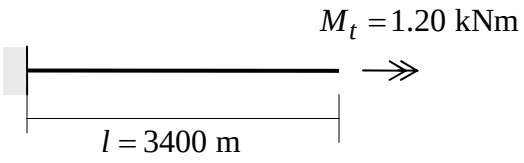
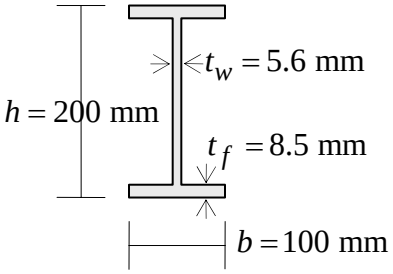
Torsion stresses in a I-section P.C.J. Hoogenboom, Delft, 29 July 2008



Cantilever loaded in torsion



Cross-section, IPE 200

Young's modulus E = 2.1e5 N/mm²

Poisson's ratio $\nu = 0.35$

Shear modulus $G = \frac{E}{2(1+\nu)} = 77777$ N/mm²

Torsion constant $J = \frac{1}{3}\left((h-t_f)t_w^3 + 2bt_f^3\right) = 52152$ mm⁴

Warping constant $C_w = \frac{1}{24}t_f\,(h-t_f)^2 b^3 = 1299\text{e}7$ mm⁶

Characteristic length $l_c = \sqrt{\frac{E\,C_w}{G\,J}} = 820.0$ mm

In general, a torsion moment causes shear stresses and it can cause normal stresses. In cross-sections that can warp freely only shear stresses occur. This occurs in the free end of the cantilever. The largest shear stress is calculated with

$$\tau_{\max} = \frac{M_t\,t_f}{J} = 196 \text{ N/mm}^2$$

The Von Mises stress is $\sqrt{3\tau_{\max}^2} = 339$ N/mm².

In cross-sections that cannot warp mostly normal stresses occur. This occurs at the fixed end of the cantilever. The largest normal stress in this section is calculated with

$$\sigma_{\max} = \frac{B}{\frac{1}{6}(h-t_f)\,t_f\,b^2} = 363 \text{ N/mm}^2$$

where B is the bi-moment. This stress occurs at the left and right of the flanges both in compression and tension. The bi-moment can be approximated accurately with

$$B = l_c M_t = 9840\text{e}5 \text{ Nmm}^2$$

provided that $l_c < \frac{1}{3}l$, which is fulfilled.

The rotation at the free end is

$$\varphi = \frac{M_t}{GJ}\left(l - l_c \tanh\frac{l}{l_c} \right) = 0.763 \text{ rad} .$$

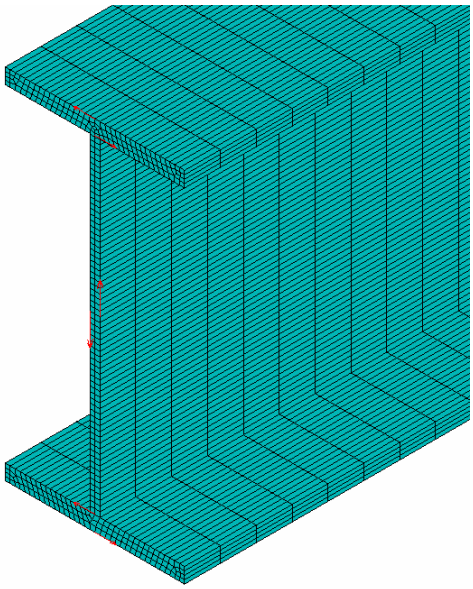
It needs to be mentioned that from a plastic point of view the stresses due to the bi-moment can be neglected because just the shear stresses are an equilibrium system that can carry the load.

References

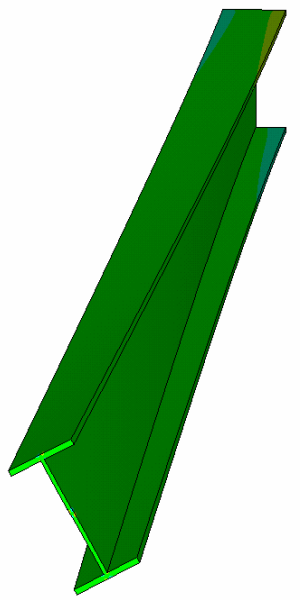
Hoogenboom PCJ, Borgart A, "Method for Including Restrained Warping in Traditional Frame Analyses", *HERON*, Vol. 50 (2005) No. 1, pp. 55-68.
Young WC, “Roark’s formulas for stress & strain”, 6th edition, McGraw-Hill, New York, 1989.

Check

A finite element analysis was made with ANSYS 11. Applied are 20 node brick shaped elements (solid95). At the fixed end all degrees of freedom have zero displacement imposed. At the free end the torsion loading is applied by 6 forces. The model consists of 959820 degrees of freedom. A linear elastic analysis was performed (31 minutes on a Pentium 4 PC).



Mesh and torsion loading (red arrows) at the free end of the cantilever

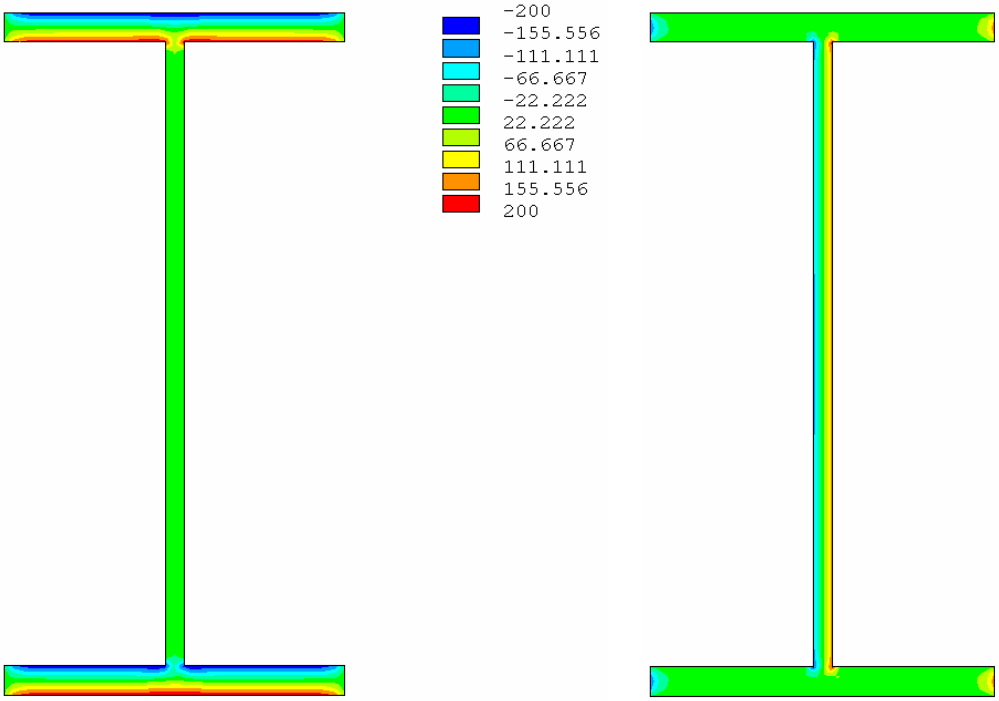


Torsion deformation

The horizontal displacement of nodes in the free ends of the flanges are –87.5 mm and 87.5 mm. Therefore, the rotation is

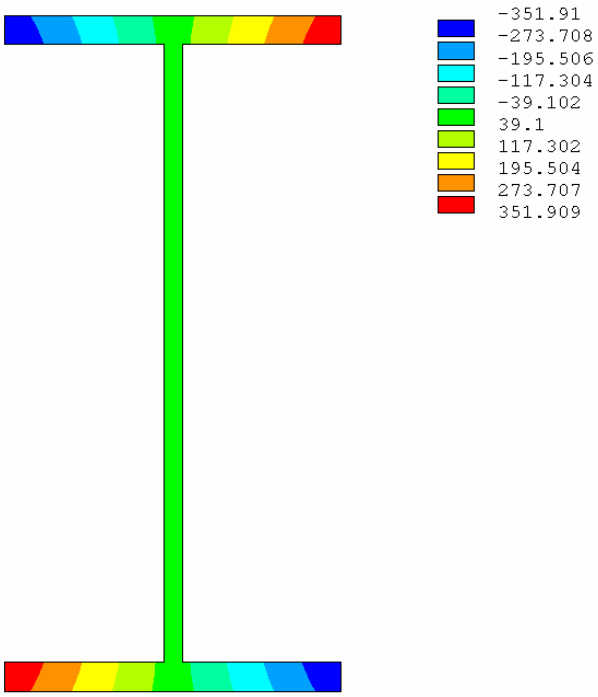
$$\varphi = \frac{87.3 + 87.3}{200} = 0.873 \text{ rad} .$$

The hand calculation result is 13% smaller. This might be caused by shear deformation of the flanges.



Horizontal and vertical shear stresses in a cross-section 300 mm from the free end

The stresses in the re-entrant corners of the cross-section are ignored. (In theory the sharp corners have infinitely large stresses.) The largest shear stress in the free end cross-section is 200 N/mm². The hand calculation result is 2% smaller.



Normal stresses in a cross-section 50 mm from the fixed end

The largest normal stress in the fixed end cross-section is 352 N/mm². The hand calculation result is 3% larger. The differences between the results of the hand calculation and the finite element analysis are acceptable for most purposes.