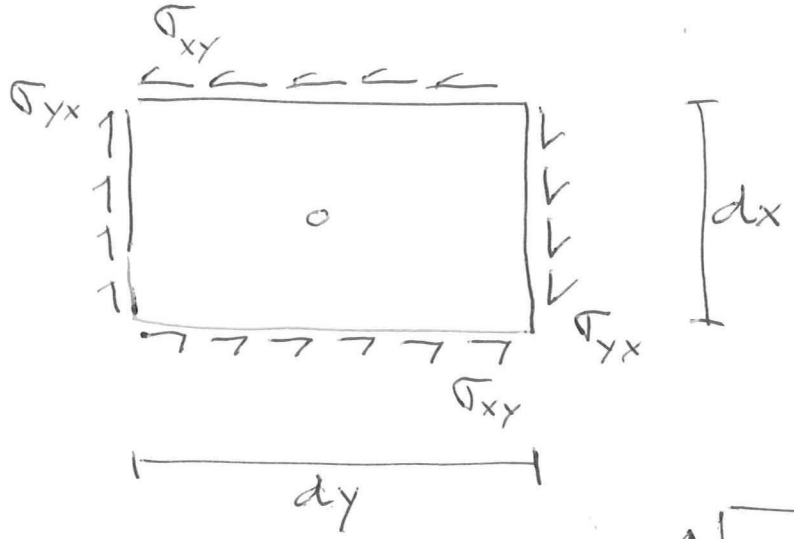
er moeten een paar grindstenen in passen, anders is het geen beton.

Dit kubusje noemen we het representatief-volume.

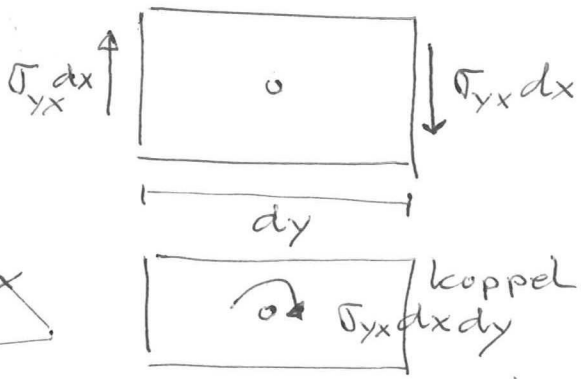
stelling

$$\sigma_{xy} = \sigma_{yx}$$

bewijs



momenten-evenwicht

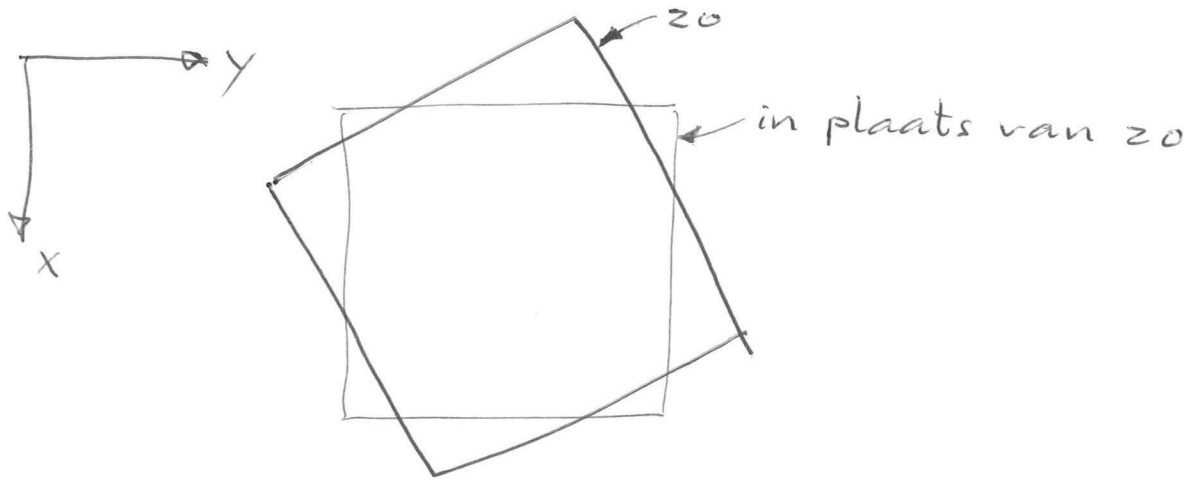


$$\sigma_{yx}\,dx\,dy = \sigma_{xy}\,dy\,dx$$

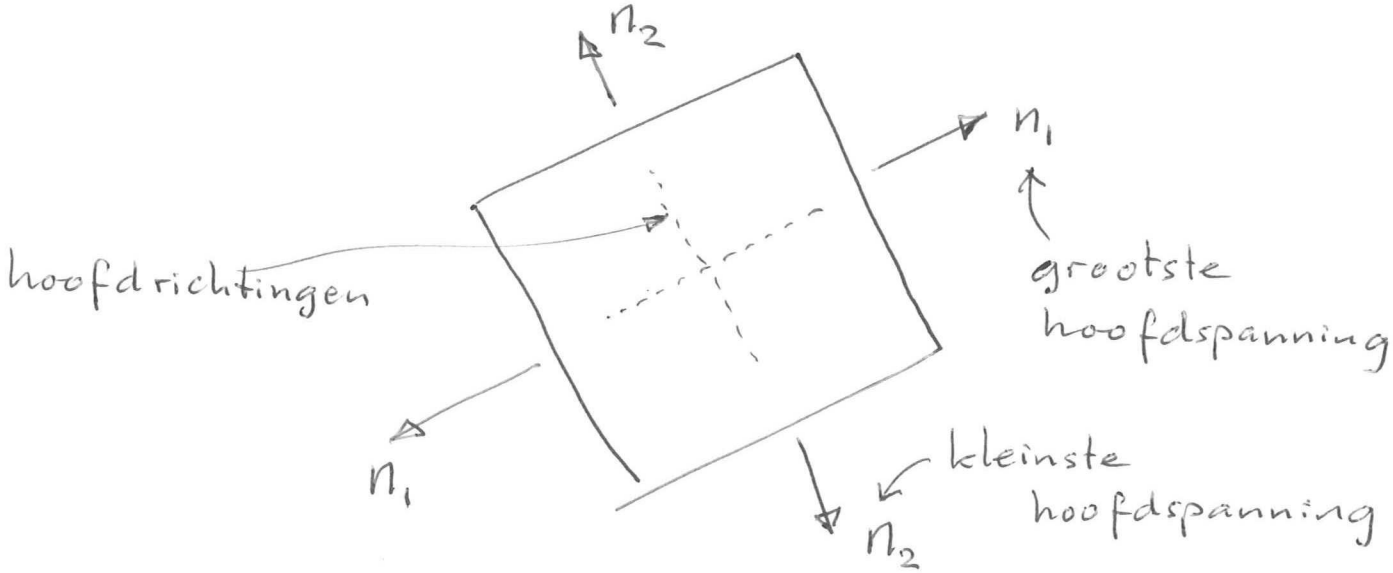
Q.E.D. (Quote Erat Demonstrandum)

↑ hetgeen te bewijzen was

Het kubusje anders uit de muur slijpen.



zodanig dat er geen schuif-spanningen zijn.

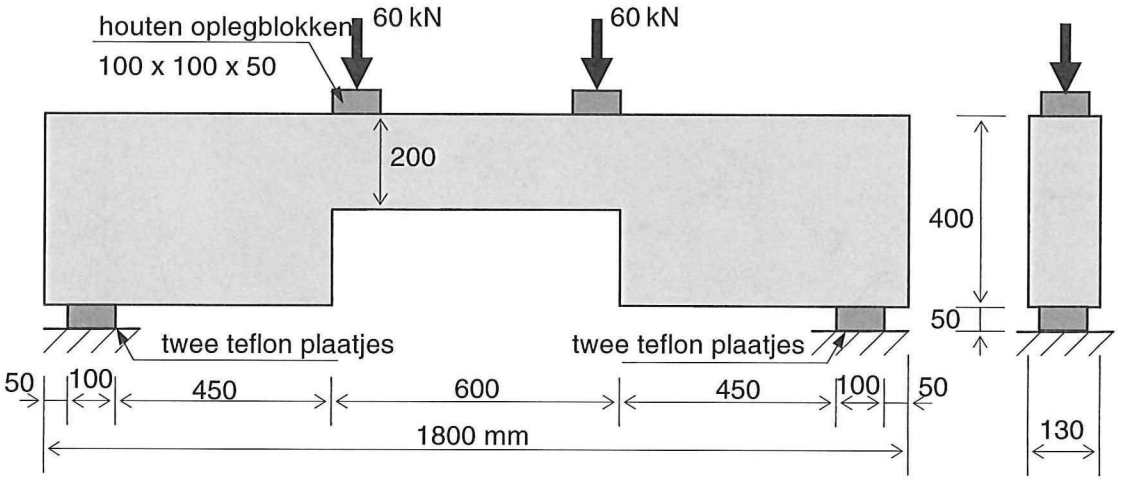


$$n_1 = \frac{n_{xx} + n_{yy}}{2} + \sqrt{\left(\frac{n_{xx} - n_{yy}}{2}\right)^2 + n_{xy}^2}$$

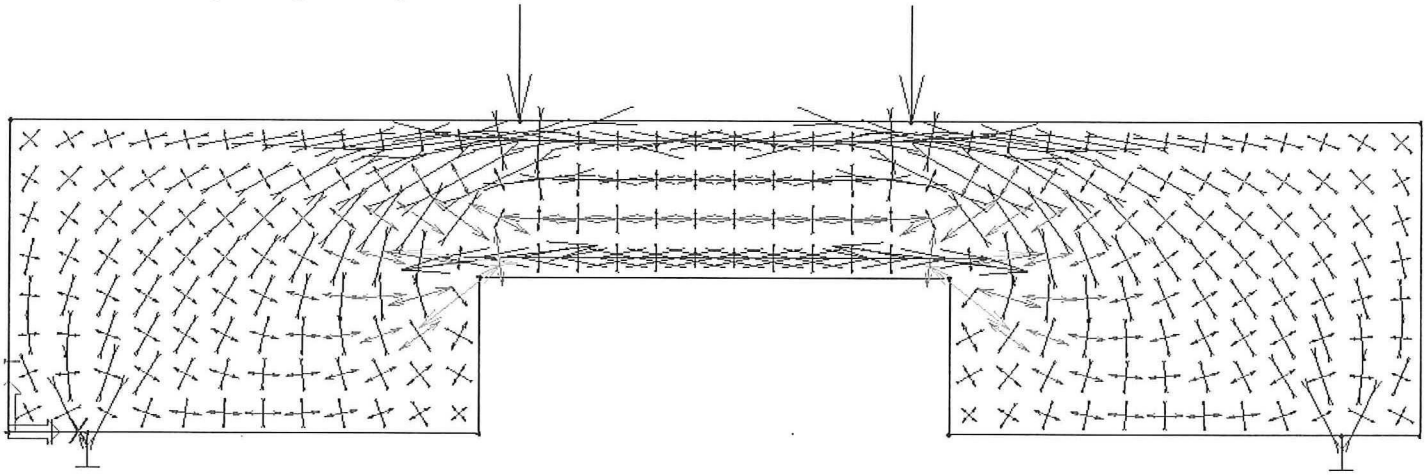
$$n_2 = \frac{n_{xx} + n_{yy}}{2} - \sqrt{\left(\frac{n_{xx} - n_{yy}}{2}\right)^2 + n_{xy}^2}$$

(Circel van Mohr)

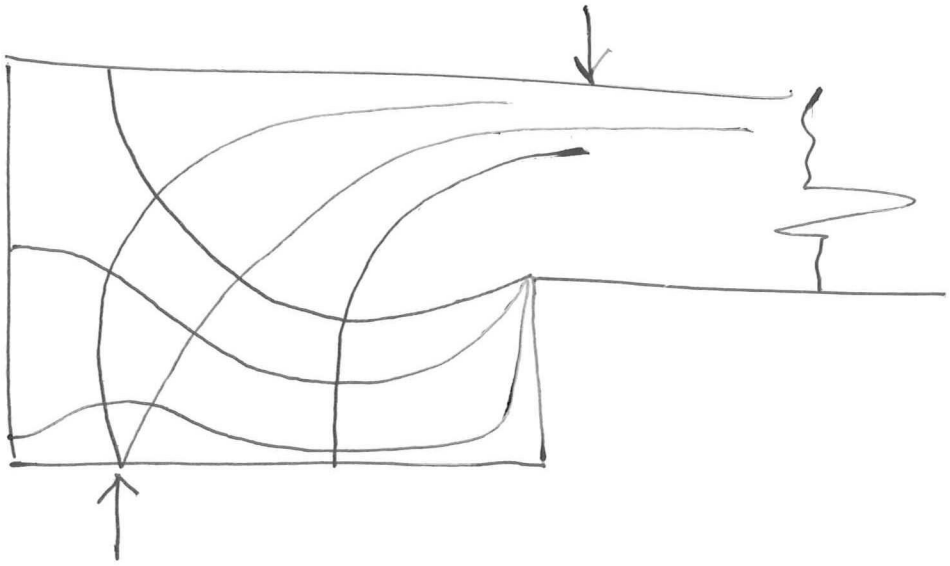
Voorbeeld



Hoofdspanningsrichtingen



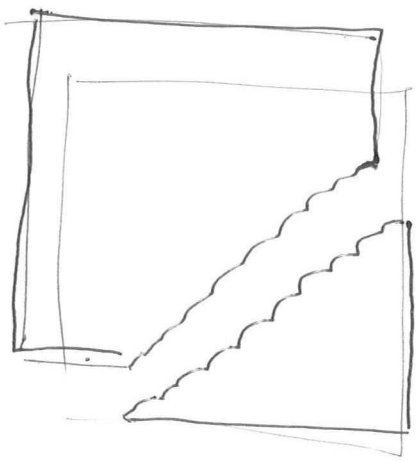
Volg met een potlood de pijlen, dan krijg je de trajectoriën.



De trajectoriën staan loodrecht op elkaar en loodrecht op de randen.

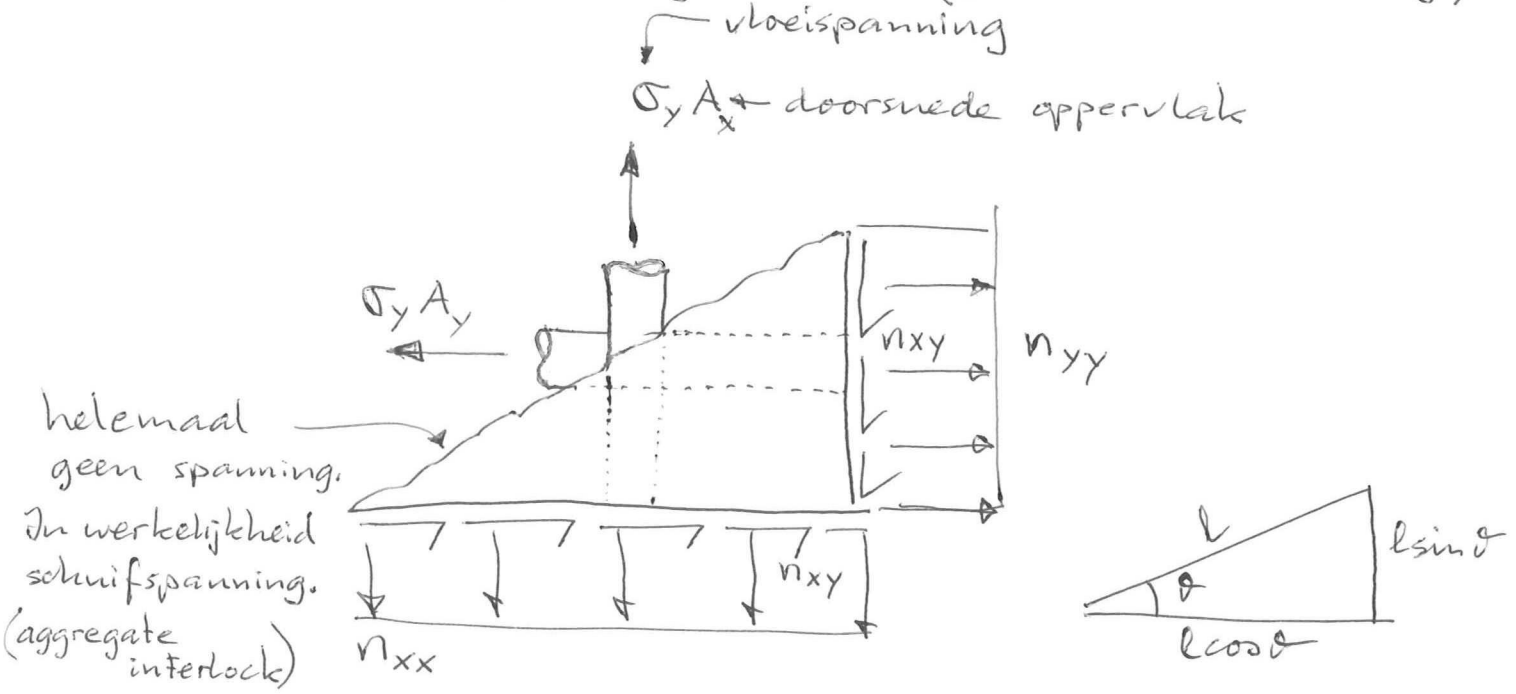
(Vierkantennet)

Het kubusje scheurt



uiterste grenstoestand (UGT)
wanneer het aan het instorten is.

Er zit wapening in (géén bewapening)



Evenwicht in de x-richting:

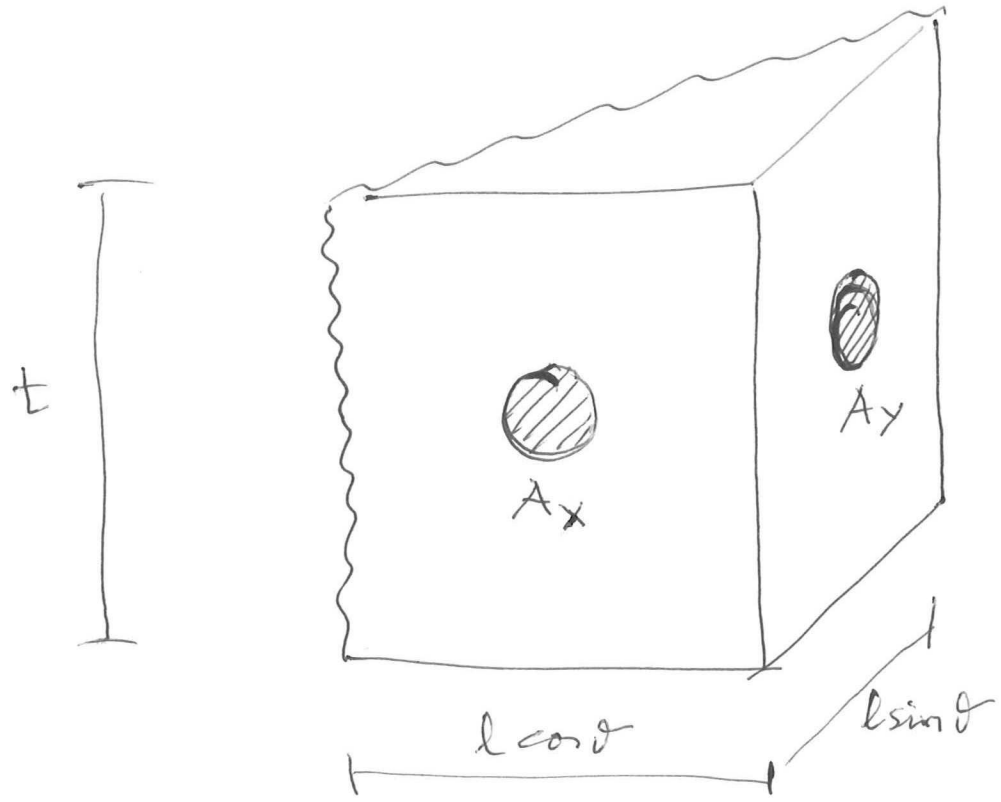
$$n_{xx}\, l\cos\vartheta + n_{xy}\, l\sin\vartheta = \sigma_y A_x \qquad (1)$$

Evenwicht in de y-richting:

$$n_{yy}\, l\sin\vartheta + n_{xy}\, l\cos\vartheta = \sigma_y A_y \qquad (2)$$

Wapeningspercentages

$$\rho_x = \frac{A_x}{t\, l\cos\vartheta} \qquad \rho_y = \frac{A_y}{t\, l\sin\vartheta} \qquad (3)\ (4)$$


t
Ax
Ay
l cos θ
l sin θ

De belasting n_{xx}, n_{yy}, n_{xy} dragen met zo min mogelijk wapening.

$$\rho = \rho_x + \rho_y = \text{minimaal} \qquad (5)$$

$$\rho_x \geq 0 \quad \rho_y \geq 0$$

substitueer (1) in (3)

$$\rho_x = \frac{n_{xx}\, l \cos\vartheta + n_{xy}\, l \sin\vartheta}{\sigma_y\, t\, l \cos\vartheta}$$

$$= \frac{n_{xx} + n_{xy} \tan\vartheta}{\sigma_y\, t} \qquad (6)$$

substitueer (2) in (4)

$$\rho_y = \frac{n_{yy}\, l \sin\vartheta + n_{xy}\, l \cos\vartheta}{\sigma_y\, t\, l \sin\vartheta}$$

$$= \frac{n_{yy} + n_{xy} \cot\vartheta}{\sigma_y\, t} \qquad (7)$$

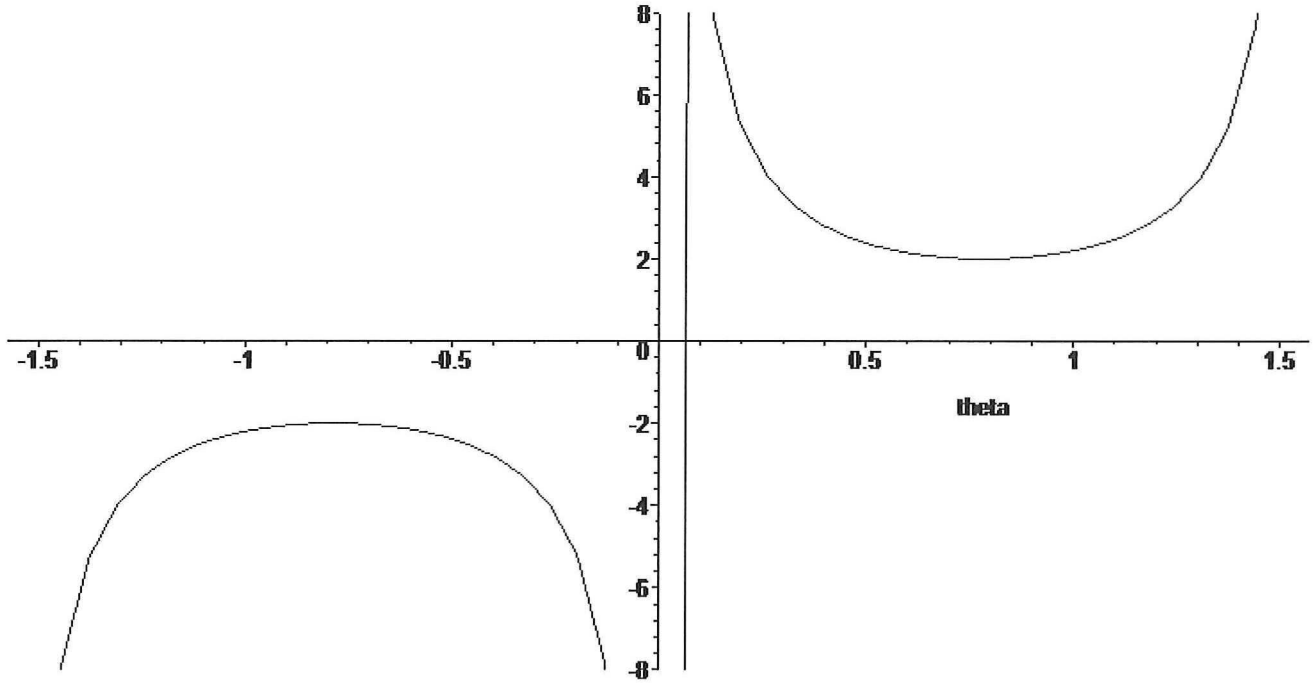
substitueer (6) en (7) in (5)

$$\rho = \frac{n_{xx} + n_{yy} + n_{xy}(\tan\vartheta + \cot\vartheta)}{\sigma_y\, t}$$

Maple 13 - [Untitled (1) - [Server 1]]

File Edit View Insert Format Spreadsheet Window Help

```
> restart:
> f1 := tan(theta) + cot(theta):
> plot( f1, theta= -Pi/2..Pi/2, -8..8 );
```




```
> f2 := diff( tan(theta)+cot(theta), theta );
```

$$f2 := \tan(\theta)^2 - \cot(\theta)^2$$

```
> solve( f2=0, theta );
```

$$\frac{\pi}{4}, -\frac{\pi}{4}$$

```
> theta:=Pi/4:
> tan(theta);
```

$$1$$

```
> theta:=-Pi/4:
> tan(theta);
```

$$-1$$

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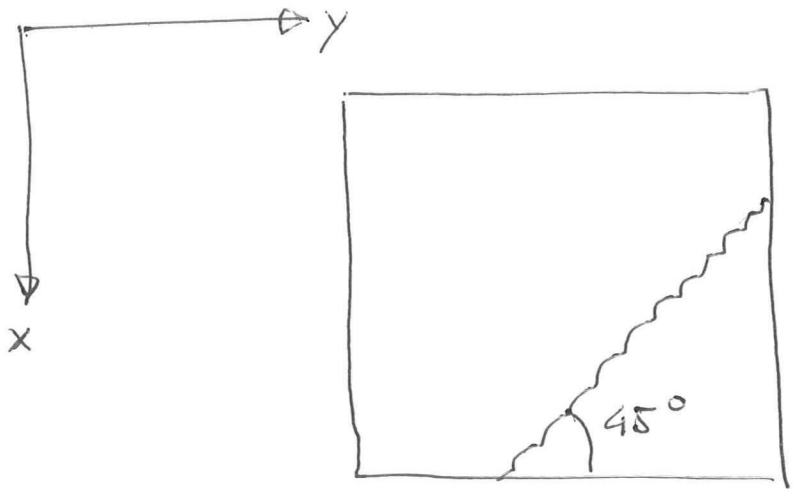
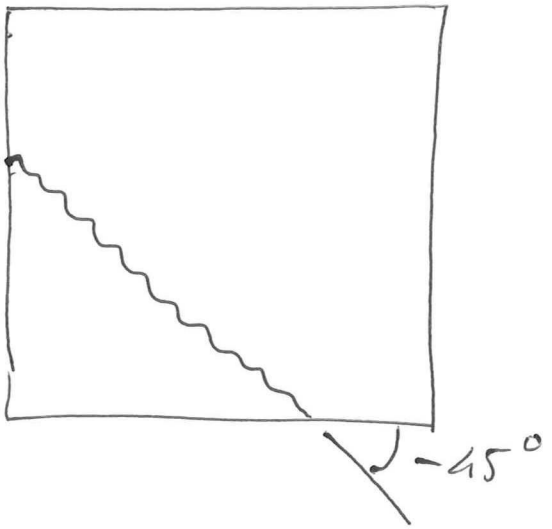
Dus:

Als we de wapening minimaal kiezen is de scheurrichting 45° of −45°





De optimale wapening is

$$\rho_x = \frac{n_{xx} + |n_{xy}|}{\sigma_y t}$$

$$\rho_y = \frac{n_{yy} + |n_{xy}|}{\sigma_y t}$$

spanning in beton

$$\sigma_c = \frac{-2|n_{xy}|}{t}$$

Tenzij $\rho_x < 0$. In dit geval is de optimale wapening (zonder bewijs)

$$\rho_x = 0$$

$$\rho_y = \frac{n_{yy} - \frac{n_{xy}^2}{n_{xx}}}{\sigma_y t}$$

$$\sigma_c = \frac{n_{xx} + \frac{n_{xy}^2}{n_{xx}}}{t}$$

Tenzij $\rho_y < 0$. In dit geval is de optimale wapening (zonder bewijs)

$$\rho_x = \frac{n_{xx} - \frac{n_{xy}^2}{n_{yy}}}{\sigma_y t}$$

$$\rho_y = 0$$

$$\sigma_c = \frac{n_{yy} + \frac{n_{xy}^2}{n_{yy}}}{t}$$

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Plasticiteit en grensspanningshypothesen

GEWAPEND BETON 2D

- Situatie

n_{xx}=1200, n_{yy}=-200, n_{xy}=-400 kN/m
f_y = 500 N/mm², f'_c= 30 N/mm²
Wanddikte = 100 mm

- Wapening

n_{sx}= 1200+400 = 1600 kN/m
n_{sy}= -200+400 = 200

- Beton

n_c= -2x400 = -800 kN/m

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Plasticiteit en grensspanningshypothesen

GEWAPEND BETON 2D

A_{sx}=1600/500 = 3.20 mm = 3200 mm²/m
2∅12–280 = 2π/4x12²x1000/280 = 3231
OK

A_{sy}=200/500 = 0.40 mm = 400 mm²/m
2∅6–500 = 2π/4x6²x1000/500 = 452 OK

σ_c = 800/100 = 8 N/mm² < f'_c OK

(veiligheidsfactoren weggelaten)

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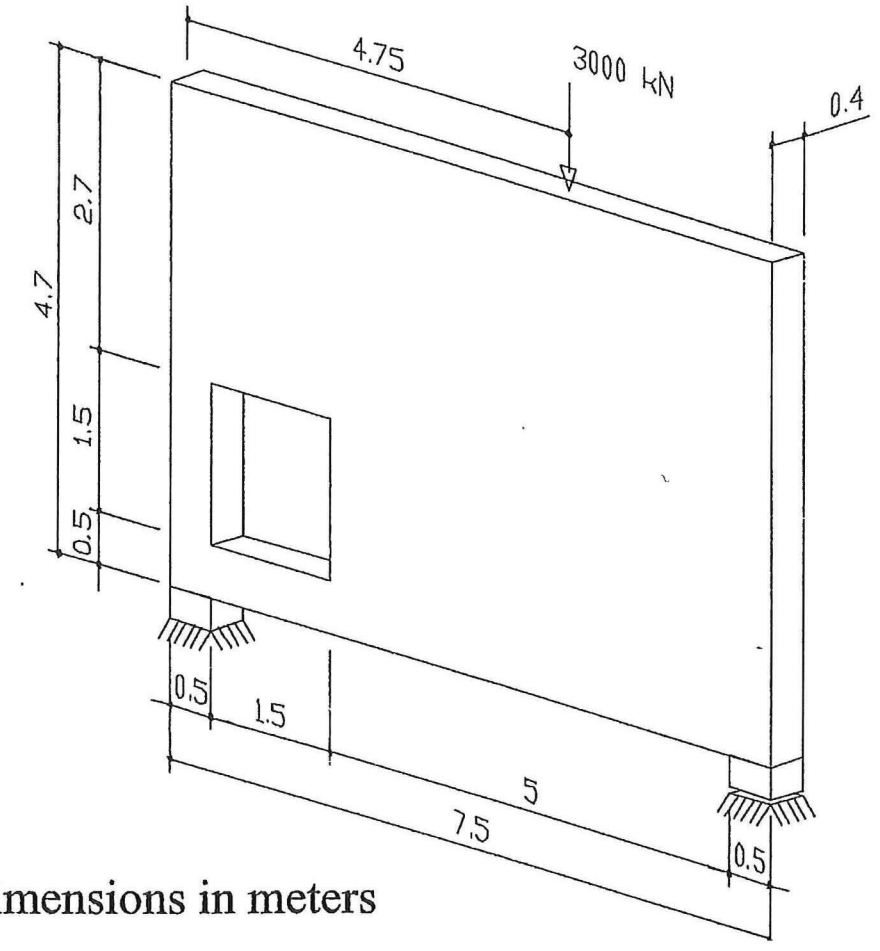
Plasticiteit en grensspanningshypothesen

GEWAPEND BETON 2D

Wandligger 4.7 x 7.5 m
2 oplegging
opening 1.5 x 1.5 m
puntlast 3000 kN
fcd = 16.67 N/mm², fyd = 435 N/mm²



Dimensions in meters

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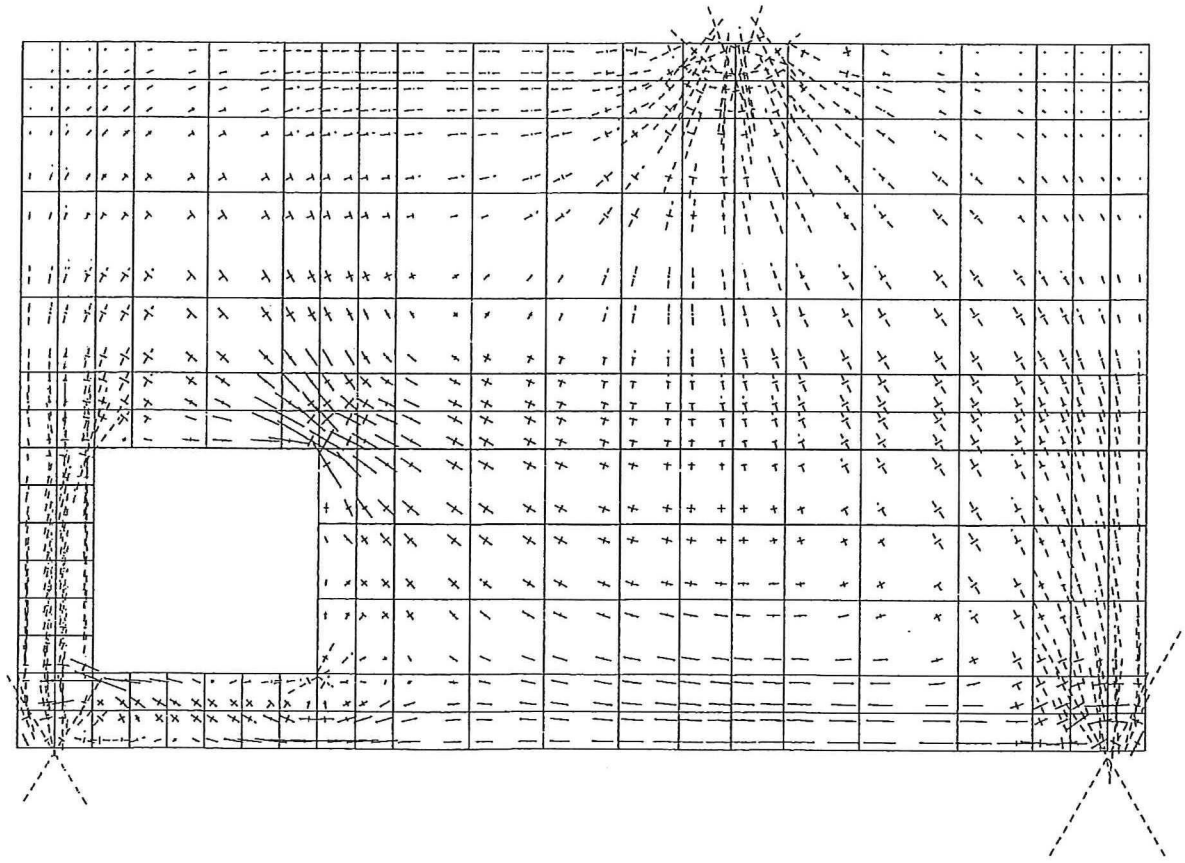
Plasticiteit en grensspanningshypothesen

GEWAPEND BETON 2D

Hoofdspanningen (lineair elastisch)



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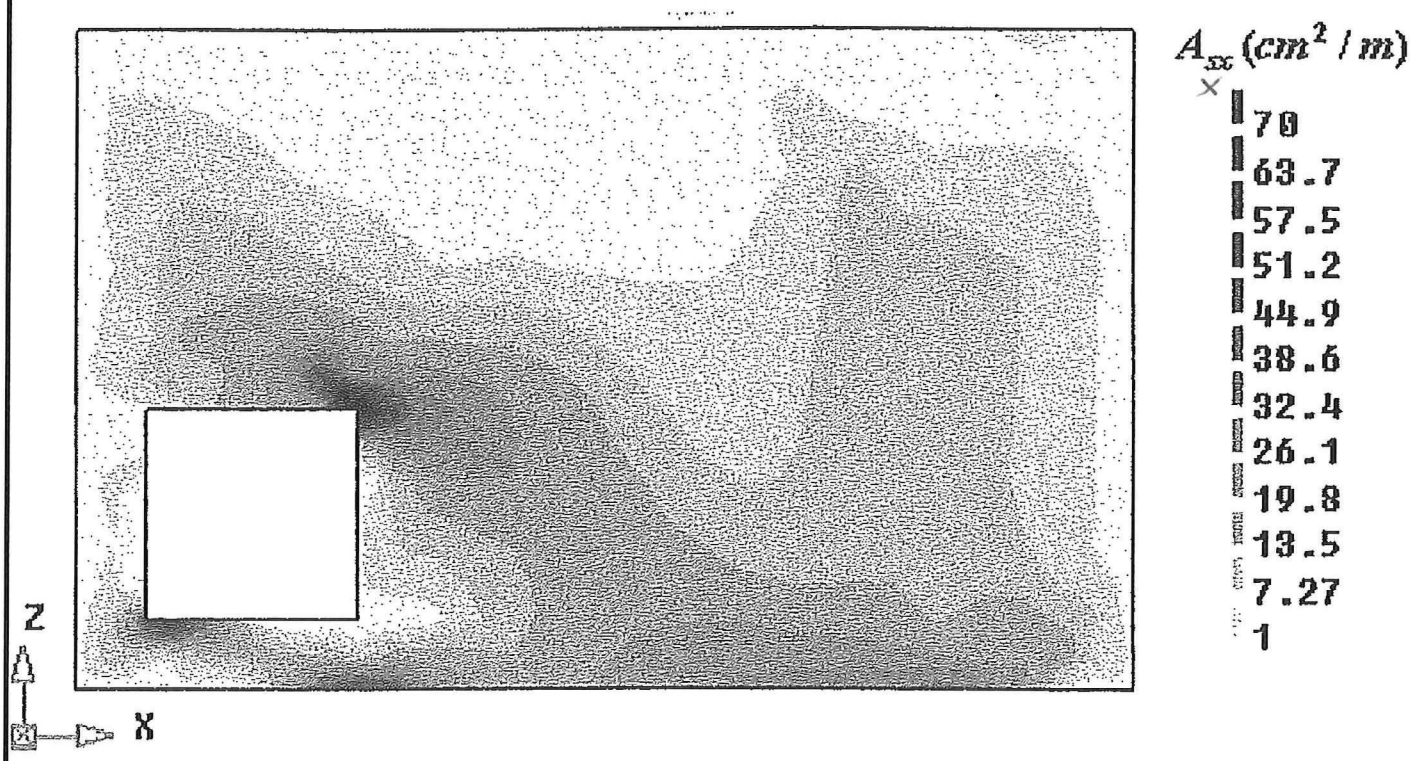
Plasticiteit en grensspanningshypothesen

GEWAPEND BETON 2D



Horizontal reinforcement

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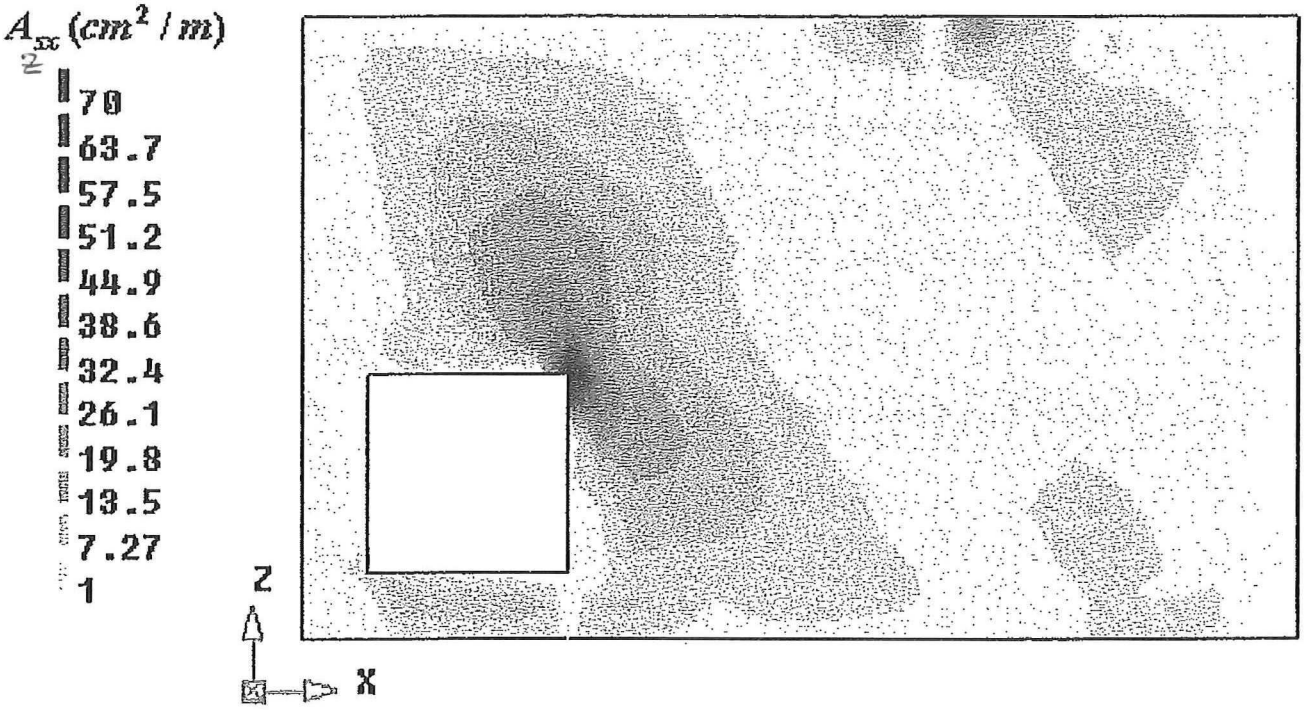
Plasticiteit en grensspanningshypothesen

GEWAPEND BETON 2D



Vertical reinforcement

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Plasticiteit en grensspanningshypothesen

GEWAPEND BETON 2D

Wapeningskeuze

