Processing experimental data P.C.J. Hoogenboom, 21 August 2006

Often we do tests on structural components. We also make a model for predicting the test results. The quotient of a test result and a model result is the model factor. It has a mean value and a standard deviation. We conclude that the model is accurate if the model factor has a mean of approximately 1 and the standard deviation is small. For example, this can show that a set of code rules - which are derived from the model - is accurate. However, this is not a scientific proof. One engineer might feel that the accuracy is sufficient while another might think that it is not.

What we need to do is calculate a number which can be compared to another number that is a requirement. A clear requirement for structural components is the allowed failure probability. It depends on the design life, the importance of the component and the importance of the structure. A failure probability less than 0.00016 ($\beta = 3.6$) is commonly accepted by codes of practice for any component. Sometimes larger probabilities are accepted for example if the component is used in a temporary structure or failure is due to wind loading.

Therefore, we use the model factor in a probabilistic computation to determine the failure probability of the structural component. Below is an example of such a computation for the ultimate limit state of high strength steel pin joints (Maple).

```
> restart:
> with(stats):
> # Monte Carlo Analysis of the failure probability of a steel pin joint
> # Maple 9.5, P.C.J. Hoogenboom, 21 August 2006
> n:=100000:   # number of simulations
> L:=30.:      # 1/2*a1+c1 [mm]
> d:=12.:      # pin diameter [mm]
> designfy:=690/1.2:
> eq1:=meanfy=designfy*1.2+1.645*stanDevfy:
> eq2:=stanDevfy=meanfy*0.08:
> assign(solve({eq1,eq2},{meanfy,stanDevfy})):
> nrTests:=32:                                        # from experiments
> meanModelFac:=1.23:                                 # "
> stanDevModelFac:=0.18:                              # "
> MRd:=1.5*Pi/32*d^3*designfy:                        # design calcuation
> FvRd:=0.6*1/4*Pi*d^2*designfy:                      # "
> FvEd:=evalf((MRd^2/(FvRd^2*L^2+MRd^2))^(1/2)*FvRd): # "
> designLoad:=2*FvEd:                                 # "
> eq1:=meanLoad=designLoad/1.35-1.645*stanDevLoad:
> eq2:=stanDevLoad=meanLoad*0.10:        # Vrdr: Wind 0.25, Verkeer 0.10
> assign(solve({eq1,eq2},{meanLoad,stanDevLoad})):
>
> nF:=0:
> for i from 1 to n do
    fy:=meanfy+stats[random,normald](1)*stanDevfy:
    t1:=0: t2:=0:
    for j from 1 to nrTests do
      modelFac:=meanModelFac+stats[random,normald](1)*stanDevModelFac:
      t1:=t1+modelFac:
      t2:=t2+modelFac*modelFac:
    end do:
    meanModelF:=t1/nrTests:
    stanDevModelF:=sqrt( (t2-nrTests*meanModelF^2) / (nrTests-1) ):
    modelFac:=meanModelF+stats[random,normald](1)*stanDevModelF:
    MRd:=1.5*Pi/32*d^3*fy:                            # model evaluation
    FvRd:=0.6*1/4*Pi*d^2*fy:                          # "
    FvEd:=evalf((MRd^2/(FvRd^2*L^2+MRd^2))^(1/2)*FvRd); # "
    R:=2*FvEd*modelFac:
    Load:=meanLoad+stats[random,normald](1)*stanDevLoad:
    S:=Load:
    if (R<S) then nF:=nF+1: end if:
  end do:
> P:=evalf(nF/n);
```

$$P := 0.00009000000000$$

In this Monte Carlo analysis one hundred thousand connections are virtually designed, constructed and loaded to see how many of them failed. The failure probability is calculated simply as the number of failures over the number of virtual connections.

For simplicity we assumed normal distributions for the material strengths, model factor and loading. The mean and standard deviation of the steel yield strength are based on material safety factor of 1.2 according to the EuroCode, a 5% characteristic value of 690 N/mm² and a variation coefficient of 0.08. The mean and standard deviation of the loading are based on a load factor of 1.35 according to the EuroCode for live load, a 5% characteristic value computed with the code regulations for pin connections and a variation coefficient of 0.10 which represents traffic loading. The computations were repeated for a variation coefficient of 0.25 which represents wind loading.

Before the experiment we determined the accurate yield strength of the used material to eliminate the material uncertainty from the tests. The mean and standard deviation of the model factor were derived from the tests. However, since these are based on just 32 experiments they are not the real mean and real standard deviation of the population – which are all high strength pin connections that will be ever made. This extra uncertainty is taken into account by assuming that the experimental mean and standard deviation are the real values (after all they are the best estimates we have) and drawing 32 numbers from the distribution from which a new mean and standard deviation are computed. This procedure can be compared to a Student distribution for the model factor.

Shear force: $½F_{Sd}$

Bending moment: $\frac{F_{Sd}}{4}(a+2c)$

Interaction condition $\left(\frac{M_{Ed}}{M_{Rd}}\right)^2+\left(\frac{F_{v,Ed}}{F_{v,Rd}}\right)^2\le 1$

From this the strength of the connection can be derived as

$$F_{v,Rd,pred}=\frac{2\cdot M_{Rd}\cdot F_{v,Rd}}{\left[\left(\frac{a_2}{2}+\frac{c_1+c_2}{2}\right)^2\cdot F_{v,Rd}{}^2+M_{Rd}{}^2\right]^{0,5}}.$$

The values of the dimensions *L* and *d* in the example do not influence the results. Therefore, the failure probability is valid for any pin joint. We could have eliminated *L* and *d* from program, however, this would make it more difficult to read. The computation for 100000 simulations takes about one hour on a modern PC.

Conclusion

The computed failure probability of 0.00009 is less than the required failure probability of 0.00016. Therefore, the connections are sufficiently safe and the code regulations with currently used material and load factors are suitable for high strength steel connections too.

It needs to be mentioned that connections dominantly loaded by dead load have not been checked as yet. Wind load has been checked and gives a smaller failure probability, however, the assumption of 5% characteristic value is not entirely correct for wind loading. It is expected that including more realistic distributions than the normal distribution will change the failure probabilities slightly.