Stresses in the edges of cold bent glass panes

Spanning in de randen van koud gevormde glasplaten

The final report of the BSc End-work

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Preface

This is the final report of the BSc End-work of Civil Engineering (CT3000-09) of TU Delft. The research is about stresses in the rims of cold bent glass panes.
I would like to thank my supervisors to sharing their knowledge and for providing assistance during the realization of this report.

Delft Jan. 2012
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TABLE OF CONTENTS

Stresses in the edges of cold bent glass panes 1
Preface 2
TABLE OF CONTENTS 3
1. Introduction 4
1.1 Problem statement 4
1.2 Objective 5
1.3 Solution method 5
1.4 Report structure 5
2. Theory 6
3. Models 8
3.1.1 Material properties 8
3.1.2 Boundary conditions 8
3.1.3 Dimensions 8
3.2 Shell model 9
3.3 Solid model 13
3.3.1 Data information: 14
3.3.2 Solid 186 17
3.3.3 Mesh size 18
3.3.4 Improve the model 20
4. Derivation 22
5. Alternative derivation 23
6. Confirmation 25
7. CONCLUSIONS 29
8. REFERENCES 30
9. Appendix 31

1. Introduction

There is much interest by architects and builders to apply curved glass. An expensive way to make curved glass is in a factory where the panes are heated and plastically deformed. A more economical way to curve glass is during construction just by just pushing it in the right shape. This introduces stresses in the glass which should not be too large. Especially the stresses in the edges need to be checked carefully because often cracks start there due to notches created by cutting and finishing. An example is the bus station at tram station Zuidpoort in Delft, created by Mick Eekhout [1]. The maximum cold twisting possibilities of laminated glass have been used which led to the maximum feasible undulating form of the glass roof. (Figure 1.1)



Figure 1.1 Tram station Zuidpoort in Delft

The engineering properties of glass, such as the failure strength of glass in bending, are essential to designers. Dr. F.A. Veer published his research in the article, the strength of glass [2]. He tested large series of specimens treated in a number of ways to obtain strength values.

1.1 Problem statement

Most stresses can be computed by a finite element program using shell elements. However, the shear stresses in the glass edges are not computed by shell elements. The edge shear stresses can be computed by volume elements but this analysis is too much work in practice.

1.2 Objective

Derive a formula for the shear stress in glass pane edges as a function of the edge shear force and the edge torsion moment.

1.3 Solution method

Model a square glass pane in ANSYS, once with shell elements and once with volume elements (solid). The glass pane models will be loaded in the corners to impose a curvature. Accurately determine the shear forces, torsion moments and the shear stress in the rim. Develop a formula for calculating the edge shear stress.

1.4 Report structure

The first section, describes the problem of this thesis. The shell element model and volume element model are given in Section 2. The calculation is presented in Section 3. And the conclusions are presented in Section 4.

2. Theory

For a small deformation a square glass pane deforms as shown in Figure 2.1. For large deformation it deforms as shown in Figure 2.2 [3].

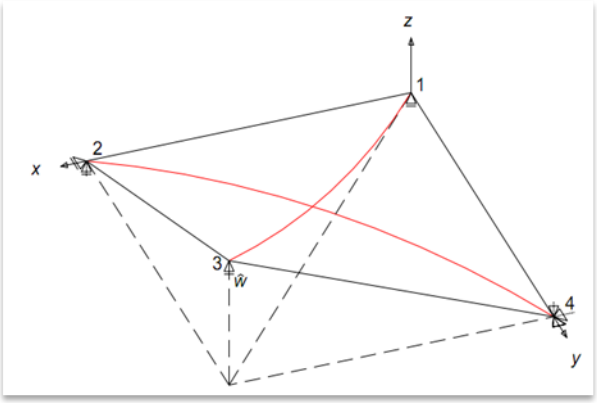


Figure 2.1: first deormation model
-curved diagonal
-straight sides

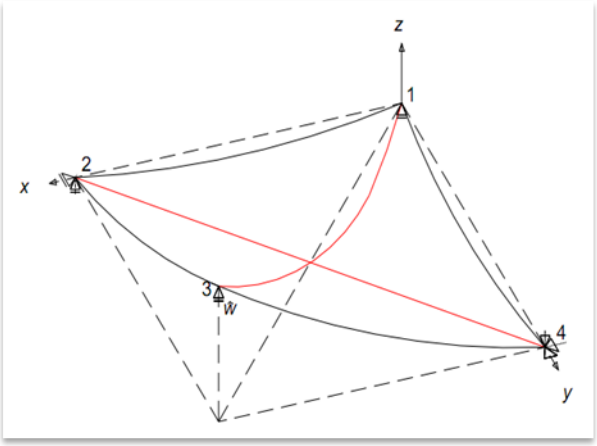


Figure 2.2: seconde deformation model
-a curved diagnoal, a nearlly straight diagonal
-curved sides

The large difference between the first and the second deformation concerns the edges. The edges of the first deformation are straight while the diagonals are curved. Following an increase in imposed corner deformation the edges become curved, one of the diagonals becomes almost straight while the other diagonal bents progressively. This kind of distortion is not a failure mechanism, but a change of deformation pattern.

In pane sections occur a distributed torsion moment m_{xx} and a distributed shear force q_x (Fig. 2.3). The torsion moment m_{xy} is constant over the section and goes to zero at the edge depending on the element size. The shear force q_x is zero over the section and goes to some value at the edge depending on the element size. The shear stress in a section flows along the pane surface and turns direction at the edge.

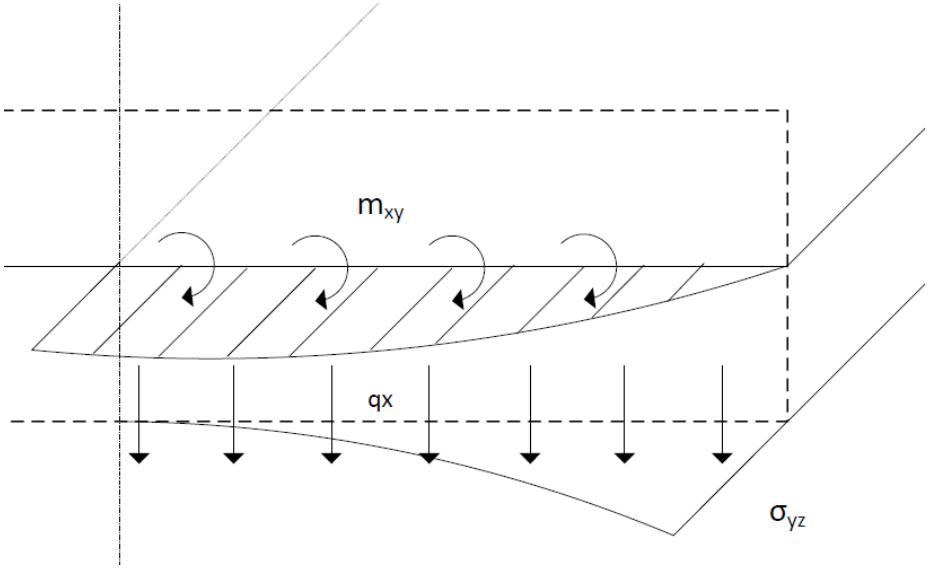


Figure 2.3 Pane section distributed torsion moment and distributed shear force for a very small shell element size.

3. Models

Two models have been made; a shell model and a solid model (3D model). Both models have been made in ANSYS using script.

3.1.1 Material properties

A linear isotropic material model has been used. The Modulus of Elasticity (Young's modulus) and Poisson's ratio are E = 72000 N/mm² and v = 0.23.

3.1.2 Boundary conditions

Three panel corners have been fixed (x, y, z-direction) and the fourth corner has been loaded by a force perpendicular to the pane (Fig. 3.1).

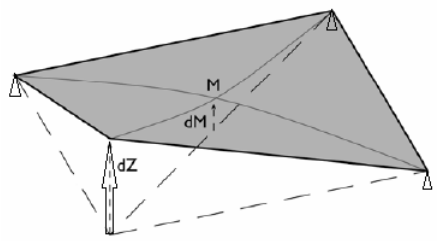


Figure 3.1 Pane boundary conditions

3.1.3 Dimensions

The pane dimensions are b x h = 500 x 500 mm or b x h = 100 x 100 mm². The pane thickness is 8 mm or 4 mm. The mesh size has been varied for both the shell model and the solid model, in order to obtain the best result.

3.2 Shell model

The shell model is a 2D model. Figure 3.2 shows the geometry of the element type “shell 181”.
SHELL181 is appropriate for analyzing thin to moderately-thick shell structures. It is really a 4-node aspect with 6 levels of flexibility at each node: translations within x, y, and z directions, and rotations regarding the x, y, and z-axes.
SHELL181 is well-suited for linear, large rotation, and/or large strain nonlinear applications. Change in shell width is accounted for in nonlinear analyses. Within the aspect domain, every little bit as full and reduced integration schemes are supported. SHELL181 records for follower (load stiffness) outcomes of submitted pressures.

Figure 181.1: SHELL181 Geometry

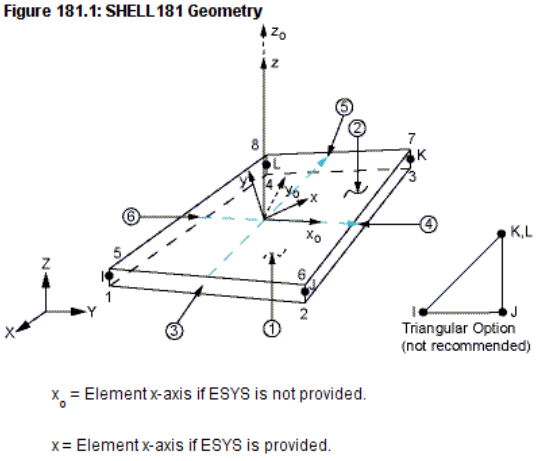


Figure 3.2 Shell 181

From ANSYS, it is not possible to get the moment of one node. So the moment of the node must be calculated. With the formula, $m_{xy} = (s_{xyt} - s_{xyb}) \times t \times t/12$ stress-Sxy of the top and the bottom are required.
With the following script, Ansys show the top and bot Stress of node 377 (node in the middle of the rim).

```
Shell,top
*Get,sxyt,node,377,s,xy
Shell,bot
*Get,sxyb,node,377,s,xy
```

Node number is 377

stress-Sxy	top	bot
s_{xy}	14.951	-14.951
t=	8	
m_{xy}=	79.73867	

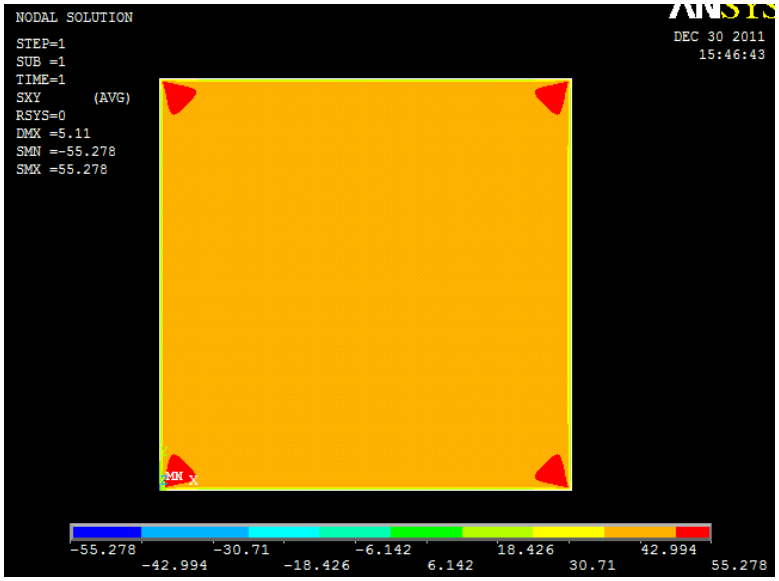


Figure 3.3 x,y shear stress op the top layer

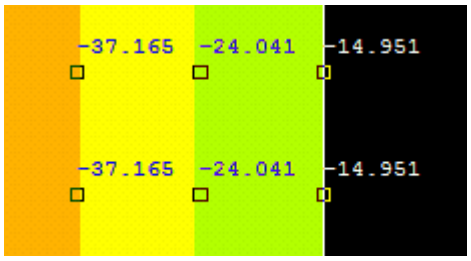


Figure 3.4 shear stress at the edge

Comments

Following the figure 3.3 and figure 3.4, the shear stress will going to zero at the rim of the glass plate. If the mesh is smaller, the data will get smaller, which is close to zero. That means the moment will also get to zero. That will match the idea that the m_{xy} become nothingness, and the q_x is get the largest at the edge.

For Transverse shear forces (per unit length) in Ansys, must follow the script under [4]

```
! For Transverse shear forces (per unit length)
/post 26
esol,3,31500,377,smisc,7,y
prvar 3
```

Then get the data from the Ansys' output window

Q13	-0.8285^{e}-4
Q23	-141.3

If changing the Poisson's ratio v=0, the thickness is 8 mm

Knop point number is 377

stress-Sxy	top	bot
s_{xy}	14.979	-14.979
t=	8	
m_{xy}=	79.888	
Q13	-0.1020e-3	
Q23	-141.6	

The thickness is 4 mm, Poisson's ratio v=0.23

Knop point number is 377

stress-Sxy	top	bot
s_{xy}	97.967	-97.967
t=	4	
m_{xy}=	130.623	
Q13	-0.6499e-4	
Q23	-210.7	

The thickness is 4 mm, Poisson's ratio v=0

Knop point number is 377

stress-Sxy	top	bot
s_{xy}	98.056	-98.056
t=	4	
m_{xy}=	130.7413	
Q13	-0.7982e-4	
Q23	-210.9	

100*100 the thickness is 4 mm, Poisson's ratio v=0.23

Knop point number is 77, element 1250

stress-Sxy	top	bot
s_{xy}	100.943	-100.943
t=	4	
m_{xy}=	134.59	
Q13	-0.7172e-2	
Q23	-216.6	

100*100 the thickness is 8 mm, Poisson's ratio v=0.23

Knop point number is 77, element 1250

stress-Sxy	top	bot
s_{xy}	15.946	-15.946
t=	8	
m_{xy}=	85.04533	
Q13	-0.8365e-2	
Q23	-149.5	

Here we tried 3-4 different sets of data, to provide information to the final calculation. More data will support the results more reasonable; it is better to reduce the error. The changes in the model are forced on the length and thickness.

3.3 Solid model

Solid model is 3-d model or Volume model, which is the key of the research. Solid model 185 has been choosing for the geometry. See the figure 3.5 below. Figure 3.6 is the figure of the Ansys, when all scripts input into the Ansys. The display is with thickness, and then you can get the shear-stress data in one assign element. The shear stress should be YZ-shear stress (σ_{yz}).

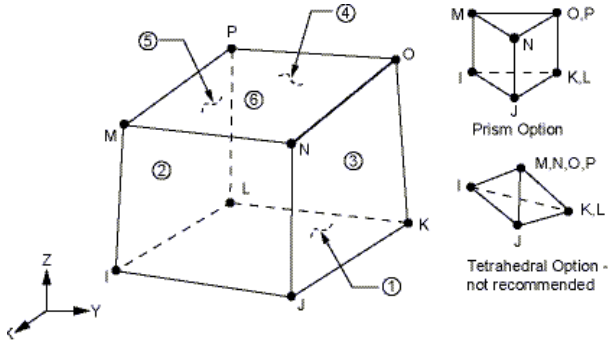


Figure 3.5 SOLID 185 Homogeneous structural solid geometry

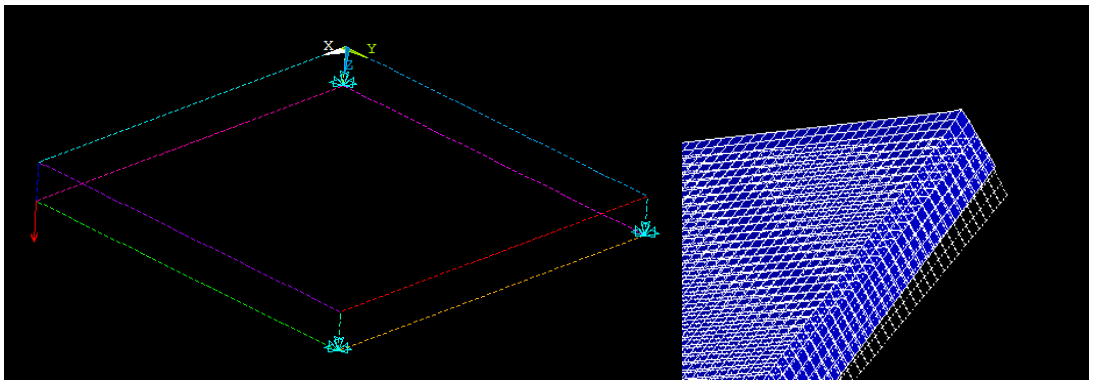


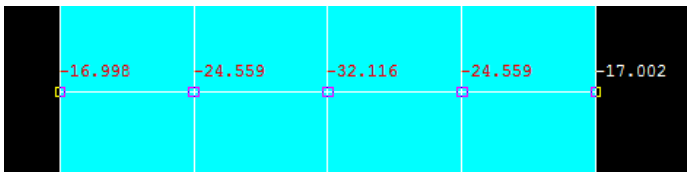
Figure 3.6 Model in Ansys

Figure 3.7 displacement after loads

3.3.1 Data information:

The follow information will be get from the Ansys. The scripts will be show at the appendix. The entire figure in this section is the screenshot from the right view of the Ansys model. The data could be point out by the software.

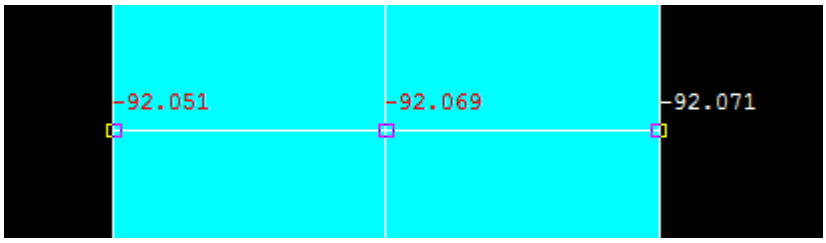
Solid 185 elements
Mesh-element: 2*2*2mm
Width: 500mm
Length: 500mm
Thickness: 8mm



X=500 Y=250

z	0	2	4	6	8
σ_{yz}	-17.0019	-24.559	-32.116	-24.559	-16.998

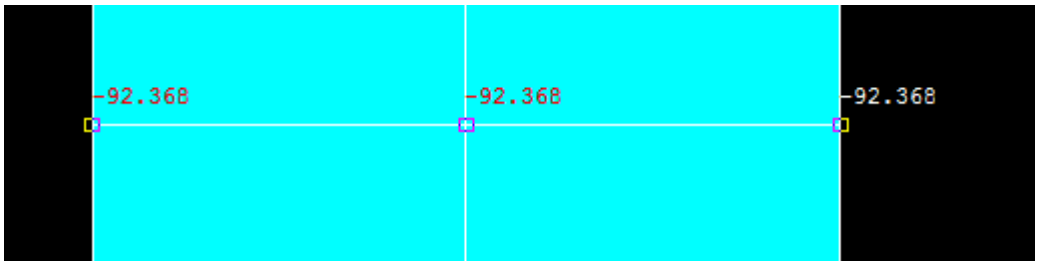
Solid 185 elements
Mesh-element: 2*2*2mm
Width: 500mm
Length: 500mm
Thickness: 4mm
PR=.23



X=500 Y=250

z	0	2	4
σ_{yz}	-92.071	-92.069	-92.051

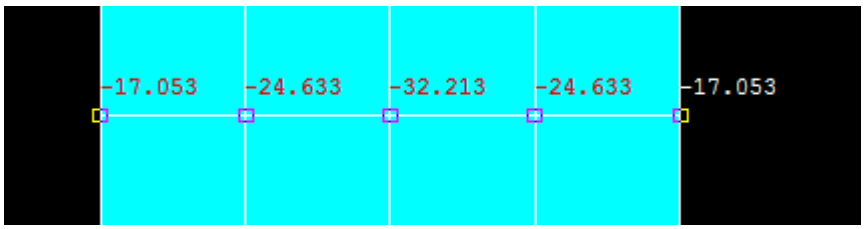
Solid 185 elements
Mesh-element: 2*2*2mm
Width: 500mm
Length: 500mm
Thickness: 4mm
PR=0



X=500 Y=250

z	0	2	4
σ_{yz}	-92.368	-92.368	-92.368

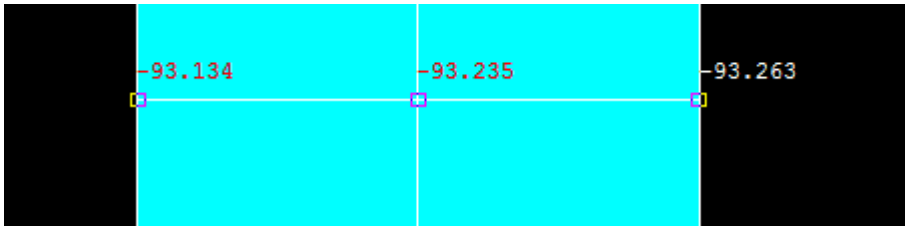
Solid 185 elements
Mesh-element: 2*2*2mm
Width: 500mm
Length: 500mm
Thickness: 8mm
PR=0



X=500 Y=250

z	0	2	4	6	8
σ_{yz}	-17.053	-24.633	-32.213	-24.633	-17.053

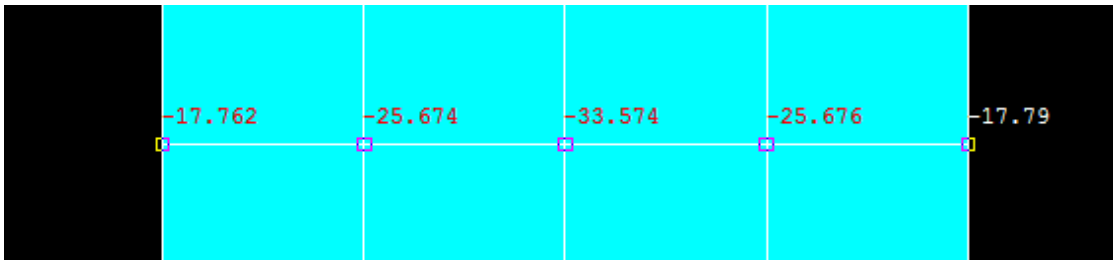
Solid 185 elements
Mesh-element: 2*2*2mm
Width: 100mm
Length: 100mm
Thickness: 4mm
PR=0.23



X=100 Y=50

z	0	2	4
σ_{yz}	-93.263	-93.235	-93.134

Solid 185 elements
Mesh-element: 2*2*2mm
Width: 100mm
Length: 100mm
Thickness: 8mm
PR=0.23



X=100 Y=50

z	0	2	4	6	8
σ_{yz}	-17.79	-25.676	-33.574	-25.674	-17.762

3.3.2 Solid 186

Solid 186 is a higher order 3-d 20-node solid element that exhibits quadratic displacement behavior. Solid 185 is a lower-order version of the solid186 element. Solid 185 element is defined by eight nodes and the orthotropic material properties. See the figure below for solid 186 element.

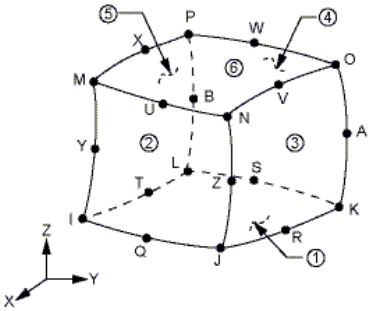


Figure 3.8 solid 186

Data:
100*100mm, mesh size = 1*1*1, solid 186

	σ_{yz}	σ_{xy}
T=4	-141.104	182.046
T=8	-36.2148	46.71

Comments

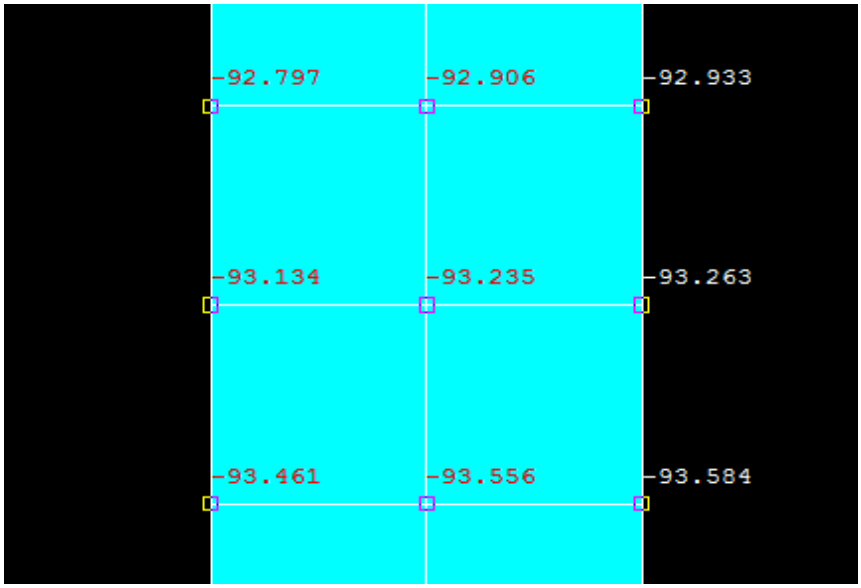
The results of σ_{yz} which test with solid 185 and of solid 186, are not same. There are two reasons. One is about the solid element. Solid 186 is higher order 3-d element than solid 185, it should be more accurate. For thickness is 8mm, the data has only 7.3% difference between each other. For thickness is 4 mm, the different is large, about 33.8%. The mesh size will be the cause of the disparity. 3.3.3 Mesh size will try to find out how the mesh size could influence the results.

3.3.3 Mesh size

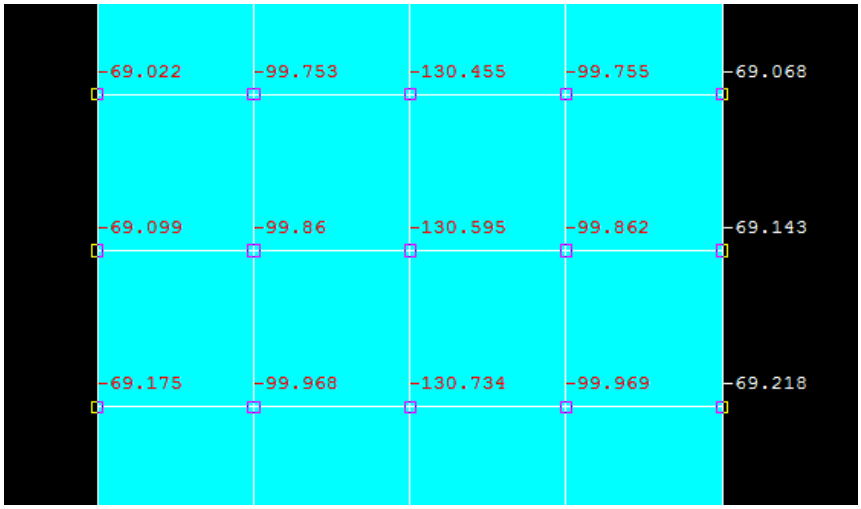
Ansys meshing technology present physics preferences that support automate the meshing process. For a first design, a mesh can usually be produced in batch with a first answer operate to find regions of interest. Even more refinement can then be produced to your mesh to improve the accuracy of the solution. You will find physics preferences for structural, fluid, explicit and electromagnetic simulation. By environment physics preferences the computer program adapts to more logical defaults inside the meshing procedure for greater solution accuracy.

In the simulation, mesh size is very important for the objective. Then we will test for the mesh size in de model" 100 x 100 mm". Figure 3 shows when the mesh size charge, how will the σ_{yz} goning.

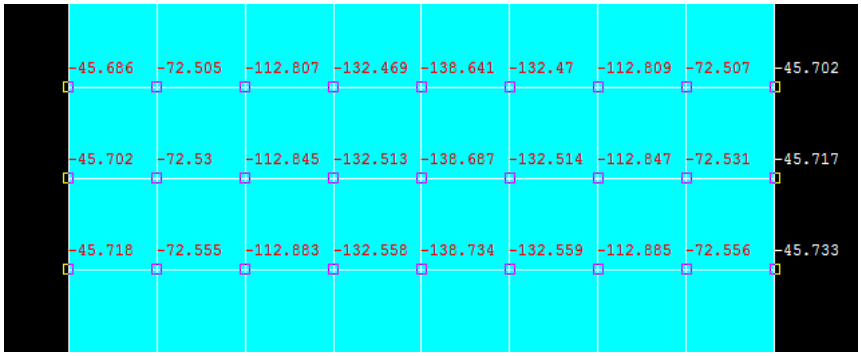
Mesh 2.0



Mesh 1.0



Mesh 0.5



σ_{yz} for z=2

Mesh size 2.0	Mesh size 1.0	Mesh size 0.5
-93.235	-130.595	-138.687

Comments

When the size of mesh changed, the stress in the middle of the thickness is not the same. The different is not small. That means the mesh size has a large influence on the data. This could impact the end results of this research.

Another test will be done for the σ_{yz} for the 100*100mm panes, and thickness is 8mm.

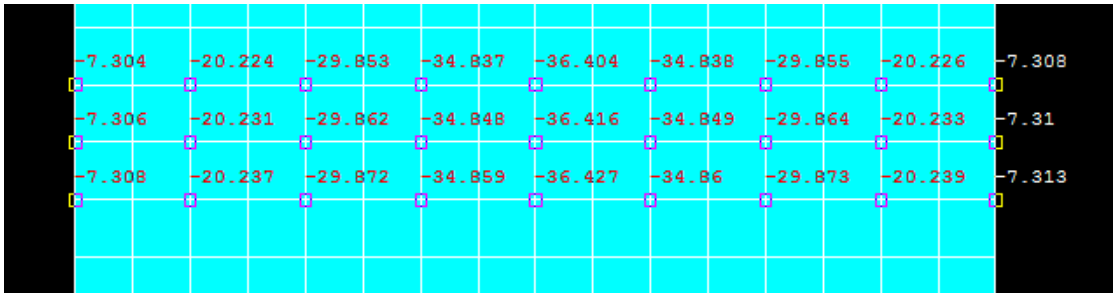


*Figure 3.9 100*100 * 8, mesh=0.5*

3.3.4 Improve the model

End results of this research are related to the good form mesh model. The model should be improved. But the computer is able to solve the problem with no more than about 32000 nodes. The new model will reduce the mesh at the area, which will get the data. And the rest part of the panes will be with relative big mesh. See the new model at the Appendix.

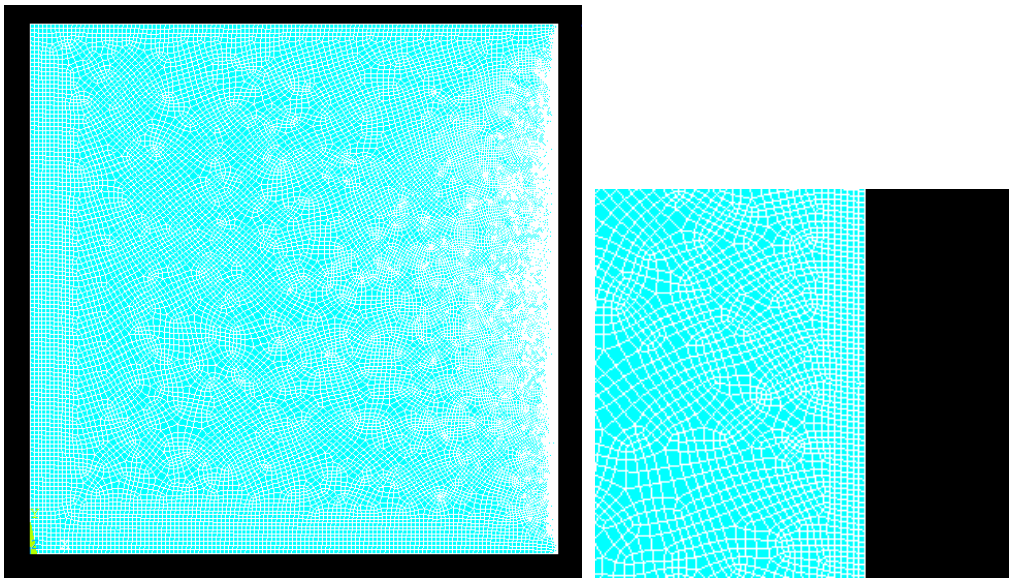
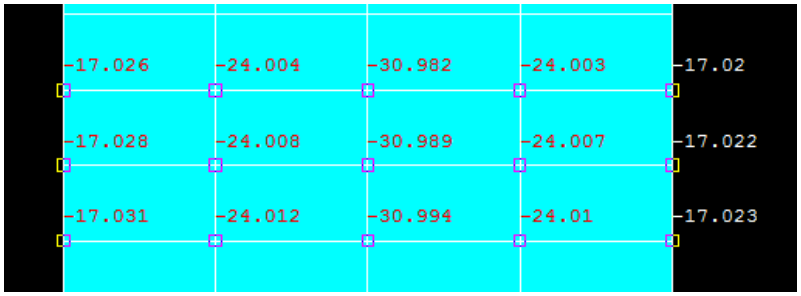


Figure 3.10 mesh size of the new solid model, detail of the mesh In the edge (left figure)

The new model have great advantage for the result. There are about 125 meshs in the three edges (up, down, right). At the left side there are 500 meshs.

Solid 185 elements
Width: 500mm
Length: 500mm
Thickness: 8mm



X=500 Y=250 Z=4

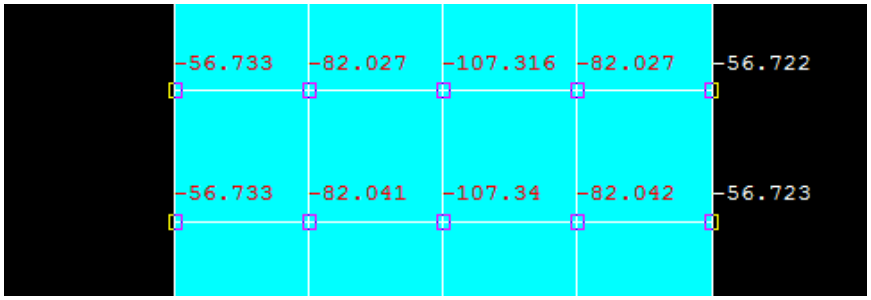
σ_{yz} = -30.989

Solid 185 elements

Width: 500mm

Length: 500mm

Thickness: 4mm



X=500 Y=250 Z=2

σ_{yz} = -107.316

4. Derivation

The derivation of the required formula is based on the data from shell-model and solid-model. Each data has a relative length and width, only the thickness changes. The data has been substituted in the calculation model, that is $\sigma_{yz} = x * \frac{m_{xy}}{t^2} + y * \frac{q_x}{t}$. Calculation is achieved through Maple (see the Appendix). The unknown parts are coefficients X and Y.

500*500 mm

σ_{yz}Read from solid model,m_{xy}, q_x read from shell model, Poisson's ratio v=0.23

	σ_{yz}	m_{xy}	q_x
t=8	-30.989	79.73867	-141.3
t=4	-107.316	130.623	-210.7

$x = -3.349035436, y = 1.518266426$

$$\sigma_{yz} = -3.349 * \frac{m_{xy}}{t^2} + 1.518 * \frac{q_x}{t}$$

100*100 mm

σ_{yz}Read from solid model,m_{xy}, q_x read from shell model, Poisson's ratio v=0.23

	σ_{yz}	m_{xy}	q_x
t=8	-36.416	85.04533	-149.5
t=4	-138.687	134.59	-216.6

After calculate with MAPLE

$\{x = -7.271030136, y = 1.431652879\}$

$$\sigma_{yz} = -7.27 * \frac{m_{xy}}{t^2} + 1.43 * \frac{q_x}{t}$$

5. Alternative derivation

An alternative derivation is an analysis with just the 3D model (solid model). This follows the relation with stress and moment. The formula, $m_{xy} = (s_{xyt} - s_{xyb}) \times t \times t/12$, is suitable for this situation. s_{xyt}, s_{xyb} are always the same number with different direction. So the formula can simplify as $m_{xy} = (s_{xyb}) \times t\text{^}2/6$. Because q_x in the edge of the panes will be zero, that means the σ_{yz} is biggest. The relation between σ_{yz} and m_{xy} will be $\sigma_{yz} = \alpha \frac{m_{xy}}{t^2}$, α is unknown.

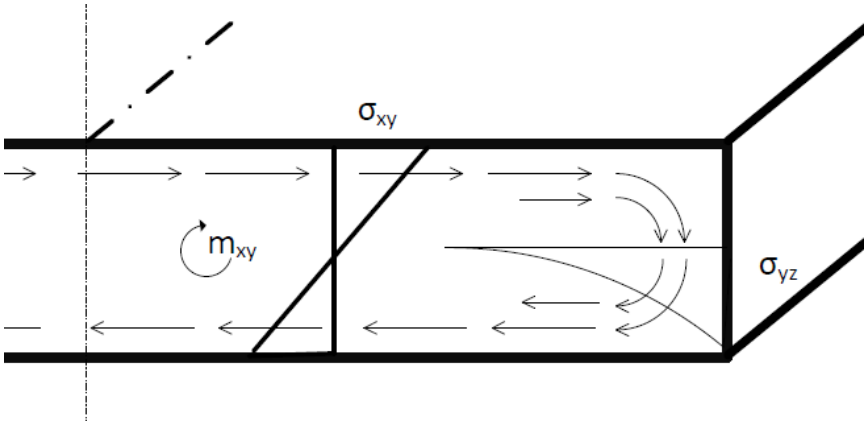


Figure5.1 Cross-section of the solid model

Figure 5.1 show the trend of σ_{xy}. At the edge it is close to zero. We should choose the element which is about 4*t distance to the edge for the veracity. s_{xyb} is choose 184 N/mm. the others necessary data is in the following table.

Test will under “solid 185”

	σ_{yz}	σ_{xy}
T=4	-138.734	184.1
T=8	-36.416	47.525

$$\begin{cases} \sigma_{yz} = \alpha \frac{m_{xy}}{t^2} \\ m_{xy} = \sigma_{xy} \times t\text{^}2/6 \end{cases} \Leftrightarrow \sigma_{yz} = \alpha \frac{\sigma_{xy}}{6}$$

	α
T=4	-4.52
T=8	-4.60

100*100 mm, mesh size = 1*1*1, solid 186

	σ_{yz}	σ_{xy}
T=4	-141.104	182.046
T=8	-36.2148	46.71

$$\begin{cases} \sigma_{yz} = \alpha \frac{m_{xy}}{t^2} \\ m_{xy} = \sigma_{xy} \times t/6 \end{cases} \Leftrightarrow \sigma_{yz} = \alpha \frac{\sigma_{xy}}{6}$$

	α
T=4	-4.6506
T=8	-4.6518

Comments:

The alternative method is also a simple way to obtain the formula, this way also reduce the uncertainty of the factor. The calculations both indicate that α exists and is about -4.60. Meanwhile the supposed relations between σ_{yz} and m_{xy} are proved.

6. Confirmation

After the derivation of chapter 5, the relation between σ_{xy} and σ_{yz} has been found out, that is the shear stress in an edge is 22% smaller than the shear stress at the glass surface. In order to prove that the relation will not change by other factors, some tests have been done which are reported below.

The following factors have been changed:
1) Young's modulus 2) Poisson's Ratio 3) Pane length and width 4) Pane length

0) Basic model
Young's modulus is 72e4
100*100 mm, mesh size = 1*1*1, solid 186

	σ_{yz}	σ_{xy}
T=4	-141.104	183.275
T=8	-36.2148	46.71

According to the basic model, α is about 4.65.

1) Young's modulus

Young's modulus changed to 64E4, σ_{xy} at punt x=80 y=50, σ_{yz} at punt x=100 y=50 z=4

	σ_{yz}	σ_{xy}
T=4	-141.104	182.046
T=8	-36.215	47.261

	α
T=4	-4.6506
T=8	-4.598

Young's modulus changed to 90e4

	σ_{yz}	σ_{xy}
T=4	-141.104	183.275
T=8	-36.2148	47.2613

	α
T=4	-4.6194
T=8	-4.5976

The result is independent of Young's modulus.

2) Poisson's ratio

Normal = 0.23

Change Poisson's ratio to 0.01

	σ_{yz}	σ_{xy}
T=4	-141.841	184.723
T=8	-36.5297	47.574

	α
T=4	-4.607
T=8	-4.607

Change Poisson's ratio to 0.49

	σ_{yz}	σ_{xy}
T=4	-140.214	180.965
T=8	-35.8773	46.744

	α
T=4	-4.6489
T=8	-4.605

The results are independent of the Poisson's ratio.

3) Pane length and width

Change Pane length and width to 80*80, σ_{xy} at x=60 y=40

	σ_{yz}	σ_{xy}
T=4	-141.997	182.978
T=8	-36.6344	47.2942

	α
T=4	-4.6562
T=8	-4.6476

Change Pane length and width to 60*60, σ_{xy} at x=40 y=30

	σ_{yz}	σ_{xy}
T=4	-143.473	185.849
T=8	-36.306	46.8675

	α
T=4	-4.63192
T=8	-4.6479

Change Pane length and width to 50*50, σ_{xy} at x=40 y=25

	σ_{yz}	σ_{xy}
T=4	-144.637	185.894
T=8	-37.8107	48.129

	α
T=4	-4.668
T=8	-4.7136

Change Pane length and width to 40*40, σ_{xy} at x=35 y=20

	σ_{yz}	σ_{xy}
T=4	-146.349	187.222
T=8	-38.4949	44.8559

	α
T=4	-4.69
T=8	-5.149

Change Pane length and width to 30*30, σ_{xy} at x=20 y=15

	σ_{yz}	σ_{xy}
T=4	-149.091	186.003
T=8	-39.3982	49.2786

	α
T=4	-4.809
T=8	-4.797

Change Pane length and width to 20*20, σ_{xy} at x=12 y=10

	σ_{yz}	σ_{xy}
T=4	-153.945	189.078
T=8	-40.574	50.2886

	α
T=4	-4.885
T=8	-4.841

4) Pane length

100*80 σ_{xy} at x=80 y=40

	σ_{yz}	σ_{xy}
T=4	-141.766	183.792
T=8	-36.5591	47.6089

	α
T=4	-4.628
T=8	-4.607

100*60 σ_{xy} at x=80 y=30

	σ_{yz}	σ_{xy}
T=4	-143.226	185.98
T=8	-37.2175	48.6372

	α
T=4	-4.6207
T=8	-4.59123

100*40 σ_{xy} at x=80 y=30

	σ_{yz}	σ_{xy}
T=4	-146.57	191.6
T=8	-38.4697	51.2506

	α
T=4	-4.5899
T=8	-4.504

The result is independent of the pane length and width, but the pane edges need to be longer than 8 times the thickness.

7. CONCLUSIONS

In a glass pane loaded in torsion the shear stress in an edge is 22% smaller than the shear stress at the glass surface at some distance of the edge. This is independent of the pane thickness, Young's modulus and Poisson's ratio. It is also valid for rectangular panes. The pane edges need to be longer than 8 times the thickness.

In computations with shell elements an obtained torsion moment at the edge is not real. It would be zero if extremely small shell elements would be used. However, these very small shell elements are not practical. The torsion moment at a distance of approximately three times the thickness from the edge does not strongly depend on the element size.

The following formula can be derived for a pane loaded in torsion $\sigma = 4.6\frac{m_{xy}}{t^2}$, where t is the pane thickness and m_{xy} is the torsion moment at some distance of the edge.

8. REFERENCES

[1] M. Eekhout, The New, Cold Bent Glass Roof of the Victoria & Albert Museum, London http://www.glassglobal.com/gpd/downloads/BuildingProjects-Eekhout.pdf

[2] F.A. Veer, *The strength of glass, a nontransparent value*, Heron, Vol. 52 (2007) No. 1/2, pp. 87-104.

[3] J. de Wit, *Computational modeling of cold bent glass panels,* MSc project, Delft University of Technology, 2009

[4] Ansys help Table 181.3

SOFTWARE

ANSYS Academic Teaching Introductory 11.0 (2011), online, http://www.ansys.com/

9. Appendix

Appendix for solid model

```
FINISH $/CLEAR
/FILNAM, MODEL    ! SECIFY JOBNAME
/PREP7    ! ENTER PREPROCESSOR

W=100 ! DIMENSIONS RECTANGLE, THE PARAMETERS CAN BE CHANGED
H=100
D=4
MW=1    ! MESHSIZE
F1=1000
F2=
E=72E4
PR=0.23

ET,1,185      !solid 185
MP,EX,1,E
MP,PRXY,1,PR

K,1,0,0 ! CREATE KEYPOINTS
K,2,W,0
K,3,0,H
K,4,W,H



a,1,2,4,3, ! CREATE AREAS

vext,1,,,,,D

lesize,all,MW

vsweep,1

FINISH


/SOLU

LPLOT
DK,5,UX,,,,UY,,,,UZ
```

```
DK,7,UX,,,,UY,,,,UZ
DK,8,UX,,,,UY,,,,UZ
FK,6,FZ,F1

SOLVE
FINISH
```

Appendix for shell model

```
FINISH $/CLEAR
/FILNAM, 2d-shell          ! SECIFY JOBNAME
/PREP7                     ! ENTER PREPROCESSOR
W=100                      ! DIMENSIONS RECTANGLE, THE PARAMETERS CAN BE CHANGED
H=100
D=8
MW=0.5                      ! MESHSIZE
F1=1000
E=72E4
PR=0.23

ET,1,181                !shell 181
MP,EX,1,E
MP,PRXY,1,PR

K,1,0,0                  ! CREATE KEYPOINTS
K,2,W,0
K,3,0,H
K,4,W,H

a,1,2,4,3,               ! CREATE AREAS

R,1,D

lesize,all,MW

ESIZE,2,0,
MSHAPE,0,2D
MSHKEY,0
!*
CM,_Y,AREA
ASEL, , , ,             1
CM,_Y1,AREA
CHKMSH,'AREA'
CMSEL,S,_Y
!*
AMESH,_Y1
!*
CMDELE,_Y
CMDELE,_Y1
CMDELE,_Y2
!*
```

```
LPLOT
DK,1,UX,,,,UY,,,,UZ
DK,4,UX,,,,UY,,,,UZ
DK,3,UX,,,,UY,,,,UZ
FK,2,FZ,F1                    !Load

/SOLU
SOLVE
FINISH


! For Transverse shear forces (per unit length)
/post 26
esol,3,31500,377,smisc,7,y
prvar 3

! for  m_xy  from the shell element
Shell, top
*get, sxyt, NODE, 31500, S, XY
Shell, bot
*get, sxyb, NODE, 31500, S, XY
Mxy=(Sxyt-Sxyb)*t^2/12
```

Calculation with Maple

> $t1 := 8$

$$8$$

$t2 := 4$

$$4$$

$mxy1 := 85.04533$

$$85.04533$$

$q231 := -149.5$

$$-149.5$$

$mxy2 := 134.59$

$$134.59$$

$q232 := -216.6$

$$-216.6$$

$o1 := -36.416$

$$-36.416$$

$o2 := -138.687$

$$-138.687$$

$$eq1 := \frac{x \cdot mxy1}{t1^2} + \frac{y \cdot q231}{t1} = o1$$

$$1.328833281\ x - 18.68750000\ y = -36.416$$

$$eq2 := \frac{x \cdot mxy2}{t2^2} + \frac{y \cdot q232}{t2} = o2$$

$$8.411875000\ x - 54.15000000\ y = -138.687$$

$solve(\{eq1, eq2\}, \{x, y\})$

$$\{x = -7.271030136,\ y = 1.431652879\}$$

>

Appendix for new solid model

```
FINISH $/CLEAR
/FILNAM, 3d    ! SECIFY JOBNAME
/PREP7    ! ENTER PREPROCESSOR

W=500 ! DIMENSIONS RECTANGLE, THE PARAMETERS CAN BE CHANGED
H=500
D=8
mw=2    ! MESHSIZE
MWA= 0.5
F1=1000
F2=
E=72E4
PR=0.23

ET,1,185        !solid 185
MP,EX,1,E
MP,PRXY,1,PR


K,1,0,0 ! CREATE KEYPOINTS
K,2,W,4D
K,3,0,H
K,4,W,H-4D
K,9,W,0
K,10,W,H

a,1,9,2,4,10,3, ! CREATE AREAS

lesize,all,,,125
lesize,3,,,500

vext,all,,,,,D

lesize,15,,,4
lesize,16,,,4


vsweep,1

FINISH
```

```
/SOLU

LPLOT
DK,10,UX,,,,UY,,,,UZ
DK,3,UX,,,,UY,,,,UZ
DK,1,UX,,,,UY,,,,UZ
FK,9,FZ,F1

SOLVE
FINISH
```