Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CIEM5210-3 Advanced mechanics of structural elements
Monday 22 June 2026, 13:30 – 16:30 hours

A lintel above a window is made of galvanised steel plate (red in the figure below). The lintel is 3 mm thick and spans 2.6 m. The lintel is supported by masonry on both sides of the window. The lintel ends stick 100 mm into the bed joints. A bricklayer lays 8 layers of bricks onto the lintel. The next day, he will continue with more layers.

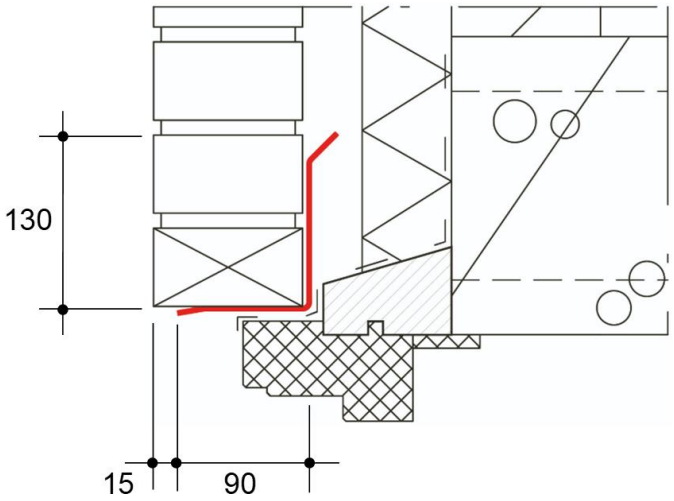


Figure 1. Lintel cross-section

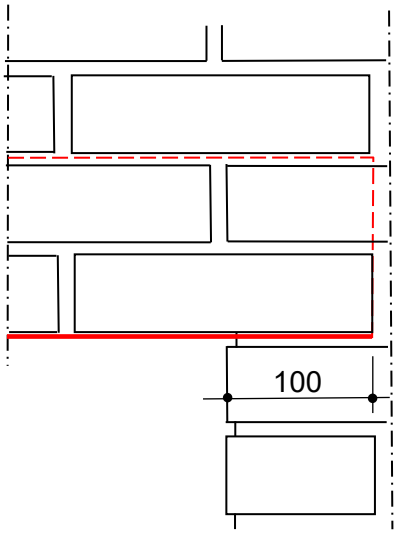
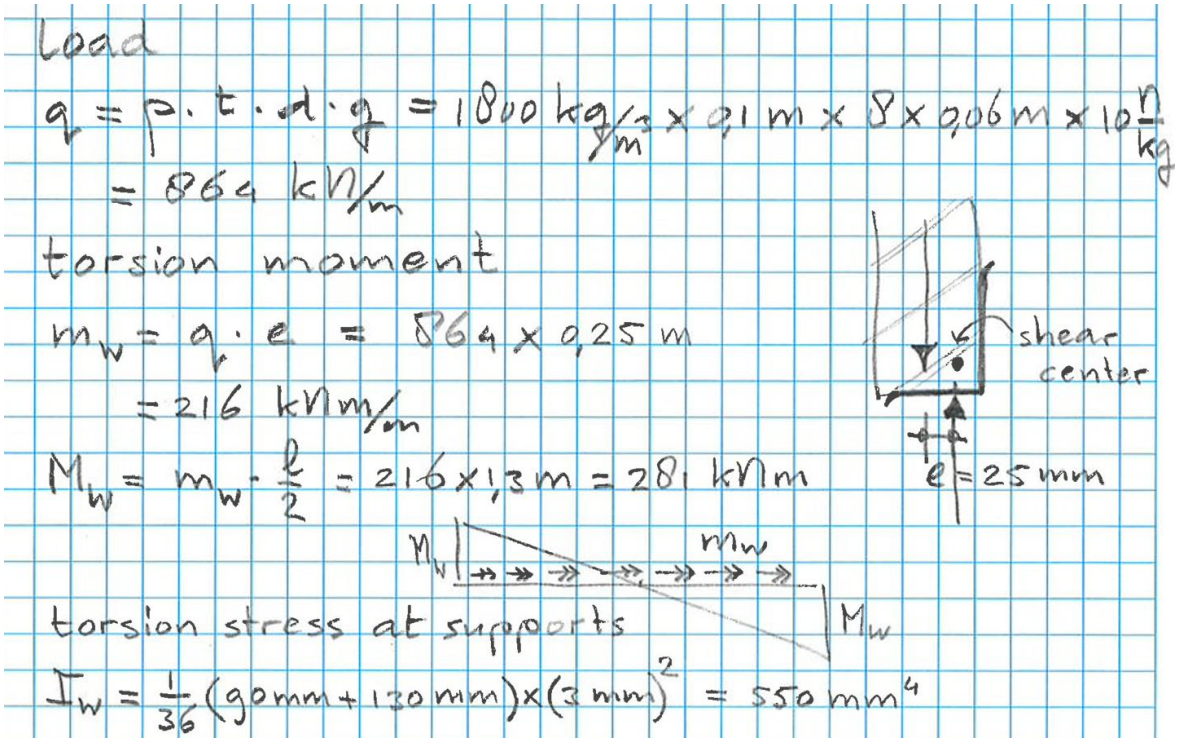
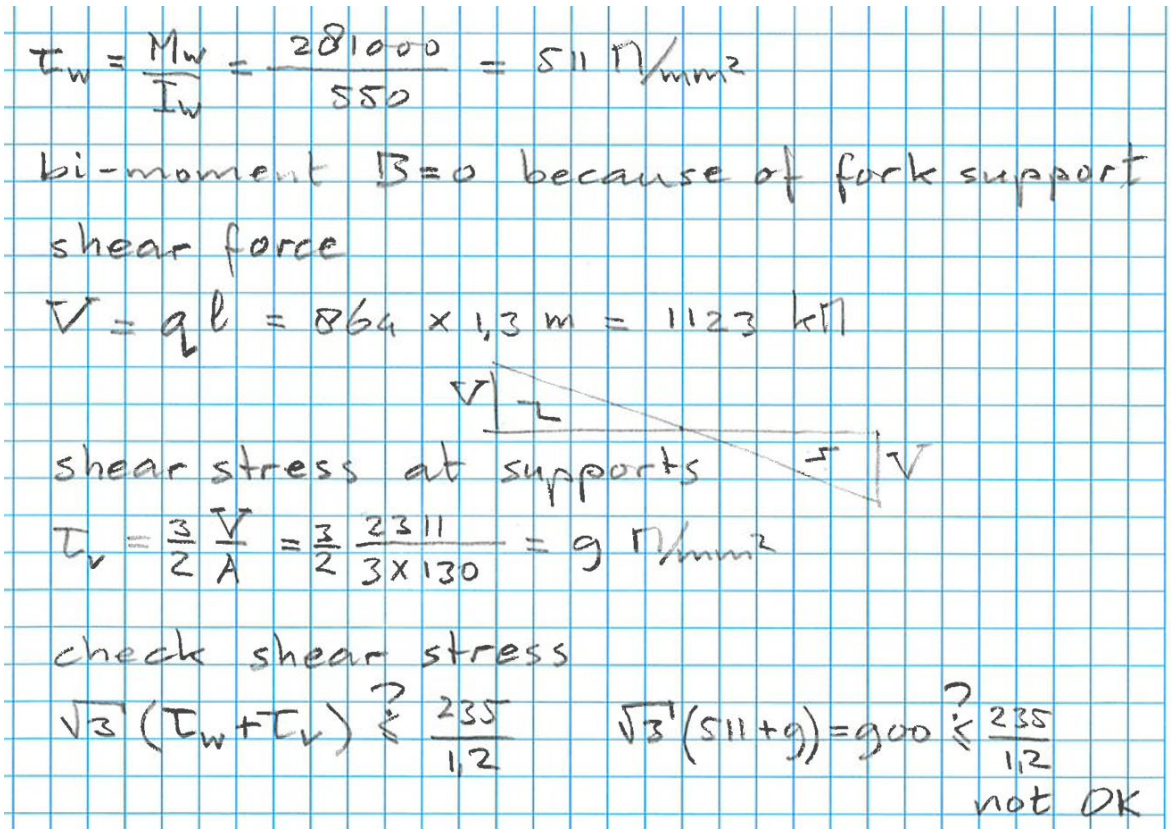


Figure 2. Lintel right support

1 Remove the mistakes from the calculation below. (1.5 points)





A reinforced concrete plate is loaded perpendicularly (Fig. 3). The plate length is 16 m and the width is 8 m. The plate thickness is 250 mm. A distributed load of 9 kN/m^2 is applied. This distributed load includes self-weight. A linear elastic finite element model is made. E = 34100 N/mm^2, v = 0.2. Applied are four node plate elements (Mindlin-Reissner theory). The element size is 500 mm. Results are shown in Figures 4 to 7.

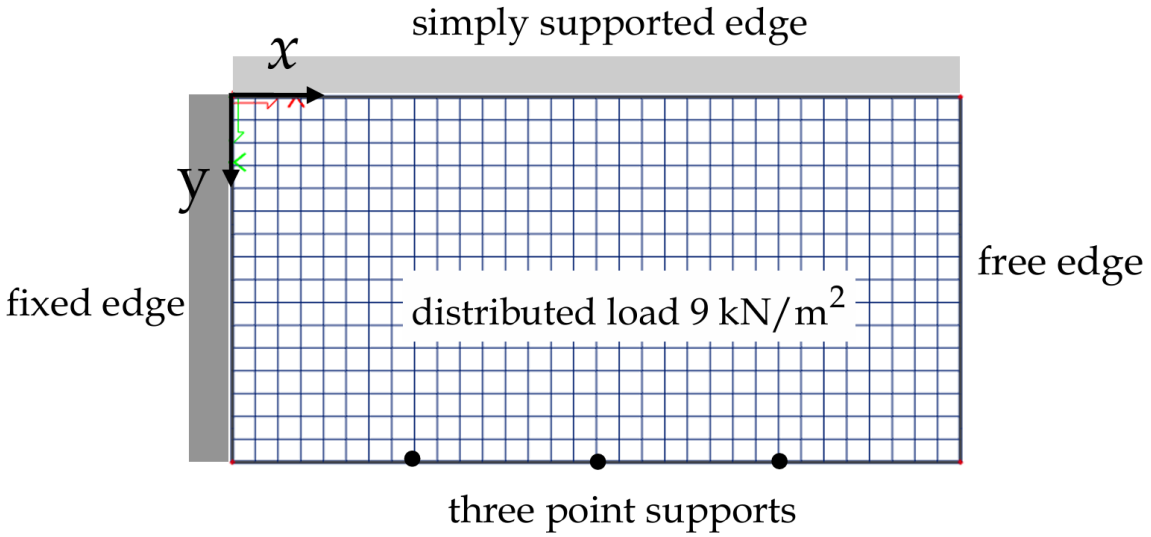


Figure 3. Reinforced concrete plate, top view.

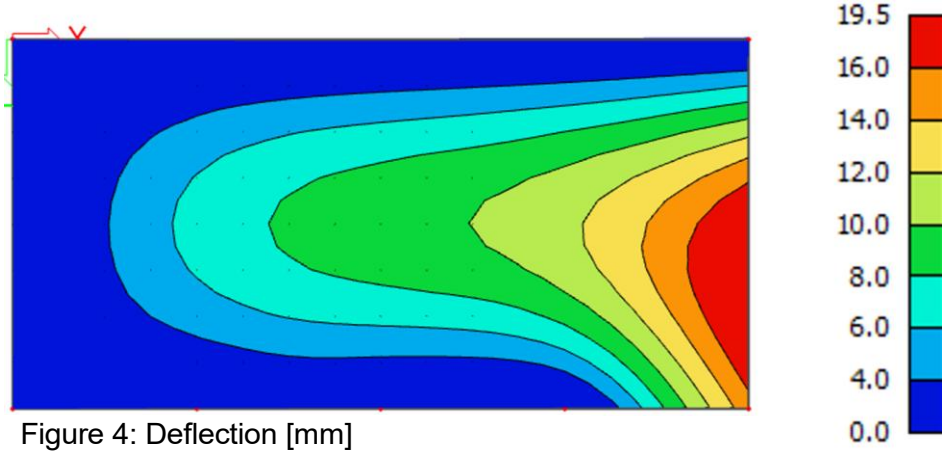


Figure 4: Deflection [mm]

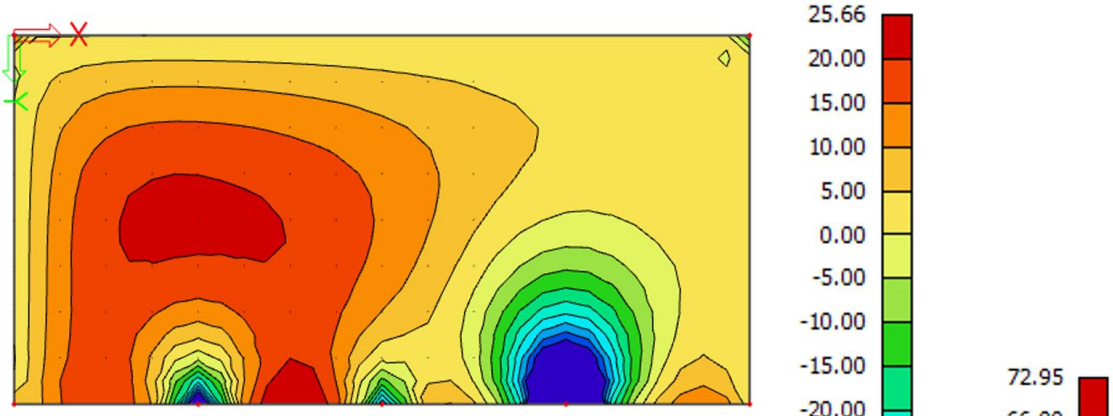


Figure 5: Bending moment mxx [kNm/m]

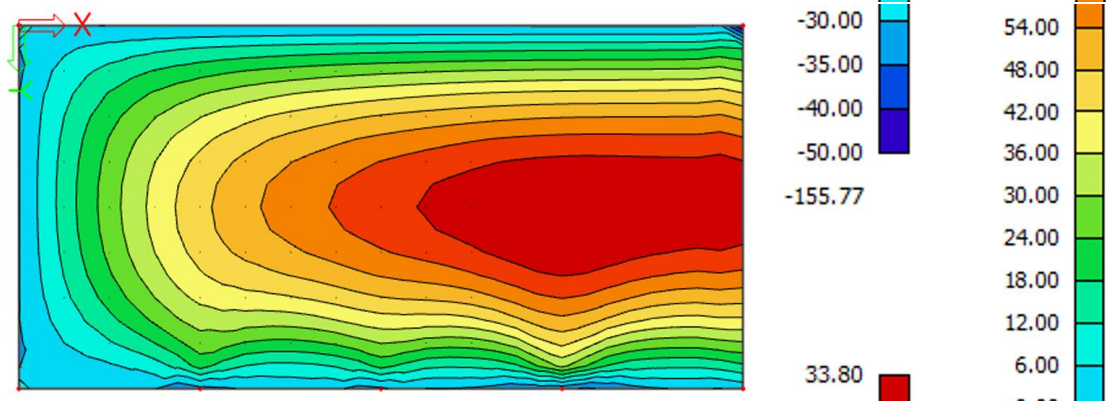


Figure 6: Bending moment myy [kNm/m]

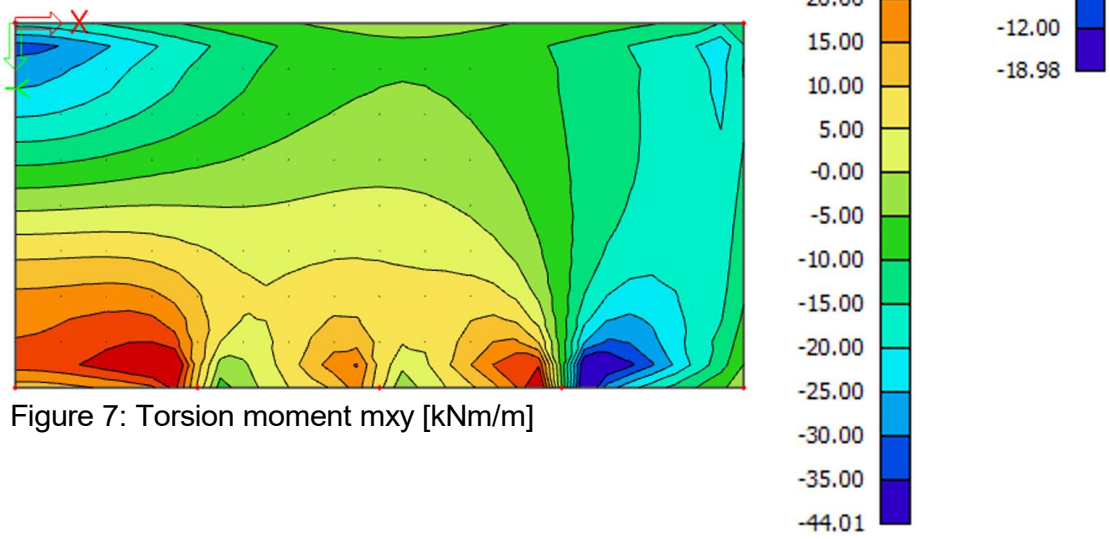


Figure 7: Torsion moment mxy [kNm/m]

2 Remove the mistakes from the text below. (1.5 points)

- This plate is thin because the thickness of 250 mm is less than span / 10 = 8000 mm / 10 = 800 mm.

- The Mindlin-Reissner theory is appropriate for this plate.

- The element size should be about two times the plate thickness. Each element is 500 mm, which is two times the plate thickness, which is okay.

- The deflection shows no mistake in the modelling of the supports.

- The moment mxx shows no mistake in the modelling of the fixed support.

- There are 6 places where singularities can occur in the finite element model.

- The largest moment in the plate is 155.77 kNm/m.

- There is a shear force of almost 20 kN in the simply supported edge.

A frame consists of three members (Fig. 8). The members have a strength M_p. The members are rigidly connected. The supports are fixed. The structure is loaded by line load q over a width 3a. The relation of Figure 9 exists between the plastic moment and the plastic normal force.

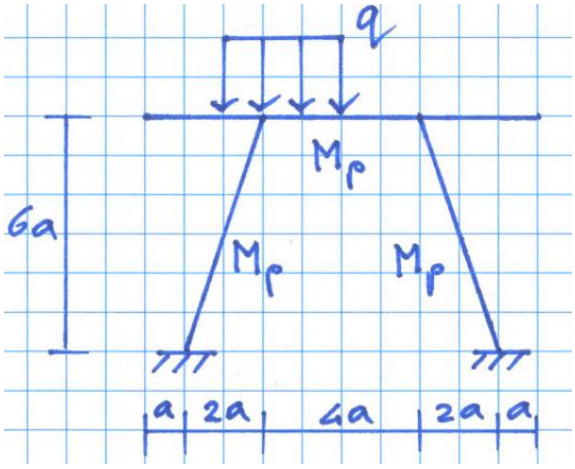


Figure 8. Frame structure

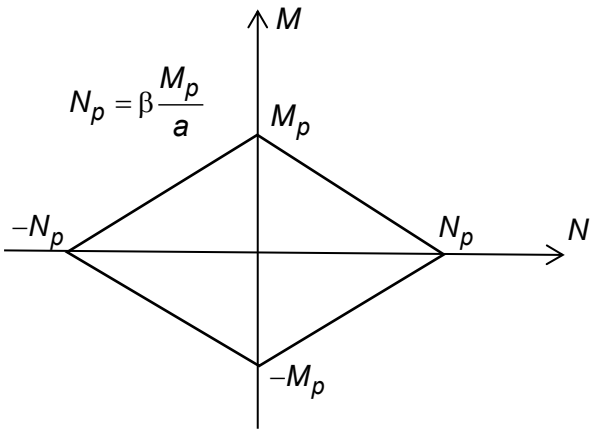


Figure 9. Yield contour

The influence of shear on the yield contour is neglected. Buckling and second order effects are not considered.

3 Assume $\beta \to \infty$. Determine the collapse load q for all possible mechanisms. Write the collapse loads as functions of M_p and a. What is the decisive collapse load? (1.5 point)

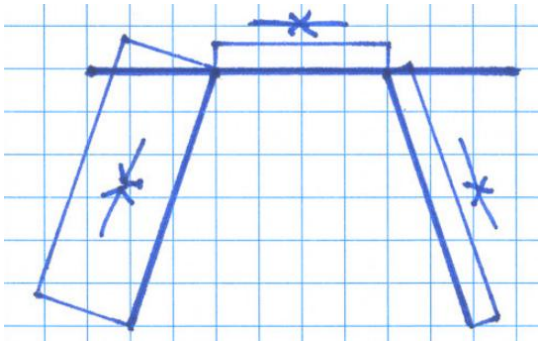


Figure 10. Normal forces of the decisive mechanism

4 Assume β = 6. Choose one of the following problems (You need not do both).
– Determine the largest lower-bound for q.
– Determine the smallest upper-bound for q.
You only need to write down the equations and not solve the equations (1.5 points).

A reinforced concrete plate has fixed edges and free edges (Fig. 11). It carries an evenly distributed load p [kN/m^2]. There is no other load on the plate. The plate is homogeneous and orthotropic.

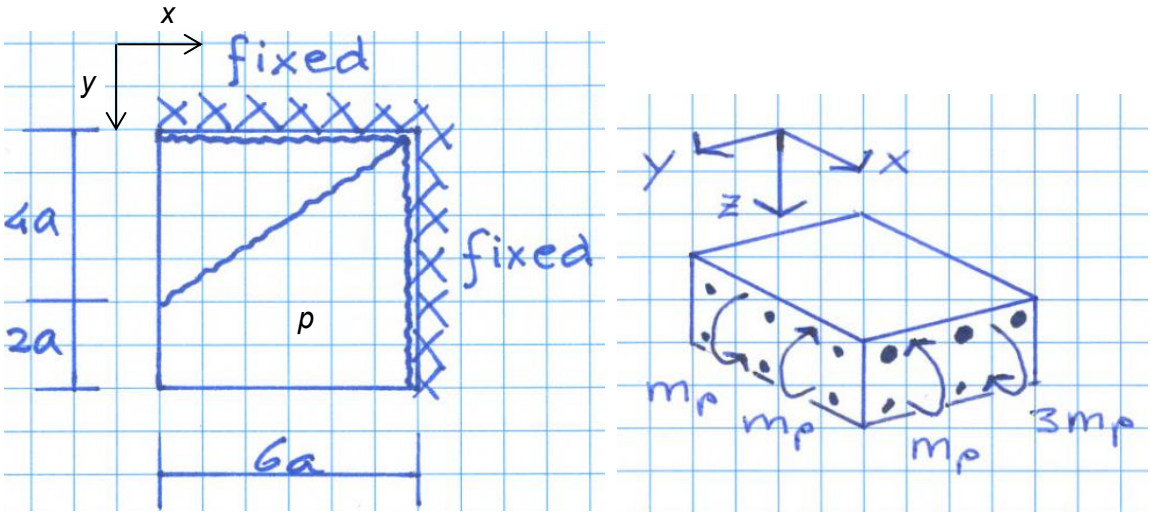


Figure 11. Plate dimensions and reinforcement

5 Consider the yield line patterns of Figure 12. Which of these patterns give kinematically possible mechanisms? (1 point)

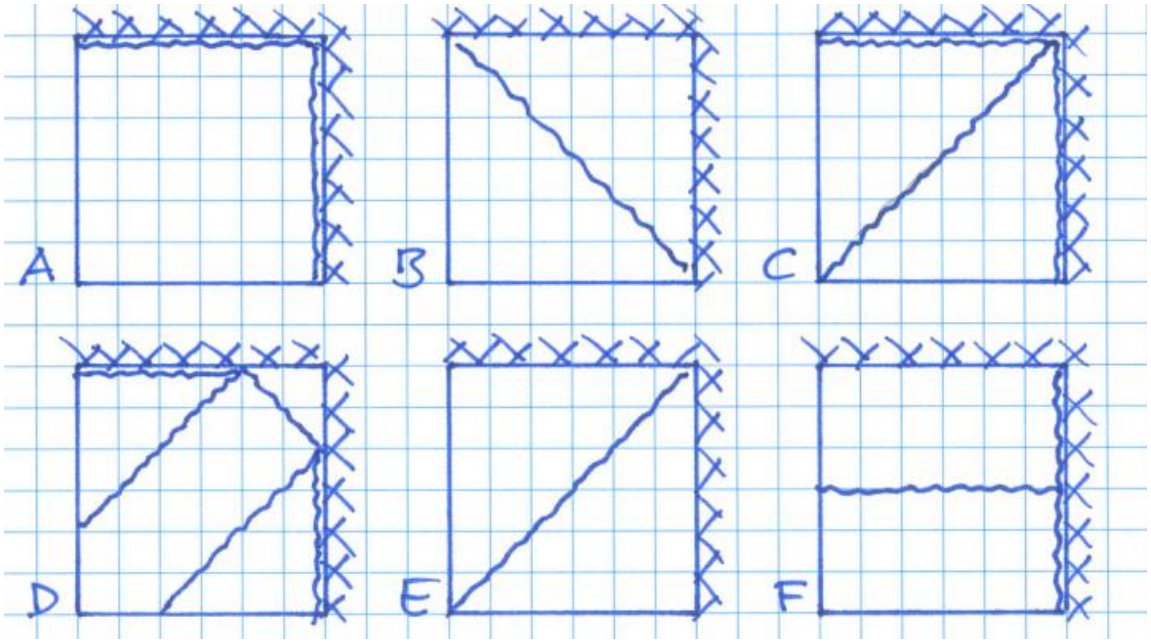
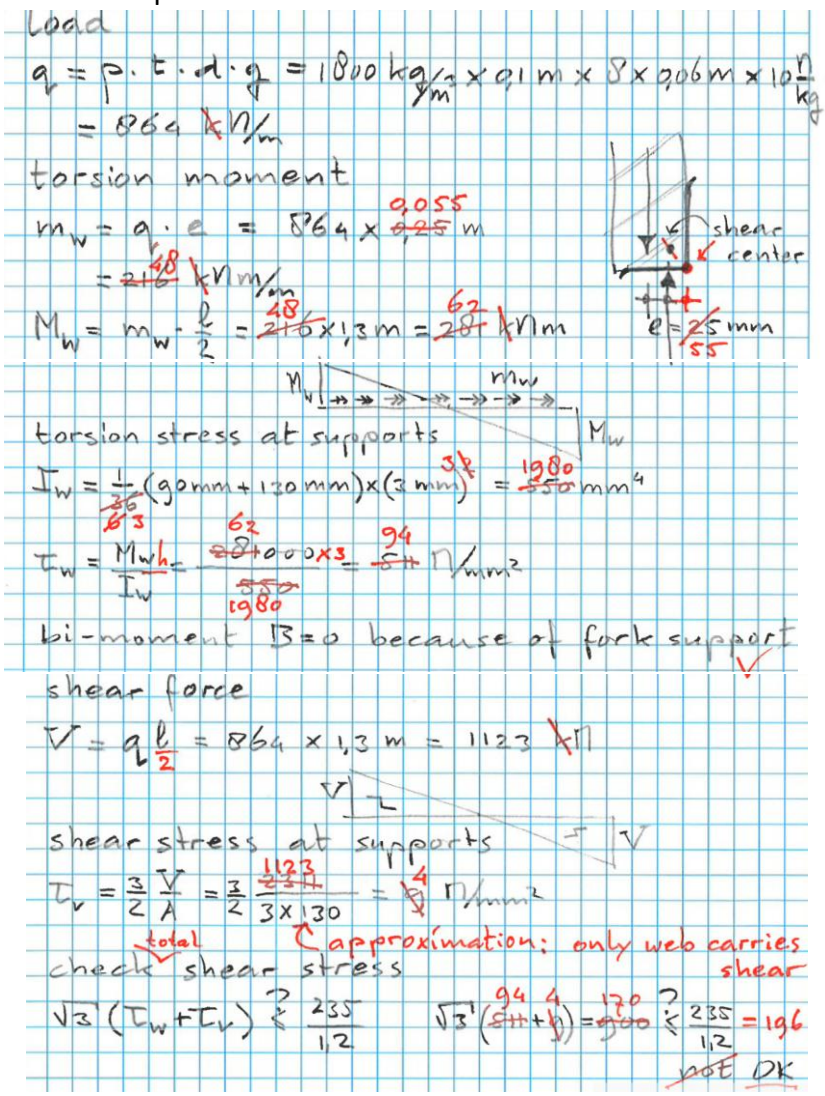


Figure 12. Yield line patterns

6 Consider the mechanism of Figure 11. Determine an upper-bound for p expressed in m_p and a (1.5 point).

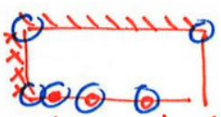
7 Determine the largest lower-bound for p using torsion free beams ($m_{xy} = 0$). (1.5 point).

Answer to question 1



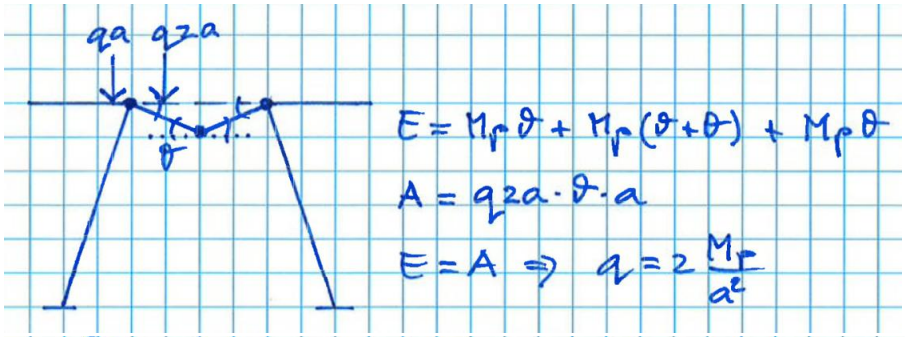
Answer to question 2

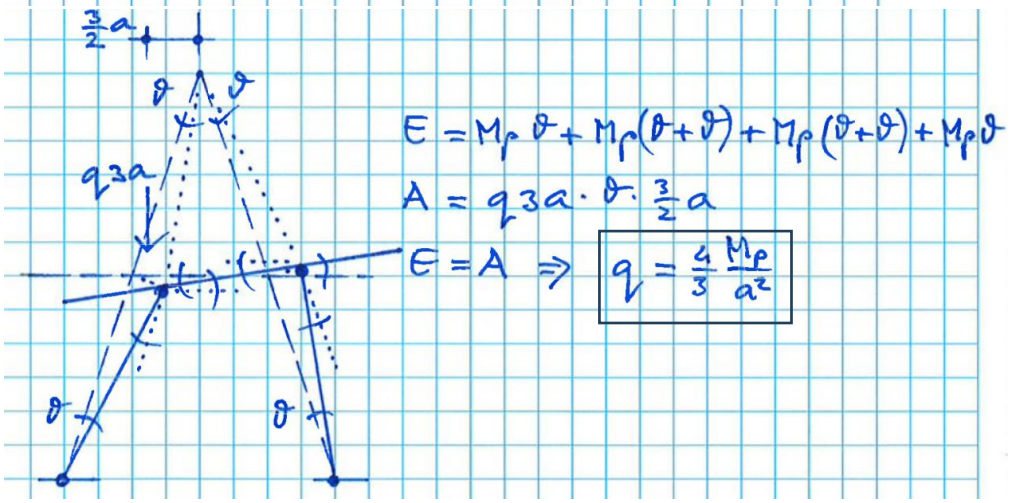
- This plate is thin because the thickness of 250 mm is less than span / 10 = 8000 mm / 10 = 800 mm. ✓
- The Mindlin-Reissner theory is not appropriate for this plate.... but perhaps Kirchhoff is not available
- The element size should be about ~~two times~~ 1/10 of the plate thickness. Each element is 500 mm, which is two times the plate thickness, which is okay. not
- The deflection shows no mistake in the modelling of the supports. ✓
- The moment mxx shows a ~~no~~ mistake in the modelling of the fixed support. ← = 0 kNm/m →
- There are 6 places where singularities can occur in the finite element model. ✓



- The largest moment in the plate ~~is 155.77 kNm/m.~~ depends on the element size.
- There is a concentrated shear force of almost 20 kN in the simply supported edge. ✓

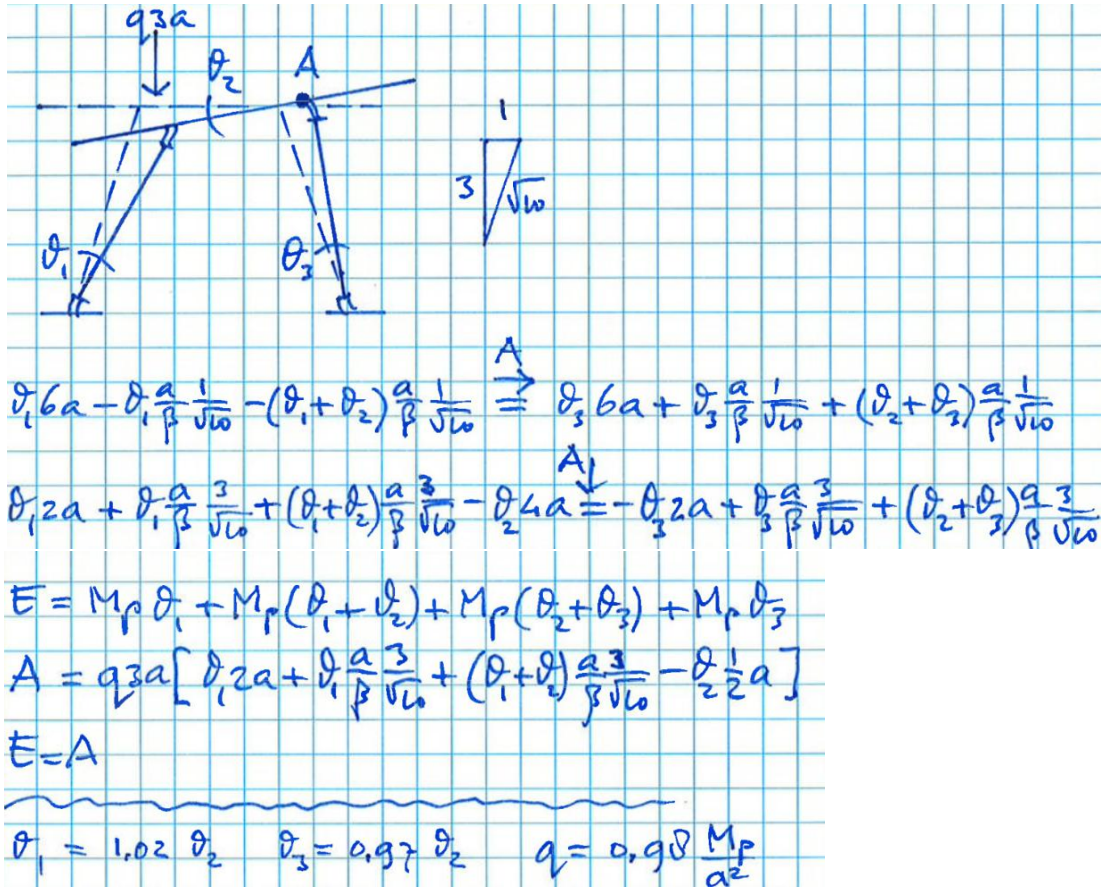
Answer to question 3



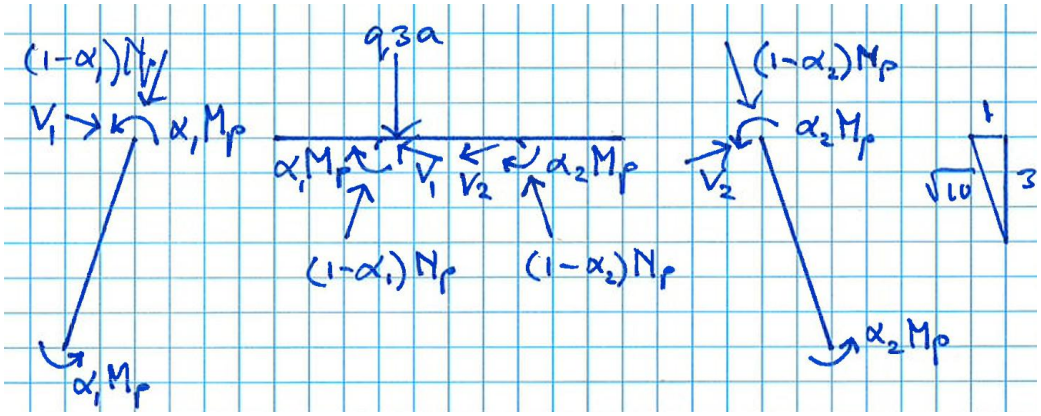


Answer to question 4

Upperbound



Lowerbound



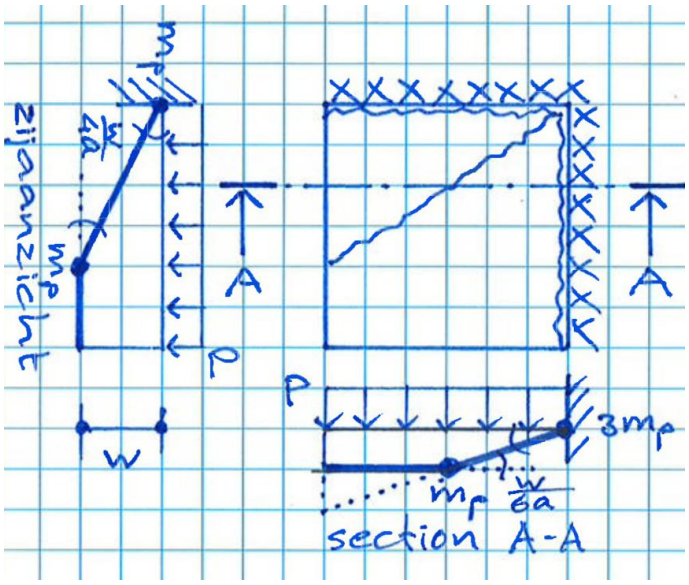

```
b:=6: e:=sqrt(10.):
Np:=b*Mp/a:
eq1:= q*3*a = (1-a1)*Np*3/e + V1*1/e + (1-a2)*Np*3/e - V2*1/e:
eq2:=     0 = (1-a1)*Np*1/e - V1*3/e - (1-a2)*Np*1/e - V2*3/e:
eq3:= q*3*a*7/2*a = (1-a1)*Np*3/e*3*a +V1*1/e*3*a +(1-a2)*Np*3/e*7*a -V2*1/e*7*a -a1*Mp -a2*Mp:
eq4:= V1*2*e*a = 2*a1*Mp:
eq5:= V2*2*e*a = 2*a2*Mp:
solve({eq1,eq2,eq3,eq4,eq5},{q,V1,V2,a1,a2});
```

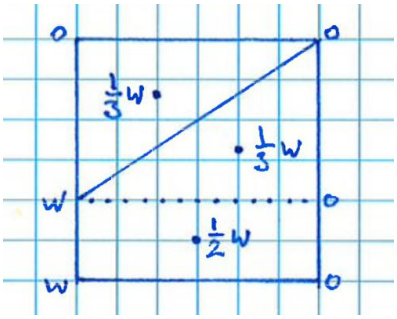
$$\left\{V1 = \frac{0.1966025523\ Mp}{a},\ V2 = \frac{0.2704500534\ Mp}{a},\ a1 = 0.6217118592,\ a2 = 0.8552381620,\ q = \frac{0.9846333477\ Mp}{a^2}\right\}$$

Answer to question 5

B, C, D …… 3 correct = 0.0
4 correct = 0.3
5 correct = 0.7
6 correct = 1.0

Answer to question 6





```
E:=mp*6*a*w/(4*a) + 3*mp*6*a*w/(6*a) + mp*4*a*w/(6*a) + mp*6*a*w/(4*a);
```

$$E := \frac{20}{3}\, mp\, w$$

```
A:=p*6*a*4*a/2*w/3 + p*6*a*4*a/2*w/3 + p*6*a*2*a*w/2;
```

$$A := 14\, p\, a^2\, w$$

```
p:=solve(E=A,p);
```

$$p := \frac{10}{21}\, \frac{mp}{a^2}$$

Answer to question 7

