

Delft University of Technology

Bachelor's thesis

# Mechanical equations for porous construction elements

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April 2, 2021

**Abstract**

Porous construction elements, with grid points in a lattice-structure, will react differently to forcing than a solid homogeneous variant. Although the reaction of a solid homogeneous construction element is often expressed in known simple equations the reaction of a porous lattice-structure will have to be calculated if needed. Therefor the goal of this research was to find an equation that expresses the reaction of a porous construction element in a simplified equation. This report shows the use of a Python script, a finite element method module, written to find the deflection of a crossed-rectangle porous cantilever beam under a concentrated force. With that script a study was done to look into the possibilities to create a simplified equation for this specific mechanical situation. A parametric analysis resulted in an equation that approaches the deflection and is similar in form as the known equation for the solid homogeneous cantilever beam. The accuracy of this equation is discussed along with the possible uses of this research. Due to the methodology used the accuracy is highly influenced by some variables and therefor not perfect. Under specific circumstances the estimation is proper.

# Mechanical equations for porous construction elements

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# 1 Introduction

## 1.1 Problem

Fabrication techniques such as selective laser melting(SLM) open up the possibility the produce lattice-structures with complex internal design that have distinctive properties. Such lattice-structures are lightweight and high strength. This makes them potentially beneficial to use in construction where these properties are much sought after.

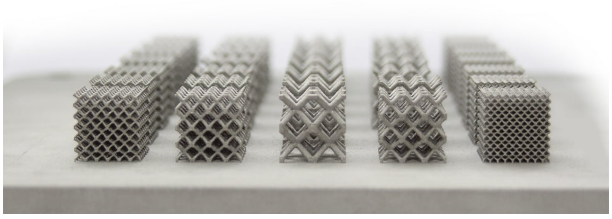

Figure 1: Lattice-structure built on Renishaw AM250 metal AM system (Renishaw, n.d.)

Although extensive research is done on the mechanical properties of such lattice-structures, mechanical equations to represent and simplify a specific lattice-structure's response can be beneficial for fast calculations and estimations. A quick calculation for an estimation is often all that is needed.

This report delivers an equation that estimates the reaction of such lattice-structure under a specific loading situation.

## 1.2 Objective

There is the need to find analytical relations for certain 'lattice-structure models', that represent porous construction elements, and express them in simplified mechanical equations. Because of the repetitive but complex structures calculations can become extensive.

The topology, anisotropy, loading direction and irregularity of the lattice-model is important(Alkhader and Vural, 2008) to determine the structure's reaction to a certain mechanical situation while this influence the mass, relative density and thus the directive elasticity, strength and strain of that element.

Due to this we need to find the mechanical equation for the lattice-structures taking into account certain input variables such as the internal angles of the connections, material properties, size and build-up of the structure.

To detail the scope of the research, the mechanical scenarios and lattice-models analysed to find mechanical equations are specific. One scenario is modelled in detail, displayed in the following figure 2:

- Lattice-structure cantilever beam with a concentrated load

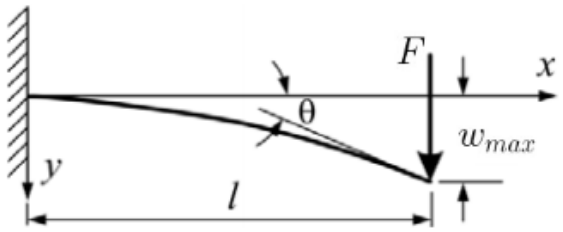


Figure 2: Cantilever deflection

A specific lattice-structure model is researched: Crossed-Rectangles (right)

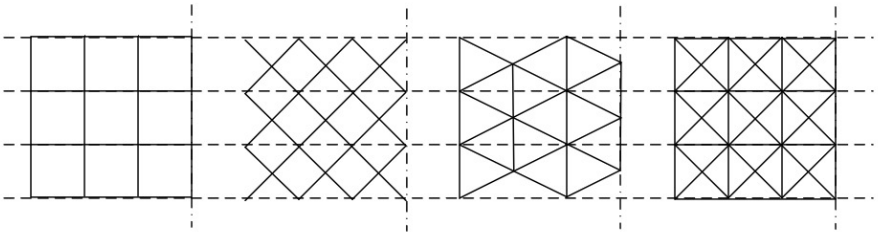

Figure 3: Visualisation lattice-structures with rectangles, diamonds, triangles and crossed rectangles (fLTR)

The research question to answer:

- How can the reaction of a porous cantilever beam under a concentrated force be expressed in a mechanical equation?

The partial questions answered and analyses done to lead to an answer to the research question are:

- Parameter study

- What equation could be created for the deflection of a porous crossed-rectangle cantilever beam?

The objective of the report is to find an equation or way to simply calculate above mentioned reaction of the porous crossed-rectangle cantilever beam with high accuracy.

## 1.3 Approach

A finite element method (FEM) results in the deflection of the scenario and that could indicate a set relation between the known mechanical equation for a solid homogeneous beam and the desired equation for the lattice-structure beam. The equation for the solid cantilever beam is known to be reliant on bending stiffness. The lattice-structure's will also have a directive bending stiffness but that will dependent on the specific model. The equation has to be accounting for this, and the input of other variables. A solid cantilever beam has the following mechanical equations, with variables:

$$w_{max} = \frac{Fl^3}{3EI}; \quad \Theta = \frac{Fl^2}{2EI} \tag{1}$$

The structure, in this research, is assumed to be slender, as the cross-sectional dimensions are much smaller than the axial lengths.

The outcome sheds light on the relation on specific input variables of a crossed-rectangle lattice-structure and the deflection of that lattice-structure in the cantilever mechanical situation.

To generate numerical results for the models a finite element method named CALFEM(Computer Aided Learning of the Finite Element Method) is used as a module for Python. These analytical results could differ from experimental results(Gümrük and Mines, 2013), while this is the case under compression. However, the analytical result might be representative for the construction element's response.

Several scripts using CALFEM are written and used to find and generate results necessary to draw a conclusion. Their function is shortly described but a more detailed description and their application can be found in the appendix.

### Crossed-Rectangle Python Script

The important goals of the script were to be able to generate a cantilever beam with the crossed-rectangle lattice-structure configuration, while remaining infinitely upscale-able. It has to be dependent on specific variables, described below.

The lattice-structure deflection is determined by several input variables in the Python model, to fulfill the set goals for the script. This establishes the following Python function that calculates the deflection of the cantilever beam: cantilever($F$,$t$,$b$,$E$,$h$,$l$,$nh$,$nl$). These variables are elaborated in figure 4 below.

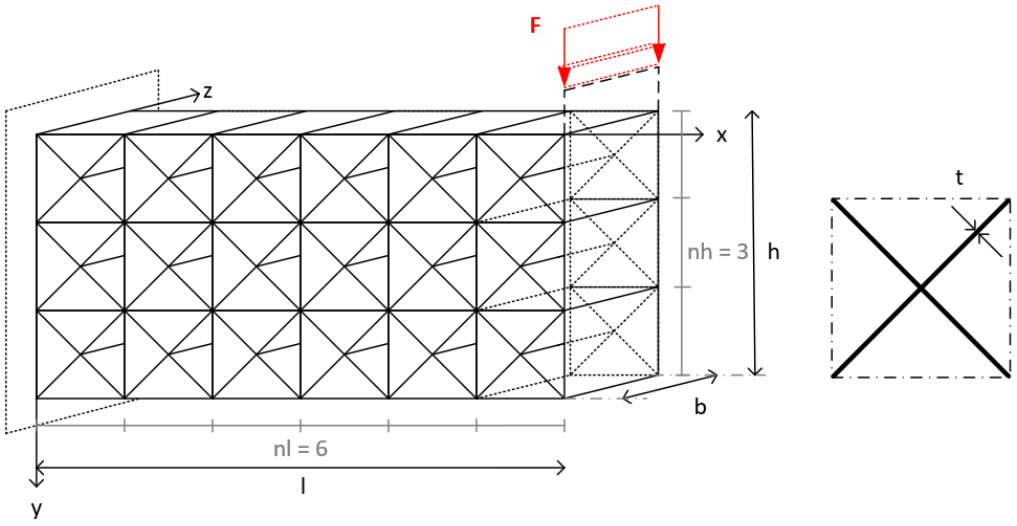


Figure 4: Rectangle with cross lattice-structure with variables

- $F$, the force on the cantilever
- $t$, the thickness of the material
- $b$, the width of the beam
- $E$, the Young's modulus
- $h$, the height of the beam
- $l$, the length of the beam
- $nh$, the number of repeated inner-structures in the y-axis
- $nl$, the number of repeated inner-structures in the x-axis

### Extensive Crossed-Rectangle Python Script

A separate script with largely the same code can be ran to not only find the deflection but find all changed data regarding the lattice-structure such as changed coordinates of the lattice and the internal forces for every connecting element. It is advised to use this script when looking at one set of specific variables. It also prints specific values such as the internal angle of the connections and states whether the chosen dimension results in a structure that is regarded slender. With this additional data it plots the deflected structure as seen in the example below. Lastly, all internal forces of every connecting element are calculated.

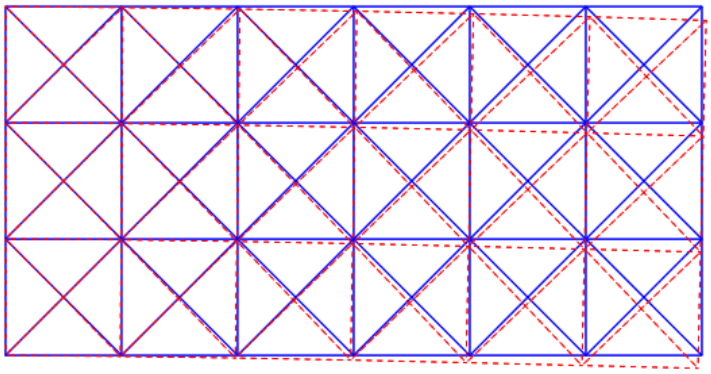

Figure 5: Example of calculated displaced beam generated by the script

# 2 Results

## 2.1 Parameter Study

### Force

The effect of the force($F$) on the deflection of the cantilever beam was researched by having every characteristic besides the force constant. By inputting the following forces on a specific beam a linear relation was quickly found with the use of the written Python function: cantilever().

$$w \sim F \tag{2}$$

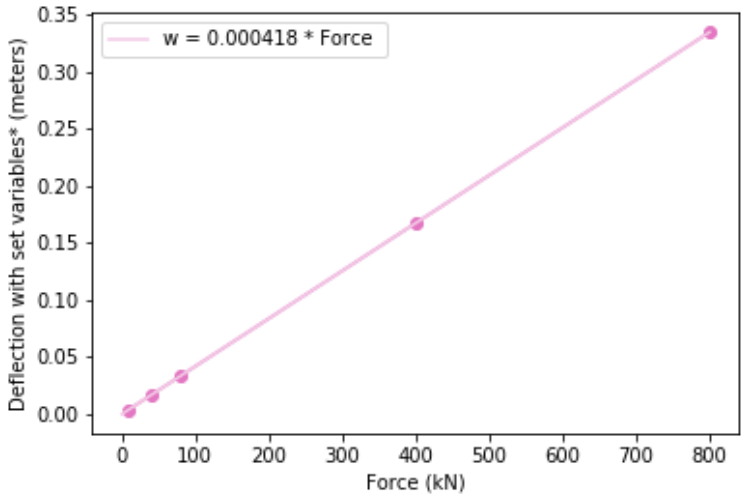


Figure 6: Relation between the applied force and cantilever beam deflection*

*set variables: cantilever(F[i], t=2.5*10**-4, b=0.2, E=210*10**10, h=0.2, l=2, nh=4, nl=6)

### Length

With the same methodology the relation of the length($l$) on the deflection was found. Which as also in line with the solid homogeneous beam as the relation was to the third power.

$$w \sim l^3 \tag{3}$$

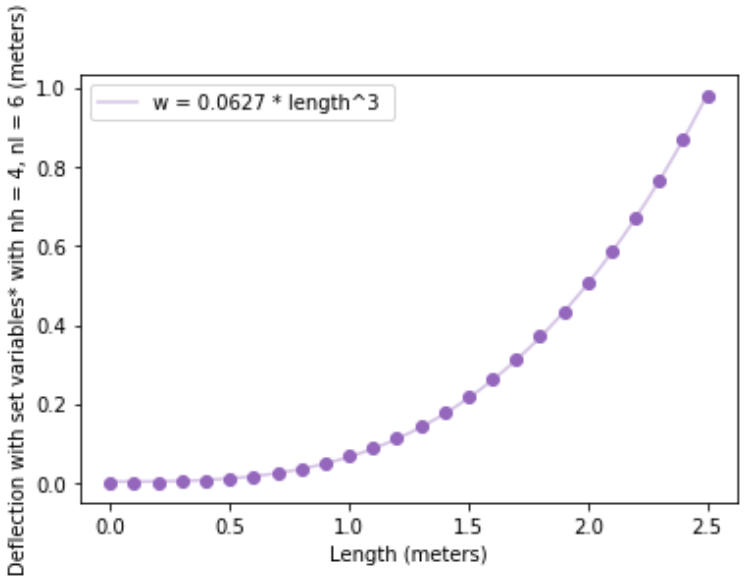


Figure 7: Relation between the length and a 2 meter crossed-rectangle cantilever beam deflection*

*set variables: cantilever(F=-800e3, t=2.5*10**-4, b=0.2, E=210*10**10, h=0.2, l=[i], nh=4, nl=6)

### Young's Modulus

The relation of the Young's modulus and the reflection is also in line with the known equation for the solid homogeneous cantilever beam:

$$w \sim \frac{1}{E} \tag{4}$$

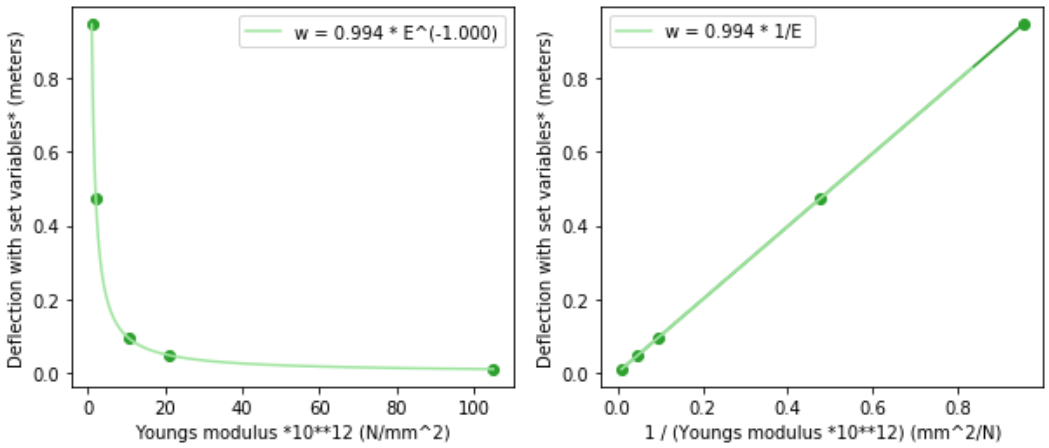


Figure 8: Relation between the Young's modulus and a 2 meter crossed-rectangle cantilever beam deflection*

*set variables: cantilever(F=-800e3, t=2.5*10**-4, b=0.2, E=[i], h=0.2, l=2, nh=4, nl=6)

### The number of repeating inner-structures

The effect that the number of repeating inner-structures has on the deflection was researched by leaving the other input variables constant. The information, 3680 separate deflection calculations, was gathered for every combination in the following sets:

$$nh = \{2 \leq nh \leq 42\}, nl = \{4 \leq nl \leq 96\} \tag{5}$$

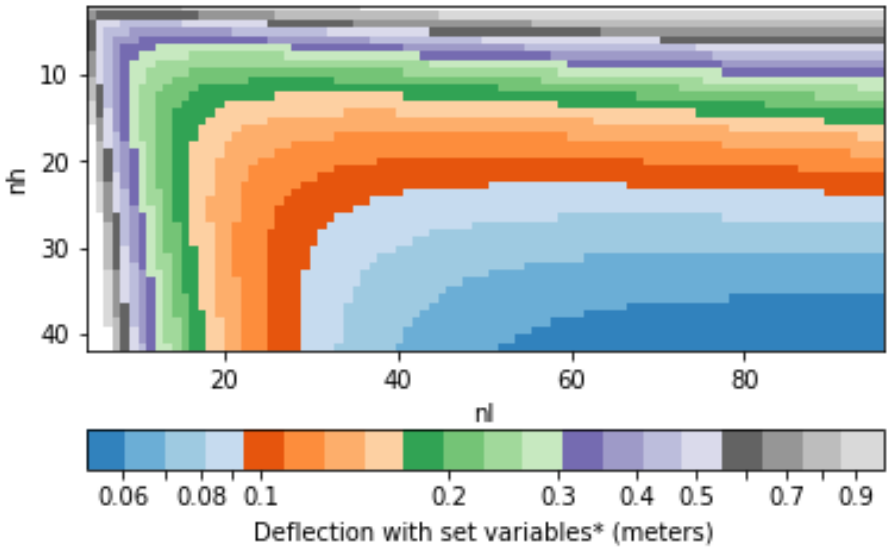


Figure 9: Deflection of 2 meter cantilever beam with variable amount of inner-structures with set variables* **

*set variables: cantilever(F=-800e3, t=2.5*10**-4, b=0.2, E=210*10**10, h=0.2, l=2, nh[i], nl[j])

**White deflections in the left bottom corner imply a deflection larger than 1m or a structural failure

Due to the fact that the more repeating inner-structures are present, the more material is used the relation that more structures give less deflection was expected. One thing about the results is significant: for a specific $nh$ or $nl$ a specific corresponding $nl$ or $nh$ gives an optimum, the least deflection. This can be seen in figure 9 above, but is elaborated in figure 10 below.

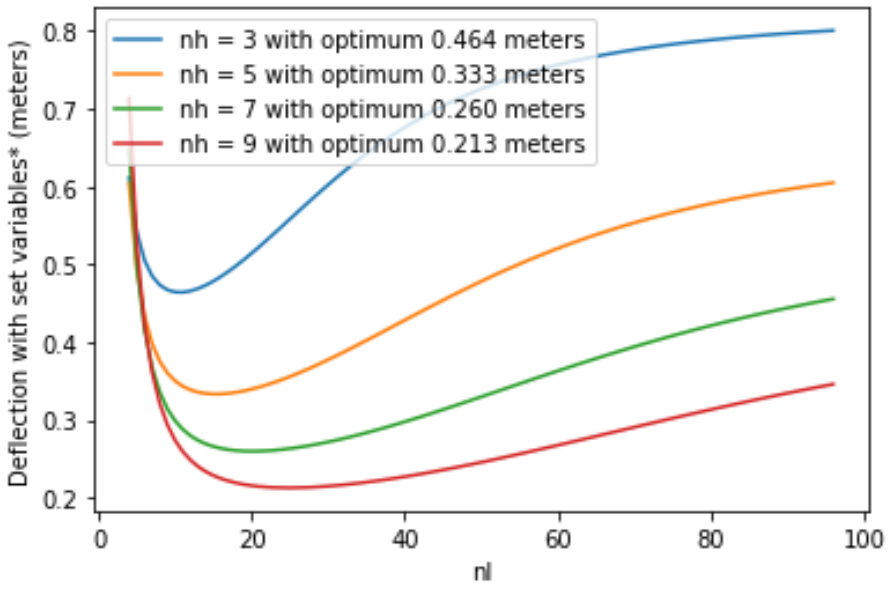


Figure 10: Deflection of 2 meter cantilever beam with variable amount of inner-structures with set variables and highlighted values of nh*

*set variables: cantilever(F=-800e3, t=2.5*10**-4, b=0.2, E=210*10**10, h=0.2, l=2, nh=[3,5,7,9], nl[j])

Four values of $nh$ are picked from the data used to create figure 10 to clearly show to optimum value of $nl$ that corresponds to the value of $nh$ to get the least deflection solution.

The inner structures and their relation to the height and length resulted in a complex relation to the deflection unable to be fitted. However, if the the inner crossed-rectangles are a square, the sides are equal in length, which means the following relation:

$$\frac{h}{nh} = \frac{l}{nl} \tag{6}$$

Then the deflection was able to be fitted and shown in figure 11 below:

$$w(\frac{h}{nh} = \frac{l}{nl}) \sim \frac{8 \cdot nh}{(nh+2) \cdot (nh+1) \cdot (2+\sqrt{2})} \tag{7}$$

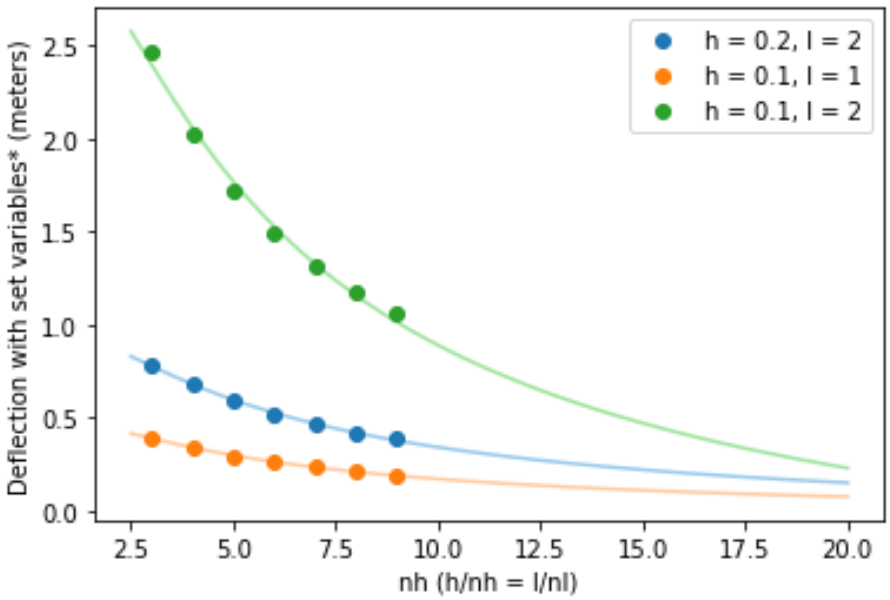


Figure 11: The relation to the deflection when h/nh = l/nl

Although this is helpful for this specific circumstance it does not help because *h, nh, l, nl* are seldom in this ratio due to the character of the chosen model.

The relation between the deflection and these variables is unable to be fitted. The deflection is influenced by a specific value which is dependent above mentioned on variables and every specific ratio that is different than shown in 8.

### Height

After fitting the data, gathered with set variables, of the effect of a changing height($h$) on the deflection the following relation became clear.

$$w \sim \frac{1}{h^2} \tag{8}$$

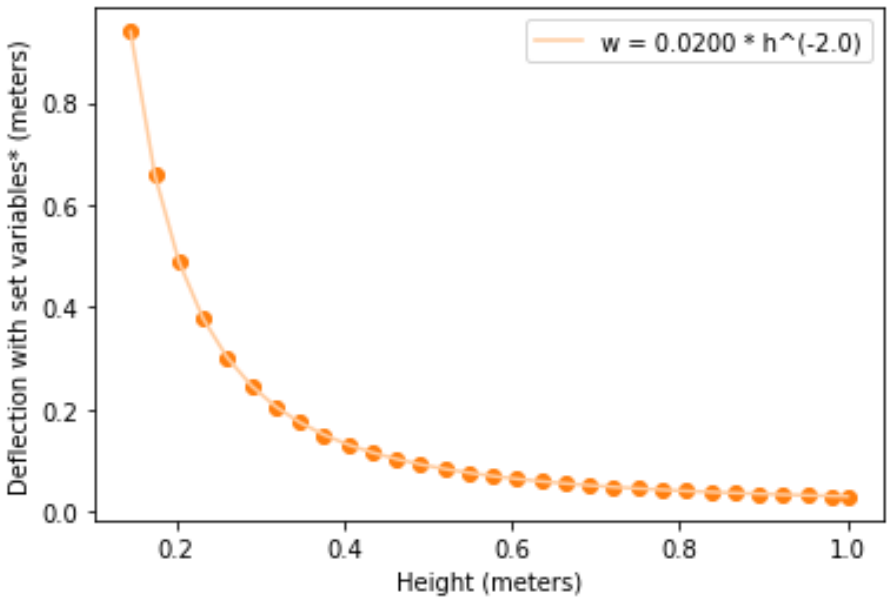


Figure 12: Relation between the Young's modulus and a 2 meter crossed-rectangle cantilever beam deflection*

*set variables: cantilever(F=-800e3, t=2.5*10**-4, b=0.2, E=210*10**10, h=[i], l=2, nh=4, nl=6)

### Thickness

After fitting the data, gathered with set variables, of the effect of a changing thickness($t$) on the deflection the following relation became clear.

$$w \sim \frac{1}{t} \tag{9}$$

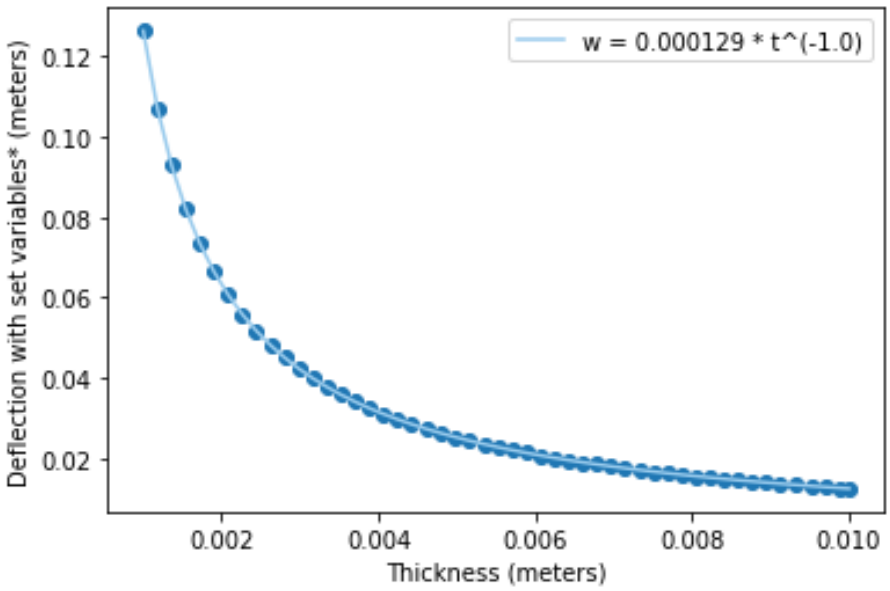


Figure 13: Relation between the thickness and a 2 meter crossed-rectangle cantilever beam deflection*

*set variables: cantilever(F=-800e3, t=[i], b=0.2, E=210*10**10, h=0.2, l=2, nh=4, nl=6)

### Width

After fitting the data, gathered with set variables, of the effect of a changing width($b$) on the deflection the following relation became clear.

$$w \sim \frac{1}{b} \tag{10}$$

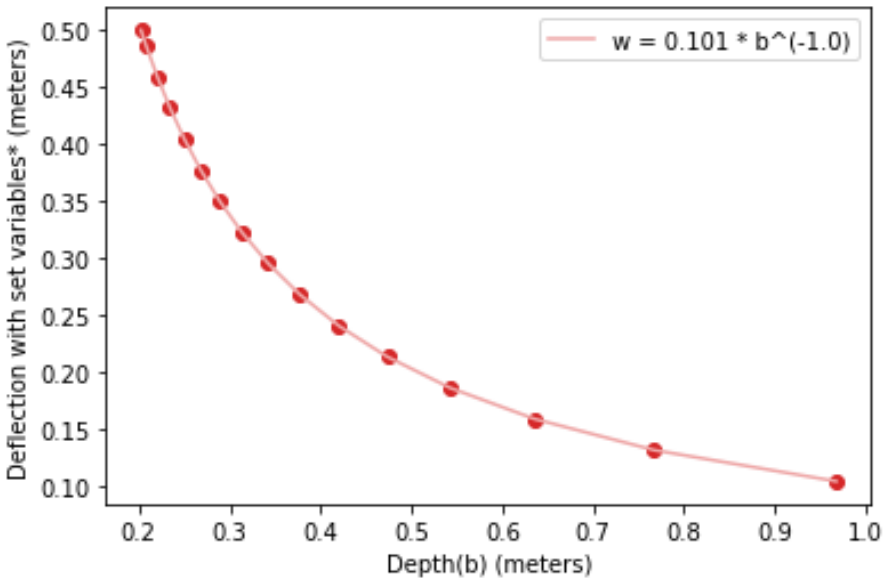


Figure 14: Relation between the thickness and a 2 meter crossed-rectangle cantilever beam deflection*

*set variables: cantilever(F=-800e3, t=2.5*10**-4, b=[i], E=210*10**10, h=0.2, l=2, nh=4, nl=6)

## 2.2 What equation could be created for the deflection of a porous crossed-rectangle cantilever beam?

Now that all relations between the deflection and the variables are made clear we can establish a function for the deflection based on those variables seen in equation 11.

$$w_{max} = \alpha \cdot \frac{F \cdot l^3}{E \cdot h^2 \cdot t \cdot b} \tag{11}$$

The component $\alpha$ is dependent on the amount of repeating inner structures, $nh$ and $nl$, their relation to the height and length and a possible constant component like in equation 1. This equation, for the porous cantilever beam, can be rewritten to be in the same form as equation 1, the function of the solid homogeneous cantilever beam. This form can be obtained with the substitution of $\hat{I}$, where $\hat{I}$ is dependent on the variables which influence the geometry.

$$w_{max} = \frac{Fl^3}{3E\hat{I}}; \quad \hat{I} = \frac{h^2 \cdot t \cdot b}{3\alpha} \tag{12}$$

Due to the nature of $\alpha$ it is a value that is specific for every combination of the variables $nh$,$nl$ and their relation to the height and length. We can therefor read the needed value of $\alpha$ from the table in subsection 5.1, Forget-me-not. The Forget-me-not is an overview of the equation used to calculate the deflection of the crossed-rectangle cantilever beam in a simplified matter and the tables necessary to use the $\alpha$.
To compute the $\alpha$ values the known values of the deflection calculated with the Python script, for a lot of combinations of $nh$ and $nl$, were divided by the right hand side of the equation which consists of the input values also inputted in the Python script.

$$\alpha_{nh,nl} = \frac{w_{nh,nl,max}}{\frac{F \cdot l^3}{E \cdot h^2 \cdot t \cdot b}} \tag{13}$$

# 3 Discussion

The results show a clear relation from the variables and the deflection which is at the origin of the created equation 11. This equation are all the variables in their found relation with the deflection in combination with the $\alpha$ value. This value was chosen as the deflection for a lot of combinations, with a specific data set used, divided by the variables with a known relation. However this equation is inaccurate. It takes into a count the amount of *nh* and *nl*. It does not take into a count the relation from the length and height with the amount of inner structures:

$$\frac{h}{nh} = p * \frac{l}{nl} \tag{14}$$

Although the script returns the exact deflections, the equation, due to the above stated reasons, can only be used as an estimation. The accuracy is represented by the deviation, which is set as a percentage that the answer from the equation is different from the FEM solution.

The following statements can be made about the (in)accuracy of the Forget-me-not(FMN).

- Accuracy is **not impacted** by *force, width and Young's modulus.*
- Accuracy is **impacted** by the *height, length and thickness and the amount of repeated inner structures.*

In figure 15 the relation of the inner structures and thickness with the deviation is highlighted in detail. A zero percent deviation implies the FEM solution equals the FMN solution. It clearly highlights the relation between the deviation due to the thickness, more deviation when the thickness is greater. The coupled lines, *red and green*, *purple and blue*, *brown and orange*, highlight the slight change in accuracy due to a change in thickness. This change in accuracy is no more than 5 percent when $t$ is less than 2 centimeters, and no more than 1.25 percent when $t$ is less than 1 cm. However the deviation from the correct deflection is significant when the above mentioned ratio, the value p, changes.

The deviation also increases if the size of the structure increases, while the p stays the same, seen in the increase in slope of the lines. Size here indicates the following: $h = 0.2,\ nh = 2,\ l = 2,\ nl = 20$ , $h = 0.4,\ nh = 4,\ l = 4,\ nl = 40$ and$h = 0.8,\ nh = 8,\ l = 8,\ nl = 80$ . The increase in p is generated with small steps of increase in the length. The last set of variables of size, $h = 0.8,\ nh = 8,\ l = 8,\ nl = 8$ has the most deviation.

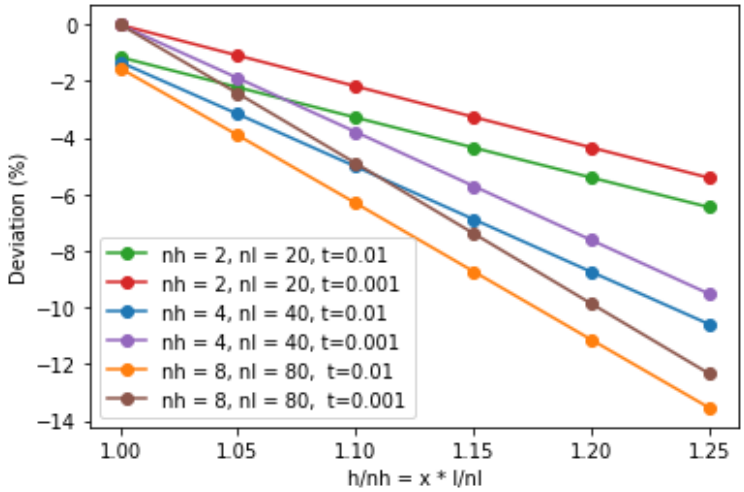


Figure 15: Accuracy of the FMN solution in comparison with the FEM solution under influence of h,nh,l,nl.

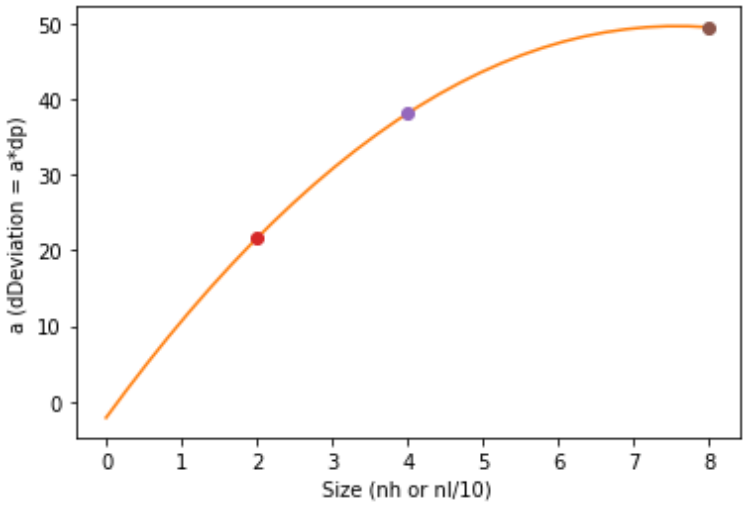


Figure 16: The increase in slope due to size, the red, purple and brown line.

All in all it is clear that the deviation is less than 5 percent if the thickness is smaller than 1 centimeter and the $p$ is less than 1.075. Independent of the sizing, because of the effect of sizing becomes significant with $p$ increases. This is clearly depicted in the top left of figure 15.

# 4 Conclusion

The goal of the research was to find out how to express the reaction of a porous construction element in a simplified mechanical equation, in particular that deflection of a porous cantilever beam with a concentrated load. If possible, the equation needed to be simple and easily usable. Comparable to the known equation 1 for a solid homogeneous cantilever beam. The lattice-structure of the porous element chosen for this research was the crossed-rectangle model depicted in figure 3.

The research, done with CALFEM, gave a clear look into all the variables and their relation to the deflection of the cantilever beam. With those relations the following equation for the deflection of the porous crossed-rectangle cantilever beam was created. A formula sheet, the Forget-me-not, was created where one can find an $\alpha$ value that corresponds to an amount of repeating inner structures. This $\alpha$ value can be used in the equation:

$$w_{max} = \alpha \cdot \frac{F \cdot l^3}{E \cdot h^2 \cdot t \cdot b} \tag{15}$$

Although the use of CALFEM gave an exact solution, the created equation did result in answers with varying accuracy. The answer of the Forget-me-not deviates less than 5 percent of the FEM solution when the thickness is less than 1 cm, $p$ is less than 1.075 and the length is shorter than 10 meters. Thus an accurate answer is only possible under very specific circumstances and it is therefor advised to stay within these boundaries or use the Python script for an exact solution.

The relation of the amount of inner structures to the deflection was able to be found for rectangles square in size, elaborated in section 2.1. It is highly likely that it is possible to find a relation for the deflection with varying size and varying values of $p$. This expectation arises due to the fact that it is possible for the square size. Also due to the linear relation of the increase in $p$ with the percentile deviation in figure 15. And lastly because of the increase in size that increases the deviation. A follow up study could reveal a more accurate, maybe exact, equation for the deflection of the crossed-rectangle cantilever beam.

A replacement was done for the moment of inertia, as seen in equation 12: $\hat{I}$. One could hypothesize that this representation of the moment of

inertia could be of use in other mechanical practices such as buckling. Due to the inaccuracy that the methodology generated this might not be feasible. Instead of researching the impact of the variables and using that information to create an equation, it could be beneficial to use the standard equation 1 and look at an estimation for the moment of inertia of a crossed-rectangle lattice structure.

Lastly, the methodology and written Python scripts followed and used in this research are also applicable to the rotation at the point of loading depicted in figure 2. The cantilever() function does also return the rotation for all input variables. The scrip can therefor be used to get the correct rotation. A Forget-me-not could be written for this relation which could give a depiction of the answer for the rotation.

# 5 Appendix

## 5.1 Forget-me-not

# Forget-me-not - Porous Crossed Rectangle Cantilever Beam

Values of α used in the deflection forget me not of a porous crossed cantilever beam dependent on the number of repeating inner structures in length(nl) and height(nh)

| NL / NH | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.007265 | 1.337352 | 1.802509 | 2.287451 | 2.779002 | 3.273539 | 3.769658 | 4.266695 | 4.764296 | 5.262259 | 5.760444 | 6.258768 | 6.757144 | 7.255566 | 7.753988 | 8.252344 | 8.750634 | 9.248859 | 9.746953 | 10.24498 |
| 2 | 0.757326 | 0.672459 | 0.707438 | 0.785466 | 0.88307 | 0.990741 | 1.104068 | 1.220783 | 1.339616 | 1.459868 | 1.581077 | 1.702975 | 1.825366 | 1.948131 | 2.071178 | 2.194434 | 2.317862 | 2.441414 | 2.565065 | 2.688794 |
| 3 | 0.711119 | 0.552148 | 0.503464 | 0.500945 | 0.520515 | 0.552187 | 0.591095 | 0.634594 | 0.681142 | 0.72977 | 0.779881 | 0.831049 | 0.883011 | 0.935563 | 0.988562 | 1.041922 | 1.09555 | 1.149409 | 1.203438 | 1.257611 |
| 4 | 0.695048 | 0.513564 | 0.433268 | 0.401223 | 0.392397 | 0.396416 | 0.408243 | 0.425135 | 0.445468 | 0.468225 | 0.492735 | 0.518544 | 0.545331 | 0.572868 | 0.600987 | 0.629565 | 0.658508 | 0.687745 | 0.717222 | 0.746891 |
| 5 | 0.687704 | 0.499693 | 0.402439 | 0.355701 | 0.333166 | 0.323972 | 0.3229 | 0.327124 | 0.334982 | 0.345427 | 0.357769 | 0.371536 | 0.386391 | 0.402095 | 0.418465 | 0.435368 | 0.4527 | 0.47038 | 0.488346 | 0.506548 |
| 6 | 0.683826 | 0.496397 | 0.387579 | 0.331838 | 0.301375 | 0.284721 | 0.276439 | 0.273614 | 0.27454 | 0.278146 | 0.283732 | 0.290813 | 0.299047 | 0.308187 | 0.318046 | 0.328485 | 0.339395 | 0.350693 | 0.36231 | 0.374194 |
| 7 | 0.681588 | 0.498701 | 0.380651 | 0.318428 | 0.282707 | 0.261303 | 0.248518 | 0.241329 | 0.237982 | 0.237384 | 0.238817 | 0.241792 | 0.245961 | 0.251071 | 0.256934 | 0.263406 | 0.270378 | 0.277762 | 0.28549 | 0.293506 |
| 8 | 0.680236 | 0.504365 | 0.37828 | 0.310778 | 0.271153 | 0.246419 | 0.230566 | 0.220451 | 0.214263 | 0.21088 | 0.20957 | 0.209835 | 0.211322 | 0.213775 | 0.217004 | 0.220862 | 0.225237 | 0.230044 | 0.235209 | 0.240678 |
| 9 | 0.679409 | 0.512149 | 0.378833 | 0.306639 | 0.263838 | 0.236566 | 0.218469 | 0.206261 | 0.198064 | 0.192727 | 0.189501 | 0.187877 | 0.187498 | 0.188103 | 0.189499 | 0.19154 | 0.194112 | 0.197126 | 0.200511 | 0.204211 |
| 10 | 0.678917 | 0.521248 | 0.381421 | 0.304824 | 0.25924 | 0.229894 | 0.21005 | 0.196257 | 0.186567 | 0.179793 | 0.175166 | 0.172167 | 0.170431 | 0.169695 | 0.169763 | 0.170488 | 0.171753 | 0.17347 | 0.175566 | 0.177985 |
| 11 | 0.678641 | 0.531084 | 0.385509 | 0.304652 | 0.256497 | 0.225354 | 0.204069 | 0.189018 | 0.178169 | 0.170292 | 0.164601 | 0.160563 | 0.157806 | 0.156063 | 0.155136 | 0.154874 | 0.155161 | 0.155907 | 0.157039 | 0.158499 |
| 12 | 0.678523 | 0.54122 | 0.390737 | 0.305709 | 0.255088 | 0.222309 | 0.199783 | 0.183686 | 0.171898 | 0.163147 | 0.156619 | 0.151771 | 0.148223 | 0.145702 | 0.144006 | 0.142984 | 0.142519 | 0.142518 | 0.142908 | 0.143632 |
| 13 | 0.678517 | 0.551325 | 0.396846 | 0.307725 | 0.254681 | 0.220362 | 0.196717 | 0.179717 | 0.167144 | 0.157674 | 0.15047 | 0.144972 | 0.140793 | 0.137654 | 0.135352 | 0.133731 | 0.132672 | 0.132083 | 0.131891 | 0.132036 |
| 14 | 0.678595 | 0.561157 | 0.40363 | 0.310514 | 0.25506 | 0.219247 | 0.194563 | 0.176756 | 0.163502 | 0.153425 | 0.145657 | 0.139626 | 0.134932 | 0.131293 | 0.1285 | 0.126396 | 0.12486 | 0.1238 | 0.123139 | 0.12282 |
| 15 | 0.67874 | 0.570546 | 0.410919 | 0.313938 | 0.256072 | 0.218786 | 0.19311 | 0.174561 | 0.160699 | 0.150094 | 0.141846 | 0.135365 | 0.130243 | 0.12619 | 0.122992 | 0.120491 | 0.118565 | 0.11712 | 0.116078 | 0.11538 |
| 16 | 0.67893 | 0.579383 | 0.418568 | 0.317894 | 0.25761 | 0.218853 | 0.192212 | 0.172961 | 0.158544 | 0.147468 | 0.138802 | 0.131934 | 0.126448 | 0.122044 | 0.118508 | 0.115677 | 0.113426 | 0.11166 | 0.110302 | 0.109291 |
| 17 | 0.67916 | 0.587606 | 0.42645 | 0.322296 | 0.259592 | 0.219359 | 0.191762 | 0.171836 | 0.156901 | 0.145395 | 0.136355 | 0.129149 | 0.123345 | 0.118643 | 0.114818 | 0.111706 | 0.10918 | 0.107145 | 0.105521 | 0.104248 |
| 18 | 0.679422 | 0.59519 | 0.434453 | 0.32707 | 0.261957 | 0.220234 | 0.191683 | 0.171099 | 0.155668 | 0.143765 | 0.134384 | 0.126874 | 0.120793 | 0.115827 | 0.111753 | 0.108399 | 0.10564 | 0.103374 | 0.101525 | 0.100028 |
| 19 | 0.679711 | 0.602138 | 0.442482 | 0.332156 | 0.264655 | 0.221427 | 0.191919 | 0.170683 | 0.154773 | 0.142492 | 0.132796 | 0.125011 | 0.11868 | 0.113483 | 0.109189 | 0.105625 | 0.102662 | 0.100198 | 0.098153 | 0.096465 |
| 20 | 0.680026 | 0.60847 | 0.450455 | 0.337494 | 0.267644 | 0.222899 | 0.192424 | 0.170537 | 0.154158 | 0.141515 | 0.131522 | 0.123483 | 0.116925 | 0.111519 | 0.10703 | 0.103281 | 0.100139 | 0.097501 | 0.095287 | 0.093434 |
| 21 | 0.680361 | 0.614219 | 0.458303 | 0.343033 | 0.27089 | 0.224616 | 0.193164 | 0.170624 | 0.153781 | 0.140785 | 0.13051 | 0.122232 | 0.115464 | 0.109869 | 0.105203 | 0.101288 | 0.097987 | 0.095197 | 0.092834 | 0.090835 |
| 22 | 0.680702 | 0.619425 | 0.46597 | 0.348727 | 0.274359 | 0.226553 | 0.194111 | 0.170912 | 0.153606 | 0.140266 | 0.129718 | 0.121213 | 0.114249 | 0.108479 | 0.103653 | 0.099588 | 0.096145 | 0.093217 | 0.090722 | 0.088594 |
| 23 | 0.681069 | 0.62413 | 0.47341 | 0.35453 | 0.278024 | 0.228687 | 0.195243 | 0.171379 | 0.153609 | 0.139926 | 0.129112 | 0.120389 | 0.113241 | 0.107307 | 0.102333 | 0.09813 | 0.094558 | 0.091508 | 0.088894 | 0.086651 |
| 24 | 0.681437 | 0.628379 | 0.48059 | 0.360403 | 0.281859 | 0.230998 | 0.196542 | 0.172004 | 0.153767 | 0.139743 | 0.128667 | 0.119733 | 0.112408 | 0.10632 | 0.101208 | 0.09688 | 0.093189 | 0.090026 | 0.087306 | 0.084959 |
| 25 | 0.681824 | 0.632215 | 0.487482 | 0.366308 | 0.285841 | 0.233469 | 0.197993 | 0.172772 | 0.154063 | 0.139698 | 0.128363 | 0.119223 | 0.111727 | 0.105492 | 0.10025 | 0.095803 | 0.092003 | 0.088738 | 0.08592 | 0.083479 |
| 26 | 0.682218 | 0.635677 | 0.494072 | 0.372214 | 0.289944 | 0.236083 | 0.199581 | 0.173669 | 0.154483 | 0.139774 | 0.128181 | 0.118838 | 0.111175 | 0.1048 | 0.099434 | 0.094877 | 0.090975 | 0.087615 | 0.084707 | 0.08218 |
| 27 | 0.682618 | 0.638805 | 0.500349 | 0.37809 | 0.29415 | 0.238828 | 0.201296 | 0.174683 | 0.155013 | 0.13996 | 0.128108 | 0.118565 | 0.110739 | 0.104227 | 0.098743 | 0.09408 | 0.090083 | 0.086634 | 0.083642 | 0.081036 |
| 28 | 0.683025 | 0.641632 | 0.506309 | 0.383908 | 0.298438 | 0.24169 | 0.203126 | 0.175807 | 0.155647 | 0.140244 | 0.128133 | 0.118389 | 0.110404 | 0.103758 | 0.09816 | 0.093396 | 0.089308 | 0.085776 | 0.082707 | 0.080028 |
| 29 | 0.683438 | 0.644191 | 0.511951 | 0.389647 | 0.302787 | 0.244654 | 0.205062 | 0.17703 | 0.156375 | 0.140617 | 0.128246 | 0.118302 | 0.110157 | 0.103381 | 0.097672 | 0.092811 | 0.088636 | 0.085026 | 0.081884 | 0.079136 |
| 30 | 0.683858 | 0.64651 | 0.517282 | 0.395286 | 0.307181 | 0.247711 | 0.207097 | 0.178346 | 0.157188 | 0.141073 | 0.128438 | 0.118294 | 0.109991 | 0.103085 | 0.097267 | 0.092313 | 0.088056 | 0.084369 | 0.081158 | 0.078346 |
| 31 | 0.684285 | 0.648615 | 0.522308 | 0.400807 | 0.311603 | 0.250848 | 0.209222 | 0.179747 | 0.158082 | 0.141604 | 0.128703 | 0.118357 | 0.109896 | 0.102863 | 0.096939 | 0.091893 | 0.087555 | 0.083797 | 0.08052 | 0.077647 |
| 32 | 0.684712 | 0.650528 | 0.527039 | 0.406197 | 0.316036 | 0.254054 | 0.211428 | 0.181229 | 0.159051 | 0.142205 | 0.129035 | 0.118487 | 0.109867 | 0.102706 | 0.096677 | 0.091541 | 0.087125 | 0.083298 | 0.079958 | 0.077027 |
| 33 | 0.685145 | 0.652271 | 0.531486 | 0.411444 | 0.320467 | 0.257319 | 0.21371 | 0.182785 | 0.160089 | 0.142872 | 0.12943 | 0.118676 | 0.109896 | 0.102609 | 0.096475 | 0.091251 | 0.08676 | 0.082865 | 0.079464 | 0.076477 |
| 34 | 0.685584 | 0.653862 | 0.535663 | 0.416538 | 0.324882 | 0.260633 | 0.216059 | 0.18441 | 0.161192 | 0.143599 | 0.129881 | 0.11892 | 0.109981 | 0.102566 | 0.096328 | 0.091017 | 0.08645 | 0.082491 | 0.079032 | 0.075992 |
| 35 | 0.686024 | 0.655316 | 0.539582 | 0.421474 | 0.329269 | 0.263986 | 0.21847 | 0.1861 | 0.162357 | 0.144384 | 0.130386 | 0.119216 | 0.110115 | 0.102573 | 0.096231 | 0.090834 | 0.086194 | 0.082169 | 0.078653 | 0.075562 |
| 36 | 0.68647 | 0.65665 | 0.543256 | 0.426245 | 0.333616 | 0.26737 | 0.220936 | 0.187849 | 0.163579 | 0.145222 | 0.130942 | 0.119559 | 0.110296 | 0.102625 | 0.09618 | 0.090697 | 0.085984 | 0.081896 | 0.078325 | 0.075183 |
| 37 | 0.686917 | 0.657871 | 0.5467 | 0.430849 | 0.337913 | 0.270775 | 0.223452 | 0.189654 | 0.164855 | 0.146111 | 0.131545 | 0.119947 | 0.11052 | 0.10272 | 0.096171 | 0.090602 | 0.085817 | 0.081667 | 0.078041 | 0.074851 |
| 38 | 0.687369 | 0.659 | 0.549928 | 0.435285 | 0.342152 | 0.274193 | 0.226011 | 0.19151 | 0.166182 | 0.147047 | 0.132192 | 0.120378 | 0.110784 | 0.102854 | 0.0962 | 0.090545 | 0.085689 | 0.081477 | 0.077797 | 0.07456 |
| 39 | 0.687822 | 0.660037 | 0.552951 | 0.439551 | 0.346324 | 0.277616 | 0.228606 | 0.193414 | 0.167556 | 0.148028 | 0.132882 | 0.120848 | 0.111086 | 0.103025 | 0.096265 | 0.090525 | 0.085596 | 0.081324 | 0.077591 | 0.074307 |
| 40 | 0.688275 | 0.660995 | 0.555783 | 0.443649 | 0.350423 | 0.281037 | 0.231234 | 0.19536 | 0.168975 | 0.149053 | 0.133612 | 0.121356 | 0.111424 | 0.103229 | 0.096364 | 0.090538 | 0.085538 | 0.081204 | 0.077419 | 0.074089 |
| 41 | 0.688734 | 0.661881 | 0.558437 | 0.447582 | 0.354442 | 0.28445 | 0.233888 | 0.197346 | 0.170437 | 0.150117 | 0.134379 | 0.121899 | 0.111795 | 0.103466 | 0.096495 | 0.090582 | 0.085509 | 0.081116 | 0.077279 | 0.073903 |
| 42 | 0.689194 | 0.662701 | 0.560924 | 0.45135 | 0.358377 | 0.287846 | 0.236564 | 0.199368 | 0.171937 | 0.15122 | 0.135183 | 0.122476 | 0.112198 | 0.103733 | 0.096654 | 0.090654 | 0.08551 | 0.081056 | 0.077167 | 0.073746 |
| 43 | 0.689653 | 0.663469 | 0.563254 | 0.45496 | 0.362223 | 0.291221 | 0.239256 | 0.201422 | 0.173474 | 0.15236 | 0.13602 | 0.123084 | 0.112631 | 0.10403 | 0.096842 | 0.090753 | 0.085537 | 0.081023 | 0.077082 | 0.073617 |
| 44 | 0.690119 | 0.664184 | 0.565439 | 0.458413 | 0.365978 | 0.29457 | 0.241959 | 0.203505 | 0.175045 | 0.153533 | 0.13689 | 0.123724 | 0.113093 | 0.104353 | 0.097055 | 0.090879 | 0.085589 | 0.081015 | 0.077022 | 0.073512 |
| 45 | 0.690585 | 0.664847 | 0.56749 | 0.461715 | 0.369637 | 0.297886 | 0.24467 | 0.205613 | 0.176647 | 0.154739 | 0.137792 | 0.124392 | 0.113582 | 0.104702 | 0.097294 | 0.091027 | 0.085665 | 0.081029 | 0.076986 | 0.073432 |
| 46 | 0.691051 | 0.66547 | 0.569413 | 0.46487 | 0.3732 | 0.301166 | 0.247383 | 0.207743 | 0.178279 | 0.155975 | 0.138723 | 0.125088 | 0.114097 | 0.105075 | 0.097555 | 0.091199 | 0.085763 | 0.081066 | 0.076971 | 0.073372 |
| 47 | 0.691517 | 0.666054 | 0.571218 | 0.467884 | 0.376665 | 0.304404 | 0.250095 | 0.209892 | 0.179938 | 0.15724 | 0.139682 | 0.125812 | 0.114637 | 0.105473 | 0.097839 | 0.091392 | 0.085882 | 0.081123 | 0.076976 | 0.073333 |
| 48 | 0.691989 | 0.666606 | 0.572915 | 0.470761 | 0.380031 | 0.307598 | 0.252801 | 0.212056 | 0.181621 | 0.158532 | 0.140668 | 0.12656 | 0.115201 | 0.105893 | 0.098144 | 0.091605 | 0.086021 | 0.0812 | 0.077001 | 0.073314 |
| 49 | 0.692455 | 0.667124 | 0.574509 | 0.473507 | 0.383299 | 0.310744 | 0.255499 | 0.214233 | 0.183326 | 0.15985 | 0.141679 | 0.127333 | 0.115787 | 0.106334 | 0.09847 | 0.091838 | 0.086177 | 0.081295 | 0.077044 | 0.073312 |
| 50 | 0.692928 | 0.66761 | 0.576009 | 0.476127 | 0.386466 | 0.313839 | 0.258184 | 0.21642 | 0.185052 | 0.161191 | 0.142715 | 0.128128 | 0.116396 | 0.106796 | 0.098815 | 0.09209 | 0.086353 | 0.081407 | 0.077103 | 0.073327 |
| 51 | 0.6934 | 0.668069 | 0.57742 | 0.478626 | 0.389536 | 0.316881 | 0.260853 | 0.218613 | 0.186795 | 0.162555 | 0.143773 | 0.128947 | 0.117026 | 0.107277 | 0.099179 | 0.092359 | 0.086546 | 0.081536 | 0.07718 | 0.073358 |
| 52 | 0.693873 | 0.668509 | 0.578749 | 0.48101 | 0.392508 | 0.319867 | 0.263503 | 0.220811 | 0.188554 | 0.163939 | 0.144853 | 0.129787 | 0.117677 | 0.107777 | 0.09956 | 0.092645 | 0.086754 | 0.081681 | 0.077271 | 0.073405 |
| 53 | 0.694345 | 0.668922 | 0.580001 | 0.483283 | 0.395384 | 0.322796 | 0.266132 | 0.223011 | 0.190328 | 0.165342 | 0.145954 | 0.130646 | 0.118346 | 0.108296 | 0.099959 | 0.092948 | 0.086978 | 0.08184 | 0.077376 | 0.073465 |
| 54 | 0.694824 | 0.669316 | 0.581182 | 0.485451 | 0.398165 | 0.325665 | 0.268736 | 0.225209 | 0.192113 | 0.166763 | 0.147074 | 0.131526 | 0.119035 | 0.108833 | 0.100375 | 0.093266 | 0.087218 | 0.082015 | 0.077497 | 0.07354 |
| 55 | 0.695297 | 0.66969 | 0.582297 | 0.487518 | 0.400853 | 0.328475 | 0.271314 | 0.227404 | 0.193907 | 0.1682 | 0.148213 | 0.132424 | 0.119741 | 0.109387 | 0.100807 | 0.0936 | 0.087471 | 0.082203 | 0.07763 | 0.073627 |
| 56 | 0.695776 | 0.670044 | 0.583349 | 0.48949 | 0.403449 | 0.331223 | 0.273863 | 0.229594 | 0.19571 | 0.169651 | 0.149369 | 0.13334 | 0.120464 | 0.109957 | 0.101254 | 0.093947 | 0.087738 | 0.082404 | 0.077776 | 0.073727 |
| 57 | 0.696255 | 0.670392 | 0.584343 | 0.49137 | 0.405957 | 0.333909 | 0.276381 | 0.231776 | 0.197519 | 0.171116 | 0.150541 | 0.134272 | 0.121204 | 0.110543 | 0.101716 | 0.09431 | 0.088019 | 0.082617 | 0.077934 | 0.073839 |
| 58 | 0.696734 | 0.670714 | 0.585283 | 0.493163 | 0.408377 | 0.336533 | 0.278866 | 0.233948 | 0.199332 | 0.172592 | 0.151728 | 0.135221 | 0.12196 | 0.111143 | 0.102192 | 0.094685 | 0.088312 | 0.082844 | 0.078105 | 0.073962 |
| 59 | 0.697213 | 0.671029 | 0.586173 | 0.494874 | 0.410712 | 0.339094 | 0.281317 | 0.236108 | 0.201148 | 0.17408 | 0.152929 | 0.136185 | 0.122731 | 0.111759 | 0.102682 | 0.095074 | 0.088619 | 0.083082 | 0.078286 | 0.074097 |
| 60 | 0.697692 | 0.671331 | 0.587016 | 0.496506 | 0.412964 | 0.341592 | 0.283732 | 0.238255 | 0.202965 | 0.175576 | 0.154143 | 0.137163 | 0.123517 | 0.112389 | 0.103186 | 0.095475 | 0.088936 | 0.083331 | 0.078478 | 0.074241 |
| 61 | 0.698171 | 0.671619 | 0.587815 | 0.498064 | 0.415137 | 0.344028 | 0.286109 | 0.240386 | 0.204782 | 0.177079 | 0.15537 | 0.138155 | 0.124316 | 0.113032 | 0.103703 | 0.095889 | 0.089266 | 0.083591 | 0.078682 | 0.074396 |
| 62 | 0.69865 | 0.671902 | 0.588573 | 0.49955 | 0.417231 | 0.346402 | 0.288449 | 0.242501 | 0.206595 | 0.17859 | 0.156607 | 0.13916 | 0.125129 | 0.113688 | 0.104232 | 0.096315 | 0.089607 | 0.083862 | 0.078894 | 0.074561 |
| 63 | 0.699136 | 0.672171 | 0.589292 | 0.500969 | 0.41925 | 0.348713 | 0.29075 | 0.244597 | 0.208406 | 0.180106 | 0.157854 | 0.140177 | 0.125955 | 0.114357 | 0.104773 | 0.096752 | 0.089959 | 0.084144 | 0.079118 | 0.074734 |
| 64 | 0.699615 | 0.672433 | 0.589975 | 0.502325 | 0.421196 | 0.350963 | 0.29301 | 0.246674 | 0.210211 | 0.181626 | 0.15911 | 0.141205 | 0.126793 | 0.115039 | 0.105326 | 0.0972 | 0.090321 | 0.084435 | 0.07935 | 0.074918 |
| 65 | 0.700101 | 0.672683 | 0.590625 | 0.503619 | 0.423071 | 0.353153 | 0.29523 | 0.248729 | 0.212011 | 0.183148 | 0.160374 | 0.142244 | 0.127642 | 0.115731 | 0.105891 | 0.097659 | 0.090694 | 0.084736 | 0.079591 | 0.075109 |
| 66 | 0.700586 | 0.672932 | 0.591243 | 0.504856 | 0.424878 | 0.355284 | 0.29741 | 0.250763 | 0.213802 | 0.184673 | 0.161646 | 0.143293 | 0.128502 | 0.116435 | 0.106465 | 0.098128 | 0.091076 | 0.085047 | 0.079842 | 0.075309 |
| 67 | 0.701065 | 0.673168 | 0.591831 | 0.506038 | 0.426619 | 0.357355 | 0.299547 | 0.252773 | 0.215585 | 0.186198 | 0.162924 | 0.144351 | 0.129373 | 0.11715 | 0.107051 | 0.098608 | 0.091468 | 0.085366 | 0.080101 | 0.075519 |
| 68 | 0.701551 | 0.673398 | 0.592392 | 0.507168 | 0.428297 | 0.359368 | 0.301643 | 0.254759 | 0.217358 | 0.187723 | 0.164207 | 0.145418 | 0.130254 | 0.117875 | 0.107647 | 0.099098 | 0.09187 | 0.085695 | 0.080369 | 0.075735 |
| 69 | 0.702037 | 0.673621 | 0.592926 | 0.508248 | 0.429913 | 0.361325 | 0.303697 | 0.25672 | 0.219119 | 0.189246 | 0.165494 | 0.146492 | 0.131144 | 0.11861 | 0.108253 | 0.099596 | 0.09228 | 0.086032 | 0.080645 | 0.07596 |
| 70 | 0.702522 | 0.673844 | 0.593437 | 0.509282 | 0.43147 | 0.363226 | 0.305708 | 0.258655 | 0.220869 | 0.190767 | 0.166786 | 0.147574 | 0.132043 | 0.119354 | 0.108869 | 0.100104 | 0.092699 | 0.086378 | 0.080929 | 0.076192 |
| 71 | 0.703008 | 0.674054 | 0.593924 | 0.510271 | 0.432971 | 0.365073 | 0.307678 | 0.260563 | 0.222605 | 0.192285 | 0.16808 | 0.148661 | 0.13295 | 0.120108 | 0.109493 | 0.100622 | 0.093127 | 0.086731 | 0.081221 | 0.076431 |
| 72 | 0.703493 | 0.674264 | 0.594389 | 0.511218 | 0.434416 | 0.366865 | 0.309605 | 0.262445 | 0.224327 | 0.193798 | 0.169376 | 0.149755 | 0.133865 | 0.120869 | 0.110126 | 0.101147 | 0.093564 | 0.087093 | 0.081519 | 0.076678 |
| 73 | 0.703979 | 0.674468 | 0.594835 | 0.512125 | 0.435809 | 0.368606 | 0.311491 | 0.264298 | 0.226035 | 0.195306 | 0.170674 | 0.150854 | 0.134787 | 0.121639 | 0.110768 | 0.101681 | 0.094008 | 0.087462 | 0.081826 | 0.076932 |
| 74 | 0.704465 | 0.674664 | 0.595261 | 0.512993 | 0.437151 | 0.370296 | 0.313335 | 0.266124 | 0.227727 | 0.196808 | 0.171972 | 0.151957 | 0.135716 | 0.122417 | 0.111417 | 0.102223 | 0.09446 | 0.087839 | 0.08214 | 0.077192 |
| 75 | 0.70495 | 0.674861 | 0.595669 | 0.513826 | 0.438445 | 0.371936 | 0.315138 | 0.267921 | 0.229402 | 0.198304 | 0.173269 | 0.153064 | 0.13665 | 0.123202 | 0.112075 | 0.102773 | 0.094919 | 0.088223 | 0.08246 | 0.077459 |
| 76 | 0.705443 | 0.675052 | 0.59606 | 0.514623 | 0.439691 | 0.373527 | 0.316901 | 0.269689 | 0.23106 | 0.199791 | 0.174566 | 0.154173 | 0.137591 | 0.123994 | 0.11274 | 0.103331 | 0.095387 | 0.088614 | 0.082788 | 0.077732 |
| 77 | 0.705928 | 0.675242 | 0.596435 | 0.515389 | 0.440892 | 0.375071 | 0.318623 | 0.271427 | 0.232701 | 0.201271 | 0.175861 | 0.155286 | 0.138536 | 0.124793 | 0.113412 | 0.103896 | 0.095861 | 0.089012 | 0.083122 | 0.078012 |
| 78 | 0.706414 | 0.675426 | 0.596795 | 0.516123 | 0.442051 | 0.376569 | 0.320304 | 0.273137 | 0.234323 | 0.202741 | 0.177154 | 0.156401 | 0.139487 | 0.125597 | 0.11409 | 0.104467 | 0.096342 | 0.089417 | 0.083462 | 0.078299 |
| 79 | 0.706906 | 0.675603 | 0.597141 | 0.516827 | 0.443168 | 0.378022 | 0.321946 | 0.274817 | 0.235927 | 0.204201 | 0.178444 | 0.157518 | 0.140441 | 0.126408 | 0.114776 | 0.105046 | 0.09683 | 0.089828 | 0.083809 | 0.07859 |
| 80 | 0.707392 | 0.675787 | 0.597474 | 0.517504 | 0.444244 | 0.379431 | 0.32355 | 0.276467 | 0.237511 | 0.205652 | 0.17973 | 0.158635 | 0.1414 | 0.127223 | 0.115467 | 0.105631 | 0.097324 | 0.090246 | 0.084161 | 0.078888 |
| 81 | 0.707884 | 0.675957 | 0.597793 | 0.518153 | 0.445283 | 0.380799 | 0.325115 | 0.278088 | 0.239076 | 0.207092 | 0.181011 | 0.159753 | 0.142361 | 0.128044 | 0.116164 | 0.106222 | 0.097825 | 0.090669 | 0.08452 | 0.079191 |
| 82 | 0.708369 | 0.676134 | 0.598101 | 0.518777 | 0.446284 | 0.382125 | 0.326643 | 0.279679 | 0.240621 | 0.20852 | 0.182289 | 0.16087 | 0.143326 | 0.128869 | 0.116867 | 0.106818 | 0.098331 | 0.091099 | 0.084885 | 0.0795 |
| 83 | 0.708862 | 0.676305 | 0.598397 | 0.519377 | 0.44725 | 0.383411 | 0.328133 | 0.281242 | 0.242146 | 0.209936 | 0.18356 | 0.161987 | 0.144292 | 0.129699 | 0.117575 | 0.107422 | 0.098844 | 0.091535 | 0.085255 | 0.079814 |
| 84 | 0.709347 | 0.676469 | 0.598683 | 0.519953 | 0.448182 | 0.384659 | 0.329586 | 0.282774 | 0.24365 | 0.211339 | 0.184826 | 0.163102 | 0.145261 | 0.130532 | 0.118288 | 0.10803 | 0.099362 | 0.091976 | 0.08563 | 0.080135 |
| 85 | 0.709839 | 0.676633 | 0.598958 | 0.520508 | 0.449082 | 0.385869 | 0.331005 | 0.284278 | 0.245133 | 0.21273 | 0.186086 | 0.164216 | 0.146231 | 0.131369 | 0.119006 | 0.108644 | 0.099886 | 0.092423 | 0.086011 | 0.08046 |
| 86 | 0.710325 | 0.676797 | 0.599225 | 0.521042 | 0.44995 | 0.387043 | 0.332387 | 0.285752 | 0.246595 | 0.214107 | 0.187338 | 0.165328 | 0.147202 | 0.132209 | 0.119728 | 0.109262 | 0.100415 | 0.092875 | 0.086397 | 0.080789 |
| 87 | 0.710817 | 0.676961 | 0.599481 | 0.521555 | 0.450788 | 0.388181 | 0.333736 | 0.287197 | 0.248036 | 0.215471 | 0.188583 | 0.166437 | 0.148174 | 0.133051 | 0.120454 | 0.109885 | 0.100949 | 0.093332 | 0.086789 | 0.081124 |
| 88 | 0.711309 | 0.677119 | 0.599729 | 0.52205 | 0.451597 | 0.389286 | 0.335051 | 0.288614 | 0.249456 | 0.216822 | 0.18982 | 0.167543 | 0.149146 | 0.133897 | 0.121184 | 0.110513 | 0.101488 | 0.093795 | 0.087185 | 0.081464 |
| 89 | 0.711802 | 0.677283 | 0.599969 | 0.522526 | 0.452379 | 0.390357 | 0.336333 | 0.290003 | 0.250854 | 0.218157 | 0.19105 | 0.168646 | 0.150119 | 0.134744 | 0.121917 | 0.111145 | 0.102031 | 0.094261 | 0.087586 | 0.081808 |
| 90 | 0.712287 | 0.677434 | 0.600201 | 0.522985 | 0.453134 | 0.391395 | 0.337582 | 0.291363 | 0.25223 | 0.219478 | 0.192271 | 0.169745 | 0.15109 | 0.135593 | 0.122654 | 0.111781 | 0.102579 | 0.094733 | 0.087991 | 0.082157 |
| 91 | 0.712779 | 0.677591 | 0.600426 | 0.523428 | 0.453863 | 0.392403 | 0.338801 | 0.292696 | 0.253586 | 0.220784 | 0.193482 | 0.170839 | 0.15206 | 0.136444 | 0.123393 | 0.112421 | 0.103132 | 0.095209 | 0.088401 | 0.08251 |
| 92 | 0.713272 | 0.677742 | 0.600644 | 0.523855 | 0.454568 | 0.393381 | 0.339988 | 0.294002 | 0.25492 | 0.222075 | 0.194684 | 0.171929 | 0.15303 | 0.137295 | 0.124136 | 0.113065 | 0.103688 | 0.095689 | 0.088816 | 0.082867 |
| 93 | 0.713764 | 0.6779 | 0.600855 | 0.524266 | 0.455249 | 0.394329 | 0.341145 | 0.29528 | 0.256232 | 0.223351 | 0.195878 | 0.173014 | 0.153998 | 0.138148 | 0.12488 | 0.113712 | 0.104248 | 0.096173 | 0.089235 | 0.08323 |
| 94 | 0.714256 | 0.678051 | 0.601059 | 0.524664 | 0.455907 | 0.39525 | 0.342272 | 0.296532 | 0.257522 | 0.224611 | 0.197061 | 0.174093 | 0.154964 | 0.139001 | 0.125627 | 0.114361 | 0.104812 | 0.096662 | 0.089658 | 0.083595 |
| 95 | 0.714748 | 0.678195 | 0.601258 | 0.525048 | 0.456544 | 0.396142 | 0.343372 | 0.297757 | 0.258792 | 0.225856 | 0.198233 | 0.175167 | 0.155928 | 0.139854 | 0.126375 | 0.115014 | 0.10538 | 0.097155 | 0.090084 | 0.083965 |
| 96 | 0.71524 | 0.678346 | 0.601452 | 0.525419 | 0.45716 | 0.397009 | 0.344443 | 0.298957 | 0.260039 | 0.227085 | 0.199395 | 0.176235 | 0.156889 | 0.140707 | 0.127125 | 0.11567 | 0.105951 | 0.097651 | 0.090515 | 0.084339 |

$$w_{max} = \alpha \cdot \frac{F \cdot l^3}{E \cdot h^2 \cdot t \cdot b}$$

# Forget-me-not - Porous Crossed Rectangle Cantilever Beam

Values of α used in the deflection forget me not of a porous crossed cantilever beam dependent on the number of repeating inner structures in length(nl) and height(nh)

| NL / NH | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 10.74288 | 11.24058 | 11.73815 | 12.23558 | 12.73283 | 13.22987 | 13.72672 | 14.22337 | 14.71982 | 15.21608 | 15.71207 | 16.20787 | 16.70347 | 17.19874 | 17.69381 | 18.18863 | 18.68318 | 19.17746 | 19.67142 | 20.16512 |
| 2 | 2.812576 | 2.936397 | 3.060251 | 3.184118 | 3.307992 | 3.431859 | 3.555727 | 3.679574 | 3.803402 | 3.927203 | 4.050972 | 4.174708 | 4.298405 | 4.422062 | 4.545673 | 4.669232 | 4.792738 | 4.916192 | 5.039587 | 5.162922 |
| 3 | 1.311903 | 1.366293 | 1.420762 | 1.475289 | 1.52987 | 1.584496 | 1.639148 | 1.693834 | 1.748539 | 1.80325 | 1.857982 | 1.912713 | 1.967444 | 2.022182 | 2.076907 | 2.131625 | 2.186336 | 2.241035 | 2.295713 | 2.350379 |
| 4 | 0.776724 | 0.806689 | 0.836758 | 0.866933 | 0.897179 | 0.927491 | 0.957863 | 0.98828 | 1.01873 | 1.049219 | 1.079741 | 1.110277 | 1.140838 | 1.171413 | 1.201994 | 1.232588 | 1.263189 | 1.29379 | 1.324398 | 1.354999 |
| 5 | 0.524946 | 0.543509 | 0.56221 | 0.581029 | 0.599946 | 0.618949 | 0.638024 | 0.657162 | 0.676351 | 0.695592 | 0.714866 | 0.73418 | 0.753519 | 0.772885 | 0.792271 | 0.811676 | 0.831095 | 0.850526 | 0.869971 | 0.889422 |
| 6 | 0.386301 | 0.398597 | 0.411053 | 0.423645 | 0.436355 | 0.449164 | 0.46206 | 0.475032 | 0.488068 | 0.50116 | 0.514302 | 0.527487 | 0.540709 | 0.553965 | 0.567249 | 0.580557 | 0.593888 | 0.607238 | 0.620604 | 0.633984 |
| 7 | 0.301765 | 0.310232 | 0.318875 | 0.327669 | 0.336594 | 0.345632 | 0.354767 | 0.363988 | 0.373284 | 0.382644 | 0.392062 | 0.401531 | 0.411044 | 0.420595 | 0.430182 | 0.439799 | 0.449443 | 0.459111 | 0.4688 | 0.478508 |
| 8 | 0.246405 | 0.252352 | 0.258487 | 0.264786 | 0.271225 | 0.277787 | 0.284455 | 0.291218 | 0.298062 | 0.304979 | 0.31196 | 0.318998 | 0.326085 | 0.333218 | 0.34039 | 0.347597 | 0.354836 | 0.362102 | 0.369393 | 0.376707 |
| 9 | 0.208178 | 0.212375 | 0.216769 | 0.221335 | 0.226049 | 0.230894 | 0.235852 | 0.240911 | 0.246059 | 0.251284 | 0.256579 | 0.261936 | 0.267346 | 0.272807 | 0.278311 | 0.283854 | 0.289433 | 0.295043 | 0.300681 | 0.306345 |
| 10 | 0.180678 | 0.183608 | 0.186743 | 0.190055 | 0.193522 | 0.197124 | 0.200846 | 0.204673 | 0.208594 | 0.212597 | 0.216674 | 0.220817 | 0.225018 | 0.229273 | 0.233574 | 0.237918 | 0.2423 | 0.246717 | 0.251165 | 0.255641 |
| 11 | 0.160241 | 0.162223 | 0.164415 | 0.166789 | 0.169323 | 0.171997 | 0.174794 | 0.177701 | 0.180705 | 0.183795 | 0.186962 | 0.190198 | 0.193497 | 0.196851 | 0.200255 | 0.203704 | 0.207194 | 0.210721 | 0.214281 | 0.217872 |
| 12 | 0.144641 | 0.145896 | 0.147364 | 0.149018 | 0.150835 | 0.152796 | 0.154884 | 0.157083 | 0.159383 | 0.161772 | 0.164241 | 0.166781 | 0.169387 | 0.17205 | 0.174765 | 0.177528 | 0.180334 | 0.183179 | 0.18606 | 0.188972 |
| 13 | 0.132469 | 0.133152 | 0.134052 | 0.13514 | 0.136394 | 0.137795 | 0.139325 | 0.140969 | 0.142717 | 0.144555 | 0.146476 | 0.14847 | 0.150531 | 0.152652 | 0.154828 | 0.157052 | 0.159322 | 0.161632 | 0.163979 | 0.16636 |
| 14 | 0.122793 | 0.123018 | 0.123462 | 0.124098 | 0.124902 | 0.125854 | 0.126938 | 0.128138 | 0.129443 | 0.130841 | 0.132324 | 0.133881 | 0.135506 | 0.137194 | 0.138937 | 0.140732 | 0.142572 | 0.144454 | 0.146375 | 0.148331 |
| 15 | 0.114978 | 0.11483 | 0.114903 | 0.115171 | 0.115608 | 0.116196 | 0.116917 | 0.117756 | 0.118701 | 0.119741 | 0.120867 | 0.122069 | 0.123341 | 0.124676 | 0.126068 | 0.127512 | 0.129004 | 0.130538 | 0.132112 | 0.133723 |
| 16 | 0.108579 | 0.108122 | 0.10789 | 0.107854 | 0.107989 | 0.108276 | 0.108697 | 0.109239 | 0.109888 | 0.110633 | 0.111463 | 0.112373 | 0.113353 | 0.114398 | 0.1155 | 0.116656 | 0.117859 | 0.119107 | 0.120396 | 0.121723 |
| 17 | 0.103275 | 0.102562 | 0.102074 | 0.101783 | 0.101666 | 0.101702 | 0.101873 | 0.102166 | 0.102567 | 0.103065 | 0.103651 | 0.104317 | 0.105053 | 0.105855 | 0.106716 | 0.107631 | 0.108595 | 0.109604 | 0.110654 | 0.111743 |
| 18 | 0.098835 | 0.097903 | 0.097199 | 0.096694 | 0.096362 | 0.096187 | 0.096147 | 0.09623 | 0.096422 | 0.096713 | 0.097092 | 0.09755 | 0.098082 | 0.098679 | 0.099336 | 0.100048 | 0.10081 | 0.101617 | 0.102467 | 0.103355 |
| 19 | 0.095083 | 0.093965 | 0.093075 | 0.092386 | 0.091874 | 0.091516 | 0.091297 | 0.091201 | 0.091215 | 0.091328 | 0.091531 | 0.091814 | 0.092171 | 0.092594 | 0.093077 | 0.093616 | 0.094205 | 0.094841 | 0.09552 | 0.096237 |
| 20 | 0.091887 | 0.090608 | 0.089558 | 0.088711 | 0.088041 | 0.087528 | 0.087154 | 0.086904 | 0.086765 | 0.086726 | 0.086777 | 0.08691 | 0.087116 | 0.087389 | 0.087724 | 0.088114 | 0.088555 | 0.089044 | 0.089576 | 0.090146 |
| 21 | 0.089146 | 0.087725 | 0.086536 | 0.085551 | 0.084746 | 0.084096 | 0.083589 | 0.083205 | 0.082934 | 0.082762 | 0.082682 | 0.082684 | 0.08276 | 0.082904 | 0.083109 | 0.083371 | 0.083684 | 0.084045 | 0.08445 | 0.084894 |
| 22 | 0.086779 | 0.085234 | 0.083923 | 0.082817 | 0.081891 | 0.081125 | 0.080499 | 0.079999 | 0.079612 | 0.079326 | 0.079131 | 0.079018 | 0.078981 | 0.079012 | 0.079105 | 0.079255 | 0.079456 | 0.079706 | 0.08 | 0.080334 |
| 23 | 0.084724 | 0.083069 | 0.08165 | 0.080438 | 0.079406 | 0.078535 | 0.077806 | 0.077203 | 0.076714 | 0.076327 | 0.076032 | 0.075819 | 0.075682 | 0.075614 | 0.075608 | 0.075659 | 0.075763 | 0.075915 | 0.076111 | 0.076349 |
| 24 | 0.082931 | 0.081177 | 0.079662 | 0.078355 | 0.07723 | 0.076266 | 0.075446 | 0.074752 | 0.074173 | 0.073696 | 0.073312 | 0.07301 | 0.072785 | 0.07263 | 0.072537 | 0.072501 | 0.072519 | 0.072585 | 0.072695 | 0.072847 |
| 25 | 0.08136 | 0.079518 | 0.077916 | 0.076524 | 0.075315 | 0.074268 | 0.073366 | 0.072592 | 0.071932 | 0.071376 | 0.070912 | 0.070532 | 0.070229 | 0.069996 | 0.069825 | 0.069713 | 0.069654 | 0.069643 | 0.069678 | 0.069753 |
| 26 | 0.079978 | 0.078056 | 0.076376 | 0.074906 | 0.073623 | 0.072502 | 0.071526 | 0.070679 | 0.069948 | 0.06932 | 0.068786 | 0.068336 | 0.067963 | 0.067659 | 0.067421 | 0.067239 | 0.067111 | 0.067033 | 0.067 | 0.067008 |
| 27 | 0.078759 | 0.076764 | 0.075012 | 0.073473 | 0.072121 | 0.070933 | 0.069891 | 0.068978 | 0.068182 | 0.06749 | 0.066892 | 0.06638 | 0.065944 | 0.065579 | 0.065277 | 0.065035 | 0.064846 | 0.064706 | 0.064612 | 0.06456 |
| 28 | 0.07768 | 0.075617 | 0.0738 | 0.072198 | 0.070783 | 0.069534 | 0.068432 | 0.067461 | 0.066606 | 0.065856 | 0.065201 | 0.064631 | 0.064139 | 0.063718 | 0.063361 | 0.063063 | 0.062819 | 0.062624 | 0.062475 | 0.062369 |
| 29 | 0.076723 | 0.074598 | 0.072721 | 0.07106 | 0.069589 | 0.068283 | 0.067127 | 0.066101 | 0.065194 | 0.064391 | 0.063684 | 0.063063 | 0.06252 | 0.062047 | 0.06164 | 0.061292 | 0.060998 | 0.060754 | 0.060555 | 0.0604 |
| 30 | 0.075873 | 0.07369 | 0.071757 | 0.070042 | 0.068518 | 0.067162 | 0.065955 | 0.06488 | 0.063924 | 0.063074 | 0.062319 | 0.061651 | 0.061061 | 0.060543 | 0.060089 | 0.059695 | 0.059356 | 0.059067 | 0.058824 | 0.058624 |
| 31 | 0.075116 | 0.072879 | 0.070894 | 0.069129 | 0.067556 | 0.066153 | 0.0649 | 0.06378 | 0.062779 | 0.061885 | 0.061087 | 0.060376 | 0.059744 | 0.059183 | 0.058688 | 0.058253 | 0.057872 | 0.057542 | 0.057259 | 0.057018 |
| 32 | 0.074442 | 0.072153 | 0.07012 | 0.068308 | 0.066691 | 0.065244 | 0.063948 | 0.062786 | 0.061744 | 0.06081 | 0.059972 | 0.059221 | 0.05855 | 0.057951 | 0.057417 | 0.056944 | 0.056526 | 0.056159 | 0.055838 | 0.05556 |
| 33 | 0.073841 | 0.071504 | 0.069425 | 0.06757 | 0.06591 | 0.064423 | 0.063087 | 0.061886 | 0.060806 | 0.059834 | 0.05896 | 0.058173 | 0.057466 | 0.056831 | 0.056263 | 0.055755 | 0.055302 | 0.0549 | 0.054545 | 0.054234 |
| 34 | 0.073306 | 0.070924 | 0.068801 | 0.066905 | 0.065205 | 0.063679 | 0.062306 | 0.06107 | 0.059954 | 0.058948 | 0.058039 | 0.057219 | 0.056479 | 0.055811 | 0.05521 | 0.05467 | 0.054186 | 0.053752 | 0.053366 | 0.053023 |
| 35 | 0.072829 | 0.070403 | 0.068239 | 0.066304 | 0.064567 | 0.063005 | 0.061598 | 0.060327 | 0.059179 | 0.05814 | 0.0572 | 0.056348 | 0.055578 | 0.05488 | 0.054249 | 0.053679 | 0.053165 | 0.052703 | 0.052288 | 0.051916 |
| 36 | 0.072405 | 0.069937 | 0.067734 | 0.065761 | 0.063989 | 0.062393 | 0.060953 | 0.059651 | 0.058472 | 0.057403 | 0.056433 | 0.055553 | 0.054753 | 0.054028 | 0.053369 | 0.052772 | 0.052231 | 0.051741 | 0.051299 | 0.050901 |
| 37 | 0.072029 | 0.069519 | 0.067279 | 0.06527 | 0.063465 | 0.061837 | 0.060366 | 0.059034 | 0.057826 | 0.056729 | 0.055732 | 0.054824 | 0.053998 | 0.053246 | 0.052562 | 0.051939 | 0.051373 | 0.050858 | 0.050391 | 0.049969 |
| 38 | 0.071695 | 0.069146 | 0.066869 | 0.064827 | 0.06299 | 0.061331 | 0.059831 | 0.058471 | 0.057235 | 0.056112 | 0.055088 | 0.054156 | 0.053305 | 0.052528 | 0.05182 | 0.051173 | 0.050583 | 0.050045 | 0.049556 | 0.04911 |
| 39 | 0.0714 | 0.068813 | 0.066501 | 0.064426 | 0.062558 | 0.060871 | 0.059343 | 0.057956 | 0.056694 | 0.055546 | 0.054498 | 0.053541 | 0.052667 | 0.051868 | 0.051137 | 0.050468 | 0.049856 | 0.049296 | 0.048785 | 0.048318 |
| 40 | 0.071141 | 0.068517 | 0.066171 | 0.064065 | 0.062167 | 0.060451 | 0.058897 | 0.057484 | 0.056199 | 0.055026 | 0.053955 | 0.052976 | 0.05208 | 0.051259 | 0.050507 | 0.049817 | 0.049184 | 0.048605 | 0.048073 | 0.047586 |
| 41 | 0.070914 | 0.068255 | 0.065875 | 0.063738 | 0.061812 | 0.060069 | 0.058489 | 0.057052 | 0.055743 | 0.054548 | 0.053456 | 0.052456 | 0.051539 | 0.050697 | 0.049925 | 0.049215 | 0.048564 | 0.047965 | 0.047414 | 0.046909 |
| 42 | 0.070718 | 0.068022 | 0.06561 | 0.063444 | 0.06149 | 0.059721 | 0.058117 | 0.056657 | 0.055325 | 0.054109 | 0.052996 | 0.051975 | 0.051039 | 0.050178 | 0.049387 | 0.048659 | 0.047989 | 0.047372 | 0.046804 | 0.046281 |
| 43 | 0.07055 | 0.067818 | 0.065375 | 0.063179 | 0.061198 | 0.059405 | 0.057777 | 0.056294 | 0.054941 | 0.053704 | 0.052571 | 0.051532 | 0.050576 | 0.049698 | 0.048889 | 0.048143 | 0.047456 | 0.046822 | 0.046237 | 0.045698 |
| 44 | 0.070406 | 0.06764 | 0.065166 | 0.062941 | 0.060934 | 0.059117 | 0.057466 | 0.055962 | 0.054588 | 0.053332 | 0.05218 | 0.051122 | 0.050149 | 0.049252 | 0.048427 | 0.047665 | 0.046961 | 0.046311 | 0.045711 | 0.045156 |
| 45 | 0.070286 | 0.067486 | 0.064981 | 0.062729 | 0.060696 | 0.058855 | 0.057182 | 0.055657 | 0.054264 | 0.052988 | 0.051818 | 0.050742 | 0.049752 | 0.04884 | 0.047998 | 0.04722 | 0.046501 | 0.045836 | 0.045221 | 0.044651 |
| 46 | 0.070189 | 0.067355 | 0.064819 | 0.06254 | 0.060482 | 0.058617 | 0.056923 | 0.055378 | 0.053965 | 0.052671 | 0.051484 | 0.050391 | 0.049385 | 0.048457 | 0.0476 | 0.046807 | 0.046073 | 0.045394 | 0.044765 | 0.044181 |
| 47 | 0.070111 | 0.067244 | 0.064678 | 0.062372 | 0.060289 | 0.058402 | 0.056686 | 0.055122 | 0.053691 | 0.05238 | 0.051175 | 0.050066 | 0.049045 | 0.048101 | 0.047229 | 0.046422 | 0.045675 | 0.044982 | 0.044339 | 0.043742 |
| 48 | 0.070053 | 0.067152 | 0.064557 | 0.062224 | 0.060117 | 0.058208 | 0.056471 | 0.054888 | 0.053439 | 0.05211 | 0.050889 | 0.049765 | 0.048729 | 0.047771 | 0.046885 | 0.046064 | 0.045303 | 0.044597 | 0.043942 | 0.043332 |
| 49 | 0.070013 | 0.067079 | 0.064454 | 0.062094 | 0.059964 | 0.058033 | 0.056276 | 0.054674 | 0.053207 | 0.051862 | 0.050626 | 0.049486 | 0.048435 | 0.047464 | 0.046564 | 0.04573 | 0.044957 | 0.044238 | 0.043571 | 0.042949 |
| 50 | 0.06999 | 0.067022 | 0.064368 | 0.061982 | 0.059828 | 0.057875 | 0.056099 | 0.054478 | 0.052995 | 0.051633 | 0.050382 | 0.049228 | 0.048163 | 0.047178 | 0.046265 | 0.045419 | 0.044633 | 0.043903 | 0.043223 | 0.042591 |
| 51 | 0.069983 | 0.066982 | 0.064298 | 0.061886 | 0.059709 | 0.057735 | 0.055939 | 0.0543 | 0.0528 | 0.051423 | 0.050156 | 0.048988 | 0.04791 | 0.046912 | 0.045987 | 0.045129 | 0.044331 | 0.043589 | 0.042899 | 0.042255 |
| 52 | 0.069991 | 0.066957 | 0.064244 | 0.061806 | 0.059605 | 0.05761 | 0.055795 | 0.054138 | 0.052622 | 0.051229 | 0.049948 | 0.048767 | 0.047675 | 0.046664 | 0.045727 | 0.044857 | 0.044049 | 0.043296 | 0.042594 | 0.04194 |
| 53 | 0.070013 | 0.066945 | 0.064203 | 0.061739 | 0.059515 | 0.0575 | 0.055666 | 0.053992 | 0.052459 | 0.051051 | 0.049756 | 0.048561 | 0.047457 | 0.046434 | 0.045485 | 0.044604 | 0.043784 | 0.043021 | 0.042309 | 0.041645 |
| 54 | 0.070049 | 0.066947 | 0.064176 | 0.061686 | 0.05944 | 0.057403 | 0.05555 | 0.053859 | 0.05231 | 0.050888 | 0.049579 | 0.048371 | 0.047254 | 0.046219 | 0.045259 | 0.044367 | 0.043537 | 0.042764 | 0.042042 | 0.041368 |
| 55 | 0.070097 | 0.066962 | 0.064161 | 0.061646 | 0.059376 | 0.057319 | 0.055448 | 0.05374 | 0.052175 | 0.050739 | 0.049416 | 0.048195 | 0.047066 | 0.04602 | 0.045049 | 0.044146 | 0.043306 | 0.042522 | 0.041791 | 0.041108 |
| 56 | 0.070157 | 0.066989 | 0.064159 | 0.061618 | 0.059325 | 0.057248 | 0.055358 | 0.053633 | 0.052053 | 0.050602 | 0.049266 | 0.048033 | 0.046892 | 0.045834 | 0.044853 | 0.04394 | 0.043089 | 0.042296 | 0.041556 | 0.040864 |
| 57 | 0.07023 | 0.067027 | 0.064168 | 0.061601 | 0.059286 | 0.057188 | 0.05528 | 0.053538 | 0.051943 | 0.050478 | 0.049128 | 0.047882 | 0.04673 | 0.045662 | 0.04467 | 0.043747 | 0.042887 | 0.042084 | 0.041335 | 0.040634 |
| 58 | 0.070313 | 0.067076 | 0.064187 | 0.061595 | 0.059257 | 0.057139 | 0.055212 | 0.053454 | 0.051843 | 0.050364 | 0.049002 | 0.047744 | 0.04658 | 0.045501 | 0.044499 | 0.043566 | 0.042697 | 0.041886 | 0.041128 | 0.040419 |
| 59 | 0.070407 | 0.067136 | 0.064217 | 0.061599 | 0.059238 | 0.0571 | 0.055155 | 0.05338 | 0.051755 | 0.050262 | 0.048887 | 0.047617 | 0.046442 | 0.045352 | 0.04434 | 0.043398 | 0.042519 | 0.041699 | 0.040933 | 0.040216 |
| 60 | 0.070511 | 0.067205 | 0.064257 | 0.061613 | 0.059229 | 0.05707 | 0.055108 | 0.053316 | 0.051676 | 0.050169 | 0.048782 | 0.0475 | 0.046314 | 0.045214 | 0.044192 | 0.043241 | 0.042353 | 0.041525 | 0.04075 | 0.040025 |
| 61 | 0.070625 | 0.067285 | 0.064307 | 0.061636 | 0.059229 | 0.057051 | 0.05507 | 0.053262 | 0.051607 | 0.050087 | 0.048686 | 0.047393 | 0.046196 | 0.045086 | 0.044054 | 0.043094 | 0.042198 | 0.041361 | 0.040578 | 0.039845 |
| 62 | 0.070749 | 0.067373 | 0.064365 | 0.061668 | 0.059239 | 0.057039 | 0.05504 | 0.053216 | 0.051546 | 0.050013 | 0.0486 | 0.047295 | 0.046088 | 0.044968 | 0.043926 | 0.042957 | 0.042052 | 0.041207 | 0.040417 | 0.039676 |
| 63 | 0.070882 | 0.06747 | 0.064432 | 0.061709 | 0.059256 | 0.057037 | 0.055019 | 0.053179 | 0.051495 | 0.049947 | 0.048522 | 0.047206 | 0.045988 | 0.044858 | 0.043808 | 0.042829 | 0.041916 | 0.041064 | 0.040265 | 0.039518 |
| 64 | 0.071023 | 0.067577 | 0.064507 | 0.061758 | 0.059282 | 0.057042 | 0.055007 | 0.05315 | 0.051451 | 0.04989 | 0.048453 | 0.047126 | 0.045897 | 0.044757 | 0.043698 | 0.04271 | 0.041789 | 0.040929 | 0.040123 | 0.039368 |
| 65 | 0.071173 | 0.067691 | 0.064591 | 0.061814 | 0.059315 | 0.057055 | 0.055001 | 0.053129 | 0.051415 | 0.049841 | 0.048391 | 0.047053 | 0.045814 | 0.044664 | 0.043596 | 0.0426 | 0.041671 | 0.040803 | 0.03999 | 0.039228 |
| 66 | 0.07133 | 0.067812 | 0.064682 | 0.061879 | 0.059356 | 0.057075 | 0.055004 | 0.053115 | 0.051386 | 0.049799 | 0.048337 | 0.046988 | 0.045738 | 0.044579 | 0.043501 | 0.042497 | 0.04156 | 0.040684 | 0.039864 | 0.039096 |
| 67 | 0.071496 | 0.067942 | 0.06478 | 0.06195 | 0.059404 | 0.057103 | 0.055013 | 0.053107 | 0.051364 | 0.049764 | 0.04829 | 0.046929 | 0.04567 | 0.044501 | 0.043415 | 0.042402 | 0.041457 | 0.040574 | 0.039747 | 0.038971 |
| 68 | 0.07167 | 0.068079 | 0.064886 | 0.062029 | 0.059459 | 0.057137 | 0.055028 | 0.053107 | 0.051349 | 0.049735 | 0.048249 | 0.046878 | 0.045608 | 0.04443 | 0.043335 | 0.042314 | 0.041362 | 0.040471 | 0.039637 | 0.038855 |
| 69 | 0.071852 | 0.068224 | 0.064998 | 0.062114 | 0.059521 | 0.057177 | 0.055051 | 0.053113 | 0.05134 | 0.049713 | 0.048215 | 0.046833 | 0.045553 | 0.044366 | 0.043262 | 0.042233 | 0.041273 | 0.040375 | 0.039534 | 0.038745 |
| 70 | 0.07204 | 0.068375 | 0.065118 | 0.062206 | 0.059589 | 0.057224 | 0.055079 | 0.053125 | 0.051337 | 0.049697 | 0.048188 | 0.046794 | 0.045504 | 0.044307 | 0.043195 | 0.042158 | 0.04119 | 0.040285 | 0.039438 | 0.038642 |
| 71 | 0.072235 | 0.068533 | 0.065244 | 0.062304 | 0.059663 | 0.057277 | 0.055114 | 0.053143 | 0.05134 | 0.049687 | 0.048165 | 0.046761 | 0.045461 | 0.044255 | 0.043134 | 0.042089 | 0.041114 | 0.040202 | 0.039348 | 0.038546 |
| 72 | 0.072438 | 0.068697 | 0.065376 | 0.062408 | 0.059743 | 0.057336 | 0.055154 | 0.053166 | 0.051349 | 0.049683 | 0.048149 | 0.046733 | 0.045424 | 0.044208 | 0.043079 | 0.042026 | 0.041043 | 0.040124 | 0.039264 | 0.038456 |
| 73 | 0.072647 | 0.068868 | 0.065514 | 0.062519 | 0.059829 | 0.057401 | 0.055199 | 0.053195 | 0.051364 | 0.049684 | 0.048138 | 0.046711 | 0.045392 | 0.044167 | 0.043029 | 0.041969 | 0.040979 | 0.040053 | 0.039185 | 0.038371 |
| 74 | 0.072862 | 0.069045 | 0.065658 | 0.062634 | 0.05992 | 0.057471 | 0.055251 | 0.05323 | 0.051383 | 0.04969 | 0.048132 | 0.046695 | 0.045365 | 0.044131 | 0.042985 | 0.041916 | 0.040919 | 0.039986 | 0.039112 | 0.038292 |
| 75 | 0.073084 | 0.069228 | 0.065808 | 0.062756 | 0.060017 | 0.057546 | 0.055307 | 0.053269 | 0.051408 | 0.049701 | 0.048131 | 0.046683 | 0.045343 | 0.0441 | 0.042945 | 0.041869 | 0.040864 | 0.039925 | 0.039045 | 0.038219 |
| 76 | 0.073312 | 0.069417 | 0.065964 | 0.062883 | 0.060119 | 0.057626 | 0.055368 | 0.053313 | 0.051437 | 0.049717 | 0.048135 | 0.046676 | 0.045326 | 0.044074 | 0.042911 | 0.041827 | 0.040815 | 0.039869 | 0.038982 | 0.03815 |
| 77 | 0.073545 | 0.069612 | 0.066125 | 0.063015 | 0.060226 | 0.057711 | 0.055434 | 0.053363 | 0.051471 | 0.049737 | 0.048143 | 0.046673 | 0.045313 | 0.044053 | 0.042881 | 0.041789 | 0.04077 | 0.039817 | 0.038924 | 0.038086 |
| 78 | 0.073785 | 0.069813 | 0.066292 | 0.063153 | 0.060338 | 0.057802 | 0.055505 | 0.053416 | 0.051509 | 0.049762 | 0.048156 | 0.046675 | 0.045305 | 0.044035 | 0.042855 | 0.041756 | 0.04073 | 0.03977 | 0.038871 | 0.038027 |
| 79 | 0.07403 | 0.070018 | 0.066463 | 0.063295 | 0.060455 | 0.057896 | 0.05558 | 0.053474 | 0.051552 | 0.049792 | 0.048173 | 0.046681 | 0.045301 | 0.044022 | 0.042834 | 0.041727 | 0.040693 | 0.039727 | 0.038822 | 0.037972 |
| 80 | 0.074282 | 0.070229 | 0.06664 | 0.063442 | 0.060577 | 0.057996 | 0.05566 | 0.053537 | 0.0516 | 0.049825 | 0.048195 | 0.046691 | 0.045302 | 0.044014 | 0.042817 | 0.041702 | 0.040661 | 0.039688 | 0.038777 | 0.037922 |
| 81 | 0.074538 | 0.070445 | 0.066822 | 0.063594 | 0.060703 | 0.058099 | 0.055744 | 0.053604 | 0.051651 | 0.049863 | 0.04822 | 0.046706 | 0.045306 | 0.044009 | 0.042803 | 0.041681 | 0.040633 | 0.039654 | 0.038736 | 0.037875 |
| 82 | 0.074799 | 0.070666 | 0.067008 | 0.063751 | 0.060834 | 0.058207 | 0.055832 | 0.053674 | 0.051706 | 0.049904 | 0.048249 | 0.046724 | 0.045314 | 0.044007 | 0.042794 | 0.041664 | 0.040609 | 0.039623 | 0.038699 | 0.037832 |
| 83 | 0.075066 | 0.070893 | 0.0672 | 0.063912 | 0.060969 | 0.05832 | 0.055925 | 0.053749 | 0.051765 | 0.04995 | 0.048282 | 0.046745 | 0.045326 | 0.04401 | 0.042788 | 0.04165 | 0.040588 | 0.039596 | 0.038666 | 0.037794 |
| 84 | 0.075339 | 0.071124 | 0.067396 | 0.064078 | 0.061108 | 0.058436 | 0.056021 | 0.053828 | 0.051828 | 0.049999 | 0.048319 | 0.046771 | 0.045341 | 0.044016 | 0.042786 | 0.04164 | 0.040571 | 0.039572 | 0.038636 | 0.037758 |
| 85 | 0.075616 | 0.07136 | 0.067596 | 0.064248 | 0.061251 | 0.058557 | 0.056121 | 0.05391 | 0.051895 | 0.050051 | 0.048359 | 0.0468 | 0.04536 | 0.044026 | 0.042787 | 0.041633 | 0.040558 | 0.039552 | 0.03861 | 0.037726 |
| 86 | 0.075897 | 0.0716 | 0.067801 | 0.064422 | 0.061399 | 0.058681 | 0.056225 | 0.053996 | 0.051965 | 0.050107 | 0.048402 | 0.046832 | 0.045382 | 0.044038 | 0.042791 | 0.04163 | 0.040547 | 0.039535 | 0.038587 | 0.037697 |
| 87 | 0.076183 | 0.071845 | 0.06801 | 0.0646 | 0.061551 | 0.058809 | 0.056333 | 0.054086 | 0.052039 | 0.050167 | 0.048449 | 0.046867 | 0.045407 | 0.044054 | 0.042799 | 0.04163 | 0.04054 | 0.039521 | 0.038567 | 0.037672 |
| 88 | 0.076475 | 0.072094 | 0.068224 | 0.064783 | 0.061706 | 0.058941 | 0.056444 | 0.054179 | 0.052116 | 0.05023 | 0.048499 | 0.046906 | 0.045435 | 0.044073 | 0.042809 | 0.041633 | 0.040536 | 0.03951 | 0.03855 | 0.03765 |
| 89 | 0.07677 | 0.072348 | 0.068442 | 0.064969 | 0.061865 | 0.059076 | 0.056559 | 0.054276 | 0.052196 | 0.050296 | 0.048552 | 0.046948 | 0.045467 | 0.044096 | 0.042823 | 0.041639 | 0.040534 | 0.039503 | 0.038536 | 0.03763 |
| 90 | 0.077071 | 0.072606 | 0.068663 | 0.065159 | 0.062028 | 0.059216 | 0.056677 | 0.054375 | 0.05228 | 0.050365 | 0.048609 | 0.046993 | 0.045501 | 0.044121 | 0.042839 | 0.041647 | 0.040536 | 0.039498 | 0.038525 | 0.037614 |
| 91 | 0.077375 | 0.072868 | 0.068889 | 0.065353 | 0.062195 | 0.059358 | 0.056799 | 0.054479 | 0.052367 | 0.050437 | 0.048668 | 0.04704 | 0.045539 | 0.044148 | 0.042859 | 0.041659 | 0.040541 | 0.039496 | 0.038517 | 0.0376 |
| 92 | 0.077683 | 0.073134 | 0.069118 | 0.065551 | 0.062365 | 0.059504 | 0.056923 | 0.054585 | 0.052457 | 0.050513 | 0.04873 | 0.047091 | 0.045579 | 0.044179 | 0.042881 | 0.041673 | 0.040548 | 0.039496 | 0.038512 | 0.037589 |
| 93 | 0.077996 | 0.073405 | 0.069352 | 0.065752 | 0.062538 | 0.059654 | 0.057052 | 0.054694 | 0.05255 | 0.050591 | 0.048796 | 0.047144 | 0.045621 | 0.044212 | 0.042905 | 0.04169 | 0.040557 | 0.039499 | 0.038509 | 0.03758 |
| 94 | 0.078312 | 0.073679 | 0.069589 | 0.065957 | 0.062715 | 0.059806 | 0.057183 | 0.054807 | 0.052645 | 0.050672 | 0.048863 | 0.047201 | 0.045667 | 0.044248 | 0.042933 | 0.04171 | 0.04057 | 0.039505 | 0.038508 | 0.037574 |
| 95 | 0.078633 | 0.073956 | 0.069829 | 0.066166 | 0.062896 | 0.059962 | 0.057317 | 0.054922 | 0.052744 | 0.050756 | 0.048934 | 0.047259 | 0.045715 | 0.044287 | 0.042963 | 0.041732 | 0.040584 | 0.039513 | 0.03851 | 0.037571 |
| 96 | 0.078957 | 0.074238 | 0.070074 | 0.066378 | 0.06308 | 0.060121 | 0.057455 | 0.05504 | 0.052846 | 0.050842 | 0.049007 | 0.047321 | 0.045766 | 0.044328 | 0.042995 | 0.041756 | 0.040601 | 0.039523 | 0.038515 | 0.037569 |

$$w_{max} = \alpha \cdot \frac{F \cdot l^3}{E \cdot h^2 \cdot t \cdot b}$$

# Forget-me-not - Porous Crossed Rectangle Cantilever Beam

Values of α used in the deflection forget me not of a porous crossed cantilever beam dependent on the number of repeating inner structures in length(nl) and height(nh)

| NL / NH | 41 | 42 | 43 | 44 | 45 |
|---|---|---|---|---|---|
| 1 | 20.65855 | 21.15173 | 21.64457 | 22.13708 | 22.62927 |
| 2 | 5.286192 | 5.409403 | 5.532542 | 5.655608 | 5.778603 |
| 3 | 2.405025 | 2.459651 | 2.514258 | 2.568838 | 2.623392 |
| 4 | 1.385606 | 1.416201 | 1.446795 | 1.477383 | 1.507958 |
| 5 | 0.90888 | 0.928344 | 0.947809 | 0.96728 | 0.986744 |
| 6 | 0.647377 | 0.660778 | 0.674192 | 0.687612 | 0.701033 |
| 7 | 0.488232 | 0.49797 | 0.507722 | 0.517484 | 0.527256 |
| 8 | 0.384041 | 0.391393 | 0.39876 | 0.406142 | 0.413536 |
| 9 | 0.312032 | 0.317739 | 0.323464 | 0.329207 | 0.334964 |
| 10 | 0.260142 | 0.264666 | 0.269211 | 0.273775 | 0.278356 |
| 11 | 0.221491 | 0.225134 | 0.2288 | 0.232488 | 0.236194 |
| 12 | 0.191914 | 0.194883 | 0.197877 | 0.200893 | 0.203929 |
| 13 | 0.168772 | 0.171212 | 0.173678 | 0.176168 | 0.178679 |
| 14 | 0.150319 | 0.152337 | 0.154382 | 0.156451 | 0.158543 |
| 15 | 0.135367 | 0.137041 | 0.138744 | 0.140472 | 0.142225 |
| 16 | 0.123083 | 0.124474 | 0.125895 | 0.127342 | 0.128814 |
| 17 | 0.112866 | 0.114022 | 0.115207 | 0.11642 | 0.117658 |
| 18 | 0.104279 | 0.105236 | 0.106223 | 0.107237 | 0.108279 |
| 19 | 0.096991 | 0.097779 | 0.098596 | 0.099444 | 0.100316 |
| 20 | 0.090754 | 0.091396 | 0.092069 | 0.092771 | 0.093501 |
| 21 | 0.085376 | 0.085891 | 0.086439 | 0.087016 | 0.087621 |
| 22 | 0.080705 | 0.081111 | 0.08155 | 0.082017 | 0.082513 |
| 23 | 0.076624 | 0.076934 | 0.077276 | 0.077648 | 0.078048 |
| 24 | 0.073037 | 0.073262 | 0.07352 | 0.073808 | 0.074124 |
| 25 | 0.069868 | 0.070018 | 0.070201 | 0.070414 | 0.070656 |
| 26 | 0.067055 | 0.067138 | 0.067254 | 0.067401 | 0.067576 |
| 27 | 0.064547 | 0.06457 | 0.064626 | 0.064713 | 0.064829 |
| 28 | 0.062302 | 0.06227 | 0.062273 | 0.062306 | 0.062369 |
| 29 | 0.060284 | 0.060204 | 0.060157 | 0.060143 | 0.060157 |
| 30 | 0.058464 | 0.05834 | 0.058249 | 0.058191 | 0.058161 |
| 31 | 0.056817 | 0.056653 | 0.056522 | 0.056424 | 0.056355 |
| 32 | 0.055322 | 0.055121 | 0.054955 | 0.05482 | 0.054715 |
| 33 | 0.053962 | 0.053728 | 0.053527 | 0.053359 | 0.053221 |
| 34 | 0.052721 | 0.052455 | 0.052225 | 0.052026 | 0.051857 |
| 35 | 0.051585 | 0.051291 | 0.051032 | 0.050806 | 0.050609 |
| 36 | 0.050544 | 0.050224 | 0.049939 | 0.049686 | 0.049464 |
| 37 | 0.049587 | 0.049242 | 0.048933 | 0.048657 | 0.04841 |
| 38 | 0.048705 | 0.048339 | 0.048007 | 0.047708 | 0.04744 |
| 39 | 0.047892 | 0.047504 | 0.047152 | 0.046832 | 0.046543 |
| 40 | 0.047141 | 0.046733 | 0.046361 | 0.046023 | 0.045714 |
| 41 | 0.046445 | 0.046019 | 0.045629 | 0.045272 | 0.044946 |
| 42 | 0.045799 | 0.045357 | 0.04495 | 0.044576 | 0.044233 |
| 43 | 0.0452 | 0.044741 | 0.044318 | 0.043929 | 0.04357 |
| 44 | 0.044643 | 0.044169 | 0.043731 | 0.043326 | 0.042953 |
| 45 | 0.044124 | 0.043635 | 0.043183 | 0.042765 | 0.042377 |
| 46 | 0.04364 | 0.043138 | 0.042672 | 0.04224 | 0.04184 |
| 47 | 0.043188 | 0.042673 | 0.042195 | 0.041751 | 0.041338 |
| 48 | 0.042766 | 0.042239 | 0.041749 | 0.041292 | 0.040868 |
| 49 | 0.042371 | 0.041832 | 0.041331 | 0.040863 | 0.040428 |
| 50 | 0.042001 | 0.041452 | 0.040939 | 0.040461 | 0.040015 |
| 51 | 0.041655 | 0.041095 | 0.040572 | 0.040083 | 0.039627 |
| 52 | 0.04133 | 0.04076 | 0.040227 | 0.039729 | 0.039263 |
| 53 | 0.041025 | 0.040445 | 0.039903 | 0.039395 | 0.03892 |
| 54 | 0.040738 | 0.040149 | 0.039598 | 0.039082 | 0.038598 |
| 55 | 0.040469 | 0.039871 | 0.039311 | 0.038787 | 0.038294 |
| 56 | 0.040216 | 0.03961 | 0.039041 | 0.038508 | 0.038008 |
| 57 | 0.039978 | 0.039364 | 0.038787 | 0.038246 | 0.037738 |
| 58 | 0.039754 | 0.039132 | 0.038547 | 0.037999 | 0.037483 |
| 59 | 0.039544 | 0.038913 | 0.038321 | 0.037766 | 0.037243 |
| 60 | 0.039345 | 0.038707 | 0.038108 | 0.037545 | 0.037016 |
| 61 | 0.039158 | 0.038513 | 0.037907 | 0.037337 | 0.036801 |
| 62 | 0.038982 | 0.03833 | 0.037717 | 0.03714 | 0.036598 |
| 63 | 0.038816 | 0.038157 | 0.037538 | 0.036955 | 0.036406 |
| 64 | 0.03866 | 0.037994 | 0.037368 | 0.036779 | 0.036224 |
| 65 | 0.038512 | 0.03784 | 0.037208 | 0.036613 | |
| 66 | 0.038374 | 0.037695 | 0.037057 | 0.036456 | |
| 67 | 0.038243 | 0.037558 | 0.036914 | 0.036307 | |
| 68 | 0.03812 | 0.037429 | 0.036779 | 0.036166 | |
| 69 | 0.038004 | 0.037308 | 0.036652 | 0.036033 | |
| 70 | 0.037895 | 0.037193 | 0.036531 | 0.035908 | |
| 71 | 0.037793 | 0.037085 | 0.036418 | 0.035789 | |
| 72 | 0.037697 | 0.036983 | 0.036311 | 0.035676 | |
| 73 | 0.037607 | 0.036887 | 0.036209 | 0.03557 | |
| 74 | 0.037522 | 0.036797 | 0.036114 | 0.03547 | |
| 75 | 0.037443 | 0.036712 | 0.036024 | 0.035375 | |
| 76 | 0.037368 | 0.036633 | 0.035939 | 0.035285 | |
| 77 | 0.037299 | 0.036558 | 0.03586 | 0.035201 | |
| 78 | 0.037234 | 0.036488 | 0.035785 | 0.035121 | |
| 79 | 0.037174 | 0.036423 | 0.035714 | 0.035046 | |
| 80 | 0.037118 | 0.036361 | 0.035648 | 0.034975 | |
| 81 | 0.037066 | 0.036304 | 0.035586 | 0.034909 | |
| 82 | 0.037018 | 0.036251 | 0.035528 | 0.034846 | |
| 83 | 0.036974 | 0.036202 | 0.035474 | 0.034788 | |
| 84 | 0.036933 | 0.036156 | 0.035424 | 0.034733 | |
| 85 | 0.036896 | 0.036114 | 0.035377 | 0.034681 | |
| 86 | 0.036862 | 0.036075 | 0.035333 | 0.034633 | |
| 87 | 0.036831 | 0.036039 | 0.035293 | 0.034589 | |
| 88 | 0.036803 | 0.036007 | 0.035256 | 0.034547 | |
| 89 | 0.036779 | 0.035977 | 0.035222 | 0.034509 | |
| 90 | 0.036757 | 0.035951 | 0.035191 | 0.034473 | |
| 91 | 0.036738 | 0.035927 | 0.035162 | 0.034441 | |
| 92 | 0.036722 | 0.035905 | 0.035136 | 0.034411 | |
| 93 | 0.036708 | 0.035887 | 0.035113 | 0.034383 | |
| 94 | 0.036697 | 0.035871 | 0.035093 | 0.034358 | |
| 95 | 0.036688 | 0.035857 | 0.035075 | 0.034336 | |
| 96 | 0.036681 | 0.035846 | 0.035059 | 0.034316 | |

$$w_{max} = \alpha \cdot \frac{F \cdot l^3}{E \cdot h^2 \cdot t \cdot b}$$

## 5.2 Python code

In this section a brief description is be made of all the code and their function. At first the entire extensive crossed-rectangle script functioning is explained and how to operate it. Secondly the cantilever function is explained. .

### Extensive Crossed-Rectangle Python Script

**Input variables** These are the specific variables, the characteristics of the cantilever beam.

```
#Force, negative for in y direction(downwards)
F=-10e3
#Materialproperties
t = 2.5*10**-4
b = 0.2
E = 210*10**9
I = 1/12*b*t**3
#Roster
h = 0.2
l = 2
A = t * b
nh   = 2
nl   = 8
```

**Dof** The degrees of freedom matrix, *Dof*, is a matrix that numbers of degrees of freedom of the entire grid. Grid point 1 will generate array line $[1, 2, 3]$ while grid point 2 will generate array line $[4, 5, 6]$.

```
#dof
dh = h/nh
dl = l/nl
n = (nl+1)*(nh+1)+(nh*nl)
n1 = (nl+1)*(nh+1)
n2 = nh*nl
dof = cfc.createdofs(n,3)
```

**Coord** The generated matrix with all the degrees of freedom will need a corresponding coordinate matrix, *Coord*, with coordinates in the XY plane as seen in figure 4.

```
42  #Coord(x,y)
43  x1 = np.arange(nl+1)*dl
44  y1 = np.arange(nh+1)*dh
45  x2 = 0.5*dl + np.arange(nl)*dl
46  y2 = 0.5*dh + np.arange(nh)*dh
47  x1,y1 = np.meshgrid(x1,y1)
48  x2,y2 = np.meshgrid(x2,y2)
49  xy1 = np.array((x1,y1))
50  xy2 = np.array((x2,y2))
51  points1 = np.reshape(xy1,(2,n1))
52  points2 = np.reshape(xy2,(2,n2))
53  points = np.hstack([points1,points2])
54  Coord = np.swapaxes(points,0,1)
```

**Edof** Now that all grid points have degrees of freedom and corresponding coordinates the topology of the lattice-structure is specified. The *Edof* matrix is an array where every line in the array corresponds to an element. $[1, 2, 3, 4, 5, 6]$ will be an element that connects grid point 1 and 2. And because of the corresponding coordinates, all elements are specified which in turn results in the entire lattice-structure.

```
#edof
nschuin = 4*nh*nl
nrecht = ((4*nl-(nl-1)) + (2*nl+1)*(nh-1))
nlijnen = nschuin + nrecht
edof = np.zeros((nlijnen,6))
#horizontals
for j in range(nh+1):
    for i in range(nl):
        edof[(j*nl)+i] = np.hstack(
            (np.array(dof[(j*(nl+1))+i,:]),
             np.array(dof[(j*(nl+1))+(i+1),:])))
#verticals
for j in range(nh):
    for i in range(nl+1):
        edof[(nl*(nh+1))+(j*(nl+1))+i] = np.hstack(
            (np.array(dof[(j*(nl+1))+i,:]),
             np.array(dof[((j+1)*(nl+1))+i,:])))
#diagonals
for j in range(nh):
    for i in range(nl):
        #top-left
        edof[nrecht+(j*nl)+i] = np.hstack(
            (np.array(dof[(j*(nl+1))+i,:]),
              np.array(dof[(nh+1)*(nl+1)+(j*nl)+i,:])))
        #top-right
        edof[nrecht+(nh*nl)+(j*nl)+i] = np.hstack(
            (np.array(dof[(j*(nl+1))+i+1,:]),
              np.array(dof[(nh+1)*(nl+1)+(j*nl)+i,:])))
        #bottom-left
        edof[nrecht+2*(nh*nl)+(j*nl)+i] = np.hstack(
            (np.array(dof[((j+1)*(nl+1))+i,:]),
              np.array(dof[(nh+1)*(nl+1)+(j*nl)+i,:])))
        #bottom-right
        edof[nrecht+3*(nh*nl)+(j*nl)+i] = np.hstack(
            (np.array(dof[((j+1)*(nl+1))+i+1,:]),
              np.array(dof[(nh+1)*(nl+1)+(j*nl)+i,:])))
```

**Print** Before the calculations are commenced certain things well be returned as a print to the user. This is information put in, gathered or checks, such as the slenderness check.

```
print('Height, y-axis: Amount of crosses and size per cross =',
      nh,'crosses and', h/nh, 'meters per cross')
print('Length, x-axis: roster crosses and size =', nl,'crosses and',
      l/nl, 'meters per cross')
print('Angle of cross connection (x to y rotation)',
      math.degrees(math.atan((Coord[(nh+1)*(nl+1)][1]) /
                             (Coord[(nh+1)*(nl+1)][0]))))

#Slenderness:
if (l / h) >= 10:
    print('Construction elements are deemed slender')
if (l / h) < 10:
    print('Construction elements are deemend non-slender')
```

**Calculations** Now that the entire structure is defined certain calculations are needed to be made to find a solution. First the element coordinate matrices ex and ey are created. Then the correct lattice point is identified as the point of the applied force.

```
#Calculations
ex,ey = cfc.coordxtr(edof, Coord, dof)
K = np.zeros(((3*len(Coord)), (3*len(Coord))))
f = np.zeros(((3*len(Coord)), 1))
f[3*((nh+1)*(nl+1))-2]   = F
ep      = np.array([E, A, I])
```

Then the boundary conditions for the structure are inserted. In this case that is the clamped connection for the origin of the beam.

```
#Boundary Conditions
bc = []
for i in range(nh+1):
    bc = np.append(bc,dof[(nl+1)*i])
bcVal = np.zeros((1,3*(nh+1)))
```

The functions *beam2e* and *assem* are then used to create the element stiffness matrix for every element and the the global stiffness matrix needed to solve the system of equations.

```
#Element Stiffness Matrix Ke, Global Stiffness Matrix K
edof_int = np.int_(edof)
for i in range(len(edof)):
    Ke = cfc.beam2e(ex[i,:],ey[i,:],ep)
    K = cfc.assem(edof_int[i,:],K,Ke)
```

The system is then solved. This returns the movement of every lattice point. The rotation of the structure is then also found.

```
a,r = cfc.solveq(K,f,np.int_(bc),bcVal)
verplaatsing = np.reshape(a,(len(Coord),3))
krachten = np.reshape(r,(len(Coord),3))
Coord2 = np.zeros((len(Coord),2))
rotation = verplaatsing[nl,2]
```

**Plot** The movement of every lattice point is used in combination with the original **Coord** array to create the displaced **Coord2** array. With this information new element coordinate matrixes are created. A plot of the displaced structure can then be made with either *cfvm* or *cfv*. Cfv is the more detailed but slower option.

```
#Plot Displacement
for i in range(len(Coord)):
     Coord2[i,:] = Coord[i,:] + verplaatsing[i,0:2]

cfu.info("Drawing results...")
ex,ey = cfc.coordxtr(edof, Coord, dof)
ex2,ey2 = cfc.coordxtr(edof, Coord2, dof)

#remove cfvm for cfv for detailed plot
cfvm.figure()
cfvm.eldraw2(ex,ey, [1,2,0])
cfvm.eldraw2(ex2,ey2, [2,4,0])
cfvm.showAndWait()
```

**Internal Forces** Some can be used to get a detailed look at the internal forces at play.

```
148 #ed = element displacement array
149 ed = cfc.extract_eldisp(edof_int,a)
150 eslr = np.zeros((len(edof),6))
151 edilr = np.zeros((len(edof),4))
152 ecilr = np.zeros((len(edof),2))
153
154 #eslr = (left coordinate(N,V,M), right coordinatet(N,V,M)) of an edof element
155 for i in range(len(eslr)):
156     es, edi, eci = cfc.beam2s(ex[i,:], ey[i,:], ep, ed[i,:])
157     eslr[i] = np.reshape(es, (1,6))
158     edilr[i] = np.reshape(edi, (1,4))
159     ecilr[i] = np.reshape(eci, (1,2))
```

**Solution Print** Lastly, a print shows the solutions.

```
161 print('The rotation at the point of the force', -F, 'N is',
162       verplaatsing[nl,2],'rad')
163 print('The displacement at the point of the force', -F, 'N is',
164       -Coord2[nl][1],'meters')
```

### Cantilever function

This function is used to gather all the data.

The cantilever function is a copy of most of the code used in the Extensive Crossed-Rectangle Python Script. However, all prints and plots are removed from the code and the script solely returns the deflection, rotation at the loading and the angle of the crosses internally. The input variables are the function's variables instead. The code that sheds light on every element's internal forces is also absent.

The function only contains **Dof**, **Coord**, **Edof** and **Calculations**

```
 19 def cantilever(F,t,b,E,h,l,nh,nl):
132     return displacement, rotation, connectionangle
```

# References

Renishaw. (n.d.). Lattice test structures built on Renishaw AM250 metal AM system at The University of Nottingham [[Online; accessed Febuari 2, 2021]]. https://resources.renishaw.com/details/ALSAM+project+aluminium+lattice+structures(254282)(93788)

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