Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CIEM5210-3 Advanced mechanics of structural elements
Tuesday 24 June 2025, 13:30 – 16:30 hours

Choose A, B, C or D.

1 What is warping? (0.23 point)

A Stresses in a beam cross-section
B Deformation of a beam cross-section
C Rotation of a beam cross-section
D Bi moment in a beam cross-section

2 Wat is different between open and closed cross-sections loaded in torsion? (0.23 point)

A Different differential equations
B Different boundary conditions
C Different software for cross-section analysis
D Different design formulas

3 What is the membrane analogy used for? (0.23 point)

A To understand the torsion properties
B To derive the Saint Venant differential equation
C To program beam cross-section software
D To measure the phi-hill

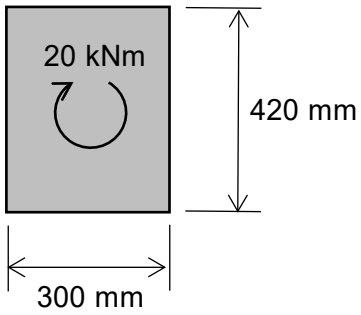
4 Which torsion theory includes a bi moment [kNm^2]? (0.23 point)

A Vlasov's
B Saint Venant's
C Circulatory
D Rankine's

5 Which boundary condition represents no warping? (0.23 point)

A $\varphi = 0$
B $\theta = 0$
C $M_t = 0$
D $B = 0$

6 See the figure of a beam cross-section with a torsion load.
The beam is made of timber with Poisson's ratio 0.30.
One beam end is fixed and cannot warp.
What is the largest normal stress? (0.23 point)



A 0.058 N/mm^2
B 0.58 N/mm^2
C 5.80 N/mm^2
D 58.00 N/mm^2

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7 What are trajectories in a plate loaded in plane? (0.23 point)

A Paths from load to supports
B Points where the linear elastic stresses are infinite
C Lines of principal directions of membrane forces
D Direction of the shear force

8 For plates loaded out of plane, we use a finite element size of two times the thickness.
Why? (0.23 point)

A The thin plate theory requires extra thickness to be accurate.
B To obtain realistic stresses under point loads.
C To compute singularities accurately.
D To compute the required reinforcement accurately.

9 A thin plate is supported at the edges and loaded by a distributed load.
Do the moments depend on Young's modulus?
Do the moments depend on Poisson's ratio? (0.23 point)

A Yes, Yes
B Yes, No
C No, Yes
D No, No

10 The Mindlin-Reissner theory is suitable for … (0.23 point)

A … thin plates.
B … thick plates.
C … shells.
D … thin plates, thick plates and shells.

11 Where can we expect a singularity in a plate? (0.23 point)

A In round corners
B In the centre of gravity
C In the middle of a fixed edge
D Where a beam element is attached

12 In a point of a plate the following moments are computed.
m_{xx} = 7 kNm/m, m_{yy} = -2 kNm/m, m_{xy} = -4 kNm/m
Which bottom reinforcement is sufficient and smallest? (0.23 point)

A m_{xb} = 9 kNm/m, m_{yb} = 9 kNm/m
B m_{xb} = 11 kNm/m, m_{yb} = 2 kNm/m
C m_{xb} = 12 kNm/m, m_{yb} = 1 kNm/m
D m_{xb} = 13 kNm/m, m_{yb} = 1 kNm/m

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13 In upperbound analyses we use the virtual work equation. Often we violate a condition of the virtual work equation. Which condition? (0.24 point)

A The virtual displacement cannot have gaps.
B The virtual displacement must fulfil the boundary conditions.
C The moment distribution must be in equilibrium with the load.
D The moments and the deformation must be related.

A frame consists of three members (Fig. 1). The members have a strength M_p. The members are rigidly connected. The supports are fixed. The structure is loaded by line load q evenly distributed per member length (self-weight). The relation of Figure 2 exists between the plastic moment and the plastic normal force.

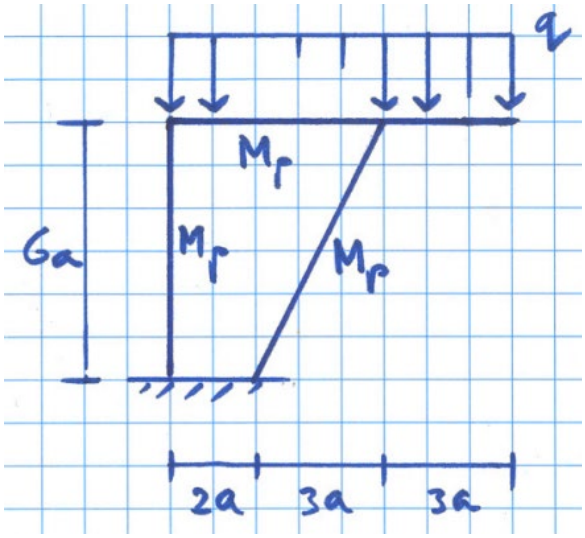


Figure 1. Frame structure

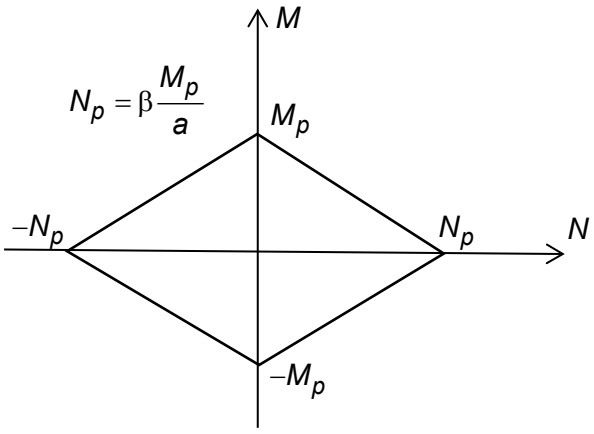


Figure 2. Yield contour

The influence of shear on the yield contour is neglected. Buckling and second order effects are not considered.

14 Assume $\beta \to \infty$. Determine the collapse load q for all possible mechanisms. Write the collapse loads as functions of M_p and a. What is the decisive collapse load? (1.5 point)

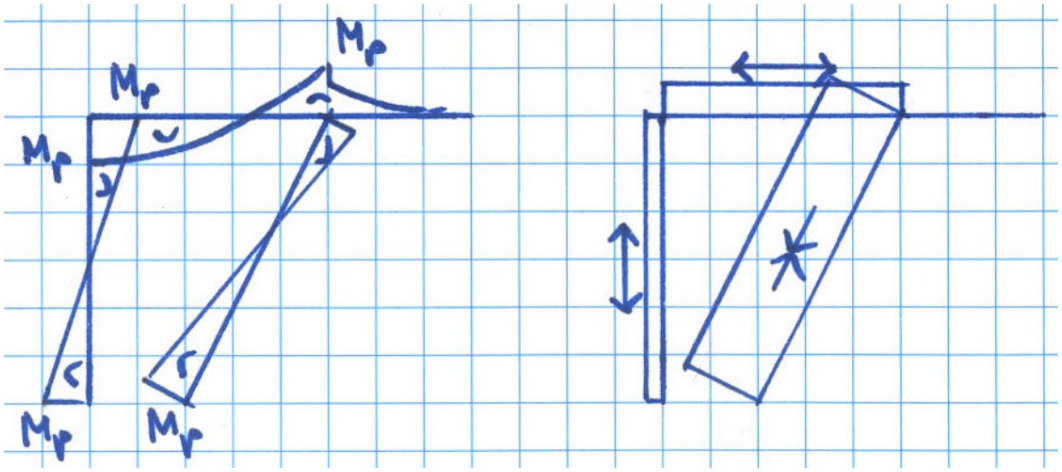


Figure 3. Moments and normal forces of the decisive mechanism

15 Assume β = 5. Choose one of the following problems (You need not do both).
– Determine the largest lower-bound for q.
– Determine the smallest upper-bound for q.
You only need to write down the equations and not solve the equations (1.5 points).

A reinforced concrete plate has simply supported edges and free edges (Fig. 4). It carries an evenly distributed load p [kN/m^2]. There is no other load on the plate. The plate is homogeneous and orthotropic.

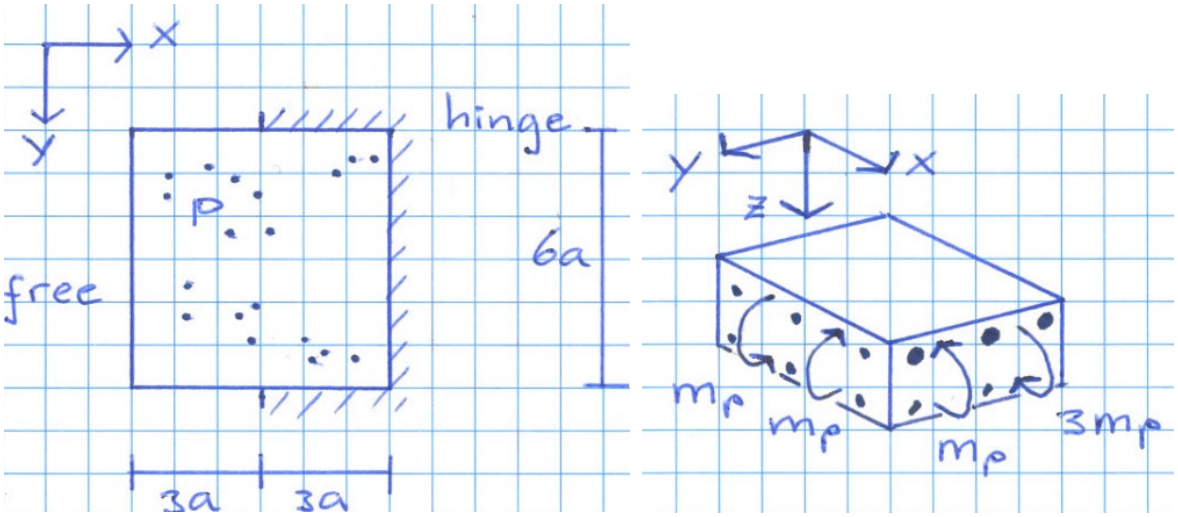


Figure 4. Plate dimensions and reinforcement

16 Consider the yield line patterns of Figure 5. Which of these patterns give kinematically possible mechanisms? (1 point)

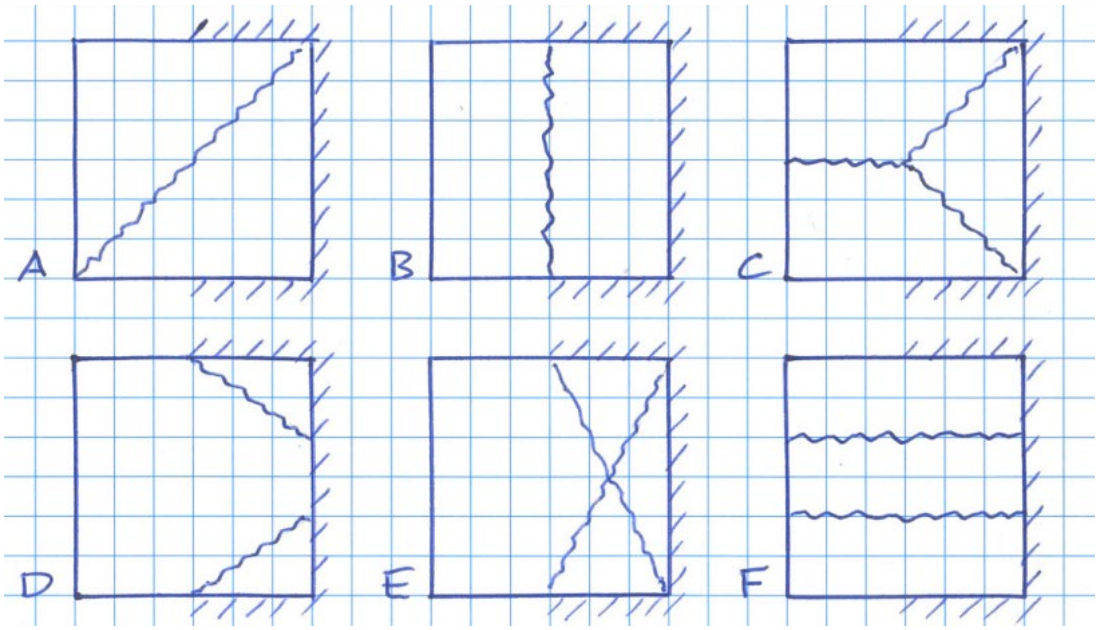


Figure 5. Yield line patterns

17 Consider the mechanism of Figure 6. Determine an upper-bound for *p* expressed in m_p and *a* (1.5 point).

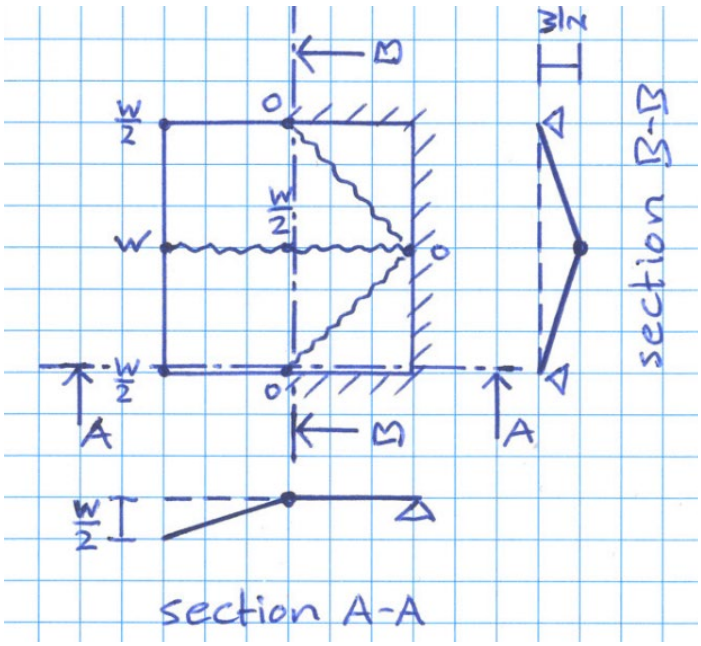


Figure 6. Plastic failure mechanism

18 Determine the largest lower-bound for *p* using torsion free beams (m_{xy} = 0). You need only write down the equations and not solve the equations (1.5 point).

Answers

1 B 100 % of the answers were correct
2 D 43 %
3 A 29 % ..... D 71 % is wrong because we do not do experiments on soap film anymore.
4 A 100 %
5 B 71 %
6 C 29 % .....

```
> Mt:=20*kN*m: h:=300.*mm: b:=420*mm: nu:=0.3:
> m:=1000*mm: kN:=1000*N: assume(mm>0): b/h;
```

$$1.400000000$$

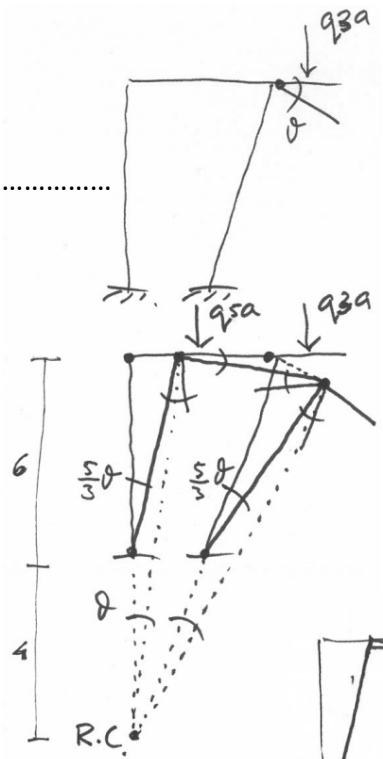
```
> G:=E/(2*(1+nu)):
> Cw:=0.838/1000*b^3*h^3:
> It:=0.187*b*h^3:
> lc:=sqrt(E*Cw/(G*It)):
> B:=lc*Mt:
> sigma:=100*B/(0.987*b^2*h^2);
```

$$\sigma := \frac{5.786406278\ N}{mm^2}$$

```
> tau:=Mt/(0.230*b*h^2);
```

$$\tau := \frac{2.300437083\ N}{mm^2}$$

7 C 57 %
8 B 57 %
9 C 29 % ..... We can see this in the plate book of Blaauwendraad on page 109.
10 B 86 %
11 D 100 %
12 B 71 %
13 C 43 %
14 63 % .....




```
> E:= Mp*t:
> A:= q*3*a *t*3/2*a:
> q:=solve(E=A,q); evalf(q);
```

$$q := \frac{2}{9}\frac{Mp}{a^2}$$

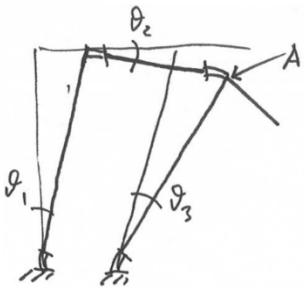
$$\frac{0.2222222222\ Mp}{a^2}$$

```
> restart:
> E:= Mp*5/3*t + Mp*(5/3*t-t) + Mp*(5/3*t-t) + Mp*5/3*t:
> A:= q*5*a *t*5/2*a + q*3*a *5/3*t*9/2*a:
> q:=solve(E=A,q); evalf(q);
```

$$q := \frac{2}{15}\frac{Mp}{a^2}$$

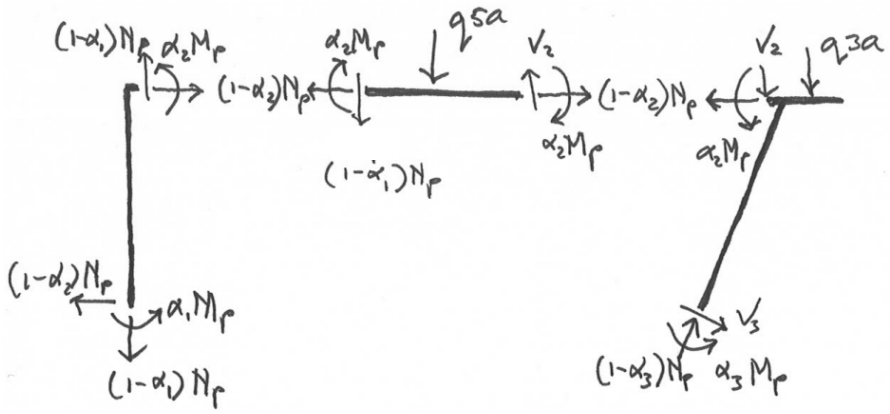
$$\frac{0.1333333333\ Mp}{a^2}$$

15 57 % .....




```
> b:=5:
> eq1:= t1*6*a + (t1-t2)*a/b + (t3-t2)*a/b = t3*6*a - t3*a/b*1/sqrt(5.):
> eq2:= -t1*a/b + t2*5*a = t3*3*a + t3*a/b*2/sqrt(5):
> E:= Mp*t1 + Mp*(t1-t2) + Mp*(t3-t2) + Mp*t3:
> A:= q*5*a *(-t1*a/b + t2*5/2*a) + q*3*a *(t3*9/2*a + t3*a/b*2/sqrt(5)):
> solve({eq1,eq2,E=A},{t1,t3,q});
```

$$\left\{q = \frac{0.1200102106\ Mp}{a^2},\ t1 = 1.430341606\ t2,\ t3 = 1.482888192\ t2\right\}$$




```
> b:=5:
> Np:=b*Mp/a:
> eq1:= a1*Mp = (1-a2)*Np*6*a - a2*Mp:
> eq2:= V2 = q*5*a + (1-a1)*Np:
> eq3:= a2*Mp = V2*5*a - q*5*a*5/2*a - a2*Mp:
> eq4:= (1-a2)*Np = V3*2/sqrt(5.) + (1-a3)*Np*1/sqrt(5):
> eq5:= q*3*a + V2 = (1-a3)*Np*2/sqrt(5) - V3*1/sqrt(5):
> eq6:= a3*Mp = q*3*a*9/2*a + V2*3*a - (1-a2)*Np*6*a - a2*Mp:
> solve({eq1,eq2,eq3,eq4,eq5,eq6},{q,V2,V3,a1,a2,a3});
```

$$\left\{V2 = \frac{0.6744109716\ Mp}{a},\ V3 = -\frac{0.1762369173\ Mp}{a},\ a1 = 0.9851280163,\ a2 = 0.9359636124,\ a3 = 0.7863155173,\ q = \frac{0.1200102106\ Mp}{a^2}\right\}$$

16 B C E ..... 3 right = 0.0 ..... 14 %
4 right = 0.3 ..... 0 %
5 right = 0.7 ..... 29 %
6 right = 1.0 ..... 57 %

17 75 % .....

```
> E:=mp*6*a*(w/(6*a)+w/(6*a)) + 2*( mp*3*a*w/(6*a) + 3*mp*3*a*w/(6*a) );
```

$$E := 6\ mp\ w$$

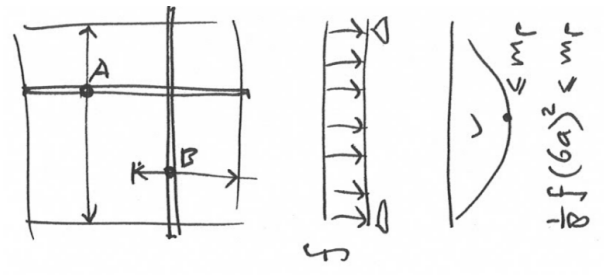
```
> A:=2*( p*3*a*3*a*w/2 + p*3*a*3*a/2*w/6 );
```

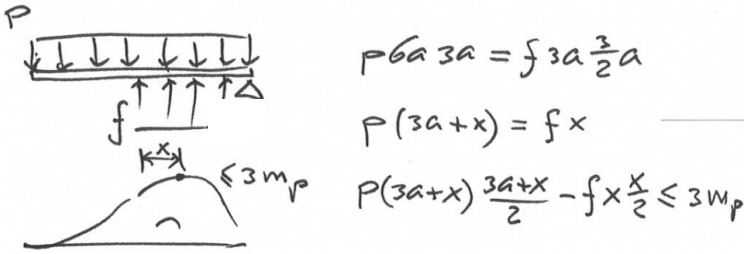
$$A := \frac{21}{2}\ p\ a^2\ w$$

```
> p:=solve(E=A,p);
```

$$p := \frac{4}{7}\ \frac{mp}{a^2}$$

18 73 % .....





```
> eq1:= 1/8*f*(6*a)^2 = mp:
> eq2:=p*6*a*3*a=f*3*a*3/2*a:
> eq3:=p*(3*a+x)=f*x:
> opl:=solve({eq1,eq2,eq3},{p,f,x}); assign(opl):
```

$$opl := \left\{ f = \frac{2}{9}\,\frac{mp}{a^2},\ p = \frac{1}{18}\,\frac{mp}{a^2},\ x = a \right\}$$

```
> eq4:=p*(3*a+x)*(3*a+x)/2-f*x*x/2<3*mp;
```

$$eq4 := \frac{1}{3}\,mp < 3\,mp$$

