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Bend And Break Shell Structures

Design exercise

Rashmi Siriwardane Arachchilage, Matija Poljanšek & Sjoerd van Eekelen

Group 10

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Summary

For the course shell structures we were supposed to create a shell structure and approximate the load it could carry. This approximation was done by making a model of the structure in the SCIA program. The first step was to come up with initial ideas. Then we went to the construction room to see what materials were available. After some trial and error we stuck to our initial idea and ordered the materials and bought the equipment (like the PVC pipe and screws) that we needed. Although facing many problems during the construction we managed to make a stiff and stable structure on the frame. Our shell structure consisted of 2 main arches and horizontal laths. With the use of our model in SCIA we approximated the structure to fail at a load of 150 kg. Eventually our structure failed at 306.6 kg, so we considerably underestimated this. Reasons for this are the great uncertainties in imperfection of the wood, the model that does not fully comply with the real shell structure and how the test up accidentally restricted buckling. We also considered the feasibility of the structure in real life. A structure that looks exactly like the model can be made in real-life, but the building process would be vastly different and therefore different material/geometric properties to consider.

Initial Idea

To start, we first did some research on what is considered a shell structure and some examples of builds on a similar scale to ours. Most of the designs online consisted of the sticks being placed in a grid pattern and then bent into shape. This, we personally did not want to make since the risk of trying to bend them all at once and hoping none break did not seem like a viable option. The other option was pre-bending them and then using glue to slowly construct the grid structure which again seemed complicated and time consuming. So, we decided to design something which would be relatively simple to build and structurally sound. The initial design consisted of 2 arcs which would cross perpendicularly at each other's peaks, with each arc being made up of 2 beams (we later increased these to 3). The arcs would be supported by horizontal lathes which would wrap around them holding them together and providing some extra support when it comes to buckling.

Initial Calculations

Once we had chosen a design, the next step was to calculate the required thickness of our main elements. This is done by comparing the calculated stress in the top of the shell and the stress that the material can withstand. The stress is calculated with the following equation:

$$n = -\frac{1}{2} * p * a$$

Where p is the load [kN/m^2], a is the radius of curvature and t is the thickness of the element.

The shell has to carry its self-weight and a snow load of 1kN/m^2. Therefore we need to know what the density of the wood is. In this early stage we only knew that the material was pine wood. The density of this material is approximately 400 kg/m^3 (Ågren, 2023) and this is equivalent to 4 kN/m^3. The thickness is assumed to be 10 cm = 0.10 m and this results in a self-weight of 0.4 kN/m^2. Thus the total load of the snow and self-weight is 1.4 kN/m^2.

The radius of curvature (a) is depicted in figure 1 and can be calculated when the height (s) and span (l) are known. We chose the height and span to be 2.50 and 8.50 meters, respectively.

$$a = \frac{1}{2}s + \frac{1}{8} * \frac{l^2}{s} = \frac{1}{2} * 2.50 + \frac{1}{8} * \frac{8.5^2}{2.50} = 4.86\, meters$$

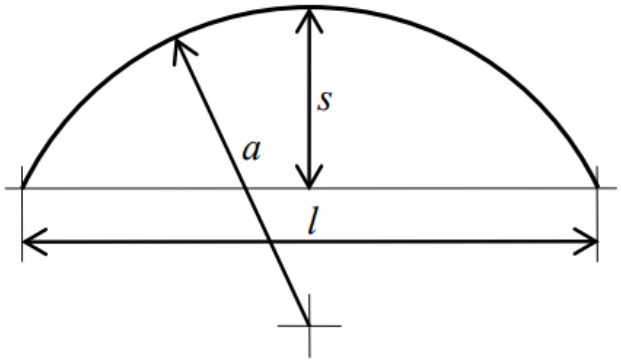

Figure 1: Radius of curvature.

Therefore

$$n = -\frac{1}{2} * p * a = -\frac{1}{2} * 1.4 * 4.86 = -3.40 \frac{kN}{m}$$

$$\sigma = \frac{n}{t} = -\frac{3.40}{0.1} = -34.0 \frac{kN}{m}$$

The compressive strength of the pine wood depends a lot on its kind. It varies from 4.5 to 8.5 psi. Hence a pine wood with a mean value of the strengths is chosen: Ponderosa Pine wood. This kind of pine wood has a compressive strength of 5.3 psi (=5.3/145.038 N/mm^2) which equals 36.54 kN/m^2 (Knecht, sd). The thickness was appropriate, because the compressive strength is higher than the stress on top of the shell structure. So we started off with a thickness of 10 centimetres for the main elements, which for our model is equal to 1 centimetre as we were working with a 1:10 scale.

Construction

Once we had our first ideas and did the initial calculations, we began playing around with the materials that were available in the pit. Quite soon, we noticed that there was no great variation in the materials that we could use. For the groups that wanted to make their shell structure out of wood, only the small laths that were available are depicted in figure 2.



Figure 2: Small sticks that were available.

We tested the bending capacity, the way we could tie and support them and the quality of the sticks. There were a lot of knots, so they broke easily. Also we tried to glue them together, but they didn't stick well as we were bending them (see figure 3).

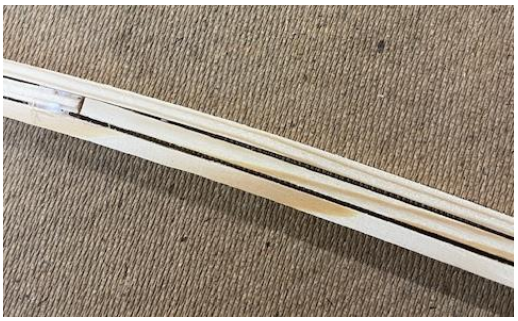

Figure 3: Poor glued connections.

Then we got the idea to pin them without the need of bending the sticks. In figure 4, a prototype can be seen. Although it seemed to be a good idea at first sight, it didn't work out as we wanted. It was very hard to cut the legs to the same size, so they were staggering when they stand on a flat surface, which we didn't want to happen.

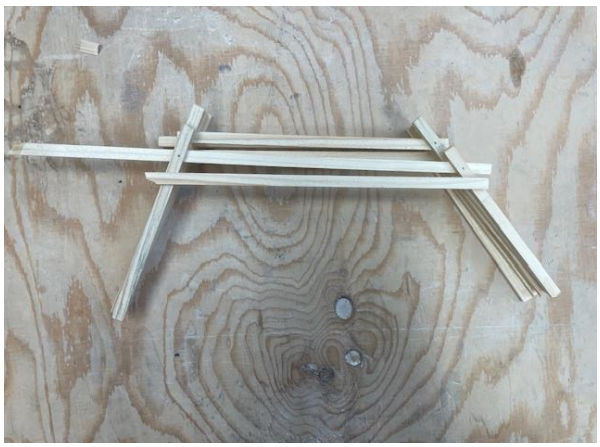

Figure 4: Design idea without bended sticks.

However, once we were able to order beams with our desired dimensions we reverted back to our initial design. To start we order 5 beams that were 1m long 2cm wide and 1cm thick. We then realised that bending them to our desired arc was going to be quite a challenge and as it turns out it was the biggest challenge we faced. We decided to use hot water for this purpose to soak the beams. We bought a PVC pipe (see figure 6) which served as our container. We experimented with multiple ways of soaking the wood and after breaking nearly all of our sticks with only 1 of the 5 making it through we decided to decrease the thickness of the beams from 1cm to 0.7cm hoping for better results. However, our problems continued realising that if we wanted a consistent bend we needed to make sure that the wooden beams did not have any knots (like in figure 5) and that the grain was consistent throughout. Luckily this seemed to be the last piece in the puzzle after soaking them in boiling water for multiple hours (which meant changing out the water every hour and pouring hot water over them before bending) we managed to bend enough beams to support our main structure. We left them to dry overnight while in tension in the mould we made for them.

Once the initial 2 arcs were done, we screwed them together using spacers in the gaps if necessary. This only left the horizontal lathes which we also bent into shape using water and while someone was holding them in place somebody else attached them to the arc using a drill and screws. The drill was essential since if we would try to screw in the screw before drilling a hole first the wood would split which would be the end of our progress.



Figure 5: One of the many knots in the sticks.



Figure 6: PVC pipe in which the sticks were soaked.

Technical drawings

Figures 7 to 9 show the model to scale as technical drawings from different perspectives.

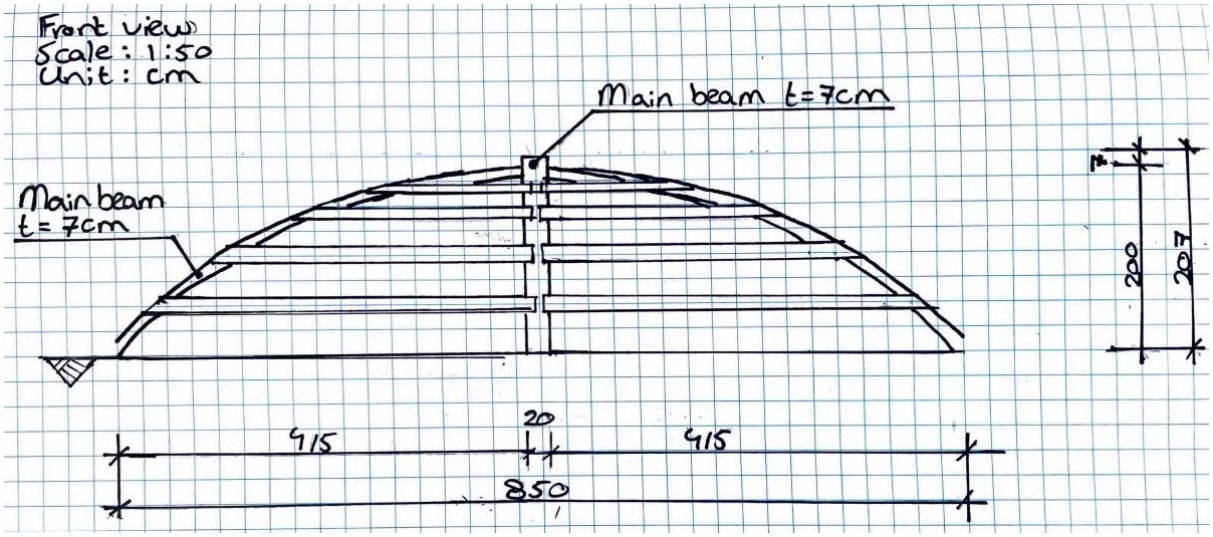

Figure 7: Technical drawing: front view.

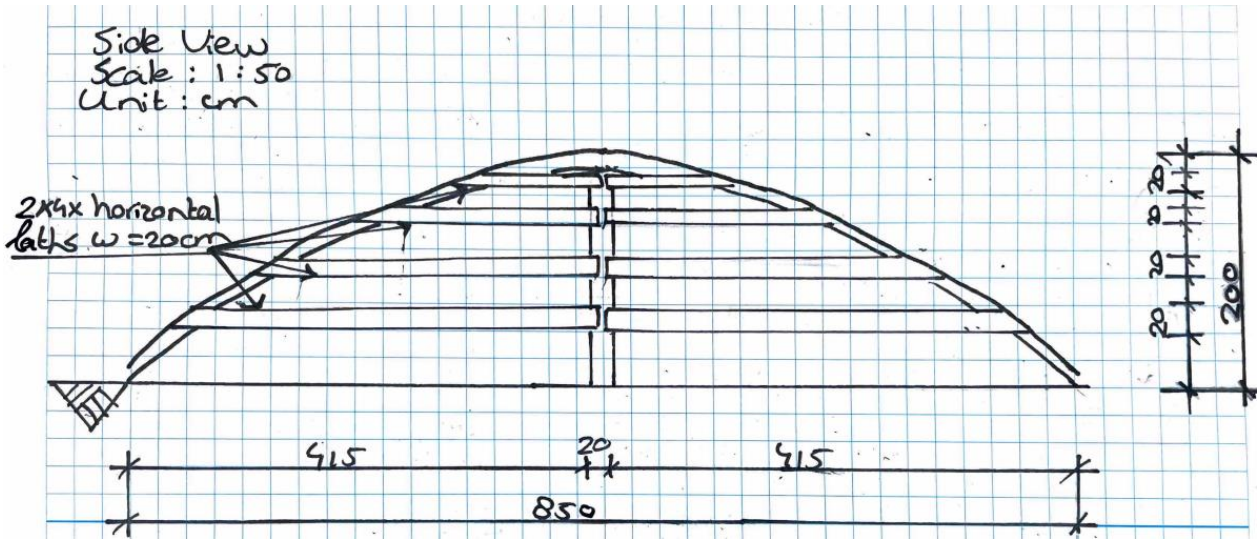

Figure 8: Technical drawing: side view.

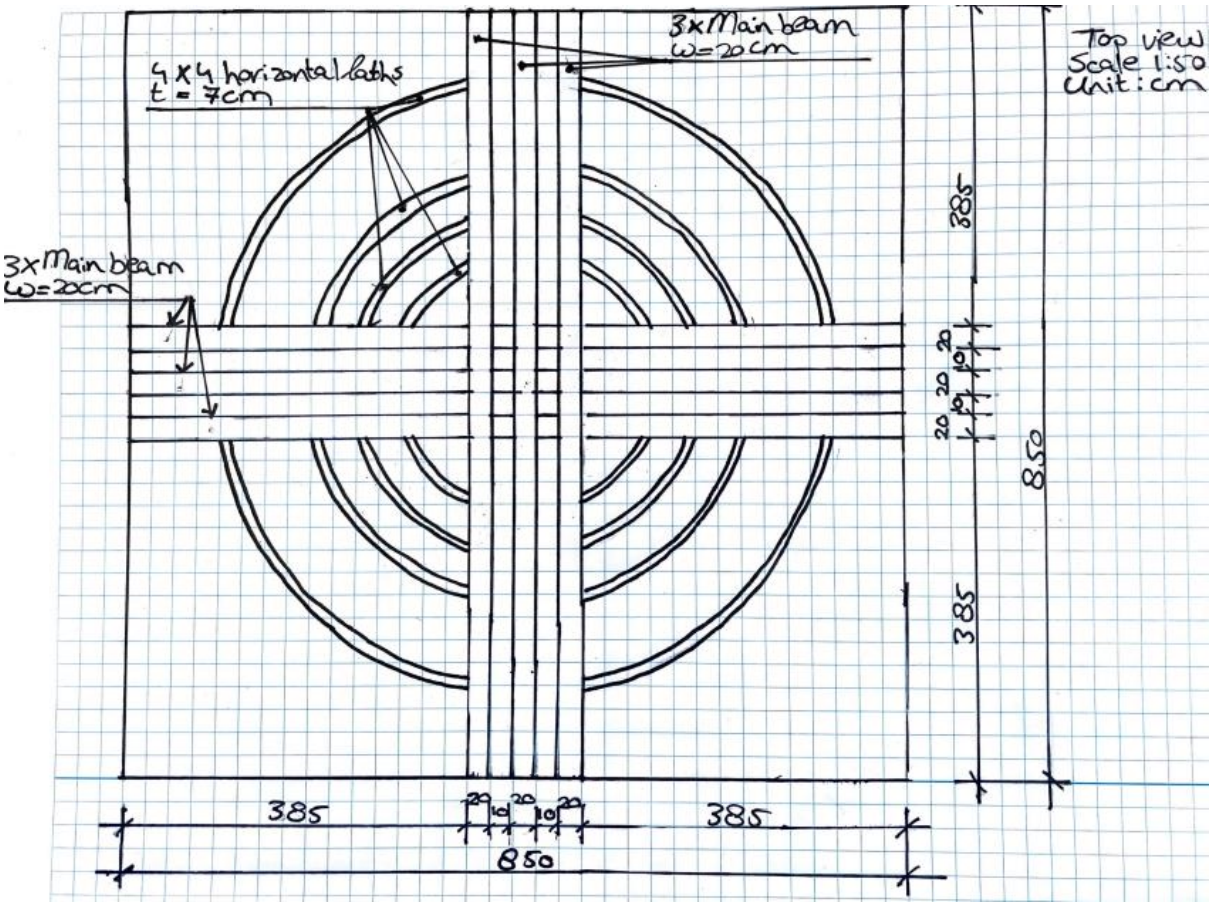

Figure 9: Technical drawing: top view.

Test set-up

Our test set up focused on the 2 arcs since the support lathes were not meant be the load bearing part of the construction. Therefore, we decided to only load the arcs as their job was to take most of the forces. We did this by placing 4 blocks on each limb and 1 main block on the top part of the construction which meant that we had a total of 17 points of loading. The loading on the arcs was equally distributed among them. Each limb of the arc had 4 loads which meant we first connected the 2 which were next to each other and then connecting those 2 to 1. Which left us with 4 blocks each one representing 1 limb of the arc. We then connected the limbs diagonally leaving use with 2 blocks for all the limbs. The final point load was the peak of the structure which was directly connected to the platform. Figures 10a and 10b show the test set up before the platform was attached. As can be seen the 17 load points were connected into 3 connection points for the platform giving us an equal distribution on the arcs of the structure.



Figure 10a: Load distribution blocks.

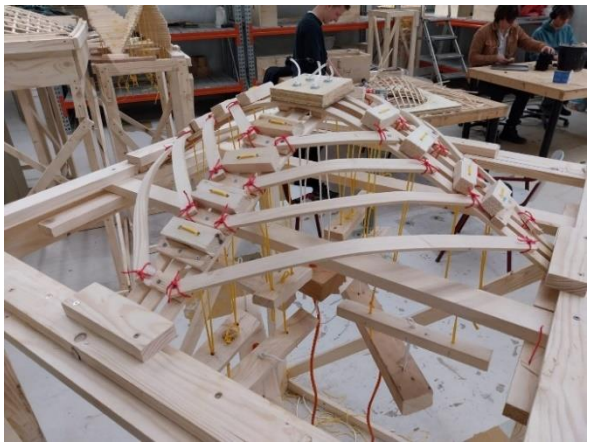

Figure 10b: Top view of load distribution.

Next to the hanging blocks and ropes also the platform with the bricks was part of our test set up (see figure 11). As mentioned before, the top of the structure was directly attached to the platform using a bar and the last two blocks were combined to 1 block, which then was connected to the platform. On this platform we could put 2 bricks simultaneously each time, so that the structure was loaded symmetrically.



Figure 11: Attached platform on which we put the bricks.

Also the deflection measurement tool was applied to our set up (see figure 12). After we put the bricks, we read the deflection on top of the shell structure. At some point, the deflection was more than the tool could measure, so at the higher loads we were not able to determine the deflection anymore.



Figure 12: Deflection measurement tool.

Prediction

The main frame of the shell structure (without the reinforcing horizontal laths) were constructed on SCIA. The test set up aimed to create a distributed load that hung from the main frame. We assumed that most of the load would be carried by this frame, especially given that the load distribution underneath the structure was attached onto the frame, not any of the horizontal laths.

Once the frame was added onto the software, the supports were introduced on as edge supports on the ends of all 12 edges on the ground. They were assumed to be rigid in all six degrees of freedom (with some negligeble tolerances). Then a distributed load was added (combination of snow load and self weight).

Determing the collapse load was the most challenging part, especially considering that the software was not truly due to its discretized nature and therefore we had to keep into account singularities and such as the mesh got finer.

The strategy was to analyse the stress disitrbution per loading value and then compare with four different criterias:

- Comparing *maximum local Von Mises stress* – with the bending stress of timber
- Comparing *maximum tensile normal stress* in the parallel direction (direction is specified due to the anisotropic nature of timber) -- with the tensile stress of timber in the parallel direction
- Comparing *maximum compressive stress* in parallel direction – with the compressive stress of timber in the parallel direction
- Comparing *maximum shear stress* – with the shear stress of timber

Then take the limiting value as the collapse load.

So we started with an initial load was 2kN per square meter load distribution. This would require around 23kg load.

$$Area\ of\ contact = 0.114m^2$$

$$2\frac{kN}{m^2} * 0.114m^2 = 228N \approx 23kg$$

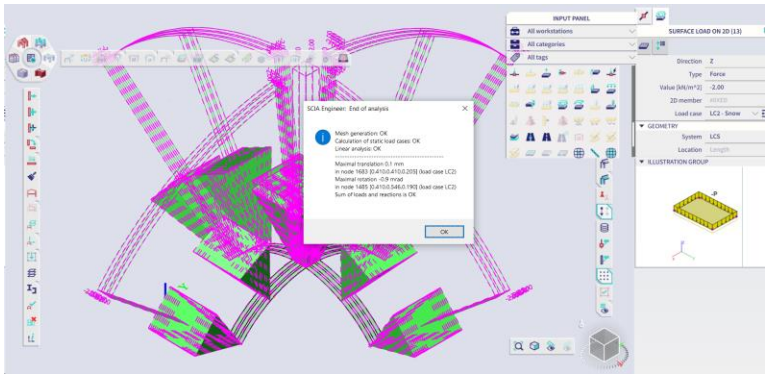

Figure 13: Initial pop-up after the calculations (-2kN/m^2).

Von Mises stresses analysis

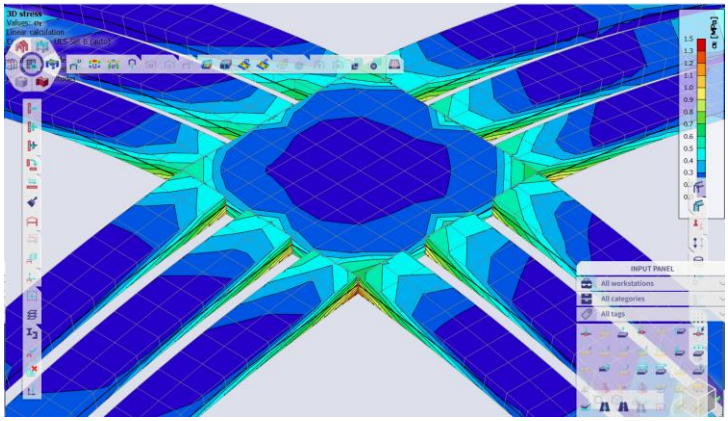

Figure 14: Von Mises stresses (-2kN/m^2).

As can be seen in figure 14, the maximum local von mises stress is 1.5MPa. According to the E338 standards (n.d.), the C24 carries a bending strength of 24 MPa. Assuming that this is in the linear region, the factor that the load needs to be increased by is

$$factor = \frac{24}{1.5} = 16$$

Tensile stress analysis (parallel) / Compressive stress analysis (parallel)

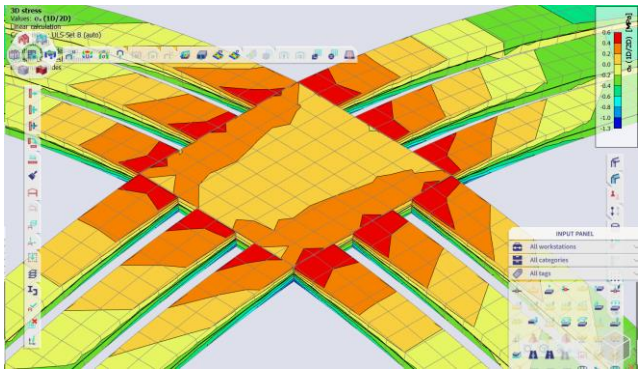

Figure 15: Tensile and compressive stresses parallel (-2kN/m^2).

As can be seen in figure 15, the maximum local tension is 0.6MPa. According to the E338 standards, the C24 carries a tensile strength (parallel) of 14 MPa. Assuming that this is in the linear region, the factor that the load needs to be increased by is

$$factor = \frac{14}{0.6} = 23.3$$

As can be seen in figure 15, the maximum local compression is 1.3MPa. According to the E338 standards, the C24 carries a compressive strength (parallel) of 21 MPa. Assuming that this is in the linear region, the factor that the load needs to be increased by is

$$factor = \frac{21}{1.3} = 16.15$$

Shear stress analysis

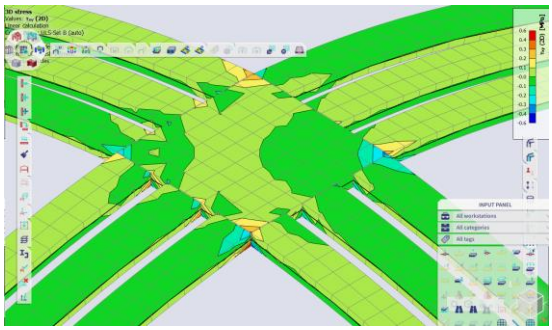

Figure 16: Shear stresses (-2kN/m^2).

As can be seen in figure 16, the maximum local shear stress is 0.6MPa. According to the E338 standards, the C24 carries a shear strength of 4 MPa. Assuming that this is in the linear region, the factor that the load needs to be increased by is

$$factor = \frac{4}{0.6} = 6.7$$

As can be seen from the four analyses, the limiting criteria is shear. Therefore the 2kN/m^2 is multiplied by a factor of 6.7 to proceed with another analysis, to check whether it does exceed the shear stress. So now this same structure is loaded with 13.4kN per square meter.

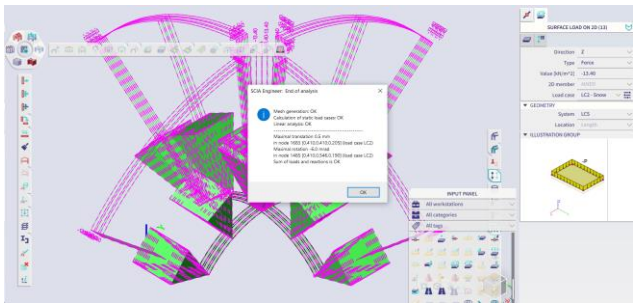

Figure 157: Initial pop-up after the calculations (-13.4kN/m^2).

Von Mises stresses analysis

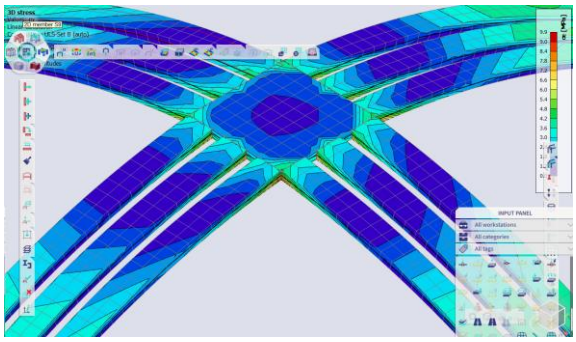

Figure 18: Von Mises stresses (-13.4kN/m^2).

As can be seen in figure 18, the maximum local von mises stress is 9.9MPa. According to the E338 standards, the C24 carries a bending strength of 24 MPa. So this would not fail due to pure bending.

Tensile stress analysis (parallel) / Compressive stress analysis (parallel)

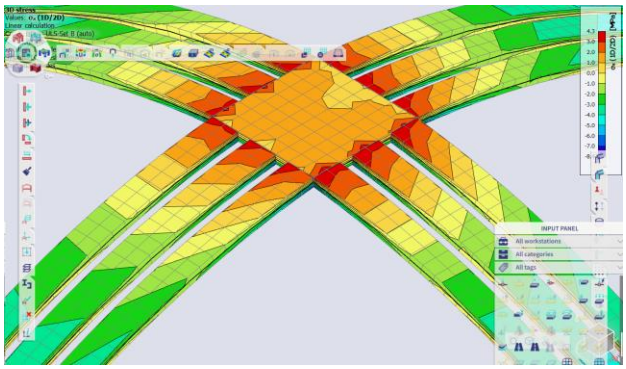

Figure 19: Tensile and compressive stress parallel (-13.4kN/m^2).

As can be seen in figure 19, the maximum local tension is 4.3MPa. According to the E338 standards, the C24 carries a tensile strength (parallel) of 14 MPa. So this would not fail due to tension.

As can be seen in figure 19, the maximum local compression is 8.4MPa. According to the E338 standards, the C24 carries a compressive strength (parallel) of 21 MPa. So this would not fail due to compression.

Shear stress analysis

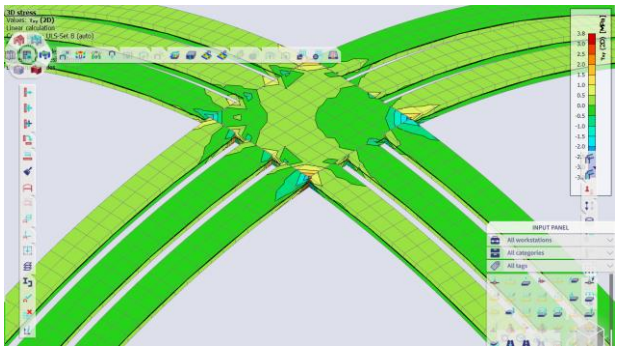

Figure 20: Shear stresses (-13.4kN/m^2).

As can be seen in figure 20, the maximum local shear stress is 3.8MPa. According to the E338 standards, the C24 carries a shear strength of 4 MPa. So this is close to failing but not completely, so the load needs to be increased further. Since the stress did not linear increase (in accordance with the increase in the load), we can conclude that we have exceeded the elastic region and now are deforming. So from now on, the load is increased in small increments to find the exact load at which the shear stress exceeds 4MPa.

As can be seen in figure 22, the shear stress exceeds 4MPa at 14kN/m^2, which equates to a load of 163kg.

$$Area\ of\ contact = 0.114m^2$$

$$14\frac{kN}{m^2} * 0.114m^2 = 1.596kN \approx 163kg$$

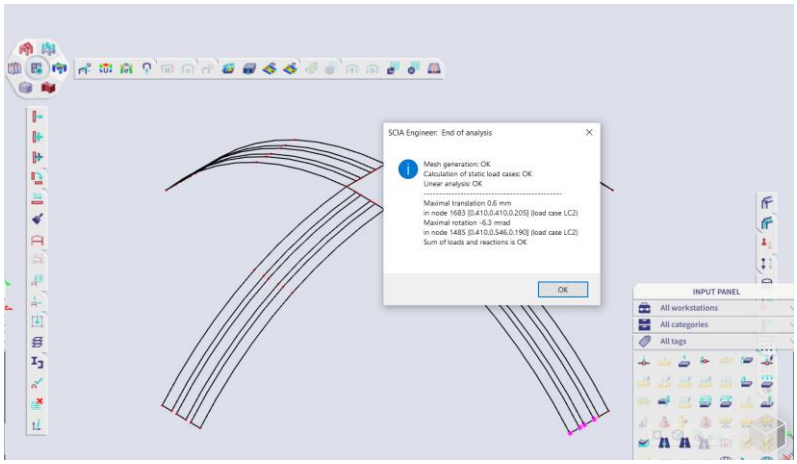

Figure 21: Initial pop-up after calculations (-14kN/m^2).

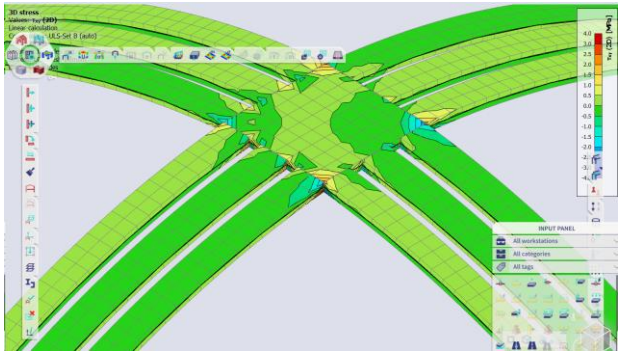

Figure 22: Shear stresses (-14kN/m^2).

Timber is a very brittle material and it has a lot of imperfections (knots). We predict that the way that the structure will break in a brittle manner at some point, especially at a point where its concentrated with knots. So the shear failure would cause the structure to snap. So the displacement at failure would be hard to determine, but according to the software (see figure 23) it is 0.6mm, which is hard to measure by hand. So we believe that it would snap off. The area of interest is the centre however in reality, the centre is heavily reinforced.

In the real life structure, the load is scaled 1:1 so the collapse load is also 14kN/m^2 but the area of contact would be different so it will not fail at 163kg.

However all of this analysis is done in the linear manner, so another analysis is done non-linearly to check for buckling, to see if they could be the critical collapse load. This is done initially with a linear buckling analysis, by computing 6 buckling modes. The linear analysis is done to compute the membrane forces, which cause buckling.

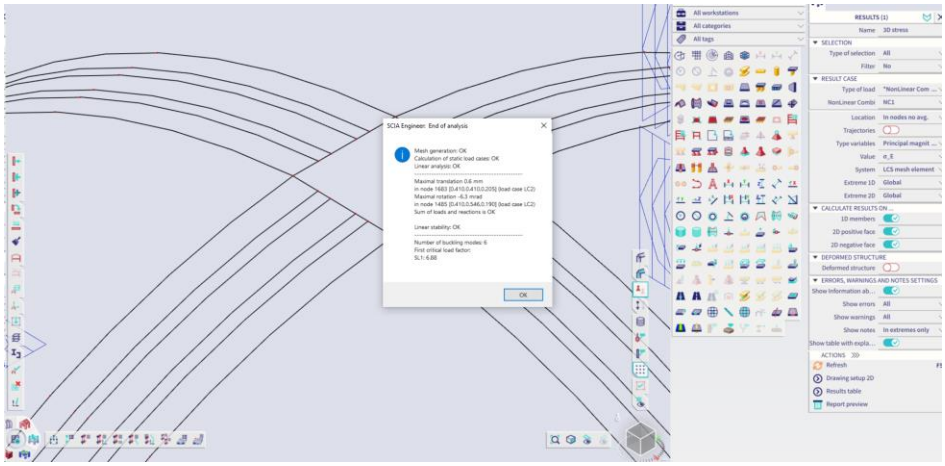

Figure 23: Initial pop-up after linear stability calculations (-14kN/m^2).

As shown in figure 23, the first critical load factor is 6.88.

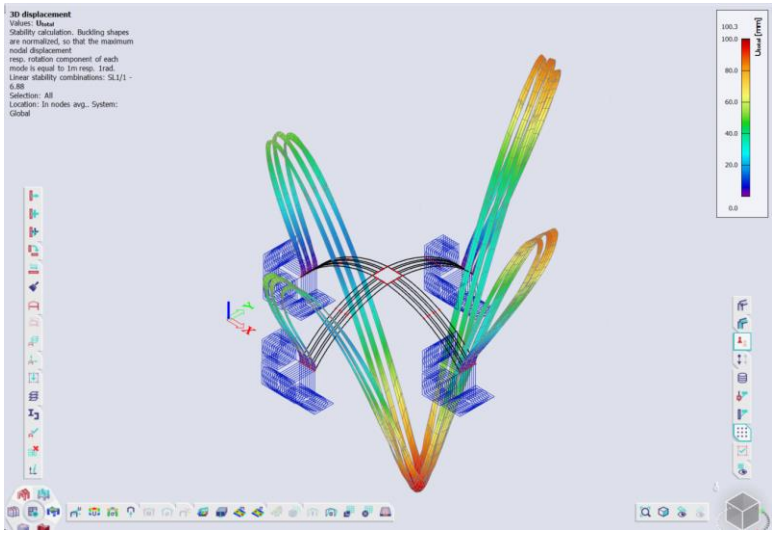

Figure 24: First critical load factor of 6.88 buckling nature.

This means that it buckles as such (see figure 24) when the load combination is multiplied with a load factor of 6.88. The subsequent buckling load factors are 6.88, 7.42, 7.44, 12.22, 12.23, 16.12, etc.

Thereby, a nonlinear analysis is conducted. We specify a shape imperfection. The chosen imperfection is an amplitude of 10mm (a little under twice the shell thickness). This imperfection is applied in the other direction.

This nonlinear analysis is done using the Newton-Raphson method, which involves few iterations until sufficient accuracy is obtained. The procedure is to increase the loads with a factor of 6.88 (buckling loading factor computed in the linear buckling analysis). Then note the increment where a singular node is detected and the structure becomes unstable. Figure 25 shows the pop-up of the software during the non-linear analysis where the structure becomes unstable.

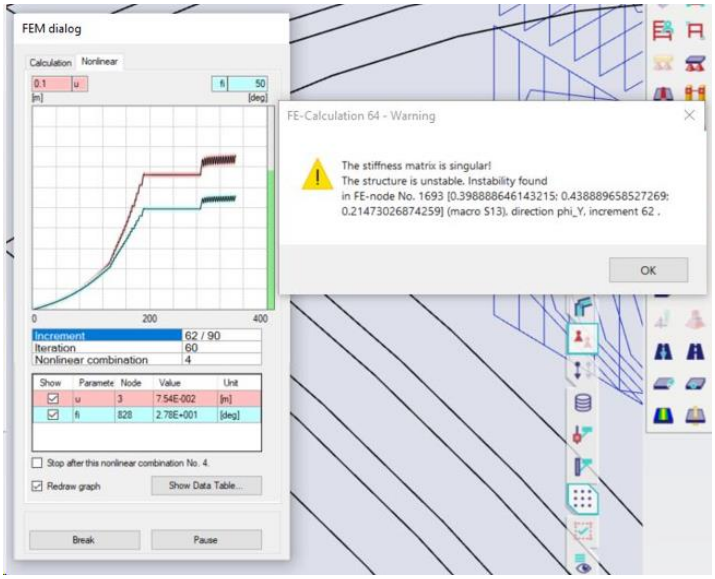

Figure 25: Pop-up during nonlinear analysis (when the structure becomes unstable).

For the amplitude of 10mm, the Newton-Raphson procedure diverges at load increment 62. This load factor then is

$$load\ factor = \frac{62}{90} * 6.88 = 4.78$$

Therefore the maximum load at which the structure becomes unstable is

$$load_{max} = 4.78 * \frac{14kN}{m^2} = \frac{66.35kN}{m^2}$$

However due to the knockdown factor, the maximum load is dropped by a factor of 6. This is due to the fact that regardless of predicted loads from numerical methods (FEM), the experimental collapse load is usually a factor of 6 smaller.

$$real\ load_{max} = \frac{66.35}{6} = \mathbf{11.06}\,\boldsymbol{\frac{kN}{m^2}}$$

This means that it carry around **129kg.**

$$Area\ of\ contact = 0.114m^2$$

$$11.06\frac{kN}{m^2} * 0.114m^2 = 1.26kN \approx 129kg$$

However, this load is assuming that it is loaded on just the main frame but the horizontal laths are not taken into account. Once these horizontal laths are taken into account, the effective area is then 0.191m^2 which gives a new predicted load of 215 kg.

$$Area\ of\ contact = 0.191m^2$$

$$11.06\frac{kN}{m^2} * 0.191m^2 = 2.11kN \approx 215kg$$

So we thought of predicting a number between these two values and leaning towards the 129kg and we settled on a prediction of **150kg.**

In addition, figure 25 shows the deflection of **7.54 cm.**

In the real life model, the collapse of that is also at 11.06kN/m^2 but since the area of contact differs, it carries a different mass than 129kg. The deflection of the real life model is on a 10 to 1 ratio. So the real life model deflects about 75cm.

Results/ Discussion

The structural analysis outlined above has provided valuable insights into the load-bearing capacity and potential failure modes of the examined shell structure. However, the actual results differ from predictions to a significant extent. Before the difference between reality and the predictions are discussed, the pure results will be described.

The shell structure experienced a “load” with the use of bricks of 2.1kg each onto the test set up stand. As the test progressed, the number of bricks increased by an increment of 2 to maintain symmetry when loading. Figure 26 shows the bricks as they are aligned on the test up.



Figure 26: Symmetrically loading of the structure.

Figure 27 shows the relationship between the mass that were loaded by the bricks and the midpoint displacement of the shell structure.

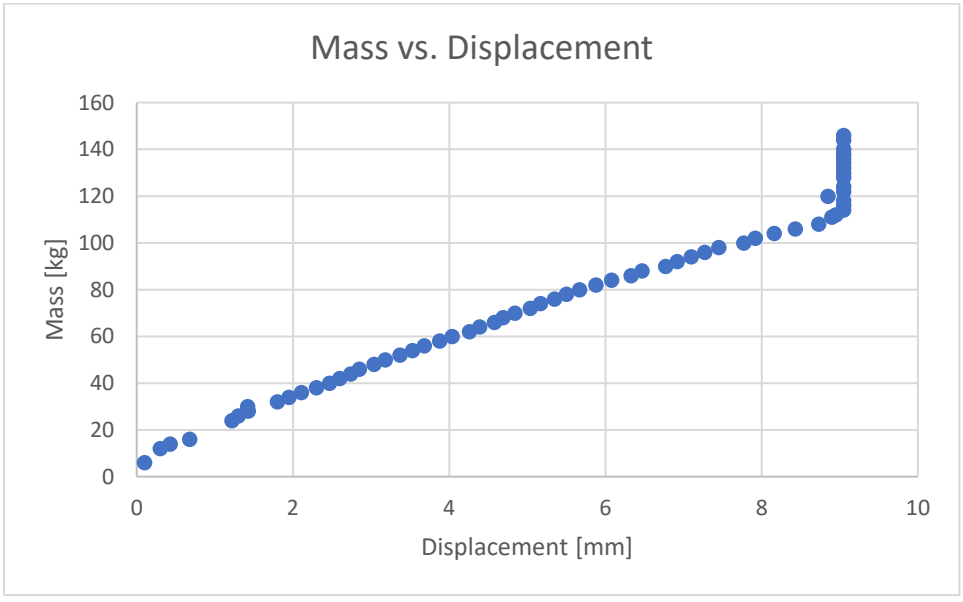

Figure 27: Relationship mass and midpoint displacement.

This mass value is converted into weight and figure 28 gives a force vs. displacement graph.

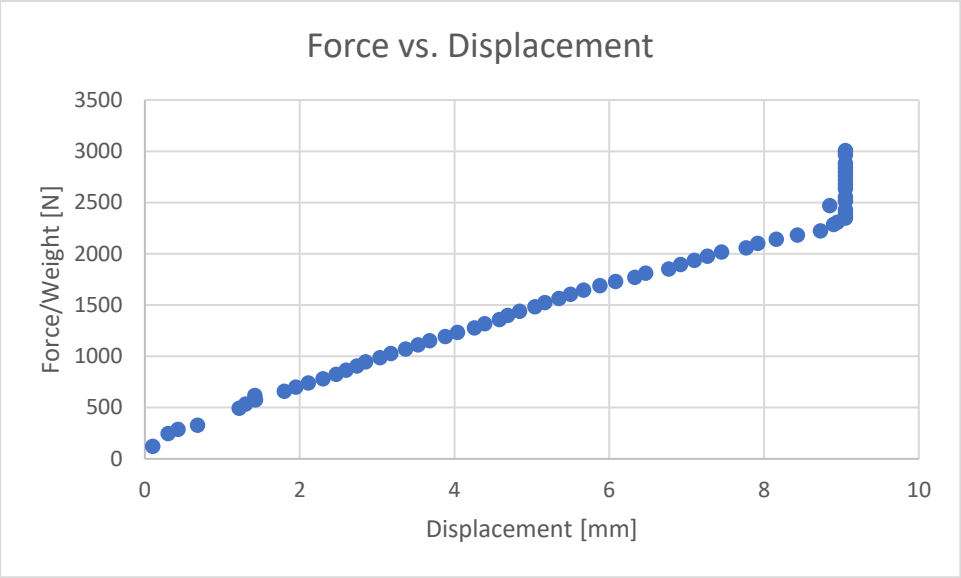

Figure 28: Relationship force and midpoint displacement.

As can be seen from figures 27 and 28, it shows stagnant point at a displacement of 9.05mm. This is because the displacement measuring device became out of range and stopped measuring the displacement. But before the device went out of range, the relationship between the force (or mass) is fairly linear indicating that the shell was still in its elastic region. The moment where the linear relation becomes non-linear is the instant where the shell may collapse however due to the device, this non-linearity could not be recorded.

Rope failure

One of the ropes of the test set up snapped at a force of 1689 N (172kg). This was then fixed before continuing to load such that the whole collapse could occur. Figure 29 shows where these ropes snapped at 1689 N.



Figure 29: Failure of a rope during testing.

The failure of ropes occurred under the tensile loading from the bricks that make up the 1689N. Especially since these ropes were knotted to interlock with the wood, the knots concentrated stress reducing the overall strength. Certain type of knots may weaken the rope more than others, and improper tying can exacerbate this.

The collapse

The shell structure finally collapsed at a load of 3007 N (306.6kg). Figures 30-33 show the collapsed structure at 3007 N.

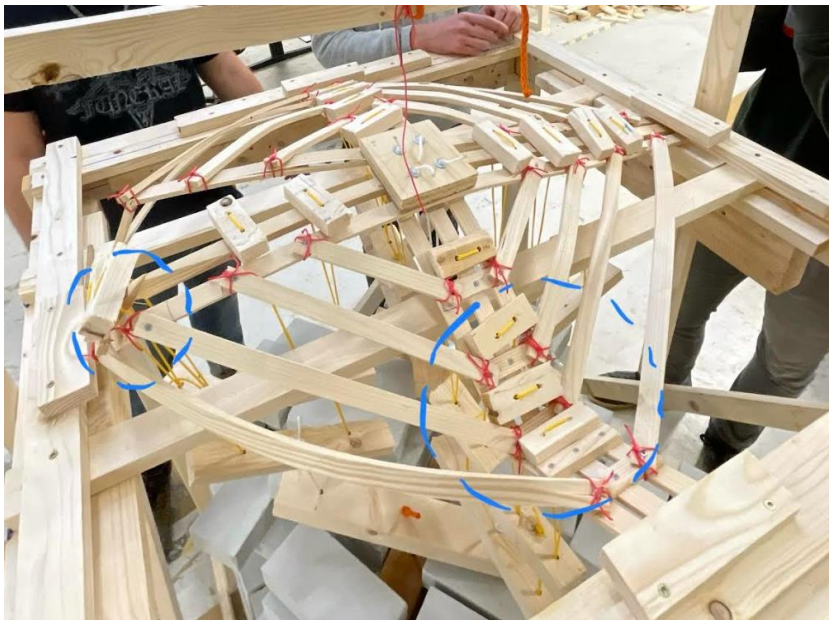

Figure 30: Overall view of destructed structure.

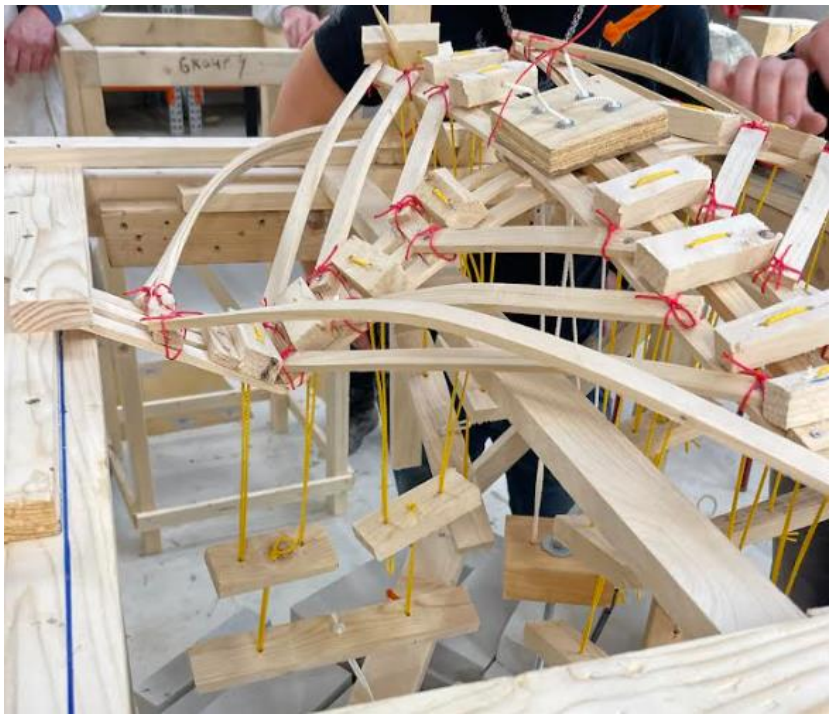

Figure 316: One of the arches that broke.



Figure 32: Splitting wood of one of the arches.



Figure 33: Another arch that broke.

Main failure mode

As can be seen the laths along the main diagonals were the ones that broke. The horizontal laths maintained a good structural integrity throughout the loading, especially because those horizontal laths were not loaded. However as can be seen, there are several snapping that occurred at the same time. This did not coincidentally occur simultaneously. The failure of one part resulted in other parts also failing. However, it can be concluded that it was a sudden catastrophic failure at 3007N. Usually, buckling does not necessarily lead to immediate failure or snapping. However, due to the brittle nature of timber, the out-of-plane buckling caused different timbers to crack and thereby cannot proceed to carry load.

Creep

The time factor of the loading should also be accounted for in the analysis. It is intuitive to see that if the same bricks were loaded constantly for a period of time, then the structure would fail earlier. Over time, the internal structure of the material can rearrange and deform under the influence of applied stress, leading to a gradual and continuous strain. But in this brittle nature of timber, cause immediate collapse much earlier.

Why different from the prediction?

The shell structure is quite strong to withstand 3007N, even though it was very different from the prediction. Why was the value of failure much stronger even though the failure mode predicted is the same?

Firstly, the predicted critical buckling load was assumed with the 1/6 knockdown factor that account for imperfections and is empirically determined. However, during the construction of the shell structure itself, the timber laths snapped most of the time due to the knots (the imperfections of timber) and so we proceeded to use the least imperfected timber laths and they were very few but those were very strong and effective when being constructed. So this reduced imperfections could explain why this knockdown factor may be too large. In addition, the prediction on FEM consisted of implementing imperfections on the structure which was purely approximated. In reality, since we chose the least imperfected laths, may have underestimated the strength of the overall structure.

Secondly, the horizontal laths were not modelled on the finite element software. This is a huge flaw, now that it is brought to light. Even though we decided not to model because the load was not distributed onto it, it still contributed to maintain a very stiff material and restricted buckling in different directions. Also, in relation to real-life, loads such as snow load or dynamic load such as rain, wind, gust will occur to all the laths that is with contact with these loads and so modelling the horizontal laths is very necessary.

Thirdly, the fact that the timber laths were bent to create their “shell” shape impacts the strength of the structure. This is because the bending or curving of an element allows redistribution of loads and stresses more efficiently which allows the same structure to carry heavier loads.

Finally, something that did not cross our mind was the size of the middle hole on the distribution wood. The size must be sufficiently big, if not it forces the deformation to be symmetric and the buckling to be restrained. Figure 34 shows this phenomenon quite well. Due to our hole being too small, it allowed the entire structure to fail at a higher load than expected.

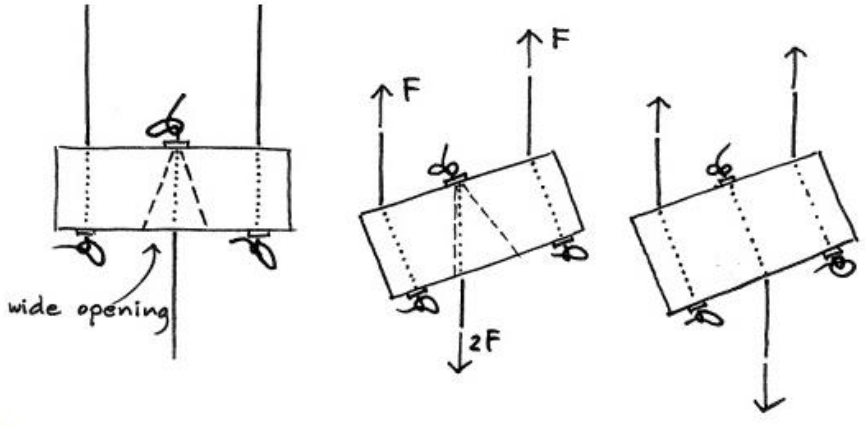

Figure 34: Wide opening of middle hole in distribution blocks (Hoogenboom, sd.).

Comparison with other groups

Most of the other groups also underestimated the collapse load of their structure. This goes to show how strong shell structures are, especially, with their pre-stressed nature. Especially since we were one of the first groups to test their structure, our contrast between the prediction and the result inspired other groups to re-think about their predictions, which goes to show that working together and sharing results like this contributes to a better development of knowledge.

The failure loads are however different, most groups experienced buckling and thereafter followed with the breaking of the laths. Some groups tested using point load and therefore experienced punching shear.

Real-Life Structure

As mentioned before, this shell structure is a model which represents $1/10^{th}$ of the actual model. In real life, this "dome-like" structure can be on top of tall buildings as roofs. Figure 35 shows a possible application of such a structure.



Figure 35: Example of an application of our structure.

Equipment

As a result of this model, we ordered several different size laths that predominantly made up the frame of the structure. Some of the laths that we ordered were of size 95x2x0.7 (cm) and in reality, it is hard to find laths of size 950x20x7 (cm). But even if these sizes can be found, what guarantees that they can be bent properly.

Construction

The model was bent with our bare hands. We bought a PVC pipe and stored the laths in and put boiling water in the hope that they became flexible to be bent. Few of them were able to be bent (least imperfected laths) by our hands. But in reality, how can it be bent? Therefore some solutions can be implemented for the real life structure. Figure 36 shows two options.

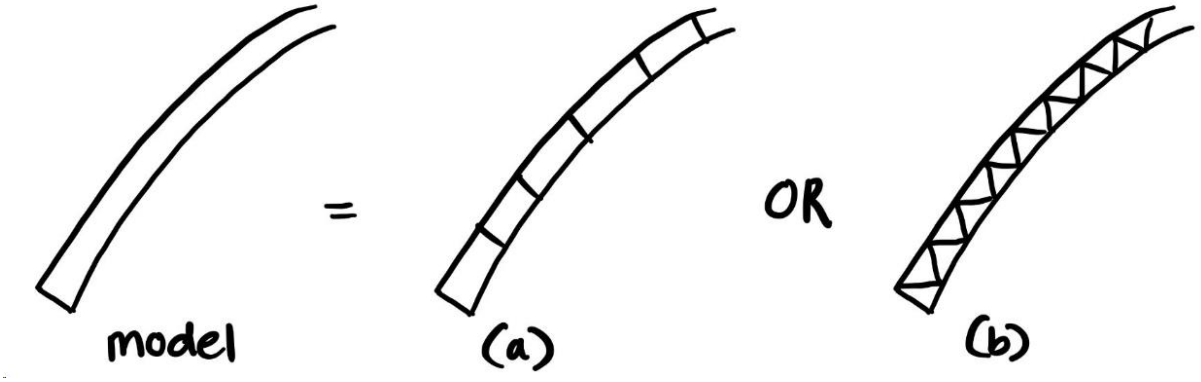

Figure 367: Possibilities to bend a long wooden beam.

As can be seen in figure 36, either the laths can be made up smaller laths (a) or the lath shape can be constructed with a grid structure (b). However, if this alternative is used for the actual structure, the model should implement this such that the loading tests can be comparable. Because this new changes influence the material properties and a new finite element model should be considered as well.

Boundary conditions

In the model, the “clamped” nature of the model was obtained as shown in figure 37.



Figure 37: Clamped support on our structure.

This can be easily implemented in the real life structure. Even an alternative can be clamping it into concrete (but then with the presence of rebars because it would not be fully loaded in compression). In either case, the model should be accounted for edge disturbance.

Test set-up

The test was conducted with the use of ropes hung under the main frame (without the horizontal laths) and we tried our best to create a uniformly distributed loading. In reality, the load would be applied to all the laths of contact, for example due to snow loading. However, a model is a model and the loading is represented sufficiently as it is not a concentrated “point load” in the centre of the structure.

Test results

The actual collapse occurred at 3007 N which translates to 15.7kN/m^2 which is the applicable parameter for the actual model. The actual area of contact will be much higher which results in a much higher collapse than 3007N. In addition, the real deflection is 10 times the model defection.

Nonetheless, the imperfections (knots) of the timber should be considered. Knots can come different sizes and they are weak spots which results in a lower strength. The real-life structure would have many more knots and that can be susceptible to an earlier failure, in comparison to the smaller model. Therefore, the design of the actual structure should consist of safety factors for reasons like this.

In conclusion, the actual structure in real life can be built to **look** similar to the model but it is not the **same** structure. Due to building constraints and equipment constraints, the mode of constructing will be altered too heavily to the point the structural properties are altered and therefore a complete alternate finite element model needs to be constructed. However the evaluated points from the results (why the load offsets from the predicted value) are useful reflective points that should be considered when predicting for this new modified real-life version.

Conclusion

This project aimed to design, construct, and test a shell structure, predicting its load-bearing capacity through calculations (by hand and also through finite element analysis). The idea consisted of intersecting arches supported by horizontal laths. Results revealed a significant deviation from approximations, with the structure failing at 306.6 kg instead of the predicted 150 kg. The main failure mode occurred at the laths along the main diagonals, leading to a sudden catastrophic collapse at 3007 N. In addition with the rope failure at 1689 N, it highlighted the importance of imperfections, knot concentrations, and the brittle nature of timber in influencing structural strength. Discrepancies from predictions were attributed to factors such as the 1/6 knockdown factor, lack of modelling for horizontal laths, the bending process, and the size of the middle hole in distribution wood.

The main conclusion that is taken from this entire project is to not underestimate the strength of shell structures. Almost every group underestimated this strength value and the shell itself turned out to be much stronger. Recommendations for real-life applications included addressing imperfections, considering safety factors, and adjusting designs to accommodate practical constraints.

References

Ågren, A. R. (2023). *Properties of softwood*. Retrieved 11 2023, from Swedish wood: https://www.swedishwood.com/wood-facts/about-wood/from-log-to-plank/properties-of-softwood/

EN 338 (n.d.). *EN338 – Table 1- Strength classes – Characteristic Values.* Retrieved 11 2023, from: EN 338 - Table 1 - Strength classes - Characteristic Values (onlinestructuraldesign.com)

Hoogenboom, d. P. (n.d.). *Wide opening of middle hole in distribution wood.* Retrieved 12 2023, from https://phoogenboom.nl/B&B_schaal_0.html

Knecht, C. (n.d.). *Wood Strengths*. Retrieved 11 2023, from Woodworkweb: https://www.woodworkweb.com/woodwork-topics/wood/146-wood-strengths.html