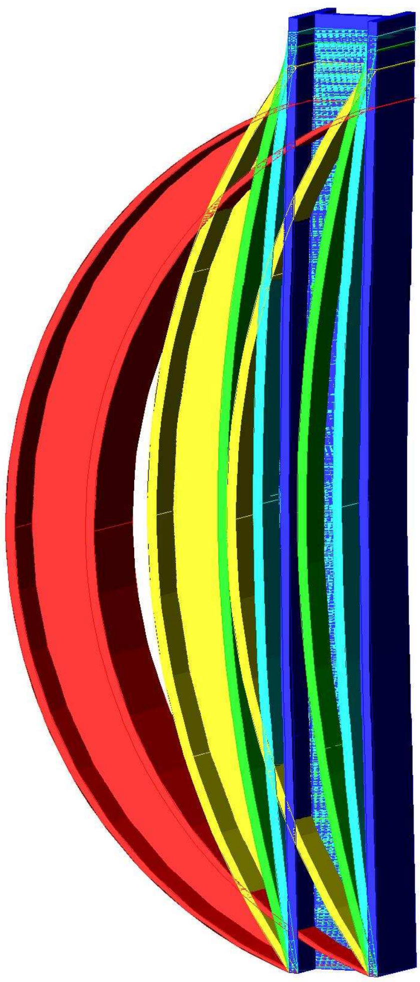
Stability design for frame type structures

Delft University of Technology
Faculty of Civil Engineering and Geoscience
Section of Structural Mechanics



Master thesis report
Appendices
Ing. R. P. Veerman

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Appendices

Master thesis report

by

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Preface

This document is the appendices report of the master thesis: Stability design for frame type structures. This report is carried out at the section structural mechanics of Delft University of Technology. This report contains the background, the analyses and the calculations. This report is the appendix of the main report. In the main report contains general information and conclusions of this thesis.

Delft, May 2009
Ing. R. P. Veerman

Index

General information appendices

The appendices in this report are the background at the ‘stability design for frame type structures’ report. The report is divided in four parts.

The following appendices are related to the different parts:

- Appendices A till F are about the stability of a single column and is related to Chapter two.
- Appendices G till L are about to the bearing capacity of an unbraced portal frame and are related to Chapter three.
- Appendices M till P are about the bearing capacity of a braced portal frame and are related to Chapter four.
- Appendices Q till U are about the bearing capacity of a braced extended frame and are related to Chapter five.

Appendix A Euler buckling load and deflection

In this appendix the Euler buckling load and the geometrical non-linear deflection formula have been derived.

A.1 Euler buckling load

Consider a column pinned at both ends and loaded by a compressive normal force *N*. At a certain load (the Euler buckling load) there are two equilibrium situations possible. In the first equilibrium situation the column remains straight. In the second equilibrium situation the column deflects in a half sine shape. The amount of deflection is unknown.

The Euler buckling load starts with the equilibrium between the internal and the external moment.

$$M_{internal} = M_{external} \rightarrow -\frac{d^2y}{dx^2} EI = F_E y$$

F_E is the Euler buckling load. This is an unknown value, but this will be derived. The expression of y and the second derivative of y are as follow:

$$y = y_0 \sin\left(\frac{\pi x}{L}\right)$$

$$\frac{d^2y}{dx^2} = -y_0 \left(\frac{\pi}{L}\right)^2 \sin\left(\frac{\pi x}{L}\right)$$

These expressions can filled in the formula what results in the following formula:

$$-\left\{-y_0 \left(\frac{\pi}{L}\right)^2 \sin\left(\frac{\pi x}{L}\right)\right\} EI = F_E \left\{y_0 \sin\left(\frac{\pi x}{L}\right)\right\}$$

The sine function can be neglected. Also the signs on the left side can be neglected.

$$\left(\frac{\pi}{L}\right)^2 EI = F_E$$

$$F_E = \frac{\pi^2 EI}{L^2}$$

A.2 Geometrical non-linear deflection

In Appendix A.1 the Euler buckling load has been derived. Euler has used a differential equation to find a theoretical buckling load. Suppose the column is loaded horizontally and deflects in a half sine shape. If the column is loaded by a normal force, the deflection increases.

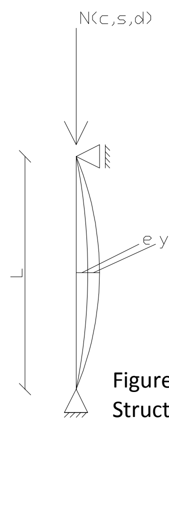


Figure A.1: Structure

The same equilibrium as Euler has used, can be used to find the total deflection. The total deflection is a summation of the original deflection and an additional deflection. The original deflection (e) is the deflection due to the horizontally load. An initial deflection (imperfection) can also be seen as an original deflection. The deflection increases if the column is loaded by a normal force. The additional deflection is called y (Fig. A.1).

$$M_i = M_u \rightarrow N_{c;s;d}(y+e) = -\frac{d^2y}{dx^2}EI$$

The expressions of y and e are the following expressions:

$$y = y_0 \sin\left(\frac{\pi x}{L}\right)$$

$$\frac{d^2y}{dx^2} = -y_0\left(\frac{\pi}{L}\right)^2 \sin\left(\frac{\pi x}{L}\right)$$

$$e = e_0 \sin\left(\frac{\pi * x}{L}\right)$$

Together with the original formula this results in:

$$N_{c;s;d}\left(\left\{y_0 \sin\left(\frac{\pi x}{L}\right)\right\} + \left\{e_0 \sin\left(\frac{\pi x}{L}\right)\right\}\right) = -\left\{-y_0\left(\frac{\pi}{L}\right)^2 \sin\left(\frac{\pi x}{L}\right)\right\}EI$$

If the sine functions are neglected.

$$N_{c;s;d}\left(y_0 + e_0\right) = -\left\{-y_0\left(\frac{\pi}{L}\right)^2\right\}EI$$

$$N_{c;s;d}\left(y_0 + e_0\right) = y_0 F_E \qquad \text{with } F_E = \frac{\pi^2 EI}{L^2}$$

Writing out the formula

$$N_{c;s;d}y_0 + N_{c;s;d}e_0 = y_0 F_E$$

On both sides an expression $e_0 F_E$ will be summed up to get the total deflection as a function of e_0, F_E and $N_{c,s,d}$.

$$N_{c;s;d}y_0 + N_{c;s;d}e_0 + F_E e_0 = y_0 F_E + e_0 F_E$$

Separate the total deflection ($e_0 + y_0$) from the rest of the equation.

$$e_0 F_E = \left(y_0 + e_0\right)\left(F_E - N_{c;s;d}\right)$$

This results in an expression for $e_0 + y_0$.

$$\left(e_0 + y_0\right) = e_0 \frac{F_E}{\left(F_E - N_{c;s;d}\right)}$$

The denominator and the numerator can be divided by the yield load.

$$\left(e_0 + y_0\right) = e_0 \frac{F_E}{\left(F_E - N_{c;s;d}\right)} \frac{\left(\frac{1}{N_{c;s;d}}\right)}{\left(\frac{1}{N_{c;s;d}}\right)}$$

Make one numerator and one denominator

$$\left(y_0 + e_0\right) = e_0 \frac{\frac{F_E}{N_{c;s;d}}}{\left(\frac{F_E}{N_{c;s;d}} - \frac{N_{c;s;d}}{N_{c;s;d}}\right)}$$

Introduce: $n = \frac{F_E}{N_{c;s;d}}$

Use the introduced n-formula and the final result is:

$$y_0 + e_0 = e_0 \frac{n}{n-1}$$

The total deflection is the original deflection multiplied by a factor.

Appendix B Relative Slenderness

In Appendix A the Euler buckling load and the geometrical non-linear deflection formula have been derived. In this appendix a buckling reduction factor will be analyzed. To find the ultimate load, the yield force must be multiplied by this reduction factor. This reduction factor depends on the length and the section properties of the structure. For relative small lengths, the influence of the buckling factor is very small. The buckling reduction factor is equal to one. Due to straight hardening it is allowed to make this assumption.

The analysis of the reduction factor is related to the Euler buckling stress. The Euler buckling stress is the Euler buckling load divided by the cross-section. If the reduction factor has been found, the buckling load is the multiplication of the reduction factor, the yield stress and the cross-section.

$$\sigma_E = \frac{F_E}{A}$$

Using the Euler buckling load this formula is:

$$\sigma_E = \frac{\left(\frac{\pi^2 EI}{L_{buc}{}^2}\right)}{A}$$

This is the same as:

$$\sigma_E = \frac{\left(\pi^2 E\right)}{\left(\frac{L_{buc}{}^2}{\left(\frac{I}{A}\right)}\right)}$$

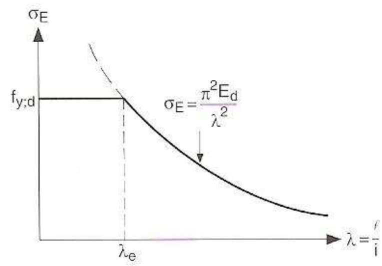


Figure B.1:
Stress / slenderness diagram

Using the gyration radius $\left(i = \sqrt{\frac{I}{A}}\right)$ the formula results in:

$$\sigma_E = \frac{\pi^2 E}{\left(\frac{L_{buc}{}^2}{i^2}\right)}$$

λ is the slenderness of a section $\left(\lambda = \frac{L}{i}\right)$. λ is used in the formula. This results in:

$$\sigma_E = \frac{\pi^2 E}{\lambda^2}$$

Consider a section with a Euler buckling stress that is equal to the yield stress. The slenderness of this section is called λ_e (Fig. B.1). The following formula can be found.

$$f_y = \frac{\pi^2 E}{\lambda_e{}^2}$$

Separate λ_e from the rest of the formula.

$$\lambda_e = \sqrt{\frac{\pi^2 E}{f_y}} \quad \text{this is equal to} \quad \lambda_e = \pi\sqrt{\frac{E}{f_y}}$$

There are many steel grades. Every steel grade has its own yield stress. It is not practical to have a different slenderness graphic for every steel grade. This problem can be solved by using dimensionless values (Fig. B.2). This results in one slenderness graphic.

On the y-axis: $\dfrac{\sigma_E}{f_y}$

On the x-axis: $\dfrac{\lambda}{\lambda_e}$

The relative slenderness is formulated as the quotient of the slenderness and the Euler slenderness.

$$\lambda_{rel} = \frac{\lambda}{\lambda_e}$$

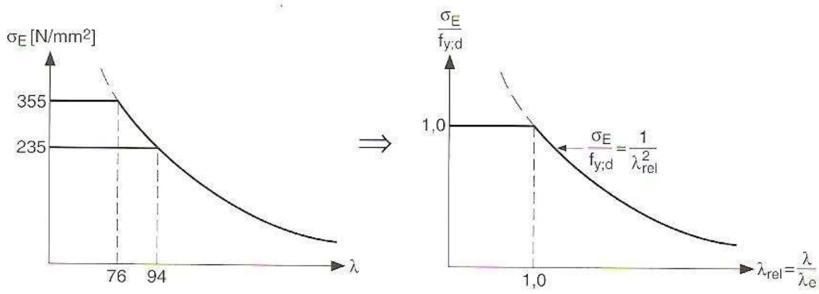


Figure B.2:
Stress / slenderness diagram

The known formulas can be used.

$$\lambda_{rel} = \frac{\frac{L}{i}}{\pi\sqrt{\frac{E}{f_y}}}$$

Using the formula of the gyration radius.

$$\lambda_{rel} = \frac{L}{\pi\sqrt{\frac{E}{f_y}}\sqrt{\frac{I}{A}}}$$

This is the same as.

$$\lambda_{rel} = \frac{L}{\pi\sqrt{\frac{EI}{f_y A}}}$$

To calculate the ultimate stress, a reduction factor must be applied. The reduction factor is called ω_{buc}. ω_{buc} is the Euler stress divided by the yield stress.

$$\omega_{buc} = \frac{\sigma_E}{f_y}$$

The value of the Euler stress is known and can be used.

$$\omega_{buc} = \frac{\left(\frac{\pi^2 E}{\lambda^2}\right)}{f_y}$$

The formula of the slenderness can be used. This results in the following formula.

$$\omega_{buc} = \frac{\pi^2 EI}{f_y L^2 A}$$

This is equal to

$$\omega_{buc} = \left(\frac{\pi \sqrt{\frac{EI}{f_y A}}}{L} \right)^2$$

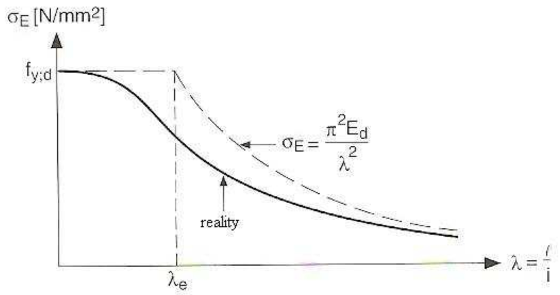


Figure B.3:
Real failure load

Using the formula of relative slenderness.

$$\omega = \frac{1}{\lambda_{rel}{}^2}$$

Only the Euler slenderness graphic has been derived. The starting position of Euler was a perfectly made section. In practice, the section is not perfectly made. The real ultimate stress is lower than the Euler buckling stress (Fig. B.3). The Euler buckling stress is an upper limit of the ultimate stress. Another limit of the ultimate stress in the yield stress. To calculate a more realistic ultimate load, the reduction factor must decrease.

The buckling stress depends on the dimensions of the section, the shape of the section and the way of construction. To make code calculations, all sections are divided in four groups. Inside a group, the influence of the residual stress is (more or less) equal. Based on the buckling analysis of Euler, the practical experience of different types of sections and different steel grades there are made four slenderness graphics (Fig. B.4). The ultimate load is the multiplication of the reduction factor, the yield stress and the cross section.

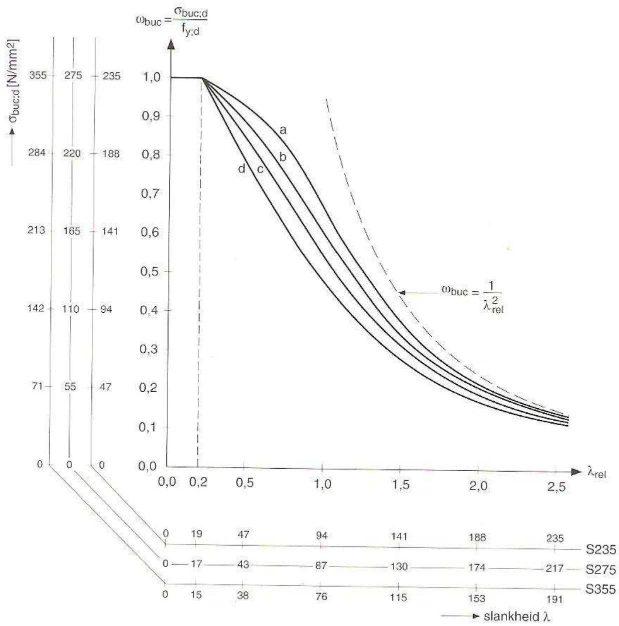


Figure B.4:
Slenderness graphics

Appendix C Differential equation buckling

If all stresses in a section are smaller than the yield stress, the $\frac{n}{n-1}$ (derived in App. B) formula can be used to calculate the geometrical non-linear deflection. If the stress increases and a part of the section starts to yield, the deflection increases and the $\frac{n}{n-1}$ formula is not valid anymore. The load on the structure what result in partial yielding is called F_1. The total deflection of the mid-span at F_1 is:

$$e_1 = e_0 \frac{F_E}{F_E - F_1} \qquad F_E = \frac{\pi^2 EI}{L_{buc}{}^2}$$

With e_1 is the total deflection at force F_1
e_0 is the initial deflection (at F = 0 N)
F_E is the Euler buckling load

Due to the chosen sine shape of the deflection, the most critical cross-section is the midspan. The stresses in the midspan have been applied over the whole length of the column. In reality the ends of the column start to yield at a larger load. This assumption is a necessary approximation for performing the mathematical evolutions.

After the first yield point, not the whole cross-section can be resists more loads. A part of the right is yielded. The stress in this part will not increase. The rest of the section is the effective (reduced) cross-section. The effective cross-section is asymmetric. The centre of gravity is shifted. Due to extending the stress at midsection over the whole column, the centre of gravity in the whole column has shifted.

The load is located in the original centre of gravity. Due to partial yielding, the original centre of gravity is not the same as the effective centre of gravity. The load is not located in the centre of gravity anymore of the effective section.

For calculations it is much easier if the force is located in the centre of gravity. To change the location of the load, an eccentric moment has been introduced. The largest eccentric moment is in the middle of the column. In practice there is no eccentric moment at the end

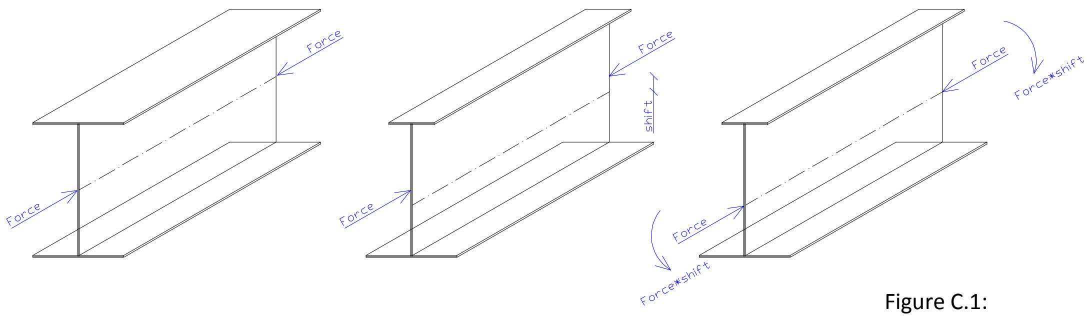


Figure C.1:
Eccentric moment

of the column. In this analysis, the eccentric moment is located at both ends of the column (Fig. C.1).

An eccentric moment at the end results in an extra deflection of the column. An extra deflection results in an extra bending moment in the midsection. Due to this extra bending moment, the deflection increases again. It is important to take the eccentric moment into account at the start of the analysis. The deflection due to this eccentric moment is called $e_{m,i}$.

Figure C.2 shows the total deflection for a column. In this figure is:

- $e_{total,i-1}$ the total deflection at load F_{i-1}
- $e_{m,i}$ deflection due to the eccentric moment
- e_i the extra deflection due to F_1
- F_i load extra load on the column ($F_{total} – F_{i-1}$)
- z_i the shift between the effective point of gravity and the original point of gravity

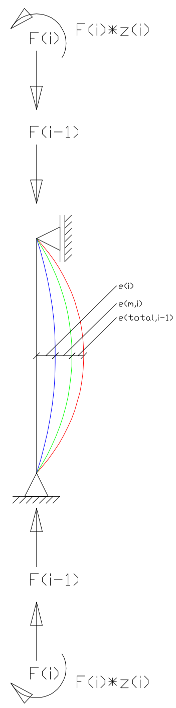


Figure C.2:
Loads on the column

The analyzed load case has subscript 'i'. The total deflection depends on the loads and on the deflection at the previous load case. The previous load case has subscript 'i-1'. The original deflection in the first load case (i=1) is e_0. This is the initial deflection.

The analysis is based on equilibrium between the internal and the external bending moments. The external moments are the load multiplied by the deflection. The internal moment is the curvature multiplies by the stiffness. The curvature is the second derivative of the deflection. The calculation is a second order differential equation.

$$M_{extern} = M_{intern}$$

$$F_{i-1}(e_{m,i} + e_i) + F_i(e_{total,i-1} + e_{m,i} + e_i) + F_i z_i = \kappa EI_i$$

$$F_{i-1}(e_{m,i} + e_i) + F_i(e_{total,i-1} + e_{m,i} + e_i) + F_i z_i = \frac{-d^2 y}{dx^2} EI_i$$

The expression of e_i has been separate from the rest of the formula

$$F_{i-1}e_{m,i} + F_{i-1}y + \Delta F_i(e_0 + e_{i-1} + e_{m,i}) + \Delta F_i y + \Delta F_i z_i = \frac{-d^2 y}{dx^2} EI_i$$

The second derivative of e_i and the functions of e must be calculated.

$$\frac{d^2 e_i}{dx^2} = -e_i \left(\frac{\pi}{L}\right)^2 \sin(\frac{\pi x}{L}) - \frac{F_i z_i}{EI_i}$$

$$\frac{de_i}{dx} = e_i\left(\frac{\pi}{L}\right)\cos(\frac{\pi x}{L}) - \frac{F_i z_i x}{EI_i} + C_1$$

$$e_i = e_i \sin(\frac{\pi x}{L}) - \frac{F_i z_i x^2}{2 * EI_i} + C_1 x + C_2 e_i$$

$$e_{total,i-1} = e_{total,i-1} \sin(\frac{\pi x}{L})$$

$$e_{m,i} = e_{m,i} \sin(\frac{\pi x}{L})$$

These functions are used in the original formula.

$$F_{i-1}e_{m,i}\sin(\frac{\pi x}{L}) + F_{i-1}\left(e_i \sin(\frac{\pi x}{L}) - \frac{F_i z_i x^2}{2EI_i} + C_1 x + C_2\right) + F_i(e_{total,i-1} + e_{m,i})\sin(\frac{\pi * x}{L})$$

$$+F_i\left(e_i \sin(\frac{\pi x}{L}) - \frac{F_i z_i x^2}{2 * EI_i} + C_1 x + C_2\right) + F_i z_i = -\left(-e_i\left(\frac{\pi}{L}\right)^2 \sin(\frac{\pi x}{L}) - \frac{F_i z_i}{EI_i}\right)EI_i$$

C_1 and C_2 are integral constants. These constants can be calculated with the following boundary conditions:
x = 0 → y = 0.
x = L → y = 0.

The first boundary condition results in:

$$F_{i-1}(C_2) + F_i(C_2) = 0 \qquad \rightarrow C_2 = 0$$

The second boundary condition results in:

$$F_{i-1}\left(-\frac{F_i z_i L^2}{2EI_i} + C_1 L\right) + F_i\left(-\frac{F_i z_i L^2}{2EI_i} + C_1 L\right) = 0$$

C_1 can be separated from the rest of the formula.

$$C_1 L(F_{i-1} + F_i) = \left(\frac{F_i z_i L^2}{2EI_i}\right)(F_{i-1} + F_i)$$

Finally this results in:

$$C_1 = \frac{F_i z_i L}{2EI_i}$$

The values C_1 and C_2 are filled in the formula.

$$F_{i-1}e_{m,1}\sin(\frac{\pi x}{L}) + F_{i-1}\left(e_i \sin(\frac{\pi x}{L}) - \frac{F_i z_i x^2}{2EI_i} + \left(\frac{F_i z_i L}{2EI_i}\right)x\right) + F_i(e_{total,i-1} + e_{m,i})\sin(\frac{\pi x}{L})$$

$$+F_i\left(e_i \sin(\frac{\pi x}{L}) - \frac{F_i z_i x^2}{2EI_i} + \left(\frac{F_i z_i L}{2EI_i}\right)x\right) + F_i z_i = -\left(-e_i\left(\frac{L}{\pi}\right)^2 \sin(\frac{\pi x}{L}) - \frac{F_i z_i}{EI_i}\right)EI_i$$

This results in the following formula:

$$F_{i-1}e_{m,i}\sin(\frac{\pi x}{L})+F_{i-1}\left(e_i\sin(\frac{\pi x}{L})+\frac{F_i z_i x(L-x)}{2EI_i}\right)+F_i(e_{total,i-1}+e_{m,i})\sin(\frac{\pi x}{L})$$
$$+F_i\left(e_i\sin(\frac{\pi x}{L})+\frac{F_i z_i x(L-x)}{2EI_i}\right)+F_i z_i=e_i F_{E,i}\sin(\frac{\pi x}{L})+Fz_i$$

Finally the total deflection must be calculated. The total deflection is $e_{total,i-1}+e_{m,i}+e_i$. The factor $e_{total,i-1}+e_{m,i}+e_i$ must be separate from the rest of the formula. To start the expression e_i is separate.

$$F_{i-1}e_{m,i}\sin(\frac{\pi x}{L})+F_{i-1}e_i\sin(\frac{\pi x}{L})+F_{i-1}\frac{F_i z_i x(L-x)}{2EI_i}+F_i(e_{total,i-1}+e_{m,i})\sin(\frac{\pi x}{L})$$
$$+F_i e_i\sin(\frac{\pi x}{L})+F_i\frac{F_i z_i x(L-x)}{2EI_i}+F_i z_i=e_i F_{E,i}\sin(\frac{\pi x}{L})+F_i z_i$$

The expressions of e_i are placed on one side of the equation. All other expressions are placed on the other side of the equation.

$$F_{i-1}e_{m,i}\sin(\frac{\pi x}{L})+F_{i-1}\frac{F_i z_i x(L-x)}{2EI_i}+F_i(e_{total,i-1}+e_{m,i})\sin(\frac{\pi x}{L})+F_i\frac{F_i z_i x(L-x)}{2EI_i}$$
$$=e_i F_{E,i}\sin(\frac{\pi x}{L})-F_{i-1}e_i\sin(\frac{\pi x}{L})-F_i e_i\sin(\frac{\pi x}{L})$$

The expressions of e_i are combined together.

$$F_{i-1}e_{m,i}\sin(\frac{\pi x}{L})+F_{i-1}\frac{F_i z_i x(L-x)}{2EI_i}+F_i(e_{total,i-1}+e_{m,i})\sin(\frac{\pi x}{L})+F_i\frac{F_i z_i x(L-x)}{2EI_i}$$
$$=e_i\left(F_{E,i}\sin(\frac{\pi x}{L})-F_{i-1}\sin(\frac{\pi x}{L})-F_i\sin(\frac{\pi x}{L})\right)$$

e_1 depends on $e_{total,i-1}$ and on $e_{m,i}$. In other words: the total expression $e_{total,i-1}+e_{m,i}+e_i$ depends on $e_{total,i}$ and on $e_{m,i}$. To realise this it is necessary to sum up $\left(e_{total,i-1}+e_{m,i}\right)\left(F_{E,i}\sin(\frac{\pi x}{L})-F_{i-1}\sin(\frac{\pi x}{L})-F_i\sin(\frac{\pi x}{L})\right)$ on both sides of the equation.

$$F_{i-1}e_{m,i}\sin(\frac{\pi x}{L})+F_{i-1}\frac{F_i z_i x(L-x)}{2EI_i}+F_i(e_{total,i-1}+e_{m,i})\sin(\frac{\pi x}{L})+F_i\frac{F_i z_i x(L-x)}{2EI_i}$$
$$+\left(e_{total,i-1}+e_{m,i}\right)\left(F_{E,i}\sin(\frac{\pi x}{L})-F_{i-1}\sin(\frac{\pi x}{L})-F_i\sin(\frac{\pi x}{L})\right)$$
$$=\left(e_{total,i-1}+e_{m,i}+e_i\right)\left(F_{E,i}\sin(\frac{\pi x}{L})-F_{i-1}\sin(\frac{\pi x}{L})-F_i\sin(\frac{\pi x}{L})\right)$$

All expressions on the left side on the equation are written out.

$$F_{i-1}e_{m,i}\sin(\frac{\pi x}{L})+F_{i-1}\frac{F_i z_i x(L-x)}{2EI_i}+F_i(e_{total,i-1}+e_{m,i})\sin(\frac{\pi x}{L})+F_i\frac{F_i z_i x(L-x)}{2EI_i}$$
$$+\left(e_{total,i-1}F_{E,i}\sin(\frac{\pi x}{L})-e_{total,i-1}F_{i-1}*\sin(\frac{\pi x}{L})-e_{total,i-1}F_i\sin(\frac{\pi x}{L})\right)$$
$$+\left(e_{m,i}F_{E,i}\sin(\frac{\pi x}{L})-e_{m,i}F_{i-1}\sin(\frac{\pi x}{L})-e_{m,i}F_i\sin(\frac{\pi x}{L})\right)$$
$$=\left(e_{total,i-1}+e_{m,i}+y_0\right)\left(F_{E,i}\sin(\frac{\pi x}{L})-F_{i-1}\sin(\frac{\pi x}{L})-F_i\sin(\frac{\pi x}{L})\right)$$

The same expressions with different signs can be neglected.

$$F_{i-1}\frac{F_i z_i x(L-x)}{2EI_i}+F_i\frac{F_i z_i x(L-x)}{2EI_i}+\left(e_{total,i-1}F_{E,i}\sin(\frac{\pi x}{L})-e_{total,i-1}F_{i-1}\sin(\frac{\pi x}{L})\right)+\left(e_{m,i}F_{E,i}\sin(\frac{\pi x}{L})\right)$$
$$=\left(e_{total,i-1}+e_{m,i}+e_i\right)\left(F_{E,i}\sin(\frac{\pi x}{L})-F_{i-1}\sin(\frac{\pi x}{L})-F_i\sin(\frac{\pi x}{L})\right)$$

The equal parts can be combined together.

$$\left(F_{i-1}+F_i\right)\frac{F_i z_i x(L-x)}{2EI_i}+\left(e_{total,i-1}+e_{m,i}\right)\left(F_{E,i}\sin(\frac{\pi x}{L})\right)-e_{total,i-1}\left(F_{i-1}\sin(\frac{\pi x}{L})\right)$$
$$=\left(e_{total,i-1}+e_{m,i}+e_i\right)\left(F_{E,i}\sin(\frac{\pi x}{L})-F_{i-1}\sin(\frac{\pi x}{L})-F_i\sin(\frac{\pi x}{L})\right)$$

$e_0+e_{i-1}+e_{m,i}+y_0$ can be expressed in a function of $e_{total,i-1}$ and on $e_{m,i}$.

$$\left(e_{total,i-1}+e_{m,i}+e_i\right)=\frac{\left(F_{i-1}+F_i\right)\frac{F_i z_i x(L-x)}{2EI_i}+\left(e_{total,i-1}+e_{m,i}\right)\left(F_{E,i}\sin(\frac{\pi x}{L})\right)-e_{total,i-1}\left(F_{i-1}\sin(\frac{\pi x}{L})\right)}{F_{E,i}\sin(\frac{\pi x}{L})-F_{i-1}\sin(\frac{\pi x}{L})-F_i\sin(\frac{\pi * x}{L})}$$

$e_{m,i}$ is the deflection in the column due to the eccentric moment ($F_i z_i$) on both sides on the column. The deflection in the midsection: $\frac{ML^2}{16EI}+\frac{ML^2}{16EI}$. This result in the following expression:

$$e_{m,i}=\frac{F_i z_i L^2}{8EI_i}$$

This expression can use in the formula:

$$\left(e_{total,i-1}+e_{m,i}+e_i\right)=\frac{\left(F_{i-1}+F_i\right)\frac{F_i z_i x(L-x)}{2EI_i}+\left(e_{total,i-1}+\frac{F_i z_i L^2}{8EI_i}\right)\left(F_{E,i}\sin(\frac{\pi x}{L})\right)-e_{total,i-1}\left(F_{i-1}\sin(\frac{\pi x}{L})\right)}{F_{E,i}\sin(\frac{\pi x}{L})-F_{i-1}\sin(\frac{\pi x}{L})-F_i\sin(\frac{\pi x}{L})}$$

The most critical cross-section (and the largest deflection) is located in the midsection (x = ½L). The calculations are based on this location. This value of x can be applied in the formula.

$$\left(e_{total,i-1}+e_{m,i}+e_i\right)=\frac{\left(F_{i-1}+F_i\right)\frac{F_i z_i L^2}{8EI_i}+\left(e_{total,i-1}+\frac{F_i z_i L^2}{8EI_i}\right)F_{E,i}-e_{total,i-1}F_{i-1}}{F_{E,i}-F_{i-1}-F_i}$$

By a simple check, big mistakes can be afforded. If there is no additional force there is no additional deflection. The total deflection should be equal to the total deflection in the previous load case.
If F= 0 than $e_{m,i}$= 0 and $e_{total,i-1}+e_{m,i}+e_i$ = $e_{total,i-1}$

$$\left(e_{total,i-1}+e_{m,i}+e_i\right)=\frac{e_{total,i-1}F_{E,i}-e_{total,i-1}F_{i-1}}{F_{E,i}-F_{i-1}}$$

$$\left(e_{total,i-1}+e_{m,i}+e_i\right)=\frac{e_{total,i-1}\left(F_{E,i}-F_{i-1}\right)}{F_{E,i}-F_{i-1}}$$

$\left(e_{total,i-1}+e_{m,i}+y_0\right)=e_{total,i-1}$ Correct

A second check is a check on the signs. A positive extra load must result in a larger deflection.

$$\left(e_{total,i-1}+e_{m,i}+e_i\right)=\frac{e_{total,i-1}\left(F_{E,i}-F_{i-1}\right)}{F_{E,i}-F_{i-1}}+\frac{\left(F_{E,i}+F_{i-1}+F_i\right)\frac{F_i z_i L^2}{8EI_i}}{F_{E,i}-F_{i-1}-F_i}$$

The Euler buckling load $\left(F_{E,i}\right)$ must be larger than the maximum load $\left(F_{i-1}+F_i\right)$. The Euler buckling load is an upper limit, so this is correct. A positive additional load results in a positive additional deflection. The second check is correct.

As a third, the dimensions can be checked. If the dimensions are not correct, the formula could not be correct.

$$\left(m+m+m\right)=\frac{\left(N+N\right)\frac{Nmm^2}{Nm^{-2}m^4}+\left(m+\frac{Nmm^2}{Nm^{-2}m^4}\right)N-mN}{N-N-N}$$

$$m=\frac{N\frac{Nm^3}{Nm^2}+\left(m+\frac{Nm^3}{Nm^2}\right)N-Nm}{N}$$ Correct

The formula is found correctly on all checks. There are no large mistakes in the formula.

Appendix D Calculation example (hand-made)

In Appendix C the differential equation for the total deflection has been made. In this Appendix the formulas (found in App. C) will be used to make some calculations. The calculations in this Appendix are made manually. More calculations will be made in Appendix E by the computer program MatLab. The computer program MatLab has been used to make many calculations. In Appendix F the same problem is solved by a computer program (Matrix Frame) based on a finite element method (FEM).

All calculations in this Appendix are based on a HE 450A section. The difference between the calculations is the length of the column. The different lengths are 30, 25, 20, 15, 10 and 5 meter. If the ultimate loads are calculated, a reduction factor curve can be made. Due to the small amount of calculations, the reduction factor curve is not very clear.. In Appendix E more lengths are calculated (by computer calculation). This results in a flowing curve.

D.1 Residual stress

The residual stress distribution has been simplified to a rectangular residual stress distribution (Fig. D.1). The moment of inertia depends mainly on the flanges. The web has just a small contribution to the moment of inertia. If (due to partial yielding) the effective surface of the flanges decreases, the effective moment of inertia decreases. If the web partial yields, the moment of inertia deceases just a little bit. This reduction is neglected. For the reduction of the cross-section the effective of both the web and the flanges must taken into account.

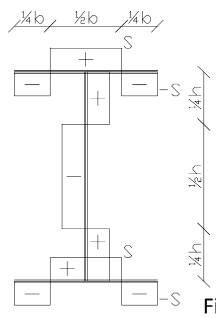


Figure D.1: Residual stress distribution

The amount of residual stress (in this report called S) is a function of the yield stress. According to the NEN 6771 the amount of residual stress in a HE 450A section is 30% of the yield stress. In other words:

S = $0.3 f_y$ = 0.3*355

S = 106.5 N/mm^2

The stress in the section cannot be larger than the yield stress. There is residual tension stress and residual compression stress. If the structure is loaded by a compression loads, the part of the section with residual compression stress yields first. This happens if $\sigma = f_y - S$. The part of the section with residual tension stress yields if $\sigma = f_y + S$. The critical stresses are:

$\sigma = f_y - S$ $\quad \sigma = f_y - 0.3 f_y$ $\quad \sigma = 0.7 f_y$ $\quad \sigma = 248.5$ N/mm^2

$\sigma = f_y + S$ $\quad \sigma = f_y + 0.3 f_y$ $\quad \sigma = 1.3 f_y$ $\quad \sigma = 461.5$ N/mm^2

D.2 Section properties

For the calculation of the ultimate load, it is important to know what the section properties are. It is also important to know the reduced section properties. These section properties will be calculated.

$I_1 = 6.372*10^8\ mm^4$

$A_1 = 17800\ mm^2$

$Z_1 = \dfrac{I_1}{½h} = \dfrac{6.372*10^8}{220}$

$Z_1 = 2.896*10^6 mm^3$

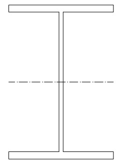


Figure D.2: Profile without reduction

$I_2 = I_1 - 2(¼b)t_f(½h)^2$

$I_2 = 6.372*10^8 - 2*75*21*220^2$

$I_2 = 4.848*10^8\ mm^4$

$A_2 = A_1 - 2*(¼b)t_f$

$A_2 = 17800 - 2*75*21$

$A_2 = 14650\ mm^2$

$Z_2 = \dfrac{I_2}{½h} = \dfrac{4.848*10^8}{220}$

$Z_2 = 2.204*10^6\ mm^3$

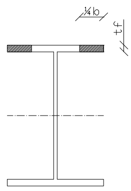


Figure D.3: Profile with reduction

$I_3 = I_2 - 2(¼b)t_f(½h)^2$

$I_3 = 4.848*10^8 - 2*75*21*220^2$

$I_3 = 3.324*10^8\ mm^4$

$A_3 = ½A_1$

$A_3 = ½*17800$

$A_3 = 8900\ mm^2$

$Z_3 = \dfrac{I_3}{½h} = \dfrac{3.324*10^8}{220}$

$Z_3 = 1.511*10^6\ mm^3$

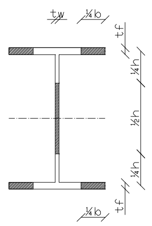


Figure D.4: Profile with reduction

In Appendix C the following formula has been derived:

$$e_{total,i} = \left(e_{total,i-1} + e_{m,i} + e_i\right) = \frac{\left(F_{i-1} + F_i\right)\dfrac{F_i z_i L^2}{8EI_i} + \left(e_{total,i-1} + \dfrac{F_i z_i}{8EI_i}\right)F_{E,i} - e_{total,i-1}F_{i-1}}{F_{E,i} - F_{i-1} - F_i}$$

For the calculation it is important to know that:

$$e_i = e_{total,i} - e_{total,i-1}$$

$$M_i = F_{i-1}e_i + Fe_{total,i}$$

$$\sigma_{right,i} = \sigma_{right,i-1} - \frac{M_i}{Z_i} - \frac{F_i}{A_i}$$

$$\sigma_{left,i} = \sigma_{left,i-1} + \frac{M_i}{Z_i} - \frac{F_i}{A_i}$$

The symbols in these formulas are:

$e_{total,i}$ = the total deflection at load case i (mm)

$e_{total,i-1}$ = the total deflection at load F_{i-1} (mm)

$e_{m,i}$ = the deflection due to the eccentric moment in load case i (mm)

e_i = the extra deflection due to the difference in normal force (mm)

F_{i-1} = the load in the previous load case (N)

F_i = the extra load in case i (N)

z_i = the difference between the original point of gravity and the effective point of gravity in case i (mm)

I_i = the moment of inertia in case i (mm^4)

$F_{E,i}$ = the Euler buckling load in case i (N)

The starting points of this analysis where:

- There is an initial deflection.
- There is a force on the section that results in partial yielding.
- There is a deflection due to the original load.
- There is an additional force.
- There is a difference between the original centre of gravity and the effective point of gravity.
- An eccentric moment has been introduced to change the location of the load. Due to the eccentric moment the load is located in the centre of gravity of the effective section.

See Appendix C for the analysis.

The ultimate load calculations are made in Appendices B.3 till B.8. Every Appendix exists in the ultimate load calculation of one column. The difference between the Appendices is the column length. For every column three calculation methods has been used:

1. Calculation according to the Dutch code.
2. Calculation of the Euler buckling load.
3. Calculation according to the formulas of the analysis in Appendix C.

D.3 Calculation for a length of 30 meter

Section properties:

$I_1 = 6.372*10^8$ mm^4 $I_2 = 4.848*10^8$ mm^4 $I_3 = 3.324*10^8$ mm^4

$A_1 = 17800$ mm^2 $A_2 = 14650$ mm^2 $A_3 = 8900$ mm^2

$Z_1 = 2.896*10^6$mm^3 $Z_2 = 2.204*10^6$ mm^3 $Z_3 = 1.511*10^6$ mm^3

Ultimate load calculation according to the Dutch code.

For the ultimate load calculation according to the Dutch code, the relative slenderness must be calculated. See Appendix B for more information about the relative slenderness.

$$\lambda_{rel} = \frac{L}{\pi\sqrt{\frac{EI_1}{f_y A_1}}} = \frac{30000}{\pi\sqrt{\frac{210000*6.372*10^8}{355*17800}}}$$

λ_{rel} = 2.08

For the ultimate load calculation of a HE 450A section stability curve a must be used (NEN 6770 art. 12.1.1).

λ_{rel} = 2.08 → ω_{buc} = 0.20

F_{max} = $\omega_{buc} f_y A_1$ = $0.20*355*17800$

F_{max} = $1264*10^3$ N

Calculation of the Euler buckling load.

$$F_{max} = \frac{\pi^2 EI_1}{L^2} = \frac{\pi^2 210000*6.372*10^8}{30000^2}$$

(I_1 is the unreduced moment of inertia

F_{max} = $1467*10^3$ N

Ultimate load calculation according to the formulas of the analysis of Appendix C.

The following equation is the general equation of the residual stress method. This formula has been derived in Appendix C.

$$e_{total,i} = \left(e_{total,i-1} + e_{m,i} + e_i\right) = \frac{\left(F_{i-1}+F_i\right)\frac{F_i z_i L^2}{8EI_i} + \left(e_{toatl,i-1} + \frac{F_i z_i}{8EI_i}\right)F_{E,i} - e_{total,i-1}F_{i-1}}{F_{E,i} - F_{i-1} - F_i}$$

According to the NEN 6771 (art. 12.2.5), the maximum deflection is one over thousand times the column length .

$$e_{1\text{-}1} = \frac{1}{1000}L = \frac{1}{1000}30000$$

$e_{1\text{-}1}$ = 30 mm

The stress in the unloaded structure is the residual stress. The deflection of the unloaded structure is the initial deflection (imperfections). Starting the calculation the following values are used:

$e_{m,i}$ = 0 mm

$F_{1\text{-}1}$ = 0 N

z_1 = 0 mm

If these values are used, the formula of the total deflection can be simplified to the following formula:

$e_{total,1} = e_1 = \frac{e_0 F_{E,1}}{F_{E,1} - F_1}$ (this is also known as the $\frac{n}{n-1}$ formula).

$F_{E,1} = \frac{\pi^2 E I_1}{L^2} = \frac{\pi^2 210000 * 6.372 * 10^8}{30000^2}$

$F_{E,1}$ = 1467*10^3 N

$e_i = \frac{e_0 F_{E,1}}{F_{E,1} - F_1} - e_0 = \frac{30 * 1467 * 10^3}{1467 * 10^3 - F_1} - 30$

$M_1 = F_1 (e_0 + e_1) = F_1 \frac{30 * 1467 * 10^3}{1467 * 10^3 - F_1}$

$\sigma_{right,1} = -\frac{M_1}{Z_1} - \frac{F_1}{A_1} = -\frac{F_1 \frac{30 * 1467 * 10^3}{1467 * 10^3 - F_1}}{2.896 * 10^6} - \frac{F_1}{17800}$ = -248.5 N/mm²

F_1 = 1348*10³ N

e_1 = 340 mm

$\sigma_{left,1} = \frac{M_1}{Z_1} - \frac{F_1}{A_1} = \frac{1348 * 10^3 \frac{30 * 1467 * 10^3}{1467 * 10^3 - 1348 * 10^3}}{2.896 * 10^6} - \frac{1348 * 10^3}{17800}$

$\sigma_{left,1}$ = 96.5 N/mm² Tension

F_1 = 0 + 1348*10³

F_1 = 1348*10³ N

$e_{total,1}$ = 30 + 340

$e_{total,1}$ = 370 mm

The section starts to yield at a load of 1348*10³ N. The total deflection is 370 mm. This is 1.2% of the length of the column. After partial yielding the moment of inertia decreases. The Euler buckling load can be calculated for the section with the reduced moment of inertia. The calculation continues with the new (reduced) Euler buckling load.

$F_{E,2} = \frac{\pi^2 E I_2}{L^2} = \frac{\pi^2 210000 * 4.848 * 10^8}{30000^2}$

$F_{E,2}$ = 1116*10³ N

The new Euler buckling load is smaller than the load on the column. After first yield, the structure becomes unstable. The column fails. The maximum load is the load that results in first yielding.

$F_{E,2} \leq F_1$ So the structure fails at $F = F_1$

F_{max} = 1348*10³ N

D.4 Calculation for a length of 25 meter

Section properties:

I_1 = 6.372*10^8 mm^4 I_2 = 4.848*10^8 mm^4 I_3 = 3.324*10^8 mm^4

A_1 = 17800 mm^2 A_2 = 14650 mm^2 A_3 = 8900 mm^2

Z_1 = 2.896*10^6mm^3 Z_2 = 2.204*10^6 mm^3 Z_3 = 1.511*10^6 mm^3

Ultimate load calculation according to the Dutch code.

$$\lambda_{rel} = \frac{L}{\pi\sqrt{\frac{EI_1}{f_y A_1}}} = \frac{25000}{\pi\sqrt{\frac{210000*6.372*10^8}{355*17800}}}$$

λ_{rel} = 1.73 → ω_{buc} = 0.28

F_{max} = $\omega_{buc} f_y A_1$ = $0.28*355*17800$

F_{max} = 1769*10^3 N

Calculation of the Euler buckling load.

$$F_{max} = \frac{\pi^2 EI_1}{L^2} = \frac{\pi^2 210000*6.372*10^8}{25000^2}$$

F_{max} = 2113*10^3 N

Ultimate load calculation according to the formula of the analysis of Appendix C.

$$e_{total,i} = \left(e_{total,i-1} + e_{m,i} + e_i\right) = \frac{\left(F_{i-1} + F_i\right)\frac{F_i z_i L^2}{8EI_i} + \left(e_{toatl,i-1} + \frac{F_i z_i}{8EI_i}\right)F_{E,i} - e_{total,i-1}F_{i-1}}{F_{E,i} - F_{i-1} - F_i}$$

$$e_{1-1} = \frac{1}{1000}L = \frac{1}{1000}25000$$

e_{1-1} = 25 mm

$e_{m,i}$ = 0 mm

F_{1-1} = 0 N

z_1 = 0 mm

If these values are used, the following formula exists:

$$e_{total,1} = \left(e_0 + e_1\right) = \frac{e_0 F_{E,1}}{F_{E,1} - F_1}$$

$F_{E,1}$ = 2113*10^3 N

$$e_1 = \frac{e_0 F_{E,1}}{F_{E,1} - F_1} - e_0 = \frac{25*2113*10^3}{2113*10^3 - F_1} - 25$$

$$M_1 = F_1\left(e_0 + e_1\right) = F_1\frac{25*2113*10^3}{2113*10^3 - F_1}$$

$$\sigma_{right,1} = -\frac{M_1}{Z_1} - \frac{F_1}{A_1} = -\frac{F_1 \frac{25*2113*10^3}{2113*10^3 - F_1}}{2.896*10^6} - \frac{F_1}{17800} = -248.5 \text{ N/mm}^2$$

F_1 = 1874*10^3 N
e_1 = 196 mm

$$\sigma_{left,1} = \frac{M_1}{Z_1} - \frac{F_1}{A_1} = \frac{1874*10^3 \frac{25*2113*10^3}{2113*10^3 - 1874*10^3}}{2.896*10^6} - \frac{1874*10^3}{17800}$$

$\sigma_{left,1}$ = 37.8 N/mm^2 Tension

F_1 = 0 + 1874*10^3
F_1 = 1874*10^3 N
$e_{total,1}$ = 25 + 196
$e_{total,1}$ = 211 mm

The section starts to yield at a load of 1874*10^3 N. The total deflection is 221 mm. This is 0.88% of the length of the column. The moment of inertia decreases. The reduced Euler buckling load becomes:

$$F_{E,2} = \frac{\pi^2 E I_2}{L^2} = \frac{\pi^2 210000*4.848*10^8}{25000^2}$$

$F_{E,2}$ = 1608*10^3 N

The new Euler buckling load is smaller than the load on the column. After first yield, the structure becomes unstable. The column fails. The maximum load is found.

$F_{E,2} \leq F_1$ So the structure fails at $F = F_1$

F_{max} = 1874*10^3 N

D.5 Calculation for a length of 20 meter

Section properties:

I_1 = 6.372*10^8 mm^4 I_2 = 4.848*10^8 mm^4 I_3 = 3.324*10^8 mm^4
A_1 = 17800 mm^2 A_2 = 14650 mm^2 A_3 = 8900 mm^2
Z_1 = 2.896*10^6mm^3 Z_2 = 2.204*10^6 mm^3 Z_3 = 1.511*10^6 mm^3

Ultimate load calculation according to the Dutch code.

$$\lambda_{rel} = \frac{L}{\pi\sqrt{\frac{EI_1}{f_y A_1}}} = \frac{20000}{\pi\sqrt{\frac{210000*6.372*10^8}{355*17800}}}$$

λ_{rel} = 1.38 → ω_{buc} = 0.43

F_{max} = $\omega_{buc} f_y A_1$ = $0.43*355*17800$

F_{max} = 2717*10^3 N

Calculation of the Euler buckling load.

$$F_{max} = \frac{\pi^2 EI_1}{L^2} = \frac{\pi^2 210000*6.372*10^8}{20000^2}$$

F_{max} = 3302*10^3 N

Ultimate load calculation according to the formulas of the analysis of Appendix C.

$$e_{total,i} = \left(e_{total,i-1} + e_{m,i} + e_i\right) = \frac{\left(F_{i-1}+F_i\right)\frac{F_i z_i L^2}{8EI_i} + \left(e_{toatl,i-1} + \frac{F_i z_i}{8EI_i}\right)F_{E,i} - e_{total,i-1}F_{i-1}}{F_{E,i} - F_{i-1} - F_i}$$

e_{1-1} = $\frac{1}{1000}L$ = $\frac{1}{1000}20000$

e_{1-1} = 20 mm

$e_{m,i}$ = 0 mm

F_{1-1} = 0 N

z_1 = 0 mm

If these values are used, the following formula exists.

$$e_{total,1} = \left(e_0 + e_1\right) = \frac{e_0 F_{E,1}}{F_{E,1} - F_1}$$

$F_{E,1}$ = 3302*10^3 N

$$e_1 = \frac{e_0 F_{E,1}}{F_{E,1} - F_1} - e_0 = \frac{20*3302*10^3}{3302*10^3 - F_1} - 20$$

$$M_1 = F_1\left(e_0 + e_1\right) = F_1\frac{20*3302*10^3}{3302*10^3 - F_1}$$

$$\sigma_{right,1} = -\frac{M_1}{Z_1} - \frac{F_1}{A_1} = -\frac{F_1 \frac{20*3302*10^3}{3302*10^3 - F_1}}{2.896*10^6} - \frac{F_1}{17800} = -248.5 \text{ N/mm}^2$$

F_1 = 2679*10^3 N
e_1 = 86 mm

$$\sigma_{left,1} = \frac{M_1}{Z_1} - \frac{F_1}{A_1} = \frac{2679*10^3 \frac{20*3302*10^3}{3302*10^3 - 2679*10^3}}{2.896*10^6} - \frac{2679*10^3}{17800}$$

$\sigma_{left,1}$ = -52.5 N/mm^2 Compression

F_1 = 0 + 2679*10^3
F_1 = 2679*10^3 N
$e_{total,1}$ = 20 + 86
$e_{total,1}$ = 106 mm

The section starts to yield at a load of 2679*10^3 N. The total deflection is 106 mm. This is 0.53% of the length of the column. The moment of inertia decreases. The reduced Euler buckling load becomes:

$$F_{E,2} = \frac{\pi^2 E I_2}{L^2} = \frac{\pi^2 * 2100004.848*10^8}{20000^2}$$

$F_{E,2}$ = 2512*10^3 N

The new Euler buckling load is smaller than the load on the column. After first yield, the structure becomes unstable. The column fails. The maximum load is found.

$F_{E,2} \leq F_1$ So the structure fails at $F = F_1$

F_{max} = 2679*10^3 N

D.6 Calculation for a length of 15 meter

Section properties:

$I_1 = 6.372*10^8\ mm^4$	$I_2 = 4.848*10^8\ mm^4$	$I_3 = 3.324*10^8\ mm^4$
$A_1 = 17800\ mm^2$	$A_2 = 14650\ mm^2$	$A_3 = 8900\ mm^2$
$Z_1 = 2.896*10^6 mm^3$	$Z_2 = 2.204*10^6\ mm^3$	$Z_3 = 1.511*10^6\ mm^3$

Ultimate load calculation according to the Dutch code.

$$\lambda_{rel} = \frac{L}{\pi\sqrt{\frac{EI_1}{f_y A_1}}} = \frac{15000}{\pi\sqrt{\frac{210000*6.372*10^8}{355*17800}}}$$

λ_{rel} = 1.04 → ω_{buc} = 0.64

F_{max} = $\omega_{buc} f_y A_1$ = $0.64*355*17800$

F_{max} = $4044*10^3$ N

Calculation of the Euler buckling load.

$$F_{max} = \frac{\pi^2 EI_1}{L^2} = \frac{\pi^2 210000*6.372*10^8}{15000^2}$$

F_{max} = $5870*10^3$ N

Ultimate load calculation according to the formulas of the analysis of Appendix C.

$$e_{total,i} = \left(e_{total,i-1} + e_{m,i} + e_i\right) = \frac{\left(F_{i-1}+F_i\right)\frac{F_i z_i L^2}{8EI_i} + \left(e_{toatl,i-1} + \frac{F_i z_i}{8EI_i}\right)F_{E,i} - e_{total,i-1}F_{i-1}}{F_{E,i} - F_{i-1} - F_i}$$

e_{1-1} = $\frac{1}{1000}L$ = $\frac{1}{1000}15000$

e_{1-1} = 15 mm

$e_{m,i}$ = 0 mm

F_{1-1} = 0 N

z_1 = 0 mm

If the values are used, the following formula exists:

$$e_{total,1} = \left(e_0 + e_1\right) = \frac{e_0 F_{E,1}}{F_{E,1} - F_1}$$

$F_{E,1}$ = $5870*10^3$ N

$$e_1 = \frac{e_0 F_{E,1}}{F_{E,1} - F_1} - e_0 = \frac{15*5870*10^3}{5870*10^3 - F_1} - 15$$

$$M_1 = F_1\left(e_0 + e_1\right) = F_1 \frac{15*5870*10^3}{5870*10^3 - F_1}$$

$$\sigma_{right,1} = -\frac{M_1}{Z_1} - \frac{F_1}{A_1} = -\frac{F_1 \frac{15*5870*10^3}{5870*10^3 - F_1}}{2.896*10^6} - \frac{F_1}{17800} = -248.5 \text{ N/mm}^2$$

F_1 = $3578*10^3$ N
e_1 = 23.4 mm

$$\sigma_{left,1} = \frac{M_1}{Z_1} - \frac{F_1}{A_1} = \frac{3578*10^3 \frac{15*5870*10^3}{5870*10^3 - 3578*10^3}}{2.896*10^6} - \frac{3578*10^3}{17800}$$

$\sigma_{left,1}$ = -153.6 N/mm^2 Compression

F_1 = 0 + $3578*10^3$
F_1 = $3578*10^3$ N
$e_{total,1}$ = 15 + 23.4
$e_{total,1}$ = 38.4 mm

The section starts to yield at a load of $3578*10^3$ N. The moment of inertia decrease. The reduced Euler buckling load becomes:

$$F_{E,2} = \frac{\pi^2 EI_2}{L^2} = \frac{\pi^2 210000*4.848*10^8}{15000^2}$$

$F_{E,2}$ = $4466*10^3$ N

The Euler buckling load is larger the load on the column. The section can resist more loads. It is possible to increase the load till another part of the section yields too. A point of attention is the shift of the centre of gravity. See Appendix C for more details.

$$e_{total,i} = \left(e_{total,i-1} + e_{m,i} + e_i\right) = \frac{\left(F_{i-1} + F_i\right)\frac{F_i z_i L^2}{8EI_i} + \left(e_{toatl,i-1} + \frac{F_i z_i}{8EI_i}\right)F_{E,i} - e_{total,i-1}F_{i-1}}{F_{E,i} - F_{i-1} - F_i}$$

$$z_2 = \frac{\sum A_k z_k}{\sum A_k} - ½h$$

$$z_2 = \frac{(b - 2*¼b)t_f ½t_f + (h - 2t_f)t_w ½h + bt_f(h - ½t_f)}{(b - 2*¼b)t_f + (h - 2t_f)t_w + bt_f} - ½h$$

$$z_2 = \frac{150*21*10.5 + 398*11.5*220 + 300*21*429.5}{150*21 + 398*11.5 + 300*21} - 220$$

z_2 = 47 mm

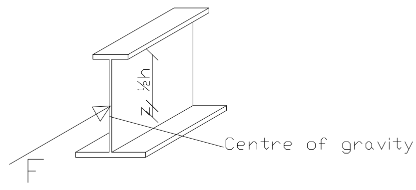


Figure D.5: Shift of centre of gravity

$$e_{total,2} = \left(e_{total,1} + e_{m,2} + e_2\right) =$$

$$\frac{\left(3578*10^3 + F_2\right)\frac{F_2 47*15000^2}{8*210000*4.848*10^8} + \left(38.4 + \frac{F_2 47}{8*210000*4.848*10^8}\right)4466*10^3 - 38.4*3578*10^3}{4466*10^3 - 3578*10^3 - F_2}$$

$$\sigma_{right,2} = \sigma_{right,1} - \frac{M_2}{Z_2} - \frac{F_2}{A_2}$$

$$\sigma_{right,2} = -248.5 - \frac{M_2}{Z_2} - \frac{F_2}{A_2} \quad = -248.5 - \frac{F_1 e_2 + F_2(e_{total,1} + e_2)}{Z_2} - \frac{F_2}{A_2} \quad = 461.5$$

$$\sigma_{right,2} = -248.5 - \frac{3578*e_2 + F_2*(38.4 + e_2)}{2.204*10^6} - \frac{F_2}{14650} \qquad = -461.5 \text{ N/mm}^2$$

F_2 = $362*10^3$ N
e_2 = 101.5 mm

$$\sigma_{left,2} = \sigma_{left,2} = \sigma_{left,1} + \frac{M_2}{Z_2} - \frac{F_2}{A_2}$$

$$\sigma_{left,2} = -153.6 + \frac{M_2}{Z_2} - \frac{F_2}{A_2} \quad = -153.6 + \frac{F_1 e_2 + F_2(e_{total,1} + e_2)}{Z_2} - \frac{F_2}{A_2}$$

$$\sigma_{left,2} = -153.6 + \frac{3578*10^3*101.5 + 362*10^3(38.4 + 101.5)}{2.204*10^6} - \frac{362*10^3}{14650}$$

$\sigma_{left,2}$ = 9.4 N/mm^2 Tension

F_2 = $3578*10^3 + 362*10^3$
F_2 = $3940*10^3$ N
$e_{total,2}$ = 38.4 + 101.5
$e_{total,2}$ = 139.9 mm

In this calculation the right flange fully yields. Only the left flange can be taken as an effective cross-section. The moment of inertia has decreased too much. If the right flange fully yields, the structure fails.

F_{max} = $3940*10^3$ N

D.7 Calculation for a length of 10 meter

Section properties:

$I_1 = 6.372*10^8$ mm^4 | $I_2 = 4.848*10^8$ mm^4 | $I_3 = 3.324*10^8$ mm^4

$A_1 = 17800$ mm^2 | $A_2 = 14650$ mm^2 | $A_3 = 8900$ mm^2

$Z_1 = 2.896*10^6$ mm^3 | $Z_2 = 2.204*10^6$ mm^3 | $Z_3 = 1.511*10^6$ mm^3

Ultimate load calculation according to the Dutch code.

$$\lambda_{rel} = \frac{L}{\pi\sqrt{\frac{EI_1}{f_y A_1}}} = \frac{10000}{\pi\sqrt{\frac{210000*6.372*10^8}{355*17800}}}$$

λ_{rel} = 0.69 → ω_{buc} = 0.85

$F_{max} = \omega_{buc} f_y A_1 = 0.85*355*17800$

F_{max} = 5371*10^3 N

Calculation of the Euler buckling load:

$$F_{max} = \frac{\pi^2 EI_1}{L^2} = \frac{\pi^2 210000*6.372*10^8}{10000^2}$$

F_{max} = 13207*10^3 N (Euler buckling load is larger than the plastic yield load)

Ultimate load calculation according to the formulas of the analysis of Appendix C.

$$e_{total,i} = \left(e_{total,i-1} + e_{m,i} + e_i\right) = \frac{\left(F_{i-1}+F_i\right)\frac{F_i z_i L^2}{8EI_i} + \left(e_{toatl,i-1} + \frac{F_i z_i}{8EI_i}\right)F_{E,i} - e_{total,i-1}F_{i-1}}{F_{E,i} - F_{i-1} - F_i}$$

$$e_{1\text{-}1} = \frac{1}{1000}L = \frac{1}{1000}10000$$

$e_{1\text{-}1}$ = 10 mm

$e_{m,i}$ = 0 mm

$F_{1\text{-}1}$ = 0 N

z_1 = 0 mm

If these values are used, the following formula exists:

$$e_{total,1} = \left(e_0 + e_1\right) = \frac{e_0 F_{E,1}}{F_{E,1} - F_1}$$

$F_{E,1}$ = 13207*10^3 N

$$e_1 = \frac{e_0 F_{E,1}}{F_{E,1} - F_1} - e_0 = \frac{10*13207*10^3}{13207*10^3 - F_1} - 10$$

$$M_1 = F_1\left(e_0 + e_1\right) = F_1 \frac{10*13207*10^3}{13207*10^3 - F_1}$$

$$\sigma_{right,1} = -\frac{M_1}{Z_1} - \frac{F_1}{A_1} = -\frac{F_1 \frac{10*13207*10^3}{13207*10^3 - F_1}}{2.896*10^6} - \frac{F_1}{17800} = -248.5 \text{ N/mm}^2$$

F_1 = 4063*10^3 N
e_1 = 4.4 mm

$$\sigma_{left,1} = \frac{M_1}{Z_1} - \frac{F_1}{A_1} = \frac{4063*10^3 \frac{10*13207*10^3}{13207*10^3 - 4063*10^3}}{2.896*10^6} - \frac{4063*10^3}{17800}$$

$\sigma_{left,1}$ = -208.1 N/mm^2 Compression

F_1 = 0 + 4063*10^3
F_1 = 4063*10^3 N
$e_{total,1}$ = 10 + 4.4
$e_{totqo,1}$ = 14.4 mm

The section starts to yield at a load of 4063*10^3 N. The moment of inertia decreases. The reduced Euler buckling load becomes:

$$F_{E,2} = \frac{\pi^2 EI_2}{L^2} = \frac{\pi^2 210000*4.848*10^8}{10000^2}$$

$F_{E,2}$ = 10048*10^3 N

The Euler buckling load is larger than the load on the column. The column can resist more loads. It is possible to increase the load till another part of the section yields too.

$$e_{total,i} = \left(e_{total,i-1} + e_{m,i} + e_i\right) = \frac{\left(F_{i-1} + F_i\right)\frac{F_i z_i L^2}{8EI_i} + \left(e_{toatl,i-1} + \frac{F_i z_i}{8EI_i}\right)F_{E,i} - e_{total,i-1}F_{i-1}}{F_{E,i} - F_{i-1} - F_i}$$

$$z_2 = \frac{\sum A_k z_k}{\sum A_k} - ½h$$

$$z_2 = \frac{(b-2*¼b)t_f ½t_f + (h-2t_f)t_w ½h + bt_f(h-½t_f)}{(b-2*¼b)t_f + (h-2t_f)t_w + bt_f} - ½h$$

$$z_2 = \frac{150*21*10.5 + 398*11.5*220 + 300*21*429.5}{150*21 + 398*11.5 + 300*21} - 220$$

z_2 = 47 mm

$$e_{total,2} = \left(e_{total,1} + e_{m,2} + e_2\right) =$$

$$\frac{\left(4063*10^3 + F_2\right)\frac{F_2 47*10000^2}{8*210000*4.848*10^8} + \left(14.4 + \frac{F_2 47}{8*210000*4.848*10^8}\right)10048*10^3 - 14.4*4063*10^3}{10048*10^3 - 4063*10^3 - F_2}$$

$$\sigma_{right,2} = \sigma_{right,1} - \frac{M_2}{Z_2} - \frac{F_2}{A_2}$$

$$\sigma_{right,2} = -248.5 - \frac{M_2}{Z_2} - \frac{F_2}{A_2} = -248.5 - \frac{F_1 e_2 + F_2(e_{total,1} + e_2)}{Z_2} - \frac{F_2}{A_2} = -461.5$$

$$\sigma_{right,2} = -248.5 - \frac{406310^3 e_2 + F_2(14.4 + e_2)}{2.204 * 10^6} - \frac{F_2}{14650} = -461.5 \text{ N/mm}^2$$

F_2 = $1571*10^3$ N
e_2 = 37.3 mm

$$\sigma_{left,2} = \sigma_{left,1} + \frac{M_2}{Z_2} - \frac{F_2}{A_2}$$

$$\sigma_{left,2} = -208.1 + \frac{M_2}{Z_2} - \frac{F_2}{A_2} = -208.1 + \frac{F_1 e_2 + F_2(e_{total,1} + e_2)}{Z_2} - \frac{F_2}{A_2}$$

$$\sigma_{left,2} = -208.1 + \frac{4063 * 10^3 * 37.3 + 1571 * 10^3 (10 + 4.4 + 37.3)}{2.204 * 10^6} - \frac{1571 * 10^3}{14650}$$

$\sigma_{left,2}$ = -209.7 N/mm^2 Compression

F_2 = $4063*10^3$ + $1571*10^3$
F_2 = $5634*10^3$ N
$e_{total,2}$ = 14.4 + 37.3
$e_{total,2}$ = 51.7 mm

In this case the right flange fully yields. The column fails.

F_{max} = $5634*10^3$ N

D.8 Calculation for a length of 5 meter

Section properties:

$I_1 = 6.372*10^8$ mm^4	$I_2 = 4.848*10^8$ mm^4	$I_3 = 3.324*10^8$ mm^4
$A_1 = 17800$ mm^2	$A_2 = 14650$ mm^2	$A_3 = 11500$ mm^2
$Z_1 = 2.896*10^6$mm^3	$Z_2 = 2.204*10^6$ mm^3	$Z_3 = 1.511*10^6$ mm^3

Ultimate load calculation according to the Dutch code.

$$\lambda_{rel} = \frac{L}{\pi\sqrt{\frac{EI_1}{f_y A_1}}} = \frac{5000}{\pi\sqrt{\frac{210000*6.372*10^8}{355*17800}}}$$

λ_{rel} = 0.35 → ω_{buc} = 0.97

F_{max} = $\omega_{buc} f_y A_1$ = $0.97*355*17800$

F_{max} = 6129*10^3 N

Calculation of the Euler buckling load:

$$F_{max} = \frac{\pi^2 EI_1}{L^2} = \frac{\pi^2 210000*6.372*10^8}{5000^2}$$

F_{max} = 52827*10^3 N (Euler buckling load is larger than the plastic yield load)

Ultimate load calculation according to the formulas of the analysis of Appendix C.

$$e_{total,i} = \left(e_{total,i-1} + e_{m,i} + e_i\right) = \frac{\left(F_{i-1}+F_i\right)\frac{F_i z_i L^2}{8EI_i} + \left(e_{toatl,i-1} + \frac{F_i z_i}{8EI_i}\right)F_{E,i} - e_{total,i-1}F_{i-1}}{F_{E,i} - F_{i-1} - F_i}$$

e_{1-1} = $\frac{1}{1000}L$ = $\frac{1}{1000}5000$

e_{1-1} = 5 mm

$e_{m,i}$ = 0 mm

F_{1-1} = 0 N

z_1 = 0 mm

If these values will be used, the following formula exists:

$$e_{total,1} = \left(e_0 + e_1\right) = \frac{e_0 F_{E,1}}{F_{E,1} - F_1}$$

$F_{E,1}$ = 52827*10^3 N

$$e_1 = \frac{e_0 F_{E,1}}{F_{E,1} - F_1} - e_0 = \frac{5*52827*10^3}{52827*10^3 - F_1} - 5$$

$$M_1 = F_1\left(e_0 + e_1\right) = F_1 \frac{5*52827*10^3}{52827*10^3 - F_1}$$

$$\sigma_{right,1} = -\frac{M_1}{Z_1} - \frac{F_1}{A_1} = -\frac{F_1 \frac{5*52827*10^3}{52827*10^3 - F_1}}{2.896*10^6} - \frac{F_1}{17800} = -248.5 \text{ N/mm}^2$$

F_1 = 4280*10³ N
e_1 = 0.44 mm

$$\sigma_{left,1} = \frac{M_1}{Z_1} - \frac{F_1}{A_1} = \frac{4280*10^3 \frac{5*52827*10^3}{52827*10^3 - 4280*10^3}}{2.896*10^6} - \frac{4280*10^3}{17800}$$

$\sigma_{left,1}$ = -232.4 N/mm² Compression

F_1 = 0 + 4280*10³
F_1 = 4208*10³ N
$e_{total,1}$ = 5 + 0.44
$e_{total,1}$ = 5.44 mm

The section starts to yield at a load of 4208*10³ N. The moment of inertia decrease. The reduced Euler buckling load becomes:

$$F_{E,2} = \frac{\pi^2 E I_2}{L^2} = \frac{\pi^2 210000*4.848*10^8}{5000^2}$$

$F_{E,2}$ = 40192*10³ N

The Euler load is larger than the load on the column. The column can resist more loads. It is possible to increase the load till another part of the section yields too.

$$e_{total,i} = \left(e_{total,i-1} + e_{m,i} + e_i\right) = \frac{\left(F_{i-1} + F_i\right)\frac{F_i z_i L^2}{8EI_i} + \left(e_{toatl,i-1} + \frac{F_i z_i}{8EI_i}\right)F_{E,i} - e_{total,i-1}F_{i-1}}{F_{E,i} - F_{i-1} - F_i}$$

$$z_2 = \frac{\sum A_k z_k}{\sum A_k} - ½h$$

$$z_2 = \frac{(b - 2*¼b)t_f ½t_f + (h - 2t_f)t_w ½h + bt_f(h - ½t_f)}{(b - 2*¼b)t_f + (h - 2t_f)t_w + bt_f} - ½h$$

$$z_2 = \frac{150*21*10.5 + 398*11.5*220 + 300*21*429.5}{150*21 + 398*11.5 + 300*21} - 220$$

z_2 = 47 mm

$$e_{total,2} = \left(e_{total,1} + e_{m,2} + e_2\right) =$$

$$\frac{\left(4280*10^3 + F_2\right)\frac{F_2 47*5000^2}{8*210000*4.848*10^8} + \left(5 + 0.44 + \frac{F_2 47}{8*210000*4.848*10^8}\right)40192*10^3 - 4280*10^3\left(5 + 0.44\right)}{40192*10^3 - 4280*10^3 - \Delta F_2}$$

$$\sigma_{left,2} = \sigma_{left,2} = \sigma_{left,1} + \frac{M_2}{Z_2} - \frac{F_2}{A_2}$$

$$\sigma_{left,2} = -232.4 + \frac{M_2}{Z_2} - \frac{F_2}{A_2} \quad = -232.4 + \frac{F_1 e_2 + F_2(e_{toatl,1} + e_2)}{Z_2} - \frac{F_2}{A_2} \quad = -248.5$$

$$\sigma_{left,2} = -232.4 + \frac{4280 e_2 + F_2(5.44 + e_2)}{2.204 * 10^6} - \frac{F_2}{14650} \quad = -248.5 \text{ N/mm}^2$$

F_2 = 261*10^3 N
e_2 = 0.51 mm

$$\sigma_{right,2} = \sigma_{right,1} - \frac{M_2}{Z_2} - \frac{F_2}{A_2}$$

$$\sigma_{right,2} = -248.5 - \frac{M}{Z_2} - \frac{F_2}{A_2} \quad = -248.5 - \frac{F_1 e_2 + F_2(e_{total,1} + e_2)}{Z_2} - \frac{F_2}{A_2}$$

$$\sigma_{right,2} = -248.5 + \frac{4280 * 10^3 * 0.51 + 261 * 10^3 * (5.44 + 0.51)}{2.204 * 10^6} - \frac{261 * 10^3}{14650}$$

$\sigma_{right,2}$ = -268.0 N/mm^2

F_2 = 4280*10^3 + 261*10^3
F_2 = 4541*10^3 N
$e_{total,2}$ = 5.44 + 0.51
$e_{total,2}$ = 5.95 mm

The left flange starts to yield if the load is 4541*10^3 N. The moment of inertia decreases again. The new reduced Euler buckling load becomes:

$$F_{E,3} = \frac{\pi^2 E I_2}{L^2} = \frac{\pi^2 210000 * 3.324 * 10^8}{5000^2}$$

$F_{E,3}$ = 27558*10^3 N

Again the Euler load is larger than the load that is on the column. The column can resist more loads. It is possible to increase the load till also a third part of the section yields.

$$e_{total,i} = \left(e_{total,i-1} + e_{m,i} + e_i\right) = \frac{\left(F_{i-1} + F_i\right)\frac{F_i z_i L^2}{8EI_i} + \left(e_{toatl,i-1} + \frac{F_i z_i}{8EI_i}\right)F_{E,i} - e_{total,i-1}F_{i-1}}{F_{E,i} - F_{i-1} - F_i}$$

Both flanges have a yield part. Due to the used residual stress distribution, the yielding areas in the midsection are the same in both flanges. The stress in the midsection has been generalized over the whole section. It is assumed that the yielding areas in the whole section are the same. The effective section become double symmetric again. The location of the centre of gravity in

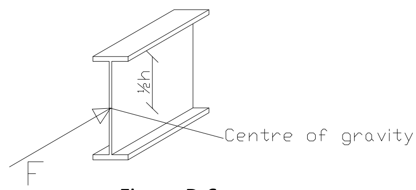


Figure D.6: New centre of gravity

the effective section is the same as the location of the centre of gravity in the original section. The shift of the centre of gravity is zero.

z_3 = 0 mm

$$e_{total,3} = \left(e_{total,2} + e_{m,3} + e_3\right) = \frac{5.95*27558*10^3 - 5.95*4541*10^3}{27558*10^3 - 4541*10^3 - F_3}$$

$$\sigma_{right,3} = \sigma_{right,2} - \frac{M_3}{Z_3} - \frac{F_3}{A_3}$$

$$\sigma_{right,3} = -268.0 - \frac{M_3}{Z_3} - \frac{F_3}{A_3} = -268.0 - \frac{F_2 e_3 + F_3(e_{total,2} + e_3)}{Z_3} - \frac{F_3}{A_3} = -461.5$$

$$\sigma_{right,3} = -268.0 - \frac{4541*10^3 e_3 + F_3(5.95 + e_3)}{1.511*10^6} - \frac{F_3}{8900} = -461.5 \text{ N/mm}^2$$

F_3 = $1648*10^3$ N
e_3 = 0.46 mm

$$\sigma_{left,3} = \sigma_{left,2} + \frac{M_3}{Z_3} - \frac{F_3}{A_3}$$

$$\sigma_{left,3} = -248.5 + \frac{M_3}{Z_3} - \frac{F_3}{A_3} = -248.5 + \frac{F_2 e_\partial + F_3(e_{total,2} + e_3)}{Z_3} - \frac{F_3}{A_3}$$

$$\sigma_{left,3} = -248.5 + \frac{4541*10^3 * 0.46 + 1648*10^3(5.95 + 0.46)}{1.511*10^6} - \frac{1648*10^3}{8900}$$

$\sigma_{left,3}$ = -425.3 N/mm^2 Compression

F_3 = $4541*10^3 + 1648*10^3$
F_3 = $6189*10^3$ N
$e_{total,3}$ = 5.95 + 0.46
$e_{total,3}$ = 6.41 mm

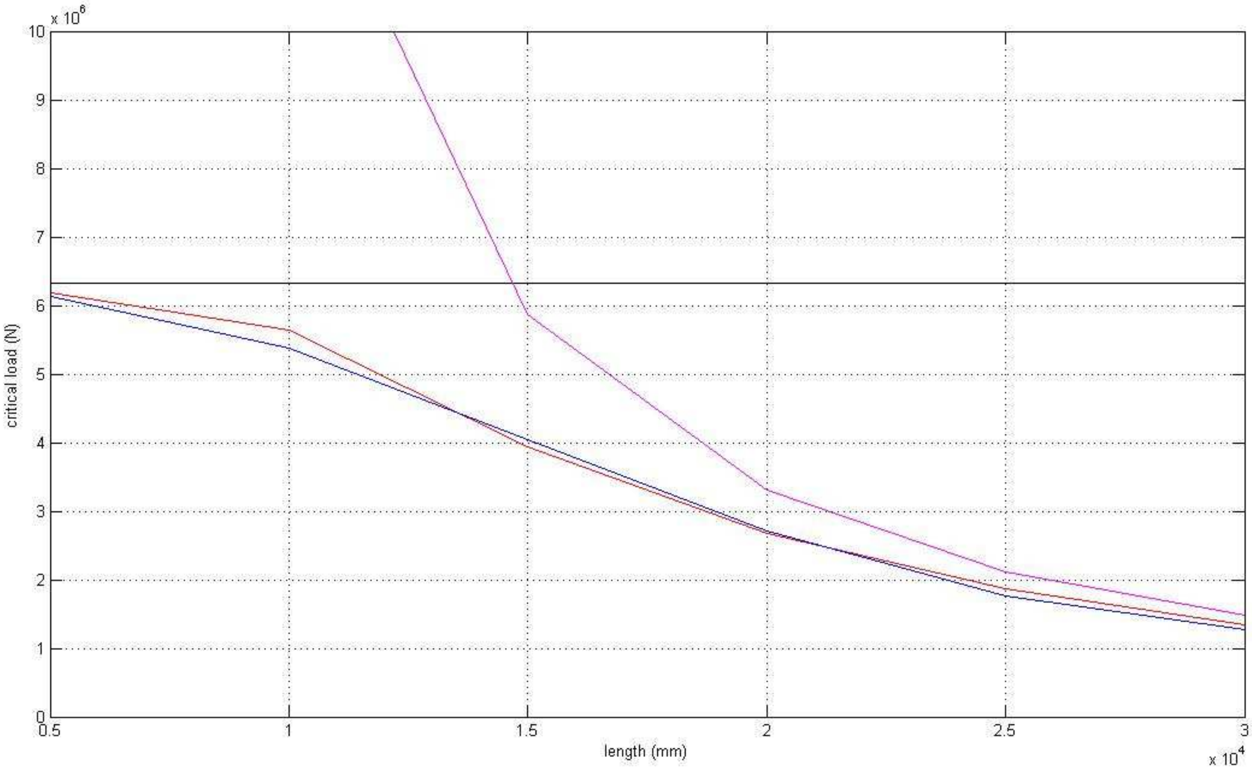
F_{max} = $6189*10^3$ N

The right flange fully yields and even the left flange has a large compression stress. The column will fail at a load of $6189*10^3$ N.

D.9 Conclusions

For several lengths is the ultimate load is calculated. There are only small differences between the ultimate load according to Dutch code and the ultimate load according to the formulas of the analysis of Appendix C. For a better result, the residual stress distribution and the yielding zone must be changed. The residual stress distribution is taken as a rectangular distribution. The real residual stress distribution is unknown, but is a more curved residual stress distribution is close to reality. The stress in the midsection has been generalized over the whole section. For a better analysis, the section must be split in several parts. Every part has his own section properties. A finite element analysis is the result.

The calculated ultimate loads are presented in Figure D.7.



- The purple line is the Euler buckling load
- The black line is the plastic yield load
- The red line is the ultimate load according to the Dutch code
- The blue line is the ultimate load according to the formulas of the analysis in Appendix C

Figure D.17: Force / length diagram

Appendix E Calculation example (MatLab)

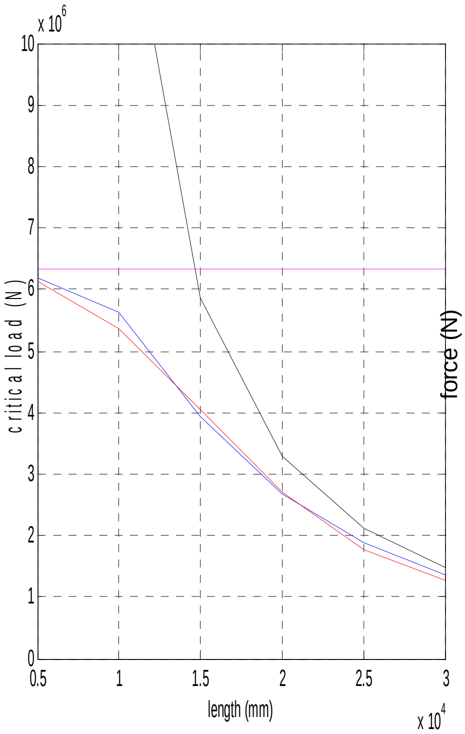
In Appendix D some hand-made calculations are made for the calculation of the ultimate load for a single column. The formulas which are used are derived in Appendix C. Appendix E exists in two parts. The first part is the general part. In this part the way of calculation and the results of the calculations are discussed. In the second part, the calculation file is inserted.

E.1 Computer calculations in general

To make calculations, the section properties are needed. The section properties of all HEA are inserted in the m-file. The correct section can be chosen by a number. For a good comparison a HE 450A section has been chosen. This is the 16th section of the list.

MatLab can calculate many iterations. Many lengths can be calculated in a small time period. There has been chosen for all lengths (with steps of one meter) between one and fifty meters. This corresponds to a relative slenderness between zero and three.

In Appendix D, in Figure D.7 a force length diagram is given. This diagram is copied to Figure E.1. As result of the computer calculations, the same force-length diagram can be made (Fig. E.2). The graphic in Figure E.2 is a more flowing line compare with the graphic in Figure E.1. Cause of this is the larger amount of ultimate load calculations. For both figures is the Euler buckling load the black line, the yield load is the purple line, the red line is according to the Dutch code and the red line is according to the formulas from the analysis of Appendix C.



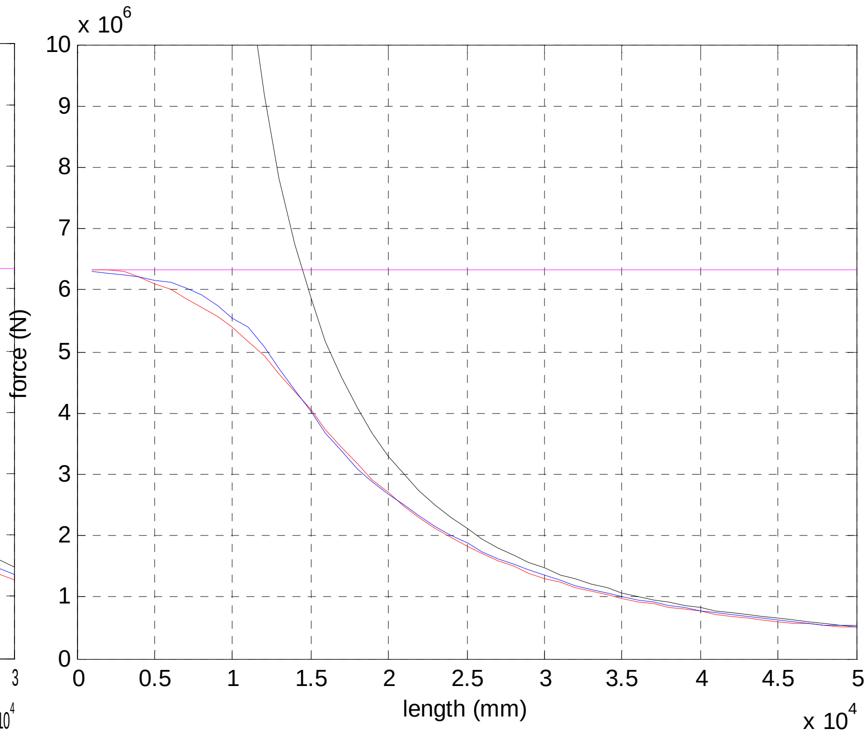


Figure E.1:
Force / length diagram
Hand –made calculations

Both Figure E.1 and E.2 shows that the ultimate load calculated by the analysis (derived in appendix C) is close to the ultimate load according to the Dutch code. One point of attention is the ultimate load for a column length between ten and eleven meters. At this length the flowing line has an interruption. The cause of this interruption is the change in load case. A more flowing residual stress distribution and a better yielding zone result in a more flowing line. If these simplifications will be made, the analysis becomes much more complex.

E.2 Calculation file

This is the calculation file for the computer program MatLab. This file is used to calculate the ultimate load for a single column.

```
clear; clf; clc; close;
E=210000;
fy=355;
FF=1e3;%belastingsstap

%input profiles
HEA=[2.124E+03  2.534E+03   3.142E+03   3.877E+03   4.525E+03   5.383E+03
6.434E+03   7.684E+03   8.682E+03   9.726E+03   1.125E+04   1.244E+04
1.335E+04   1.428E+04   1.590E+04   1.780E+04   1.975E+04   2.118E+04
2.265E+04   2.416E+04   2.605E+04   2.858E+04   3.205E+04   3.468E+04;%A
(cross-section)

3.492E+06   6.062E+06   1.033E+07   1.673E+07   2.510E+07   3.692E+07
5.410E+07   7.763E+07   1.046E+08   1.367E+08   1.826E+08   2.293E+08
2.769E+08   3.309E+08   4.507E+08   6.372E+08   8.698E+08   1.119E+09
1.412E+09   1.752E+09   2.153E+09   3.034E+09   4.221E+09   5.538E+09;%I
(moment of inertia)

8.301E+04   1.195E+05   1.735E+05   2.451E+05   3.249E+05   4.295E+05
5.685E+05   7.446E+05   9.198E+05   1.112E+06   1.383E+06   1.628E+06
1.850E+06   2.088E+06   2.562E+06   3.216E+06   3.949E+06   4.622E+06
5.350E+06   6.136E+06   7.032E+06   8.699E+06   1.081E+07   1.282E+07;%Z
plastic (Section modulus)

7.276E+04   1.063E+05   1.554E+05   2.201E+05   2.936E+05   3.886E+05
5.152E+05   6.751E+05   8.364E+05   1.013E+06   1.260E+06   1.479E+06
1.678E+06   1.891E+06   2.311E+06   2.896E+06   3.550E+06   4.146E+06
4.787E+06   5.474E+06   6.241E+06   7.682E+06   9.485E+06   1.119E+07;%Z
elastic (Section modulus)

4.055E+01   4.891E+01   5.734E+01   6.569E+01   7.448E+01   8.282E+01
9.170E+01   1.005E+02   1.097E+02   1.186E+02   1.274E+02   1.358E+02
1.440E+02   1.522E+02   1.684E+02   1.892E+02   2.099E+02   2.299E+02
2.497E+02   2.693E+02   2.875E+02   3.258E+02   3.629E+02   3.996E+02;%i
(Gyration radius)

96 114 133 152 171 190 210 230 250 270 290 310 330 350 390 440 490 540 590
640 690 790 890 990;%h height

100 120 140 160 180 200 220 240 260 280 300 300 300 300 300 300 300 300 300
300 300 300 300 300;%b width
```

```
8 8 8.5 9 9.5 10 11 12 12.5 13 14 15.5 16.5 17.5 19 21 23 24 25 26 27 28 30
31;%tf thickness flange

5 5 5.5 6 6 6.5 7 7.5 7.5 8 8.5 9 9.5 10 11 11.5 12 12.5 13 13.5 14.5 15 16
16.5];%tw thickness web

m=16;
    A(1,1)=HEA(1,m);
    I(1,1)=HEA(2,m);
    Z(1,1)=HEA(4,m);
    h=HEA(6,m);
    i=HEA(5,m);
    b=HEA(7,m);
    tf=HEA(8,m);
    tw=HEA(9,m);

    Np=A*fy;
    Mp=Z*fy;

     if m<=14 ;
         S=0.5;
         ak=0.34;
     else
         if m<=24;
             S=0.3;
             ak=0.21;
         else
             if m<=38;
                 S=0.5;
                 ak=0.49;
             else
                 S=0.3;
                 ak=0.34;
             end% if
         end% if
     end% if
    yield1=-(1-S)*fy;
    yield2=-(1+S)*fy;

A(1,2)=A(1,1)-2*(0.25*b)*tf;
A(1,3)=A(1,2)-0.5*tw*(h-2*tf)-2*(1-0.25*pi)*27^2;
A(1,4)=A(1,3)-2*(0.25*b)*tf;
I(1,2)=I(1,1)-2*(0.25*b)*tf*(0.5*h)^2;
I(1,3)=I(1,2);
I(1,4)=I(1,3)-2*(0.25*b)*tf*(0.5*h)^2;
Z(1,2)=2*I(1,2)/h;
Z(1,3)=2*I(1,3)/h;
Z(1,4)=2*I(1,4)/h;

for k=1:50;
    L(k,1)=1000*k;
    e0(k,1)=L(k,1)/1000;
    fy1(k)=fy*A(1,1);

% According to NEN 6771
    labda(k,1)=L(k,1)/(pi*sqrt(E*I(1,1)/(A(1,1)*fy)));
    omega(k,1)=((1+ak*(labda(k,1)-0.2)+labda(k,1)^2)-
sqrt((1+ak*(labda(k,1)-0.2)+labda(k,1)^2)^2-
4*labda(k,1)^2))/(2*labda(k,1)^2);
    if omega(k,1)>=1;
```

```
        omega(k,1)=1;
    else
        omega(k,1)=omega(k,1);
    end% if omega
    Fnen(k,1)=A(1,1)*fy*omega(k,1);

% According to Euler
    FE(k,1)=pi^2*E*I(1,1)/(L(k,1)^2);
    FE(k,2)=pi^2*E*I(1,2)/(L(k,1)^2);
    FE(k,3)=pi^2*E*I(1,3)/(L(k,1)^2);
    FE(k,4)=pi^2*E*I(1,4)/(L(k,1)^2);

% According to residual stress method
    sigmatop(k,1)=0;
    f=0;
    while sigmatop(k,1)>yield1;
        f=f+1;
        deltaF1(k,f)=FF*f;
        F1(k,1)=max(deltaF1(k,f));
        Ftotal(k,f)=F1(k,1);
        f1(k,1)=F1(k,1)/FF;
        deltae1(k,f)=e0(k,1)*FE(k,1)/(FE(k,1)-deltaF1(k,f))-e0(k,1);
        etotal(k,f)=deltae1(k,f)+e0(k,1);
        emax(k,1)=max(etotal(k,f));
        e1(k,1)=max(deltae1(k,:));
        deltaM1(k,f)=deltaF1(k,f)*(e0(k,1)+deltae1(k,f));
        sigmatop(k,1)=-deltaM1(k,f)/Z(1,1)-deltaF1(k,f)/A(1,1);
        sigmatop1(k,1)=sigmatop(k,1);
        sigmabottom(k,1)=deltaM1(k,f)/Z(1,1)-deltaF1(k,f)/A(1,1);
        sigmabottom1(k,1)=sigmabottom(k,1);
        sigmacentre(k,1)=-deltaF1(k,f)/A(1,1);
        sigmacentre1(k,1)=sigmacentre(k,1);
        Frs1(k,1)=max(deltaF1(k,f));
        Frs(k,1)=Frs1(k,1);
    end% while

    Frs2(k,1)=0;
    if FE(k,2)>Frs(k,1)+2*FF;
        f=0;
        z=(0.5*b*tf*0.5*tf+(h-2*tf)*tw*0.5*h+b*tf*(h-0.5*tf))/(0.5*b*tf+(h-
2*tf)*tw+b*tf)-0.5*h;

        while sigmatop(k,1)>yield2 & sigmacentre(k,1)>yield1;
        f=f+1;
        deltaF2(k,f)=f*FF;
        F2(k,1)=F1(k,1)+max(deltaF2(k,f));
        Ftotal(k,f+f1(k,1))=F1(k,1)+max(deltaF2(k,f));
        f2(k,1)=F2(k,1)/FF;
        hulp=z/(8*E*I(1,2));

etotal(k,f+f1(k,1))=(FE(k,2)*(e0(k,1)+e1(k,1)+deltaF2(k,f)*hulp*L(k,1)^2)-
F1(k,1)*(e0(k,1)+e1(k,1)))/(FE(k,2)-F1(k,1)-deltaF2(k,f));
        emax(k,1)=max(etotal(k,f+f1(k,1)));
        deltae2(k,f)=etotal(k,f+f1(k,1))-e0(k,1)-e1(k,1);
        e2(k,1)=max(deltae2(k,:))+e1(k,1);
        deltaM2(k,f)=F1(k,1)*deltae2(k,f)+deltaF2(k,f)*etotal(k,f+f1(k,1));
        sigmatop(k,1)=sigmatop1(k,1)-deltaM2(k,f)/Z(1,2)-
deltaF2(k,f)/A(1,2);
        sigmatop2(k,1)=sigmatop(k,1);
        sigmabottom(k,1)=sigmabottom1(k,1)+deltaM2(k,f)/Z(1,2)-
deltaF2(k,f)/A(1,2);
```

```
        sigmabottom2(k,1)=sigmabottom(k,1);
        sigmacentre(k,1)=sigmacentre1(k,1)-deltaF2(k,f)/A(1,2);
        sigmacentre2(k,1)=sigmacentre(k,1);
        Frs2(k,1)=max(deltaF2(k,f));
        end% while
        Frs(k,1)=Frs1(k,1)+Frs2(k,1);
    else
        Frs(k,1)=Frs(k,1);
    end% if FE(k,2)

        Frs3(k,1)=0;

        if FE(k,3)>Frs(k,1)+2*FF;
        f=0;
        z=(0.5*b*tf*0.5*tf+(h-2*tf)*tw*0.5*h+b*tf*(h-0.5*tf))/(0.5*b*tf+(h-
2*tf)*tw+b*tf)-0.5*h;

        while sigmatop(k,1)>yield2 & sigmabottom(k,1)>yield1;
        f=f+1;
        deltaF3(k,f)=f*FF;
        F3(k,1)=F2(k,1)+max(deltaF3(k,f));
        Ftotal(k,f+f2(k,1))=F2(k,1)+max(deltaF3(k,f));
        f3(k,1)=F3(k,1)/FF;
        hulp=z/(8*E*I(1,3));

etotal(k,f+f2(k,1))=(FE(k,3)*(e0(k,1)+e2(k,1)+deltaF3(k,f)*hulp*L(k,1)^2)-
F2(k,1)*(e0(k,1)+e2(k,1)))/(FE(k,3)-F2(k,1)-deltaF3(k,f));
        emax(k,1)=max(etotal(k,f+f2(k,1)));
        deltae3(k,f)=etotal(k,f+f2(k,1))-e0(k,1)-e2(k,1);
        e3(k,1)=max(deltae3(k,:))+e2(k,1);
        deltaM3(k,f)=F2(k,1)*deltae3(k,f)+deltaF3(k,f)*etotal(k,f+f2(k,1));
        sigmatop(k,1)=sigmatop2(k,1)-deltaM3(k,f)/Z(1,3)-
deltaF3(k,f)/A(1,3);
        sigmatop3(k,1)=sigmatop(k,1);
        sigmabottom(k,1)=sigmabottom2(k,1)+deltaM3(k,f)/Z(1,3)-
deltaF3(k,f)/A(1,3);
        sigmabottom3(k,1)=sigmabottom(k,1);
        sigmacentre(k,1)=sigmacentre2(k,1)-deltaF3(k,f)/A(1,3);
        sigmacentre3(k,1)=sigmacentre(k,1);
        Frs3(k,1)=max(deltaF3(k,f));
        end% while
        Frs(k,1)=Frs1(k,1)+Frs2(k,1)+Frs3(k,1);
    else
        Frs(k,1)=Frs(k,1);
    end% if FE(k,2)


Frs4(k,1)=0;
%z=0;
    if FE(k,4)>Frs(k,1)+2*FF;
        f=0;
        z=0;

        while sigmatop(k,1)>yield2;
        f=f+1;
        deltaF4(k,f)=f*FF;
        F4(k,1)=F3(k,1)+max(deltaF4(k,f));
        Ftotal(k,f+f3(k,1))=F3(k,1)+max(deltaF4(k,f));
        f4(k,1)=F4(k,1)/FF;
        hulp=z/(8*E*I(1,3));
```

```
etotal(k,f+f3(k,1))=(FE(k,4)*(e0(k,1)+e3(k,1)+deltaF4(k,f)*hulp*L(k,1)^2)-
F3(k,1)*(e0(k,1)+e3(k,1)))/(FE(k,4)-F3(k,1)-deltaF4(k,f));
        emax(k,1)=max(etotal(k,f+f3(k,1)));
        deltae4(k,f)=etotal(k,f+f3(k,1))-e0(k,1)-e3(k,1);
        e4(k,1)=max(deltae4(k,:))+e3(k,1);
        deltaM4(k,f)=F3(k,1)*deltae4(k,f)+deltaF4(k,f)*etotal(k,f+f3(k,1));
        sigmatop(k,1)=sigmatop3(k,1)-deltaM4(k,f)/Z(1,4)-
deltaF4(k,f)/A(1,4);
        sigmatop4(k,1)=sigmatop(k,1);
        sigmabottom(k,1)=sigmabottom3(k,1)+deltaM4(k,f)/Z(1,4)-
deltaF4(k,f)/A(1,4);
        sigmabottom4(k,1)=sigmatop(k,1);
        sigmacentre(k,1)=sigmacentre3(k,1)-deltaF4(k,f)/A(1,4);
        sigmacentre4(k,1)=sigmacentre(k,1);
        Frs4(k,1)=max(deltaF4(k,f));
        end% while
            Frs(k,1)=Frs1(k,1)+Frs2(k,1)+Frs3(k,1)+Frs4(k,1);
    else
        Frs(k,1)=Frs(k,1);
    end% if FE(k,2)

    for kk=1:(Frs1(1,1)+Frs2(1,1)+Frs3(1,1)+Frs4(1,1))/FF;
        if etotal(k,kk)==0;
            etotal(k,kk)=emax(k,1);
        else
            etotal(k,kk)=etotal(k,kk);
        end% if etotal
        if Ftotal(k,kk)==0;
            Ftotal(k,kk)=Frs(k,1);
        else
            Ftotal(k,kk)=Ftotal(k,kk);
        end% if Ftotal
    end% for kk

    MF=[6070000; 6042000; 6015000; 5978000; 5930000; 5879000; 5815000;
5749000; 5657000; 5545000; 5399000; 5220000; 4990000; 4710000; 4395000;
4065000; 3740000; 3429000; 3145000; 2886000; 2650000; 2436000; 2251000;
2080000; 1928000; 1790000; 1668000; 1555000; 1455000; 1361000; 1278000;
1202000; 1132000; 1069000; 1011000; 956000; 906000; 860000; 818000; 778000;
741000; 707000; 675000; 645000; 617000; 591000; 566000; 543000; 522000;
502000];

    omegars(k,1)=Frs(k,1)/(A(1,1)*fy);
    omegaeuler(k,1)=1/(labda(k,1)^2);
    omegaMF(k,1)=MF(k,1)/(A(1,1)*fy);


end% for k

    %handcalculations
Lhand=[5000 10000 15000 20000 25000 30000];
fyhand=[6319e3 6319e3 6319e3 6319e3 6319e3 6319e3];
FEhand=[52827e3 13207e3  5870e3 3302e3 2113e3 1476e3];
Frshand=[6189e3 5634e3 3940e3 2679e3 1874e3 1348e3];
FNenhand=[6129e3 5371e3 4044e3 2717e3 1769e3 1264e3];
labdahand=[5000/(pi*sqrt(E*I(1,1)/(fy*A(1,1))))
10000/(pi*sqrt(E*I(1,1)/(fy*A(1,1)))) 15000/(pi*sqrt(E*I(1,1)/(fy*A(1,1))))
20000/(pi*sqrt(E*I(1,1)/(fy*A(1,1)))) 25000/(pi*sqrt(E*I(1,1)/(fy*A(1,1))))
30000/(pi*sqrt(E*I(1,1)/(fy*A(1,1))))];
```

```
omegahand=Frshand/(fy*A(1,1));

%plot(L,Fnen,'red');hold
on;plot(L,FE(:,1),'black');plot(L,Frs,'blue');axis([0 50000 0
1e7]);xlabel('length (mm)');ylabel('force (N)');grid;
%plot(labda,omegaMF,'green');hold
on;plot(labda,omegaeuler,'black');plot(labda,omega,'red');plot(labda,omegar
s,'blue');axis([0 3 0 1.1]);xlabel('relative
slenderness');ylabel('reduction factor');grid;
%plot(Lhand,fyhand,'black');hold on;
plot(Lhand,FEhand,'m');plot(Lhand,Frshand,'r');plot(Lhand,FNenhand,'blue');
axis([5000 30000 0 1e7]);xlabel('length (mm)');ylabel('critical load
(N)');grid


clear k;
%clear kk;

%for x=2:6295;if Ftotal(5,x)<Ftotal(5,(x-1));Ftotal(5,x)=Ftotal(5,(x-
1));end;end
%for x=2:6295;if etotal(5,x)<etotal(5,(x-1));etotal(5,x)=etotal(5,(x-
1));end;end
%plot(etotal(5,:),Ftotal(5,:))
%plot(etotal(25,:),Ftotal(25,:));xlabel('deflection in the midsection
(mm)');ylabel('critical load (N)');axis([0 250 0 2e6]);title('Length is 25
meter');grid
```

Appendix F Calculation example (Matrix Frame)

In Appendix D and in Appendix E some ultimate load calculations are made. Manually calculations in Appendix D and computer calculations (with the program MatLab) in Appendix E. All these ultimate load calculations are based on the formulas derived in Appendix C. In this Appendix the ultimate load of the same structures are made by the computer program Matrix Frame. Matrix Frame is a computer program based on the finite element method (FEM). The calculations in this appendix are made as a reference.

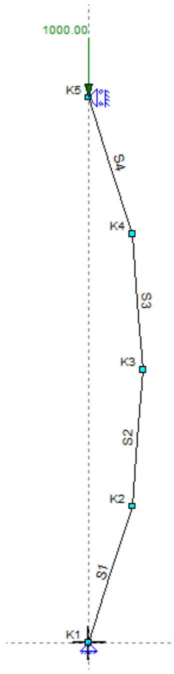


Figure F.1
Input column

F.1 Calculations in general

Matrix Frame can only calculate straight sections. To calculate with an initial deflection the column is split in four parts. The middle of the column (K3) has a horizontal displacement of $\frac{L}{1000}$. The horizontal displacement of point K2 and point K4 is $\frac{0.8L}{1000}$ (Fig. F.1, the horizontal displacement is elongate to get a clear view). These horizontal deflections correspond to a curved column.

Matrix Frame has a four calculations method. These calculations methods are:

- Geometrical Linear elastic
- Geometrical Non-Linear elastic
- Physical Non-Linear
- Physical and Geometrical Non-Linear

F.2 Different calculations

Four different calculations have been made for a column with a length of ten meters and a HE450 A section. The different calculations are: geometrical linear elastic, geometrical non-linear elastic, physical non-linear and physical and geometrical non-linear.

The geometrical linear elastic calculations results in a linear relation between the load on the column and the deflection of the column. The linear elastic calculation does not take yielding into account. There is no load limit in this calculation method.

The geometric non-linear elastic calculation is based on the $\frac{n}{n-1}$ formula. This formula is derived in Appendix A.2. There is a non-linear relation between the load and the deflection. The limit of the load is the Euler buckling load (F_E=13207*103 N).

The physical non-linear calculation (method 3) does take yielding into account. Due to partial the effective section properties decreases. The deflection increases. The calculation stops if the whole mid-section yields.

The physical and geometrical non-linear calculation methods take both the geometrical and yielding influence into account. The normal force multiplied by the deflection results in a bending moment. The section fails due to a combination between the normal force and the bending moment.

Matrix Frame uses a load-control calculation method. If the load increases, the deflection increases. The calculation cannot continue if the ultimate load has been reached. Matrix Frame cannot calculate the post buckling behaviour.

The results of the four calculation methods are displayed in Figure F.2 and Figure F.3. Figure F.2 is the general load-deflection graphic. Figure F.3 is the same graphic zoomed in on the critical values.

- The red line is the Linear elastic analysis.
- The blue line is the Geometrical Non-Linear elastic analysis.
- The green line is the Physical Non-Linear analysis.
- The black line is the Physical and Geometrical Non-Linear

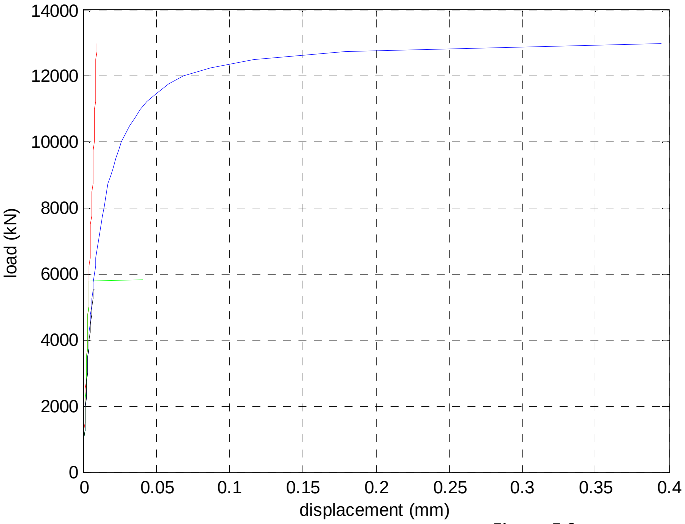


Figure F.2
Calculation methods

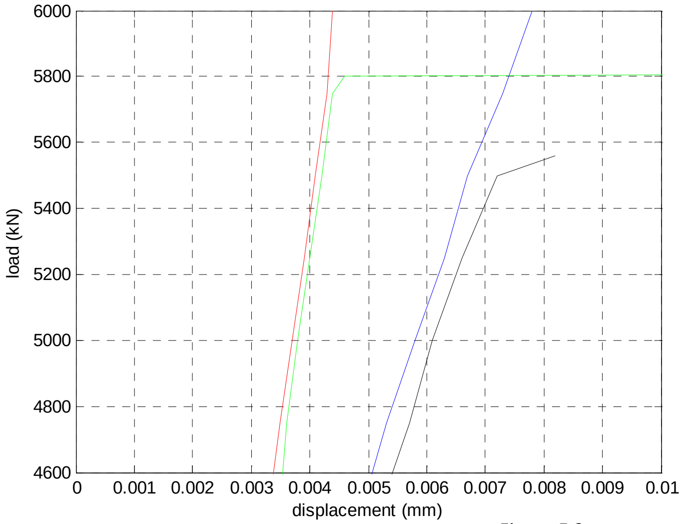


Figure F.3
Calculation methods

F.3 Maximum load calculation

The physical non-linear and the physical and geometrical non-linear calculation method have the possibility to calculate the ultimate load. The ultimate load is calculated on several lengths. With these ultimate loads it is possible to draw a reduction factor curve. The results can be found in Figure F.4. In the graphic is horizontal axis is the relative slenderness and vertical axis is the reduction factor. The green line is the line according to Matrix-Frame calculation, the Dutch code results (red line) and the calculations of Appendix E (blue line) are drawn in the same Figure. As reference also the Euler buckling load (black line) is inserted in Figure F.4.

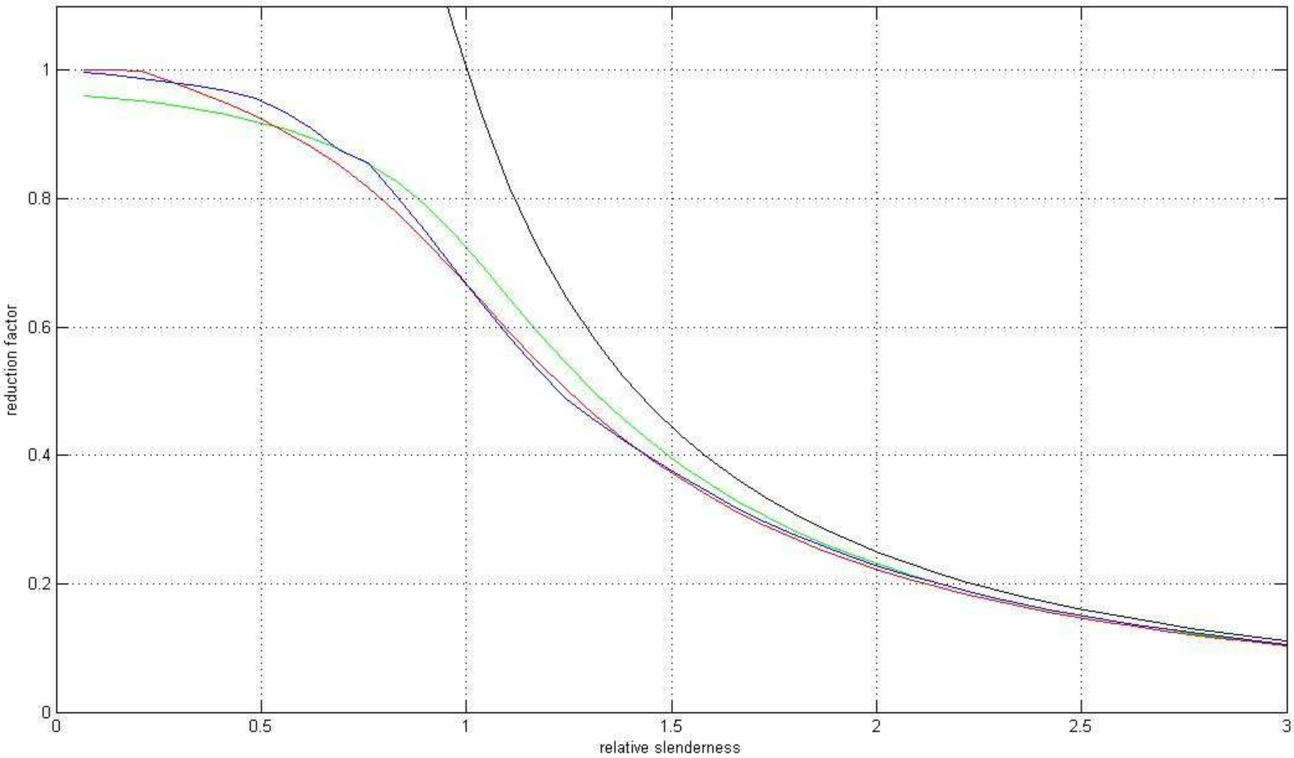


Figure F.4
Instabilitycurve

F.4 Residual stress in Matrix Frame

In the buckling calculation of Appendix D and of Appendix E the residual stress distribution of Figure F.5 is taken into account. Three possibilities are worked out to make calculations with residual stresses. These possibilities are clearly described in Chapter 2.9 of the main report.

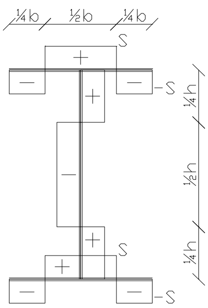


Figure F.5: Residual stress distribution

To take the residual stresses into account, two calculations are made. The section parameters of all sections have been halved. The first calculation is the calculation with steel grade S235. The second calculation is the calculation with steel grade S460. The section properties and the construction are the same in both calculations. In other words: the geometrical deflections are equal in both calculations. The only difference is the physical non-linear deflection. These differences are very small and can be neglected.

F.5 Matrix Frame file

This part of the Analysis exists of the calculation file. In this file the starting positions as well as the results are given.

L

Appendix G Calculation according to the Dutch code

This Appendix is about the ultimate load calculation of the column of an unbraced portal frame according to the Dutch code. Important for the ultimate load calculation of the length of a portal frame is the buckling length. The buckling length is not the same as the length of the of the column of the portal frame The buckling length depends on different aspects. First (at the most important) is the type of the portal frame. Is the portal frame braced or not. The buckling length of a braced portal frame is maximum the column length. The buckling length of an unbraced portal frame is at least twice the column length. The second criterion for the buckling length is the rotation freedom of the column.

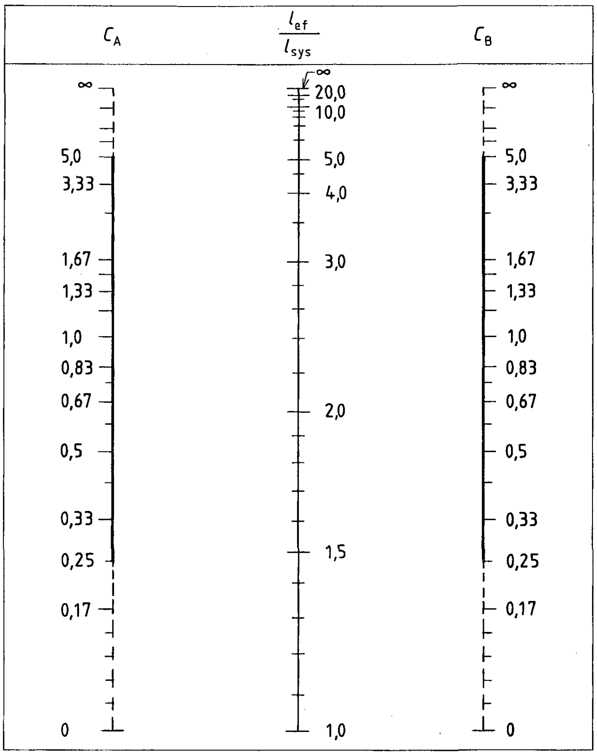


Figure G.1: Buckling length (NEN 6770, art. 12.1.1.3)

The rotation freedom depends on the stiffness of the column and the stiffness of the beam. A very stiff beam will not rotate. A very soft beam rotates very easily.

The portal frame of Chapter three is an unbraced portal frame supported by two hinges. A hinge can rotate freely. The rotation freedom is infinity. The other end of the column is connected with the beam and the rotation freedom must be calculated by the following formula:

$$C = \frac{\sum \frac{I_{cln}}{L_{cln}}}{\sum \mu \frac{I_{bm}}{L_{bm}}} \quad \text{(NEN 6770 art. 12.1.1.3)}$$

In witch μ is a correction factor. This correction factor depends on the boundary conditions at the other end of the beam. In the case of a portal frame μ=3.

The value of both rotation freedoms must be filled in a graphic (Fig. G.1) to find the buckling length.

The calculation of a column loaded by a normal force only is discussed in Appendix D. Talking about a portal frame, the column is loaded by a normal force only and bending moment. The bending moment results in extra deflection and in extra stresses. Due to the bending moment, the ultimate load decreases. The following formula must be used to calculate the buckling load.

$$\frac{N_{c;s;d}}{N_{c;u;d}} + \frac{n_y}{n_y - 1}\frac{M_{y;equ;s;d} + F_{y;tot;s;d}e_y^*}{\omega_{lb} M_{y;u;d}} + \frac{n_z}{n_z - 1}\frac{\chi_y M_{y;equ;s;d}}{M_{z;u;d}} \leq 1.0 \qquad \text{(NEN 6771 art. 12.3.1.2)}$$

The stability in this study is based on a single bending moment. The bending moment $M_{y;equ;s;d}$ is neglected. It is assumed that the columns are critical. Failure mechanism of the beam is neglected. In other words: $\omega_{lb} = 0$. If these assumptions are used, the formula can be simplified. The following formula exists:

$$\frac{N_{c;s;d}}{N_{c;u;d}} + \frac{n_y}{n_y - 1} \frac{M_{y;equ;s;d} + F_{y;tot;s;d} e_y^*}{M_{y;u;d}} \leq 1.0$$

The following expressions must be used in the formula.

$$n_y = \frac{F_E}{F_{y;tot;s;d}}$$

$$e_y^* = \alpha_k \left(\lambda_{y;rel} - \lambda_0 \right) \frac{M_{y;u;d}}{N_{c;u;d}}$$

$M_{y;equ;s;d}$ depends on the different moments on the structure and the type of structure. The following formulas must be used.

$$M_{y;equ;s;d} = \max \begin{pmatrix} \left| 0.1\left(M_{y;2;s;d} - M_{y;1;s;d} \right) + M_{y;mid;s;d} \right| \\ \left| 0.6 M_{y;2;s;d} \right| \end{pmatrix}$$ For a braced structure

$$M_{y;equ;s;d} = \max \begin{pmatrix} \left| M_{y;mid;s;d} \right| \\ \left| 0.85 M_{y;2;s;d} \right| \end{pmatrix}$$ For an unbraced structure

By these formulas the buckling load can be calculated.

Appendix H Linear analysis portal frame

The main subject of this Appendix is an unbraced portal frame. See Figure H.1 for a schematisation of the structure. For this structure several analyses have been made. This Appendix is about the linear analysis. See Appendix I,J and K for other analyses. The analysed portal frame is out of square and supported on two hinges. The portal frame is loaded by a uniformly distributed load. It is assumed that the structure fails if one of the columns fails.

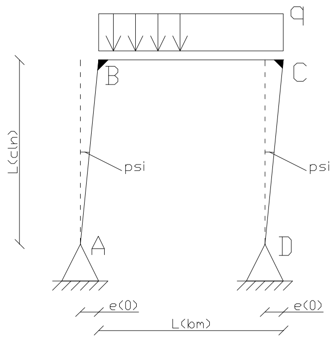


Figure H.1: Structure

H.1 In general

The result of the linear analysis is a straight line in the load-deflection graphic. This line is displayed in Figure H.2.

To start the analysis the equilibrium formulas will be used. The result of moment equilibrium is:

$$\sum T \mid A = 0$$

$$qL_{bm}\left(\tfrac{1}{2}L_{bm} + e_0\right) - V_D L_{bm} = 0$$

The starting deflection is very small compare with the length of the beam. The influence of this deflection can be neglected at the calculation of the vertical reaction force. This results in:

$$\tfrac{1}{2}qL_{bm}{}^2 - V_D L_{bm} = 0$$

$$V_D = \frac{qL_{bm}\left(\tfrac{1}{2}L_{bm}\right)}{L_{bm}}$$

$$V_D = \tfrac{1}{2}qL_{bm}$$

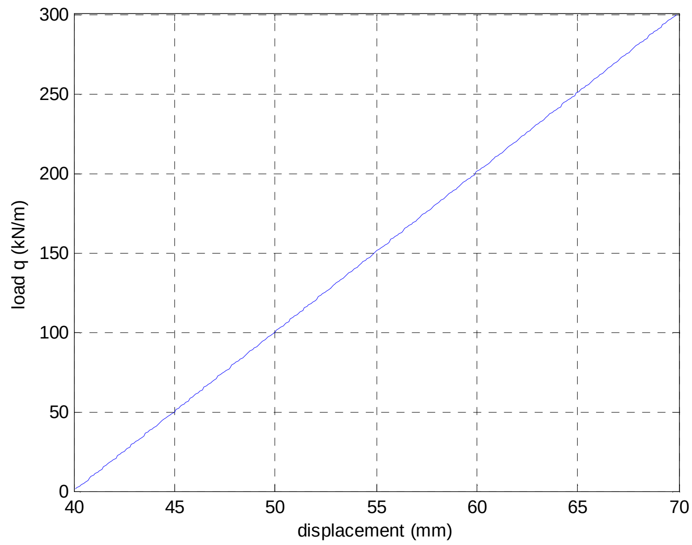


Figure H.2: Load-deflection Graphic

Colum:	Length 10 m HE 360A
Beam:	Length 5 m HE 900A

Also the translation equilibriums are calculated:

Vertical equilibrium:

$$\sum F_{vert} = 0$$

$$qL_{bm} - V_A - V_D = 0$$

$$qL_{bm} - V_A - \tfrac{1}{2}qL_{bm} = 0$$

$$V_A = \tfrac{1}{2}qL_{bm}$$

Horizontal equilibrium:

$$\sum F_{hor} = 0$$

$$H_A - H_D = 0$$

$$H_A = H_D$$

The portal frame is a one degree statically undetermined structure. The three equilibriums are not enough to calculate all reaction forces and the deflections. A fourth formula must be found. This formula is related to the deflections. Both column AB and column CD will deflect. The elongation of the beam is very small compare with other deflections. Te elongation of the beam is neglected. The deflection of both columns must be the same.

The total deflection depends on the rotation of the beam and on the bending of the column (Fig. H.3). The floor of an unbraced structure is not supported horizontally. The displacement of the floor will only be limited by the columns. The supports of the portal frame are hinges. These hinges can rotate without limitations. The rotation freedom of the supports and the translation freedom of the floor can be schematized as a cantilever beam with one free end. The loads on the cantilever beam are the reaction forces of the supports.

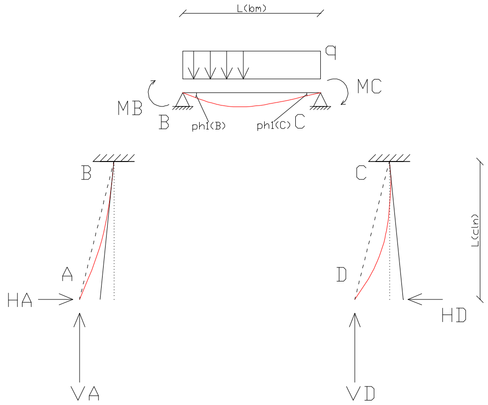


Figure H.3: Deformation

Important for the stability of the portal frame are the bending moments in the connection of the columns with the beam. These moments can be calculated as follow:

$$M_B = V_A e_0 - H_A L_{cln} \qquad M_C = V_D e_0 + H_A L_{cln}$$
$$M_B = ½ q L_{bm} \psi L_{cln} - H_A L_{cln} \qquad M_C = ½ q L_{bm} \psi L_{cln} + H_A L_{cln}$$
$$M_B = \left(½ \psi q L_{bm} - H_A\right) L_{cln} \qquad M_C = \left(½ \psi q L_{bm} + H_A\right) L_{cln}$$

The portal frame is out of square. The horizontal and vertical reaction forces are not parallel and perpendicular to the working line of the cantilever beam. The remaining force is the load perpendicular to the working line (Fig. H.4). The remaining force depends on both the horizontal and the vertical reaction force. The remaining force depends on the deflection too. In the linear analysis is this deflection is the initial deflection.

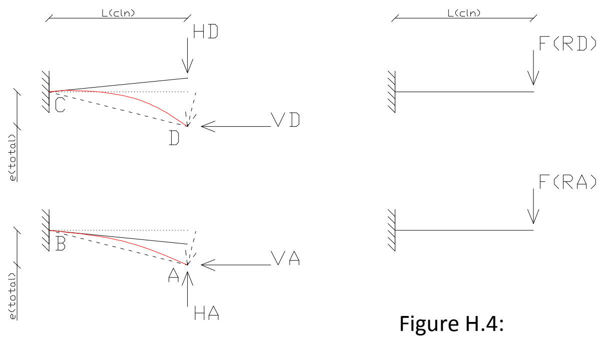


Figure H.4: Remaining force

The remaining forces at support A and D are:

$$F_{R,A} = \frac{V_A e_0}{L_{cln}} - H_A \qquad F_{R,D} = \frac{V_D e_0}{L_{cln}} + H_A$$
$$F_{R,A} = \frac{½ q L_{bm} \psi L_{cln}}{L_{cln}} - H_A \qquad F_{R,D} = \frac{½ q L_{bm} \psi L_{cln}}{L_{cln}} + H_A$$
$$F_{R,A} = ½ \psi q L_{bm} - H_A \qquad F_{R,D} = ½ \psi q L_{bm} + H_A$$

H.2 Analysis

The analysis starts with the rotation of the beam.

$$\varphi_B = \frac{-q L_{bm}^{\ 3}}{24 EI_{bm}} - \frac{M_B L_{bm}}{3 EI_{bm}} + \frac{M_C L_{bm}}{6 EI_{bm}}$$

The total deflection of the column AB is as follow:

$$e_{total}^{AB} = -\varphi_B L_{cln} + \psi L_{cln} + \frac{F_{R,A} L_{cln}^{3}}{3EI_{cln}}$$

The formula of the rotation in point B can be used. This results in:

$$e_{total}^{AB} = -\left(\frac{-qL_{bm}^{3}}{24EI_{bm}} - \frac{M_B L_{bm}}{3EI_{bm}} + \frac{M_C L_{bm}}{6EI_{bm}} \right) L_{cln} + \psi L_{cln} + \frac{F_{R,A} L_{cln}^{3}}{3EI_{cln}}$$

This results in:

$$e_{total}^{AB} = \frac{qL_{bm}^{3} L_{cln}}{24EI_{bm}} + \frac{M_B L_{bm} L_{cln}}{3EI_{bm}} - \frac{M_C L_{bm} L_{cln}}{6EI_{bm}} + \psi L_{cln} + \frac{F_{R,A} L_{cln}^{3}}{3EI_{cln}}$$

The formulas of the bending moments and the remaining force are known. These formulas can be applied.

$$e_{total}^{AB} = \frac{qL_{bm}^{3} L_{cln}}{24EI_{bm}} + \frac{\left(\left(½\psi qL_{bm} - H_A\right) L_{cln}\right) L_{bm} L_{cln}}{3EI_{bm}} - \frac{\left(\left(½\psi qL_{bm} + H_A\right) L_{cln}\right) L_{bm} L_{cln}}{6EI_{bm}} + \psi L_{cln}$$
$$+ \frac{\left(½\psi qL_{bm} - H_A\right) L_{cln}^{3}}{3EI_{cln}}$$

Some brackets can be removed.

$$e_{total}^{AB} = \frac{qL_{bm}^{3} L_{cln}}{24EI_{bm}} + \frac{\left(½\psi qL_{bm} - H_A\right) L_{bm} L_{cln}^{2}}{3EI_{bm}} - \frac{\left(½\psi qL_{bm} + H_A\right) L_{bm} L_{cln}^{2}}{6EI_{bm}} + \psi L_{cln} + \frac{\left(½\psi qL_{bm} - H_A\right) L_{cln}^{3}}{3EI_{cln}}$$

The formulas are written out. Some expressions can be combined together. This results in the following formula:

$$e_{total}^{AB} = \frac{qL_{bm}^{3} L_{cln}}{24EI_{bm}} + \frac{\psi qL_{bm}^{2} L_{cln}^{2}}{12EI_{bm}} - \frac{H_A L_{bm} L_{cln}^{2}}{2EI_{bm}} + \psi L_{cln} + \frac{\psi qL_{bm} L_{cln}^{3}}{6EI_{cln}} - \frac{H_A L_{cln}^{3}}{3EI_{cln}}$$

A formula is found to calculate the total deflection in column AB. The same analysis can be made for column CD. This analysis also starts at the rotation of the beam.

$$\varphi_C = \frac{qL_{bm}^{3}}{24EI_{bm}} + \frac{M_B L_{bm}}{6EI_{bm}} - \frac{M_C L_{bm}}{3EI_{bm}}$$

The total deflection is:

$$e_{total}^{CD} = -\varphi_C L_{cln} + \psi L_{cln} + \frac{F_{R,D} L_{cln}^{3}}{3EI_{cln}}$$

The formula of the rotation can be used.

$$e_{total}^{CD} = -\left(\frac{qL_{bm}^{3}}{24EI_{bm}} + \frac{M_B L_{bm}}{6EI_{bm}} - \frac{M_C L_{bm}}{3EI_{bm}} \right) L_{cln} + \psi L_{cln} + \frac{F_{R,D} L_{cln}^{3}}{3EI_{cln}}$$

This results in:

$$e_{total}{}^{CD} = -\frac{qL_{bm}{}^3L_{cln}}{24EI_{bm}} - \frac{M_BL_{bm}L_{cln}}{6EI_{bm}} + \frac{M_CL_{bm}L_{cln}}{3EI_{bm}} + \psi L_{cln} + \frac{F_{R,D}L_{cln}{}^3}{3EI_{cln}}$$

The formulas of the bending moments and the remaining forces are known. These formulas can be applied.

$$e_{total}{}^{CD} = -\frac{qL_{bm}{}^3L_{cln}}{24EI_{bm}} - \frac{\left(\left(½\psi qL_{bm} - H_A\right)L_{cln}\right)L_{bm}L_{cln}}{6EI_{bm}} + \frac{\left(\left(½\psi qL_{bm} + H_A\right)L_{cln}\right)L_{bm}L_{cln}}{3EI_{bm}} + \psi L_{cln}$$
$$+\frac{\left(½\psi qL_{bm} + H_A\right)L_{cln}{}^3}{3EI_{cln}}$$

Some brackets can be removed.

$$e_{total}{}^{CD} = -\frac{qL_{bm}{}^3L_{cln}}{24EI_{bm}} - \frac{\left(½\psi qL_{bm} - H_A\right)L_{bm}L_{cln}{}^2}{6EI_{bm}} + \frac{\left(½\psi qL_{bm} + H_A\right)L_{bm}L_{cln}{}^2}{3EI_{bm}} + \psi L_{cln} + \frac{\left(½\psi qL_{bm} + H_A\right)L_{cln}{}^3}{3EI_{cln}}$$

The formulas are written out. Some expressions are the same and can be combined together. This results in the following formula:

$$e_{total}{}^{CD} = -\frac{qL_{bm}{}^3L_{cln}}{24EI_{bm}} + \frac{\psi qL_{bm}{}^2L_{cln}{}^2}{12EI_{bm}} + \frac{H_AL_{bm}L_{cln}{}^2}{2EI_{bm}} + \psi L_{cln} + \frac{\psi qL_{bm}L_{cln}{}^3}{6EI_{cln}} + \frac{H_AL_{cln}{}^3}{3EI_{cln}}$$

Two formulas are found to calculate the total deflection. One formula for the total deflection for column AB and one formula for the total deflection for column CD. Both formulas have two unknowns. The first unknown is the total deflection and the second unknown is the horizontal reaction force. The total deflection in both columns is the same $\left(e_{total}{}^{AB} = e_{total}{}^{CD}\right)$.

The following formula is created.

$$\frac{qL_{bm}{}^3L_{cln}}{24EI_{bm}} + \frac{\psi qL_{bm}{}^2L_{cln}{}^2}{12EI_{bm}} - \frac{H_AL_{bm}L_{cln}{}^2}{2EI_{bm}} + \psi L_{cln} + \frac{\psi qL_{bm}L_{cln}{}^3}{6EI_{cln}} - \frac{H_AL_{cln}{}^3}{3EI_{cln}}$$
$$=$$
$$-\frac{qL_{bm}{}^3L_{cln}}{24EI_{bm}} + \frac{\psi qL_{bm}{}^2L_{cln}{}^2}{12EI_{bm}} + \frac{H_AL_{bm}L_{cln}{}^2}{2EI_{bm}} + \psi L_{cln} + \frac{\psi qL_{bm}L_{cln}{}^3}{6EI_{cln}} + \frac{H_AL_{cln}{}^3}{3EI_{cln}}$$

The same expressions can be combined or neglected. This results in the following formula:

$$\frac{qL_{bm}{}^3L_{cln}}{12EI_{bm}} - \frac{H_AL_{bm}L_{cln}{}^2}{EI_{bm}} - \frac{2H_AL_{cln}{}^3}{3EI_{cln}} = 0$$

The only unknown value is H_A. This value has been separate from the rest of the formula.

$$H_A\left(\frac{L_{bm}L_{cln}{}^2}{EI_{bm}} + \frac{2L_{cln}{}^3}{3EI_{cln}}\right) = \frac{qL_{bm}{}^3L_{cln}}{12EI_{bm}}$$

The formula is not clear enough. All expressions are multiplied to get the same denominator.

$$H_A\left(\frac{12L_{bm}L_{cln}{}^{2}EI_{cln}}{12EI_{bm}EI_{cln}}+\frac{8L_{cln}{}^{3}EI_{bm}}{12EI_{bm}EI_{cln}}\right)=\frac{qL_{bm}{}^{3}L_{cln}EI_{cln}}{12EI_{bm}EI_{cln}}$$

The denominator can be neglected. The formula of the horizontal reaction force is created.

$$H_A=\frac{qL_{bm}{}^{3}L_{cln}EI_{cln}}{12L_{bm}L_{cln}{}^{2}EI_{cln}+8L_{cln}{}^{3}EI_{bm}}$$

The formula that is found can be checked on the dimensions.

$$N=\frac{Nm^{-1}m^{3}mNm^{-2}m^{4}}{mm^{2}Nm^{-2}m^{4}+m^{3}Nm^{-2}m^{4}}$$

$$N=\frac{N^{2}m^{5}}{Nm^{5}+Nm^{5}}$$

The dimensions of the formula are correct. The formula does not have big mistakes.

The last part of this Appendix is a list of all formulas. All forces and the maximum deflection can be calculated by these formulas.

$$V_D=\tfrac{1}{2}qL_{bm}$$

$$V_A=\tfrac{1}{2}qL_{bm}$$

$$H_A=\frac{qL_{bm}{}^{3}L_{cln}EI_{cln}}{12L_{bm}L_{cln}{}^{2}EI_{cln}+8L_{cln}{}^{3}EI_{bm}}$$

$$H_D=\frac{qL_{bm}{}^{3}L_{cln}EI_{cln}}{12L_{bm}L_{cln}{}^{2}EI_{cln}+8L_{cln}{}^{3}EI_{bm}}$$

$$M_B=\left(\tfrac{1}{2}\psi qL_{bm}-H_A\right)L_{cln}$$

$$M_C=\left(\tfrac{1}{2}\psi qL_{bm}+H_A\right)L_{cln}$$

$$e_{total}=-\frac{qL_{bm}{}^{3}L_{cln}}{24EI_{bm}}+\frac{\psi qL_{bm}{}^{2}L_{cln}{}^{2}}{12EI_{bm}}+\frac{H_AL_{bm}L_{cln}{}^{2}}{2EI_{bm}}+\psi L_{cln}+\frac{\psi qL_{bm}L_{cln}{}^{3}}{6EI_{cln}}+\frac{H_AL_{cln}{}^{3}}{3EI_{cln}}$$

Appendix I Residual stress in the linear analysis

In Appendix H the linear analysis of a portal frame has been analyzed. The analysis in Appendix H does not take residual stress into account. Due to residual stress, a loaded structure will yield in parts. Due to partial yielding the effective stiffness decrease. The deflection increase. The residual stress distribution is described clearly in Chapter 1 and in Appendix D. The residual stress distribution can be found in Figure I.1.

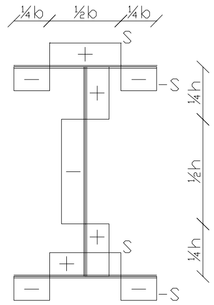


Figure I.1: Residual stress distribution

It is assumed that the beam will not fail. The portal frame has two columns. Both columns can fail. The portal frame is out of square and the columns are not loaded equally. One column is heavier loaded than the other column. In this analysis only one column yields and the other column keep the original stiffness. The assumption that the beam will not fail must be checked afterwards.

I.1 *In general*

Due to the used residual stress distribution, the column of the portal frame can partial yield in three parts. The first part is the half of the right flange. The second part is the half of the left flange. The third (and last) part is the second half of the right flange. If the third part is yielded, the stiffness has decreased too much. The structure cannot resist more loads. The structure fails. The order of the analysis in this Appendix is also related to the loading part.

In Figure I.2 the load-deflection graphic is displayed. The blue line is the load displacement graphic is the residual stress is taken into account. The red line is the original displacement graphic (without residual stress). The green line is the deflection if the second part (left flange) does not yield.

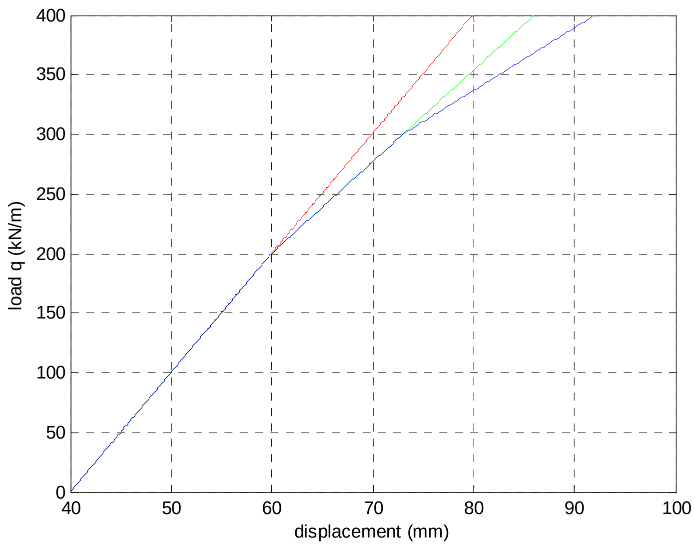


Figure I.2: Load-deflection Graphic

Column: Length 10 m HE 360A
Beam: Length 5 m HE 900A

I.2 *Analysis if one part yields*

In this part of the Appendix the structure is loaded by an original load. The effective stiffness of the column is decreased. The half of the right flange is already yielding. This is the first yield part. The second load case starts. The first load case is the linear analysis according to Appendix H.

Before the analysis starts it is important to know what the symbols are.

q_1 original load
e_1 original deformation
$H_{A,1}$ original horizontal reaction force in point A
$V_{A,1}$ original vertical reaction force in point A
$F_{R,A,1}$ original remaining force in point A
$H_{D,1}$ original horizontal reaction force in point D
$V_{D,1}$ original vertical reaction force in point D
$F_{R,D,1}$ original remaining force in point D

The following formulas are known from Appendix H:

$$V_{D,total,2} = ½ q_{total,2} L_{bm}$$

$$V_{A,total,2} = ½ q_{total,2} L_{bm}$$

$$H_{A,total,2} = H_{D,total,2}$$

$$M_{B,total,2} = \left(½ \psi q_{total,2} L_{bm} - H_{A,total,2} \right) L_{cln}$$

$$M_{C,total,2} = \left(½ \psi q_{total,2} L_{bm} + H_{A,total,2} \right) L_{cln}$$

$$\varphi_{B,total,2} = \frac{-q_{total,2} L_{bm}^{3}}{24 EI_{bm}} - \frac{M_{B,total,2} L_{bm}}{3 EI_{bm}} + \frac{M_{C,total,2} L_{bm}}{6 EI_{bm}}$$

$$\varphi_{C,total,2} = \frac{q_{total,2} L_{bm}^{3}}{24 EI_{bm}} + \frac{M_{B,total,2} L_{bm}}{6 EI_{bm}} - \frac{M_{C,total,2} L_{bm}}{3 EI_{bm}}$$

Al formulas above do not depend on the stiffness of the column. These formulas are still valid. If the formulas depends on the stiffness of the column the formula must be separate in an original part and an additional part. For the original part, the original stiffness must be used. For the additional part the reduced stiffness must be applied.

$$e_{total,2}^{AB} = -\varphi_{B,total,2} L_{cln} + \psi L_{cln} + \frac{F_{R,A,1} L_{cln}^{3}}{3 EI_{cln,1}} + \frac{F_{R,A,2} L_{cln}^{3}}{3 EI_{cln,2}}$$

$$e_{total,2}^{CD} = -\varphi_{C,total,2} L_{cln} + \psi L_{cln} + \frac{F_{R,D,1} L_{cln}^{3}}{3 EI_{cln,1}} + \frac{F_{R,D,2} L_{cln}^{3}}{3 EI_{cln,2}}$$

The loads $F_{R,A}$ and $F_{R,D}$ are split in the formulas. The formulas of this expression must be separated too. The formulas are relative easy because of the linearity of this analysis.

$$F_{R,A,1} = ½ \psi q_1 L_{bm} - H_{A,1}$$

$$F_{R,A,2} = ½ \psi q_2 L_{bm} - H_{A,2}$$

$$F_{R,D,1} = ½ \psi q_1 L_{bm} + H_{A,1}$$

$$F_{R,D,2} = ½ \psi q_2 L_{bm} + H_{A,2}$$

The total deflection of column AB can be analysed. The analysis starts with the following formula:

$$e_{total,2}^{AB} = -\varphi_{B,total,2} L_{cln} + \psi L_{cln} + \frac{F_{R,A,1} L_{cln}^{3}}{3 EI_{cln,1}} + \frac{F_{R,A,2} L_{cln}^{3}}{3 EI_{cln,2}}$$

Filled in the formula of $\varphi_{B,total,2}$ results in:

$$e_{total,2}^{AB} = -\left(\frac{-q_{total,2} L_{bm}^{3}}{24EI_{bm}} - \frac{M_{B,total,2} L_{bm}}{3EI_{bm}} + \frac{M_{C,total,2} L_{bm}}{6EI_{bm}} \right) L_{cln} + \psi L_{cln} + \frac{F_{R,A,1} L_{cln}^{3}}{3EI_{cln,1}} + \frac{F_{R,A,2} L_{cln}^{3}}{3EI_{cln,2}}$$

The formulas of $M_{B,total,2}$, $M_{C,total,2}$, $F_{R,A,1}$ and $F_{R,A,2}$ are known and can be used.

$$e_{total,2}^{AB} = -\left(\frac{-q_{total,2} L_{bm}^{3}}{24EI_{bm}} - \frac{\left(\left(\tfrac{1}{2}\psi q_{total,2} L_{bm} - H_{A,total,2}\right) L_{cln}\right) L_{bm}}{3EI_{bm}} + \frac{\left(\left(\tfrac{1}{2}\psi q_{total,2} L_{bm} + H_{A,total,2}\right) L_{cln}\right) L_{bm}}{6EI_{bm}} \right) L_{cln}$$
$$+\psi L_{cln} + \frac{\left(\tfrac{1}{2}\psi q_{1} L_{bm} - H_{A,1}\right) L_{cln}^{3}}{3EI_{cln,1}} + \frac{\left(\tfrac{1}{2}\psi q_{2} L_{bm} - H_{A,2}\right) L_{cln}^{3}}{3EI_{cln,2}}$$

All expressions can be written out.

$$e_{total,2}^{AB} = \frac{q_{total,2} L_{bm}^{3} L_{cln}}{24EI_{bm}} + \frac{\psi q_{total,2} L_{bm}^{2} L_{cln}^{2}}{6EI_{bm}} - \frac{H_{A,total,2} L_{bm} L_{cln}^{2}}{3EI_{bm}} - \frac{\psi q_{total,2} L_{bm}^{2} L_{cln}^{2}}{12EI_{bm}} - \frac{H_{A,total,2} L_{bm} L_{cln}^{2}}{6EI_{bm}}$$
$$+\psi L_{cln} + \frac{\psi q_{1} L_{bm} L_{cln}^{3}}{6EI_{cln,1}} - \frac{H_{A,1} L_{cln}^{3}}{3EI_{cln,1}} + \frac{\psi q_{2} L_{bm} L_{cln}^{3}}{6EI_{cln,2}} - \frac{H_{A,2} L_{cln}^{3}}{3EI_{cln,2}}$$

Some expressions can be combined together. This results in the following formula:

$$e_{total,2}^{AB} = \frac{q_{total,2} L_{bm}^{3} L_{cln}}{24EI_{bm}} + \frac{\psi q_{total,2} L_{bm}^{2} L_{cln}^{2}}{12EI_{bm}} - \frac{H_{A,total,2} L_{bm} L_{cln}^{2}}{2EI_{bm}} + \psi L_{cln}$$
$$+ \frac{\psi q_{1} L_{bm} L_{cln}^{3}}{6EI_{cln,1}} - \frac{H_{A,1} L_{cln}^{3}}{3EI_{cln,1}} + \frac{\psi q_{2} L_{bm} L_{cln}^{3}}{6EI_{cln,2}} - \frac{H_{A,2} L_{cln}^{3}}{3EI_{cln,2}}$$

In the formula above two parameters are unknown. These unknowns are: the total deflection and the difference in horizontal reaction force. The same analysis can be used for the total deflection in column CD. The total deflection of column CD can be calculated by the following formula:

$$e_{total,2}^{CD} = -\varphi_{C,total,2} L_{cln} + \psi L_{cln} + \frac{F_{R,D,1} L_{cln}^{3}}{3EI_{cln,1}} + \frac{F_{R,D,2} L_{cln}^{3}}{3EI_{cln,2}}$$

Filled in the formula of $\varphi_{C,total,2}$ results in:

$$e_{total,2}^{CD} = -\left(\frac{q_{total,2} L_{bm}^{3}}{24EI_{bm}} + \frac{M_{B,total,2} L_{bm}}{6EI_{bm}} - \frac{M_{C,total,2} L_{bm}}{3EI_{bm}} \right) L_{cln} + \psi L_{cln} + \frac{F_{R,D,1} L_{cln}^{3}}{3EI_{cln,1}} + \frac{F_{R,D,2} L_{cln}^{3}}{3EI_{cln,2}}$$

The formulas of $M_{B,total,2}$, $M_{C,total,2}$, $F_{R,D,1}$ and $F_{R,D,2}$ are known and can be used.

$$e_{total,2}{}^{CD} = -\left(\frac{q_{total,2} L_{bm}{}^{3}}{24EI_{bm}} + \frac{\left(\left(½\psi q_{total,2} L_{bm} - H_{A,total,2}\right) L_{cln}\right) L_{bm}}{6EI_{bm}} - \frac{\left(\left(½\psi q_{total,2} L_{bm} + H_{A,total,2}\right) L_{cln}\right) L_{bm}}{3EI_{bm}} \right) L_{cln}$$

$$+\psi L_{cln} + \frac{\left(½\psi q_{1} L_{bm} + H_{A,1}\right) L_{cln}{}^{3}}{3EI_{cln,1}} + \frac{\left(½\psi \Delta q_{2} L_{bm} + H_{A,2}\right) L_{cln}{}^{3}}{3EI_{cln,2}}$$

All expressions can be written out.

$$e_{total,2}{}^{CD} = -\frac{q_{total,2} L_{bm}{}^{3} L_{cln}}{24EI_{bm}} - \frac{\psi q_{total,2} L_{bm}{}^{2} L_{cln}{}^{2}}{12EI_{bm}} + \frac{H_{A,total,2} L_{bm} L_{cln}{}^{2}}{6EI_{bm}} + \frac{\psi q_{total,2} L_{bm}{}^{2} L_{cln}{}^{2}}{6EI_{bm}} + \frac{H_{A,total,2} L_{bm} L_{cln}{}^{2}}{3EI_{bm}}$$

$$+\psi L_{cln} + \frac{\psi q_{1} L_{bm} L_{cln}{}^{3}}{6EI_{cln,1}} + \frac{H_{A,1} L_{cln}{}^{3}}{3EI_{cln,1}} + \frac{\psi q_{2} L_{bm} L_{cln}{}^{3}}{6EI_{cln,2}} + \frac{H_{A,2} L_{cln}{}^{3}}{3EI_{cln,2}}$$

Some expressions can be combined together. This results in the following formula:

$$e_{total,2}{}^{CD} = -\frac{q_{total,2} L_{bm}{}^{3} L_{cln}}{24EI_{bm}} + \frac{\psi q_{total,2} L_{bm}{}^{2} L_{cln}{}^{2}}{12EI_{bm}} + \frac{H_{A,total,2} L_{bm} L_{cln}{}^{2}}{2EI_{bm}} + \psi L_{cln}$$

$$+\frac{\psi q_{1} L_{bm} L_{cln}{}^{3}}{6EI_{cln,1}} + \frac{H_{A,1} L_{cln}{}^{3}}{3EI_{cln,1}} + \frac{\psi q_{2} L_{bm} L_{cln}{}^{3}}{6EI_{cln,2}} + \frac{H_{A,2} L_{cln}{}^{3}}{3EI_{cln,2}}$$

For column AB and for column CD a formula is found to calculate the total deflection. Both formulas have two unknowns. The first unknown is the total deflection and the second unknown is the extra horizontal reaction force. The total deflection in both columns is the same $\left(e_{total,2}{}^{AB} = e_{total,2}{}^{CD}\right)$.

The following formula is created.

$$\frac{q_{total,2} L_{bm}{}^{3} L_{cln}}{24EI_{bm}} + \frac{\psi q_{total,2} L_{bm}{}^{2} L_{cln}{}^{2}}{12EI_{bm}} - \frac{H_{A,total,2} L_{bm} L_{cln}{}^{2}}{2EI_{bm}} + \psi L_{cln}$$

$$+\frac{\psi q_{1} L_{bm} L_{cln}{}^{3}}{6EI_{cln,1}} - \frac{H_{A,1} L_{cln}{}^{3}}{3EI_{cln,1}} + \frac{\psi q_{2} L_{bm} L_{cln}{}^{3}}{6EI_{cln,2}} - \frac{H_{A,2} L_{cln}{}^{3}}{3EI_{cln,2}}$$

$$=$$

$$-\frac{q_{total,2} L_{bm}{}^{3} L_{cln}}{24EI_{bm}} + \frac{\psi q_{total,2} L_{bm}{}^{2} L_{cln}{}^{2}}{12EI_{bm}} + \frac{H_{A,total,2} L_{bm} L_{cln}{}^{2}}{2EI_{bm}} + \psi L_{cln}$$

$$+\frac{\psi q_{1} L_{bm} L_{cln}{}^{3}}{6EI_{cln,1}} + \frac{H_{A,1} L_{cln}{}^{3}}{3EI_{cln,1}} + \frac{\psi q_{2} L_{bm} L_{cln}{}^{3}}{6EI_{cln,2}} + \frac{H_{A,2} L_{cln}{}^{3}}{3EI_{cln,2}}$$

Some expressions are the same. These expressions can be combined or neglected.

$$\frac{q_{total,2} L_{bm}{}^{3} L_{cln}}{12EI_{bm}} - \frac{H_{A,total,2} L_{bm} L_{cln}{}^{2}}{EI_{bm}} - \frac{2H_{A,1} L_{cln}{}^{3}}{3EI_{cln,1}} - \frac{H_{A,2} L_{cln}{}^{3}}{3EI_{cln,2}} - \frac{H_{A,2} L_{cln}{}^{3}}{3EI_{cln,2}} = 0$$

$H_{A,2}$ is the desired expression. This expression will be separated from the rest of the formula. Attention: in the expression of $H_{A,total,2}$ is also a $H_{A,2}$ expression hided.

$$\frac{H_{A,2}L_{bm}L_{cln}^{2}}{EI_{bm}}+\frac{2\Delta H_{A,2}L_{cln}^{3}}{3EI_{cln,2}}=\frac{q_{total,2}L_{bm}^{3}L_{cln}}{12EI_{bm}}-\frac{H_{A,1}L_{bm}L_{cln}^{2}}{EI_{bm}}-\frac{2H_{A,1}L_{cln}^{3}}{3EI_{cln,1}}$$

To make the formula more clear all expressions must have the same denominator. This results in:

$$\frac{12H_{A,2}L_{bm}L_{cln}^{2}EI_{cln,1}EI_{cln,2}}{12EI_{bm}EI_{cln,1}EI_{cln,2}}+\frac{8H_{A,2}L_{cln}^{3}EI_{bm}EI_{cln,1}}{12EI_{bm}EI_{cln,1}EI_{cln,2}}=$$

$$\frac{q_{total,2}L_{bm}^{3}L_{cln}EI_{cln,1}EI_{cln,2}}{12EI_{bm}EI_{cln,1}EI_{cln,2}}-\frac{12H_{A,1}L_{bm}L_{cln}^{2}EI_{cln,1}EI_{cln,2}}{12EI_{bm}EI_{cln,1}EI_{cln,2}}-\frac{8H_{A,1}L_{cln}^{3}EI_{bm}EI_{cln,2}}{12EI_{bm}EI_{cln,1}EI_{cln,2}}$$

Every expression has the same denominator. This denominator can be neglected.

$$12H_{A,2}L_{bm}L_{cln}^{2}EI_{cln,1}EI_{cln,2}+8H_{A,2}L_{cln}^{3}EI_{bm}EI_{cln,1}$$
$$=q_{total,2}L_{bm}^{3}L_{cln}EI_{cln,1}EI_{cln,2}-12H_{A,1}L_{bm}L_{cln}^{2}EI_{cln,1}EI_{cln,2}-8H_{A,1}L_{cln}^{3}EI_{bm}EI_{cln,2}$$

This formula can be used to find the formula of $H_{A,2}$.

$$H_{A,2}=\frac{q_{total,2}L_{bm}^{3}L_{cln}EI_{cln,1}EI_{cln,2}-12H_{A,1}L_{bm}L_{cln}^{2}EI_{cln,1}EI_{cln,2}-8H_{A,1}L_{cln}^{3}EI_{bm}EI_{cln,2}}{12L_{bm}L_{cln}^{2}EI_{cln,1}EI_{cln,2}+8L_{cln}^{3}EI_{bm}EI_{cln,1}}$$

All parameters of this formula are known. The formula of $H_{A,1}$ is known and can be used. It is also possible to calculate with the numerical value of $H_{A,1}$.

Two checks are made to avoid mistakes. First a check on the dimension.

$$N=\frac{Nm^{-1}m^{3}mNm^{-2}m^{4}Nm^{-2}m^{4}-Nmm^{2}Nm^{-2}m^{4}Nm^{-2}m^{4}-Nm^{3}Nm^{-2}m^{4}Nm^{-2}m^{4}}{mm^{2}Nm^{-2}m^{4}Nm^{-2}m^{4}+m^{3}Nm^{-2}m^{4}Nm^{-2}m^{4}}$$

$$N=\frac{N^{3}m^{7}-N^{3}m^{7}-N^{3}m^{7}}{N^{2}m^{7}+N^{2}m^{7}}$$

The dimensions are correct.

The second check is the check on the stiffness. If $EI_{cln,1}=EI_{cln,2}=EI_{cln}$ the horizontal reaction force must be equal to the formula found in appendix H.

$$H_{A,2}=\frac{q_{total,2}L_{bm}^{3}L_{cln}EI_{cln}EI_{cln}-12H_{A,1}L_{bm}L_{cln}^{2}EI_{cln}EI_{cln}-8H_{A,1}L_{cln}^{3}EI_{bm}EI_{cln}}{12L_{bm}L_{cln}^{2}EI_{cln}EI_{cln}+8L_{cln}^{3}EI_{bm}EI_{cln}}$$

All expressions van divided by EI_{cln}.

$$H_{A,2}=\frac{q_{total,2}L_{bm}^{3}L_{cln}EI_{cln}-12H_{A,1}L_{bm}L_{cln}^{2}EI_{cln}-8H_{A,1}L_{cln}^{3}EI_{bm}}{12L_{bm}L_{cln}^{2}EI_{cln}+8L_{cln}^{3}EI_{bm}}$$

The formula can be split in two expressions. The first expression depends on the total load and the second expression depends on the original horizontal reaction force.

$$H_{A,2} = \frac{q_{total,2} L_{bm}{}^3 L_{cln} EI_{cln}}{12 L_{bm} L_{cln}{}^2 EI_{cln} + 8 L_{cln}{}^3 EI_{bm}} - \frac{H_{A,1}\left(12 L_{bm} L_{cln}{}^2 EI_{cln} + 8 L_{cln}{}^3 EI_{bm}\right)}{12 L_{bm} L_{cln}{}^2 EI_{cln} + 8 L_{cln}{}^3 EI_{bm}}$$

The numerator and the denominator of the second expression are the same and can be neglected. This results in the following formula.

$$H_{A,1} + H_{A,2} = \frac{q_{total,2} L_{bm}{}^3 L_{cln} EI_{cln}}{12 L_{bm} L_{cln}{}^2 EI_{cln} + 8 L_{cln}{}^3 EI_{bm}}$$

This results in the same formula as found in Appendix H. The second check is found correctly.

It is possible to simplify the formula.

$$H_{A,2} = \frac{q_{total,2} L_{bm}{}^3 L_{cln} EI_{cln,1} EI_{cln,2} - 12 H_{A,1} L_{bm} L_{cln}{}^2 EI_{cln,1} EI_{cln,2} - 8 H_{A,1} L_{cln}{}^3 EI_{bm} EI_{cln,2}}{12 L_{bm} L_{cln}{}^2 EI_{cln,1} EI_{cln,2} + 8 L_{cln}{}^3 EI_{cln,1} EI_{bm}}$$

The total load $\left(q_{total,2}\right)$ can be separate in the original load $\left(q_1\right)$ and an extra load $\left(q_2\right)$.

$$H_{A,2} = \frac{\left(q_1 + q_2\right) L_{bm}{}^3 L_{cln} EI_{cln,1} EI_{cln,2} - 12 H_{A,1} L_{bm} L_{cln}{}^2 EI_{cln,1} EI_{cln,2} - 8 H_{A,1} L_{cln}{}^3 EI_{bm} EI_{cln,2}}{12 L_{bm} L_{cln}{}^2 EI_{cln,1} EI_{cln,2} + 8 L_{cln}{}^3 EI_{cln,1} EI_{bm}}$$

If the order of the formula is changed, the following formula exists:

$$H_{A,2} = q_1 \frac{L_{bm}{}^3 L_{cln} EI_{cln,1} EI_{cln,2}}{12 L_{bm} L_{cln}{}^2 EI_{cln,1} EI_{cln,2} + 8 L_{cln}{}^3 EI_{cln,1} EI_{bm}} - H_{A,1} \frac{12 L_{bm} L_{cln}{}^2 EI_{cln,1} EI_{cln,2} + 8 L_{cln}{}^3 EI_{bm} EI_{cln,2}}{12 L_{bm} L_{cln}{}^2 EI_{cln,1} EI_{cln,2} + 8 L_{cln}{}^3 EI_{cln,1} EI_{bm}}$$
$$+ q_2 \frac{L_{bm}{}^3 L_{cln} EI_{cln,1} EI_{cln,2}}{12 L_{bm} L_{cln}{}^2 EI_{cln,1} EI_{cln,2} + 8 L_{cln}{}^3 EI_{cln,1} EI_{bm}}$$

The formula of $H_{A,1}$ is known (App. H); $\left(H_{A,1} = \frac{q_1 L_{bm}{}^3 L_{cln} EI_{cln,1}}{12 L_{bm} L_{cln}{}^2 EI_{cln,1} + 8 L_{cln}{}^3 EI_{bm}}\right)$. This formula can be used in this formula.

$$H_{A,2} = q_1 \frac{L_{bm}{}^3 L_{cln} EI_{cln,1} EI_{cln,2}}{12 L_{bm} L_{cln}{}^2 EI_{cln,1} EI_{cln,2} + 8 L_{cln}{}^3 EI_{cln,1} EI_{bm}}$$
$$- q_1 \frac{L_{bm}{}^3 L_{cln} EI_{cln,1}}{\left(12 L_{bm} L_{cln}{}^2 EI_{cln,1} + 8 L_{cln}{}^3 EI_{bm}\right)} \frac{\left(12 L_{bm} L_{cln}{}^2 EI_{cln,1} EI_{cln,2} + 8 L_{cln}{}^3 EI_{bm} EI_{cln,2}\right)}{12 L_{bm} L_{cln}{}^2 EI_{cln,1} EI_{cln,2} + 8 L_{cln}{}^3 EI_{cln,1} EI_{bm}}$$
$$+ q_2 \frac{L_{bm}{}^3 L_{cln} EI_{cln,1} EI_{cln,2}}{12 L_{bm} L_{cln}{}^2 EI_{cln,1} EI_{cln,2} + 8 L_{cln}{}^3 EI_{cln,1} EI_{bm}}$$

In the second expression a part of the numerator and the denominator is the same. This can be neglected. This results in the following formula:

$$H_{A,2} = q_1 \frac{L_{bm}{}^3 L_{cln} EI_{cln,1} EI_{cln,2}}{12 L_{bm} L_{cln}{}^2 EI_{cln,1} EI_{cln,2} + 8 L_{cln}{}^3 EI_{cln,1} EI_{bm}} - q_1 \frac{L_{bm}{}^3 L_{cln} EI_{cln,1} EI_{cln,2}}{12 L_{bm} L_{cln}{}^2 EI_{cln,1} EI_{cln,2} + 8 L_{cln}{}^3 EI_{cln,1} EI_{bm}}$$

$$+ q_2 \frac{L_{bm}{}^3 L_{cln} EI_{cln,2}}{12 L_{bm} L_{cln}{}^2 EI_{cln,2} + 8 L_{cln}{}^3 EI_{bm}}$$

Two expressions are the same and can be neglected. The final result is relative easy formula:

$$H_{A,2} = q_2 \frac{L_{bm}{}^3 L_{cln} EI_{cln,2}}{12 L_{bm} L_{cln}{}^2 EI_{cln,2} + 8 L_{cln}{}^3 EI_{bm}}$$

I.3 Analysis if two parts yield

The formula found in Appendix I.2 is only valid at the second load case. The formula of the first load case is already known (linear analysis without residual stress, App. H). The formula of the third load case can be analyzed on the same way as the second load case. This analysis is only a replay of the formulas. Because of this the analysis is not included in this Appendix. The results of the analysis are:

$$H_{A,1} = \frac{q_1 L_{bm}{}^3 L_{cln} EI_{cln,1}}{12 L_{bm} L_{cln}{}^2 EI_{cln,1} + 8 L_{cln}{}^3 EI_{bm}}$$

$$H_{A,2} = \frac{q_2 L_{bm}{}^3 L_{cln} EI_{cln,2}}{12 L_{bm} L_{cln}{}^2 EI_{cln,2} + 8 L_{cln}{}^3 EI_{bm}}$$

$$H_{A,3} = \frac{q_3 L_{bm}{}^3 L_{cln} EI_{cln,3}}{12 L_{bm} L_{cln}{}^2 EI_{cln,3} + 8 L_{cln}{}^3 EI_{bm}}$$

There is a logical relation between the load case and the horizontal reaction force. These formulas can be generalized in the following formula:

$$H_{A,i} = \frac{q_i L_{bm}{}^3 L_{cln} EI_{cln,i}}{12 L_{bm} L_{cln}{}^2 EI_{cln,i} + 8 L_{cln}{}^3 EI_{bm}}$$

The subscript i is the related to the load case. The first load case is before yielding. The second load case starts if the half of the top flange yields. The third load case starts if both the top flange as well as the bottom flanges is half yielded.

The formula of the extra horizontal reaction force is known. This formula can be used to find the total deflection. In Appendix H and in Appendix I.2 two formulas for the total deflection are found. These formulas are:

$$e_{total,1} = -\frac{q_1 L_{bm}{}^3 L_{cln}}{24 EI_{bm}} + \frac{\psi q_1 L_{bm}{}^2 L_{cln}{}^2}{12 EI_{bm}} + \frac{H_{A,1} L_{bm} L_{cln}{}^2}{2 EI_{bm}} + \psi L_{cln} + \frac{\psi q_1 L_{bm} L_{cln}{}^3}{6 EI_{cln,1}} + \frac{H_{A,1} L_{cln}{}^3}{3 EI_{cln,1}};$$

$$e_{total,2} = -\frac{q_{total,2} L_{bm}{}^3 L_{cln}}{24 EI_{bm}} + \frac{\psi q_{total,2} L_{bm}{}^2 L_{cln}{}^2}{12 EI_{bm}} + \frac{H_{A,total,2} L_{bm} L_{cln}{}^2}{2 EI_{bm}} + \psi L_{cln}$$

$$+ \frac{\psi q_1 L_{bm} L_{cln}{}^3}{6 EI_{cln,1}} + \frac{H_{A,1} L_{cln}{}^3}{3 EI_{cln,1}} + \frac{\psi q_2 L_{bm} L_{cln}{}^3}{6 EI_{cln,2}} + \frac{H_{A,2} L_{cln}{}^3}{3 EI_{cln,2}}$$

The formulas of $H_{A,1}$ and $H_{A,2}$ are known. These formulas can be used in the formulas.

$$e_{total,1} = -\frac{q_1 L_{bm}{}^3 L_{cln}}{24EI_{bm}} + \frac{\psi q_1 L_{bm}{}^2 L_{cln}{}^2}{12EI_{bm}} + q_1 \frac{L_{bm}{}^3 L_{cln} EI_{cln,1}}{12L_{bm}L_{cln}{}^2 EI_{cln,1} + 8L_{cln}{}^3 EI_{bm}} \frac{L_{bm}L_{cln}{}^2}{2EI_{bm}} + \psi L_{cln}$$

$$+\frac{\psi q_1 L_{bm} L_{cln}{}^3}{6EI_{cln,1}} + q_1 \frac{L_{bm}{}^3 L_{cln} EI_{cln,1}}{12L_{bm}L_{cln}{}^2 EI_{cln,1} + 8L_{cln}{}^3 EI_{bm}} \frac{L_{cln}{}^3}{3EI_{cln,1}}$$

$$e_{total,2} = -\frac{q_{total,2} L_{bm}{}^3 L_{cln}}{24EI_{bm}} + \frac{\psi q_{total,2} L_{bm}{}^2 L_{cln}{}^2}{12EI_{bm}}$$

$$+\left(q_1 \frac{L_{bm}{}^3 L_{cln} EI_{cln,1}}{12L_{bm}L_{cln}{}^2 EI_{cln,1} + 8L_{cln}{}^3 EI_{bm}} + \Delta q_2 \frac{L_{bm}{}^3 L_{cln} EI_{cln,2}}{12L_{bm}L_{cln}{}^2 EI_{cln,2} + 8L_{cln}{}^3 EI_{bm}} \right) \frac{L_{bm}L_{cln}{}^2}{2EI_{bm}} + \psi L_{cln}$$

$$+\frac{\psi q_1 L_{bm} L_{cln}{}^3}{6EI_{cln,1}} + q_1 \frac{L_{bm}{}^3 L_{cln} EI_{cln,1}}{12L_{bm}L_{cln}{}^2 EI_{cln,1} + 8L_{cln}{}^3 EI_{bm}} \frac{L_{cln}{}^3}{3EI_{cln,1}} + \frac{\psi q_2 L_{bm} L_{cln}{}^3}{6EI_{cln,2}} + q_2 \frac{L_{bm}{}^3 L_{cln} EI_{cln,2}}{12L_{bm}L_{cln}{}^2 EI_{cln,2} + 8L_{cln}{}^3 EI_{bm}} \frac{L_{cln}{}^3}{3EI_{cln,2}}$$

In both formulas some simplifications can be made. Some expressions in the numerator and the denominator are the same and can be neglected. The order of the expressions has changed.

$$e_{total,1} = \psi L_{cln} - q_1 \frac{L_{bm}{}^3 L_{cln}}{24EI_{bm}} + q_1 \frac{\psi L_{bm}{}^2 L_{cln}{}^2}{12EI_{bm}}$$

$$+q_1 \frac{\psi L_{bm} L_{cln}{}^3}{6EI_{cln,1}} + q_1 \frac{L_{bm}{}^3 L_{cln}{}^2}{36L_{bm}EI_{cln,1} + 24L_{cln}EI_{bm}} + q_1 \frac{L_{bm}{}^4 L_{cln} EI_{cln,1}}{24L_{bm}EI_{bm}EI_{cln,1} + 16L_{cln}\left(EI_{bm}\right)^2}$$

$$e_{total,2} = \psi L_{cln} - q_{total,2} \frac{L_{bm}{}^3 L_{cln}}{24EI_{bm}} + q_{total,2} \frac{\psi L_{bm}{}^2 L_{cln}{}^2}{12EI_{bm}}$$

$$+q_1 \frac{\psi L_{bm} L_{cln}{}^3}{6EI_{cln,1}} + q_1 \frac{L_{bm}{}^3 L_{cln}{}^2}{36L_{bm}EI_{cln,1} + 24L_{cln}EI_{bm}} + q_1 \frac{L_{bm}{}^4 L_{cln} EI_{cln,1}}{24L_{bm}EI_{bm}EI_{cln,1} + 16L_{cln}\left(EI_{bm}\right)^2}$$

$$+q_2 \frac{\psi L_{bm} L_{cln}{}^3}{6EI_{cln,2}} + q_2 \frac{L_{bm}{}^3 L_{cln}{}^2}{36L_{bm}EI_{cln,2} + 24L_{cln}EI_{bm}} + q_2 \frac{L_{bm}{}^4 L_{cln} EI_{cln,2}}{24L_{bm}EI_{bm}EI_{cln,2} + 16L_{cln}\left(EI_{bm}\right)^2}$$

The second formula is an extended version of the first formula. All expressions in the first formula can also be found in the second formula. The relation between the formulas is clear enough to make a general formula for the total deflection. This general formula is:

$$e_{total,i} = \psi L_{cln} + q_{total,i} \frac{L_{bm}{}^3 L_{cln}}{24EI_{bm}} + q_{total,i} \frac{\psi L_{bm}{}^2 L_{cln}{}^2}{12EI_{bm}}$$

$$\sum_i q_i \frac{\psi L_{bm} L_{cln}{}^3}{6EI_{cln,i}} - -q_i \frac{L_{bm}{}^3 L_{cln}{}^4}{36L_{bm}L_{cln}{}^2 EI_{cln,i} + 24L_{cln}{}^3 EI_{bm}} q_i \frac{L_{bm}{}^4 L_{cln}{}^3 EI_{cln,i}}{24L_{bm}L_{cln}{}^2 EI_{bm}EI_{cln,i} + 16L_{cln}{}^3\left(EI_{bm}\right)^2}$$

As summary of this Appendix the formulas of the reaction forces and the deflection are summed up.

$$V_{D,total,i} = ½ q_{total,i} L_{bm}$$

$$V_{A,total,i} = ½ q_{total,i} L_{bm}$$

$$M_{B,total,i} = \left(½\psi q_{total,i} L_{bm} - H_{A,total,i}\right) L_{cln}$$

$$M_{C,total,i} = \left(½\psi q_{total,i} L_{bm} + H_{A,total,i}\right) L_{cln}$$

$$H_{A,i} = \frac{q_i L_{bm}{}^3 L_{cln} EI_{cln,i}}{12 L_{bm} L_{cln}{}^2 EI_{cln,i} + 8 L_{cln}{}^3 EI_{bm}}$$

$$e_{total,i} = \psi L_{cln} + q_{total,i} \frac{L_{bm}{}^3 L_{cln}}{24 EI_{bm}} + q_{total,i} \frac{\psi L_{bm}{}^2 L_{cln}{}^2}{12 EI_{bm}}$$

$$+\sum_i q_i \frac{\psi L_{bm} L_{cln}{}^3}{6 EI_{cln,i}} - -q_i \frac{L_{bm}{}^3 L_{cln}{}^4}{36 L_{bm} L_{cln}{}^2 EI_{cln,i} + 24 L_{cln}{}^3 EI_{bm}} q_i \frac{L_{bm}{}^4 L_{cln}{}^3 EI_{cln,i}}{24 L_{bm} L_{cln}{}^2 EI_{bm} EI_{cln,i} + 16 L_{cln}{}^3 \left(EI_{bm}\right)^2}$$

Appendix J Non-Linear analysis portal frame

The linear analyses are made in Appendix H and in Appendix. Appendix H was a linear analysis without residual stresses and the linear analysis in Appendix I was including residual stresses. At a linear analysis the deflections do not influence the reaction forces. At a non-linear analysis the deflection does influence the reaction forces. The reaction force influence the total deflection and the total deflection influence the reaction forces. Because of this relation the formulas become very complex.

The initial deflection is 0.4 percent of the column length (NEN 6771 art. 10.2.5.1.3). This value is very small and could be neglected (and is neglected in the linear analyses). The total deflection can increase till 10% of the column length or even larger. The influence of the total deflection cannot be neglected.

In this Appendix the same portal frame will be analyzed as in Appendices H and I (Fig. J.1).

The formulas in this Appendix are related to a section without residual stress. In practice, the ultimate load is slammer than the ultimate load found in this Appendix. Due to the residual stress, the deflection increases and the ultimate load decreases. The influence of the residual stress will be discussed in Appendix K.

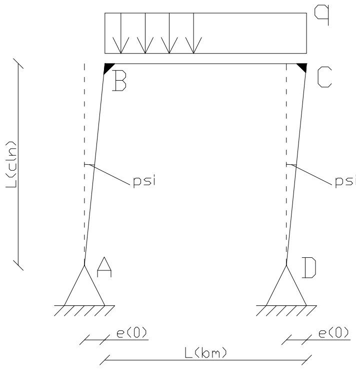


Figure J.1:
Structure

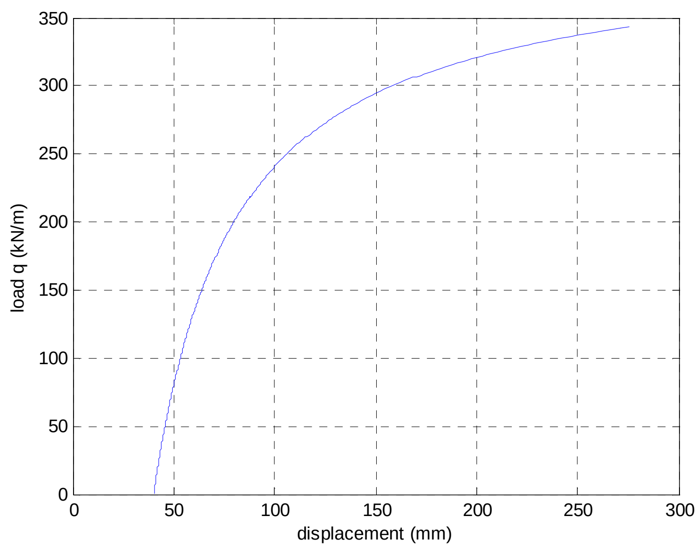


Figure J.2:
Load-deflection
Graphic

Column: Length 10 m
HE 360A
Beam: Length 5 m
HE 900A

J.1 In general

The basic principle of the non-linear analysis is the same as the linear analyses and starts with the equilibrium formulas.

Moment equilibrium:

$$\sum T \mid A = 0$$
$$qL_{bm}\left(½L_{bm} + e_{total}\right) - V_D L_{bm} = 0$$
$$V_D = \frac{qL_{bm}\left(½L_{bm} + e_{total}\right)}{L_{bm}}$$
$$V_D = q\left(½L_{bm} + e_{total}\right)$$

Vertical equilibrium:

$$\sum F_{vert} = 0$$
$$qL_{bm} - V_A - V_D = 0$$
$$qL_{bm} - V_A - q\left(½L_{bm} + e_{total}\right) = 0$$
$$V_A = q\left(½L_{bm} - e_{total}\right)$$

Horizontal equilibrium:

$$\sum F_{hor} = 0$$
$$H_A - H_D = 0$$
$$H_A = H_D$$

As same as the linear analysis the stability of the portal frame is based on the bending moments in the intersection point of the column and the beam. The rotation in the beam and remaining force in the supports are important to calculate the total deflection of the portal frame (Fig. J.3 and Fig. J.4).

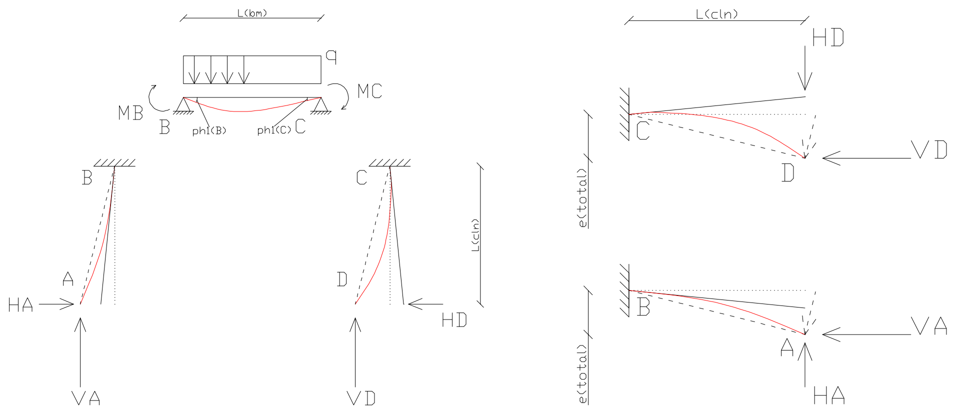


Figure J.3:
Deflection

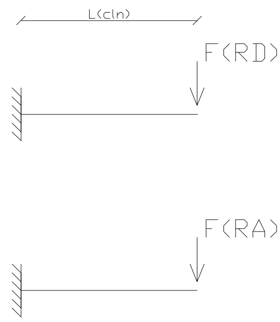


Figure J.4:
Remaining force

Bending moment in B

$$M_B = V_A e_{total} - H_A L_{cln}$$
$$M_B = q\left(½L_{bm} - e_{total}\right)e_{total} - H_A L_{cln}$$

Bending moment in C

$$M_C = V_D e_{total} + H_D L_{cln}$$
$$M_C = q\left(½L_{bm} + e_{total}\right)e_{total} + H_A L_{cln}$$

It is assumed that the columns are critical. The beam is strong enough and will not fail. Lateral buckling and other failure mechanism in the beam does not occur. Column CD is the heaviest loaded column fails before column AB fails. Column CD is the critical element of the structure. The analysis is about the failure of column CD. If the column fails, the structure collapses.

The portal frame is out of square. The columns are deflected. The reaction forces are not parallel or perpendicular to the working line of the column. Both the horizontal as well as the vertical reaction force have influence on the total deflection of the column (Fig. J.4).

$F_{R,A}$ and $F_{R,D}$ are the remaining loads in point A respective point D. These loads depend on the reaction forces and on the deflections. The relation between the deflection and the length of the column is the same as the relation between the remaining load and the reaction forces (See Fig. J.5 for angle α).

$$\sin(\alpha) = \frac{e_{total}}{L_{cln}}$$

$$\cos(\alpha) = \frac{\sqrt{L_{cln}{}^2 - e_{total}{}^2}}{L_{cln}} \approx 1$$

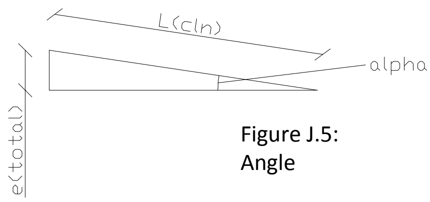


Figure J.5: Angle

The remaining force in point D:

$$F_{R,D} = V_D \sin(\alpha) + H_D \cos(\alpha)$$

$$F_{R,D} = V_D \frac{e_{total}}{L_{cln}} + H_D$$

$$F_{R,D} = q\left(½L_{bm} + e_{total}\right)\frac{e_{total}}{L_{cln}} + H_A$$

The remaining force in point A:

$$F_{R,A} = V_A \sin(\alpha) - H_A \cos(\alpha)$$

$$F_{R,A} = V_A \frac{e_{total}}{L_{cln}} - H_A$$

$$F_{R,A} = q\left(½L_{bm} - e_{total}\right)\frac{e_{total}}{L_{cln}} - H_A$$

The follow formulas are found:

$$V_D = q\left(½L_{bm} + e_{total}\right)$$

$$V_A = q\left(½L_{bm} - e_{total}\right)$$

$$H_A = H_D$$

$$M_B = q\left(½L_{bm} - e_{total}\right)e_{total} - H_A L_{cln}$$

$$M_C = q\left(½L_{bm} + e_{total}\right)e_{total} + H_A L_{cln}$$

$$F_{R,D} = q\left(½L_{bm} + e_{total}\right)\frac{e_{total}}{L_{cln}} + H_A$$

$$F_{R,A} = q\left(½L_{bm} - e_{total}\right)\frac{e_{total}}{L_{cln}} - H_A$$

The angles in the points B and C can be calculated by standard formulas for a beam on two supports. The angles φ_B and φ_C can be calculated as follow:

$$\varphi_B = \frac{-qL_{bm}{}^3}{24EI_{bm}} - \frac{M_B L_{bm}}{3EI_{bm}} + \frac{M_C L_{bm}}{6EI_{bm}}$$

$$\varphi_C = \frac{qL_{bm}{}^3}{24EI_{bm}} + \frac{M_B L_{bm}}{6EI_{bm}} - \frac{M_C L_{bm}}{3EI_{bm}}$$

The total deflection is the summation of the initial deflection, the deflection due to the rotation and the deflection due to the remaining force. This is the same as the linear analysis in Appendix H. The deflection due to the remaining force is also the results of a standard formula. The total deflections are:

$$e_{total}{}^{AB} = -\varphi_B L_{cln} + \psi L_{cln} + \frac{F_{R,A} L_{cln}{}^3}{3EI_{cln}}$$

$$e_{total}{}^{CD} = -\varphi_C L_{cln} + \psi L_{cln} + \frac{F_{R,D} L_{cln}{}^3}{3EI_{cln}}$$

$e_{total}{}^{AB}$ is the total deflection in column AB.

$e_{total}{}^{CD}$ is the total deflection in column CD.

The total deflection of column AB $\left(e_{total}{}^{AB}\right)$ is the same as the deflection of column CD$\left(e_{total}{}^{CD}\right)$.

J.2 Deflection of column the column

First the deflection of column AB has been analyzed. To start the formula the basic formula has been written.

$$e_{total}{}^{AB} = -\varphi_B L_{cln} + \psi L_{cln} + \frac{F_{R,A} L_{cln}{}^3}{3EI_{cln}}$$

φ_B is a known expression. This expression is filled in the formula:

$$e_{total}{}^{AB} = \left(\frac{qL_{bm}{}^3}{24EI_{bm}} + \frac{M_B L_{bm}}{3EI_{bm}} - \frac{M_C L_{bm}}{6EI_{bm}}\right) L_{cln} + \psi L_{cln} + F_{R,A} \frac{L_{cln}{}^3}{3EI_{cln}}$$

The expressions of M_B, M_C and $F_{R,A}$ are filled in the formula. This results in:

$$e_{total}{}^{AB} = \left(\frac{qL_{bm}{}^3}{24EI_{bm}} + \left(q\left(½L_{bm} - e_{total}\right)e_{total} - H_A L_{cln}\right)\frac{L_{bm}}{3EI_{bm}} - \left(q\left(½L_{bm} + e_{total}\right)e_{total} + H_A L_{cln}\right)\frac{L_{bm}}{6EI_{bm}}\right) L_{cln}$$
$$+\psi L_{cln} + \left(q\left(½L_{bm} - e_{total}\right)\frac{e_{total}}{L_{cln}} - H_A\right)\frac{L_{cln}{}^3}{3EI_{cln}}$$

This formula is a function of two unknowns. These unknowns are e_{total} and H_A. Finally both unknowns must be found. In the linear analysis it was easy to separate e_{total}. In this non-linear analysis there are more expressions with e_{total}. Not all expressions of e_{total} are linear but also quadratic expressions are included. The quadratic expressions results in complex formulas. Chosen is to separate H_A. This is easier for the analysis.

$$H_A L_{cln} \frac{L_{bm}}{3EI_{bm}} * L_{cln} + H_A L_{cln} \frac{L_{bm}}{6EI_{bm}} L_{cln} + H_A \frac{L_{cln}{}^3}{3EI_{cln}} =$$
$$\left(\frac{qL_{bm}{}^3}{24EI_{bm}} + \left(q\left(½L_{bm} - e_{total}\right)e_{total}\right)\frac{L_{bm}}{3EI_{bm}} - \left(q\left(½L_{bm} + e_{total}\right)e_{total}\right)\frac{L_{bm}}{6EI_{bm}}\right) L_{cln}$$
$$+\psi L_{cln} + \left(q\left(½L_{bm} - e_{total}\right)\frac{e_{total}}{L_{cln}}\right)\frac{L_{cln}{}^3}{3EI_{cln}} - e_{total}$$

The left part of the formula exists in expressions of H_A and the right part in expressions of e_{total}. Some expressions are the same and can be combined together.

$$H_A \frac{L_{bm} L_{cln}{}^2}{2EI_{bm}} + H_A \frac{L_{cln}{}^3}{3EI_{cln}} = \frac{qL_{bm}{}^3 L_{cln}}{24EI_{bm}} + \frac{qL_{bm}{}^2 L_{cln} e_{total}}{12EI_{bm}} - \frac{qL_{bm} L_{cln} e_{total}{}^2}{2EI_{bm}} + \frac{qL_{bm} L_{cln}{}^2 e_{total}}{6EI_{cln}} - \frac{qL_{cln}{}^2 e_{total}{}^2}{3EI_{cln}} + \psi L_{cln} - e_{total}$$

The denominator is not the same for all expressions. For a clear calculations it is useful to multiply all expression to create everywhere the same denominator.

$$H_A \frac{12L_{bm}L_{cln}^2 EI_{cln} + 8L_{cln}^3 EI_{bm}}{24EI_{bm}EI_{cln}} = \frac{qL_{bm}^3 L_{cln}EI_{cln}}{24EI_{bm}EI_{cln}} + \frac{2qL_{bm}^2 L_{cln}e_{total}EI_{cln}}{24EI_{bm}EI_{cln}} - \frac{12qL_{bm}L_{cln}e_{total}^2 EI_{cln}}{24EI_{bm}EI_{cln}}$$

$$+\frac{4qL_{bm}L_{cln}^2 e_{total}EI_{bm}}{24EI_{bm}EI_{cln}} - \frac{8qL_{cln}^2 e_{total}^2 EI_{bm}}{24EI_{bm}EI_{cln}} + \frac{24\psi L_{cln}EI_{bm}EI_{cln}}{24EI_{bm}EI_{cln}} - \frac{24e_{total}EI_{bm}EI_{cln}}{24EI_{bm}EI_{cln}}$$

All denominators are the same and can be neglected. After neglect the denominators, H_A can be taken as a function of e_{total}.

$$H_A^{AB} = \frac{\begin{pmatrix} qL_{bm}^3 L_{cln}EI_{cln} + 2qL_{bm}^2 L_{cln}e_{total}EI_{cln} - 12qL_{bm}L_{cln}e_{total}^2 EI_{cln} + 4qL_{bm}L_{cln}^2 e_{total}EI_{bm} \\ -8qL_{cln}^2 e_{total}^2 EI_{bm} + 24\psi L_{cln}EI_{bm}EI_{cln} - 24e_{total}EI_{bm}EI_{cln} \end{pmatrix}}{12L_{bm}L_{cln}^2 EI_{cln} + 8L_{cln}^3 EI_{bm}}$$

Secondly the deflection of CD has been calculated. Again starting with the following formula:

$$e_{total}^{CD} = -\varphi_C L_{cln} + \psi L_{cln} + \frac{F_{R,D}L_{cln}^3}{3EI_{cln}}$$

Use the formula of φ_B in the formula:

$$e_{total}^{CD} = \left(-\frac{qL_{bm}^3}{24EI_{bm}} - \frac{M_B L_{bm}}{6EI_{bm}} + \frac{M_C L_{bm}}{3EI_{bm}}\right)L_{cln} + \psi L_{cln} + \frac{F_{R,D}L_{cln}^3}{3EI_{cln}}$$

The formulas of M_B, M_C and $F_{R,D}$ filled in the formula:

$$e_{total}^{CD} = \left(-\frac{qL_{bm}^3}{24EI_{bm}} - \left(q\left(½L_{bm} - e_{total}\right)e_{total} - H_A L_{cln}\right)\frac{L_{bm}}{6EI_{bm}} + \left(q\left(½L_{bm} + e_{total}\right)e_{total} + H_A L_{cln}\right)\frac{L_{bm}}{3EI_{bm}}\right)L_{cln}$$

$$+\psi L_{cln} + \left(q\left(½L_{bm} + e_{total}\right)\frac{e_{total}}{L_{cln}} + H_A\right)\frac{L_{cln}^3}{3EI_{cln}}$$

Again there are two unknowns: e_{total} and H_A. H_A will be separated from the rest of the formula.

$$-H_A L_{cln}\frac{L_{bm}}{6EI_{bm}}L_{cln} - H_A L_{cln}\frac{L_{bm}}{3EI_{bm}}L_{cln} - H_A\frac{L_{cln}^3}{3EI_{cln}} =$$

$$\left(-\frac{qL_{bm}^3}{24EI_{bm}} - \left(q\left(½L_{bm} - e_{total}\right)e_{total}\right)\frac{L_{bm}}{6EI_{bm}} + \left(q\left(½L_{bm} + e_{total}\right)e_{total}\right)\frac{L_{bm}}{3EI_{bm}}\right)L_{cln}$$

$$+\psi L_{cln} + \left(q\left(½L_{bm} + e_{total}\right)\frac{e_{total}}{L_{cln}}\right)\frac{L_{cln}^3}{3EI_{cln}} - e_{total}$$

The left part of the formula exists in expressions of H_A and the right part in expressions of e_{total}. Some expressions are the same and be can combined together.

$$-H_A\frac{L_{bm}L_{cln}^2}{2EI_{bm}} - H_A\frac{L_{cln}^3}{3EI_{cln}}$$

$$= -\frac{qL_{bm}^3 L_{cln}}{24EI_{bm}} + \frac{qL_{bm}^2 L_{cln}e_{total}}{12EI_{bm}} + \frac{qL_{bm}L_{cln}e_{total}^2}{2EI_{bm}} + \frac{qL_{bm}L_{cln}^2 e_{total}}{6EI_{cln}} + \frac{qL_{cln}^2 e_{total}^2}{3EI_{cln}} + \psi L_{cln} - e_{total}$$

The denominator is not the same for all expressions. For a clear calculations it is useful to multiply all expression to create everywhere the same denominator.

$$-H_A \frac{12L_{bm}L_{cln}^2EI_{cln}+8L_{cln}^3EI_{cln}}{24EI_{bm}EI_{cln}}=-\frac{qL_{bm}^3L_{cln}EI_{cln}}{24EI_{bm}EI_{cln}}+\frac{2qL_{bm}^2L_{cln}e_{total}EI_{cln}}{24EI_{bm}EI_{cln}}+\frac{12qL_{bm}L_{cln}e_{total}^2EI_{cln}}{24EI_{bm}EI_{cln}}$$

$$+\frac{4qL_{bm}L_{cln}^2e_{total}EI_{bm}}{24EI_{bm}EI_{cln}}+\frac{8qL_{cln}^2e_{total}^2EI_{bm}}{24EI_{bm}EI_{cln}}+\frac{24\psi L_{cln}EI_{bm}EI_{cln}}{24EI_{bm}EI_{cln}}-\frac{24e_{total}EI_{bm}EI_{cln}}{24EI_{bm}EI_{cln}}$$

All denominators are the same and can be neglected. After neglecting the denominators, H_A can be taken as a function of e_{total}.

$$H_A^{CD}=\frac{\begin{pmatrix}qL_{bm}^3L_{cln}EI_{cln}-2qL_{bm}^2L_{cln}e_{total}EI_{cln}-12qL_{bm}L_{cln}e_{total}^2EI_{cln}-4qL_{bm}L_{cln}^2e_{total}EI_{bm}\\-8qL_{cln}^2e_{total}^2EI_{bm}-24\psi L_{cln}EI_{bm}EI_{cln}+24e_{total}EI_{bm}EI_{cln}\end{pmatrix}}{12L_{bm}L_{cln}^2EI_{cln}+8L_{cln}^3EI_{cln}}$$

J.3 Calculation of the deflections

As discussed before the total deflection of column AB and the total deflection of column CD are the same. In other words: $e_{total}^{AB}=e_{total}^{CD}$. This is also the case for the horizontal forces: $H_A^{AB}=H_A^{CD}$. The two expressions found before are equal together. The following formula is created.

$$\frac{\begin{pmatrix}qL_{bm}^3L_{cln}EI_{cln}+2qL_{bm}^2L_{cln}e_{total}EI_{cln}-12qL_{bm}L_{cln}e_{total}^2EI_{cln}+4qL_{bm}L_{cln}^2e_{total}EI_{bm}\\-8qL_{cln}^2e_{total}^2EI_{bm}+24\psi L_{cln}EI_{bm}EI_{cln}-24e_{total}EI_{bm}EI_{cln}\end{pmatrix}}{12L_{bm}L_{cln}^2EI_{cln}+8L_{cln}^3EI_{bm}}$$

$$=$$

$$\frac{\begin{pmatrix}qL_{bm}^3L_{cln}EI_{cln}-2qL_{bm}^2L_{cln}e_{total}EI_{cln}-12qL_{bm}L_{cln}e_{total}^2EI_{cln}-4qL_{bm}L_{cln}^2e_{total}EI_{bm}\\-8qL_{cln}^2e_{total}^2EI_{bm}-24\psi L_{cln}EI_{bm}EI_{cln}+24e_{total}EI_{bm}EI_{cln}\end{pmatrix}}{12L_{bm}L_{cln}^2EI_{cln}+8L_{cln}^3EI_{cln}}$$

Both expressions have the same denominator. The denominator can be neglected.

$$\begin{pmatrix}qL_{bm}^3L_{cln}EI_{cln}+2qL_{bm}^2L_{cln}e_{total}EI_{cln}-12qL_{bm}L_{cln}e_{total}^2EI_{cln}+4qL_{bm}L_{cln}^2e_{total}EI_{bm}\\-8qL_{cln}^2e_{total}^2EI_{bm}+24\psi L_{cln}EI_{bm}EI_{cln}-24e_{total}EI_{bm}EI_{cln}\end{pmatrix}$$

$$=$$

$$\begin{pmatrix}qL_{bm}^3L_{cln}EI_{cln}-2qL_{bm}^2L_{cln}e_{total}EI_{cln}-12qL_{bm}L_{cln}e_{total}^2EI_{cln}-4qL_{bm}L_{cln}^2e_{total}EI_{bm}\\-8qL_{cln}^2e_{total}^2EI_{bm}-24\psi L_{cln}EI_{bm}EI_{cln}+24e_{total}EI_{bm}EI_{cln}\end{pmatrix}$$

The same expressions on both sides of the formula can be neglected or combined together. This results in the following formula:

$$4qL_{bm}^2L_{cln}e_{total}EI_{cln}+8qL_{bm}L_{cln}^2e_{total}EI_{bm}+48\psi L_{cln}EI_{bm}EI_{cln}-48e_{total}EI_{bm}EI_{cln}=0$$

The expressions of e_{total} can be separated from the rest of the formula.

$$4e_{total}\left(12EI_{bm}EI_{cln} - qL_{bm}{}^{2}L_{cln}EI_{cln} - 2qL_{bm}L_{cln}{}^{2}EI_{bm}\right) = 4\left(12\psi L_{cln}EI_{bm}EI_{cln}\right)$$

Finally this result is the following formula:

$$e_{total} = \frac{12\psi L_{cln}EI_{bm}EI_{cln}}{12EI_{bm}EI_{cln} - qL_{bm}{}^{2}L_{cln}EI_{cln} - 2qL_{bm}L_{cln}{}^{2}EI_{bm}}$$

This formula will be checked on large mistakes. The first check is the check on dimensions:

$$m = \frac{mNm^{-2}m^{4}Nm^{-2}m^{4}}{Nm^{-2}m^{4}Nm^{-2}m^{4} - Nm^{-1}m^{2}mNm^{-2}m^{4} - Nm^{-1}mm^{2}Nm^{-2}m^{4}}$$

$$m = \frac{N^{2}m^{5}}{N^{2}m^{4} - N^{2}m^{4} - N^{2}m^{4}}$$

The dimensions of the formula are correct.

The second check: if there is no load (q=0) there is only the initial deflection ($e_{total} = \psi L_{cln}$).

$$e_{total} = \frac{12\psi L_{cln}EI_{bm}EI_{cln}}{12EI_{bm}EI_{cln} - 0L_{bm}{}^{2}L_{cln}EI_{cln} - 0L_{bm}L_{cln}{}^{2}EI_{bm}}$$

$$e_{total} = \psi L_{cln}\frac{12EI_{bm}EI_{cln}}{12EI_{bm}EI_{cln}}$$

The total deflection is the same as the initial deflection, so this check is also correct.

The last check is the shape of the formula. Is the formula logical? Already known is the buckling calculation of a column and the Euler buckling load. These formulas have the same shape: $e_{total} = \frac{\textit{function} \text{ of stiffness}}{\textit{function} \text{ of stiffness - } \textit{function} \text{ of load}}$. The formula found in this Appendix has the same shape and could be correct.

The non-linear analysis results in the following formula:

$$e_{total} = \frac{12\psi L_{cln}EI_{bm}EI_{cln}}{12EI_{bm}EI_{cln} - qL_{bm}{}^{2}L_{cln}EI_{cln} - 2qL_{bm}L_{cln}{}^{2}EI_{bm}}$$

The total deflection is known. The forces in the portal frame depend on the reaction forces. The reaction forces depend on the total deflection. The numerical value of the total deflection can be used to calculate the horizontal reaction force, but it is also possible to use the formula of the total deflection to find a formula for the horizontal reaction force. This is what happens next. The formula of the horizontal reaction force is:

$$H_{A} = \frac{\begin{pmatrix} qL_{bm}{}^{3}L_{cln}EI_{cln} + 2qL_{bm}{}^{2}L_{cln}e_{total}EI_{cln} - 12qL_{bm}L_{cln}e_{total}{}^{2}EI_{cln} + 4qL_{bm}L_{cln}{}^{2}e_{total}EI_{bm} \\ -8qL_{cln}{}^{2}e_{total}{}^{2}EI_{bm} + 24\psi L_{cln}EI_{bm}EI_{cln} - 24e_{total}EI_{bm}EI_{cln} \end{pmatrix}}{12L_{bm}L_{cln}{}^{2}EI_{cln} + 8L_{cln}{}^{3}EI_{bm}}$$

If the formula of the total deflection is used the following expression exists:

$$H_A = \frac{\begin{pmatrix} qL_{bm}{}^3 L_{cln} EI_{cln} + 2qL_{bm}{}^2 L_{cln}\left(\frac{12\psi L_{cln} EI_{bm} EI_{cln}}{12EI_{bm}EI_{cln} - qL_{bm}{}^2 L_{cln} EI_{cln} - 2qL_{bm} L_{cln}{}^2 EI_{bm}}\right) EI_{cln} \\ -12qL_{bm} L_{cln}\left(\frac{12\psi L_{cln} EI_{bm} EI_{cln}}{12EI_{bm}EI_{cln} - qL_{bm}{}^2 L_{cln} EI_{cln} - 2qL_{bm} L_{cln}{}^2 EI_{bm}}\right)^2 EI_{cln} \\ +4qL_{bm} L_{cln}{}^2\left(\frac{12\psi L_{cln} EI_{bm} EI_{cln}}{12EI_{bm}EI_{cln} - qL_{bm}{}^2 L_{cln} EI_{cln} - 2qL_{bm} L_{cln}{}^2 EI_{bm}}\right) EI_{bm} \\ -8qL_{cln}{}^2\left(\frac{12\psi L_{cln} EI_{bm} EI_{cln}}{12EI_{bm}EI_{cln} - qL_{bm}{}^2 L_{cln} EI_{cln} - 2qL_{bm} L_{cln}{}^2 EI_{bm}}\right)^2 EI_{bm} \\ -24\left(\frac{12\psi L_{cln} EI_{bm} EI_{cln}}{12EI_{bm}EI_{cln} - qL_{bm}{}^2 L_{cln} EI_{cln} - 2qL_{bm} L_{cln}{}^2 EI_{bm}}\right) EI_{bm} EI_{cln} + 24\psi L_{cln} EI_{bm} EI_{cln} \end{pmatrix}}{12L_{bm} L_{cln}{}^2 EI_{cln} + 8L_{cln}{}^3 EI_{bm}}$$

This formula is very worse. To get a clearer formula some expressions are combined together.

$$H_A = \frac{\begin{pmatrix} qL_{bm}{}^3 L_{cln} EI_{cln} \\ +\left(2qL_{bm}{}^2 L_{cln} EI_{cln} + 4qL_{bm} L_{cln}{}^2 EI_{bm} - 24EI_{bm} EI_{cln}\right)\left(\frac{12\psi L_{cln} EI_{bm} EI_{cln}}{12EI_{bm}EI_{cln} - qL_{bm}{}^2 L_{cln} EI_{cln} - 2qL_{bm} L_{cln}{}^2 EI_{bm}}\right) \\ -\left(12qL_{bm} L_{cln} EI_{cln} + 8qL_{cln}{}^2 EI_{bm}\right)\left(\frac{12\psi L_{cln} EI_{bm} EI_{cln}}{12EI_{bm}EI_{cln} - qL_{bm}{}^2 L_{cln} EI_{cln} - 2qL_{bm} L_{cln}{}^2 EI_{bm}}\right)^2 \\ +24\psi L_{cln} EI_{bm} EI_{cln} \end{pmatrix}}{12L_{bm} L_{cln}{}^2 EI_{cln} + 8L_{cln}{}^3 EI_{bm}}$$

The order can be changed.

$$H_A = \frac{\begin{pmatrix} qL_{bm}{}^3 L_{cln} EI_{cln} \\ -2\left(12EI_{bm} EI_{cln} - qL_{bm}{}^2 L_{cln} EI_{cln} - 2qL_{bm} L_{cln}{}^2 EI_{bm}\right)\left(\frac{12\psi L_{cln} EI_{bm} EI_{cln}}{12EI_{bm}EI_{cln} - qL_{bm}{}^2 L_{cln} EI_{cln} - 2qL_{bm} L_{cln}{}^2 EI_{bm}}\right) \\ -\left(12qL_{bm} L_{cln} EI_{cln} + 8qL_{cln}{}^2 EI_{bm}\right)\left(\frac{12\psi L_{cln} EI_{bm} EI_{cln}}{12EI_{bm}EI_{cln} - qL_{bm}{}^2 L_{cln} EI_{cln} - 2qL_{bm} L_{cln}{}^2 EI_{bm}}\right)^2 \\ +24\psi L_{cln} EI_{bm} EI_{cln} \end{pmatrix}}{12L_{bm} L_{cln}{}^2 EI_{cln} + 8L_{cln}{}^3 EI_{bm}}$$

If the numerator and the denominator have the same expressions, these expressions can be neglected. The result is as follow:

$$H_A = \frac{\begin{pmatrix} qL_{bm}{}^3 L_{cln} EI_{cln} - 24\psi L_{cln} EI_{bm} EI_{cln} \\ -\left(12qL_{bm} L_{cln} EI_{cln} + 8qL_{cln}{}^2 EI_{bm}\right)\left(\frac{12\psi L_{cln} EI_{bm} EI_{cln}}{12EI_{bm} EI_{cln} - qL_{bm}{}^2 L_{cln} EI_{cln} - 2qL_{bm} L_{cln}{}^2 EI_{bm}}\right)^2 \\ +24\psi L_{cln} EI_{bm} EI_{cln} \end{pmatrix}}{12L_{bm} L_{cln}{}^2 EI_{cln} + 8L_{cln}{}^3 EI_{bm}}$$

Two expressions are the same and can be neglected. This results in the following formula:

$$H_A = \frac{\left(qL_{bm}{}^3 L_{cln} EI_{cln} - L_{cln}\left(12qL_{bm} L_{cln}{}^2 EI_{cln} + 8qL_{cln}{}^3 EI_{bm}\right)\left(\frac{12\psi EI_{bm} EI_{cln}}{12EI_{bm} EI_{cln} - qL_{bm}{}^2 L_{cln} EI_{cln} - 2qL_{bm} L_{cln}{}^2 EI_{bm}}\right)^2\right)}{12L_{bm} L_{cln}{}^2 EI_{cln} + 8L_{cln}{}^3 EI_{bm}}$$

There are two expressions in this formula. If the expressions are split, the formula results in:

$$H_A = q\frac{L_{bm}{}^3 L_{cln} EI_{cln}}{12L_{bm} L_{cln}{}^2 EI_{cln} + 8L_{cln}{}^3 EI_{bm}} - qL_{cln}\left(\frac{12\psi EI_{bm} EI_{cln}}{12EI_{bm} EI_{cln} - qL_{bm}{}^2 L_{cln} EI_{cln} - 2qL_{bm} L_{cln}{}^2 EI_{bm}}\right)^2$$

This formula is much better than the original one. It is possible to make calculations with this formula.

As summery all formulas of the forces and the formula for the total deflection are summed up.

$$V_D = q\left(½L_{bm} + e_{total}\right)$$
$$V_A = q\left(½L_{bm} - e_{total}\right)$$
$$M_B = q\left(½L_{bm} - e_{total}\right)e_{total} - H_A L_{cln}$$
$$M_C = q\left(½L_{bm} + e_{total}\right)e_{total} + H_A L_{cln}$$
$$e_{total} = \frac{12\psi L_{cln} EI_{bm} EI_{cln}}{12EI_{bm} EI_{cln} - qL_{bm}{}^2 L_{cln} EI_{cln} - 2qL_{bm} L_{cln}{}^2 EI_{bm}}$$
$$H_A = q\frac{L_{bm}{}^3 L_{cln} EI_{cln}}{12L_{bm} L_{cln}{}^2 EI_{cln} + 8L_{cln}{}^3 EI_{bm}} - qL_{cln}\left(\frac{12\psi EI_{bm} EI_{cln}}{12EI_{bm} EI_{cln} - qL_{bm}{}^2 L_{cln} EI_{cln} - 2qL_{bm} L_{cln}{}^2 EI_{bm}}\right)^2$$

Appendix K Residual stress in a non-Linear analysis

In Appendix J a non-linear analysis is made. In Appendix I the influence of residual stress has been introduced at a linear analysis. This Appendix is about the combination of these two: the influence of residual stress in a non-linear analysis.

If the structure partial yields, the reaction forces and the deflections changes. The influence of partial yielding to the final deflection and the internal stress distribution will be discussed in this Appendix.

The structure is loaded by a load q_1. This results in the reaction forces $H_{A,1}$, $H_{D,1}$, $V_{A,1}$ and $V_{D,1}$.The total deflection at this load is called e_1. e1 is the blue line in Figure K.1. If the load increase the total deflection increase (red line in Fig. K.1). The reaction forces changes too.

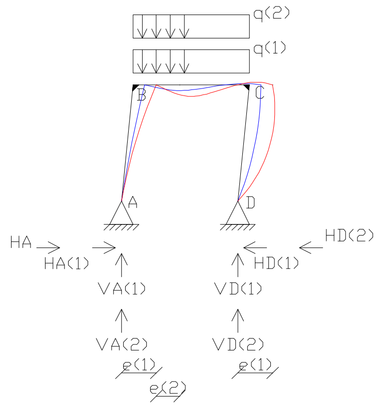


Figure K.1: Structure

The two columns in the Portal Frame are not equally loaded. In other words: the two columns do not yield at the same load. An assumption is that one of the columns (the left one) does not yield at all. The only column that yields is the right column. This assumption is realistic because both the bending moment and the normal force are larger in the righter column.

An I-shaped section is double symmetric. If an I-shaped section partial yields, an asymmetric section can exist. There can be a shift of the centre of gravity in the effective section. To take this effect into account an extra deflection z_i has been introduced. The deflection z_i is the shift of the centre of gravity in the i^{th} load case.

This Appendix is split up in two parts. These parts are based on the different load cases. In this non-linear analysis it is not possible to find one general formula for all loading (yield) cases.

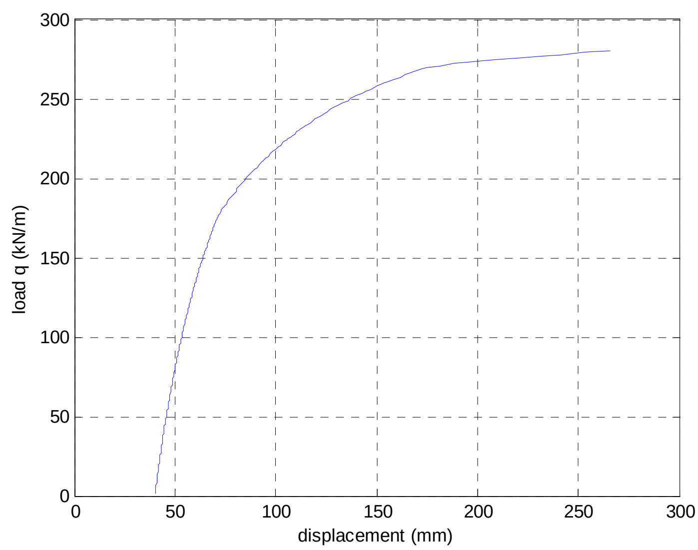


Figure K.2: Load-deflection Graphic

Column: Length 10m
HE 360A
Beam: Length 5m
HE 900A

The total deflection increases if the stiffness decreases. This has influence on the ultimate load. The load-deflection graphic of the non-linear analysis with residual stress can be found in Figure K.2.

K.1 Analysis if one part yields

To start, the formula of the second load case will be analysed. The second load case is the load after first yield. The half of the right flange yields. If this happens the section is not double symmetric anymore. The centre of gravity has been shift. This shift is taken into account at the analysis. As same as discussed before, the structure must be in equilibrium. The input of the equilibrium formulas is the total load and the total reaction forces. The following equilibriums are calculated:

Moment equilibrium:

$$\sum M \mid A = 0$$

$$\left(q_{total,2}\right) L_{bm} \left(\tfrac{1}{2} L_{bm} + e_{total,2}\right) - V_{D,total,2} L_{bm} = 0$$

The original load and the additional load are split.

$$\left(q_1 + q_2\right) L_{bm} \left(\tfrac{1}{2} L_{bm} + e_1 + e_2\right) - \left(V_{D,1} + V_{D,2}\right) L_{bm} = 0$$

All expressions can be divided by L_{bm}. There is only interest in the additional reaction force. The original reaction force is already discussed in Appendix J.

$$V_{D,1} + V_{D,2} = q_1 \left(\tfrac{1}{2} L_{bm} + e_1\right) + q_1 e_2 + q_2 \left(\tfrac{1}{2} L_{bm} + e_1 + e_2\right)$$

The original reaction force is known: $V_{D,1} = q_1 \left(\tfrac{1}{2} L_{bm} + e_1\right)$. This can be neglected. What is left is the following formula:

$$V_{D,2} = q_1 e_2 + q_2 \left(\tfrac{1}{2} L_{bm} + e_1 + e_2\right)$$

Vertical equilibrium.

$$\sum vert = 0$$

$$\left(q_{total,2}\right) L_{bm} - V_{A,total,2} - V_{D,total,2} = 0$$

The original load and the additional load are split.

$$\left(q_1 + q_2\right) L_{bm} - V_{A,1} - V_{A,2} - V_{D,1} - V_{D,2} = 0$$

Writing out some expressions

$$\left(q_1 L_{bm} - V_{A,1} - V_{D,1}\right) + q_2 L_{bm} - V_{A,2} - V_{D,2} = 0$$

Some expressions are discussed before. Known are the original equilibrium $\left(q_1 L_{bm} = V_{A,1} + V_{D,1}\right)$ and the extra load in point D $\left(V_{D,1} = q_1 e_2 + q_2 \left(\tfrac{1}{2} L_{bm} + e_1 + e_2\right)\right)$. These expressions are filled in and results in:

$$q_2 L_{bm} - V_{A,2} - \left(q_1 e_2 + q_2 \left(\tfrac{1}{2} L_{bm} + e_1 + e_2\right)\right) = 0$$

The same expressions can be taken together.

$$-V_{A,2} - q_1 e_2 + q_2 \left(\tfrac{1}{2} L_{bm} - e_1 - e_2\right) = 0$$

This results in a formula of $V_{A,2}$:

$$V_{A,2} = -q_1 e_2 + q_2 \left(\tfrac{1}{2} L_{bm} - e_1 - e_2 \right)$$

Moment in point C.

To calculate the additional deflection it is necessary to calculate the additional bending moments. The additional bending moment in point C $\left(M_{C,2}\right)$ has been determined. In this formula is the shift of the centre of gravity taken into account.

$$M_{C,total,2} = V_{D,total,2} e_{total,2} + H_{A,total,2} L_{cln} + V_{D,2} z_2$$

The bending moment and the reaction forces can be split is the original force and the additional force.

$$M_{C,1} + M_{C,2} = \left(V_{D,1} + V_{D,2}\right)\left(e_1 + \Delta e_2\right) + \left(H_{A,1} + H_{A,2}\right) L_{cln} + V_{D,2} z_2$$

Some expressions must be split to get the original expressions.

$$M_{C,1} + M_{C,2} = V_{D,1} e_1 + V_{D,1} e_2 + V_{D,2}\left(e_1 + e_2\right) + H_{A,1} L_{cln} + H_{A,2} L_{cln} + V_{D,2} z_2$$

The original moment $\left(M_{C,1} = V_{D,1} e_1 + H_{A,1} L_{cln}\right)$ can be filled in the formula.

$$M_{C,2} = V_{D,1} e_2 + V_{D,2}\left(e_1 + e_2 + z_2\right) + H_{A,2} L_{cln}$$

The expression of $V_{D,2}$ $\left(V_{D,2} = q_1 e_2 + q_2\left(\tfrac{1}{2} L_{bm} + e_1 + e_2\right)\right)$ is found before and can be used in the formula. Also the formula of the original reaction force $\left(V_{D,1} = q_1\left(\tfrac{1}{2} L_{bm} + e_1\right)\right)$ can be used in the formula.

$$M_{C,2} = q_1\left(\tfrac{1}{2} L_{bm} + e_1\right) e_2 + \left(q_1 e_2 + q_2\left(\tfrac{1}{2} L_{bm} + e_1 + e_2\right)\right)\left(e_1 + e_2 + z_2\right) + H_{A,2} L_{cln}$$

This formula can be made clearer. This results in the following formula:

$$M_{C,2} = q_2\left(\tfrac{1}{2} L_{bm} + e_1 + e_2\right)\left(e_1 + e_2 + z_2\right) + q_1 e_2\left(\tfrac{1}{2} L_{bm} + 2e_1 + e_2 + z_2\right) + H_{A,2} L_{cln}$$

Moment in point B.

The moment difference in point C has been formulated. The moment difference in point B will be analysed. Column AB will not yield. The point of gravity remains on the same location. To analysis starts with the following formula:

$$M_{B,total,2} = V_{A,total,2} e_{total,2} - H_{A,total,2} L_{cln}$$

Again the loads had been split in the original load and an additional load.

$$M_{B,1} + M_{B,2} = \left(V_{A,1} + V_{A,2}\right)\left(e_1 + e_2\right) - \left(H_{A,1} + H_{A,2}\right) L_{cln}$$

Some expressions are split to get the original formula.

$$M_{B,1} + M_{B,2} = V_{A,1} e_1 + V_{A,1} e_2 + V_{A,2}\left(e_1 + e_2\right) - H_{A,1} L_{cln} - H_{A,2} L_{cln}$$

Known is the original bending moment $\left(M_{B,1} = V_{A,1}e_1 - H_{A,1}L_{cln}\right)$. This moment is neglected from the formula.

$$M_{B,2} = V_{A,1}e_2 + V_{A,2}\left(e_1 + e_2\right) - H_{A,2}L_{cln}$$

Also known is the formula for the extra vertical reaction force in support A $\left(V_{A,2} = -q_1e_2 + q_2\left(\tfrac{1}{2}L_{bm} - e_1 - e_2\right)\right)$. This formula can be used in the formula. Also the formula for the original vertical reaction force $\left(V_{A,1} = q_1\left(\tfrac{1}{2}L_{bm} - e_1\right)\right)$ can be used.

$$M_{B,2} = q_1\left(\tfrac{1}{2}L_{bm} - e_1\right)e_2 + \left(-q_1e_2 + q_2\left(\tfrac{1}{2}L_{bm} - e_1 - e_2\right)\right)\left(e_1 + e_2\right) - H_{A,2}L_{cln}$$

The formula has been changed to the following one.

$$M_{B,2} = q_2\left(\tfrac{1}{2}L_{bm} - e_1 - e_2\right)\left(e_1 + e_2\right) + q_1e_2\left(\tfrac{1}{2}L_{bm} - 2e_1 - e_2\right) - H_{A,2}L_{cln}$$

The remaining forces at the supports are important to calculate the total deflection. This is also discussed in Appendix H, I and J. In these Appendices the total remaining force was important. In this case the increase of the remaining force is interesting.

$$F_{R,D,total,2} = V_{D,total,2}\frac{e_{total,2}}{L_{cln}} + H_{D,total,2}$$

'Total' has been split in an original part and an additional part.

$$F_{R,D,1} + F_{R,D,2} = \left(V_{D,1} + V_{D,2}\right)\frac{e_1 + e_2}{L_{cln}} + H_{D,1} + H_{D,2}$$

Some expressions are written out to get the original formula.

$$F_{R,D,1} + F_{R,D,2} = V_{D,1}\frac{e_1}{L_{cln}} + V_{D,1}\frac{e_2}{L_{cln}} + V_{D,2}\frac{e_1 + e_2}{L_{cln}} + H_{D,1} + H_{D,2}$$

The formula of the original remaining force $\left(F_{R,D,1} = V_{D,1}\frac{e_1}{L_{cln}} + H_{D,1}\right)$ can be neglected from the formula.

$$F_{R,D,2} = V_{D,1}\frac{e_2}{L_{cln}} + V_{D,2}\frac{e_1 + e_2}{L_{cln}} + H_{D,2}$$

The known formulas of the vertical reaction forces $\left(V_{D,2} = q_1e_2 + q_2\left(\tfrac{1}{2}L_{bm} + e_1 + e_2\right)\right)$ and $\left(V_{D,1} = q_1\left(\tfrac{1}{2}L_{bm} + e_1\right)\right)$ can be used.

$$F_{R,D,2} = q_1\left(\tfrac{1}{2}L_{bm} + e_1\right)\frac{e_2}{L_{cln}} + \left(q_1e_2 + q_2\left(\tfrac{1}{2}L_{bm} + e_1 + e_2\right)\right)\frac{e_1 + e_2}{L_{cln}} + H_{D,2}$$

This can be made clearer.

$$F_{R,D,2} = q_2\left(\tfrac{1}{2}L_{bm} + e_1 + e_2\right)\frac{e_1 + e_2}{L_{cln}} + q_1 e_2 \frac{\tfrac{1}{2}L_{bm} + 2e_1 + e_2}{L_{cln}} + H_{D,2}$$

The same analysis can be made for the remaining force in point A. Starting with the following formula.

$$F_{R,A,total,2} = V_{A,total,2}\frac{e_{total,2}}{L_{cln}} - H_{A,total,2}$$

Total has been split in an original part and an additional part.

$$F_{R,A,1} + F_{A,D,2} = \left(V_{A,1} + V_{A,2}\right)\frac{e_1 + e_2}{L_{cln}} - H_{A,1} - H_{A,2}$$

Some expressions are written out to get the original formulas.

$$F_{R,A,1} + F_{R,A,2} = V_{A,1}\frac{e_1}{L_{cln}} + V_{A,1}\frac{e_2}{L_{cln}} + V_{A,2}\frac{e_1 + e_2}{L_{cln}} - H_{A,1} - H_{A,2}$$

The original formula of the remaining force $\left(F_{R,A,1} = V_{A,1}\frac{e_1}{L_{cln}} - H_{A,1}\right)$ can be neglected from the formula.

$$F_{R,A,2} = V_{A,1}\frac{e_2}{L_{cln}} + V_{A,2}\frac{e_1 + e_2}{L_{cln}} - H_{A,2}$$

The known formulas of the vertical reaction forces $\left(V_{A,2} = -q_1 e_2 + q_2\left(\tfrac{1}{2}L_{bm} - e_1 - e_2\right)\right)$ and $\left(V_{A,1} = q_1\left(\tfrac{1}{2}L_{bm} - e_1\right)\right)$ can be used.

$$F_{R,A,2} = q_1\left(\tfrac{1}{2}L_{bm} - e_1\right)\frac{e_2}{L_{cln}} + \left(-q_1 e_2 + q_2\left(\tfrac{1}{2}L_{bm} - e_1 - e_2\right)\right)\frac{e_1 + e_2}{L_{cln}} - H_{A,2}$$

This can be made clearer.

$$F_{R,A,2} = q_2\left(\tfrac{1}{2}L_{bm} - e_1 - e_2\right)\frac{e_1 + e_2}{L_{cln}} + q_1 e_2 \frac{\tfrac{1}{2}L_{bm} - 2e_1 - e_2}{L_{cln}} - H_{A,2}$$

A list of all analysed formula is made. These formulas can be used to make an analysis to find a formula of the total deflection.

$$V_{D,2} = q_1 e_2 + q_2\left(\tfrac{1}{2}L_{bm} + e_1 + e_2\right)$$
$$V_{A,2} = -q_1 e_2 + q_2\left(\tfrac{1}{2}L_{bm} - e_1 - e_2\right)$$
$$M_{C,2} = q_2\left(\tfrac{1}{2}L_{bm} + e_1 + e_2\right)\left(e_1 + e_2 + z_2\right) + q_1 e_2\left(\tfrac{1}{2}L_{bm} + 2e_1 + e_2 + z_2\right) + H_{A,2}L_{cln}$$
$$M_{B,2} = q_2\left(\tfrac{1}{2}L_{bm} - e_1 - e_2\right)\left(e_1 + e_2\right) + q_1 e_2\left(\tfrac{1}{2}L_{bm} - 2e_1 - e_2\right) - H_{A,2}L_{cln}$$
$$F_{R,D,2} = q_2\left(\tfrac{1}{2}L_{bm} + e_1 + e_2\right)\frac{e_1 + e_2}{L_{cln}} + q_1 e_2 \frac{\tfrac{1}{2}L_{bm} + 2e_1 + e_2}{L_{cln}} + H_{D,2}$$
$$F_{R,A,2} = q_2\left(\tfrac{1}{2}L_{bm} - e_1 - e_2\right)\frac{e_1 + e_2}{L_{cln}} + q_1 e_2 \frac{\tfrac{1}{2}L_{bm} - 2e_1 - e_2}{L_{cln}} - H_{A,2}$$

As same as before the rotations in the point B and C can be calculated.

$$\varphi_{C,2} = \frac{q_2 L_{bm}{}^3}{24EI_{bm}} + \frac{M_{B,2}L_{bm}}{6EI_{bm}} - \frac{M_{C,2}L_{bm}}{3EI_{bm}}$$
$$\varphi_{B,2} = -\frac{q_2 L_{bm}{}^3}{24EI_{bm}} - \frac{M_{B,2}L_{bm}}{3EI_{bm}} + \frac{M_{C,2}L_{bm}}{6EI_{bm}}$$

An important issue is that column CD yields and column AB does not yield. The stiffness of column CD decreases while the stiffness of column AB remain constant (the original stiffness).

The difference in deflection of column AB can be expressed in the following formula:

$$e_2{}^{AB} = -\varphi_{B,2}L_{cln} + \frac{F_{R,A,2}L_{cln}{}^3}{3EI_{cln,1}}$$

The additional angle is a known formula and can be used in the formula.

$$e_2{}^{AB} = -\left(-\frac{q_2 L_{bm}{}^3}{24EI_{bm}} - \frac{M_{B,2}L_{bm}}{3EI_{bm}} + \frac{M_{C,2}L_{bm}}{6EI_{bm}}\right)L_{cln} + \frac{F_{R,A,2}L_{cln}{}^3}{3EI_{cln,1}}$$

There formulas of $M_{B,2}$, $M_{C,2}$ and $F_{R,A,2}$ are known. These formulas can be used in the formula.

$$e_2{}^{AB} = \frac{q_2 L_{bm}{}^3 L_{cln}}{24EI_{bm}}$$
$$+\left[q_2\left(\tfrac{1}{2}L_{bm} - e_1 - e_2\right)\left(e_1 + e_2\right) + q_1 e_2\left(\tfrac{1}{2}L_{bm} - 2e_1 - e_2\right) - H_{A,2}L_{cln}\right]\frac{L_{bm}L_{cln}}{3EI_{bm}}$$
$$-\left[q_2\left(\tfrac{1}{2}L_{bm} + e_1 + e_2\right)\left(e_1 + e_2 + z_2\right) + q_1 e_2\left(\tfrac{1}{2}L_{bm} + 2e_1 + e_2 + z_2\right) + H_{A,2}L_{cln}\right]\frac{L_{bm}L_{cln}}{6EI_{bm}}$$
$$+\left[q_2\left(\tfrac{1}{2}L_{bm} - e_1 - e_2\right)\frac{e_1 + e_2}{L_{cln}} + q_1 e_2 \frac{\tfrac{1}{2}L_{bm} - 2e_1 - e_2}{L_{cln}} - H_{A,2}\right]\frac{L_{cln}{}^3}{3EI_{cln,1}}$$

In the non linear analysis without residual stress (App. J) the horizontal reaction force has been separated. It was easier to separate the horizontal force than to separate the total deflection. The difference in the total horizontal reaction force will be separate from the rest of the formula.

$$H_{A,2}L_{cln}\frac{L_{bm}L_{cln}}{3EI_{bm}}+H_{A,2}L_{cln}\frac{L_{bm}L_{cln}}{6EI_{bm}}+H_{A,2}\frac{L_{cln}^{3}}{3EI_{cln,1}}=\frac{q_2L_{bm}^{3}L_{cln}}{24EI_{bm}}$$

$$+\left[q_2\left(\tfrac{1}{2}L_{bm}-e_1-e_2\right)\left(e_1+e_2\right)+q_1e_2\left(\tfrac{1}{2}L_{bm}-2e_1-e_2\right)\right]\frac{L_{bm}L_{cln}}{3EI_{bm}}$$

$$-\left[q_2\left(\tfrac{1}{2}L_{bm}+e_1+e_2\right)\left(e_1+e_2+z_2\right)+q_1e_2\left(\tfrac{1}{2}L_{bm}+2e_1+e_2+z_2\right)\right]\frac{L_{bm}L_{cln}}{6EI_{bm}}$$

$$+\left[q_2\left(\tfrac{1}{2}L_{bm}-e_1-e_2\right)\frac{e_1+e_2}{L_{cln}}+q_1e_2\frac{\tfrac{1}{2}L_{bm}-2e_1-e_2}{L_{cln}}\right]\frac{L_{cln}^{3}}{3EI_{cln,1}}-e_2$$

Combine the same expression of $H_{A,2}$.

$$H_{A,2}\left(\frac{L_{bm}L_{cln}^{2}}{2EI_{bm}}+\frac{L_{cln}^{3}}{3EI_{cln,1}}\right)=\frac{q_2L_{bm}^{3}L_{cln}}{24EI_{bm}}$$

$$+\left[q_2\left(\tfrac{1}{2}L_{bm}-e_1-e_2\right)\left(e_1+e_2\right)+q_1e_2\left(\tfrac{1}{2}L_{bm}-2e_1-e_2\right)\right]\frac{L_{bm}L_{cln}}{3EI_{bm}}$$

$$-\left[q_2\left(\tfrac{1}{2}L_{bm}+e_1+e_2\right)\left(e_1+e_2+z_2\right)+q_1e_2\left(\tfrac{1}{2}L_{bm}+2e_1+e_2+z_2\right)\right]\frac{L_{bm}L_{cln}}{6EI_{bm}}$$

$$+\left[q_2\left(\tfrac{1}{2}L_{bm}-e_1-e_2\right)\frac{e_1+e_2}{L_{cln}}+q_1e_2\frac{\tfrac{1}{2}L_{bm}-2e_1-e_2}{L_{cln}}\right]\frac{L_{cln}^{3}}{3EI_{cln,1}}-e_2$$

To continue the analysis it is important to write out all expressions. Write out all expressions in ones results in an unclear situation. The different expressions are written out in parts. There are three expressions written out. The last three expressions of the right part of the formula.

To write out the first part:

$$\left[q_2\left(\tfrac{1}{2}L_{bm}-e_1-e_2\right)\left(e_1+e_2\right)+q_1e_2\left(\tfrac{1}{2}L_{bm}-2e_1-e_2\right)\right]\frac{L_{bm}L_{cln}}{3EI_{bm}}$$

$$=\frac{q_2L_{bm}^{2}L_{cln}e_1}{6EI_{bm}}+\frac{q_2L_{bm}^{2}L_{cln}e_2}{6EI_{bm}}-\frac{q_2L_{bm}L_{cln}e_1^{2}}{3EI_{bm}}-\frac{q_2L_{bm}L_{cln}e_1e_2}{3EI_{bm}}-\frac{q_2L_{bm}L_{cln}e_1e_2}{3EI_{bm}}-\frac{q_2L_{bm}L_{cln}e_2^{2}}{3EI_{bm}}$$

$$+\frac{q_1L_{bm}^{2}L_{cln}e_2}{6EI_{bm}}-\frac{2q_1L_{bm}L_{cln}e_1e_2}{3EI_{bm}}-\frac{q_1L_{bm}L_{cln}e_2^{2}}{3EI_{bm}}$$

The second part:

$$-\left[q_2\left(\tfrac{1}{2}L_{bm}+e_1+e_2\right)\left(e_1+e_2+z_2\right)+q_1e_2\left(\tfrac{1}{2}L_{bm}+2e_1+e_2+z_2\right)\right]\frac{L_{bm}L_{cln}}{6EI_{bm}}$$

$$=-\frac{q_2L_{bm}{}^2L_{cln}e_1}{12EI_{bm}}-\frac{q_2L_{bm}{}^2L_{cln}e_2}{12EI_{bm}}-\frac{q_2L_{bm}{}^2L_{cln}z_2}{12EI_{bm}}-\frac{q_2L_{bm}L_{cln}e_1^2}{6EI_{bm}}-\frac{q_2L_{bm}L_{cln}e_1e_2}{6EI_{bm}}$$

$$-\frac{q_2L_{bm}L_{cln}e_1z_2}{6EI_{bm}}-\frac{q_2L_{bm}L_{cln}e_1e_2}{6EI_{bm}}-\frac{q_2L_{bm}L_{cln}e_2{}^2}{6EI_{bm}}-\frac{q_2L_{bm}L_{cln}z_2e_2}{6EI_{bm}}$$

$$-\frac{q_1L_{bm}{}^2L_{cln}e_2}{12EI_{bm}}-\frac{q_1L_{bm}L_{cln}e_1e_2}{3EI_{bm}}-\frac{q_1L_{bm}L_{cln}e_2{}^2}{6EI_{bm}}-\frac{q_1L_{bm}L_{cln}z_2e_2}{6EI_{bm}}$$

And the third part:

$$\left[q_2\left(\tfrac{1}{2}L_{bm}-e_1-e_2\right)\frac{e_1+e_2}{L_{cln}}+q_1e_2\frac{\tfrac{1}{2}L_{bm}-2e_1-e_2}{L_{cln}}\right]\frac{L_{cln}{}^3}{3EI_{cln,1}}$$

$$=\frac{q_2L_{bm}L_{cln}{}^2e_1}{6EI_{cln,1}}+\frac{q_2L_{bm}L_{cln}{}^2e_2}{6EI_{cln,1}}-\frac{q_2L_{cln}{}^2e_1^2}{3EI_{cln,1}}-\frac{q_2L_{cln}{}^2e_1e_2}{3EI_{cln,1}}-\frac{q_2L_{cln}{}^2e_1e_2}{3EI_{cln,1}}-\frac{q_2L_{cln}{}^2e_2{}^2}{3EI_{cln,1}}$$

$$+\frac{q_1L_{bm}L_{cln}{}^2e_2}{6EI_{cln,1}}-\frac{2q_1L_{cln}{}^2e_1e_2}{3EI_{cln,1}}-\frac{q_1L_{cln}{}^2e_2{}^2}{3EI_{cln,1}}$$

The three worked out expressions can be used in the formula. This results in the following formula:

$$H_{A,2}\left(\frac{L_{bm}L_{cln}{}^2}{2EI_{bm}}+\frac{L_{cln}{}^3}{3EI_{cln,1}}\right)=\frac{q_2L_{bm}{}^3L_{cln}}{24EI_{bm}}$$

$$+\frac{q_2L_{bm}{}^2L_{cln}e_1}{6EI_{bm}}+\frac{q_2L_{bm}{}^2L_{cln}e_2}{6EI_{bm}}-\frac{q_2L_{bm}L_{cln}e_1^2}{3EI_{bm}}-\frac{q_2L_{bm}L_{cln}e_1e_2}{3EI_{bm}}-\frac{q_2L_{bm}L_{cln}e_1e_2}{3EI_{bm}}-\frac{q_2L_{bm}L_{cln}e_2{}^2}{3EI_{bm}}$$

$$+\frac{q_1L_{bm}{}^2L_{cln}e_2}{6EI_{bm}}-\frac{2q_1L_{bm}L_{cln}e_1e_2}{3EI_{bm}}-\frac{q_1L_{bm}L_{cln}e_2{}^2}{3EI_{bm}}$$

$$-\frac{q_2L_{bm}{}^2L_{cln}e_1}{12EI_{bm}}-\frac{q_2L_{bm}{}^2L_{cln}e_2}{12EI_{bm}}-\frac{q_2L_{bm}{}^2L_{cln}z_2}{12EI_{bm}}-\frac{q_2L_{bm}L_{cln}e_1^2}{6EI_{bm}}-\frac{q_2L_{bm}L_{cln}e_1e_2}{6EI_{bm}}$$

$$-\frac{q_2L_{bm}L_{cln}e_1z_2}{6EI_{bm}}-\frac{q_2L_{bm}L_{cln}e_1e_2}{6EI_{bm}}-\frac{q_2L_{bm}L_{cln}e_2{}^2}{6EI_{bm}}-\frac{q_2L_{bm}L_{cln}z_2e_2}{6EI_{bm}}$$

$$-\frac{q_1L_{bm}{}^2L_{cln}e_2}{12EI_{bm}}-\frac{q_1L_{bm}L_{cln}e_1e_2}{3EI_{bm}}-\frac{q_1L_{bm}L_{cln}e_2{}^2}{6EI_{bm}}-\frac{q_1L_{bm}L_{cln}z_2e_2}{6EI_{bm}}$$

$$+\frac{q_2L_{bm}L_{cln}{}^2e_1}{6EI_{cln,1}}+\frac{q_2L_{bm}L_{cln}{}^2e_2}{6EI_{cln,1}}-\frac{q_2L_{cln}{}^2e_1^2}{3EI_{cln,1}}-\frac{q_2L_{cln}{}^2e_1e_2}{3EI_{cln,1}}-\frac{q_2L_{cln}{}^2e_1e_2}{3EI_{cln,1}}-\frac{q_2L_{cln}{}^2e_2{}^2}{3EI_{cln,1}}$$

$$+\frac{q_1L_{bm}L_{cln}{}^2e_2}{6EI_{cln,1}}-\frac{2q_1L_{cln}{}^2e_1e_2}{3EI_{cln,1}}-\frac{q_1L_{cln}{}^2e_2{}^2}{3EI_{cln,1}}-e_2$$

Combine the same expressions.

$$H_{A,2}\left(\frac{L_{bm}L_{cln}{}^2}{2EI_{bm}}+\frac{L_{cln}{}^3}{3EI_{cln,1}}\right)=\frac{q_2L_{bm}{}^3L_{cln}}{24EI_{bm}}+\frac{q_2L_{bm}{}^2L_{cln}e_1}{12EI_{bm}}+\frac{q_2L_{bm}{}^2L_{cln}e_2}{12EI_{bm}}-\frac{q_2L_{bm}L_{cln}e_1^2}{2EI_{bm}}-\frac{q_2L_{bm}L_{cln}e_1e_2}{EI_{bm}}$$
$$-\frac{q_2L_{bm}L_{cln}e_2{}^2}{2EI_{bm}}+\frac{q_1L_{bm}{}^2L_{cln}e_2}{12EI_{bm}}-\frac{q_1L_{bm}L_{cln}e_1e_2}{EI_{bm}}-\frac{q_1L_{bm}L_{cln}e_2{}^2}{2EI_{bm}}-\frac{q_2L_{bm}{}^2L_{cln}z_2}{12EI_{bm}}-\frac{q_2L_{bm}L_{cln}e_1z_2}{6EI_{bm}}-\frac{q_2L_{bm}L_{cln}z_2e_2}{6EI_{bm}}$$
$$-\frac{q_1L_{bm}L_{cln}z_2e_2}{6EI_{bm}}+\frac{q_2L_{bm}L_{cln}{}^2e_1}{6EI_{cln,1}}+\frac{q_2L_{bm}L_{cln}{}^2e_2}{6EI_{cln,1}}-\frac{q_2L_{cln}{}^2e_1^2}{3EI_{cln,1}}-\frac{2q_2L_{cln}{}^2e_1e_2}{3EI_{cln,1}}-\frac{q_2L_{cln}{}^2e_2{}^2}{3EI_{cln,1}}+\frac{q_1L_{bm}L_{cln}{}^2e_2}{6EI_{cln,1}}$$
$$-\frac{2q_1L_{cln}{}^2e_1e_2}{3EI_{cln,1}}-\frac{q_1L_{cln}{}^2e_2{}^2}{3EI_{cln,1}}-e_2$$

To find a formula for the horizontal force it is necessary to make one denominator.

$$H_{A,2}\left(\frac{12L_{bm}L_{cln}{}^2EI_{cln,1}+8L_{cln}{}^3EI_{bm}}{24EI_{cln,1}EI_{bm}}\right)=\frac{q_2L_{bm}{}^3L_{cln}EI_{cln,1}}{24EI_{bm}EI_{cln,1}}+\frac{2q_2L_{bm}{}^2L_{cln}e_1EI_{cln,1}}{24EI_{bm}EI_{cln,1}}+\frac{2q_2L_{bm}{}^2L_{cln}e_2EI_{cln,1}}{24EI_{bm}EI_{cln,1}}$$
$$-\frac{12q_2L_{bm}L_{cln}e_1^2EI_{cln,1}}{24EI_{bm}EI_{cln,1}}-\frac{24q_2L_{bm}L_{cln}e_1e_2EI_{cln,1}}{24EI_{bm}EI_{cln,1}}-\frac{12q_2L_{bm}L_{cln}e_2{}^2EI_{cln,1}}{24EI_{bm}EI_{cln,1}}+\frac{2q_1L_{bm}{}^2L_{cln}e_2EI_{cln,1}}{24EI_{bm}EI_{cln,1}}$$
$$-\frac{24q_1L_{bm}L_{cln}e_1e_2EI_{cln,1}}{24EI_{bm}EI_{cln,1}}-\frac{12q_1L_{bm}L_{cln}e_2{}^2EI_{cln,1}}{24EI_{bm}EI_{cln,1}}-\frac{2q_2L_{bm}{}^2L_{cln}z_2EI_{cln,1}}{24EI_{bm}EI_{cln,1}}-\frac{4q_2L_{bm}L_{cln}e_1z_2EI_{cln,1}}{24EI_{bm}EI_{cln,1}}$$
$$-\frac{4q_2L_{bm}L_{cln}z_2e_2EI_{cln,1}}{24EI_{bm}EI_{cln,1}}-\frac{4q_1L_{bm}L_{cln}z_2e_2EI_{cln,1}}{24EI_{bm}EI_{cln,1}}+\frac{4q_2L_{bm}L_{cln}{}^2e_1EI_{bm}}{24EI_{bm}EI_{cln,1}}+\frac{4q_2L_{bm}L_{cln}{}^2e_2EI_{bm}}{24EI_{bm}EI_{cln,1}}$$
$$-\frac{8q_2L_{cln}{}^2e_1^2EI_{bm}}{24EI_{bm}EI_{cln,1}}-\frac{16q_2L_{cln}{}^2e_1e_2EI_{bm}}{24EI_{bm}EI_{cln,1}}-\frac{8q_2L_{cln}{}^2e_2{}^2EI_{bm}}{24EI_{bm}EI_{cln,1}}+\frac{4q_1L_{bm}L_{cln}{}^2e_2EI_{bm}}{24EI_{bm}EI_{cln,1}}-\frac{16q_1L_{cln}{}^2e_1e_2EI_{bm}}{24EI_{bm}EI_{cln,1}}$$
$$-\frac{8q_1L_{cln}{}^2e_2{}^2EI_{bm}}{24EI_{bm}EI_{cln,1}}-\frac{24e_2EI_{bm}EI_{cln,1}}{24EI_{bm}EI_{cln,1}}$$

All denominators are the same and can be neglected. After neglect these denominators it is possible to make a formula of $H_{A,2}$. The result is the following formula:

$$H_{A,2}=\frac{\begin{pmatrix}q_2L_{bm}{}^3L_{cln}EI_{cln,1}+2q_2L_{bm}{}^2L_{cln}e_1EI_{cln,1}+2q_2L_{bm}{}^2L_{cln}e_2EI_{cln,1}-12q_2L_{bm}L_{cln}e_1^2EI_{cln,1}\\-24q_2L_{bm}L_{cln}e_1e_2EI_{cln,1}-12q_2L_{bm}L_{cln}e_2{}^2EI_{cln,1}+2q_1L_{bm}{}^2L_{cln}e_2EI_{cln,1}-24q_1L_{bm}L_{cln}e_1e_2EI_{cln,1}\\-12q_1L_{bm}L_{cln}e_2{}^2EI_{cln,1}-2q_2L_{bm}{}^2L_{cln}z_2EI_{cln,1}-4q_2L_{bm}L_{cln}e_1z_2EI_{cln,1}-4q_2L_{bm}L_{cln}z_2e_2EI_{cln,1}\\-4q_1L_{bm}L_{cln}z_2e_2EI_{cln,1}+4q_2L_{bm}L_{cln}{}^2e_1EI_{bm}+4q_2L_{bm}L_{cln}{}^2e_2EI_{bm}-8q_2L_{cln}{}^2e_1^2EI_{bm}-16q_2L_{cln}{}^2e_1e_2EI_{bm}\\-8q_2L_{cln}{}^2e_2{}^2EI_{bm}+4q_1L_{bm}L_{cln}{}^2e_2EI_{bm}-16q_1L_{cln}{}^2e_1e_2EI_{bm}-8q_1L_{cln}{}^2e_2{}^2EI_{bm}-24e_2EI_{bm}EI_{cln,1}\end{pmatrix}}{12L_{bm}L_{cln}{}^2EI_{cln,1}+8L_{cln}{}^3EI_{bm}}$$

This formula will be used later on. The formula above is the result of the difference of deflection in column AB. The same analysis can be made to find a formula for the difference of deflection in column CD. Column CD is partial yielded and has a reduced stiffness. The starting formula of this analysis is:

$$e_2{}^{CD}=-\varphi_{C,2}L_{cln}+F_{R,D,2}\frac{L_{cln}{}^3}{3EI_{cln,2}}$$

$\varphi_{C,2}$ is known and can be used.

$$e_2^{CD} = -\left(\frac{q_2 L_{bm}^3}{24EI_{bm}} + \frac{M_{B,2} L_{bm}}{6EI_{bm}} - \frac{M_{C,2} L_{bm}}{3EI_{bm}} \right) L_{cln} + F_{R,D,2} \frac{L_{cln}^3}{3EI_{cln,2}}$$

Also the expressions for $M_{B,2}$, $M_{C,2}$ and $F_{R,D,2}$ are known. These expressions can be filled in the formula.

$$e_2^{CD} = -\frac{q_2 L_{bm}^3 L_{cln}}{24EI_{bm}}$$

$$-\left[q_2 \left(\tfrac{1}{2} L_{bm} - e_1 - e_2 \right)\left(e_1 + e_2 \right) + q_1 e_2 \left(\tfrac{1}{2} L_{bm} - 2e_1 - e_2 \right) - H_{A,2} L_{cln} \right] \frac{L_{bm} L_{cln}}{6EI_{bm}}$$

$$+\left[q_2 \left(\tfrac{1}{2} L_{bm} + e_1 + e_2 \right)\left(e_1 + e_2 + z_2 \right) + q_1 e_2 \left(\tfrac{1}{2} L_{bm} + 2e_1 + e_2 + z_2 \right) + H_{A,2} L_{cln} \right] \frac{L_{bm} L_{cln}}{3EI_{bm}}$$

$$+\left[q_2 \left(\tfrac{1}{2} L_{bm} + e_1 + e_2 \right) \frac{e_1 + e_2}{L_{cln}} + q_1 e_2 \frac{\tfrac{1}{2} L_{bm} + 2e_1 + e_2}{L_{cln}} + H_{D,2} \right] \frac{L_{cln}^3}{3EI_{cln,2}}$$

$H_{D,2}$ can be separate from the rest of the formula.

$$-H_{A,2} L_{cln} \frac{L_{bm} L_{cln}}{6EI_{bm}} - H_{A,2} L_{cln} \frac{L_{bm} L_{cln}}{3EI_{bm}} - H_{D,2} \frac{L_{cln}^3}{3EI_{cln,2}} = -\frac{q_2 L_{bm}^3 L_{cln}}{24EI_{bm}}$$

$$-\left[q_2 \left(\tfrac{1}{2} L_{bm} - e_1 - e_2 \right)\left(e_1 + e_2 \right) + q_1 e_2 \left(\tfrac{1}{2} L_{bm} - 2e_1 - e_2 \right) \right] \frac{L_{bm} L_{cln}}{6EI_{bm}}$$

$$+\left[q_2 \left(\tfrac{1}{2} L_{bm} + e_1 + e_2 \right)\left(e_1 + e_2 + z_2 \right) + q_1 e_2 \left(\tfrac{1}{2} L_{bm} + 2e_1 + e_2 + z_2 \right) \right] \frac{L_{bm} L_{cln}}{3EI_{bm}}$$

$$+\left[q_2 \left(\tfrac{1}{2} L_{bm} + e_1 + e_2 \right) \frac{e_1 + e_2}{L_{cln}} + q_1 e_2 \frac{\tfrac{1}{2} L_{bm} + 2e_1 + e_2}{L_{cln}} \right] \frac{L_{cln}^3}{3EI_{cln,2}} - e_2$$

Combine the expressions of $H_{A,2}$.

$$-H_{A,2} \left(\frac{L_{bm} L_{cln}^2}{2EI_{bm}} + \frac{L_{cln}^3}{3EI_{cln,2}} \right) = -\frac{q_2 L_{bm}^3 L_{cln}}{24EI_{bm}}$$

$$-\left[q_2 \left(\tfrac{1}{2} L_{bm} - e_1 - e_2 \right)\left(e_1 + e_2 \right) + q_1 e_2 \left(\tfrac{1}{2} L_{bm} - 2e_1 - e_2 \right) \right] \frac{L_{bm} L_{cln}}{6EI_{bm}}$$

$$+\left[q_2 \left(\tfrac{1}{2} L_{bm} + e_1 + e_2 \right)\left(e_1 + e_2 + z_2 \right) + q_1 e_2 \left(\tfrac{1}{2} L_{bm} + 2e_1 + e_2 + z_2 \right) \right] \frac{L_{bm} L_{cln}}{3EI_{bm}}$$

$$+\left[q_2 \left(\tfrac{1}{2} L_{bm} + e_1 + e_2 \right) \frac{e_1 + e_2}{L_{cln}} + q_1 e_2 \frac{\tfrac{1}{2} L_{bm} + 2e_1 + e_2}{L_{cln}} \right] \frac{L_{cln}^3}{3EI_{cln,2}} - e_2$$

To continue the analysis it is important to write out all expressions. Write out all expressions in ones results in an unclear situation. The different expressions are written out in parts. There are three expressions written out. The last three expressions of the right part of the formula.

Write out the first part:

$$-\left[q_2\left(½L_{bm}-e_1-e_2\right)\left(e_1+e_2\right)+q_1e_2\left(½L_{bm}-2e_1-e_2\right)\right]\frac{L_{bm}L_{cln}}{6EI_{bm}}$$

$$=-\frac{q_2L_{bm}{}^2L_{cln}e_1}{12EI_{bm}}-\frac{q_2L_{bm}{}^2L_{cln}e_2}{12EI_{bm}}+\frac{q_2L_{bm}L_{cln}e_1{}^2}{6EI_{bm}}+\frac{q_2L_{bm}L_{cln}e_1e_2}{6EI_{bm}}+\frac{q_2L_{bm}L_{cln}e_1e_2}{6EI_{bm}}+\frac{\Delta q_2L_{bm}L_{cln}e_2{}^2}{6EI_{bm}}$$

$$-\frac{q_1L_{bm}{}^2L_{cln}e_2}{12EI_{bm}}+\frac{q_1L_{bm}L_{cln}e_1e_2}{3EI_{bm}}+\frac{q_1L_{bm}L_{cln}e_2{}^2}{6EI_{bm}}$$

The second part:

$$\left[q_2\left(½L_{bm}+e_1+e_2\right)\left(e_1+e_2+z_2\right)+q_1e_2\left(½L_{bm}+2e_1+e_2+z_2\right)\right]\frac{L_{bm}L_{cln}}{3EI_{bm}}$$

$$=\frac{q_2L_{bm}{}^2L_{cln}e_1}{6EI_{bm}}+\frac{q_2L_{bm}{}^2L_{cln}\Delta e_2}{6EI_{bm}}+\frac{q_2L_{bm}{}^2L_{cln}z_2}{6EI_{bm}}+\frac{q_2L_{bm}L_{cln}e_1{}^2}{3EI_{bm}}+q_2\frac{L_{bm}L_{cln}e_1e_2}{3EI_{bm}}+\frac{q_2L_{bm}L_{cln}e_1z_2}{3EI_{bm}}$$

$$+\frac{q_2L_{bm}L_{cln}e_1e_2}{3EI_{bm}}+\frac{q_2L_{bm}L_{cln}e_2{}^2}{3EI_{bm}}+\frac{q_2L_{bm}L_{cln}z_2e_2}{3EI_{bm}}+\frac{q_1L_{bm}{}^2L_{cln}e_2}{6EI_{bm}}+\frac{2q_1L_{bm}L_{cln}e_1e_2}{3EI_{bm}}+\frac{q_1L_{bm}L_{cln}e_2{}^2}{3EI_{bm}}+\frac{q_1L_{bm}L_{cln}z_2e_2}{3EI_{bm}}$$

The third part:

$$\left[q_2\left(½L_{bm}+e_1+e_2\right)\frac{e_1+e_2}{L_{cln}}+q_1e_2\frac{½L_{bm}+2e_1+e_2}{L_{cln}}\right]\frac{L_{cln}{}^3}{3EI_{cln,2}}$$

$$=\frac{q_2L_{bm}L_{cln}{}^2e_1}{6EI_{cln,2}}+\frac{q_2L_{bm}L_{cln}{}^2e_2}{6EI_{cln,2}}+\frac{q_2L_{cln}{}^2e_1{}^2}{3EI_{cln,2}}+\frac{q_2L_{cln}{}^2e_1e_2}{3EI_{cln,2}}+\frac{q_2L_{cln}{}^2e_1e_2}{3EI_{cln,2}}+\frac{q_2L_{cln}{}^2e_2{}^2}{3EI_{cln,2}}$$

$$+\frac{q_1L_{bm}L_{cln}{}^2e_2}{6EI_{cln,2}}+\frac{2q_1L_{cln}{}^2e_1e_2}{3EI_{cln,2}}+\frac{q_1L_{cln}{}^2e_2{}^2}{3EI_{cln,2}}$$

If all these expressions are used, the following formula exists.

$$-H_{A,2}\left(\frac{L_{bm}L_{cln}^{2}}{2EI_{bm}}+\frac{L_{cln}^{3}}{3EI_{cln,2}}\right)=-\frac{q_2L_{bm}^{3}L_{cln}}{24EI_{bm}}$$

$$-\frac{q_2L_{bm}^{2}L_{cln}e_1}{12EI_{bm}}-\frac{q_2L_{bm}^{2}L_{cln}e_2}{12EI_{bm}}+\frac{q_2L_{bm}L_{cln}e_1^{2}}{6EI_{bm}}+\frac{q_2L_{bm}L_{cln}e_1e_2}{6EI_{bm}}+\frac{q_2L_{bm}L_{cln}e_1e_2}{6EI_{bm}}+\frac{q_2L_{bm}L_{cln}e_2^{2}}{6EI_{bm}}$$

$$-\frac{q_1L_{bm}^{2}L_{cln}e_2}{12EI_{bm}}+\frac{q_1L_{bm}L_{cln}e_1e_2}{3EI_{bm}}+\frac{q_1L_{bm}L_{cln}e_2^{2}}{6EI_{bm}}$$

$$+\frac{q_2L_{bm}^{2}L_{cln}e_1}{6EI_{bm}}+\frac{q_2L_{bm}^{2}L_{cln}e_2}{6EI_{bm}}+\frac{q_2L_{bm}^{2}L_{cln}z_2}{6EI_{bm}}+\frac{q_2L_{bm}L_{cln}e_1^{2}}{3EI_{bm}}+\frac{q_2L_{bm}L_{cln}e_1e_2}{3EI_{bm}}+\frac{q_2L_{bm}L_{cln}e_1z_2}{3EI_{bm}}$$

$$+\frac{q_2L_{bm}L_{cln}e_1e_2}{3EI_{bm}}+\frac{q_2L_{bm}L_{cln}e_2^{2}}{3EI_{bm}}+\frac{q_2L_{bm}L_{cln}z_2e_2}{3EI_{bm}}$$

$$+\frac{q_1L_{bm}^{2}L_{cln}e_2}{6EI_{bm}}+\frac{2q_1L_{bm}L_{cln}e_1e_2}{3EI_{bm}}+\frac{q_1L_{bm}L_{cln}e_2^{2}}{3EI_{bm}}+\frac{q_1L_{bm}L_{cln}z_2e_2}{3EI_{bm}}$$

$$+\frac{q_2L_{bm}L_{cln}^{2}e_1}{6EI_{cln,2}}+\frac{q_2L_{bm}L_{cln}^{2}e_2}{6EI_{cln,2}}+\frac{q_2L_{cln}^{2}e_1^{2}}{3EI_{cln,2}}+\frac{q_2L_{cln}^{2}e_1e_2}{3EI_{cln,2}}+\frac{q_2L_{cln}^{2}e_1e_2}{3EI_{cln,2}}+\frac{q_2L_{cln}^{2}e_2^{2}}{3EI_{cln,2}}$$

$$+\frac{q_1L_{bm}L_{cln}^{2}e_2}{6EI_{cln,2}}+\frac{2q_1L_{cln}^{2}e_1e_2}{3EI_{cln,2}}+\frac{q_1L_{cln}^{2}e_2^{2}}{3EI_{cln,2}}-e_2$$

Combine the same expressions.

$$-H_{A,2}\left(\frac{L_{bm}L_{cln}^{2}}{2EI_{bm}}+\frac{L_{cln}^{3}}{3EI_{cln,2}}\right)=-\frac{q_2L_{bm}^{3}L_{cln}}{24EI_{bm}}+\frac{q_2L_{bm}^{2}L_{cln}e_1}{12EI_{bm}}+\frac{q_2L_{bm}^{2}L_{cln}e_2}{12EI_{bm}}+\frac{q_2L_{bm}L_{cln}e_1^{2}}{2EI_{bm}}+\frac{q_2L_{bm}L_{cln}e_1e_2}{EI_{bm}}$$

$$+\frac{q_2L_{bm}L_{cln}e_2^{2}}{2EI_{bm}}+\frac{q_1L_{bm}^{2}L_{cln}e_2}{12EI_{bm}}+\frac{q_1L_{bm}L_{cln}e_1e_2}{EI_{bm}}+\frac{q_1L_{bm}L_{cln}e_2^{2}}{2EI_{bm}}+\frac{q_2L_{bm}^{2}L_{cln}z_2}{6EI_{bm}}+\frac{q_2L_{bm}L_{cln}e_1z_2}{3EI_{bm}}+\frac{q_2L_{bm}L_{cln}z_2e_2}{3EI_{bm}}$$

$$+\frac{q_1L_{bm}L_{cln}z_2e_2}{3EI_{bm}}+\frac{q_2L_{bm}L_{cln}^{2}e_1}{6EI_{cln,2}}+\frac{q_2L_{bm}L_{cln}^{2}e_2}{6EI_{cln,2}}+\frac{q_2L_{cln}^{2}e_1^{2}}{3EI_{cln,2}}+\frac{2q_2L_{cln}^{2}e_1e_2}{3EI_{cln,2}}+\frac{q_2L_{cln}^{2}e_2^{2}}{3EI_{cln,2}}+\frac{q_1L_{bm}L_{cln}^{2}e_2}{6EI_{cln,2}}$$

$$+\frac{2q_1L_{cln}^{2}e_1e_2}{3EI_{cln,2}}+\frac{q_1L_{cln}^{2}e_2^{2}}{3EI_{cln,2}}-e_2$$

Make the same denominator for all expressions.

$$-H_{A,2}\left(\frac{12L_{bm}L_{cln}^{2}EI_{cln,2}+8L_{cln}^{3}EI_{bm}}{24EI_{bm}EI_{cln,2}}\right)=-\frac{q_2L_{bm}^{3}L_{cln}EI_{cln,2}}{24EI_{bm}EI_{cln,2}}+\frac{2q_2L_{bm}^{2}L_{cln}e_1EI_{cln,2}}{24EI_{bm}EI_{cln,2}}+\frac{2q_2L_{bm}^{2}L_{cln}\Delta e_2EI_{cln,2}}{24EI_{bm}EI_{cln,2}}$$
$$+\frac{12q_2L_{bm}L_{cln}e_1^{2}EI_{cln,2}}{24EI_{bm}EI_{cln,2}}+\frac{24q_2L_{bm}L_{cln}e_1e_2EI_{cln,2}}{24EI_{bm}EI_{cln,2}}+\frac{12q_2L_{bm}L_{cln}e_2^{2}EI_{cln,2}}{24EI_{bm}EI_{cln,2}}+\frac{2q_1L_{bm}^{2}L_{cln}e_2EI_{cln,2}}{24EI_{bm}EI_{cln,2}}$$
$$+\frac{24q_1L_{bm}L_{cln}e_1e_2EI_{cln,2}}{24EI_{bm}EI_{cln,2}}+\frac{12q_1L_{bm}L_{cln}e_2^{2}EI_{cln,2}}{24EI_{bm}EI_{cln,2}}+\frac{4q_2L_{bm}^{2}L_{cln}z_2EI_{cln,2}}{24EI_{bm}EI_{cln,2}}+\frac{8q_2L_{bm}L_{cln}e_1z_2EI_{cln,2}}{24EI_{bm}EI_{cln,2}}$$
$$+\frac{8q_2L_{bm}L_{cln}z_2e_2EI_{cln,2}}{24EI_{bm}EI_{cln,2}}+\frac{8q_1L_{bm}L_{cln}z_2e_2EI_{cln,2}}{24EI_{bm}EI_{cln,2}}+\frac{4q_2L_{bm}L_{cln}^{2}e_1EI_{bm}}{24EI_{bm}EI_{cln,2}}+\frac{4q_2L_{bm}L_{cln}^{2}e_2EI_{bm}}{24EI_{bm}EI_{cln,2}}$$
$$+\frac{8q_2L_{cln}^{2}e_1^{2}EI_{bm}}{24EI_{bm}EI_{cln,2}}+\frac{16q_2L_{cln}^{2}e_1e_2EI_{bm}}{24EI_{bm}EI_{cln,2}}+\frac{8q_2L_{cln}^{2}e_2^{2}EI_{bm}}{24EI_{bm}EI_{cln,2}}+\frac{4q_1L_{bm}L_{cln}^{2}e_2EI_{bm}}{24EI_{bm}EI_{cln,2}}+\frac{16q_1L_{cln}^{2}e_1e_2EI_{bm}}{24EI_{bm}EI_{cln,2}}$$
$$+\frac{8q_1L_{cln}^{2}e_2^{2}EI_{bm}}{24EI_{bm}EI_{cln,2}}-\frac{24e_2EI_{bm}EI_{cln,2}}{24EI_{bm}EI_{cln,2}}$$

All denominators are the same and can be neglected. After neglect these denominators it is possible to make a formula of H_A. The result is the following formula:

$$H_{A,2}=\frac{\begin{pmatrix}q_2L_{bm}^{3}L_{cln}EI_{cln,2}-2q_2L_{bm}^{2}L_{cln}e_1EI_{cln,2}-2q_2L_{bm}^{2}L_{cln}e_2EI_{cln,2}-12q_2L_{bm}L_{cln}e_1^{2}EI_{cln,2}\\-24q_2L_{bm}L_{cln}e_1e_2EI_{cln,2}-12q_2L_{bm}L_{cln}e_2^{2}EI_{cln,2}-2q_1L_{bm}^{2}L_{cln}e_2EI_{cln,2}-24q_1L_{bm}L_{cln}e_1e_2EI_{cln,2}\\-12q_1L_{bm}L_{cln}e_2^{2}EI_{cln,2}-4q_2L_{bm}^{2}L_{cln}z_2EI_{cln,2}-8q_2L_{bm}L_{cln}e_1z_2EI_{cln,2}-8q_2L_{bm}L_{cln}z_2e_2EI_{cln,2}\\-8q_1L_{bm}L_{cln}z_2e_2EI_{cln,2}-4q_2L_{bm}L_{cln}^{2}e_1EI_{bm}-4q_2L_{bm}L_{cln}^{2}e_2EI_{bm}-8q_2L_{cln}^{2}e_1^{2}EI_{bm}-16q_2L_{cln}^{2}e_1e_2EI_{bm}\\-8q_2L_{cln}^{2}e_2^{2}EI_{bm}-4q_1L_{bm}L_{cln}^{2}e_2EI_{bm}-16q_1L_{cln}^{2}e_1e_2EI_{bm}-8q_1L_{cln}^{2}e_2^{2}EI_{bm}+24e_2EI_{bm}EI_{cln,2}\end{pmatrix}}{12L_{bm}L_{cln}^{2}EI_{cln,2}+8L_{cln}^{3}EI_{bm}}$$

Two formulas for the difference in horizontal reaction force are known. One is based on column AB and one based on column CD. The horizontal reaction force in column AB is the same as the horizontal reaction force in column CD. The results of the two different formulas are the same. The two formulas are equal to each other.

$$\frac{\left(\begin{array}{l} q_2 L_{bm}^{\;3} L_{cln} EI_{cln,1} + 2q_2 L_{bm}^{\;2} L_{cln} e_1 EI_{cln,1} + 2q_2 L_{bm}^{\;2} L_{cln} \Delta e_2 EI_{cln,1} - 12q_2 L_{bm} L_{cln} e_1^{\;2} EI_{cln,1} \\ -24q_2 L_{bm} L_{cln} e_1 e_2 EI_{cln,1} - 12q_2 L_{bm} L_{cln} e_2^{\;2} EI_{cln,1} + 2q_1 L_{bm}^{\;2} L_{cln} e_2 EI_{cln,1} - 24q_1 L_{bm} L_{cln} e_1 e_2 EI_{cln,1} \\ -12q_1 L_{bm} L_{cln} e_2^{\;2} EI_{cln,1} - 2q_2 L_{bm}^{\;2} L_{cln} z_2 EI_{cln,1} - 4q_2 L_{bm} L_{cln} e_1 z_2 EI_{cln,1} - 4q_2 L_{bm} L_{cln} z_2 e_2 EI_{cln,1} \\ -4q_1 L_{bm} L_{cln} z_2 e_2 EI_{cln,1} + 4q_2 L_{bm} L_{cln}^{\;2} e_1 EI_{bm} + 4q_2 L_{bm} L_{cln}^{\;2} e_2 EI_{bm} - 8q_2 L_{cln}^{\;2} e_1^{\;2} EI_{bm} - 16q_2 L_{cln}^{\;2} e_1 e_2 EI_{bm} \\ -8q_2 L_{cln}^{\;2} e_2^{\;2} EI_{bm} + 4q_1 L_{bm} L_{cln}^{\;2} e_2 EI_{bm} - 16q_1 L_{cln}^{\;2} e_1 e_2 EI_{bm} - 8q_1 L_{cln}^{\;2} e_2^{\;2} EI_{bm} - 24e_2 EI_{bm} EI_{cln,1} \end{array}\right)}{12L_{bm} L_{cln}^{\;2} EI_{cln,1} + 8L_{cln}^{\;3} EI_{bm}}$$

$$=$$

$$\frac{\left(\begin{array}{l} q_2 L_{bm}^{\;3} L_{cln} EI_{cln,2} - 2q_2 L_{bm}^{\;2} L_{cln} e_1 EI_{cln,2} - 2q_2 L_{bm}^{\;2} L_{cln} e_2 EI_{cln,2} - 12q_2 L_{bm} L_{cln} e_1^{\;2} EI_{cln,2} \\ -24q_2 L_{bm} L_{cln} e_1 e_2 EI_{cln,2} - 12q_2 L_{bm} L_{cln} e_2^{\;2} EI_{cln,2} - 2q_1 L_{bm}^{\;2} L_{cln} e_2 EI_{cln,2} - 24q_1 L_{bm} L_{cln} e_1 e_2 EI_{cln,2} \\ -12q_1 L_{bm} L_{cln} e_2^{\;2} EI_{cln,2} - 4q_2 L_{bm}^{\;2} L_{cln} z_2 EI_{cln,2} - 8q_2 L_{bm} L_{cln} e_1 z_2 EI_{cln,2} - 8q_2 L_{bm} L_{cln} z_2 e_2 EI_{cln,2} \\ -8q_1 L_{bm} L_{cln} z_2 e_2 EI_{cln,2} - 4q_2 L_{bm} L_{cln}^{\;2} e_1 EI_{bm} - 4q_2 L_{bm} L_{cln}^{\;2} e_2 EI_{bm} - 8q_2 L_{cln}^{\;2} e_1^{\;2} EI_{bm} - 16q_2 L_{cln}^{\;2} e_1 e_2 EI_{bm} \\ -8q_2 L_{cln}^{\;2} e_2^{\;2} EI_{bm} - 4q_1 L_{bm} L_{cln}^{\;2} e_2 EI_{bm} - 16q_1 L_{cln}^{\;2} e_1 e_2 EI_{bm} - 8q_1 L_{cln}^{\;2} e_2^{\;2} EI_{bm} + 24e_2 EI_{bm} EI_{cln,2} \end{array}\right)}{12L_{bm} L_{cln}^{\;2} EI_{cln,2} + 8L_{cln}^{\;3} EI_{bm}}$$

In Appendix H,I and J the denominators are the same. This is not the case in this formula, there is a difference in stiffness. One column is partial yielded and the other column is not yielded. The stiffness of one section is decreased and the stiffness of the other section is remain constant. These values are not the same, so the denominators of both expression are not the same. All expressions must be multiplied by both denominators.

$$\left(12L_{bm} L_{cln}^{\;2} EI_{cln,2} + 8L_{cln}^{\;3} EI_{bm}\right)$$
$$\left(\begin{array}{l} q_2 L_{bm}^{\;3} L_{cln} EI_{cln,1} + 2q_2 L_{bm}^{\;2} L_{cln} e_1 EI_{cln,1} + 2q_2 L_{bm}^{\;2} L_{cln} e_2 EI_{cln,1} - 12q_2 L_{bm} L_{cln} e_1^{\;2} EI_{cln,1} \\ -24q_2 L_{bm} L_{cln} e_1 e_2 EI_{cln,1} - 12q_2 L_{bm} L_{cln} e_2^{\;2} EI_{cln,1} + 2q_1 L_{bm}^{\;2} L_{cln} e_2 EI_{cln,1} - 24q_1 L_{bm} L_{cln} e_1 e_2 EI_{cln,1} \\ -12q_1 L_{bm} L_{cln} e_2^{\;2} EI_{cln,1} - 2q_2 L_{bm}^{\;2} L_{cln} z_2 EI_{cln,1} - 4q_2 L_{bm} L_{cln} e_1 z_2 EI_{cln,1} - 4q_2 L_{bm} L_{cln} z_2 e_2 EI_{cln,1} \\ -4q_1 L_{bm} L_{cln} z_2 e_2 EI_{cln,1} + 4q_2 L_{bm} L_{cln}^{\;2} e_1 EI_{bm} + 4q_2 L_{bm} L_{cln}^{\;2} e_2 EI_{bm} - 8q_2 L_{cln}^{\;2} e_1^{\;2} EI_{bm} - 16q_2 L_{cln}^{\;2} e_1 e_2 EI_{bm} \\ -8q_2 L_{cln}^{\;2} e_2^{\;2} EI_{bm} + 4q_1 L_{bm} L_{cln}^{\;2} e_2 EI_{bm} - 16q_1 L_{cln}^{\;2} e_1 e_2 EI_{bm} - 8q_1 L_{cln}^{\;2} e_2^{\;2} EI_{bm} - 24e_2 EI_{bm} EI_{cln,1} \end{array}\right)$$

$$=$$

$$\left(12L_{bm} L_{cln}^{\;2} EI_{cln,1} + 8L_{cln}^{\;3} EI_{bm}\right)$$
$$\left(\begin{array}{l} q_2 L_{bm}^{\;3} L_{cln} EI_{cln,2} - 2q_2 L_{bm}^{\;2} L_{cln} e_1 EI_{cln,2} - 2q_2 L_{bm}^{\;2} L_{cln} e_2 EI_{cln,2} - 12q_2 L_{bm} L_{cln} e_1^{\;2} EI_{cln,2} \\ -24q_2 L_{bm} L_{cln} e_1 e_2 EI_{cln,2} - 12q_2 L_{bm} L_{cln} e_2^{\;2} EI_{cln,2} - 2q_1 L_{bm}^{\;2} L_{cln} e_2 EI_{cln,2} - 24q_1 L_{bm} L_{cln} e_1 e_2 EI_{cln,2} \\ -12q_1 L_{bm} L_{cln} e_2^{\;2} EI_{cln,2} - 4q_2 L_{bm}^{\;2} L_{cln} z_2 EI_{cln,2} - 8q_2 L_{bm} L_{cln} e_1 z_2 EI_{cln,2} - 8q_2 L_{bm} L_{cln} z_2 e_2 EI_{cln,2} \\ -8q_1 L_{bm} L_{cln} z_2 e_2 EI_{cln,2} - 4q_2 L_{bm} L_{cln}^{\;2} e_1 EI_{bm} - 4q_2 L_{bm} L_{cln}^{\;2} e_2 EI_{bm} - 8q_2 L_{cln}^{\;2} e_1^{\;2} EI_{bm} - 16q_2 L_{cln}^{\;2} e_1 e_2 EI_{bm} \\ -8q_2 L_{cln}^{\;2} e_2^{\;2} EI_{bm} - 4q_1 L_{bm} L_{cln}^{\;2} e_2 EI_{bm} - 16q_1 L_{cln}^{\;2} e_1 e_2 EI_{bm} - 8q_1 L_{cln}^{\;2} e_2^{\;2} EI_{bm} + 24e_2 EI_{bm} EI_{cln,2} \end{array}\right)$$

It is (nearly) impossible to calculate with this formula. To create a realistic (and manageable) formula the expression must write out. This results in the following formula:

$$
\left(
\begin{array}{l}
12q_2 L_{bm}{}^4 L_{cln}{}^3 EI_{cln,1} EI_{cln,2} + 24q_2 L_{bm}{}^3 L_{cln}{}^3 e_1 EI_{cln,1} EI_{cln,2} + 24q_2 L_{bm}{}^3 L_{cln}{}^3 e_2 EI_{cln,1} EI_{cln,2} \\
-144q_2 L_{bm}{}^2 L_{cln}{}^3 e_1{}^2 EI_{cln,1} EI_{cln,2} - 288q_2 L_{bm}{}^2 L_{cln}{}^3 e_1 e_2 EI_{cln,1} EI_{cln,2} - 144q_2 L_{bm}{}^2 L_{cln}{}^3 e_2{}^2 EI_{cln,1} EI_{cln,2} \\
+24q_1 L_{bm}{}^3 L_{cln}{}^3 e_2 EI_{cln,1} EI_{cln,2} - 288q_1 L_{bm}{}^2 L_{cln}{}^3 e_1 e_2 EI_{cln,1} EI_{cln,2} - 144q_1 L_{bm}{}^2 L_{cln}{}^3 e_2{}^2 EI_{cln,1} EI_{cln,2} \\
-24q_2 L_{bm}{}^3 L_{cln}{}^3 z_2 EI_{cln,1} EI_{cln,2} - 48q_2 L_{bm}{}^2 L_{cln}{}^3 e_1 z_2 EI_{cln,1} EI_{cln,2} - 48q_2 L_{bm}{}^2 L_{cln}{}^3 z_2 e_2 EI_{cln,1} EI_{cln,2} \\
-48q_1 L_{bm}{}^2 L_{cln}{}^3 z_2 e_2 EI_{cln,1} EI_{cln,2} + 48q_2 L_{bm}{}^2 L_{cln}{}^4 e_1 EI_{bm} EI_{cln,2} + 48q_2 L_{bm}{}^2 L_{cln}{}^4 e_2 EI_{bm} EI_{cln,2} \\
-96q_2 L_{bm} L_{cln}{}^4 e_1{}^2 EI_{bm} EI_{cln,2} - 192q_2 L_{bm} L_{cln}{}^4 e_1 e_2 EI_{bm} EI_{cln,2} - 96q_2 L_{bm} L_{cln}{}^4 e_2{}^2 EI_{bm} EI_{cln,2} \\
+48q_1 L_{bm}{}^2 L_{cln}{}^4 e_2 EI_{bm} EI_{cln,2} - 192q_1 L_{bm} L_{cln}{}^4 e_1 e_2 EI_{bm} EI_{cln,2} - 96q_1 L_{bm} L_{cln}{}^4 e_2{}^2 EI_{bm} EI_{cln,2} \\
-288 L_{bm} L_{cln}{}^2 e_2 EI_{bm} EI_{cln,1} EI_{cln,2}
\end{array}
\right)
$$

$$+$$

$$
\left(
\begin{array}{l}
8q_2 L_{bm}{}^3 L_{cln}{}^4 EI_{bm} EI_{cln,1} + 16q_2 L_{bm}{}^2 L_{cln}{}^4 e_1 EI_{bm} EI_{cln,1} + 16q_2 L_{bm}{}^2 L_{cln}{}^4 e_2 EI_{bm} EI_{cln,1} \\
-96q_2 L_{bm} L_{cln}{}^4 e_1{}^2 EI_{bm} EI_{cln,1} - 192q_2 L_{bm} L_{cln}{}^4 e_1 e_2 EI_{bm} EI_{cln,1} - 96q_2 L_{bm} L_{cln}{}^4 e_2{}^2 EI_{bm} EI_{cln,1} \\
+16q_1 L_{bm}{}^2 L_{cln}{}^4 e_2 EI_{bm} EI_{cln,1} - 192q_1 L_{bm} L_{cln}{}^4 e_1 e_2 EI_{bm} EI_{cln,1} - 96q_1 L_{bm} L_{cln}{}^4 e_2{}^2 EI_{bm} EI_{cln,1} \\
-16q_2 L_{bm}{}^2 L_{cln}{}^4 z_2 EI_{bm} EI_{cln,1} - 32q_2 L_{bm} L_{cln}{}^4 e_1 z_2 EI_{bm} EI_{cln,1} - 32q_2 L_{bm} L_{cln}{}^4 z_2 e_2 EI_{bm} EI_{cln,1} \\
-32q_1 L_{bm} L_{cln}{}^4 z_2 e_2 EI_{bm} EI_{cln,1} + 32q_2 L_{bm} L_{cln}{}^5 e_1 \left(EI_{bm} \right)^2 + 32q_2 L_{bm} L_{cln}{}^5 e_2 \left(EI_{bm} \right)^2 \\
-64q_2 L_{cln}{}^5 e_1{}^2 \left(EI_{bm} \right)^2 - 128q_2 L_{cln}{}^5 e_1 e_2 \left(EI_{bm} \right)^2 - 64q_2 L_{cln}{}^5 e_2{}^2 \left(EI_{bm} \right)^2 + 32q_1 L_{bm} L_{cln}{}^5 e_2 \left(EI_{bm} \right)^2 \\
-128q_1 L_{cln}{}^5 e_1 e_2 \left(EI_{bm} \right)^2 - 64q_1 L_{cln}{}^5 e_2{}^2 \left(EI_{bm} \right)^2 - 192 L_{cln}{}^3 e_2 \left(EI_{bm} \right)^2 EI_{cln,1}
\end{array}
\right)
$$

$$=$$

$$
\left(
\begin{array}{l}
12q_2 L_{bm}{}^4 L_{cln}{}^3 EI_{cln,1} EI_{cln,2} - 24q_2 L_{bm}{}^3 L_{cln}{}^3 e_1 EI_{cln,1} EI_{cln,2} - 24q_2 L_{bm}{}^3 L_{cln}{}^3 e_2 EI_{cln,1} EI_{cln,2} \\
-144q_2 L_{bm}{}^2 L_{cln}{}^3 e_1{}^2 EI_{cln,1} EI_{cln,2} - 288q_2 L_{bm}{}^2 L_{cln}{}^3 e_1 e_2 EI_{cln,1} EI_{cln,2} - 144q_2 L_{bm}{}^2 L_{cln}{}^3 e_2{}^2 EI_{cln,1} EI_{cln,2} \\
-24q_1 L_{bm}{}^3 L_{cln}{}^3 e_2 EI_{cln,1} EI_{cln,2} - 288q_1 L_{bm}{}^2 L_{cln}{}^3 e_1 e_2 EI_{cln,1} EI_{cln,2} - 144q_1 L_{bm}{}^2 L_{cln}{}^3 e_2{}^2 EI_{cln,1} EI_{cln,2} \\
-48q_2 L_{bm}{}^3 L_{cln}{}^3 z_2 EI_{cln,1} EI_{cln,2} - 96q_2 L_{bm}{}^2 L_{cln}{}^3 e_1 z_2 EI_{cln,1} EI_{cln,2} - 96q_2 L_{bm}{}^2 L_{cln}{}^3 z_2 e_2 EI_{cln,1} EI_{cln,2} \\
-96q_1 L_{bm}{}^2 L_{cln}{}^3 z_2 e_2 EI_{cln,1} EI_{cln,2} - 48q_2 L_{bm}{}^2 L_{cln}{}^4 e_1 EI_{bm} EI_{cln,1} - 48q_2 L_{bm}{}^2 L_{cln}{}^4 e_2 EI_{bm} EI_{cln,1} \\
-96q_2 L_{bm} L_{cln}{}^4 e_1{}^2 EI_{bm} EI_{cln,1} - 192q_2 L_{bm} L_{cln}{}^4 e_1 e_2 EI_{bm} EI_{cln,1} - 96q_2 L_{bm} L_{cln}{}^4 e_2{}^2 EI_{bm} EI_{cln,1} \\
-48q_1 L_{bm}{}^2 L_{cln}{}^4 e_2 EI_{bm} EI_{cln,1} - 192q_1 L_{bm} L_{cln}{}^4 e_1 e_2 EI_{bm} EI_{cln,1} - 96q_1 L_{bm} L_{cln}{}^4 e_2{}^2 EI_{bm} EI_{cln,1} \\
+288 L_{bm} L_{cln}{}^2 e_2 EI_{bm} EI_{cln,1} EI_{cln,2}
\end{array}
\right)
$$

$$+$$

$$
\left(
\begin{array}{l}
8q_2 L_{bm}{}^3 L_{cln}{}^4 EI_{bm} EI_{cln,2} - 16q_2 L_{bm}{}^2 L_{cln}{}^4 e_1 EI_{bm} EI_{cln,2} - 16q_2 L_{bm}{}^2 L_{cln}{}^4 e_2 EI_{bm} EI_{cln,2} \\
-96q_2 L_{bm} L_{cln}{}^4 e_1{}^2 EI_{bm} EI_{cln,2} - 192q_2 L_{bm} L_{cln}{}^4 e_1 e_2 EI_{bm} EI_{cln,2} - 96q_2 L_{bm} L_{cln}{}^4 e_2{}^2 EI_{bm} EI_{cln,2} \\
-16q_1 L_{bm}{}^2 L_{cln}{}^4 e_2 EI_{bm} EI_{cln,2} - 192q_1 L_{bm} L_{cln}{}^4 e_1 e_2 EI_{bm} EI_{cln,2} - 96q_1 L_{bm} L_{cln}{}^4 e_2{}^2 EI_{bm} EI_{cln,2} \\
-32q_2 L_{bm}{}^2 L_{cln}{}^4 z_2 EI_{bm} EI_{cln,2} - 64q_2 L_{bm} L_{cln}{}^4 e_1 z_2 EI_{bm} EI_{cln,2} - 64q_2 L_{bm} L_{cln}{}^4 z_2 e_2 EI_{bm} EI_{cln,2} \\
-64q_1 L_{bm} L_{cln}{}^4 z_2 e_2 EI_{bm} EI_{cln,2} - 32q_2 L_{bm} L_{cln}{}^5 e_1 \left(EI_{bm} \right)^2 - 32q_2 L_{bm} L_{cln}{}^5 e_2 \left(EI_{bm} \right)^2 \\
-64q_2 L_{cln}{}^5 e_1{}^2 \left(EI_{bm} \right)^2 - 128q_2 L_{cln}{}^5 e_1 e_2 \left(EI_{bm} \right)^2 - 64q_2 L_{cln}{}^5 e_2{}^2 \left(EI_{bm} \right)^2 - 32q_1 L_{bm} L_{cln}{}^5 e_2 \left(EI_{bm} \right)^2 \\
-128q_1 L_{cln}{}^5 e_1 e_2 \left(EI_{bm} \right)^2 - 64q_1 L_{cln}{}^5 e_2{}^2 \left(EI_{bm} \right)^2 + 192 L_{cln}{}^3 e_2 \left(EI_{bm} \right)^2 EI_{cln,2}
\end{array}
\right)
$$

The first step to simplify this formula is to find the same expressions on both sides of the equation. The same expressions are neglected or combined together.

$$\begin{pmatrix} 48q_2L_{bm}{}^3L_{cln}{}^3e_1EI_{cln,1}EI_{cln,2}+48q_2L_{bm}{}^3L_{cln}{}^3e_2EI_{cln,1}EI_{cln,2}+48q_1L_{bm}{}^3L_{cln}{}^3e_2EI_{cln,1}EI_{cln,2} \\ +24q_2L_{bm}{}^3L_{cln}{}^3z_2EI_{cln,1}EI_{cln,2}+48q_2L_{bm}{}^2L_{cln}{}^3e_1z_2EI_{cln,1}EI_{cln,2}+48q_2L_{bm}{}^2L_{cln}{}^3z_2e_2EI_{cln,1}EI_{cln,2} \\ +48q_1L_{bm}{}^2L_{cln}{}^3z_2e_2EI_{cln,1}EI_{cln,2}+48q_2L_{bm}{}^2L_{cln}{}^4e_1EI_{bm}EI_{cln,2}+48q_2L_{bm}{}^2L_{cln}{}^4e_2EI_{bm}EI_{cln,2} \\ -96q_2L_{bm}L_{cln}{}^4e_1{}^2EI_{bm}EI_{cln,2}-192q_2L_{bm}L_{cln}{}^4e_1e_2EI_{bm}EI_{cln,2}-96q_2L_{bm}L_{cln}{}^4e_2{}^2EI_{bm}EI_{cln,2} \\ +48q_1L_{bm}{}^2L_{cln}{}^4e_2EI_{bm}EI_{cln,2}-192q_1L_{bm}L_{cln}{}^4e_1e_2EI_{bm}EI_{cln,2}-96q_1L_{bm}L_{cln}{}^4e_2{}^2EI_{bm}EI_{cln,2} \\ -576L_{bm}L_{cln}{}^2e_2EI_{bm}EI_{cln,1}EI_{cln,2} \end{pmatrix}$$

$$+$$

$$\begin{pmatrix} 8q_2L_{bm}{}^3L_{cln}{}^4EI_{bm}EI_{cln,1}+16q_2L_{bm}{}^2L_{cln}{}^4e_1EI_{bm}EI_{cln,1}+16q_2L_{bm}{}^2L_{cln}{}^4e_2EI_{bm}EI_{cln,1} \\ -96q_2L_{bm}L_{cln}{}^4e_1{}^2EI_{bm}EI_{cln,1}-192q_2L_{bm}L_{cln}{}^4e_1e_2EI_{bm}EI_{cln,1}-96q_2L_{bm}L_{cln}{}^4e_2{}^2EI_{bm}EI_{cln,1} \\ +16q_1L_{bm}{}^2L_{cln}{}^4e_2EI_{bm}EI_{cln,1}-192q_1L_{bm}L_{cln}{}^4e_1e_2EI_{bm}EI_{cln,1}-96q_1L_{bm}L_{cln}{}^4e_2{}^2EI_{bm}EI_{cln,1} \\ -16q_2L_{bm}{}^2L_{cln}{}^4z_2EI_{bm}EI_{cln,1}-32q_2L_{bm}L_{cln}{}^4e_1z_2EI_{bm}EI_{cln,1}-32q_2L_{bm}L_{cln}{}^4z_2e_2EI_{bm}EI_{cln,1} \\ -32q_1L_{bm}L_{cln}{}^4z_2e_2EI_{bm}EI_{cln,1}+64q_2L_{bm}L_{cln}{}^5e_1\left(EI_{bm}\right)^2+64q_2L_{bm}L_{cln}{}^5e_2\left(EI_{bm}\right)^2 \\ +64q_1L_{bm}L_{cln}{}^5e_2\left(EI_{bm}\right)^2-192L_{cln}{}^3e_2\left(EI_{bm}\right)^2EI_{cln,1} \end{pmatrix}$$

$$=$$

$$\begin{pmatrix} -48q_2L_{bm}{}^2L_{cln}{}^4e_1EI_{bm}EI_{cln,1}-48q_2L_{bm}{}^2L_{cln}{}^4e_2EI_{bm}EI_{cln,1}-96q_2L_{bm}L_{cln}{}^4e_1{}^2EI_{bm}EI_{cln,1} \\ -192q_2L_{bm}L_{cln}{}^4e_1e_2EI_{bm}EI_{cln,1}-96q_2L_{bm}L_{cln}{}^4e_2{}^2EI_{bm}EI_{cln,1}-48q_1L_{bm}{}^2L_{cln}{}^4e_2EI_{bm}EI_{cln,1} \\ -192q_1L_{bm}L_{cln}{}^4e_1e_2EI_{bm}EI_{cln,1}-96q_1L_{bm}L_{cln}{}^4e_2{}^2EI_{bm}EI_{cln,1} \end{pmatrix}$$

$$+$$

$$\begin{pmatrix} 8q_2L_{bm}{}^3L_{cln}{}^4EI_{bm}EI_{cln,2}-16q_2L_{bm}{}^2L_{cln}{}^4e_1EI_{bm}EI_{cln,2}-16q_2L_{bm}{}^2L_{cln}{}^4e_2EI_{bm}EI_{cln,2} \\ -96q_2L_{bm}L_{cln}{}^4e_1{}^2EI_{bm}EI_{cln,2}-192q_2L_{bm}L_{cln}{}^4e_1e_2EI_{bm}EI_{cln,2}-96q_2L_{bm}L_{cln}{}^4e_2{}^2EI_{bm}EI_{cln,2} \\ -16q_1L_{bm}{}^2L_{cln}{}^4e_2EI_{bm}EI_{cln,2}-192q_1L_{bm}L_{cln}{}^4e_1e_2EI_{bm}EI_{cln,2}-96q_1L_{bm}L_{cln}{}^4e_2{}^2EI_{bm}EI_{cln,2} \\ -32q_2L_{bm}{}^2L_{cln}{}^4z_2EI_{bm}EI_{cln,2}-64q_2L_{bm}L_{cln}{}^4e_1z_2EI_{bm}EI_{cln,2}-64q_2L_{bm}L_{cln}{}^4z_2e_2EI_{bm}EI_{cln,2} \\ -64q_1L_{bm}L_{cln}{}^4z_2e_2EI_{bm}EI_{cln,2}+192L_{cln}{}^3e_2\left(EI_{bm}\right)^2EI_{cln,2} \end{pmatrix}$$

The second step to simplify the formula is to combine the same expressions. All expressions of the right part of the formula are inserted in the left part of the formula.

$$\begin{pmatrix} 48q_2L_{bm}{}^3L_{cln}{}^3e_1EI_{cln,1}EI_{cln,2} + 48q_2L_{bm}{}^3L_{cln}{}^3e_2EI_{cln,1}EI_{cln,2} + 48q_1L_{bm}{}^3L_{cln}{}^3e_2EI_{cln,1}EI_{cln,2} \\ +24q_2L_{bm}{}^3L_{cln}{}^3z_2EI_{cln,1}EI_{cln,2} + 48q_2L_{bm}{}^2L_{cln}{}^3e_1z_2EI_{cln,1}EI_{cln,2} + 48q_2L_{bm}{}^2L_{cln}{}^3z_2e_2EI_{cln,1}EI_{cln,2} \\ +48q_1L_{bm}{}^2L_{cln}{}^3z_2e_2EI_{cln,1}EI_{cln,2} + 48q_2L_{bm}{}^2L_{cln}{}^4e_1EI_{bm}\left(EI_{cln,1} + EI_{cln,2}\right) \\ +48q_2L_{bm}{}^2L_{cln}{}^4e_2EI_{bm}\left(EI_{cln,1} + EI_{cln,2}\right) + 96q_2L_{bm}L_{cln}{}^4e_1{}^2EI_{bm}\left(EI_{cln,1} - EI_{cln,2}\right) \\ +192q_2L_{bm}L_{cln}{}^4e_1e_2EI_{bm}\left(EI_{cln,1} - EI_{cln,2}\right) + 96q_2L_{bm}L_{cln}{}^4e_2{}^2EI_{bm}\left(EI_{cln,1} - EI_{cln,2}\right) \\ +48q_1L_{bm}{}^2L_{cln}{}^4e_2EI_{bm}\left(EI_{cln,1} + EI_{cln,2}\right) + 192q_1L_{bm}L_{cln}{}^4e_1e_2EI_{bm}\left(EI_{cln,1} - EI_{cln,2}\right) \\ +96q_1L_{bm}L_{cln}{}^4e_2{}^2EI_{bm}\left(EI_{cln,1} - EI_{cln,2}\right) - 576L_{bm}L_{cln}{}^2e_2EI_{bm}EI_{cln,1}EI_{cln,2} \end{pmatrix}$$

$$+$$

$$\begin{pmatrix} 8q_2L_{bm}{}^3L_{cln}{}^4EI_{bm}\left(EI_{cln,1} - EI_{cln,2}\right) + 16q_2L_{bm}{}^2L_{cln}{}^4e_1EI_{bm}\left(EI_{cln,1} + EI_{cln,2}\right) \\ +16q_2L_{bm}{}^2L_{cln}{}^4e_2EI_{bm}\left(EI_{cln,1} + EI_{cln,2}\right) + 96q_2L_{bm}L_{cln}{}^4e_1{}^2EI_{bm}\left(-EI_{cln,1} + EI_{cln,2}\right) \\ +192q_2L_{bm}L_{cln}{}^4e_1e_2EI_{bm}\left(-EI_{cln,1} + EI_{cln,2}\right) + 96q_2L_{bm}L_{cln}{}^4e_2{}^2EI_{bm}\left(-EI_{cln,1} + EI_{cln,2}\right) \\ +16q_1L_{bm}{}^2L_{cln}{}^4e_2EI_{bm}\left(EI_{cln,1} + EI_{cln,2}\right) + 192q_1L_{bm}L_{cln}{}^4e_1e_2EI_{bm}\left(-EI_{cln,1} + EI_{cln,2}\right) \\ +96q_1L_{bm}L_{cln}{}^4e_2{}^2EI_{bm}\left(-EI_{cln,1} + EI_{cln,2}\right) + 16q_2L_{bm}{}^2L_{cln}{}^4z_2EI_{bm}\left(-EI_{cln,1} + 2EI_{cln,2}\right) \\ +32q_2L_{bm}L_{cln}{}^4e_1z_2EI_{bm}\left(-EI_{cln,1} + 2EI_{cln,2}\right) + 32q_2L_{bm}L_{cln}{}^4z_2e_2EI_{bm}\left(-EI_{cln,1} + 2EI_{cln,2}\right) \\ +32q_1L_{bm}L_{cln}{}^4z_2e_2EI_{bm}\left(-EI_{cln,1} + 2EI_{cln,2}\right) + 64q_2L_{bm}L_{cln}{}^5e_1\left(EI_{bm}\right)^2 + 64q_2L_{bm}L_{cln}{}^5e_2\left(EI_{bm}\right)^2 \\ +64q_1L_{bm}L_{cln}{}^5e_2\left(EI_{bm}\right)^2 - 192L_{cln}{}^3e_2\left(EI_{bm}\right)^2\left(EI_{cln,1} + EI_{cln,2}\right) \end{pmatrix}$$

$$= 0$$

Some expressions are the same and these expressions can be combined together. This results in the following formula:

$$\begin{aligned} &48q_2L_{bm}{}^3L_{cln}{}^3e_1EI_{cln,1}EI_{cln,2} + 48q_2L_{bm}{}^3L_{cln}{}^3e_2EI_{cln,1}EI_{cln,2} + 48q_1L_{bm}{}^3L_{cln}{}^3e_2EI_{cln,1}EI_{cln,2} \\ &+24q_2L_{bm}{}^3L_{cln}{}^3z_2EI_{cln,1}EI_{cln,2} + 48q_2L_{bm}{}^2L_{cln}{}^3e_1z_2EI_{cln,1}EI_{cln,2} + 48q_2L_{bm}{}^2L_{cln}{}^3z_2e_2EI_{cln,1}EI_{cln,2} \\ &+48q_1L_{bm}{}^2L_{cln}{}^3z_2e_2EI_{cln,1}EI_{cln,2} + 64q_2L_{bm}{}^2L_{cln}{}^4e_1EI_{bm}\left(EI_{cln,1} + EI_{cln,2}\right) \\ &+64q_2L_{bm}{}^2L_{cln}{}^4e_2EI_{bm}\left(EI_{cln,1} + EI_{cln,2}\right) + 64q_1L_{bm}{}^2L_{cln}{}^4e_2EI_{bm}\left(EI_{cln,1} + EI_{cln,2}\right) \\ &-576L_{bm}L_{cln}{}^2e_2EI_{bm}EI_{cln,1}EI_{cln,2} + 8q_2L_{bm}{}^3L_{cln}{}^4EI_{bm}\left(EI_{cln,1} - EI_{cln,2}\right) \\ &+16q_2L_{bm}{}^2L_{cln}{}^4z_2EI_{bm}\left(-EI_{cln,1} + 2EI_{cln,2}\right) + 32q_2L_{bm}L_{cln}{}^4e_1z_2EI_{bm}\left(-EI_{cln,1} + 2EI_{cln,2}\right) \\ &+32q_2L_{bm}L_{cln}{}^4z_2e_2EI_{bm}\left(-EI_{cln,1} + 2EI_{cln,2}\right) + 32q_1L_{bm}L_{cln}{}^4z_2e_2EI_{bm}\left(-EI_{cln,1} + 2EI_{cln,2}\right) \\ &+64q_2L_{bm}L_{cln}{}^5e_1\left(EI_{bm}\right)^2 + 64q_2L_{bm}L_{cln}{}^5e_2\left(EI_{bm}\right)^2 + 64q_1L_{bm}L_{cln}{}^5e_2\left(EI_{bm}\right)^2 \\ &-192L_{cln}{}^3e_2\left(EI_{bm}\right)^2\left(EI_{cln,1} + EI_{cln,2}\right) = 0 \end{aligned}$$

The unknown value is e_2. All expression of e_2 will be separated from the rest of the formula.

$$-48q_2L_{bm}{}^3L_{cln}{}^3e_2EI_{cln,1}EI_{cln,2}-48q_1L_{bm}{}^3L_{cln}{}^3e_2EI_{cln,1}EI_{cln,2}-48q_2L_{bm}{}^2L_{cln}{}^3z_2\Delta e_2EI_{cln,1}EI_{cln,2}$$
$$-48q_1L_{bm}{}^2L_{cln}{}^3z_2e_2EI_{cln,1}EI_{cln,2}-64q_1L_{bm}{}^2L_{cln}{}^4e_2EI_{bm}\left(EI_{cln,1}+EI_{cln,2}\right)$$
$$-64q_2L_{bm}{}^2L_{cln}{}^4e_2EI_{bm}\left(EI_{cln,1}+EI_{cln,2}\right)+576L_{bm}L_{cln}{}^2e_2EI_{bm}EI_{cln,1}EI_{cln,2}$$
$$-32q_2L_{bm}L_{cln}{}^4z_2e_2EI_{bm}\left(-EI_{cln,1}+2EI_{cln,2}\right)-32q_1L_{bm}L_{cln}{}^4z_2e_2EI_{bm}\left(-EI_{cln,1}+2EI_{cln,2}\right)$$
$$-64q_2L_{bm}L_{cln}{}^5e_2\left(EI_{bm}\right)^2-64q_1L_{bm}L_{cln}{}^5e_2\left(EI_{bm}\right)^2+192L_{cln}{}^3e_2\left(EI_{bm}\right)^2\left(EI_{cln,1}+EI_{cln,2}\right)$$
$$=$$
$$48q_2L_{bm}{}^3L_{cln}{}^3e_1EI_{cln,1}EI_{cln,2}+24q_2L_{bm}{}^3L_{cln}{}^3z_2EI_{cln,1}EI_{cln,2}+48q_2L_{bm}{}^2L_{cln}{}^3e_1z_2EI_{cln,1}EI_{cln,2}$$
$$+8q_2L_{bm}{}^3L_{cln}{}^4EI_{bm}\left(EI_{cln,1}-EI_{cln,2}\right)$$
$$+16q_2L_{bm}{}^2L_{cln}{}^4z_2EI_{bm}\left(-EI_{cln,1}+2EI_{cln,2}\right)+32q_2L_{bm}L_{cln}{}^4e_1z_2EI_{bm}\left(-EI_{cln,1}+2EI_{cln,2}\right)$$
$$+64q_2L_{bm}L_{cln}{}^5e_1\left(EI_{bm}\right)^2+64q_2L_{bm}{}^2L_{cln}{}^4e_1EI_{bm}\left(EI_{cln,1}+EI_{cln,2}\right)$$

Again some expressions are (partial) the same and can be combined together.

$$16L_{cln}{}^2e_2\begin{bmatrix}-3L_{bm}{}^3L_{cln}\left(q_1+q_2\right)EI_{cln,1}EI_{cln,2}-3L_{bm}{}^2L_{cln}z_2\left(q_1+q_2\right)EI_{cln,1}EI_{cln,2}\\-4L_{bm}{}^2L_{cln}{}^2\left(q_1+q_2\right)EI_{bm}\left(EI_{cln,1}+EI_{cln,2}\right)-2L_{bm}L_{cln}{}^2z_2\left(q_1+q_2\right)EI_{bm}\left(-EI_{cln,1}+2EI_{cln,2}\right)\\-4L_{bm}L_{cln}{}^3\left(q_1+q_2\right)\left(EI_{bm}\right)^2+12L_{cln}\left(EI_{bm}\right)^2\left(EI_{cln,1}+EI_{cln,2}\right)+36L_{bm}EI_{bm}EI_{cln,1}EI_{cln,2}\end{bmatrix}$$
$$=$$
$$8L_{bm}L_{cln}{}^3q_2\begin{bmatrix}6L_{bm}{}^2e_1EI_{cln,1}EI_{cln,2}+3L_{bm}{}^2z_2EI_{cln,1}EI_{cln,2}+6L_{bm}e_1z_2EI_{cln,1}EI_{cln,2}\\+8L_{bm}L_{cln}e_1EI_{bm}\left(EI_{cln,1}+EI_{cln,2}\right)+L_{bm}{}^2L_{cln}EI_{bm}\left(EI_{cln,1}-EI_{cln,2}\right)+8L_{cln}{}^2e_1\left(EI_{bm}\right)^2\\+2L_{cln}z_2\left(L_{bm}+2e_1\right)EI_{bm}\left(-EI_{cln,1}+2EI_{cln,2}\right)\end{bmatrix}$$

The total load (q_{total}) is split in two loads q_1 and q_2. These loads are already combined but now the term q_{total} is used.

$$16L_{cln}{}^2e_2\begin{bmatrix}-3q_{total}L_{bm}{}^3L_{cln}EI_{cln,1}EI_{cln,2}-3q_{total}L_{bm}{}^2L_{cln}z_2EI_{cln,1}EI_{cln,2}\\-4q_{total}L_{bm}{}^2L_{cln}{}^2EI_{bm}\left(EI_{cln,1}+EI_{cln,2}\right)-2q_{total}L_{bm}L_{cln}{}^2z_2EI_{bm}\left(-EI_{cln,1}+2EI_{cln,2}\right)\\-4q_{total}L_{bm}L_{cln}{}^3\left(EI_{bm}\right)^2+12L_{cln}\left(EI_{bm}\right)^2\left(EI_{cln,1}+EI_{cln,2}\right)+36L_{bm}EI_{bm}EI_{cln,1}EI_{cln,2}\end{bmatrix}$$
$$=$$
$$8L_{bm}L_{cln}{}^3q_2\begin{bmatrix}6L_{bm}{}^2e_1EI_{cln,1}EI_{cln,2}+3L_{bm}{}^2z_2EI_{cln,1}EI_{cln,2}+6L_{bm}e_1z_2EI_{cln,1}EI_{cln,2}\\+8L_{bm}L_{cln}e_1EI_{bm}\left(EI_{cln,1}+EI_{cln,2}\right)+L_{bm}{}^2L_{cln}EI_{bm}\left(EI_{cln,1}-EI_{cln,2}\right)+8L_{cln}{}^2e_1\left(EI_{bm}\right)^2\\+2L_{cln}z_2\left(L_{bm}+2e_1\right)EI_{bm}\left(-EI_{cln,1}+2EI_{cln,2}\right)\end{bmatrix}$$

The formula of e_2 is as follow.

$$e_2 = \frac{q_2 L_{bm} L_{cln} \begin{bmatrix} 6L_{bm}{}^2 e_1 EI_{cln,1} EI_{cln,2} + 3L_{bm}{}^2 z_2 EI_{cln,1} EI_{cln,2} + 6L_{bm} e_1 z_2 EI_{cln,1} EI_{cln,2} \\ +8L_{bm} L_{cln} e_1 EI_{bm} \left(EI_{cln,1} + EI_{cln,2} \right) + L_{bm}{}^2 L_{cln} EI_{bm} \left(EI_{cln,1} - EI_{cln,2} \right) + 8L_{cln}{}^2 e_1 \left(EI_{bm} \right)^2 \\ +2L_{cln} z_2 \left(L_{bm} + 2e_1 \right) EI_{bm} \left(-EI_{cln,1} + 2EI_{cln,2} \right) \end{bmatrix}}{2 \begin{bmatrix} -3q_{total} L_{bm}{}^3 L_{cln} EI_{cln,1} EI_{cln,2} - 3q_{total} L_{bm}{}^2 L_{cln} z_2 EI_{cln,1} EI_{cln,2} \\ -4q_{total} L_{bm}{}^2 L_{cln}{}^2 EI_{bm} \left(EI_{cln,1} + EI_{cln,2} \right) - 2q_{total} L_{bm} L_{cln}{}^2 z_2 EI_{bm} \left(-EI_{cln,1} + 2EI_{cln,2} \right) \\ -4q_{total} L_{bm} L_{cln}{}^3 \left(EI_{bm} \right)^2 + 12L_{cln} \left(EI_{bm} \right)^2 \left(EI_{cln,1} + EI_{cln,2} \right) + 36L_{bm} EI_{bm} EI_{cln,1} EI_{cln,2} \end{bmatrix}}$$

Check on dimensions

$$m = \frac{Nm^{-1} mm \begin{bmatrix} m^2 mNm^{-2} m^4 Nm^{-2} m^4 + m^2 mNm^{-2} m^4 Nm^{-2} m^4 + mmmNm^{-2} m^4 Nm^{-2} m^4 \\ +mmmNm^{-2} m^4 \left(Nm^{-2} m^4 + Nm^{-2} m^4 \right) + m^2 mNm^{-2} m^4 \left(Nm^{-2} m^4 - Nm^{-2} m^4 \right) + m^2 m \left(Nm^{-2} m^4 \right)^2 \\ +mm \left(m + m \right) Nm^{-2} m^4 \left(Nm^{-2} m^4 + Nm^{-2} m^4 \right) \end{bmatrix}}{\begin{bmatrix} Nm^{-1} m^3 mNm^{-2} m^4 Nm^{-2} m^4 - Nm^{-1} m^2 mmNm^{-2} m^4 Nm^{-2} m^4 \\ -Nm^{-1} m^2 m^2 Nm^{-2} m^4 \left(Nm^{-2} m^4 + Nm^{-2} m^4 \right) - Nm^{-1} mm^2 mNm^{-2} m^4 \left(Nm^{-2} m^4 + Nm^{-2} m^4 \right) \\ -Nm^{-1} mm^3 \left(Nm^{-2} m^4 \right)^2 + m \left(Nm^{-2} m^4 \right)^2 \left(Nm^{-2} m^4 + Nm^{-2} m^4 \right) + mNm^{-2} m^4 Nm^{-2} m^4 Nm^{-2} m^4 \end{bmatrix}}$$

$$m = \frac{Nm \begin{bmatrix} N^2 m^7 + N^2 m^7 + N^2 m^7 + Nm^5 \left(Nm^2 + Nm^2 \right) + Nm^5 \left(Nm^2 - Nm^2 \right) \\ +m^3 \left(N^2 m^4 \right) + Nm^5 \left(Nm^2 + Nm^2 \right) \end{bmatrix}}{\begin{bmatrix} N^3 m^7 - N^3 m^7 - N^2 m^5 \left(Nm^2 + Nm^2 \right) - N^2 m^5 \left(Nm^2 + Nm^2 \right) \\ -Nm^3 \left(N^2 m^4 \right) + m \left(N^2 m^4 \right) \left(Nm^2 + Nm^2 \right) + N^3 m^7 \end{bmatrix}}$$

$$m = \frac{N^3 m^8}{N^3 m^7}$$ Correct

The formula of the total deflection in the second load case is found. This formula is very complex. It is very complex to find a formula for the additional horizontal reaction force using this formula of the total deflection. The numerical value of the total deflection will be used for the calculation of the horizontal reaction force.

The formula of the additional horizontal reaction force can be simplified. The formula of the extra horizontal reaction force is:

$$H_{A,2}=\frac{\begin{pmatrix} q_2 L_{bm}{}^3 L_{cln} EI_{cln,2} - 2q_2 L_{bm}{}^2 L_{cln} e_1 EI_{cln,2} - 2q_2 L_{bm}{}^2 L_{cln} e_2 EI_{cln,2} - 12q_2 L_{bm} L_{cln} e_1{}^2 EI_{cln,2} \\ -24q_2 L_{bm} L_{cln} e_1 e_2 EI_{cln,2} - 12q_2 L_{bm} L_{cln} e_2{}^2 EI_{cln,2} - 2q_1 L_{bm}{}^2 L_{cln} e_2 EI_{cln,2} - 24q_1 L_{bm} L_{cln} e_1 e_2 EI_{cln,2} \\ -12q_1 L_{bm} L_{cln} e_2{}^2 EI_{cln,2} - 4q_2 L_{bm}{}^2 L_{cln} z_2 EI_{cln,2} - 8q_2 L_{bm} L_{cln} e_1 z_2 EI_{cln,2} - 8q_2 L_{bm} L_{cln} z_2 e_2 EI_{cln,2} \\ -8q_1 L_{bm} L_{cln} z_2 e_2 EI_{cln,2} - 4q_2 L_{bm} L_{cln}{}^2 e_1 EI_{bm} - 4q_2 L_{bm} L_{cln}{}^2 e_2 EI_{bm} - 8q_2 L_{cln}{}^2 e_1{}^2 EI_{bm} - 16q_2 L_{cln}{}^2 e_1 e_2 EI_{bm} \\ -8q_2 L_{cln}{}^2 e_2{}^2 EI_{bm} - 4q_1 L_{bm} L_{cln}{}^2 e_2 EI_{bm} - 16q_1 L_{cln}{}^2 e_1 e_2 EI_{bm} - 8q_1 L_{cln}{}^2 e_2{}^2 EI_{bm} + 24 e_2 EI_{bm} EI_{cln,2} \end{pmatrix}}{12 L_{bm} L_{cln}{}^2 EI_{cln,2} + 8 L_{cln}{}^3 EI_{bm}}$$

The simplified the formula the order is changed.

$$H_{A,2}=\frac{\begin{pmatrix} -12q_1 L_{bm} L_{cln} e_2{}^2 EI_{cln,2} - 12q_2 L_{bm} L_{cln} e_2{}^2 EI_{cln,2} - 24q_1 L_{bm} L_{cln} e_1 e_2 EI_{cln,2} - 24q_2 L_{bm} L_{cln} e_1 e_2 EI_{cln,2} \\ -8q_1 L_{bm} L_{cln} z_2 e_2 EI_{cln,2} - 8q_2 L_{bm} L_{cln} z_2 e_2 EI_{cln,2} - 2q_1 L_{bm}{}^2 L_{cln} e_2 EI_{cln,2} - 2q_2 L_{bm}{}^2 L_{cln} e_2 EI_{cln,2} \\ -4q_1 L_{bm} L_{cln}{}^2 e_2 EI_{bm} - 4q_2 L_{bm} L_{cln}{}^2 e_2 EI_{bm} - 16q_1 L_{cln}{}^2 e_1 e_2 EI_{bm} - 16\Delta q_2 L_{cln}{}^2 e_1 e_2 EI_{bm} \\ -8q_1 L_{cln}{}^2 e_2{}^2 EI_{bm} - 8q_2 L_{cln}{}^2 e_2{}^2 EI_{bm} \\ +24 e_2 EI_{bm} EI_{cln,2} + q_2 L_{bm}{}^3 L_{cln} EI_{cln,2} - 2q_2 L_{bm}{}^2 L_{cln} e_1 EI_{cln,2} - 12q_2 L_{bm} L_{cln} e_1{}^2 EI_{cln,2} \\ -4q_2 L_{bm}{}^2 L_{cln} z_2 EI_{cln,2} - 8q_2 L_{bm} L_{cln} e_1 z_2 EI_{cln,2} - 4q_2 L_{bm} L_{cln}{}^2 e_1 EI_{bm} - 8q_2 L_{cln}{}^2 e_1{}^2 EI_{bm} \end{pmatrix}}{12 L_{bm} L_{cln}{}^2 EI_{cln,2} + 8 L_{cln}{}^3 EI_{bm}}$$

If possible the total load is used instead of the two different loads separated.

$$H_{A,2}=\frac{\begin{pmatrix} -12q_{total,2} L_{bm} L_{cln} e_2{}^2 EI_{cln,2} - 24q_{total,2} L_{bm} L_{cln} e_1 e_2 EI_{cln,2} - 8q_{total,2} L_{bm} L_{cln} z_2 e_2 EI_{cln,2} \\ -2q_{total,2} L_{bm}{}^2 L_{cln} e_2 EI_{cln,2} 4q_{total,2} L_{bm} L_{cln}{}^2 e_2 EI_{bm} - 16q_{total,2} L_{cln}{}^2 e_1 e_2 EI_{bm} - 8q_{total,2} L_{cln}{}^2 e_2{}^2 EI_{bm} \\ +24 e_2 EI_{bm} EI_{cln,2} + q_2 L_{bm}{}^3 L_{cln} EI_{cln,2} - 2q_2 L_{bm}{}^2 L_{cln} e_1 EI_{cln,2} - 12q_2 L_{bm} L_{cln} e_1{}^2 EI_{cln,2} \\ -4q_2 L_{bm}{}^2 L_{cln} z_2 EI_{cln,2} - 8q_2 L_{bm} L_{cln} e_1 z_2 EI_{cln,2} - 4q_2 L_{bm} L_{cln}{}^2 e_1 EI_{bm} - 8q_2 L_{cln}{}^2 e_1{}^2 EI_{bm} \end{pmatrix}}{12 L_{bm} L_{cln}{}^2 EI_{cln,2} + 8 L_{cln}{}^3 EI_{bm}}$$

Some expressions are combined together.

$$H_{A,2}=\frac{\begin{pmatrix} -q_{total,2} L_{bm} L_{cln} e_2 \left[12e_2 + 24e_1 + 8z_2 + 2L_{bm}\right] EI_{cln,2} + q_{total,2} L_{cln}{}^2 e_2 \left[4L_{bm} - 16e_1 - 8e_2\right] EI_{bm} \\ +24 e_2 EI_{bm} EI_{cln,2} + q_2 L_{bm} L_{cln} \left[L_{bm}{}^2 - 2L_{bm} e_1 - 12e_1{}^2 - 4L_{bm} z_2 - 8e_1 z_2\right] EI_{cln,2} - q_2 L_{cln}{}^2 e_1 \left[4L_{bm} + 8e_1\right] EI_{bm} \end{pmatrix}}{12 L_{bm} L_{cln}{}^2 EI_{cln,2} + 8 L_{cln}{}^3 EI_{bm}}$$

The following formulas can be used to find the loads and the deflection in the second load case (the additional load after first yielding).

$$V_{D,2} = q_1 e_2 + q_2\left(\tfrac{1}{2}L_{bm} + e_1 + e_2\right)$$
$$V_{A,2} = -q_1 e_2 + q_2\left(\tfrac{1}{2}L_{bm} - e_1 - e_2\right)$$
$$M_{C,2} = q_2\left(\tfrac{1}{2}L_{bm} + e_1 + e_2\right)\left(e_1 + e_2 + z_2\right) + q_1 e_2\left(\tfrac{1}{2}L_{bm} + 2e_1 + e_2 + z_2\right) + H_{A,2} L_{cln}$$
$$M_{B,2} = q_2\left(\tfrac{1}{2}L_{bm} - e_1 - e_2\right)\left(e_1 + e_2\right) + q_1 e_2\left(\tfrac{1}{2}L_{bm} - 2e_1 - e_2\right) - H_{A,2} L_{cln}$$

$$H_{A,2}=\frac{\begin{pmatrix}-q_{total,2}L_{bm}L_{cln}e_2\left[12e_2+24e_1+8z_2+2L_{bm}\right]EI_{cln,2}+q_{total,2}L_{cln}{}^2e_2\left[4L_{bm}-16e_1-8e_2\right]EI_{bm}\\+24e_2EI_{bm}EI_{cln,2}+q_2L_{bm}L_{cln}\left[L_{bm}{}^2-2L_{bm}e_1-12e_1{}^2-4L_{bm}z_2-8e_1z_2\right]EI_{cln,2}-q_2L_{cln}{}^2e_1\left[4L_{bm}+8e_1\right]EI_{bm}\end{pmatrix}}{12L_{bm}L_{cln}{}^2EI_{cln,2}+8L_{cln}{}^3EI_{bm}}$$

$$e_2=\frac{q_2L_{bm}L_{cln}\begin{bmatrix}6L_{bm}{}^2e_1EI_{cln,1}EI_{cln,2}+3L_{bm}{}^2z_2EI_{cln,1}EI_{cln,2}+6L_{bm}e_1z_2EI_{cln,1}EI_{cln,2}\\+8L_{bm}L_{cln}e_1EI_{bm}\left(EI_{cln,1}+EI_{cln,2}\right)+L_{bm}{}^2L_{cln}EI_{bm}\left(EI_{cln,1}-EI_{cln,2}\right)+8L_{cln}{}^2e_1\left(EI_{bm}\right)^2\\+2L_{cln}z_2\left(L_{bm}+2e_1\right)EI_{bm}\left(-EI_{cln,1}+2EI_{cln,2}\right)\end{bmatrix}}{2\begin{bmatrix}-3q_{total}L_{bm}{}^3L_{cln}EI_{cln,1}EI_{cln,2}-3q_{total}L_{bm}{}^2L_{cln}z_2EI_{cln,1}EI_{cln,2}\\-4q_{total}L_{bm}{}^2L_{cln}{}^2EI_{bm}\left(EI_{cln,1}+EI_{cln,2}\right)-2q_{total}L_{bm}L_{cln}{}^2z_2EI_{bm}\left(-EI_{cln,1}+2EI_{cln,2}\right)\\-4q_{total}L_{bm}L_{cln}{}^3\left(EI_{bm}\right)^2+12L_{cln}\left(EI_{bm}\right)^2\left(EI_{cln,1}+EI_{cln,2}\right)+36L_{bm}EI_{bm}EI_{cln,1}EI_{cln,2}\end{bmatrix}}$$

K.2 Analysis if two parts yield

The formulas of the second load case are known. The same analysis as for the second load case can be made for the third load case. The third load case starts if the stress in the left flange reaches the yield level too. The stiffness decrease again, but the section is double symmetric again. It is not necessary to take a shift of the centre of gravity into account. The analysis starts again with the formulas of equilibrium.

Moment equilibrium:

$$\sum M \mid A=0$$

$$\left(q_{total,2}\right)L_{bm}\left(\tfrac{1}{2}L_{bm}+e_{total,2}\right)-V_{D,total,2}L_{bm}=0$$

The original load and the additional load are split.

$$\left(q_1+q_2+q_3\right)L_{bm}\left(\tfrac{1}{2}L_{bm}+e_1+e_2+e_3\right)-\left(V_{D,1}+V_{D,2}+V_{D,3}\right)L_{bm}=0$$

All expressions can be divided by L_{bm}. There is only interest in the additional reaction force.

$$V_{D,1}+V_{D,2}+V_{D,3}=q_1\left(\tfrac{1}{2}L_{bm}+e_1\right)+q_1\left(e_2+e_3\right)+q_2\left(\tfrac{1}{2}L_{bm}+e_1+e_2\right)+q_2e_3+q_3\left(\tfrac{1}{2}L_{bm}+e_1+e_2+e_3\right)$$

The original reaction forces in the first load case and in the second load case are known $\left(V_{D,1}=q_1\left(\tfrac{1}{2}L_{bm}+e_1\right);V_{D,2}=q_1e_2+q_2\left(\tfrac{1}{2}L_{bm}+e_1+e_2\right)\right)$. These expressions can be neglected of the formula. What is left is the following formula:

$$V_{D,3}=\left(q_1+q_2\right)e_3+q_3\left(\tfrac{1}{2}L_{bm}+e_1+e_2+e_3\right)$$

Vertical equilibrium.

$$\sum vert=0$$

$$\left(q_{total,2}\right)L_{bm}-V_{A,total,2}-V_{D,total,2}=0$$

The original load and the additional load are split.

$$\left(q_1 + q_2 + q_3\right) L_{bm} - V_{A,1} - V_{A,2} - V_{A,3} - V_{D,1} - V_{D,2} - V_{D,3} = 0$$

Write out some expressions and change the order results in the following formula:

$$\left(q_1 L_{bm} + q_2 L_{bm} - V_{A,1} - V_{A,2} - V_{D,1} - V_{D,2}\right) + q_3 L_{bm} - V_{A,3} - V_{D,3} = 0$$

Some expressions are discussed before. From Appendix K.1 is known that $q_1 L_{bm} + q_2 L_{bm} = V_{A,1} + V_{A,2} + V_{D,1} + V_{D,2}$. Also known is the change in the vertical reaction force in point D $\left(V_{D,3} = \left(q_1 + q_2\right) e_3 + q_3 \left(\frac{1}{2} L_{bm} + e_1 + e_2 + e_3\right)\right)$. These expressions are filled in the formula.

$$q_3 L_{bm} - V_{A,3} - \left[\left(q_1 + q_2\right) e_3 + q_3 \left(\frac{1}{2} L_{bm} + e_1 + e_2 + e_3\right)\right] = 0$$

The same expressions can be combined together.

$$-V_{A,3} - \left(q_1 + q_2\right) e_3 + q_3 \left(\frac{1}{2} L_{bm} - e_1 - e_2 - e_3\right) = 0$$

This results in a formula of $V_{A,3}$:

$$V_{A,3} = -\left(q_1 + q_2\right) e_3 + q_3 \left(\frac{1}{2} L_{bm} - e_1 - e_2 - e_3\right)$$

Moment in point C.

To calculate the additional deflection it is necessary to calculate the extra moments. The additional moment in point C $\left(M_{C,3}\right)$ will be analyzed first. In the third load case both the right flange and the left flange are partial yielded. The effective section is double symmetric again. There is no shift in the centre of gravity.

$$M_{C,total,3} = V_{D,total,3} e_{total,3} + H_{A,total,3} L_{cln}$$

The bending moments and the reaction forces can be split in an original part and an additional part.

$$M_{C,1} + M_{C,2} + M_{C,3} = \left(V_{D,1} + V_{D,2} + V_{D,3}\right)\left(e_1 + e_2 + e_3\right) + \left(H_{A,1} + H_{A,2} + H_{A,3}\right) L_{cln}$$

Some expressions must be split to get the original expressions.

$$M_{C,1} + M_{C,2} + M_{C,3}$$
$$= V_{D,1} e_1 + V_{D,1} e_2 + V_{D,1} e_3 + V_{D,2}\left(e_1 + e_2\right) + V_{D,2} e_3 + V_{D,3}\left(e_1 + e_2 + e_3\right) + H_{A,1} L_{cln} + H_{A,2} L_{cln} + H_{A,3} L_{cln}$$

The original moments $\left(M_{C,1} = V_{D,1} e_1 + H_{A,1} L_{cln}\right)$ and $\left(M_{C,2} = V_{D,1} e_2 + V_{D,2}\left(e_1 + e_2\right) + H_{A,2} L_{cln}\right)$ can be neglected from the formula.

$$M_{C,3} = \left(V_{D,1} + V_{D,2}\right) e_3 + V_{D,3}\left(e_1 + e_2 + e_3\right) + H_{A,3} L_{cln}$$

The expression of $V_{D,3}$ $\left(V_{D,3} = \left(q_1 + q_2\right) e_3 + q_3 \left(\frac{1}{2} L_{bm} + e_1 + e_2 + e_3\right)\right)$ is found before and can used in the formula. Also the formulas of the original reaction forces $\left(V_{D,1} = q_1 \left(\frac{1}{2} L_{bm} + e_1\right)\right)$ and $\left(V_{D,2} = q_1 e_2 + q_2 \left(\frac{1}{2} L_{bm} + e_1 + e_2\right)\right)$ can be used in the formula. This results in:

$$M_{C,3} = \left(\left(V_{D,1} = q_1\left(½L_{bm} + e_1\right)\right) + \left(V_{D,2} = q_1 e_2 + q_2\left(½L_{bm} + e_1 + e_2\right)\right)\right)e_3$$
$$+\left(\left(q_1 + q_2\right)e_3 + q_3\left(½L_{bm} + e_1 + e_2 + e_3\right)\right)\left(e_1 + e_2 + e_3\right) + H_{A,3}L_{cln}$$

This formula can be made clearer. This results in the following formula:

$$M_{C,3} = e_3\left(q_1 + q_2\right)\left(½L_{bm} + 2e_1 + 2e_2 + e_3\right) + q_3\left(½L_{bm} + e_1 + e_2 + e_3\right)\left(e_1 + e_2 + e_3\right) + H_{A,3}L_{cln}$$

Moment in point B.

The moment difference in point C has been formulated. The moment difference in point B will be analysed. Starting with the following formula:

$$M_{B,total,3} = V_{A,total,3}e_{total,3} - H_{A,total,3}L_{cln}$$

Again the loads had been split in the original load and the additional load.

$$M_{B,1} + M_{B,2} + M_{B,3} = \left(V_{A,1} + V_{A,2} + V_{A,3}\right)\left(e_1 + e_2 + e_3\right) - \left(H_{A,1} + H_{A,2} + H_{A,3}\right)L_{cln}$$

Some expressions are split to get the original formula.

$$M_{B,1} + M_{B,2} + M_{B,3}$$
$$= V_{A,1}e_1 + V_{A,1}e_2 + V_{A,1}e_3 + V_{A,2}\left(e_1 + e_2\right) + V_{A,2}e_3 + V_{A,3}\left(e_1 + e_2 + e_3\right) - H_{A,1}L_{cln} - H_{A,2}L_{cln} - H_{A,3}L_{cln}$$

Known are the original bending moments $\left(M_{B,1} = V_{A,1}e_1 - H_{A,1}L_{cln}\right)$ and $\left(M_{B,2} = V_{A,1}e_2 + V_{A,2}\left(e_1 + e_2\right) - H_{A,2}L_{cln}\right)$. These moments are neglected from the formula. The following formula is left.

$$M_{B,3} = V_{A,1}e_3 + V_{A,2}e_3 + V_{A,3}\left(e_1 + e_2 + e_3\right) - H_{A,3}L_{cln}$$

Also known is the formula for the additional vertical force in support A. $\left(V_{A,3} = -\left(q_1 + q_2\right)e_3 + q_3\left(½L_{bm} - e_1 - e_2 - e_3\right)\right)$ This expression can be used in the formula. Also the formula for the original vertical reaction forces $\left(V_{A,1} = q_1\left(½L_{bm} - e_1\right)\right)$ and $\left(V_{A,2} = -q_1 e_2 + q_2\left(½L_{bm} - e_1 - e_2\right)\right)$ can be used.

$$M_{B,3} = \left(q_1\left(½L_{bm} - e_1\right)\right)e_3 + \left(-q_1 e_2 + q_2\left(½L_{bm} - e_1 - e_2\right)\right)e_3$$
$$+\left(-\left(q_1 + q_2\right)e_3 + q_3\left(½L_{bm} - e_1 - e_2 - e_3\right)\right)\left(e_1 + e_2 + e_3\right) - H_{A,3}L_{cln}$$

The formula has been changed to the following formula.

$$M_{B,3} = e_3\left(q_1 + q_2\right)\left(½L_{bm} - 2e_1 - 2e_2 - e_3\right) + q_3\left(½L_{bm} - e_1 - e_2 - e_3\right)\left(e_1 + e_2 + e_3\right) - H_{A,3}L_{cln}$$

Remaining force in point D.

As discussed in Appendix I, the deflection depends on the remaining forces on the columns too. These forces depend on the horizontal en vertical reaction force. First the difference in the remaining force $F_{R,D,2}$ is analyzed.

$$F_{R,D,total,3} = V_{D,total,3}\frac{e_{total,3}}{L_{cln}} + H_{D,total,3}$$

Change the term 'total' in the three different load cases.

$$F_{R,D,1}+F_{R,D,2}+F_{R,D,3}=\left(V_{D,1}+V_{D,2}+V_{D,3}\right)\frac{e_1+e_2+e_3}{L_{cln}}+H_{D,1}+H_{D,2}+H_{D,3}$$

Some expressions are written out to get the original formulas.

$$F_{R,D,1}+F_{R,D,2}+F_{R,D,3}$$
$$=V_{D,1}\frac{e_1}{L_{cln}}+V_{D,1}\frac{e_2}{L_{cln}}+V_{D,1}\frac{e_3}{L_{cln}}+V_{D,2}\frac{e_1+e_2}{L_{cln}}+V_{D,2}\frac{e_3}{L_{cln}}+V_{D,3}\frac{e_1+e_2+e_3}{L_{cln}}+H_{D,1}+H_{D,2}+H_{D,3}$$

The original formulas $\left(F_{R,D,1}=V_{D,1}\frac{e_1}{L_{cln}}+H_{D,1}\right)$ and $\left(F_{R,D,2}=V_{D,1}\frac{e_2}{L_{cln}}+V_{D,2}\frac{e_1+e_2}{L_{cln}}+H_{D,2}\right)$ can be neglected from this formula. This result in:

$$F_{R,D,3}=V_{D,1}\frac{e_3}{L_{cln}}+V_{D,2}\frac{e_3}{L_{cln}}+V_{D,3}\frac{e_1+e_2+e_3}{L_{cln}}+H_{D,3}$$

The known formulas $\left(V_{D,3}=\left(q_1+q_2\right)e_3+q_3\left(\tfrac{1}{2}L_{bm}+e_1+e_2+e_3\right)\right)$, $\left(V_{D,1}=q_1\left(\tfrac{1}{2}L_{bm}+e_1\right)\right)$ and $\left(V_{D,2}=q_1e_2+q_2\left(\tfrac{1}{2}L_{bm}+e_1+e_2\right)\right)$ can be used.

$$F_{R,D,3}=\left(q_1\left(\tfrac{1}{2}L_{bm}+e_1\right)\right)\frac{e_3}{L_{cln}}+\left(q_1e_2+q_2\left(\tfrac{1}{2}L_{bm}+e_1+e_2\right)\right)\frac{e_3}{L_{cln}}$$
$$+\left(\left(q_1+q_2\right)e_3+q_3\left(\tfrac{1}{2}L_{bm}+e_1+e_2+e_3\right)\right)\frac{e_1+e_2+e_3}{L_{cln}}+H_{D,3}$$

Combine the same expressions results in the following formula.

$$F_{R,D,3}=\left(q_1+q_2\right)\left(\tfrac{1}{2}L_{bm}+2e_1+2e_2+e_3\right)\frac{e_3}{L_{cln}}+q_3\left(\tfrac{1}{2}L_{bm}+e_1+e_2+e_3\right)\frac{e_1+e_2+e_3}{L_{cln}}+H_{D,3}$$

Remaining force in point A.

The formula of the additional remaining force in point D is known. The same analysis can be made for the additional remaining force in point D. This analysis starts with the following formula.

$$F_{R,A,total,3}=V_{A,total,3}\frac{e_{total,3}}{L_{cln}}-H_{A,total,3}$$

Total has been split in the different load cases.

$$F_{R,A,1}+F_{R,A,2}+F_{R,A,3}=\left(V_{A,1}+V_{A,2}+V_{A,3}\right)\frac{e_1+e_2+e_2}{L_{cln}}-H_{A,1}-H_{A,2}-H_{A,3}$$

Some expressions are written out to get the original expressions.

$$F_{R,A,1}+F_{R,A,2}+F_{R,A,3}$$
$$=V_{A,1}\frac{e_1}{L_{cln}}+V_{A,1}\frac{e_2}{L_{cln}}+V_{A,1}\frac{e_2}{L_{cln}}+V_{A,2}\frac{e_1+e_2}{L_{cln}}+V_{A,2}\frac{e_2}{L_{cln}}+V_{A,3}\frac{e_1+e_2+e_2}{L_{cln}}-H_{A,1}-H_{A,2}-H_{A,3}$$

The original formulas $\left(F_{R,A,1}=V_{A,1}\frac{e_1}{L_{cln}}-H_{A,1}\right)$ and $\left(F_{R,A,2}=V_{A,1}\frac{e_2}{L_{cln}}+V_{A,2}\frac{e_1+e_2}{L_{cln}}-H_{D,2}\right)$ can be neglected from the formula.

$$F_{R,A,3}=V_{A,1}\frac{e_3}{L_{cln}}+V_{A,2}\frac{e_3}{L_{cln}}+V_{A,3}\frac{e_1+e_2+e_3}{L_{cln}}-H_{A,3}$$

The known formulas $\left(V_{A,3}=-(q_1+q_2)e_3+q_3\left(\tfrac{1}{2}L_{bm}-e_1-e_2-e_3\right)\right)$, $\left(V_{A,1}=q_1\left(\tfrac{1}{2}L_{bm}-e_1\right)\right)$ and $\left(V_{A,2}=-q_1e_2+q_2\left(\tfrac{1}{2}L_{bm}-e_1-e_2\right)\right)$ can be used.

$$F_{R,A,3}=\left(q_1\left(\tfrac{1}{2}L_{bm}-e_1\right)\right)\frac{e_3}{L_{cln}}+\left(-q_1e_2+q_2\left(\tfrac{1}{2}L_{bm}-e_1-e_3\right)\right)\frac{e_3}{L_{cln}}$$
$$+\left(-(q_1+q_2)e_3+q_3\left(\tfrac{1}{2}L_{bm}-e_1-e_2-e_3\right)\right)\frac{e_1+e_2+e_3}{L_{cln}}-H_{A,3}$$

This can be made clearer.
Combine the same expressions results in the following formula.

$$F_{R,A,3}=(q_1+q_2)\left(\tfrac{1}{2}L_{bm}-2e_1-2e_2-e_3\right)\frac{e_3}{L_{cln}}+q_3\left(\tfrac{1}{2}L_{bm}-e_1-e_2-e_3\right)\frac{e_1+e_2+e_3}{L_{cln}}-H_{A,3}$$

A list of all analyzed formulas is made. These formulas can be used to make an analysis to find a formula of the total deflection.

$$V_{D,3}=(q_1+q_2)e_3+q_3\left(\tfrac{1}{2}L_{bm}+e_1+e_2+e_3\right)$$
$$V_{A,3}=-(q_1+q_2)e_3+q_3\left(\tfrac{1}{2}L_{bm}-e_1-e_2-e_3\right)$$
$$M_{C,3}=e_3(q_1+q_2)\left(\tfrac{1}{2}L_{bm}+2e_1+2e_2+e_3\right)+q_3\left(\tfrac{1}{2}L_{bm}+e_1+e_2+e_3\right)(e_1+e_2+e_3)+H_{A,3}L_{cln}$$
$$M_{B,3}=e_3(q_1+q_2)\left(\tfrac{1}{2}L_{bm}-2e_1-2e_2-e_3\right)+q_3\left(\tfrac{1}{2}L_{bm}-e_1-e_2-e_3\right)(e_1+e_2+e_3)-H_{A,3}L_{cln}$$
$$F_{R,D,3}=(q_1+q_2)\left(\tfrac{1}{2}L_{bm}+2e_1+2e_2+e_3\right)\frac{e_3}{L_{cln}}+q_3\left(\tfrac{1}{2}L_{bm}+e_1+e_2+e_3\right)\frac{e_1+e_2+e_3}{L_{cln}}+H_{D,3}$$
$$F_{R,A,3}=(q_1+q_2)\left(\tfrac{1}{2}L_{bm}-2e_1-2e_2-e_3\right)\frac{e_3}{L_{cln}}+q_3\left(\tfrac{1}{2}L_{bm}-e_1-e_2-e_3\right)\frac{e_1+e_2+e_3}{L_{cln}}-H_{A,3}$$

As same as before the rotations in the point B and C can be calculated.

$$\varphi_{C,3}=\frac{q_3L_{bm}{}^3}{24EI_{bm}}+\frac{M_{B,3}L_{bm}}{6EI_{bm}}-\frac{M_{C,3}L_{bm}}{3EI_{bm}}$$
$$\varphi_{B,3}=-\frac{q_3L_{bm}{}^3}{24EI_{bm}}-\frac{M_{B,3}L_{bm}}{3EI_{bm}}+\frac{M_{C,3}L_{bm}}{6EI_{bm}}$$

The additional deflection of column AB will be analysed. The analysis starts with the following formula:

$$e_3^{AB} = -\varphi_B L_{cln} + \frac{F_{R,A,3} L_{cln}^3}{3EI_{cln,1}}$$

The additional rotation is a known expression. This expression can be used in the formula.

$$e_3^{AB} = -\left(-\frac{q_3 L_{bm}^3}{24EI_{bm}} - \frac{M_{B,3} L_{bm}}{3EI_{bm}} + \frac{M_{C,3} L_{bm}}{6EI_{bm}} \right) L_{cln} + \frac{F_{R,A,3} L_{cln}^3}{3EI_{cln,1}}$$

There are also expressions known for $M_{B,3}$, $M_{C,3}$ and $F_{R,A,3}$. These expressions can be used in the formula too.

$$e_3^{AB} = \frac{q_3 L_{bm}^3 L_{cln}}{24EI_{bm}} +$$

$$\left[e_3 (q_1 + q_2)(\tfrac{1}{2} L_{bm} - 2e_1 - 2e_2 - e_3) + q_3 (\tfrac{1}{2} L_{bm} - e_1 - e_2 - e_3)(e_1 + e_2 + e_3) - H_{A,3} L_{cln} \right] \frac{L_{bm} L_{cln}}{3EI_{bm}}$$

$$-\left[e_3 (q_1 + q_2)(\tfrac{1}{2} L_{bm} + 2e_1 + 2e_2 + e_3) + q_3 (\tfrac{1}{2} L_{bm} + e_1 + e_2 + e_3)(e_1 + e_2 + e_3) + H_{A,3} L_{cln} \right] \frac{L_{bm} L_{cln}}{6EI_{bm}}$$

$$+\left[(q_1 + q_2)(\tfrac{1}{2} L_{bm} - 2e_1 - 2e_2 - e_3) \frac{e_3}{L_{cln}} + q_3 (\tfrac{1}{2} L_{bm} - e_1 - e_2 - e_3) \frac{e_1 + e_2 + e_3}{L_{cln}} - H_{A,3} \right] \frac{L_{cln}^3}{3EI_{cln,1}}$$

All expressions with $H_{A,3}$ will be separate from the rest of the formula. This results in:

$$H_{A,3} L_{cln} \frac{L_{bm} L_{cln}}{3EI_{bm}} + H_{A,3} L_{cln} \frac{L_{bm} L_{cln}}{6EI_{bm}} + H_{A,3} \frac{L_{cln}^3}{3EI_{cln,1}} = \frac{q_3 L_{bm}^3 L_{cln}}{24EI_{bm}} +$$

$$\left[e_3 (q_1 + q_2)(\tfrac{1}{2} L_{bm} - 2e_1 - 2e_2 - e_3) + q_3 (\tfrac{1}{2} L_{bm} - e_1 - e_2 - e_3)(e_1 + e_2 + e_3) \right] \frac{L_{bm} L_{cln}}{3EI_{bm}}$$

$$-\left[e_3 (q_1 + q_2)(\tfrac{1}{2} L_{bm} + 2e_1 + 2e_2 + e_3) + q_3 (\tfrac{1}{2} L_{bm} + e_1 + e_2 + e_3)(e_1 + e_2 + e_3) \right] \frac{L_{bm} L_{cln}}{6EI_{bm}}$$

$$+\left[(q_1 + q_2)(\tfrac{1}{2} L_{bm} - 2e_1 - 2e_2 - e_3) \frac{e_3}{L_{cln}} + q_3 (\tfrac{1}{2} L_{bm} - e_1 - e_2 - e_3) \frac{e_1 + e_2 + e_3}{L_{cln}} \right] \frac{L_{cln}^3}{3EI_{cln,1}} - e_3$$

Combine all expressions of $H_{A,3}$.

$$H_{A,3} \left(\frac{L_{bm} L_{cln}^2}{2EI_{bm}} + \frac{L_{cln}^3}{3EI_{cln,1}} \right) = \frac{q_3 L_{bm}^3 L_{cln}}{24EI_{bm}} +$$

$$\left[e_3 (q_1 + q_2)(\tfrac{1}{2} L_{bm} - 2e_1 - 2e_2 - e_3) + q_3 (\tfrac{1}{2} L_{bm} - e_1 - e_2 - e_3)(e_1 + e_2 + e_3) \right] \frac{L_{bm} L_{cln}}{3EI_{bm}}$$

$$-\left[e_3 (q_1 + q_2)(\tfrac{1}{2} L_{bm} + 2e_1 + 2e_2 + e_3) + q_3 (\tfrac{1}{2} L_{bm} + e_1 + e_2 + e_3)(e_1 + e_2 + e_3) \right] \frac{L_{bm} L_{cln}}{6EI_{bm}}$$

$$+\left[(q_1 + q_2)(\tfrac{1}{2} L_{bm} - 2e_1 - 2e_2 - e_3) \frac{e_3}{L_{cln}} + q_3 (\tfrac{1}{2} L_{bm} - e_1 - e_2 - e_3) \frac{e_1 + e_2 + e_3}{L_{cln}} \right] \frac{L_{cln}^3}{3EI_{cln,1}} - e_3$$

To continue the analysis it is important to write out all expressions. To write out all expressions in ones results in an unclear situation. The different expressions are written out in parts. There are three expressions written out. The last three expressions of the right part of the formula.

Write out the first part:

$$\left[e_3\left(q_1+q_2\right)\left(\tfrac{1}{2}L_{bm}-2e_1-2e_2-e_3\right)+q_3\left(\tfrac{1}{2}L_{bm}-e_1-e_2-e_3\right)\left(e_1+e_2+e_3\right)\right]\frac{L_{bm}L_{cln}}{3EI_{bm}}$$

$$=\frac{q_1L_{bm}{}^2L_{cln}e_3}{6EI_{bm}}-\frac{2q_1L_{bm}L_{cln}e_1e_3}{3EI_{bm}}-\frac{2q_1L_{bm}L_{cln}e_2e_3}{3EI_{bm}}-\frac{q_1L_{bm}L_{cln}e_3{}^2}{3EI_{bm}}+\frac{q_2L_{bm}{}^2L_{cln}e_3}{6EI_{bm}}-\frac{2q_2L_{bm}L_{cln}e_1e_3}{3EI_{bm}}$$

$$-\frac{2q_2L_{bm}L_{cln}e_2e_3}{3EI_{bm}}-\frac{q_2L_{bm}L_{cln}e_3{}^2}{3EI_{bm}}+\frac{q_3L_{bm}{}^2L_{cln}e_1}{6EI_{bm}}+\frac{q_3L_{bm}{}^2L_{cln}e_2}{6EI_{bm}}+\frac{q_3L_{bm}{}^2L_{cln}e_3}{6EI_{bm}}-\frac{q_3L_{bm}L_{cln}e_1{}^2}{3EI_{bm}}-\frac{q_3L_{bm}L_{cln}e_1e_2}{3EI_{bm}}$$

$$-\frac{q_3L_{bm}L_{cln}e_1e_3}{3EI_{bm}}-\frac{q_3L_{bm}L_{cln}e_1e_2}{3EI_{bm}}-\frac{q_3L_{bm}L_{cln}e_2{}^2}{3EI_{bm}}-\frac{q_3L_{bm}L_{cln}e_2e_3}{3EI_{bm}}-\frac{q_3L_{bm}L_{cln}e_1e_3}{3EI_{bm}}-\frac{q_3L_{bm}L_{cln}e_2e_3}{3EI_{bm}}-\frac{q_3L_{bm}L_{cln}e_3{}^2}{3EI_{bm}}$$

The second part:

$$-\left[e_3\left(q_1+q_2\right)\left(\tfrac{1}{2}L_{bm}+2e_1+2e_2+e_3\right)+q_3\left(\tfrac{1}{2}L_{bm}+e_1+e_2+e_3\right)\left(e_1+e_2+e_3\right)\right]\frac{L_{bm}L_{cln}}{6EI_{bm}}$$

$$=-\frac{e_3q_1L_{bm}{}^2L_{cln}}{12EI_{bm}}-\frac{e_3q_1L_{bm}L_{cln}e_1}{3EI_{bm}}-\frac{e_3q_1L_{bm}L_{cln}e_2}{3EI_{bm}}-\frac{e_3q_1L_{bm}L_{cln}e_3}{6EI_{bm}}-\frac{e_3q_2L_{bm}{}^2L_{cln}}{12EI_{bm}}-\frac{e_3q_2L_{bm}L_{cln}e_1}{3EI_{bm}}$$

$$-\frac{e_3q_2L_{bm}L_{cln}e_2}{3EI_{bm}}-\frac{e_3q_2L_{bm}L_{cln}e_3}{6EI_{bm}}-\frac{q_3L_{bm}{}^2L_{cln}e_1}{12EI_{bm}}-\frac{q_3L_{bm}{}^2L_{cln}e_2}{12EI_{bm}}-\frac{q_3L_{bm}{}^2L_{cln}e_3}{12EI_{bm}}-\frac{q_3L_{bm}L_{cln}e_1{}^2}{6EI_{bm}}-\frac{q_3L_{bm}L_{cln}e_1e_2}{6EI_{bm}}$$

$$-\frac{q_3L_{bm}L_{cln}e_1e_3}{6EI_{bm}}-\frac{q_3L_{bm}L_{cln}e_1e_2}{6EI_{bm}}-\frac{q_3L_{bm}L_{cln}e_2{}^2}{6EI_{bm}}-\frac{q_3L_{bm}L_{cln}e_2e_3}{6EI_{bm}}-\frac{q_3L_{bm}L_{cln}e_1e_3}{6EI_{bm}}-\frac{q_3L_{bm}L_{cln}e_2e_3}{6EI_{bm}}-\frac{q_3L_{bm}L_{cln}e_3{}^2}{6EI_{bm}}$$

And the third part:

$$\left[\left(q_1+q_2\right)\left(\tfrac{1}{2}L_{bm}-2e_1-2e_2-e_3\right)\frac{e_3}{L_{cln}}+q_3\left(\tfrac{1}{2}L_{bm}-e_1-e_2-e_3\right)\frac{e_1+e_2+e_2}{L_{cln}}\right]\frac{L_{cln}{}^3}{3EI_{cln,1}}$$

$$=\frac{q_1L_{bm}L_{cln}{}^2e_3}{6EI_{cln,1}}-\frac{2q_1L_{cln}{}^2e_1e_3}{3EI_{cln,1}}-\frac{2q_1L_{cln}{}^2e_2e_3}{3EI_{cln,1}}-\frac{q_1L_{cln}{}^2e_3{}^2}{3EI_{cln,1}}+\frac{q_2L_{bm}L_{cln}{}^2e_3}{6EI_{cln,1}}-\frac{2q_2L_{cln}{}^2e_1e_3}{3EI_{cln,1}}$$

$$-\frac{2q_2L_{cln}{}^2e_2e_3}{3EI_{cln,1}}-\frac{q_2L_{cln}{}^2e_3{}^2}{3EI_{cln,1}}+\frac{q_3L_{bm}L_{cln}{}^2e_1}{6EI_{cln,1}}+\frac{q_3L_{bm}L_{cln}{}^2e_2}{6EI_{cln,1}}+\frac{q_3L_{bm}L_{cln}{}^2e_3}{6EI_{cln,1}}-\frac{q_3L_{cln}{}^2e_1{}^2}{3EI_{cln,1}}-\frac{q_3L_{cln}{}^2e_1e_2}{3EI_{cln,1}}$$

$$-\frac{q_3L_{cln}{}^2e_1e_3}{3EI_{cln,1}}-\frac{q_3L_{cln}{}^3e_1e_2}{3EI_{cln,1}}-\frac{q_3L_{cln}{}^3e_2{}^2}{3EI_{cln,1}}-\frac{q_3L_{cln}{}^2e_2e_3}{3EI_{cln,1}}-\frac{q_3L_{cln}{}^2e_1e_3}{3EI_{cln,1}}-\frac{q_3L_{cln}{}^2e_2e_3}{3EI_{cln,1}}-\frac{q_3L_{cln}{}^2e_3{}^2}{3EI_{cln,1}}$$

The three expressions which are written out can be used in the formula. This results in the following formula:

$$H_{A,3}\left(\frac{L_{bm}L_{cln}{}^{2}}{2EI_{bm}}+\frac{L_{cln}{}^{3}}{3EI_{cln,1}}\right)=\frac{q_3L_{bm}{}^{3}L_{cln}}{24EI_{bm}}$$

$$+\frac{q_1L_{bm}{}^{2}L_{cln}e_3}{6EI_{bm}}-\frac{2q_1L_{bm}L_{cln}e_1e_3}{3EI_{bm}}-\frac{2q_1L_{bm}L_{cln}e_2e_3}{3EI_{bm}}-\frac{q_1L_{bm}L_{cln}e_3{}^{2}}{3EI_{bm}}+\frac{q_2L_{bm}{}^{2}L_{cln}e_3}{6EI_{bm}}-\frac{2q_2L_{bm}L_{cln}e_1e_3}{3EI_{bm}}$$

$$-\frac{2q_2L_{bm}L_{cln}e_2e_3}{3EI_{bm}}-\frac{q_2L_{bm}L_{cln}e_3{}^{2}}{3EI_{bm}}+\frac{q_3L_{bm}{}^{2}L_{cln}e_1}{6EI_{bm}}+\frac{q_3L_{bm}{}^{2}L_{cln}e_2}{6EI_{bm}}+\frac{q_3L_{bm}{}^{2}L_{cln}e_3}{6EI_{bm}}-\frac{q_3L_{bm}L_{cln}e_1{}^{2}}{3EI_{bm}}-\frac{q_3L_{bm}L_{cln}e_1e_2}{3EI_{bm}}$$

$$-\frac{q_3L_{bm}L_{cln}e_1e_3}{3EI_{bm}}-\frac{q_3L_{bm}L_{cln}e_1e_2}{3EI_{bm}}-\frac{q_3L_{bm}L_{cln}e_2{}^{2}}{3EI_{bm}}-\frac{q_3L_{bm}L_{cln}e_2e_3}{3EI_{bm}}-\frac{q_3L_{bm}L_{cln}e_1e_3}{3EI_{bm}}-\frac{q_3L_{bm}L_{cln}e_2e_3}{3EI_{bm}}-\frac{q_3L_{bm}L_{cln}e_3{}^{2}}{3EI_{bm}}$$

$$-\frac{q_1L_{bm}{}^{2}L_{cln}e_3}{12EI_{bm}}-\frac{q_1L_{bm}L_{cln}e_1e_3}{3EI_{bm}}-\frac{q_1L_{bm}L_{cln}e_2e_3}{3EI_{bm}}-\frac{q_1L_{bm}L_{cln}e_3{}^{2}}{6EI_{bm}}-\frac{q_2L_{bm}{}^{2}L_{cln}e_3}{12EI_{bm}}-\frac{q_2L_{bm}L_{cln}e_1e_3}{3EI_{bm}}$$

$$-\frac{q_2L_{bm}L_{cln}e_2e_3}{3EI_{bm}}-\frac{q_2L_{bm}L_{cln}e_3{}^{2}}{6EI_{bm}}-\frac{q_3L_{bm}{}^{2}L_{cln}e_1}{12EI_{bm}}-\frac{q_3L_{bm}{}^{2}L_{cln}e_2}{12EI_{bm}}-\frac{q_3L_{bm}{}^{2}L_{cln}e_3}{12EI_{bm}}-\frac{q_3L_{bm}L_{cln}e_1{}^{2}}{6EI_{bm}}-\frac{q_3L_{bm}L_{cln}e_1e_2}{6EI_{bm}}$$

$$-\frac{q_3L_{bm}L_{cln}e_1e_3}{6EI_{bm}}-\frac{q_3L_{bm}L_{cln}e_1e_2}{6EI_{bm}}-\frac{q_3L_{bm}L_{cln}e_2{}^{2}}{6EI_{bm}}-\frac{q_3L_{bm}L_{cln}e_2e_3}{6EI_{bm}}-\frac{q_3L_{bm}L_{cln}e_1e_3}{6EI_{bm}}-\frac{q_3L_{bm}L_{cln}e_2e_3}{6EI_{bm}}-\frac{q_3L_{bm}L_{cln}e_3{}^{2}}{6EI_{bm}}$$

$$+\frac{q_1L_{bm}L_{cln}{}^{2}e_3}{6EI_{cln,1}}-\frac{2q_1L_{cln}{}^{2}e_1e_3}{3EI_{cln,1}}-\frac{2q_1L_{cln}{}^{2}e_2e_3}{3EI_{cln,1}}-\frac{q_1L_{cln}{}^{2}e_3{}^{2}}{3EI_{cln,1}}+\frac{q_2L_{bm}L_{cln}{}^{2}e_3}{6EI_{cln,1}}-\frac{2q_2L_{cln}{}^{2}e_1e_3}{3EI_{cln,1}}$$

$$-\frac{2q_2L_{cln}{}^{2}e_2e_3}{3EI_{cln,1}}-\frac{q_2L_{cln}{}^{2}e_3{}^{2}}{3EI_{cln,1}}+\frac{q_3L_{bm}L_{cln}{}^{2}e_1}{6EI_{cln,1}}+\frac{q_3L_{bm}L_{cln}{}^{2}e_2}{6EI_{cln,1}}+\frac{q_3L_{bm}L_{cln}{}^{2}e_3}{6EI_{cln,1}}-\frac{q_3L_{cln}{}^{2}e_1{}^{2}}{3EI_{cln,1}}-\frac{q_3L_{cln}{}^{2}e_1e_2}{3EI_{cln,1}}$$

$$-\frac{q_3L_{cln}{}^{2}e_1e_3}{3EI_{cln,1}}-\frac{q_3L_{cln}{}^{2}e_1e_2}{3EI_{cln,1}}-\frac{q_3L_{cln}{}^{2}e_2{}^{2}}{3EI_{cln,1}}-\frac{q_3L_{cln}{}^{2}e_2e_3}{3EI_{cln,1}}-\frac{q_3L_{cln}{}^{2}e_1e_3}{3EI_{cln,1}}-\frac{q_3L_{cln}{}^{2}e_2e_3}{3EI_{cln,1}}-\frac{q_3L_{cln}{}^{2}e_3{}^{2}}{3EI_{cln,1}}-e_3$$

Some expressions are the same and can be combined together.

$$H_{A,3}\left(\frac{L_{bm}L_{cln}{}^{2}}{2EI_{bm}}+\frac{L_{cln}{}^{3}}{3EI_{cln,1}}\right)=\frac{q_3L_{bm}{}^{3}L_{cln}}{24EI_{bm}}+\frac{q_1L_{bm}{}^{2}L_{cln}e_3}{12EI_{bm}}-\frac{q_1L_{bm}L_{cln}e_1e_3}{EI_{bm}}-\frac{q_1L_{bm}L_{cln}e_2e_3}{EI_{bm}}-\frac{q_1L_{bm}L_{cln}e_3{}^{2}}{2EI_{bm}}$$

$$+\frac{q_2L_{bm}{}^{2}L_{cln}e_3}{12EI_{bm}}-\frac{q_2L_{bm}L_{cln}e_1e_3}{EI_{bm}}-\frac{q_2L_{bm}L_{cln}e_2e_3}{EI_{bm}}-\frac{q_2L_{bm}L_{cln}e_3{}^{2}}{2EI_{bm}}+\frac{q_3L_{bm}{}^{2}L_{cln}e_1}{12EI_{bm}}+\frac{q_3L_{bm}{}^{2}L_{cln}e_2}{12EI_{bm}}$$

$$+\frac{q_3L_{bm}{}^{2}L_{cln}e_3}{12EI_{bm}}-\frac{q_3L_{bm}L_{cln}e_1{}^{2}}{2EI_{bm}}-\frac{q_3L_{bm}L_{cln}e_1e_2}{EI_{bm}}-\frac{q_3L_{bm}L_{cln}e_1e_3}{EI_{bm}}-\frac{q_3L_{bm}L_{cln}e_2{}^{2}}{2EI_{bm}}-\frac{q_3L_{bm}L_{cln}e_2e_3}{EI_{bm}}$$

$$-\frac{q_3L_{bm}L_{cln}e_3{}^{2}}{2EI_{bm}}+\frac{q_1L_{bm}L_{cln}{}^{2}e_3}{6EI_{cln,1}}-\frac{2q_1L_{cln}{}^{2}e_1e_3}{3EI_{cln,1}}-\frac{2q_1L_{cln}{}^{2}e_2e_3}{3EI_{cln,1}}-\frac{q_1L_{cln}{}^{2}e_3{}^{2}}{3EI_{cln,1}}+\frac{q_2L_{bm}L_{cln}{}^{2}e_3}{6EI_{cln,1}}-\frac{2q_2L_{cln}{}^{2}e_1e_3}{3EI_{cln,1}}$$

$$-\frac{2q_2L_{cln}{}^{2}e_2e_3}{3EI_{cln,1}}-\frac{q_2L_{cln}{}^{2}e_3{}^{2}}{3EI_{cln,1}}+\frac{q_3L_{bm}L_{cln}{}^{2}e_1}{6EI_{cln,1}}+\frac{q_3L_{bm}L_{cln}{}^{2}e_2}{6EI_{cln,1}}+\frac{q_3L_{bm}L_{cln}{}^{2}e_3}{6EI_{cln,1}}-\frac{q_3L_{cln}{}^{2}e_1{}^{2}}{3EI_{cln,1}}$$

$$-\frac{2q_3L_{cln}{}^{2}e_1e_2}{3EI_{cln,1}}-\frac{2q_3L_{cln}{}^{2}e_1e_3}{3EI_{cln,1}}-\frac{q_3L_{cln}{}^{2}e_2{}^{2}}{3EI_{cln,1}}-\frac{2q_3L_{cln}{}^{2}e_2e_3}{3EI_{cln,1}}-\frac{q_3L_{cln}{}^{2}e_3{}^{2}}{3EI_{cln,1}}-e_3$$

Every expression in the formula is multiplied to get the same denominator. This results in:

$$H_{A,3}\left(\frac{12L_{bm}L_{cln}{}^{2}EI_{cln,1}+8L_{cln}{}^{3}EI_{bm}}{24EI_{bm}EI_{cln,1}}\right)=\frac{q_3L_{bm}{}^{3}L_{cln}EI_{cln,1}}{24EI_{bm}EI_{cln,1}}+\frac{2q_1L_{bm}{}^{2}L_{cln}e_3EI_{cln,1}}{24EI_{bm}EI_{cln,1}}-\frac{24q_1L_{bm}L_{cln}e_1e_3EI_{cln,1}}{24EI_{bm}EI_{cln,1}}$$

$$-\frac{24q_1L_{bm}L_{cln}e_2e_3EI_{cln,1}}{24EI_{bm}EI_{cln,1}}-\frac{12q_1L_{bm}L_{cln}e_3{}^{2}EI_{cln,1}}{24EI_{bm}EI_{cln,1}}+\frac{2q_2L_{bm}{}^{2}L_{cln}e_3EI_{cln,1}}{24EI_{bm}EI_{cln,1}}-\frac{24q_2L_{bm}L_{cln}e_1e_3EI_{cln,1}}{24EI_{bm}EI_{cln,1}}$$

$$-\frac{24q_2L_{bm}L_{cln}e_2e_3EI_{cln,1}}{24EI_{bm}EI_{cln,1}}-\frac{12q_2L_{bm}L_{cln}e_3{}^{2}EI_{cln,1}}{24EI_{bm}EI_{cln,1}}+\frac{2q_3L_{bm}{}^{2}L_{cln}e_1EI_{cln,1}}{24EI_{bm}EI_{cln,1}}+\frac{2q_3L_{bm}{}^{2}L_{cln}e_2EI_{cln,1}}{24EI_{bm}EI_{cln,1}}$$

$$+\frac{2q_3L_{bm}{}^{2}L_{cln}e_3EI_{cln,1}}{24EI_{bm}EI_{cln,1}}-\frac{12q_3L_{bm}L_{cln}e_1{}^{2}EI_{cln,1}}{24EI_{bm}EI_{cln,1}}-\frac{24q_3L_{bm}L_{cln}e_1e_2EI_{cln,1}}{24EI_{bm}EI_{cln,1}}-\frac{24q_3L_{bm}L_{cln}e_1e_3EI_{cln,1}}{24EI_{bm}EI_{cln,1}}$$

$$-\frac{12q_3L_{bm}L_{cln}e_2{}^{2}EI_{cln,1}}{24EI_{bm}EI_{cln,1}}-\frac{24q_3L_{bm}L_{cln}e_2e_3EI_{cln,1}}{24EI_{bm}EI_{cln,1}}-\frac{12q_3L_{bm}L_{cln}e_3{}^{2}EI_{cln,1}}{24EI_{bm}EI_{cln,1}}+\frac{4q_1L_{bm}L_{cln}{}^{2}e_3EI_{bm}}{24EI_{bm}EI_{cln,1}}$$

$$-\frac{16q_1L_{cln}{}^{2}e_1e_3EI_{bm}}{24EI_{bm}EI_{cln,1}}-\frac{16q_1L_{cln}{}^{2}e_2e_3EI_{bm}}{24EI_{bm}EI_{cln,1}}-\frac{8q_1L_{cln}{}^{2}e_3{}^{2}EI_{bm}}{24EI_{bm}EI_{cln,1}}+\frac{4q_2L_{bm}L_{cln}{}^{2}e_3EI_{bm}}{24EI_{bm}EI_{cln,1}}$$

$$-\frac{16q_2L_{cln}{}^{2}e_1e_3EI_{bm}}{24EI_{bm}EI_{cln,1}}-\frac{16q_2L_{cln}{}^{2}e_2e_3EI_{bm}}{24EI_{bm}EI_{cln,1}}-\frac{8q_2L_{cln}{}^{2}e_3{}^{2}EI_{bm}}{24EI_{bm}EI_{cln,1}}+\frac{4q_3L_{bm}L_{cln}{}^{2}e_1EI_{bm}}{24EI_{bm}EI_{cln,1}}$$

$$+\frac{4q_3L_{bm}L_{cln}{}^{2}e_2EI_{bm}}{24EI_{bm}EI_{cln,1}}+\frac{4q_3L_{bm}L_{cln}{}^{2}\Delta e_3EI_{bm}}{24EI_{bm}EI_{cln,1}}-\frac{8q_3L_{cln}{}^{2}e_1{}^{2}EI_{bm}}{24EI_{bm}EI_{cln,1}}-\frac{16q_3L_{cln}{}^{2}e_1e_2EI_{bm}}{24EI_{bm}EI_{cln,1}}$$

$$-\frac{16q_3L_{cln}{}^{2}e_1e_3EI_{bm}}{24EI_{bm}EI_{cln,1}}-\frac{8q_3L_{cln}{}^{2}e_2{}^{2}EI_{bm}}{24EI_{bm}EI_{cln,1}}-\frac{16q_3L_{cln}{}^{2}e_2e_3EI_{bm}}{24EI_{bm}EI_{cln,1}}-\frac{8q_3L_{cln}{}^{2}e_3{}^{2}EI_{bm}}{24EI_{bm}EI_{cln,1}}-\frac{24e_3EI_{cln,1}EI_{bm}}{24EI_{bm}EI_{cln,1}}$$

All denominators are the same and can be neglected. After neglect the denominators it is possible to make a formula of $H_{A,3}$.

$$H_{A,3}=\frac{\left(\begin{aligned}&q_3L_{bm}{}^{3}L_{cln}EI_{cln,1}+2q_1L_{bm}{}^{2}L_{cln}e_3EI_{cln,1}-24q_1L_{bm}L_{cln}e_1e_3EI_{cln,1}-24q_1L_{bm}L_{cln}e_2e_3EI_{cln,1}\\&-12q_1L_{bm}L_{cln}e_3{}^{2}EI_{cln,1}+2q_2L_{bm}{}^{2}L_{cln}e_3EI_{cln,1}-24q_2L_{bm}L_{cln}e_1e_3EI_{cln,1}-24q_2L_{bm}L_{cln}e_2e_3EI_{cln,1}\\&-12q_2L_{bm}L_{cln}e_3{}^{2}EI_{cln,1}+2q_3L_{bm}{}^{2}L_{cln}e_1EI_{cln,1}+2q_3L_{bm}{}^{2}L_{cln}e_2EI_{cln,1}+2q_3L_{bm}{}^{2}L_{cln}e_3EI_{cln,1}\\&-12q_3L_{bm}L_{cln}e_1{}^{2}EI_{cln,1}-24q_3L_{bm}L_{cln}e_1e_2EI_{cln,1}-24q_3L_{bm}L_{cln}e_1e_3EI_{cln,1}-12q_3L_{bm}L_{cln}e_2{}^{2}EI_{cln,1}\\&-24q_3L_{bm}L_{cln}e_2\Delta e_3EI_{cln,1}-12q_3L_{bm}L_{cln}e_3{}^{2}EI_{cln,1}+4q_1L_{bm}L_{cln}{}^{2}e_3EI_{bm}-16q_1L_{cln}{}^{2}e_1e_3EI_{bm}\\&-16q_1L_{cln}{}^{2}e_2e_3EI_{bm}-8q_1L_{cln}{}^{2}e_3{}^{2}EI_{bm}+4q_2L_{bm}L_{cln}{}^{2}e_3EI_{bm}-16q_2L_{cln}{}^{2}e_1e_3EI_{bm}-16q_2L_{cln}{}^{2}e_2e_3EI_{bm}\\&-8q_2L_{cln}{}^{2}e_3{}^{2}EI_{bm}+4q_3L_{bm}L_{cln}{}^{2}e_1EI_{bm}+4q_3L_{bm}L_{cln}{}^{2}e_2EI_{bm}+4q_3L_{bm}L_{cln}{}^{2}e_3EI_{bm}-8q_3L_{cln}{}^{2}e_1{}^{2}EI_{bm}\\&-16q_3L_{cln}{}^{2}e_1e_2EI_{bm}-16q_3L_{cln}{}^{2}e_1e_3EI_{bm}-8q_3L_{cln}{}^{2}e_2{}^{2}EI_{bm}-16q_3L_{cln}{}^{2}e_2e_3EI_{bm}-8q_3L_{cln}{}^{2}e_3{}^{2}EI_{bm}\\&-24e_3EI_{cln,1}EI_{bm}\end{aligned}\right)}{12L_{bm}L_{cln}{}^{2}EI_{cln,1}+8L_{cln}{}^{3}EI_{bm}}$$

The formula above is the result of the difference of deflection in column AB. It is also possible to make the same analysis for the difference of deflection in column CD. Column CD is partial yielded and has a reduced stiffness. The analysis starts with the following formula.

$$e_3^{CD} = -\varphi_{C,3} L_{cln} + F_{R,D,3} \frac{L_{cln}^{\ 3}}{3EI_{cln,3}}$$

$\varphi_{C,3}$ is known and can be used in:

$$e_3^{CD} = -\left(\frac{q_3 L_{bm}^{\ 3}}{24EI_{bm}} + \frac{M_{B,3} L_{bm}}{6EI_{bm}} - \frac{M_{C,3} L_{bm}}{3EI_{bm}} \right) L_{cln} + F_{R,D,3} \frac{L_{cln}^{\ 3}}{3EI_{cln,3}}$$

Also the expressions for $M_{B,3}$, $M_{C,3}$ and $F_{R,D,3}$ are known. These expressions can be used in the formula too.

$$e_3^{CD} = -\frac{q_3 L_{bm}^{\ 3} L_{cln}}{24EI_{bm}}$$

$$-\left[e_3 (q_1 + q_2)(½L_{bm} - 2e_1 - 2e_2 - e_3) + q_3 (½L_{bm} - e_1 - e_2 - e_3)(e_1 + e_2 + e_3) - H_{A,3} L_{cln} \right] \frac{L_{bm} L_{cln}}{6EI_{bm}}$$

$$+\left[e_3 (q_1 + q_2)(½L_{bm} + 2e_1 + 2e_2 + e_3) + q_3 (½L_{bm} + e_1 + e_2 + e_3)(e_1 + e_2 + e_3) + H_{A,3} L_{cln} \right] \frac{L_{bm} L_{cln}}{3EI_{bm}}$$

$$+\left[(q_1 + q_2)(½L_{bm} + 2e_1 + 2e_2 + e_3) \frac{e_3}{L_{cln}} + q_3 (½L_{bm} + e_1 + e_2 + e_3) \frac{e_1 + e_2 + e_3}{L_{cln}} + H_{D,3} \right] \frac{L_{cln}^{\ 3}}{3EI_{cln,3}}$$

$H_{A,3}$ and $H_{D,3}$ are equal together (horizontal equilibrium). Put all expressions of $H_{A,3}$ to one side of the equation.

$$-H_{A,3} L_{cln} \frac{L_{bm} L_{cln}}{6EI_{bm}} - H_{A,3} L_{cln} \frac{L_{bm} L_{cln}}{3EI_{bm}} - H_{D,3} \frac{L_{cln}^{\ 3}}{3EI_{cln,3}} = -\frac{q_3 L_{bm}^{\ 3} L_{cln}}{24EI_{bm}}$$

$$-\left[e_3 (q_1 + q_2)(½L_{bm} - 2e_1 - 2e_2 - e_3) + q_3 (½L_{bm} - e_1 - e_2 - e_3)(e_1 + e_2 + e_3) \right] \frac{L_{bm} L_{cln}}{6EI_{bm}}$$

$$+\left[e_3 (q_1 + q_2)(½L_{bm} + 2e_1 + 2e_2 + e_3) + q_3 (½L_{bm} + e_1 + e_2 + e_3)(e_1 + e_2 + e_3) \right] \frac{L_{bm} L_{cln}}{3EI_{bm}}$$

$$+\left[(q_1 + q_2)(½L_{bm} + 2e_1 + 2e_2 + e_3) \frac{e_3}{L_{cln}} + q_3 (½L_{bm} + e_1 + e_2 + e_3) \frac{e_1 + e_2 + e_3}{L_{cln}} \right] \frac{L_{cln}^{\ 3}}{3EI_{cln,3}} - e_3$$

Combine all expressions of $H_{A,3}$.

$$-H_{A,3} \left(\frac{L_{bm} L_{cln}^{\ 2}}{2EI_{bm}} + \frac{L_{cln}^{\ 3}}{3EI_{cln,3}} \right) = -\frac{q_3 L_{bm}^{\ 3} L_{cln}}{24EI_{bm}}$$

$$-\left[e_3 (q_1 + q_2)(½L_{bm} - 2e_1 - 2e_2 - e_3) + q_3 (½L_{bm} - e_1 - e_2 - e_3)(e_1 + e_2 + e_3) \right] \frac{L_{bm} L_{cln}}{6EI_{bm}}$$

$$+\left[e_3 (q_1 + q_2)(½L_{bm} + 2e_1 + 2e_2 + e_3) + q_3 (½L_{bm} + e_1 + e_2 + e_3)(e_1 + e_2 + e_3) \right] \frac{L_{bm} L_{cln}}{3EI_{bm}}$$

$$+\left[(q_1 + q_2)(½L_{bm} + 2e_1 + 2e_2 + e_3) \frac{e_3}{L_{cln}} + q_3 (½L_{bm} + e_1 + e_2 + e_3) \frac{e_1 + e_2 + e_3}{L_{cln}} \right] \frac{L_{cln}^{\ 3}}{3EI_{cln,3}} - e_3$$

To continue the analysis it is important to write out all expressions. To write out all expressions in ones results in an unclear situation. The different expressions are written out in parts. There are three expressions written out. The last three expressions of the right part of the formula.

Write out the first part:

$$-\left[e_3\left(q_1+q_2\right)\left(\tfrac{1}{2}L_{bm}-2e_1-2e_2-e_3\right)+q_3\left(\tfrac{1}{2}L_{bm}-e_1-e_2-e_3\right)\left(e_1+e_2+e_3\right)\right]\frac{L_{bm}L_{cln}}{6EI_{bm}}$$

$$=-\frac{q_1L_{bm}{}^2L_{cln}e_3}{12EI_{bm}}+\frac{q_1L_{bm}L_{cln}e_1e_3}{3EI_{bm}}+\frac{q_1L_{bm}L_{cln}e_2e_3}{3EI_{bm}}+\frac{q_1L_{bm}L_{cln}e_3{}^2}{6EI_{bm}}-\frac{q_2L_{bm}{}^2L_{cln}e_3}{12EI_{bm}}+\frac{q_2L_{bm}L_{cln}e_1e_3}{3EI_{bm}}$$

$$+\frac{q_2L_{bm}L_{cln}e_2e_3}{3EI_{bm}}+\frac{q_2L_{bm}L_{cln}e_3{}^2}{6EI_{bm}}-\frac{q_3L_{bm}{}^2L_{cln}e_1}{12EI_{bm}}-\frac{q_3L_{bm}{}^2L_{cln}e_2}{12EI_{bm}}-\frac{q_3L_{bm}{}^2L_{cln}e_3}{12EI_{bm}}+\frac{q_3L_{bm}L_{cln}e_1{}^2}{6EI_{bm}}+\frac{q_3L_{bm}L_{cln}e_1e_2}{6EI_{bm}}$$

$$+\frac{q_3L_{bm}L_{cln}e_1e_3}{6EI_{bm}}+\frac{q_3L_{bm}L_{cln}e_1e_2}{6EI_{bm}}+\frac{q_3L_{bm}L_{cln}e_2{}^2}{6EI_{bm}}+\frac{q_3L_{bm}L_{cln}e_2e_3}{6EI_{bm}}+\frac{q_3L_{bm}L_{cln}e_1e_3}{6EI_{bm}}+\frac{q_3L_{bm}L_{cln}e_2e_3}{6EI_{bm}}+\frac{q_3L_{bm}L_{cln}e_3{}^2}{6EI_{bm}}$$

The second part:

$$\left[e_3\left(q_1+q_2\right)\left(\tfrac{1}{2}L_{bm}+2e_1+2e_2+e_3\right)+q_3\left(\tfrac{1}{2}L_{bm}+e_1+e_2+e_3\right)\left(e_1+e_2+e_3\right)\right]\frac{L_{bm}L_{cln}}{3EI_{bm}}$$

$$=\frac{q_1L_{bm}{}^2L_{cln}e_3}{6EI_{bm}}+\frac{2q_1L_{bm}L_{cln}e_1e_3}{3EI_{bm}}+\frac{2q_1L_{bm}L_{cln}e_2e_3}{3EI_{bm}}+\frac{q_1L_{bm}L_{cln}e_3{}^2}{3EI_{bm}}+\frac{q_2L_{bm}{}^2L_{cln}e_3}{6EI_{bm}}+\frac{2q_2L_{bm}L_{cln}e_1e_3}{3EI_{bm}}$$

$$+\frac{2q_2L_{bm}L_{cln}e_2e_3}{3EI_{bm}}+\frac{q_2L_{bm}L_{cln}e_3{}^2}{3EI_{bm}}+\frac{q_3L_{bm}{}^2L_{cln}e_1}{6EI_{bm}}+\frac{q_3L_{bm}{}^2L_{cln}e_2}{6EI_{bm}}+\frac{q_3L_{bm}{}^2L_{cln}e_3}{6EI_{bm}}+\frac{q_3L_{bm}L_{cln}e_1{}^2}{3EI_{bm}}+\frac{q_3L_{bm}L_{cln}e_1e_2}{3EI_{bm}}$$

$$+\frac{q_3L_{bm}L_{cln}e_1e_3}{3EI_{bm}}+\frac{q_3L_{bm}L_{cln}e_1e_2}{3EI_{bm}}+\frac{q_3L_{bm}L_{cln}e_2{}^2}{3EI_{bm}}+\frac{q_3L_{bm}L_{cln}e_2e_3}{3EI_{bm}}+\frac{q_3L_{bm}L_{cln}e_1e_3}{3EI_{bm}}+\frac{q_3L_{bm}L_{cln}e_2e_3}{3EI_{bm}}+\frac{q_3L_{bm}L_{cln}e_3{}^2}{3EI_{bm}}$$

The third part:

$$\left[\left(q_1+q_2\right)\left(\tfrac{1}{2}L_{bm}+2e_1+2e_2+e_3\right)\frac{e_3}{L_{cln}}+q_3\left(\tfrac{1}{2}L_{bm}+e_1+e_2+e_3\right)\frac{e_1+e_2+e_3}{L_{cln}}\right]\frac{L_{cln}{}^3}{3EI_{cln,3}}$$

$$=\frac{q_1L_{bm}L_{cln}{}^2e_3}{6EI_{cln,3}}+\frac{2q_1L_{cln}{}^2e_3e_1}{3EI_{cln,3}}+\frac{2q_1L_{cln}{}^2e_3e_2}{3EI_{cln,3}}+\frac{q_1L_{cln}{}^2e_3{}^2}{3EI_{cln,3}}+\frac{q_2L_{bm}L_{cln}{}^2e_3}{6EI_{cln,3}}+\frac{2q_2L_{cln}{}^2e_3e_1}{3EI_{cln,3}}$$

$$+\frac{2q_2L_{cln}{}^2e_3e_2}{3EI_{cln,3}}+\frac{q_2L_{cln}{}^2e_3{}^2}{3EI_{cln,3}}+\frac{q_3L_{bm}L_{cln}{}^2e_1}{6EI_{cln,3}}+\frac{q_3L_{bm}L_{cln}{}^2e_2}{6EI_{cln,3}}+\frac{q_3L_{bm}L_{cln}{}^2e_3}{6EI_{cln,3}}+\frac{q_3L_{cln}{}^2e_1{}^2}{3EI_{cln,3}}+\frac{q_3L_{cln}{}^2e_1e_2}{3EI_{cln,3}}$$

$$+\frac{q_3L_{cln}{}^2e_1e_3}{3EI_{cln,3}}+\frac{q_3L_{cln}{}^2e_1e_2}{3EI_{cln,3}}+\frac{q_3L_{cln}{}^2e_2{}^2}{3EI_{cln,3}}+\frac{q_3L_{cln}{}^2e_2e_3}{3EI_{cln,3}}+\frac{q_3L_{cln}{}^2e_1e_3}{3EI_{cln,3}}+\frac{q_3L_{cln}{}^2e_2e_3}{3EI_{cln,3}}+\frac{q_3L_{cln}{}^2e_3{}^2}{3EI_{cln,3}}$$

The three expressions which are written out can be used in the formula. This results in the following formula.

$$-H_{A,3}\left(\frac{L_{bm}L_{cln}{}^2}{2EI_{bm}}+\frac{L_{cln}{}^3}{3EI_{cln,3}}\right)=$$

$$-\frac{q_3L_{bm}{}^3L_{cln}}{24EI_{bm}}-\frac{q_1L_{bm}{}^2L_{cln}e_3}{12EI_{bm}}+\frac{q_1L_{bm}L_{cln}e_1e_3}{3EI_{bm}}+\frac{q_1L_{bm}L_{cln}e_2e_3}{3EI_{bm}}+\frac{q_1L_{bm}L_{cln}e_3{}^2}{6EI_{bm}}-\frac{q_2L_{bm}{}^2L_{cln}e_3}{12EI_{bm}}+\frac{q_2L_{bm}L_{cln}e_1e_3}{3EI_{bm}}$$

$$+\frac{q_2L_{bm}L_{cln}e_2e_3}{3EI_{bm}}+\frac{q_2L_{bm}L_{cln}e_3{}^2}{6EI_{bm}}-\frac{q_3L_{bm}{}^2L_{cln}e_1}{12EI_{bm}}-\frac{q_3L_{bm}{}^2L_{cln}e_2}{12EI_{bm}}-\frac{q_3L_{bm}{}^2L_{cln}e_3}{12EI_{bm}}+\frac{q_3L_{bm}L_{cln}e_1{}^2}{6EI_{bm}}+\frac{q_3L_{bm}L_{cln}e_1e_2}{6EI_{bm}}$$

$$+\frac{q_3L_{bm}L_{cln}e_1e_3}{6EI_{bm}}+\frac{q_3L_{bm}L_{cln}e_1e_2}{6EI_{bm}}+\frac{q_3L_{bm}L_{cln}e_2{}^2}{6EI_{bm}}+\frac{q_3L_{bm}L_{cln}e_2e_3}{6EI_{bm}}+\frac{q_3L_{bm}L_{cln}e_1e_3}{6EI_{bm}}+\frac{q_3L_{bm}L_{cln}e_2e_3}{6EI_{bm}}+\frac{q_3L_{bm}L_{cln}e_3{}^2}{6EI_{bm}}$$

$$+\frac{q_1L_{bm}{}^2L_{cln}e_3}{6EI_{bm}}+\frac{2q_1L_{bm}L_{cln}e_1e_3}{3EI_{bm}}+\frac{2q_1L_{bm}L_{cln}e_2e_3}{3EI_{bm}}+\frac{q_1L_{bm}L_{cln}e_3{}^2}{3EI_{bm}}+\frac{q_2L_{bm}{}^2L_{cln}e_3}{6EI_{bm}}+\frac{2q_2L_{bm}L_{cln}e_1e_3}{3EI_{bm}}$$

$$+\frac{2q_2L_{bm}L_{cln}e_2e_3}{3EI_{bm}}+\frac{q_2L_{bm}L_{cln}e_3{}^2}{3EI_{bm}}+\frac{q_3L_{bm}{}^2L_{cln}e_1}{6EI_{bm}}+\frac{q_3L_{bm}{}^2L_{cln}e_2}{6EI_{bm}}+\frac{q_3L_{bm}{}^2L_{cln}e_3}{6EI_{bm}}+\frac{q_3L_{bm}L_{cln}e_1{}^2}{3EI_{bm}}+\frac{q_3L_{bm}L_{cln}e_1e_2}{3EI_{bm}}$$

$$+\frac{q_3L_{bm}L_{cln}e_1e_3}{3EI_{bm}}+\frac{q_3L_{bm}L_{cln}e_1e_2}{3EI_{bm}}+\frac{q_3L_{bm}L_{cln}e_2{}^2}{3EI_{bm}}+\frac{q_3L_{bm}L_{cln}e_2e_3}{3EI_{bm}}+\frac{q_3L_{bm}L_{cln}e_1e_3}{3EI_{bm}}+\frac{q_3L_{bm}L_{cln}e_2e_3}{3EI_{bm}}+\frac{q_3L_{bm}L_{cln}e_3{}^2}{3EI_{bm}}$$

$$+\frac{q_1L_{bm}L_{cln}{}^2e_3}{6EI_{cln,3}}+\frac{2q_1L_{cln}{}^2e_3e_1}{3EI_{cln,3}}+\frac{2q_1L_{cln}{}^2e_3e_2}{3EI_{cln,3}}+\frac{q_1L_{cln}{}^2e_3{}^2}{3EI_{cln,3}}+\frac{q_2L_{bm}L_{cln}{}^2e_3}{6EI_{cln,3}}+\frac{2q_2L_{cln}{}^2e_3e_1}{3EI_{cln,3}}$$

$$+\frac{2q_2L_{cln}{}^2e_3e_2}{3EI_{cln,3}}+\frac{q_2L_{cln}{}^2e_3{}^2}{3EI_{cln,3}}+\frac{q_3L_{bm}L_{cln}{}^2e_1}{6EI_{cln,3}}+\frac{q_3L_{bm}L_{cln}{}^2e_2}{6EI_{cln,3}}+\frac{q_3L_{bm}L_{cln}{}^2e_3}{6EI_{cln,3}}+\frac{q_3L_{cln}{}^2e_1{}^2}{3EI_{cln,3}}+\frac{q_3L_{cln}{}^2e_1e_2}{3EI_{cln,3}}$$

$$+\frac{q_3L_{cln}{}^2e_1e_3}{3EI_{cln,3}}+\frac{q_3L_{cln}{}^2e_1e_2}{3EI_{cln,3}}+\frac{q_3L_{cln}{}^2e_2{}^2}{3EI_{cln,3}}+\frac{q_3L_{cln}{}^2e_2e_3}{3EI_{cln,3}}+\frac{q_3L_{cln}{}^2e_1e_3}{3EI_{cln,3}}+\frac{q_3L_{cln}{}^2e_2e_3}{3EI_{cln,3}}+\frac{q_3L_{cln}{}^2e_3{}^2}{3EI_{cln,3}}-e_3$$

Some expressions are the same and can be combined together.

$$-H_{A,3}\left(\frac{L_{bm}L_{cln}{}^2}{2EI_{bm}}+\frac{L_{cln}{}^3}{3EI_{cln,3}}\right)=-\frac{q_3L_{bm}{}^3L_{cln}}{24EI_{bm}}+\frac{q_1L_{bm}{}^2L_{cln}e_3}{12EI_{bm}}+\frac{q_1L_{bm}L_{cln}e_1e_3}{EI_{bm}}+\frac{q_1L_{bm}L_{cln}e_2e_3}{EI_{bm}}+\frac{q_1L_{bm}L_{cln}e_3{}^2}{2EI_{bm}}$$

$$+\frac{q_2L_{bm}{}^2L_{cln}e_3}{12EI_{bm}}+\frac{q_2L_{bm}L_{cln}e_1e_3}{EI_{bm}}+\frac{q_2L_{bm}L_{cln}e_2e_3}{EI_{bm}}+\frac{q_2L_{bm}L_{cln}e_3{}^2}{2EI_{bm}}+\frac{q_3L_{bm}{}^2L_{cln}e_1}{12EI_{bm}}+\frac{q_3L_{bm}{}^2L_{cln}e_2}{12EI_{bm}}$$

$$+\frac{q_3L_{bm}{}^2L_{cln}e_3}{12EI_{bm}}+\frac{q_3L_{bm}L_{cln}e_1{}^2}{2EI_{bm}}+\frac{q_3L_{bm}L_{cln}e_1e_2}{EI_{bm}}+\frac{q_3L_{bm}L_{cln}e_1e_3}{EI_{bm}}+\frac{q_3L_{bm}L_{cln}e_2{}^2}{2EI_{bm}}+\frac{q_3L_{bm}L_{cln}e_2e_3}{EI_{bm}}$$

$$+\frac{q_3L_{bm}L_{cln}e_3{}^2}{2EI_{bm}}+\frac{q_1L_{bm}L_{cln}{}^2e_3}{6EI_{cln,3}}+\frac{2q_1L_{cln}{}^2e_3e_1}{3EI_{cln,3}}+\frac{2q_1L_{cln}{}^2e_3e_2}{3EI_{cln,3}}+\frac{q_1L_{cln}{}^2e_3{}^2}{3EI_{cln,3}}+\frac{q_2L_{bm}L_{cln}{}^2e_3}{6EI_{cln,3}}+\frac{2q_2L_{cln}{}^2e_3e_1}{3EI_{cln,3}}$$

$$+\frac{2q_2L_{cln}{}^2e_3e_2}{3EI_{cln,3}}+\frac{q_2L_{cln}{}^2e_3{}^2}{3EI_{cln,3}}+\frac{q_3L_{bm}L_{cln}{}^2e_1}{6EI_{cln,3}}+\frac{q_3L_{bm}L_{cln}{}^2e_2}{6EI_{cln,3}}+\frac{q_3L_{bm}L_{cln}{}^2e_3}{6EI_{cln,3}}+\frac{q_3L_{cln}{}^2e_1{}^2}{3EI_{cln,3}}$$

$$+\frac{2q_3L_{cln}{}^2e_1e_2}{3EI_{cln,3}}+\frac{2q_3L_{cln}{}^2e_1e_3}{3EI_{cln,3}}+\frac{q_3L_{cln}{}^2e_2{}^2}{3EI_{cln,3}}+\frac{2q_3L_{cln}{}^2e_2e_3}{3EI_{cln,3}}+\frac{q_3L_{cln}{}^2e_3{}^2}{3EI_{cln,3}}-e_3$$

Make every where the same denominator.

$$-H_{A,3}\left(\frac{12L_{bm}L_{cln}^{2}EI_{cln,3}+8L_{cln}^{3}EI_{bm}}{24EI_{bm}EI_{cln,3}}\right)=-\frac{q_3L_{bm}^{3}L_{cln}EI_{cln,3}}{24EI_{bm}EI_{cln,3}}+\frac{2q_1L_{bm}^{2}L_{cln}e_3EI_{cln,3}}{24EI_{bm}EI_{cln,3}}+\frac{24q_1L_{bm}L_{cln}e_1e_3EI_{cln,3}}{24EI_{bm}EI_{cln,3}}$$

$$+\frac{24q_1L_{bm}L_{cln}e_2e_3EI_{cln,3}}{24EI_{bm}EI_{cln,3}}+\frac{12q_1L_{bm}L_{cln}e_3^{2}EI_{cln,3}}{24EI_{bm}EI_{cln,3}}+\frac{2q_2L_{bm}^{2}L_{cln}e_3EI_{cln,3}}{24EI_{bm}EI_{cln,3}}+\frac{24q_2L_{bm}L_{cln}e_1e_3EI_{cln,3}}{24EI_{bm}EI_{cln,3}}$$

$$+\frac{24q_2L_{bm}L_{cln}e_2e_3EI_{cln,3}}{24EI_{bm}EI_{cln,3}}+\frac{12q_2L_{bm}L_{cln}e_3^{2}EI_{cln,3}}{24EI_{bm}EI_{cln,3}}+\frac{2q_3L_{bm}^{2}L_{cln}e_1EI_{cln,3}}{24EI_{bm}EI_{cln,3}}+\frac{2q_3L_{bm}^{2}L_{cln}e_2EI_{cln,3}}{24EI_{bm}EI_{cln,3}}$$

$$+\frac{2q_3L_{bm}^{2}L_{cln}e_3EI_{cln,3}}{24EI_{bm}EI_{cln,3}}+\frac{12q_3L_{bm}L_{cln}e_1^{2}EI_{cln,3}}{24EI_{bm}EI_{cln,3}}+\frac{24q_3L_{bm}L_{cln}e_1e_2EI_{cln,3}}{24EI_{bm}EI_{cln,3}}+\frac{24q_3L_{bm}L_{cln}e_1e_3EI_{cln,3}}{24EI_{bm}EI_{cln,3}}$$

$$+\frac{12q_3L_{bm}L_{cln}e_2^{2}EI_{cln,3}}{24EI_{bm}EI_{cln,3}}+\frac{24q_3L_{bm}L_{cln}e_2e_3EI_{cln,3}}{24EI_{bm}EI_{cln,3}}+\frac{12q_3L_{bm}L_{cln}e_3^{2}EI_{cln,3}}{24EI_{bm}EI_{cln,3}}+\frac{4q_1L_{bm}L_{cln}^{2}e_3EI_{bm}}{24EI_{bm}EI_{cln,3}}$$

$$+\frac{16q_1L_{cln}^{2}e_3e_1EI_{bm}}{24EI_{bm}EI_{cln,3}}+\frac{16q_1L_{cln}^{2}e_3e_2EI_{bm}}{24EI_{bm}EI_{cln,3}}+\frac{8q_1L_{cln}^{2}e_3^{2}EI_{bm}}{24EI_{bm}EI_{cln,3}}+\frac{4q_2L_{bm}L_{cln}^{2}e_3EI_{bm}}{24EI_{bm}EI_{cln,3}}$$

$$+\frac{16q_2L_{cln}^{2}e_3e_1EI_{bm}}{24EI_{bm}EI_{cln,3}}+\frac{16q_2L_{cln}^{2}e_3e_2EI_{bm}}{24EI_{bm}EI_{cln,3}}+\frac{8q_2L_{cln}^{2}e_3^{2}EI_{bm}}{24EI_{bm}EI_{cln,3}}+\frac{4q_3L_{bm}L_{cln}^{2}e_1EI_{bm}}{24EI_{bm}EI_{cln,3}}$$

$$+\frac{4q_3L_{bm}L_{cln}^{2}e_2EI_{bm}}{24EI_{bm}EI_{cln,3}}+\frac{4q_3L_{bm}L_{cln}^{2}e_3EI_{bm}}{24EI_{bm}EI_{cln,3}}+\frac{8q_3L_{cln}^{2}e_1^{2}EI_{bm}}{24EI_{bm}EI_{cln,3}}+\frac{16q_3L_{cln}^{2}e_1e_2EI_{bm}}{24EI_{bm}EI_{cln,3}}$$

$$+\frac{16q_3L_{cln}^{2}e_1e_3EI_{bm}}{24EI_{bm}EI_{cln,3}}+\frac{8q_3L_{cln}^{2}e_2^{2}EI_{bm}}{24EI_{bm}EI_{cln,3}}+\frac{16q_3L_{cln}^{2}e_2e_3EI_{bm}}{24EI_{bm}EI_{cln,3}}+\frac{8q_3L_{cln}^{2}e_3^{2}EI_{bm}}{24EI_{bm}EI_{cln,3}}-\frac{24e_3EI_{bm}EI_{cln,3}}{24EI_{bm}EI_{cln,3}}$$

All denominators are the same and can be neglected. After neglecting the denominators, it is possible to make a formula of $H_{A,3}$.

$$H_{A,3}=\frac{\left(\begin{array}{l}q_3L_{bm}^{3}L_{cln}EI_{cln,3}-2q_1L_{bm}^{2}L_{cln}e_3EI_{cln,3}-24q_1L_{bm}L_{cln}e_1e_3EI_{cln,3}-24q_1L_{bm}L_{cln}e_2e_3EI_{cln,3}\\-12q_1L_{bm}L_{cln}e_3^{2}EI_{cln,3}-2q_2L_{bm}^{2}L_{cln}e_3EI_{cln,3}-24q_2L_{bm}L_{cln}e_1e_3EI_{cln,3}-24q_2L_{bm}L_{cln}e_2e_3EI_{cln,3}\\-12q_2L_{bm}L_{cln}e_3^{2}EI_{cln,3}-2q_3L_{bm}^{2}L_{cln}e_1EI_{cln,3}-2q_3L_{bm}^{2}L_{cln}e_2EI_{cln,3}-2q_3L_{bm}^{2}L_{cln}e_3EI_{cln,3}\\-12q_3L_{bm}L_{cln}e_1^{2}EI_{cln,3}-24q_3L_{bm}L_{cln}e_1e_2EI_{cln,3}-24q_3L_{bm}L_{cln}e_1e_3EI_{cln,3}-12q_3L_{bm}L_{cln}e_2^{2}EI_{cln,3}\\-24q_3L_{bm}L_{cln}e_2e_3EI_{cln,3}-12q_3L_{bm}L_{cln}e_3^{2}EI_{cln,3}-4q_1L_{bm}L_{cln}^{2}e_3EI_{bm}-16q_1L_{cln}^{2}e_3e_1EI_{bm}\\-16q_1L_{cln}^{2}e_3e_2EI_{bm}-8q_1L_{cln}^{2}e_3^{2}EI_{bm}-4q_2L_{bm}L_{cln}^{2}e_3EI_{bm}-16q_2L_{cln}^{2}e_3e_1EI_{bm}-16q_2L_{cln}^{2}e_3e_2EI_{bm}\\-8q_2L_{cln}^{2}e_3^{2}EI_{bm}-4q_3L_{bm}L_{cln}^{2}e_1EI_{bm}-4q_3L_{bm}L_{cln}^{2}e_2EI_{bm}-4q_3L_{bm}L_{cln}^{2}e_3EI_{bm}-8q_3L_{cln}^{2}e_1^{2}EI_{bm}\\-16q_3L_{cln}^{2}e_1e_2EI_{bm}-16q_3L_{cln}^{2}e_1e_3EI_{bm}-8q_3L_{cln}^{2}e_2^{2}EI_{bm}-16q_3L_{cln}^{2}e_2e_3EI_{bm}-8q_3L_{cln}^{2}e_3^{2}EI_{bm}\\+24e_3EI_{bm}EI_{cln,3}\end{array}\right)}{12L_{bm}L_{cln}^{2}EI_{cln,3}+8L_{cln}^{3}EI_{bm}}$$

There are two formulas for the difference in horizontal reaction force. These formulas can be compared together. The following formula is created:

$$\frac{\left(\begin{array}{l} q_3 L_{bm}{}^3 L_{cln} EI_{cln,1} + 2q_1 L_{bm}{}^2 L_{cln} e_3 EI_{cln,1} - 24q_1 L_{bm} L_{cln} e_1 e_3 EI_{cln,1} - 24q_1 L_{bm} L_{cln} e_2 e_3 EI_{cln,1} \\ -12q_1 L_{bm} L_{cln} e_3{}^2 EI_{cln,1} + 2q_2 L_{bm}{}^2 L_{cln} e_3 EI_{cln,1} - 24q_2 L_{bm} L_{cln} e_1 e_3 EI_{cln,1} - 24q_2 L_{bm} L_{cln} e_2 e_3 EI_{cln,1} \\ -12q_2 L_{bm} L_{cln} e_3{}^2 EI_{cln,1} + 2q_3 L_{bm}{}^2 L_{cln} e_1 EI_{cln,1} + 2q_3 L_{bm}{}^2 L_{cln} e_2 EI_{cln,1} + 2q_3 L_{bm}{}^2 L_{cln} e_3 EI_{cln,1} \\ -12q_3 L_{bm} L_{cln} e_1{}^2 EI_{cln,1} - 24q_3 L_{bm} L_{cln} e_1 e_2 EI_{cln,1} - 24q_3 L_{bm} L_{cln} e_1 e_3 EI_{cln,1} - 12q_3 L_{bm} L_{cln} e_2{}^2 EI_{cln,1} \\ -24q_3 L_{bm} L_{cln} e_2 e_3 EI_{cln,1} - 12q_3 L_{bm} L_{cln} e_3{}^2 EI_{cln,1} + 4q_1 L_{bm} L_{cln}{}^2 e_3 EI_{bm} - 16q_1 L_{cln}{}^2 e_1 e_3 EI_{bm} \\ -16q_1 L_{cln}{}^2 e_2 e_3 EI_{bm} - 8q_1 L_{cln}{}^2 e_3{}^2 EI_{bm} + 4q_2 L_{bm} L_{cln}{}^2 e_3 EI_{bm} - 16q_2 L_{cln}{}^2 e_1 e_3 EI_{bm} - 16q_2 L_{cln}{}^2 e_2 e_3 EI_{bm} \\ -8q_2 L_{cln}{}^2 e_3{}^2 EI_{bm} + 4q_3 L_{bm} L_{cln}{}^2 e_1 EI_{bm} + 4q_3 L_{bm} L_{cln}{}^2 e_2 EI_{bm} + 4q_3 L_{bm} L_{cln}{}^2 e_3 EI_{bm} - 8q_3 L_{cln}{}^2 e_1{}^2 EI_{bm} \\ -16q_3 L_{cln}{}^2 e_1 e_2 EI_{bm} - 16q_3 L_{cln}{}^2 e_1 e_3 EI_{bm} - 8q_3 L_{cln}{}^2 e_2{}^2 EI_{bm} - 16q_3 L_{cln}{}^2 e_2 e_3 EI_{bm} - 8q_3 L_{cln}{}^2 e_3{}^2 EI_{bm} \\ -24e_3 EI_{cln,1} EI_{bm} \end{array}\right)}{12L_{bm} L_{cln}{}^2 EI_{cln,1} + 8L_{cln}{}^3 EI_{bm}}$$

$$=$$

$$\frac{\left(\begin{array}{l} q_3 L_{bm}{}^3 L_{cln} EI_{cln,3} - 2q_1 L_{bm}{}^2 L_{cln} e_3 EI_{cln,3} - 24q_1 L_{bm} L_{cln} e_1 e_3 EI_{cln,3} - 24q_1 L_{bm} L_{cln} e_2 e_3 EI_{cln,3} \\ -12q_1 L_{bm} L_{cln} e_3{}^2 EI_{cln,3} - 2q_2 L_{bm}{}^2 L_{cln} e_3 EI_{cln,3} - 24q_2 L_{bm} L_{cln} e_1 e_3 EI_{cln,3} - 24q_2 L_{bm} L_{cln} e_2 e_3 EI_{cln,3} \\ -12q_2 L_{bm} L_{cln} e_3{}^2 EI_{cln,3} - 2q_3 L_{bm}{}^2 L_{cln} e_1 EI_{cln,3} - 2q_3 L_{bm}{}^2 L_{cln} e_2 EI_{cln,3} - 2q_3 L_{bm}{}^2 L_{cln} e_3 EI_{cln,3} \\ -12q_3 L_{bm} L_{cln} e_1{}^2 EI_{cln,3} - 24q_3 L_{bm} L_{cln} e_1 e_2 EI_{cln,3} - 24q_3 L_{bm} L_{cln} e_1 e_3 EI_{cln,3} - 12q_3 L_{bm} L_{cln} e_2{}^2 EI_{cln,3} \\ -24q_3 L_{bm} L_{cln} e_2 e_3 EI_{cln,3} - 12q_3 L_{bm} L_{cln} e_3{}^2 EI_{cln,3} - 4q_1 L_{bm} L_{cln}{}^2 e_3 EI_{bm} - 16q_1 L_{cln}{}^2 e_3 e_1 EI_{bm} \\ -16q_1 L_{cln}{}^2 e_3 e_2 EI_{bm} - 8q_1 L_{cln}{}^2 e_3{}^2 EI_{bm} - 4q_2 L_{bm} L_{cln}{}^2 e_3 EI_{bm} - 16q_2 L_{cln}{}^2 e_3 e_1 EI_{bm} - 16q_2 L_{cln}{}^2 e_3 e_2 EI_{bm} \\ -8q_2 L_{cln}{}^2 e_3{}^2 EI_{bm} - 4q_3 L_{bm} L_{cln}{}^2 e_1 EI_{bm} - 4q_3 L_{bm} L_{cln}{}^2 e_2 EI_{bm} - 4q_3 L_{bm} L_{cln}{}^2 e_3 EI_{bm} - 8q_3 L_{cln}{}^2 e_1{}^2 EI_{bm} \\ -16q_3 L_{cln}{}^2 e_1 e_2 EI_{bm} - 16q_3 L_{cln}{}^2 e_1 e_3 EI_{bm} - 8q_3 L_{cln}{}^2 e_2{}^2 EI_{bm} - 16q_3 L_{cln}{}^2 e_2 e_3 EI_{bm} - 8q_3 L_{cln}{}^2 e_3{}^2 EI_{bm} \\ +24e_3 EI_{bm} EI_{cln,3} \end{array}\right)}{12L_{bm} L_{cln}{}^2 EI_{cln,3} + 8L_{cln}{}^3 EI_{bm}}$$

The denominators are not the same. This has been seen before (in the analysis of the second load case in Appendix K.1). The expressions must be multiplied by the both denominators.

$$\left(12L_{bm}L_{cln}{}^{2}EI_{cln,3}+8L_{cln}{}^{3}EI_{bm}\right)$$

$$\left(\begin{array}{l}
q_3L_{bm}{}^{3}L_{cln}EI_{cln,1}+2q_1L_{bm}{}^{2}L_{cln}e_3EI_{cln,1}-24q_1L_{bm}L_{cln}e_1e_3EI_{cln,1}-24q_1L_{bm}L_{cln}e_2e_3EI_{cln,1}\\
-12q_1L_{bm}L_{cln}e_3{}^{2}EI_{cln,1}+2q_2L_{bm}{}^{2}L_{cln}e_3EI_{cln,1}-24q_2L_{bm}L_{cln}e_1e_3EI_{cln,1}-24q_2L_{bm}L_{cln}e_2e_3EI_{cln,1}\\
-12q_2L_{bm}L_{cln}e_3{}^{2}EI_{cln,1}+2q_3L_{bm}{}^{2}L_{cln}e_1EI_{cln,1}+2q_3L_{bm}{}^{2}L_{cln}e_2EI_{cln,1}+2q_3L_{bm}{}^{2}L_{cln}\Delta e_3EI_{cln,1}\\
-12q_3L_{bm}L_{cln}e_1{}^{2}EI_{cln,1}-24q_3L_{bm}L_{cln}e_1e_2EI_{cln,1}-24q_3L_{bm}L_{cln}e_1e_3EI_{cln,1}-12q_3L_{bm}L_{cln}e_2{}^{2}EI_{cln,1}\\
-24q_3L_{bm}L_{cln}e_2e_3EI_{cln,1}-12q_3L_{bm}L_{cln}e_3{}^{2}EI_{cln,1}+4q_1L_{bm}L_{cln}{}^{2}e_3EI_{bm}-16q_1L_{cln}{}^{2}e_1e_3EI_{bm}\\
-16q_1L_{cln}{}^{2}e_2e_3EI_{bm}-8q_1L_{cln}{}^{2}e_3{}^{2}EI_{bm}+4q_2L_{bm}L_{cln}{}^{2}e_3EI_{bm}-16q_2L_{cln}{}^{2}e_1e_3EI_{bm}-16q_2L_{cln}{}^{2}e_2e_3EI_{bm}\\
-8q_2L_{cln}{}^{2}e_3{}^{2}EI_{bm}+4q_3L_{bm}L_{cln}{}^{2}e_1EI_{bm}+4q_3L_{bm}L_{cln}{}^{2}e_2EI_{bm}+4q_3L_{bm}L_{cln}{}^{2}e_3EI_{bm}-8q_3L_{cln}{}^{2}e_1{}^{2}EI_{bm}\\
-16q_3L_{cln}{}^{2}e_1e_2EI_{bm}-16q_3L_{cln}{}^{2}e_1e_3EI_{bm}-8q_3L_{cln}{}^{2}e_2{}^{2}EI_{bm}-16q_3L_{cln}{}^{2}e_2e_3EI_{bm}-8q_3L_{cln}{}^{2}e_3{}^{2}EI_{bm}\\
-24e_3EI_{cln,1}EI_{bm}
\end{array}\right)$$

$$=$$

$$\left(12L_{bm}L_{cln}{}^{2}EI_{cln,1}+8L_{cln}{}^{3}EI_{bm}\right)$$

$$\left(\begin{array}{l}
q_3L_{bm}{}^{3}L_{cln}EI_{cln,3}-2q_1L_{bm}{}^{2}L_{cln}e_3EI_{cln,3}-24q_1L_{bm}L_{cln}e_1e_3EI_{cln,3}-24q_1L_{bm}L_{cln}e_2e_3EI_{cln,3}\\
-12q_1L_{bm}L_{cln}e_3{}^{2}EI_{cln,3}-2q_2L_{bm}{}^{2}L_{cln}e_3EI_{cln,3}-24q_2L_{bm}L_{cln}e_1e_3EI_{cln,3}-24q_2L_{bm}L_{cln}e_2e_3EI_{cln,3}\\
-12q_2L_{bm}L_{cln}e_3{}^{2}EI_{cln,3}-2q_3L_{bm}{}^{2}L_{cln}e_1EI_{cln,3}-2q_3L_{bm}{}^{2}L_{cln}e_2EI_{cln,3}-2q_3L_{bm}{}^{2}L_{cln}e_3EI_{cln,3}\\
-12q_3L_{bm}L_{cln}e_1{}^{2}EI_{cln,3}-24q_3L_{bm}L_{cln}e_1e_2EI_{cln,3}-24q_3L_{bm}L_{cln}e_1e_3EI_{cln,3}-12q_3L_{bm}L_{cln}e_2{}^{2}EI_{cln,3}\\
-24q_3L_{bm}L_{cln}e_2e_3EI_{cln,3}-12q_3L_{bm}L_{cln}e_3{}^{2}EI_{cln,3}-4q_1L_{bm}L_{cln}{}^{2}e_3EI_{bm}-16q_1L_{cln}{}^{2}e_3e_1EI_{bm}\\
-16q_1L_{cln}{}^{2}e_3e_2EI_{bm}-8q_1L_{cln}{}^{2}e_3{}^{2}EI_{bm}-4q_2L_{bm}L_{cln}{}^{2}e_3EI_{bm}-16q_2L_{cln}{}^{2}e_3e_1EI_{bm}-16q_2L_{cln}{}^{2}e_3e_2EI_{bm}\\
-8q_2L_{cln}{}^{2}e_3{}^{2}EI_{bm}-4q_3L_{bm}L_{cln}{}^{2}e_1EI_{bm}-4q_3L_{bm}L_{cln}{}^{2}e_2EI_{bm}-4q_3L_{bm}L_{cln}{}^{2}e_3EI_{bm}-8q_3L_{cln}{}^{2}e_1{}^{2}EI_{bm}\\
-16q_3L_{cln}{}^{2}e_1e_2EI_{bm}-16q_3L_{cln}{}^{2}e_1e_3EI_{bm}-8q_3L_{cln}{}^{2}e_2{}^{2}EI_{bm}-16q_3L_{cln}{}^{2}e_2e_3EI_{bm}-8q_3L_{cln}{}^{2}e_3{}^{2}EI_{bm}\\
+24e_3EI_{bm}EI_{cln,3}
\end{array}\right)$$

Write out the formula.

$$\left(\begin{array}{l}
12q_3L_{bm}{}^4L_{cln}{}^3EI_{cln,1}EI_{cln,3}+24q_1L_{bm}{}^3L_{cln}{}^3e_3EI_{cln,1}EI_{cln,3}-288q_1L_{bm}{}^2L_{cln}{}^3e_1e_3EI_{cln,1}EI_{cln,3}-288q_1L_{bm}{}^2L_{cln}{}^3e_2e_3EI_{cln,1}EI_{cln,3}\\
-144q_1L_{bm}{}^2L_{cln}{}^3e_3{}^2EI_{cln,1}EI_{cln,3}+24q_2L_{bm}{}^3L_{cln}{}^3e_3EI_{cln,1}EI_{cln,3}-288q_2L_{bm}{}^2L_{cln}{}^3e_1e_3EI_{cln,1}EI_{cln,3}-288q_2L_{bm}{}^2L_{cln}{}^3e_2e_3EI_{cln,1}EI_{cln,3}\\
-144q_2L_{bm}{}^2L_{cln}{}^3e_3{}^2EI_{cln,1}EI_{cln,3}+24q_3L_{bm}{}^3L_{cln}{}^3e_1EI_{cln,1}EI_{cln,3}+24q_3L_{bm}{}^3L_{cln}{}^3e_2EI_{cln,1}EI_{cln,3}+24q_3L_{bm}{}^3L_{cln}{}^3e_3EI_{cln,1}EI_{cln,3}\\
-144q_3L_{bm}{}^2L_{cln}{}^3e_1{}^2EI_{cln,1}EI_{cln,3}-288q_3L_{bm}{}^2L_{cln}{}^3e_1e_2EI_{cln,1}EI_{cln,3}-288q_3L_{bm}{}^2L_{cln}{}^3e_1e_3EI_{cln,1}EI_{cln,3}-144q_3L_{bm}{}^2L_{cln}{}^3e_2{}^2EI_{cln,1}EI_{cln,3}\\
-288q_3L_{bm}{}^2L_{cln}{}^3e_2e_3EI_{cln,1}EI_{cln,3}-144q_3L_{bm}{}^2L_{cln}{}^3e_3{}^2EI_{cln,1}EI_{cln,3}+48q_1L_{bm}{}^2L_{cln}{}^4e_3EI_{bm}EI_{cln,3}-192q_1L_{bm}L_{cln}{}^4e_1e_3EI_{bm}EI_{cln,3}\\
-192q_1L_{bm}L_{cln}{}^4e_2e_3EI_{bm}EI_{cln,3}-96q_1L_{bm}L_{cln}{}^4e_3{}^2EI_{bm}EI_{cln,3}+48q_2L_{bm}{}^2L_{cln}{}^4e_3EI_{bm}EI_{cln,3}-192q_2L_{bm}L_{cln}{}^4e_1e_3EI_{bm}EI_{cln,3}\\
-192q_2L_{bm}L_{cln}{}^4e_2e_3EI_{bm}EI_{cln,3}-96q_2L_{bm}L_{cln}{}^4e_3{}^2EI_{bm}EI_{cln,3}+48q_3L_{bm}{}^2L_{cln}{}^4e_1EI_{bm}EI_{cln,3}+48q_3L_{bm}{}^2L_{cln}{}^4e_2EI_{bm}EI_{cln,3}\\
+48q_3L_{bm}{}^2L_{cln}{}^4e_3EI_{bm}EI_{cln,3}-96q_3L_{bm}L_{cln}{}^4e_1{}^2EI_{bm}EI_{cln,3}-192q_3L_{bm}L_{cln}{}^4e_1e_2EI_{bm}EI_{cln,3}-192q_3L_{bm}L_{cln}{}^4e_1e_3EI_{bm}L_{cln}{}^2EI_{cln,3}\\
-96q_3L_{bm}L_{cln}{}^4e_2{}^2EI_{bm}EI_{cln,3}-192q_3L_{bm}L_{cln}{}^4e_2e_3EI_{bm}EI_{cln,3}-96q_3L_{bm}L_{cln}{}^4e_3{}^2EI_{bm}EI_{cln,3}-288L_{bm}L_{cln}{}^2e_3EI_{cln,1}EI_{bm}EI_{cln,3}
\end{array}\right)$$

$$+$$

$$\left(\begin{array}{l}
8q_3L_{bm}{}^3L_{cln}{}^4EI_{bm}EI_{cln,1}+16q_1L_{bm}{}^2L_{cln}{}^4e_3EI_{bm}EI_{cln,1}-192q_1L_{bm}L_{cln}{}^4e_1e_3EI_{bm}EI_{cln,1}-192q_1L_{bm}L_{cln}{}^4e_2e_3EI_{bm}EI_{cln,1}\\
-96q_1L_{bm}L_{cln}{}^4e_3{}^2EI_{bm}EI_{cln,1}+16q_2L_{bm}{}^2L_{cln}{}^4e_3EI_{bm}EI_{cln,1}-192q_2L_{bm}L_{cln}{}^4e_1e_3EI_{bm}EI_{cln,1}-192q_2L_{bm}L_{cln}{}^4e_2e_3EI_{bm}EI_{cln,1}\\
-96q_2L_{bm}L_{cln}{}^4e_3{}^2EI_{bm}EI_{cln,1}+16q_3L_{bm}{}^2L_{cln}{}^4e_1EI_{bm}EI_{cln,1}+16q_3L_{bm}{}^2L_{cln}{}^4e_2EI_{bm}EI_{cln,1}+16q_3L_{bm}{}^2L_{cln}{}^4e_3EI_{bm}EI_{cln,1}\\
-96q_3L_{bm}L_{cln}{}^4e_1{}^2EI_{bm}EI_{cln,1}-192q_3L_{bm}L_{cln}{}^4e_1e_2EI_{bm}EI_{cln,1}-192q_3L_{bm}L_{cln}{}^4e_1e_3EI_{bm}EI_{cln,1}-96q_3L_{bm}L_{cln}{}^4e_2{}^2EI_{bm}EI_{cln,1}\\
-192q_3L_{bm}L_{cln}{}^4e_2e_3EI_{bm}EI_{cln,1}-96q_3L_{bm}L_{cln}{}^4e_3{}^2EI_{bm}EI_{cln,1}+32q_1L_{bm}L_{cln}{}^5e_3\left(EI_{bm}\right)^2-128q_1L_{cln}{}^5e_1e_3\left(EI_{bm}\right)^2\\
-128q_1L_{cln}{}^5e_2e_3\left(EI_{bm}\right)^2-64q_1L_{cln}{}^5e_3{}^2\left(EI_{bm}\right)^2+32q_2L_{bm}L_{cln}{}^5e_3\left(EI_{bm}\right)^2-128q_2L_{cln}{}^5e_1e_3\left(EI_{bm}\right)^2\\
-128q_2L_{cln}{}^5e_2e_3\left(EI_{bm}\right)^2-64q_2L_{cln}{}^5e_3{}^2\left(EI_{bm}\right)^2+32q_3L_{bm}L_{cln}{}^5e_1\left(EI_{bm}\right)^2+32q_3L_{bm}L_{cln}{}^5e_2\left(EI_{bm}\right)^2\\
+32q_3L_{bm}L_{cln}{}^5e_3\left(EI_{bm}\right)^2-64q_3L_{cln}{}^5e_1{}^2\left(EI_{bm}\right)^2-128q_3L_{cln}{}^5e_1e_2\left(EI_{bm}\right)^2-128q_3L_{cln}{}^5e_1e_3\left(EI_{bm}\right)^2\\
-64q_3L_{cln}{}^5e_2{}^2\left(EI_{bm}\right)^2-128q_3L_{cln}{}^5e_2e_3\left(EI_{bm}\right)^2-64q_3L_{cln}{}^5e_3{}^2\left(EI_{bm}\right)^2-192e_3L_{cln}{}^3EI_{cln,1}\left(EI_{bm}\right)^2
\end{array}\right)$$

$$=$$

$$\left(\begin{array}{l}
12q_3L_{bm}{}^4L_{cln}{}^3EI_{cln,1}EI_{cln,3}-24q_1L_{bm}{}^3L_{cln}{}^3e_3EI_{cln,1}EI_{cln,3}-288q_1L_{bm}{}^2L_{cln}{}^3e_1e_3EI_{cln,1}EI_{cln,3}-288q_1L_{bm}{}^2L_{cln}{}^3e_2e_3EI_{cln,1}EI_{cln,3}\\
-144q_1L_{bm}{}^2L_{cln}{}^3e_3{}^2EI_{cln,1}EI_{cln,3}-24q_2L_{bm}{}^3L_{cln}{}^3e_3EI_{cln,1}EI_{cln,3}-288q_2L_{bm}{}^2L_{cln}{}^3e_1e_{3EI_{cln,1}}EI_{cln,3}-288q_2L_{bm}{}^2L_{cln}{}^3e_2e_3EI_{cln,1}EI_{cln,3}\\
-144q_2L_{bm}{}^2L_{cln}{}^3e_3{}^2EI_{cln,1}EI_{cln,3}-24q_3L_{bm}{}^3L_{cln}{}^3e_1EI_{cln,1}EI_{cln,3}-24q_3L_{bm}{}^3L_{cln}{}^3e_2EI_{cln,1}EI_{cln,3}-24q_3L_{bm}{}^3L_{cln}{}^3e_3EI_{cln,1}EI_{cln,3}\\
-144q_3L_{bm}{}^2L_{cln}{}^3e_1{}^2EI_{cln,1}EI_{cln,3}-288q_3L_{bm}{}^2L_{cln}{}^3e_1e_2EI_{cln,1}EI_{cln,3}-288q_3L_{bm}{}^2L_{cln}{}^3e_1e_3EI_{cln,1}EI_{cln,3}-144q_3L_{bm}{}^2L_{cln}{}^3e_2{}^2EI_{cln,1}EI_{cln,3}\\
-288q_3L_{bm}{}^2L_{cln}{}^3e_2e_3EI_{cln,1}EI_{cln,3}-144q_3L_{bm}{}^2L_{cln}{}^3e_3{}^2EI_{cln,1}EI_{cln,3}-48q_1L_{bm}{}^2L_{cln}{}^4e_3EI_{cln,1}EI_{bm}-192q_1L_{bm}L_{cln}{}^4e_3e_1EI_{cln,1}EI_{bm}\\
-192q_1L_{bm}L_{cln}{}^4e_3e_2EI_{cln,1}EI_{bm}-96q_1L_{bm}L_{cln}{}^4e_3{}^2EI_{cln,1}EI_{bm}-48q_2L_{bm}{}^2L_{cln}{}^4e_3EI_{cln,1}EI_{bm}-192q_2L_{bm}L_{cln}{}^4e_3e_1EI_{cln,1}EI_{bm}\\
-192q_2L_{bm}L_{cln}{}^4e_3e_2EI_{cln,1}EI_{bm}-96q_2L_{bm}L_{cln}{}^4e_3{}^2EI_{cln,1}EI_{bm}-48q_3L_{bm}{}^2L_{cln}{}^4e_1EI_{cln,1}EI_{bm}-48q_3L_{bm}{}^2L_{cln}{}^4e_2EI_{cln,1}EI_{bm}\\
-48q_3L_{bm}{}^2L_{cln}{}^4e_3EI_{cln,1}EI_{bm}-96q_3L_{bm}L_{cln}{}^4e_1{}^2EI_{cln,1}EI_{bm}-192q_3L_{bm}L_{cln}{}^4e_1e_2EI_{cln,1}EI_{bm}-192q_3L_{bm}L_{cln}{}^4e_1e_3EI_{cln,1}EI_{bm}\\
-96q_3L_{bm}L_{cln}{}^4e_2{}^2EI_{cln,1}EI_{bm}-192q_3L_{bm}L_{cln}{}^4e_2e_3EI_{cln,1}EI_{bm}-96q_3L_{bm}L_{cln}{}^4e_3{}^2EI_{cln,1}EI_{bm}+288e_3L_{bm}L_{cln}{}^2EI_{bm}EI_{cln,1}EI_{cln,3}
\end{array}\right)$$

$$+$$

$$\left(\begin{array}{l}
8q_3L_{bm}{}^3L_{cln}{}^4EI_{bm}EI_{cln,3}-16q_1L_{bm}{}^2L_{cln}{}^4e_3EI_{bm}EI_{cln,3}-192q_1L_{bm}L_{cln}{}^4e_1e_3EI_{bm}EI_{cln,3}-192q_1L_{bm}L_{cln}{}^4e_2e_3EI_{bm}EI_{cln,3}\\
-96q_1L_{bm}L_{cln}{}^4e_3{}^2EI_{bm}EI_{cln,3}-16q_2L_{bm}{}^2L_{cln}{}^4e_3EI_{bm}EI_{cln,3}-192q_2L_{bm}L_{cln}{}^4e_1e_3EI_{bm}EI_{cln,3}-192q_2L_{bm}L_{cln}{}^4e_2e_3EI_{bm}EI_{cln,3}\\
-96q_2L_{bm}L_{cln}{}^4e_3{}^2EI_{bm}EI_{cln,3}-16q_3L_{bm}{}^2L_{cln}{}^4e_1EI_{bm}EI_{cln,3}-16q_3L_{bm}{}^2L_{cln}{}^4e_2EI_{bm}EI_{cln,3}-16q_3L_{bm}{}^2L_{cln}{}^4e_3EI_{bm}EI_{cln,3}\\
-96q_3L_{bm}L_{cln}{}^4e_1{}^2EI_{bm}EI_{cln,3}-192q_3L_{bm}L_{cln}{}^4e_1e_2EI_{bm}EI_{cln,3}-192q_3L_{bm}L_{cln}{}^4e_1e_3EI_{bm}EI_{cln,3}-96q_3L_{bm}L_{cln}{}^4e_2{}^2EI_{bm}EI_{cln,3}\\
-192q_3L_{bm}L_{cln}{}^4e_2e_3EI_{bm}EI_{cln,3}-96q_3L_{bm}L_{cln}{}^4e_3{}^2EI_{bm}EI_{cln,3}-32q_1L_{bm}L_{cln}{}^5e_3\left(EI_{bm}\right)^2-128q_1L_{cln}{}^5e_3e_1\left(EI_{bm}\right)^2\\
-128q_1L_{cln}{}^5e_3e_2\left(EI_{bm}\right)^2-64q_1L_{cln}{}^5e_3{}^2\left(EI_{bm}\right)^2-16q_2L_{bm}L_{cln}{}^5e_3\left(EI_{bm}\right)^2-128q_2L_{cln}{}^5e_3e_1\left(EI_{bm}\right)^2\\
-128q_2L_{cln}{}^5e_3e_2\left(EI_{bm}\right)^2-64q_2L_{cln}{}^5e_3{}^2\left(EI_{bm}\right)^2-32q_3L_{bm}L_{cln}{}^5e_1\left(EI_{bm}\right)^2-32q_3L_{bm}L_{cln}{}^5e_2\left(EI_{bm}\right)^2\\
-32q_3L_{bm}L_{cln}{}^5e_3\left(EI_{bm}\right)^2-64q_3L_{cln}{}^5e_1{}^2\left(EI_{bm}\right)^2-128q_3L_{cln}{}^5e_1e_2\left(EI_{bm}\right)^2-128q_3L_{cln}{}^5e_1e_3\left(EI_{bm}\right)^2\\
-64q_3L_{cln}{}^5e_2{}^2\left(EI_{bm}\right)^2-128q_3L_{cln}{}^5e_2e_3\left(EI_{bm}\right)^2-64q_3L_{cln}{}^5e_3{}^2\left(EI_{bm}\right)^2+192e_3L_{cln}{}^3\left(EI_{bm}\right)^2EI_{cln,3}
\end{array}\right)$$

The first step to simplify this formula is to find the same expressions on both sides of the equation. The same expressions are neglected or combined together.

$$\left(\begin{array}{l}
48q_1L_{bm}{}^3L_{cln}{}^3e_3EI_{cln,1}EI_{cln,3}+48q_2L_{bm}{}^3L_{cln}{}^3e_3EI_{cln,1}EI_{cln,3}\\
+48q_3L_{bm}{}^3L_{cln}{}^3e_1EI_{cln,1}EI_{cln,3}+48q_3L_{bm}{}^3L_{cln}{}^3e_2EI_{cln,1}EI_{cln,3}+48q_3L_{bm}{}^3L_{cln}{}^3e_3EI_{cln,1}EI_{cln,3}\\
+48q_1L_{bm}{}^2L_{cln}{}^4e_3EI_{bm}EI_{cln,3}-192q_1L_{bm}L_{cln}{}^4e_1e_3EI_{bm}EI_{cln,3}\\
-192q_1L_{bm}L_{cln}{}^4e_2e_3EI_{bm}EI_{cln,3}-96q_1L_{bm}L_{cln}{}^4e_3{}^2EI_{bm}EI_{cln,3}+48q_2L_{bm}{}^2L_{cln}{}^4e_3EI_{bm}EI_{cln,3}-192q_2L_{bm}L_{cln}{}^4e_1e_3EI_{bm}EI_{cln,3}\\
-192q_2L_{bm}L_{cln}{}^4e_2e_3EI_{bm}EI_{cln,3}-96q_2L_{bm}L_{cln}{}^4e_3{}^2EI_{bm}EI_{cln,3}+48q_3L_{bm}{}^2L_{cln}{}^4e_1EI_{bm}EI_{cln,3}+48q_3L_{bm}{}^2L_{cln}{}^4e_2EI_{bm}EI_{cln,3}\\
+48q_3L_{bm}{}^2L_{cln}{}^4e_3EI_{bm}EI_{cln,3}-96q_3L_{bm}L_{cln}{}^4e_1{}^2EI_{bm}EI_{cln,3}-192q_3L_{bm}L_{cln}{}^4e_1e_2EI_{bm}EI_{cln,3}-192q_3L_{bm}L_{cln}{}^4e_1e_3EI_{bm}L_{cln}{}^2EI_{cln,3}\\
-96q_3L_{bm}L_{cln}{}^4e_2{}^2EI_{bm}EI_{cln,3}-192q_3L_{bm}L_{cln}{}^4e_2e_3EI_{bm}EI_{cln,3}-96q_3L_{bm}L_{cln}{}^4e_3{}^2EI_{bm}EI_{cln,3}-576L_{bm}L_{cln}{}^2e_3EI_{cln,1}EI_{bm}EI_{cln,3}
\end{array}\right)$$

$$+$$

$$\left(\begin{array}{l}
8q_3L_{bm}{}^3L_{cln}{}^4EI_{bm}EI_{cln,1}+16q_1L_{bm}{}^2L_{cln}{}^4e_3EI_{bm}EI_{cln,1}-192q_1L_{bm}L_{cln}{}^4e_1e_3EI_{bm}EI_{cln,1}-192q_1L_{bm}L_{cln}{}^4e_2e_3EI_{bm}EI_{cln,1}\\
-96q_1L_{bm}L_{cln}{}^4e_3{}^2EI_{bm}EI_{cln,1}+16q_2L_{bm}{}^2L_{cln}{}^4e_3EI_{bm}EI_{cln,1}-192q_2L_{bm}L_{cln}{}^4e_1e_3EI_{bm}EI_{cln,1}-192q_2L_{bm}L_{cln}{}^4e_2e_3EI_{bm}EI_{cln,1}\\
-96q_2L_{bm}L_{cln}{}^4e_3{}^2EI_{bm}EI_{cln,1}+16q_3L_{bm}{}^2L_{cln}{}^4e_1EI_{bm}EI_{cln,1}+16q_3L_{bm}{}^2L_{cln}{}^4e_2EI_{bm}EI_{cln,1}+16q_3L_{bm}{}^2L_{cln}{}^4e_3EI_{bm}EI_{cln,1}\\
-96q_3L_{bm}L_{cln}{}^4e_1{}^2EI_{bm}EI_{cln,1}-192q_3L_{bm}L_{cln}{}^4e_1e_2EI_{bm}EI_{cln,1}-192q_3L_{bm}L_{cln}{}^4e_1e_3EI_{bm}EI_{cln,1}-96q_3L_{bm}L_{cln}{}^4e_2{}^2EI_{bm}EI_{cln,1}\\
-192q_3L_{bm}L_{cln}{}^4e_2e_3EI_{bm}EI_{cln,1}-96q_3L_{bm}L_{cln}{}^4e_3{}^2EI_{bm}EI_{cln,1}\\
+64q_1L_{bm}L_{cln}{}^5e_3\left(EI_{bm}\right)^2+64q_2L_{bm}L_{cln}{}^5e_3\left(EI_{bm}\right)^2+64q_3L_{bm}L_{cln}{}^5e_1\left(EI_{bm}\right)^2+64q_3L_{bm}L_{cln}{}^5e_2\left(EI_{bm}\right)^2\\
+64\Delta q_3L_{bm}L_{cln}{}^5e_3\left(EI_{bm}\right)^2-192L_{cln}{}^3e_3EI_{cln,1}\left(EI_{bm}\right)^2
\end{array}\right)$$

$$=$$

$$\left(\begin{array}{l}
-48q_1L_{bm}{}^2L_{cln}{}^4e_3EI_{cln,1}EI_{bm}-192q_1L_{bm}L_{cln}{}^4e_3e_1EI_{cln,1}EI_{bm}\\
-192q_1L_{bm}L_{cln}{}^4e_3e_2EI_{cln,1}EI_{bm}-96q_1L_{bm}L_{cln}{}^4e_3{}^2EI_{cln,1}EI_{bm}-48q_2L_{bm}{}^2L_{cln}{}^4e_3EI_{cln,1}EI_{bm}-192q_2L_{bm}L_{cln}{}^4e_3e_1EI_{cln,1}EI_{bm}\\
-192q_2L_{bm}L_{cln}{}^4e_3e_2EI_{cln,1}EI_{bm}-96q_2L_{bm}L_{cln}{}^4e_3{}^2EI_{cln,1}EI_{bm}-48q_3L_{bm}{}^2L_{cln}{}^4e_1EI_{cln,1}EI_{bm}-48q_3L_{bm}{}^2L_{cln}{}^4e_2EI_{cln,1}EI_{bm}\\
-48q_3L_{bm}{}^2L_{cln}{}^4e_3EI_{cln,1}EI_{bm}-96q_3L_{bm}L_{cln}{}^4e_1{}^2EI_{cln,1}EI_{bm}-192q_3L_{bm}L_{cln}{}^4e_1e_2EI_{cln,1}EI_{bm}-192q_3L_{bm}L_{cln}{}^4e_1e_3EI_{cln,1}EI_{bm}\\
-96q_3L_{bm}L_{cln}{}^4e_2{}^2EI_{cln,1}EI_{bm}-192q_3L_{bm}L_{cln}{}^4e_2e_3EI_{cln,1}EI_{bm}-96q_3L_{bm}L_{cln}{}^4e_3{}^2EI_{cln,1}EI_{bm}
\end{array}\right)$$

$$+$$

$$\left(\begin{array}{l}
8q_3L_{bm}{}^3L_{cln}{}^4EI_{bm}EI_{cln,3}-16q_1L_{bm}{}^2L_{cln}{}^4e_3EI_{bm}EI_{cln,3}-192q_1L_{bm}L_{cln}{}^4e_1e_3EI_{bm}EI_{cln,3}-192q_1L_{bm}L_{cln}{}^4e_2e_3EI_{bm}EI_{cln,3}\\
-96q_1L_{bm}L_{cln}{}^4e_3{}^2EI_{bm}EI_{cln,3}-16q_2L_{bm}{}^2L_{cln}{}^4e_3EI_{bm}EI_{cln,3}-192q_2L_{bm}L_{cln}{}^4e_1e_3EI_{bm}EI_{cln,3}-192q_2L_{bm}L_{cln}{}^4e_2e_3EI_{bm}EI_{cln,3}\\
-96q_2L_{bm}L_{cln}{}^4e_3{}^2EI_{bm}EI_{cln,3}-16q_3L_{bm}{}^2L_{cln}{}^4e_1EI_{bm}EI_{cln,3}-16q_3L_{bm}{}^2L_{cln}{}^4e_2EI_{bm}EI_{cln,3}-16q_3L_{bm}{}^2L_{cln}{}^4e_3EI_{bm}EI_{cln,3}\\
-96q_3L_{bm}L_{cln}{}^4e_1{}^2EI_{bm}EI_{cln,3}-192q_3L_{bm}L_{cln}{}^4e_1e_2EI_{bm}EI_{cln,3}-192q_3L_{bm}L_{cln}{}^4e_1e_3EI_{bm}EI_{cln,3}-96q_3L_{bm}L_{cln}{}^4e_2{}^2EI_{bm}EI_{cln,3}\\
-192q_3L_{bm}L_{cln}{}^4e_2e_3EI_{bm}EI_{cln,3}-96q_3L_{bm}L_{cln}{}^4e_3{}^2EI_{bm}EI_{cln,3}+192e_3L_{cln}{}^3\left(EI_{bm}\right)^2EI_{cln,3}
\end{array}\right)$$

The second step to simplify the formula is to combine the same expressions. All expressions of the right part of the formula are inserted in the left part of the formula.

$$
\begin{pmatrix}
48q_1L_{bm}{}^3L_{cln}{}^3e_3EI_{cln,1}EI_{cln,3}+48q_2L_{bm}{}^3L_{cln}{}^3e_3EI_{cln,1}EI_{cln,3}+48q_3L_{bm}{}^3L_{cln}{}^3e_1EI_{cln,1}EI_{cln,3} \\
+48q_3L_{bm}{}^3L_{cln}{}^3e_2EI_{cln,1}EI_{cln,3}+48q_3L_{bm}{}^3L_{cln}{}^3e_3EI_{cln,1}EI_{cln,3}+48q_1L_{bm}{}^2L_{cln}{}^4e_3EI_{bm}\left(EI_{cln,1}+EI_{cln,3}\right) \\
+192q_1L_{bm}L_{cln}{}^4e_1e_3EI_{bm}\left(EI_{cln,1}-EI_{cln,3}\right)+192q_1L_{bm}L_{cln}{}^4e_2e_3EI_{bm}\left(EI_{cln,1}-EI_{cln,3}\right) \\
+96q_1L_{bm}L_{cln}{}^4e_3{}^2EI_{bm}\left(EI_{cln,1}-EI_{cln,3}\right)+48q_2L_{bm}{}^2L_{cln}{}^4e_3EI_{bm}\left(EI_{cln,1}+EI_{cln,3}\right) \\
+192q_2L_{bm}L_{cln}{}^4e_1e_3EI_{bm}\left(EI_{cln,1}-EI_{cln,3}\right)+192q_2L_{bm}L_{cln}{}^4e_2e_3EI_{bm}\left(EI_{cln,1}-EI_{cln,3}\right) \\
+96q_2L_{bm}L_{cln}{}^4e_3{}^2EI_{bm}\left(EI_{cln,1}-EI_{cln,3}\right)+48q_3L_{bm}{}^2L_{cln}{}^4e_1EI_{bm}\left(EI_{cln,1}+EI_{cln,3}\right) \\
+48q_3L_{bm}{}^2L_{cln}{}^4e_2EI_{bm}\left(EI_{cln,1}+EI_{cln,3}\right)+48q_3L_{bm}{}^2L_{cln}{}^4e_3EI_{bm}\left(EI_{cln,1}+EI_{cln,3}\right) \\
+96q_3L_{bm}L_{cln}{}^4e_1{}^2EI_{bm}\left(EI_{cln,1}-EI_{cln,3}\right)+192q_3L_{bm}L_{cln}{}^4e_1e_2EI_{bm}\left(EI_{cln,1}-EI_{cln,3}\right) \\
+192q_3L_{bm}L_{cln}{}^4e_1e_3EI_{bm}L_{cln}{}^2\left(EI_{cln,1}-EI_{cln,3}\right)+96q_3L_{bm}L_{cln}{}^4e_2{}^2EI_{bm}\left(EI_{cln,1}-EI_{cln,3}\right) \\
+96q_3L_{bm}L_{cln}{}^4e_2e_3EI_{bm}\left(2EI_{cln,1}-EI_{cln,3}\right)+96q_3L_{bm}L_{cln}{}^4e_2e_3EI_{bm}\left(EI_{cln,1}-2EI_{cln,3}\right) \\
-576L_{bm}L_{cln}{}^2e_3EI_{bm}EI_{cln,1}EI_{cln,3}
\end{pmatrix}
$$

$$+$$

$$
\begin{pmatrix}
8q_3L_{bm}{}^3L_{cln}{}^4EI_{bm}\left(EI_{cln,1}-EI_{cln,3}\right)+16q_1L_{bm}{}^2L_{cln}{}^4e_3EI_{bm}\left(EI_{cln,1}+EI_{cln,3}\right) \\
+192q_1L_{bm}L_{cln}{}^4e_1e_3EI_{bm}\left(-EI_{cln,1}+EI_{cln,3}\right)+192q_1L_{bm}L_{cln}{}^4e_2e_3EI_{bm}\left(-EI_{cln,1}+EI_{cln,3}\right) \\
+96q_1L_{bm}L_{cln}{}^4e_3{}^2EI_{bm}\left(-EI_{cln,1}+EI_{cln,3}\right)+16q_2L_{bm}{}^2L_{cln}{}^4e_3EI_{bm}\left(EI_{cln,1}+EI_{cln,3}\right) \\
+192q_2L_{bm}L_{cln}{}^4e_1e_3EI_{bm}\left(-EI_{cln,1}+EI_{cln,3}\right)+192q_2L_{bm}L_{cln}{}^4e_2e_3EI_{bm}\left(-EI_{cln,1}+EI_{cln,3}\right) \\
+96q_2L_{bm}L_{cln}{}^4e_3{}^2EI_{bm}\left(-EI_{cln,1}+EI_{cln,3}\right)+16q_3L_{bm}{}^2L_{cln}{}^4e_1EI_{bm}\left(EI_{cln,1}+EI_{cln,3}\right) \\
+16q_3L_{bm}{}^2L_{cln}{}^4e_2EI_{bm}\left(EI_{cln,1}+EI_{cln,3}\right)+16q_3L_{bm}{}^2L_{cln}{}^4e_3EI_{bm}\left(EI_{cln,1}+EI_{cln,3}\right) \\
+96q_3L_{bm}L_{cln}{}^4e_1{}^2EI_{bm}\left(-EI_{cln,1}+EI_{cln,3}\right)+192q_3L_{bm}L_{cln}{}^4e_1e_2EI_{bm}\left(-EI_{cln,1}+EI_{cln,3}\right) \\
+192q_3L_{bm}L_{cln}{}^4e_1e_3EI_{bm}\left(-EI_{cln,1}+EI_{cln,3}\right)+96q_3L_{bm}L_{cln}{}^4e_2{}^2EI_{bm}\left(-EI_{cln,1}+EI_{cln,3}\right) \\
+192q_3L_{bm}L_{cln}{}^4e_2e_3EI_{bm}\left(-EI_{cln,1}+EI_{cln,3}\right)+96q_3L_{bm}L_{cln}{}^4e_3{}^2EI_{bm}\left(-EI_{cln,1}+EI_{cln,3}\right) \\
+64q_1L_{bm}L_{cln}{}^5e_3\left(EI_{bm}\right)^2+64q_2L_{bm}L_{cln}{}^5e_3\left(EI_{bm}\right)^2+64q_3L_{bm}L_{cln}{}^5e_1\left(EI_{bm}\right)^2 \\
+64q_3L_{bm}L_{cln}{}^5e_2\left(EI_{bm}\right)^2+64q_3L_{bm}L_{cln}{}^5e_3\left(EI_{bm}\right)^2-192L_{cln}{}^3e_3\left(EI_{bm}\right)^2\left(EI_{cln,1}+EI_{cln,3}\right)
\end{pmatrix}
$$

$$=0$$

Some equal expressions can be combined together. This results in the following formula:

$$
\begin{aligned}
&48q_1L_{bm}{}^3L_{cln}{}^3e_3EI_{cln,1}EI_{cln,3}+48q_2L_{bm}{}^3L_{cln}{}^3e_3EI_{cln,1}EI_{cln,3}+48q_3L_{bm}{}^3L_{cln}{}^3e_1EI_{cln,1}EI_{cln,3}\\
&+48q_3L_{bm}{}^3L_{cln}{}^3e_2EI_{cln,1}EI_{cln,3}+48q_3L_{bm}{}^3L_{cln}{}^3e_3EI_{cln,1}EI_{cln,3}+64q_1L_{bm}{}^2L_{cln}{}^4e_3EI_{bm}\left(EI_{cln,1}+EI_{cln,3}\right)\\
&+64q_2L_{bm}{}^2L_{cln}{}^4e_3EI_{bm}\left(EI_{cln,1}+EI_{cln,3}\right)+64q_3L_{bm}{}^2L_{cln}{}^4e_1EI_{bm}\left(EI_{cln,1}+EI_{cln,3}\right)\\
&+64q_3L_{bm}{}^2L_{cln}{}^4e_2EI_{bm}\left(EI_{cln,1}+EI_{cln,3}\right)+64q_3L_{bm}{}^2L_{cln}{}^4e_3EI_{bm}\left(EI_{cln,1}+EI_{cln,3}\right)\\
&-576L_{bm}L_{cln}{}^2e_3EI_{bm}EI_{cln,1}EI_{cln,3}+8q_3L_{bm}{}^3L_{cln}{}^4EI_{bm}\left(EI_{cln,1}-EI_{cln,3}\right)+64q_1L_{bm}L_{cln}{}^5e_3\left(EI_{bm}\right)^2\\
&+64q_2L_{bm}L_{cln}{}^5e_3\left(EI_{bm}\right)^2+64q_3L_{bm}L_{cln}{}^5e_1\left(EI_{bm}\right)^2+64q_3L_{bm}L_{cln}{}^5e_2\left(EI_{bm}\right)^2+64q_3L_{bm}L_{cln}{}^5e_3\left(EI_{bm}\right)^2\\
&-192L_{cln}{}^3e_3\left(EI_{bm}\right)^2\left(EI_{cln,1}+EI_{cln,3}\right)=0
\end{aligned}
$$

Put all expressions of e_3 to the left side of the equation.

$$
\begin{aligned}
&-48q_2L_{bm}{}^3L_{cln}{}^3e_3EI_{cln,1}EI_{cln,3}-48q_1L_{bm}{}^3L_{cln}{}^3e_3EI_{cln,1}EI_{cln,3}-48q_3L_{bm}{}^3L_{cln}{}^3e_3EI_{cln,1}EI_{cln,3}\\
&-64q_1L_{bm}{}^2L_{cln}{}^4e_3EI_{bm}\left(EI_{cln,1}+EI_{cln,3}\right)-64q_2L_{bm}{}^2L_{cln}{}^4e_3EI_{bm}\left(EI_{cln,1}+EI_{cln,3}\right)\\
&-64q_3L_{bm}{}^2L_{cln}{}^4e_3EI_{bm}\left(EI_{cln,1}+EI_{cln,3}\right)+576L_{bm}L_{cln}{}^2e_3EI_{bm}EI_{cln,1}EI_{cln,3}\\
&-64q_1L_{bm}L_{cln}{}^5e_3\left(EI_{bm}\right)^2-64q_2L_{bm}L_{cln}{}^5e_3\left(EI_{bm}\right)^2\\
&-64q_3L_{bm}L_{cln}{}^5e_3\left(EI_{bm}\right)^2+192L_{cln}{}^3e_3\left(EI_{bm}\right)^2\left(EI_{cln,1}+EI_{cln,3}\right)\\
&=\\
&48q_3L_{bm}{}^3L_{cln}{}^3e_1EI_{cln,1}EI_{cln,3}+48q_3L_{bm}{}^3L_{cln}{}^3e_2EI_{cln,1}EI_{cln,3}\\
&+64q_3L_{bm}{}^2L_{cln}{}^4e_1EI_{bm}\left(EI_{cln,1}+EI_{cln,3}\right)+64q_3L_{bm}{}^2L_{cln}{}^4e_2EI_{bm}\left(EI_{cln,1}+EI_{cln,3}\right)\\
&+8q_3L_{bm}{}^3L_{cln}{}^4EI_{bm}\left(EI_{cln,1}-EI_{cln,3}\right)+64q_3L_{bm}L_{cln}{}^5e_1\left(EI_{bm}\right)^2+64q_3L_{bm}L_{cln}{}^5e_2\left(EI_{bm}\right)^2
\end{aligned}
$$

Combine some expressions

$$
16L_{cln}{}^2e_3\begin{bmatrix}-3L_{bm}{}^3L_{cln}\left(q_2+q_1+q_3\right)EI_{cln,1}EI_{cln,3}-4L_{bm}{}^2L_{cln}{}^2\left(q_1+q_2+q_3\right)EI_{bm}\left(EI_{cln,1}+EI_{cln,3}\right)\\-4L_{bm}L_{cln}{}^3\left(q_1+q_2+q_3\right)\left(EI_{bm}\right)^2+36L_{bm}EI_{bm}EI_{cln,1}EI_{cln,3}\\+12L_{cln}\left(EI_{bm}\right)^2\left(EI_{cln,1}+EI_{cln,3}\right)\end{bmatrix}
$$

$$=$$

$$
8L_{bm}L_{cln}{}^3q_3\begin{bmatrix}6L_{bm}{}^2\left(e_1+e_2\right)EI_{cln,1}EI_{cln,3}+8L_{bm}L_{cln}\left(e_1+e_2\right)EI_{bm}\left(EI_{cln,1}+EI_{cln,3}\right)\\+8L_{cln}{}^2\left(e_1+e_2\right)\left(EI_{bm}\right)^2+L_{bm}{}^2L_{cln}EI_{bm}\left(EI_{cln,1}-EI_{cln,3}\right)\end{bmatrix}
$$

The term total load (q_{total}) can be used instead of the different terms q_1, q_2 and q_3.

$$16L_{cln}{}^{2}e_3\begin{bmatrix}-3L_{bm}{}^{3}L_{cln}q_{total}EI_{cln,1}EI_{cln,3}-4L_{bm}{}^{2}L_{cln}{}^{2}q_{total}EI_{bm}\left(EI_{cln,1}+EI_{cln,3}\right)-4L_{bm}L_{cln}{}^{3}q_{total}\left(EI_{bm}\right)^{2}\\+36L_{bm}EI_{bm}EI_{cln,1}EI_{cln,3}+12L_{cln}\left(EI_{bm}\right)^{2}\left(EI_{cln,1}+EI_{cln,3}\right)\end{bmatrix}$$

$$=$$

$$8L_{bm}L_{cln}{}^{3}q_3\begin{bmatrix}6L_{bm}{}^{2}\left(e_1+e_2\right)EI_{cln,1}EI_{cln,3}+8L_{bm}L_{cln}\left(e_1+e_2\right)EI_{bm}\left(EI_{cln,1}+EI_{cln,3}\right)\\+8L_{cln}{}^{2}\left(e_1+e_2\right)\left(EI_{bm}\right)^{2}+L_{bm}{}^{2}L_{cln}EI_{bm}\left(EI_{cln,1}-EI_{cln,3}\right)\end{bmatrix}$$

The formula of e_3 can be found.

$$e_3=\frac{q_3L_{bm}L_{cln}\begin{bmatrix}6L_{bm}{}^{2}\left(e_1+e_2\right)EI_{cln,1}EI_{cln,3}+8L_{bm}L_{cln}\left(e_1+e_2\right)EI_{bm}\left(EI_{cln,1}+EI_{cln,3}\right)\\+8L_{cln}{}^{2}\left(e_1+e_2\right)\left(EI_{bm}\right)^{2}+L_{bm}{}^{2}L_{cln}EI_{bm}\left(EI_{cln,1}-EI_{cln,3}\right)\end{bmatrix}}{2\begin{bmatrix}-3L_{bm}{}^{3}L_{cln}q_{total}EI_{cln,1}EI_{cln,3}-4L_{bm}{}^{2}L_{cln}{}^{2}q_{total}EI_{bm}\left(EI_{cln,1}+EI_{cln,3}\right)-4L_{bm}L_{cln}{}^{3}q_{total}\left(EI_{bm}\right)^{2}\\+36L_{bm}EI_{bm}EI_{cln,1}EI_{cln,3}+12L_{cln}\left(EI_{bm}\right)^{2}\left(EI_{cln,1}+EI_{cln,3}\right)\end{bmatrix}}$$

Check the dimensions

$$m=\frac{Nm^{-1}mm\begin{bmatrix}m^{2}\left(m+m\right)Nm^{-2}m^{4}Nm^{-2}m^{4}+mm\left(m+m\right)Nm^{-2}m^{4}\left(Nm^{-2}m^{4}+Nm^{-2}m^{4}\right)\\+m^{2}\left(m+m\right)\left(Nm^{-2}m^{4}\right)^{2}+m^{2}mNm^{-2}m^{4}\left(Nm^{-2}m^{4}-Nm^{-2}m^{4}\right)\end{bmatrix}}{\begin{bmatrix}m^{3}mNm^{-1}Nm^{-2}m^{4}Nm^{-2}m^{4}-m^{2}m^{2}Nm^{-1}Nm^{-2}m^{4}\left(Nm^{-2}m^{4}+Nm^{-2}m^{4}\right)-mm^{3}Nm^{-1}\left(Nm^{-2}m^{4}\right)^{2}\\+mNm^{-2}m^{4}Nm^{-2}m^{4}Nm^{-2}m^{4}+m\left(Nm^{-2}m^{4}\right)^{2}\left(Nm^{-2}m^{4}+Nm^{-2}m^{4}\right)\end{bmatrix}}$$

$$m=\frac{Nm\left[N^{2}m^{7}+N^{2}m^{7}+N^{2}m^{7}+N^{2}m^{7}\right]}{\left[N^{3}m^{7}-N^{3}m^{7}-N^{3}m^{7}+N^{3}m^{7}+N^{3}m^{7}\right]}$$

$$m=\frac{N^{3}m^{8}}{N^{3}m^{7}}$$

The dimensions are correct.

In Appendix K.1 a formula is found for the total deflection in the second load case. In Appendix K.2 a formula for the total deflection in the third load case is found. To check the formulas the formulas will be compared together. The following formulas were found in Appendix K.1:

$$e_2 = \frac{q_2 L_{bm} L_{cln} \begin{bmatrix} 6L_{bm}^2 e_1 EI_{cln,1} EI_{cln,2} + 3L_{bm}^2 z_2 EI_{cln,1} EI_{cln,2} + 6L_{bm} e_1 z_2 EI_{cln,1} EI_{cln,2} \\ +8L_{bm} L_{cln} e_1 EI_{bm} \left(EI_{cln,1} + EI_{cln,2}\right) + L_{bm}^2 L_{cln} EI_{bm} \left(EI_{cln,1} - EI_{cln,2}\right) + 8L_{cln}^2 e_1 \left(EI_{bm}\right)^2 \\ +2L_{cln} z_2 \left(L_{bm} + 2e_1\right) EI_{bm} \left(-EI_{cln,1} + 2EI_{cln,2}\right) \end{bmatrix}}{2 \begin{bmatrix} -3q_{total} L_{bm}^3 L_{cln} EI_{cln,1} EI_{cln,2} - 3q_{total} L_{bm}^2 L_{cln} z_2 EI_{cln,1} EI_{cln,2} \\ -4q_{total} L_{bm}^2 L_{cln}^2 EI_{bm} \left(EI_{cln,1} + EI_{cln,2}\right) - 2q_{total} L_{bm} L_{cln}^2 z_2 EI_{bm} \left(-EI_{cln,1} + 2EI_{cln,2}\right) \\ -4q_{total} L_{bm} L_{cln}^3 \left(EI_{bm}\right)^2 + 12L_{cln} \left(EI_{bm}\right)^2 \left(EI_{cln,1} + EI_{cln,2}\right) + 36L_{bm} EI_{bm} EI_{cln,1} EI_{cln,2} \end{bmatrix}}$$

One big difference between the second load case and the third load case is the shift of the centre of gravity. Suppose this shift is equal to zero. The following formula remains.

$$e_2 = \frac{q_2 L_{bm} L_{cln} \begin{bmatrix} 6L_{bm}^2 e_1 EI_{cln,1} EI_{cln,2} + 8L_{bm} L_{cln} e_1 EI_{bm} \left(EI_{cln,1} + EI_{cln,2}\right) \\ +L_{bm}^2 L_{cln} EI_{bm} \left(EI_{cln,1} - EI_{cln,2}\right) + 8L_{cln}^2 e_1 \left(EI_{bm}\right)^2 \end{bmatrix}}{2 \begin{bmatrix} -3q_{total} L_{bm}^3 L_{cln} EI_{cln,1} EI_{cln,2} - 4q_{total} L_{bm}^2 L_{cln}^2 EI_{bm} \left(EI_{cln,1} + EI_{cln,2}\right) \\ -4q_{total} L_{bm} L_{cln}^3 \left(EI_{bm}\right)^2 + 12L_{cln} \left(EI_{bm}\right)^2 \left(EI_{cln,1} + EI_{cln,2}\right) + 36L_{bm} EI_{bm} EI_{cln,1} EI_{cln,2} \end{bmatrix}}$$

The formula of the additional deflection in the third load case is the following formula.

$$e_3 = \frac{q_3 L_{bm} L_{cln} \begin{bmatrix} 6L_{bm}^2 \left(e_1 + e_2\right) EI_{cln,1} EI_{cln,3} + 8L_{bm} L_{cln} \left(e_1 + e_2\right) EI_{bm} \left(EI_{cln,1} + EI_{cln,3}\right) \\ +8L_{cln}^2 \left(e_1 + e_2\right)\left(EI_{bm}\right)^2 + L_{bm}^2 L_{cln} EI_{bm} \left(EI_{cln,1} - EI_{cln,3}\right) \end{bmatrix}}{2 \begin{bmatrix} -3L_{bm}^3 L_{cln} q_{total} EI_{cln,1} EI_{cln,3} - 4L_{bm}^2 L_{cln}^2 q_{total} EI_{bm} \left(EI_{cln,1} + EI_{cln,3}\right) - 4L_{bm} L_{cln}^3 q_{total} \left(EI_{bm}\right)^2 \\ +36L_{bm} EI_{bm} EI_{cln,1} EI_{cln,3} + 12L_{cln} \left(EI_{bm}\right)^2 \left(EI_{cln,1} + EI_{cln,3}\right) \end{bmatrix}}$$

If the formula of e_2 and the formula of e_3 are compared together, the following conclusions can be made:

- The signs and the numerical parameters are the same in both formulas.
- Somme differences between the formulas are the stiffness ($EI_{cln,2}$ compare with $EI_{cln,3}$), the load (q_2 become q_3) and the starting deflection (e_1 compare with e_1+e_2).

If there is an error in one formula, the error is including the other formula too. The probability of making twice the same mistake is small and this concludes that the signs and the characters in both formulas are correct.

The formula of the total deflection in the second load case is found. This formula is very complex. It is very complex to find a formula for the additional horizontal reaction force using this formula of the total deflection. The numerical value of the total deflection will be used for the calculation of the horizontal reaction force.

The formula of the additional horizontal reaction force can be simplified. The formula of the additional horizontal reaction force is:

$$H_{A,3} = \frac{\left(\begin{array}{l} q_3 L_{bm}{}^3 L_{cln} EI_{cln,3} - 2q_1 L_{bm}{}^2 L_{cln} e_3 EI_{cln,3} - 24q_1 L_{bm} L_{cln} e_1 e_3 EI_{cln,3} - 24q_1 L_{bm} L_{cln} e_2 e_3 EI_{cln,3} \\ -12q_1 L_{bm} L_{cln} e_3{}^2 EI_{cln,3} - 2q_2 L_{bm}{}^2 L_{cln} e_3 EI_{cln,3} - 24q_2 L_{bm} L_{cln} e_1 e_3 EI_{cln,3} - 24q_2 L_{bm} L_{cln} e_2 e_3 EI_{cln,3} \\ -12q_2 L_{bm} L_{cln} e_3{}^2 EI_{cln,3} - 2q_3 L_{bm}{}^2 L_{cln} e_1 EI_{cln,3} - 2q_3 L_{bm}{}^2 L_{cln} e_2 EI_{cln,3} - 2q_3 L_{bm}{}^2 L_{cln} e_3 EI_{cln,3} \\ -12q_3 L_{bm} L_{cln} e_1{}^2 EI_{cln,3} - 24q_3 L_{bm} L_{cln} e_1 e_2 EI_{cln,3} - 24q_3 L_{bm} L_{cln} e_1 e_3 EI_{cln,3} - 12q_3 L_{bm} L_{cln} e_2{}^2 EI_{cln,3} \\ -24q_3 L_{bm} L_{cln} e_2 e_3 EI_{cln,3} - 12q_3 L_{bm} L_{cln} e_3{}^2 EI_{cln,3} - 4q_1 L_{bm} L_{cln}{}^2 e_3 EI_{bm} - 16q_1 L_{cln}{}^2 e_3 e_1 EI_{bm} \\ -16q_1 L_{cln}{}^2 e_3 e_2 EI_{bm} - 8q_1 L_{cln}{}^2 e_3{}^2 EI_{bm} - 4q_2 L_{bm} L_{cln}{}^2 e_3 EI_{bm} - 16q_2 L_{cln}{}^2 e_3 e_1 EI_{bm} - 16q_2 L_{cln}{}^2 e_3 e_2 EI_{bm} \\ -8q_2 L_{cln}{}^2 e_3{}^2 EI_{bm} - 4q_3 L_{bm} L_{cln}{}^2 e_1 EI_{bm} - 4q_3 L_{bm} L_{cln}{}^2 e_2 EI_{bm} - 4q_3 L_{bm} L_{cln}{}^2 e_3 EI_{bm} - 8q_3 L_{cln}{}^2 e_1{}^2 EI_{bm} \\ -16q_3 L_{cln}{}^2 e_1 e_2 EI_{bm} - 16q_3 L_{cln}{}^2 e_1 e_3 EI_{bm} - 8q_3 L_{cln}{}^2 e_2{}^2 EI_{bm} - 16q_3 L_{cln}{}^2 e_2 e_3 EI_{bm} - 8q_3 L_{cln}{}^2 e_3{}^2 EI_{bm} \\ +24 e_3 EI_{bm} EI_{cln,3} \end{array}\right)}{12 L_{bm} L_{cln}{}^2 EI_{cln,3} + 8 L_{cln}{}^3 EI_{bm}}$$

To simplify the formula the order is changed.

$$H_{A,3} = \frac{\left(\begin{array}{l} -2q_1 L_{bm}{}^2 L_{cln} e_3 EI_{cln,3} - 2q_2 L_{bm}{}^2 L_{cln} e_3 EI_{cln,3} - 2q_3 L_{bm}{}^2 L_{cln} e_1 EI_{cln,3} \\ -24q_1 L_{bm} L_{cln} e_1 e_3 EI_{cln,3} - 24q_2 L_{bm} L_{cln} e_1 e_3 EI_{cln,3} - 24q_3 L_{bm} L_{cln} e_1 e_3 EI_{cln,3} \\ -24q_1 L_{bm} L_{cln} e_2 e_3 EI_{cln,3} - 24q_2 L_{bm} L_{cln} e_2 e_3 EI_{cln,3} - 24q_3 L_{bm} L_{cln} e_2 e_3 EI_{cln,3} \\ -12q_1 L_{bm} L_{cln} e_3{}^2 EI_{cln,3} - 12q_2 L_{bm} L_{cln} e_3{}^2 EI_{cln,3} - 12q_3 L_{bm} L_{cln} e_3{}^2 EI_{cln,3} \\ -4q_1 L_{bm} L_{cln}{}^2 e_3 EI_{bm} - 4q_2 L_{bm} L_{cln}{}^2 e_3 EI_{bm} - 4q_3 L_{bm} L_{cln}{}^2 e_3 EI_{bm} \\ -16q_1 L_{cln}{}^2 e_3 e_1 EI_{bm} - 16q_2 L_{cln}{}^2 e_3 e_1 EI_{bm} - 16q_3 L_{cln}{}^2 e_1 e_3 EI_{bm} \\ -16q_1 L_{cln}{}^2 e_3 e_2 EI_{bm} - 16q_2 L_{cln}{}^2 e_3 e_2 EI_{bm} - 16q_3 L_{cln}{}^2 e_2 e_3 EI_{bm} \\ -8q_1 L_{cln}{}^2 e_3{}^2 EI_{bm} - 8q_2 L_{cln}{}^2 e_3{}^2 EI_{bm} - 8q_3 L_{cln}{}^2 e_3{}^2 EI_{bm} \\ -2q_3 L_{bm}{}^2 L_{cln} e_2 EI_{cln,3} - 2q_3 L_{bm}{}^2 L_{cln} e_3 EI_{cln,3} - 12q_3 L_{bm} L_{cln} e_1{}^2 EI_{cln,3} - 24q_3 L_{bm} L_{cln} e_1 e_2 EI_{cln,3} \\ -12q_3 L_{bm} L_{cln} e_2{}^2 EI_{cln,3} - 4q_3 L_{bm} L_{cln}{}^2 e_1 EI_{bm} - 4q_3 L_{bm} L_{cln}{}^2 e_2 EI_{bm} - 8q_3 L_{cln}{}^2 e_1{}^2 EI_{bm} \\ -16q_3 L_{cln}{}^2 e_1 e_2 EI_{bm} - 8q_3 L_{cln}{}^2 e_2{}^2 EI_{bm} + 24 e_3 EI_{bm} EI_{cln,3} + q_3 L_{bm}{}^3 L_{cln} EI_{cln,3} \end{array}\right)}{12 L_{bm} L_{cln}{}^2 EI_{cln,3} + 8 L_{cln}{}^3 EI_{bm}}$$

If possible the total load is used instead of the three different loads separated.

$$H_{A,3} = \frac{\left(\begin{array}{l} -2q_{total,3} L_{bm}{}^2 L_{cln} e_3 EI_{cln,3} - 24q_{total,3} L_{bm} L_{cln} e_1 e_3 EI_{cln,3} - 24q_{total,3} L_{bm} L_{cln} e_2 e_3 EI_{cln,3} \\ -12q_{total,3} L_{bm} L_{cln} e_3{}^2 EI_{cln,3} - 4q_{total,3} L_{bm} L_{cln}{}^2 e_3 EI_{bm} - 16q_{total,3} L_{cln}{}^2 e_3 e_1 EI_{bm} - 16q_{total,3} L_{cln}{}^2 e_3 e_2 EI_{bm} \\ -8q_{total,3} L_{cln}{}^2 e_3{}^2 EI_{bm} - 2q_3 L_{bm}{}^2 L_{cln} e_2 EI_{cln,3} - 2q_3 L_{bm}{}^2 L_{cln} e_3 EI_{cln,3} - 12q_3 L_{bm} L_{cln} e_1{}^2 EI_{cln,3} \\ -24q_3 L_{bm} L_{cln} e_1 e_2 EI_{cln,3} - 12q_3 L_{bm} L_{cln} e_2{}^2 EI_{cln,3} - 4q_3 L_{bm} L_{cln}{}^2 e_1 EI_{bm} - 4q_3 L_{bm} L_{cln}{}^2 e_2 EI_{bm} - 8q_3 L_{cln}{}^2 e_1{}^2 EI_{bm} \\ -16q_3 L_{cln}{}^2 e_1 e_2 EI_{bm} - 8q_3 L_{cln}{}^2 e_2{}^2 EI_{bm} + 24 e_3 EI_{bm} EI_{cln,3} + q_3 L_{bm}{}^3 L_{cln} EI_{cln,3} \end{array}\right)}{12 L_{bm} L_{cln}{}^2 EI_{cln,3} + 8 L_{cln}{}^3 EI_{bm}}$$

Some expressions are combined together.

$$H_{A,3}=\frac{\left(\begin{array}{l} -q_{total,3}L_{bm}L_{cln}\left[2L_{bm}e_3+24e_1e_3+24e_2e_3+12e_3^2\right]EI_{cln,3}\\ -q_{total,3}L_{cln}^2\left[4L_{bm}e_3+16e_3e_1+16e_3e_2+8e_3^2\right]EI_{bm}\\ +q_3L_{bm}L_{cln}\left[L_{bm}^2-2L_{bm}e_2-2L_{bm}e_3-12e_1^2-24e_1e_2-12e_2^2\right]EI_{cln,3}\\ -q_3L_{cln}^2\left[4L_{bm}e_1+4L_{bm}e_2+8e_1^2+16e_1e_2+8e_2^2\right]EI_{bm}+24e_3EI_{bm}EI_{cln,3}\end{array}\right)}{12L_{bm}L_{cln}^2EI_{cln,3}+8L_{cln}^3EI_{bm}}$$

The following formulas can be used to find the loads and the deflection in the second and in the third load case.

$$V_{D,2}=q_1e_2+q_2\left(\tfrac{1}{2}L_{bm}+e_1+e_2\right)$$
$$V_{A,2}=-q_1e_2+q_2\left(\tfrac{1}{2}L_{bm}-e_1-e_2\right)$$
$$M_{C,2}=q_2\left(\tfrac{1}{2}L_{bm}+e_1+e_2\right)\left(e_1+e_2+z_2\right)+q_1e_2\left(\tfrac{1}{2}L_{bm}+2e_1+e_2+z_2\right)+H_{A,2}L_{cln}$$
$$M_{B,2}=q_2\left(\tfrac{1}{2}L_{bm}-e_1-e_2\right)\left(e_1+e_2\right)+q_1e_2\left(\tfrac{1}{2}L_{bm}-2e_1-e_2\right)-H_{A,2}L_{cln}$$
$$H_{A,2}=\frac{\left(\begin{array}{l} -q_{total,2}L_{bm}L_{cln}e_2\left[12e_2+24e_1+8z_2+2L_{bm}\right]EI_{cln,2}+q_{total,2}L_{cln}^2e_2\left[4L_{bm}-16e_1-8e_2\right]EI_{bm}\\ +24e_2EI_{bm}EI_{cln,2}+q_2L_{bm}L_{cln}\left[L_{bm}^2-2L_{bm}e_1-12e_1^2-4L_{bm}z_2-8e_1z_2\right]EI_{cln,2}-q_2L_{cln}^2e_1\left[4L_{bm}+8e_1\right]EI_{bm}\end{array}\right)}{12L_{bm}L_{cln}^2EI_{cln,2}+8L_{cln}^3EI_{bm}}$$

$$e_2=\frac{q_2L_{bm}L_{cln}\left[\begin{array}{l} 6L_{bm}^2e_1EI_{cln,1}EI_{cln,2}+3L_{bm}^2z_2EI_{cln,1}EI_{cln,2}+6L_{bm}e_1z_2EI_{cln,1}EI_{cln,2}\\ +8L_{bm}L_{cln}e_1EI_{bm}\left(EI_{cln,1}+EI_{cln,2}\right)+L_{bm}^2L_{cln}EI_{bm}\left(EI_{cln,1}-EI_{cln,2}\right)+8L_{cln}^2e_1\left(EI_{bm}\right)^2\\ +2L_{cln}z_2\left(L_{bm}+2e_1\right)EI_{bm}\left(-EI_{cln,1}+2EI_{cln,2}\right)\end{array}\right]}{2\left[\begin{array}{l} -3q_{total}L_{bm}^3L_{cln}EI_{cln,1}EI_{cln,2}-3q_{total}L_{bm}^2L_{cln}z_2EI_{cln,1}EI_{cln,2}\\ -4q_{total}L_{bm}^2L_{cln}^2EI_{bm}\left(EI_{cln,1}+EI_{cln,2}\right)-2q_{total}L_{bm}L_{cln}^2z_2EI_{bm}\left(-EI_{cln,1}+2EI_{cln,2}\right)\\ -4q_{total}L_{bm}L_{cln}^3\left(EI_{bm}\right)^2+12L_{cln}\left(EI_{bm}\right)^2\left(EI_{cln,1}+EI_{cln,2}\right)+36L_{bm}EI_{bm}EI_{cln,1}EI_{cln,2}\end{array}\right]}$$

$$V_{D,3}=\left(q_1+q_2\right)e_3+q_3\left(\tfrac{1}{2}L_{bm}+e_1+e_2+e_3\right)$$
$$V_{A,3}=-\left(q_1+q_2\right)e_3+q_3\left(\tfrac{1}{2}L_{bm}-e_1-e_2-e_3\right)$$
$$M_{C,3}=e_3\left(q_1+q_2\right)\left(\tfrac{1}{2}L_{bm}+2e_1+2e_2+e_3\right)+q_3\left(\tfrac{1}{2}L_{bm}+e_1+e_2+e_3\right)\left(e_1+e_2+e_3\right)+H_{A,3}L_{cln}$$
$$M_{B,3}=e_3\left(q_1+q_2\right)\left(\tfrac{1}{2}L_{bm}-2e_1-2e_2-e_3\right)+q_3\left(\tfrac{1}{2}L_{bm}-e_1-e_2-e_3\right)\left(e_1+e_2+e_3\right)-H_{A,3}L_{cln}$$

$$H_{A,3}=\frac{\left(\begin{array}{l} -q_{total,3}L_{bm}L_{cln}\left[2L_{bm}e_3+24e_1e_3+24e_2e_3+12e_3^2\right]EI_{cln,3}\\ -q_{total,3}L_{cln}^2\left[4L_{bm}e_3+16e_3e_1+16e_3e_2+8e_3^2\right]EI_{bm}\\ +q_3L_{bm}L_{cln}\left[L_{bm}^2-2L_{bm}e_2-2L_{bm}e_3-12e_1^2-24e_1e_2-12e_2^2\right]EI_{cln,3}\\ -q_3L_{cln}^2\left[4L_{bm}e_1+4L_{bm}e_2+8e_1^2+16e_1e_2+8e_2^2\right]EI_{bm}+24e_3EI_{bm}EI_{cln,3}\end{array}\right)}{12L_{bm}L_{cln}^2EI_{cln,3}+8L_{cln}^3EI_{bm}}$$

$$e_3 = \frac{q_3 L_{bm} L_{cln} \begin{bmatrix} 6L_{bm}{}^2 (e_1 + e_2) EI_{cln,1} EI_{cln,3} + 8L_{bm} L_{cln} (e_1 + e_2) EI_{bm} \left(EI_{cln,1} + EI_{cln,3} \right) \\ +8L_{cln}{}^2 (e_1 + e_2) (EI_{bm})^2 + L_{bm}{}^2 L_{cln} EI_{bm} \left(EI_{cln,1} - EI_{cln,3} \right) \end{bmatrix}}{2 \begin{bmatrix} -3L_{bm}{}^3 L_{cln} q_{total} EI_{cln,1} EI_{cln,3} - 4L_{bm}{}^2 L_{cln}{}^2 q_{total} EI_{bm} \left(EI_{cln,1} + EI_{cln,3} \right) - 4L_{bm} L_{cln}{}^3 q_{total} (EI_{bm})^2 \\ +36L_{bm} EI_{bm} EI_{cln,1} EI_{cln,3} + 12L_{cln} (EI_{bm})^2 \left(EI_{cln,1} + EI_{cln,3} \right) \end{bmatrix}}$$

Appendix L Calculation example of an unbraced portal frame

In this Appendix a calculation example of an unbraced portal frame has been made. This calculation is based on formulas of the non-linear analysis from Appendices J and Appendix K. The linear analysis has been made to understand the non-linear analysis better. The linear analysis will not be used in this calculation. Because of the complication of the formulas, a computer program has been used to make the iterations. The calculation file can be found in Appendix L.2. Appendix L.1 is about the use of the formulas.

L.1 Calculation example.

This Appendix is a calculation example for an unbraced portal frame. In this Appendix it becomes clear what to do with the formulas found in Appendices J and K. Because of the complexity of the formulas not every step will be made manually. The calculations are made by the computer program MatLab. The calculation file can be found in Appendix L.2.

The column of this calculation example has a length ten meters. The section of the column is a HE 360A section. The beam has a length of five meters and has section HE 900A (Fig. L.1).

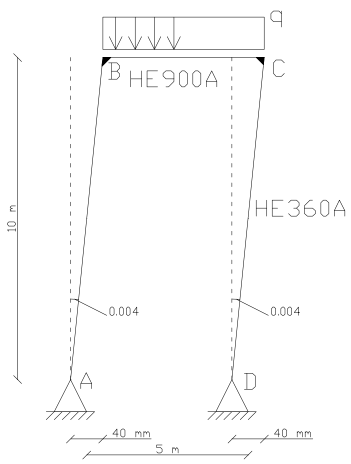


Figure L.1: Structure

The section properties are:

L_{cln} = 10000 mm
L_{bm} = 5000 mm
e_0 = 10 mm
$EI_{cln,1}$ = 6.959*10^{13} Nmm^2
$EI_{cln,2}$ = 5.263*10^{13} Nmm^2
$EI_{cln,3}$ = 3.573*10^{13} Nmm^2
$Z_{cln,1}$ = 1.891*10^{6} mm^3
$Z_{cln,2}$ = 1.432*10^{6} mm^3
$Z_{cln,3}$ = 9.721*10^{5} mm^3
EI_{bm} = 8.864*10^{14} Nmm^2
$f_{c,1}$ = 177.5 N/mm^2 (first critical stress)
$f_{c,2}$ = 532.5 N/mm^2 (second critical stress)
$A_{cln,1}$ = 14280 mm^2
$A_{cln,2}$ = 11655 mm^2
$A_{cln,3}$ = 7280 mm^2
z_2 = 39.6 mm

The first load is:

y_1 = 479 N/mm (kN/m)

The deflection can be calculated.

$$e_1 = \frac{12\psi L_{cln} EI_{bm} EI_{cln,1}}{12EI_{bm}EI_{cln,1} - q_1 L_{bm}{}^2 L_{cln} EI_{cln,1} - 2q_1 L_{bm} L_{cln}{}^2 EI_{bm}}$$

$$e_1 = \frac{12*0.004*10000*8.864*10^{14}*6.959*10^{13}}{\begin{pmatrix}12*8.864*10^{14}*6.959*10^{13} - 479*5000^2*10000*6.959*10^{13} \\ -2*479*5000*10000^2*8.864*10^{14}\end{pmatrix}}$$

$$e_1 = \frac{2.961*10^{31}}{3.073*10^{29}}$$

$$e_1 = 96.5mm$$

The reaction forces can be calculated:

$$V_{D,1} = q_1\left(½L_{bm} + e_1\right)$$

$$V_{D,1} = 479\left(½*5000 + 96.5\right)$$

$$V_{D,1} = 1244*10^3 N$$

$$H_{D,1} = q_1 \frac{L_{bm}{}^3 L_{cln} EI_{cln,1}}{12L_{bm}L_{cln}{}^2 EI_{cln,1} + 8L_{cln}{}^3 EI_{bm}} - q_1 L_{cln}\left(\frac{12\psi EI_{bm}EI_{cln,1}}{12EI_{bm}EI_{cln,1} - q_1 L_{bm}{}^2 L_{cln}EI_{cln,1} - 2q_1 L_{bm} L_{cln}{}^2 EI_{bm}}\right)^2$$

$$H_{D,1} = 479\frac{5000^3*10000*6.959*10^{13}}{12*5000*10000^2*6.959*10^{13} + 8*10000^3*8.864*10^{14}}$$

$$-479*10000\left(\frac{12*0.004*8.864*10^{14}*6.959*10^{13}}{\begin{pmatrix}12*8.864*10^{14}*6.959*10^{13} - 479*5000^2*10000*6.959*10^{13} \\ -2*479*5000*10000^2*8.864*10^{14}\end{pmatrix}}\right)^2$$

$$H_{D,1} = \frac{4.167*10^{31}}{7.509*10^{27}} - 4.79*10^6\left(\frac{2.961*10^{27}}{3.073*10^{29}}\right)^2$$

$$H_{D,1} = 5104N$$

Calculate the bending moment in point C.

$$M_{C,1} = q_1\left(½L_{bm} + e_1\right)e_1 + H_{A,1}L_{cln}$$

$$M_{C,1} = 479\left(½*5000 + 96.5\right)96.5 + 5104*10000$$

$$M_{C,1} = 171.1*10^6 Nmm$$

Calculate the stresses

$$\sigma_{right,1} = -\frac{V_{D,1}}{A_{cln,1}} - \frac{M_{C,1}}{Z_{cln,1}}$$

$$\sigma_{right,1} = -\frac{1244*10^3}{14280} - \frac{171.1*10^6}{1.891*10^6}$$

$\sigma_{right,1} = -177.6 N/mm^2$ first critical stress

$$\sigma_{left,1} = -\frac{V_{D,1}}{A_{cln,1}} + \frac{M_{C,1}}{Z_{cln,1}}$$

$$\sigma_{left,1} = -\frac{1244*10^3}{14280} + \frac{171.1*10^6}{1.891*10^6}$$

$\sigma_{left,1} = 3.4 N/mm^2$ Tension

$$\sigma_{centre,1} = -\frac{V_{D,1}}{A_{cln,1}}$$

$$\sigma_{centre,1} = -\frac{1244*10^3}{14280}$$

$$\sigma_{centre,1} = -87.1 N/mm^2$$

The second load is:

y_2 = 153 N/mm (kN/m)

The additional deflection can be calculated.

$$e_2 = \frac{q_2 L_{bm} L_{cln} \begin{bmatrix} 6L_{bm}{}^2 e_1 EI_{cln,1} EI_{cln,2} + 3L_{bm}{}^2 z_2 EI_{cln,1} EI_{cln,2} + 6L_{bm} e_1 z_2 EI_{cln,1} EI_{cln,2} \\ +8L_{bm} L_{cln} e_1 EI_{bm}\left(EI_{cln,1} + EI_{cln,2}\right) + L_{bm}{}^2 L_{cln} EI_{bm}\left(EI_{cln,1} - EI_{cln,2}\right) + 8L_{cln}{}^2 e_1 \left(EI_{bm}\right)^2 \\ +2L_{cln} z_2 \left(L_{bm} + 2e_1\right) EI_{bm}\left(-EI_{cln,1} + 2EI_{cln,2}\right) \end{bmatrix}}{2\begin{bmatrix} -3q_{total} L_{bm}{}^3 L_{cln} EI_{cln,1} EI_{cln,2} - 3q_{total} L_{bm}{}^2 L_{cln} z_2 EI_{cln,1} EI_{cln,2} \\ -4q_{total} L_{bm}{}^2 L_{cln}{}^2 EI_{bm}\left(EI_{cln,1} + EI_{cln,2}\right) - 2q_{total} L_{bm} L_{cln}{}^2 z_2 EI_{bm}\left(-EI_{cln,1} + 2EI_{cln,2}\right) \\ -4q_{total} L_{bm} L_{cln}{}^3 \left(EI_{bm}\right)^2 + 12L_{cln}\left(EI_{bm}\right)^2\left(EI_{cln,1} + EI_{cln,2}\right) + 36L_{bm} EI_{bm} EI_{cln,1} EI_{cln,2} \end{bmatrix}}$$

$$e_2 = \frac{153*5000*10000\begin{bmatrix} 6*5000^2*96.5*6.959*10^{13}*5.263*10^{13} \\ +3*5000^2*39.6*6.959*10^{13}*5.263*10^{13} \\ +6*5000*96.5*39.6*6.959*10^{13}*5.263*10^{13} \\ +8*5000*10000*96.5*8.864*10^{14}\left(6.959*10^{13}+5.263*10^{13}\right) \\ +5000^2*10000*8.864*10^{14}\left(6.959*10^{13}-5.263*10^{13}\right) \\ +8*10000^2*96.5\left(8.864*10^{14}\right)^2 \\ +2*10000*39.6\left(5000+2*96.5\right)8.864*10^{14}\left(-6.959*10^{13}+2*5.263*10^{13}\right) \end{bmatrix}}{2\begin{bmatrix} -3*632*5000^3*10000*6.959*10^{13}*5.263*10^{13} \\ -3*632*5000^2*10000*39.6*6.959*10^{13}*5.263*10^{13} \\ -4*632*5000^2*10000^2*8.864*10^{14}\left(6.959*10^{13}+5.263*10^{13}\right) \\ -2*632*5000*10000^2*39.6*8.864*10^{14}\left(-6.959*10^{13}+2*5.263*10^{13}\right) \\ -4*632*5000*10000^3\left(8.864*10^{14}\right)^2 \\ +12*10000\left(8.864*10^{14}\right)^2\left(6.959*10^{13}+5.263*10^{13}\right) \\ +36*5000*8.864*10^{14}*6.959*10^{13}*5.263*10^{13} \end{bmatrix}}$$

$$e_2 = \frac{5.263*10^{50}}{2.941*10^{48}}$$

$$e_2 = 179mm$$

The reaction forces can be calculated:

$$V_{D,2} = q_1 e_2 + q_2\left(\tfrac{1}{2}L_{bm} + e_1 + e_2\right)$$

$$V_{D,2} = 479*179 + 153*\left(\tfrac{1}{2}*5000 + 96.5 + 179\right)$$

$$V_{D,2} = 510*10^3 N$$

$$H_{D,2} = \frac{\begin{pmatrix} -q_{total,2}L_{bm}L_{cln}e_2\left[12e_2+24e_1+8z_2+2L_{bm}\right]EI_{cln,2} + q_{total,2}L_{cln}{}^2e_2\left[4L_{bm}-16e_1-8e_2\right]EI_{bm} \\ +24e_2EI_{bm}EI_{cln,2} + q_2L_{bm}L_{cln}\left[L_{bm}{}^2-2L_{bm}e_1-12e_1{}^2-4L_{bm}z_2-8e_1z_2\right]EI_{cln,2} \\ -q_2L_{cln}{}^2e_1\left[4L_{bm}+8e_1\right]EI_{bm} \end{pmatrix}}{12L_{bm}L_{cln}{}^2EI_{cln,2}+8L_{cln}{}^3EI_{bm}}$$

$$H_{D,2} = \frac{\begin{pmatrix} -632*5000*10000*179[12*179+24*96.5+8*39.6+2*5000]5.263*10^{13} \\ +632*10000^2*179[4*5000-16*96.5-8*179]8.864*10^{14} \\ +24*179*8.864*10^{14}*5.263*10^{13} \\ +153*5000*10000\begin{bmatrix}5000^2-2*5000*96.54-12*96.5^2 \\ -4*5000*39.6-8*96.5*39.6\end{bmatrix}5.263*10^{13} \\ -153*10000^2*96.5[4*5000+8*96.5]*8.864*1014 \end{pmatrix}}{12*5000*10000^2*5.263*10^{13}+8*10000^3*8.864*10^{14}}$$

$$H_{D,2} = \frac{-5.238*10^{31}}{7.407*10^{27}}$$

$$H_{D,2} = -7072N$$

Calculate the bending moment in point C.

$$M_{C,2} = q_2\left(\tfrac{1}{2}L_{bm}+e_1+e_2\right)\left(e_1+e_2+z_2\right)+q_1e_2\left(\tfrac{1}{2}L_{bm}+2e_1+e_2+z_2\right)+H_{A,2}L_{cln}$$

$$M_{C,2} = 153\left(\tfrac{1}{2}*5000+96.5+179\right)\left(96.5+179+39.6\right)$$

$$+479*179\left(\tfrac{1}{2}*5000+2*96.5+179+39.6\right)-7072*10000$$

$$M_{C,2} = 312.7*10^6\,Nmm$$

Calculate the stresses

$$\sigma_{right,2} = \sigma_{right,1}-\frac{V_{D,2}}{A_{cln,2}}-\frac{M_{C,2}}{Z_{cln,2}}$$

$$\sigma_{right,2} = -177.6-\frac{510*10^3}{11655}-\frac{312.7*10^6}{1.432*10^6}$$

$$\sigma_{right,2} = -439.7N/mm^2$$

$$\sigma_{left,2} = \sigma_{left,1}-\frac{V_{D,2}}{A_{cln,2}}+\frac{M_{C,2}}{Z_{cln,2}}$$

$$\sigma_{left,2} = 3.4-\frac{510*10^3}{11655}+\frac{312.7*10^6}{1.432*10^6}$$

$\sigma_{left,2} = 178.0N/mm^2$ Tension, first critical stress

$$\sigma_{centre,2} = \sigma_{centre,1}-\frac{V_{D,2}}{A_{cln,2}}$$

$$\sigma_{centre,2} = -87.1-\frac{510*10^3}{11655}$$

$$\sigma_{centre,2} = -130.9N/mm^2$$

Interesting is the left flange. Yielding due to tension stress. The tension stress due to the bending moment is larger than the compression stress due to the normal force. Both the right flange as the left flanges are partial yielded.

The third load case:
The calculation of this third load case starts at the calculation of the denominator of the formula of the extra deflection. The extra load of the third load case is not jet taken into account.

$$e_3 = \frac{numerator}{2\begin{bmatrix} -3L_{bm}{}^3L_{cln}q_{total}EI_{cln,1}EI_{cln,3} - 4L_{bm}{}^2L_{cln}{}^2q_{total}EI_{bm}\left(EI_{cln,1}+EI_{cln,3}\right) - 4L_{bm}L_{cln}{}^3q_{total}\left(EI_{bm}\right)^2 \\ +36L_{bm}EI_{bm}EI_{cln,1}EI_{cln,3} + 12L_{cln}\left(EI_{bm}\right)^2\left(EI_{cln,1}+EI_{cln,3}\right) \end{bmatrix}}$$

$$e_3 = \frac{numerator}{2\begin{bmatrix} -3*5000^3*10000*632*6.959*10^{13}*3.573*10^{13} \\ -4*5000^2*10000^2*632*8.864*10^{14}\left(6.959*1013+3.573*10^{13}\right) \\ -4*5000*10000^3*632\left(8.864*10^{14}\right)^2 \\ +36*5000*8.864*10^{14}*6.959*10^{13}*3.573*10^{13} \\ +12*10000\left(8.864*10^{14}\right)^2\left(6.959*10^{13}+3.573*10^{13}\right) \end{bmatrix}}$$

$$e_3 = \frac{numerator}{-4.01*10^{47}}$$

The denominator of the additional deflection formula is negative. A positive denominator means that an extra positive load results in an extra positive deflection. If the denominator becomes zero, the deflection of the column will be infinity. The column fails. If the denominator is negative, the column cannot resist the load. The denominator can only be negative at the shift to another (reduced) stiffness. The reduced column should fail at a lower load. The reduction of the stiffness is too much. The column fails at the maximum load in the second load case.

The ultimate load is 632 N/mm (=kN/m)

It is assumed that the column is critical. To check this assumption, the stress of the beam must be calculated.

$$f_y \leq \frac{\frac{1}{8}\left(q_1+q_2\right)L_{bm}{}^2}{Z_{bm}} - \frac{M_{top,2}}{Z_{bm}}$$

$$f_y \leq \frac{\frac{1}{8}*632*5000^2}{9.485*10^6} - \frac{\left(171.1+312.7\right)*10^6}{9.485*10^6}$$

$f_y \leq 157.2N/mm^2$ The assumption is correct.

L.2 *Computer calculation file*

The formulas of Appendix J and Appendix K are very complex. It is very difficult and time-consuming to make these calculations manually. The computer program MatLab has been used to use the make iterations and to calculate the ultimate load. The calculation file can been found in this Appendix.

```
clear; clf; clc; close;
E=210000;
fy=355;
qq=1;%belastingsstap
psi=0.004;
Lbm=5000;
Lcln=10000;

%input profiles
HEA=[2.124E+03  2.534E+03   3.142E+03   3.877E+03   4.525E+03   5.383E+03
6.434E+03   7.684E+03   8.682E+03   9.726E+03   1.125E+04   1.244E+04
1.335E+04   1.428E+04   1.590E+04   1.780E+04   1.975E+04   2.118E+04
2.265E+04   2.416E+04   2.605E+04   2.858E+04   3.205E+04   3.468E+04;%A
(cross-section)

3.492E+06   6.062E+06   1.033E+07   1.673E+07   2.510E+07   3.692E+07
5.410E+07   7.763E+07   1.046E+08   1.367E+08   1.826E+08   2.293E+08
2.769E+08   3.309E+08   4.507E+08   6.372E+08   8.698E+08   1.119E+09
1.412E+09   1.752E+09   2.153E+09   3.034E+09   4.221E+09   5.538E+09;%I
(moment of inertia)

8.301E+04   1.195E+05   1.735E+05   2.451E+05   3.249E+05   4.295E+05
5.685E+05   7.446E+05   9.198E+05   1.112E+06   1.383E+06   1.628E+06
1.850E+06   2.088E+06   2.562E+06   3.216E+06   3.949E+06   4.622E+06
5.350E+06   6.136E+06   7.032E+06   8.699E+06   1.081E+07   1.282E+07;%Z
plastic (Section modulus)

7.276E+04   1.063E+05   1.554E+05   2.201E+05   2.936E+05   3.886E+05
5.152E+05   6.751E+05   8.364E+05   1.013E+06   1.260E+06   1.479E+06
1.678E+06   1.891E+06   2.311E+06   2.896E+06   3.550E+06   4.146E+06
4.787E+06   5.474E+06   6.241E+06   7.682E+06   9.485E+06   1.119E+07;%Z
elastic (Section modulus)

4.055E+01   4.891E+01   5.734E+01   6.569E+01   7.448E+01   8.282E+01
9.170E+01   1.005E+02   1.097E+02   1.186E+02   1.274E+02   1.358E+02
1.440E+02   1.522E+02   1.684E+02   1.892E+02   2.099E+02   2.299E+02
2.497E+02   2.693E+02   2.875E+02   3.258E+02   3.629E+02   3.996E+02;%i
(Gyration radius)

96 114 133 152 171 190 210 230 250 270 290 310 330 350 390 440 490 540 590
640 690 790 890 990;%h height

100 120 140 160 180 200 220 240 260 280 300 300 300 300 300 300 300 300 300
300 300 300 300 300;%b width

8 8 8.5 9 9.5 10 11 12 12.5 13 14 15.5 16.5 17.5 19 21 23 24 25 26 27 28 30
31;%tf thickness flange

5 5 5.5 6 6 6.5 7 7.5 7.5 8 8.5 9 9.5 10 11 11.5 12 12.5 13 13.5 14.5 15 16
16.5];%tw thickness web

cln=14;
    Acln(1,1)=HEA(1,cln);
    Icln(1,1)=HEA(2,cln);
    Zcln(1,1)=HEA(4,cln);
    hcln=HEA(6,cln);
    bcln=HEA(7,cln);
    tfcln=HEA(8,cln);
    twcln=HEA(9,cln);
```

```
bm=23;
    Abm(1,1)=HEA(1,bm);
    Ibm(1,1)=HEA(2,bm);
    Zbm(1,1)=HEA(4,bm);

    Npcln=Acln(1,1)*fy;
    Mpcln=Zcln(1,1)*fy;

     if cln<=14 ;
         S=0.5;
         ak=0.34;
     else
         if cln<=24;
             S=0.3;
             ak=0.21;
         else
             if cln<=38;
                 S=0.5;
                 ak=0.49;
             else
                 S=0.3;
                 ak=0.34;
             end% if
         end% if
     end% if
    yield1=-(1-S)*fy;
    yield2=-(1+S)*fy;

Acln(1,2)=Acln(1,1)-2*(0.25*bcln)*tfcln;
Acln(1,3)=Acln(1,2)-2*(0.25*bcln)*tfcln;
Icln(1,2)=Icln(1,1)-2*(0.25*bcln)*tfcln*(0.5*hcln)^2;
Icln(1,3)=Icln(1,2)-2*(0.25*bcln)*tfcln*(0.5*hcln)^2;
Zcln(1,2)=2*Icln(1,2)/hcln;
Zcln(1,3)=2*Icln(1,3)/hcln;

z(1,1)=0;
z(1,2)=(0.5*bcln*tfcln*0.5*tfcln+(hcln-
2*tfcln)*twcln*0.5*hcln+bcln*tfcln*(hcln-0.5*tfcln))/(0.5*bcln*tfcln+(hcln-
2*tfcln)*twcln+bcln*tfcln)-0.5*hcln;
z(1,3)=0;

sigmatop(1,1)=0;
q1=0;
while sigmatop(1,1)>yield1;
    q1=q1+1;
    Q1(q1)=q1*qq;
    Q(q1)=Q1(q1);
    Qmax(1,1)=max(Q1);
     hulp1(q1)=12*psi*Lcln*E*Ibm*E*Icln(1,1);
     hulp2(q1)=12*E*Ibm*E*Icln(1,1)-Q(q1)*Lbm^2*Lcln*E*Icln(1,1)-
2*Q(q1)*Lbm*Lcln^2*E*Ibm;
    e1(q1)=hulp1(q1)/hulp2(q1);
    e(q1)=e1(q1);
    emax(1,1)=max(e);


hulp3(q1)=Q(q1)*Lbm^3*Lcln*E*Icln(1,1)/(12*Lbm*Lcln^2*E*Icln(1,1)+8*Lcln^3*
E*Ibm(1,1));
```

```
    hulp4(q1)=12*psi*E*Ibm(1,1)*E*Icln(1,1)/(12*E*Ibm(1,1)*E*Icln(1,1)-
Q(q1)*Lbm^2*Lcln*E*Icln(1,1)-2*Q(q1)*Lbm*Lcln^2*E*Ibm(1,1));
    HD(q1)=hulp3(q1)-Q(q1)*Lcln*hulp4(q1)^2;

    VD(q1)=Q(q1)*(0.5*Lbm+e(q1));
    VA(q1)=Q(q1)*(0.5*Lbm-e(q1));
    MC(q1)=VD(q1)*e(q1)+HD(q1)*Lcln;

    sigmatop(1,1)=-MC(q1)/Zcln(1,1)-VD(q1)/Acln(1,1);
    sigmatop1(1,1)=sigmatop(1,1);
    sigmabottom(1,1)=MC(q1)/Zcln(1,1)-VD(q1)/Acln(1,1);
    sigmabottom1(1,1)=sigmabottom(1,1);

end% while

hulpa=12*Lcln*E*Ibm(1,1)*E*Ibm(1,1)*E*(Icln(1,1)+Icln(1,2)) +
36*Lbm*E*Ibm(1,1)*E*Icln(1,1)*E*Icln(1,2) -
(Qmax+2*qq)*Lcln*(3*Lbm^2*(Lbm+z(1,2))*E*Icln(1,1)*E*Icln(1,2) +
2*Lbm*Lcln*(2*Lbm*E*Icln(1,1)+2*Lbm*E*Icln(1,2)+2*Lcln*E*Ibm(1,1)-
z(1,2)*E*Icln(1,1)+2*z(1,2)*E*Icln(1,2))*E*Ibm(1,1));

q2=0;
if hulpa>0;
    while sigmabottom(1,1)<-yield1 & sigmatop(1,1)>yield2;
        q2=q2+1;
        Q2(q2)=q2*qq;
        Q(q1+q2)=Q1(q1)+Q2(q2);
        Qmax(1,1)=max(Q);
         hulp5(q2)=Q2(q2)*Lbm*Lcln*(6*Lbm^2*e(q1)*E*Icln(1,1)*E*Icln(1,2) +
3*Lbm^2*z(1,2)*E*Icln(1,1)*E*Icln(1,2) +
+6*Lbm*e(q1)*z(1,2)*E*Icln(1,1)*E*Icln(1,2) +
8*Lbm*Lcln*e(q1)*E*Ibm(1,1)*E*(Icln(1,1)+Icln(1,2)) +
Lbm^2*Lcln*E*Ibm(1,1)*E*(Icln(1,1)-Icln(1,2)) +
8*Lcln^2*e(q1)*E*Ibm(1,1)*E*Ibm(1,1) +
2*Lcln*z(1,2)*(Lbm+2*e(q1))*E*Ibm*E*(-Icln(1,1)+2*Icln(1,2)));
         hulp6(q2)=2*(-3*Q(q1+q2)*Lbm^3*Lcln*E*Icln(1,1)*E*Icln(1,2) -
3*Q(q1+q2)*Lbm^2*Lcln*z(1,2)*E*Icln(1,1)*E*Icln(1,2) -
4*Q(q1+q2)*Lbm^2*Lcln^2*E*Ibm(1,1)*E*(Icln(1,1)+Icln(1,2)) -
2*Q(q1+q2)*Lbm*Lcln^2*z(1,2)*E*Ibm(1,1)*E*(-Icln(1,1)+2*Icln(1,2)) -
4*Q(q1+q2)*Lbm*Lcln^3*E*Ibm(1,1)*E*Ibm(1,1) +
12*Lcln*E*Ibm(1,1)*E*Ibm(1,1)*E*(Icln(1,1)+Icln(1,2)) +
36*Lbm*E*Ibm(1,1)*E*Icln(1,1)*E*Icln(1,2));
        e2(q2)=hulp5(q2)/hulp6(q2);
        e(q1+q2)=e1(q1)+e2(q2);
        emax(1,1)=max(e);

         hulp7(q2)=Q2(q2)*Lbm^3*Lcln*E*Icln(1,2) -
2*Q2(q2)*Lbm^2*Lcln*e(q1)*E*Icln(1,2) -
2*Q2(q2)*Lbm^2*Lcln*e2(q2)*E*Icln(1,2) -
12*Q2(q2)*Lbm*Lcln*e(q1)*e(q1)*E*Icln(1,2)         -
24*Q2(q2)*Lbm*Lcln*e(q1)*e2(q2)*E*Icln(1,2) -
12*Q2(q2)*Lbm*Lcln*e2(q2)*e2(q2)*E*Icln(1,2) -
2*Q(q1)*Lbm^2*Lcln*e2(q2)*E*Icln(1,2) -
24*Q(q1)*Lbm*Lcln*e(q1)*e2(q2)*E*Icln(1,2)          -
12*Q(q1)*Lbm*Lcln*e2(q2)*e2(q2)*E*Icln(1,2) -
4*Q2(q2)*Lbm^2*Lcln*z(1,2)*E*Icln(1,2) -
8*Q2(q2)*Lbm*Lcln*e(q1)*z(1,2)*E*Icln(1,2) -
8*Q2(q2)*Lbm*Lcln*z(1,2)*e2(q2)*E*Icln(1,2)          -
8*Q(q1)*Lbm*Lcln*z(1,2)*e2(q2)*E*Icln(1,2) -
4*Q2(q2)*Lbm*Lcln^2*e(q1)*E*Ibm(1,1) -
```

```
4*Q2(q2)*Lbm*Lcln^2*e2(q2)*E*Ibm(1,1) -
8*Q2(q2)*Lcln^2*e(q1)*e(q1)*E*Ibm(1,1) -
16*Q2(q2)*Lcln^2*e(q1)*e2(q2)*E*Ibm(1,1)          -
8*Q2(q2)*Lcln^2*e2(q2)*e2(q2)*E*Ibm(1,1) -
4*Q(q1)*Lbm*Lcln^2*e2(q2)*E*Ibm(1,1) -
16*Q(q1)*Lcln^2*e(q1)*e2(q2)*E*Ibm(1,1) -
8*Q(q1)*Lcln^2*e2(q2)*e2(q2)*E*Ibm(1,1) + 24*e2(q2)*E*Ibm(1,1)*E*Icln(1,2);
         hulp8(q2)=12*Lbm*Lcln^2*E*Icln(1,2)+8*Lcln^3*E*Ibm;
        deltaHD(q2)=hulp7(q2)/hulp8(q2);
        HD(q1+q2)=HD(q1)+deltaHD(q2);
        deltaVD(q2)=Q(q1)*e2(q2)+Q2(q2)*(0.5*Lbm+e(q1+q2));
        VD(q1+q2)=VD(q1)+deltaVD(q2);
        deltaVA(q2)=-Q(q1)*e2(q2)+Q2(q2)*(0.5*Lbm-e(q1+q2));
        VA(q1+q2)=VA(q1)+deltaVA(q2);
        deltaMC(q2)=Q2(q2)*(0.5*Lbm+e(q1+q2))*(e(q1+q2)+z(1,2)) +
Q(q1)*e2(q2)*(0.5*Lbm+2*e(q1)+e2(q2)+z(1,2)) + deltaHD(q2)*Lcln;
        MC(q1+q2)=MC(q1)+deltaMC(q2);

        sigmatop(1,1)=sigmatop1(1,1)-deltaMC(q2)/Zcln(1,2)-
deltaVD(q2)/Acln(1,2);
        sigmatop2(1,1)=sigmatop(1,1);
        sigmabottom(1,1)=sigmabottom1(1,1)+deltaMC(q2)/Zcln(1,2)-
deltaVD(q2)/Acln(1,2);
        sigmabottom2(1,1)=sigmabottom(1,1);

    end%while

else
    Q2=0;
    e2=0;
    sigmatop2=sigmatop1;
    sigmabottom2=sigmabottom1;
    Qmax=Qmax;
end%if

hulpb=-3*Lbm^3*Lcln*(Q(q1+q2)+2*qq)*E*Icln(1,1)*E*Icln(1,3) -
4*Lbm^2*Lcln^2*(Q(q1+q2)+2*qq)*E*Ibm(1,1)*E*(Icln(1,1)+Icln(1,3)) -
4*Lbm*Lcln^3*(Q(q1+q2)+2*qq)*E*Ibm(1,1)*E*Ibm(1,1) +
36*Lbm*E*Ibm(1,1)*E*Icln(1,1)*E*Icln(1,3) +
12*Lcln*E*Ibm(1,1)*E*Ibm(1,1)*E*(Icln(1,1)+Icln(1,3)) ;

q3=0;
if hulpb>0;
    while sigmatop(1,1)>yield2;
        q3=q3+1;
        Q3(q3)=q3*qq;
        Q(q1+q2+q3)=Q1(q1)+Q2(q2)+Q3(q3);
        Qmax(1,1)=max(Q);

hulp9(q3)=Q3(q3)*Lbm*Lcln*(6*Lbm^2*e(q1+q2)*E*Icln(1,1)*E*Icln(1,3) +
8*Lbm*Lcln*e(q1+q2)*E*Ibm(1,1)*E*(Icln(1,1)+Icln(1,3)) +
8*Lcln^2*e(q1+q2)*E*Ibm(1,1)*E*Ibm(1,1) +
Lbm^2*Lcln*E*Ibm(1,1)*E*(Icln(1,1)-Icln(1,3)));
         hulp10(q3)=2*(-3*Lbm^3*Lcln*Q(q1+q2+q3)*E*Icln(1,1)*E*Icln(1,3) -
4*Lbm^2*Lcln^2*Q(q1+q2+q3)*E*Ibm(1,1)*E*(Icln(1,1)+Icln(1,3)) -
4*Lbm*Lcln^3*Q(q1+q2+q3)*E*Ibm(1,1)*E*Ibm(1,1) +
36*Lbm*E*Ibm(1,1)*E*Icln(1,1)*E*Icln(1,3) +
12*Lcln*E*Ibm(1,1)*E*Ibm(1,1)*E*(Icln(1,1)+Icln(1,3)));
        e3(q3)=hulp9(q3)/hulp10(q3);
        e(q1+q2+q3)=e1(q1)+e2(q2)+e3(q3);
```

```
        emax(1,1)=max(e);

         hulp11(q3)=-2*Q(q1+q2+q3)*Lbm^2*Lcln*e3(q3)*E*Icln(1,3) -
24*Q(q1+q2+q3)*Lbm*Lcln*e(q1+q2)*e3(q3)*E*Icln(1,3) -
12*Q(q1+q2+q3)*Lbm*Lcln*e3(q3)*e3(q3)*E*Icln(1,3)          -
2*Q3(q3)*Lbm^2*Lcln*e(q1+q2)*E*Icln(1,3) -
12*Q3(q3)*Lbm*Lcln*e(q1+q2)*e(q1+q2)*E*Icln(1,3) +
Q3(q3)*Lbm^3*Lcln*E*Icln(1,3)          -
4*Q(q1+q2+q3)*Lbm*Lcln^2*e3(q3)*E*Ibm(1,1) -
16*Q(q1+q2+q3)*Lcln^2*e3(q3)*e(q1+q2)*E*Ibm(1,1) -
8*Q(q1+q2+q3)*Lcln^2&e3(q3)*e3(q3)*E*Ibm(1,1)         -
4*Q3(q3)*Lbm*Lcln^2*e(q1+q2)*E*Ibm(1,1) -
8*Q3(q3)*Lcln^2*e(q1+q2)*e(q1+q2)*E*Ibm(1,1) +
24*e3(q3)*E*Ibm(1,1)*E*Icln(1,3);
         hulp12(q3)=12*Lbm*Lcln^2*E*Icln(1,3)+8*Lcln^3*E*Ibm;
        deltaHD(q3)=hulp11(q3)/hulp12(q3);
        HD(q1+q2+q3)=HD(q1+q2)+deltaHD(q3);
        deltaVD(q3)=Q(q1+q2)*e3(q3)+Q3(q3)*(0.5*Lbm+e(q1+q2+q3));
        VD(q1+q2+q3)=VD(q1+q2)+deltaVD(q3);
        deltaMC(q3)=e3(q3)*Q(q1+q2)*(0.5*Lbm+2*e(q1+q2)+e3(q3)) +
Q3(q3)*(0.5*Lbm+e(q1+q2+q3))*e(q1+q2+q3) + deltaHD(q3)*Lcln;
        MC(q1+q2+q3)=MC(q1+q2)+deltaMC(q3);
        sigmatop(1,1)=sigmatop2(1,1)-deltaMC(q3)/Zcln(1,3)-
deltaVD(q3)/Acln(1,3);
        sigmatop3(1,1)=sigmatop(1,1);
        sigmabottom(1,1)=sigmabottom2(1,1)+deltaMC(q3)/Zcln(1,3)-
deltaVD(q3)/Acln(1,3);
        sigmabottom3(1,1)=sigmabottom(1,1);

    end%while


else
    Q3=0;
    e3=0;
    sigmatop3=sigmatop2;
    sigmabottom3=sigmabottom2;
    Qmax=Qmax;
end%if


clear hulp1;clear hulp2;clear hulp3;clear hulp4;clear hulp5;clear hulp6;
clear hulp7;clear hulp8;clear hulp9;clear hulp10;clear hulp11;clear hulp12;
clear hulpa;clear hulpb;clear qq;
clear ak;clear cln;clear bm;
clear deltaHD;clear deltaMC;clear deltaVA;clear deltaVD;
clear Q1;clear Q2;clear Q3;clear e1;clear e2;clear e3;

Qmax
%plot(e,Q);xlabel('deflection in the midsection (mm)');ylabel('load q
(kN/m)');title('Length is 10 meter');grid
```

L.3 Calculation according to Matrix Frame

The calculation has also been made by another computer calculation. This calculation is based on the finite element method. This computer program (Matrix Frame) has been used as reference. The calculation file of Matrix Frame can be found in this Appendix.

Appendix M Linear analysis braced portal frame

Appendices G till L were related to an unbraced portal frame. The main subject of this Appendix is a braced portal frame. The main difference between a braced and an unbraced portal frame is the support in lateral direction. This Appendix is about the linear analysis of a braced portal frame. The load-deflection graphic of the linear analysis can be found in Figure M.1.

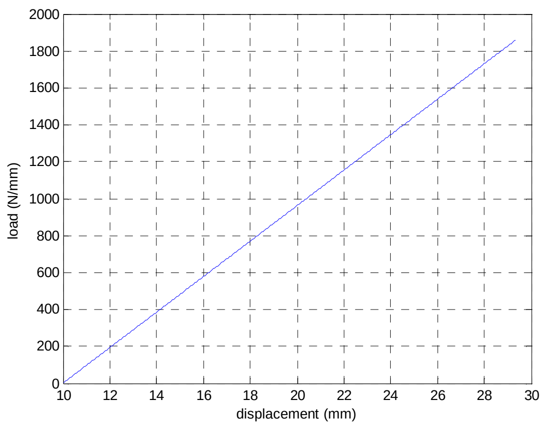


Figure M.1: Load-deflection Graphic

Column: Length 5 m
HE 360A
Beam: Length 10 m
HE 900A

The portal frame is a two degrees statically undetermined structure. Fortunately the portal frame is symmetric. This is the case in a straight situation but also in the deformed situation (Fig. M.2). Because of symmetry the horizontal reaction force in support C is zero. The bending moments in point B and in point C are equal.

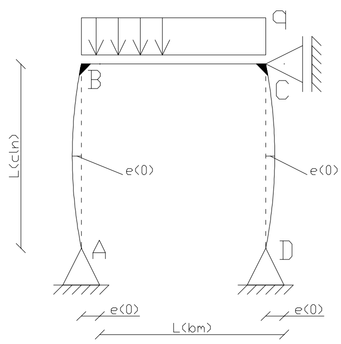


Figure M.2: Structure

The beam of the portal frame can be schematized as a beam supported on two supports. This beam is loaded by an uniformly distributed load and a bending moment at both ends (Fig. M.3). The rotation of both ends can be calculated by the following formulas:

$$\varphi_1 = \frac{qL_{bm}{}^3 L_{cln}}{24EI_{bm}}$$ (rotation due to uniformly distributed load)

$$\varphi_2 = -\frac{ML_{bm}}{3EI_{bm}} - \frac{ML_{bm}}{6EI_{bm}}$$ (rotation due to

$$\varphi_2 = -\frac{ML_{bm}}{2EI_{bm}}$$ bending moment)

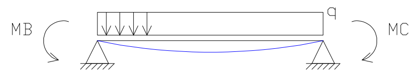


Figure M.3: Beam

The rotation due to the uniformly distributed load can be calculated easily. The rotation due to the bending moments cannot be calculated that easy. The bending moments depend on the stiffness of the column and the stiffness of the beam. The bending moments are unknown (and desired) values. To calculate these bending moments a better look at the column must be made (Fig. M.4). The column is connected to the beam. The bending moment in B is the multiplication of the horizontal reaction force in A and the column length. Point B is connected to the beam and cannot rotate free. The influence of the beam is taken into account by a rotation spring. The stiffness of this spring is $\frac{2EI_{bm}}{L_{bm}}$ (according to φ_2).

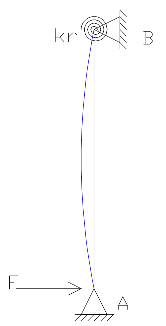


Figure M.4: Column

To make calculations, the support is schematized as a free end with a horizontal force. The total displacement of point A must be zero.

The deformation of point A (called δ) has been split in three parts. The summation of these parts is zero.

$\delta_1 = \frac{qL_{bm}{}^3 L_{cln}}{24EI_{bm}}$ Deflection due to the rotation of the beam.

$\delta_2 = -\frac{FL_{cln}{}^3}{3EI_{cln}}$ Deflection due to the force F.

$\delta_3 = -\frac{ML_{cln}}{k_r} = -\frac{FL_{bm}L_{cln}{}^2}{2EI_{bm}}$ Deflection due to the rotation spring.

$$\frac{qL_{bm}{}^3 L_{cln}}{24EI_{bm}} - \frac{FL_{bm}L_{cln}{}^2}{2EI_{bm}} - \frac{FL_{cln}{}^3}{3EI_{cln}} = 0$$

The unknown value is F. This value will be placed to the left side of the equation. All expressions without the value F will be placed to the right side.

$$\frac{FL_{bm}L_{cln}{}^2}{2EI_{bm}} + \frac{FL_{cln}{}^3}{3EI_{cln}} = \frac{qL_{bm}{}^3 L_{cln}}{24EI_{bm}}$$

Make everywhere the same denominator.

$$\frac{12FL_{bm}L_{cln}{}^2 EI_{cln} + 8FL_{cln}{}^3 EI_{bm}}{24EI_{cln}EI_{bm}} = \frac{qL_{bm}{}^3 L_{cln}EI_{cln}}{24EI_{cln}EI_{bm}}$$

Neglect the denominator.

$$F\left(12L_{bm}L_{cln}{}^2 EI_{cln} + 8L_{cln}{}^3 EI_{bm}\right) = qL_{bm}{}^3 L_{cln}EI_{cln}$$

The formula of F can be found.

$$F = \frac{qL_{bm}{}^3 EI_{cln}}{4L_{cln}\left(3L_{bm}EI_{cln} + 2L_{cln}EI_{bm}\right)}$$

F is the horizontal reaction force of support A. The portal frame is symmetric and this can be used to determine all reaction forces.

$H_A = \frac{qL_{bm}{}^3 EI_{cln}}{4L_{cln}\left(3L_{bm}EI_{cln} + 2L_{cln}EI_{bm}\right)}$

$V_A = 0.5qL_{bm}$

$H_D = \frac{qL_{bm}{}^3 EI_{cln}}{4L_{cln}\left(3L_{bm}EI_{cln} + 2L_{cln}EI_{bm}\right)}$

$V_D = 0.5qL_{bm}$

With these reaction forces all values can be found.

$M_B = H_A L_{cln}$ (moment in B)

M_B = $\dfrac{qL_{bm}{}^3 EI_{cln}}{4L_{cln}\left(3L_{bm}EI_{cln} + 2L_{cln}EI_{bm}\right)} L_{cln}$

M_B = $\dfrac{qL_{bm}{}^3 EI_{cln}}{4\left(3L_{bm}EI_{cln} + 2L_{cln}EI_{bm}\right)}$

e = $\dfrac{M_B L_{cln}{}^2}{16EI_{cln}}$ (total deflection, in the middle of the column)

e = $\dfrac{qL_{bm}{}^3 EI_{cln}}{4\left(3L_{bm}EI_{cln} + 2L_{cln}EI_{bm}\right)} \dfrac{L_{cln}{}^2}{16EI_{cln}}$

e = $\dfrac{qL_{bm}{}^3 L_{cln}{}^2}{64\left(3L_{bm}EI_{cln} + 2L_{cln}EI_{bm}\right)}$

In the linear analysis, the total deflection does not have any influence on the reaction forces or to the internal moment distribution. This influence is taken into account at the second order analysis. The second order analysis is made in Appendix N. In the linear analysis the critical cross-section is point B. At this point both the bending moment and the normal force are largest. The unity check must be made as this point.

Appendix N Non-linear analysis braced portal frame

Appendix M was a linear analysis of a portal frame. In this appendix a non-linear analyses has been made. The non-linear analysis is an extension of the linear analysis. The load-deflection graphic of the non-linear analysis can be found in Figure N.1.

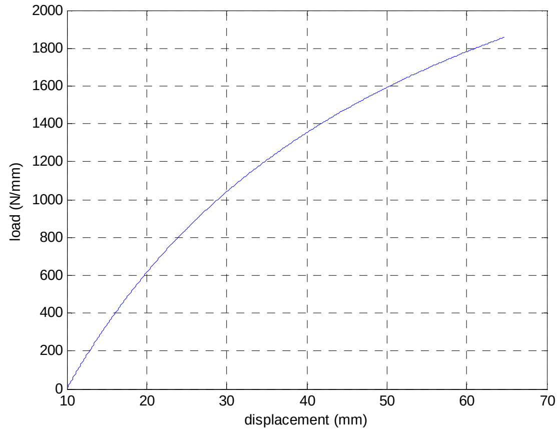


Figure N.1: Load-deflection Graphic

Column: Length 10m HE 360A
Beam: Length 5 m HE 900A

In the linear analysis the largest bending moment was in point B. Point B was the critical cross-section of the portal frame. In the non-linear analysis, the internal moments depend on the deflection of the structure. The displacement of point B is zero. It is possible that the maximum bending moment is located elsewhere in the column. There are two logical location for the maximum bending moment: point B and the middle of the column. It is not clear which is critical so both locations must be calculated.

At the non-linear analysis the deflection influences the reaction forces. To start the non-linear analysis the formulas of the deflection has been derived. The total deflection of the column is split in four parts. These four parts are:

1. Deflection due to moment (linear analysis)
2. Starting deflection
3. Deflection due to the rotation spring
4. Additional deflection

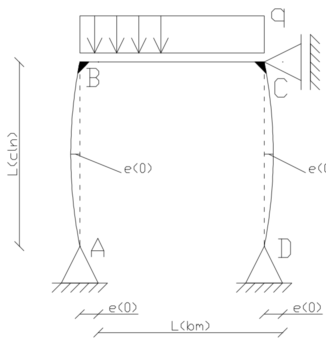


Figure N.2: Structure

The calculation of the reaction force (as result of the linear analysis) has been analysed in Appendix M. The result of this analyse is:

$$H_A = F = \frac{qL_{bm}{}^3 EI_{cln}}{4L_{cln}\left(3L_{bm}EI_{cln} + 2L_{cln}EI_{bm}\right)}$$

This reaction force is the linear reaction force. The influence of the additional deformation is not (yet) taken into account.

The four deflection parts are analysed.

Deflection due to moment

$$M_1 = F\left(L_{cln} - x\right)$$

$$\kappa_1 = \frac{-F\left(L_{cln} - x\right)}{EI_{cln}}$$

$$\varphi_1 = \frac{-F\left(2L_{cln}x - x^2\right)}{2EI_{cln}} + C_1$$

$$y_1 = \frac{-F\left(3L_{cln}x^2 - x^3\right)}{6EI_{cln}} + C_1 x + C_2$$

Use the boundary conditions to solve the integral constants.

$y_{1,x=0} = 0$ → $C_2 = 0$

$y_{1,x=L_{cln}} = 0$ → $C_1 = \frac{2FL_{cln}{}^2}{6EI_{cln}}$

The formula of the deflection is:

$$y_1 = \frac{F\left(2L_{cln}{}^2 x - 3L_{cln}x^2 + x^3\right)}{6EI_{cln}}$$

The formula of F is known and can be used.

$$y_1 = \frac{qL_{bm}{}^3 EI_{cln}}{4L_{cln}\left(3L_{bm}EI_{cln} + 2L_{cln}EI_{bm}\right)} \frac{\left(2L_{cln}{}^2 x - 3L_{cln}x^2 + x^3\right)}{6EI_{cln}}$$

$$y_1 = \frac{qL_{bm}{}^3\left(2L_{cln}{}^2 x - 3L_{cln}x^2 + x^3\right)}{24L_{cln}\left(3L_{bm}EI_{cln} + 2L_{cln}EI_{bm}\right)}$$

The deflection of this load type is positive for: $0 \le x \le L_{cln}$

Starting deflection

$$y_2 = e_0 \sin\left(\frac{\pi x}{L_{cln}}\right)$$

Deflection due to rotation spring:

$$M_3 = -\varphi k_r$$

The deflection of the rotation spring is influenced by the aditional deflection. The original deflection is already taken into account at the calculation of the force F.

$$M_{3,x=0} = -\varphi_{extra,x=0} k_r$$

The moment depends on the position in the column.

$$M_3 = \frac{-\varphi_{extra,x=0} k_r \left(L_{cln} - x\right)}{L_{cln}}$$

$$\kappa_3 = \frac{\varphi_{extra,x=0} k_r \left(L_{cln} - x\right)}{L_{cln}EI_{cln}}$$

$$\varphi_3 = \frac{\varphi_{extra,x=0} k_r \left(2L_{cln}x - x^2\right)}{2L_{cln}EI_{cln}} + C_1$$

$$y_3 = \frac{\varphi_{extra,x=0} k_r \left(3L_{cln}x^2 - x^3\right)}{6L_{cln}EI_{cln}} + C_1 x + C_2$$

Use the boundary conditions to solve the integral constants.

$y_{3,x=0} = 0$ → $C_2 = 0$

$y_{3,x=L_{cln}} = 0$ → $C_1 = \frac{-2\varphi_{extra,x=0} k_r L_{cln}^{\ 2}}{6L_{cln}EI_{cln}}$

$$y_3 = \frac{\varphi_{extra,x=0} k_r \left(-2L_{cln}^{\ 2}x + 3L_{cln}x^2 - x^3\right)}{6L_{cln}EI_{cln}}$$

Use the formula of k_r in this formula.

$$y_3 = \frac{\varphi_{extra,x=0} \frac{2EI_{bm}}{L_{bm}} \left(-2L_{cln}^{\ 2}x + 3L_{cln}x^2 - x^3\right)}{6L_{cln}EI_{cln}}$$

$$y_3 = \frac{\varphi_{extra,x=0} EI_{bm} \left(-2L_{cln}^{\ 2}x + 3L_{cln}x^2 - x^3\right)}{3L_{bm}L_{cln}EI_{cln}}$$

This deflection is negative.

Extra deflection

$$y_4 = e_1 \sin\left(\frac{\pi x}{L_{cln}}\right)$$

Summation of all parts of the total deflection.

$$y_1 = \frac{qL_{bm}^{\ 3}\left(2L_{cln}^{\ 2}x - 3L_{cln}x^2 + x^3\right)}{24L_{cln}\left(3L_{bm}EI_{cln} + 2L_{cln}EI_{bm}\right)}$$

$$y_2 = e_0 \sin\left(\frac{\pi x}{L_{cln}}\right)$$

$$y_3 = \frac{\varphi_{extra,x=0} EI_{bm} \left(-2L_{cln}^{\ 2}x + 3L_{cln}x^2 - x^3\right)}{3L_{bm}L_{cln}EI_{cln}}$$

$$y_4 = e_1 \sin\left(\frac{\pi x}{L_{cln}}\right)$$

The deflections y_1 and y_2 are the known deflections. These deflections are the same as deflection y_3 and $y_4$in the linear analysis. The additional deflections y_3 and y_4 are important to calculate the additional rotation. The derivatives of these deflections are:

$$y_3 = \frac{\varphi_{extra,x=0} EI_{bm}\left(-2L_{cln}{}^2 x + 3L_{cln}x^2 - x^3\right)}{3L_{bm}L_{cln}EI_{cln}}$$

$$\varphi_3 = \frac{\varphi_{extra,x=0} EI_{bm}\left(-2L_{cln}{}^2 + 6L_{cln}x - 3x^2\right)}{3L_{bm}L_{cln}EI_{cln}}$$

$$\kappa_3 = \frac{\varphi_{extra,x=0} EI_{bm}\left(6L_{cln} - 6x\right)}{3L_{bm}L_{cln}EI_{cln}}$$

$$y_4 = e_1 \sin\left(\frac{\pi x}{L_{cln}}\right)$$

$$\varphi_4 = e_1 \frac{\pi}{L_{cln}} \cos\left(\frac{\pi x}{L_{cln}}\right)$$

$$\kappa_4 = -e_1 \frac{\pi^2}{L_{cln}{}^2} \sin\left(\frac{\pi x}{L_{cln}}\right)$$

The additional rotation can be calculated.

$$\varphi_{extra,x=0} = \varphi_{3,x=0} + \varphi_{4,x=0}$$

$$\varphi_{extra,x=0} = \frac{\varphi_{extra,x=0} EI_{bm}\left(-2L_{cln}{}^2 + 6L_{cln} * 0 - 3 * 0^2\right)}{3L_{bm}L_{cln}EI_{cln}} + e_1 \frac{\pi}{L_{cln}} \cos\left(\frac{\pi 0}{L_{cln}}\right)$$

Neglect some expressions.

$$\varphi_{extra,x=0} = \frac{-2L_{cln}{}^2 \varphi_{extra,x=0} EI_{bm}}{3L_{bm}L_{cln}EI_{cln}} + e_1 \frac{\pi}{L_{cln}}$$

There are two unknown values ($\varphi_{extra,x=0}$ and e_1). The expressions of $\varphi_{extra,x=0}$ will be placed on the left side of the equation and the expression of e_1 will be placed on the right side.

$$\varphi_{extra,x=0} + \frac{2L_{cln}{}^2 \varphi_{extra,x=0} EI_{bm}}{3L_{bm}L_{cln}EI_{cln}} = e_1 \frac{\pi}{L_{cln}}$$

Make everywhere the same denominator.

$$\varphi_{extra,x=0}\left(\frac{3L_{bm}L_{cln}EI_{cln} + 2L_{cln}{}^2 EI_{bm}}{3L_{bm}L_{cln}EI_{cln}}\right) = \frac{3\pi L_{bm} e_1 EI_{cln}}{3L_{bm}L_{cln}EI_{cln}}$$

The formula of $\varphi_{extra,x=0}$ can be described as function of e_1.

$$\varphi_{extra,x=0} = \frac{3\pi L_{bm} e_1 EI_{cln}}{L_{cln}\left(3L_{bm}EI_{cln} + 2L_{cln}EI_{bm}\right)}$$

This formula can be used for the deflection y_3.

$$y_3 = \frac{\varphi_{extra,x=0} EI_{bm}\left(-2L_{cln}^{\ 2}x + 3L_{cln}x^2 - x^3\right)}{3L_{bm}L_{cln}EI_{cln}}$$

$$y_3 = \frac{3\pi L_{bm} e_1 EI_{cln}}{L_{cln}\left(3L_{bm}EI_{cln} + 2L_{cln}EI_{bm}\right)} \frac{EI_{bm}\left(-2L_{cln}^{\ 2}x + 3L_{cln}x^2 - x^3\right)}{3L_{bm}L_{cln}EI_{cln}}$$

This results in the following formulas.

$$y_3 = \frac{\pi e_1 EI_{bm}\left(-2L_{cln}^{\ 2}x + 3L_{cln}x^2 - x^3\right)}{L_{cln}^{\ 2}\left(3L_{bm}EI_{cln} + 2L_{cln}EI_{bm}\right)}$$

$$\varphi_3 = \frac{\pi e_1 EI_{bm}\left(-2L_{cln}^{\ 2} + 6L_{cln}x - 3x^2\right)}{L_{cln}^{\ 2}\left(3L_{bm}EI_{cln} + 2L_{cln}EI_{bm}\right)}$$

$$\kappa_3 = \frac{\pi e_1 EI_{bm}\left(6L_{cln} - 6x\right)}{L_{cln}^{\ 2}\left(3L_{bm}EI_{cln} + 2L_{cln}EI_{bm}\right)}$$

With the formulas found before a differential equation can be made. The differential equation is based on equilibrium between the internal moments and the external moments. The internal moments are the stiffness multiplied by the second order bending. The external moments are the vertical load multiplied by the total deflection and the bending moments at the end of the column.

$$M_{intern} = M_{extern}$$

$$-\kappa EI_{cln} = Ny_{total} + \frac{M_B x}{L_{cln}} - \frac{k_r \varphi_{extra,x=0} x}{L_{clen}}$$

$$-\left(\kappa_3 + \kappa_4\right)EI_{cln} = N\left(y_1 + y_2 + y_3 + y_4\right) + \frac{M_B x}{L_{cln}} - \frac{k_r \varphi_{extra,x=0} x}{L_{cln}}$$

Use the known formulas in this formula.

$$-\left(\frac{\pi e_1 EI_{bm}\left(6L_{cln} - 6x\right)}{L_{cln}^{\ 2}\left(3L_{bm}EI_{cln} + 2L_{cln}EI_{bm}\right)} - e_1\frac{\pi^2}{L_{cln}^{\ 2}}\sin\left(\frac{\pi x}{L_{cln}}\right)\right)EI_{cln} =$$

$$N\left(\frac{qL_{bm}^{\ 3}\left(2L_{cln}^{\ 2}x - 3L_{cln}x^2 + x^3\right)}{24L_{cln}\left(3L_{bm}EI_{cln} + 2L_{cln}EI_{bm}\right)} + e_0\sin\left(\frac{\pi x}{L_{cln}}\right) + \frac{\pi e_1 EI_{bm}\left(-2L_{cln}^{\ 2}x + 3L_{cln}x^2 - x^3\right)}{L_{cln}^{\ 2}\left(3L_{bm}EI_{cln} + 2L_{cln}EI_{bm}\right)} + e_1\sin\left(\frac{\pi x}{L_{cln}}\right)\right)$$

$$+\frac{M_B x}{L_{cln}} - \frac{k_r \varphi_{extra,x=0} x}{L_{cln}}$$

Write out some expressions to neglect the brackets.

$$-\frac{\pi e_1 EI_{bm} EI_{cln}\left(6L_{cln}-6x\right)}{L_{cln}^{\ 2}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)}+e_1\frac{\pi^2 EI_{cln}}{L_{cln}^{\ 2}}\sin\left(\frac{\pi x}{L_{cln}}\right)=$$
$$\frac{qNL_{bm}^{\ 3}\left(2L_{cln}^{\ 2}x-3L_{cln}x^2+x^3\right)}{24L_{cln}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)}+Ne_0\sin\left(\frac{\pi x}{L_{cln}}\right)+\frac{\pi Ne_1 EI_{bm}\left(-2L_{cln}^{\ 2}x+3L_{cln}x^2-x^3\right)}{L_{cln}^{\ 2}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)}+Ne_1\sin\left(\frac{\pi x}{L_{cln}}\right)$$
$$+\frac{qL_{bm}^{\ 3}EI_{cln}x}{4L_{cln}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)}-\frac{6\pi e_1 EI_{bm}EI_{cln}x}{L_{cln}^{\ 2}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)}$$

The portal frame is symmetric. The portal frame is loaded by a uniformly distributed load. The vertical reaction forces of support A and support D are equal together. The load on the column is the half the load on the portal frame. $N=\frac{1}{2}qL_{bm}$

$$-\frac{\pi e_1 EI_{bm} EI_{cln}\left(6L_{cln}-6x\right)}{L_{cln}^{\ 2}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)}+e_1\frac{\pi^2 EI_{cln}}{L_{cln}^{\ 2}}\sin\left(\frac{\pi x}{L_{cln}}\right)=$$
$$\frac{q^2L_{bm}^{\ 4}\left(2L_{cln}^{\ 2}x-3L_{cln}x^2+x^3\right)}{48L_{cln}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)}+\frac{qL_{bm}e_0\sin\left(\frac{\pi x}{L_{cln}}\right)}{2}+\frac{\pi qL_{bm}e_1 EI_{bm}\left(-2L_{cln}^{\ 2}x+3L_{cln}x^2-x^3\right)}{2L_{cln}^{\ 2}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)}+\frac{qL_{bm}e_1\sin\left(\frac{\pi x}{L_{cln}}\right)}{2}$$
$$+\frac{qL_{bm}^{\ 3}EI_{cln}x}{4L_{cln}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)}-\frac{6\pi e_1 EI_{bm}EI_{cln}x}{L_{cln}^{\ 2}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)}$$

The value e_1 is the additional deflection in the middle of the column. To find e_1, all expressions must be calculated in the middle of the column $\left(x=0.5L_{cln}\right)$.

$$-\frac{3\pi L_{cln}e_1 EI_{bm}EI_{cln}}{L_{cln}^{\ 2}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)}+e_1\frac{\pi^2 EI_{cln}}{L_{cln}^{\ 2}}$$
$$=\frac{3q^2L_{bm}^{\ 4}L_{cln}^{\ 3}}{384L_{cln}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)}+\frac{qL_{bm}e_0}{2}-\frac{3\pi qL_{bm}L_{cln}^{\ 3}e_1 EI_{bm}}{16L_{cln}^{\ 2}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)}+\frac{qL_{bm}e_1}{2}$$
$$+\frac{qL_{bm}^{\ 3}EI_{cln}}{8\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)}-\frac{3\pi e_1 EI_{bm}EI_{cln}}{L_{cln}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)}$$

The only unknown value is the value of e_1. To calculate this value, all expressions of e_1 must be separate from the rest of the formula. All expressions of e_1 will be placed on one side of the equation.

$$-\frac{3\pi L_{cln}e_1 EI_{bm}EI_{cln}}{L_{cln}^{\ 2}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)}+\frac{3\pi qL_{bm}L_{cln}^{\ 3}e_1 EI_{bm}}{16L_{cln}^{\ 2}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)}+e_1\frac{\pi^2 EI_{cln}}{L_{cln}^{\ 2}}-\frac{qL_{bm}e_1}{2}$$
$$+\frac{3\pi e_1 EI_{bm}EI_{cln}}{L_{cln}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)}=\frac{3q^2L_{bm}^{\ 4}L_{cln}^{\ 3}}{384L_{cln}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)}+\frac{qL_{bm}e_0}{2}+\frac{qL_{bm}^{\ 3}EI_{cln}}{8\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)}$$

Two expressions are equal but have a different sign. These expressions can be neglected.

$$\frac{3\pi q L_{bm} L_{cln}{}^3 e_1 EI_{bm}}{16 L_{cln}{}^2 \left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)} + e_1 \frac{\pi^2 EI_{cln}}{L_{cln}{}^2} - \frac{q L_{bm} e_1}{2}$$
$$= \frac{3q^2 L_{bm}{}^4 L_{cln}{}^3}{384 L_{cln} \left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)} + \frac{q L_{bm} e_0}{2} + \frac{q L_{bm}{}^3 EI_{cln}}{8\left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)}$$

To calculate the value e_1 it is necessary to have one denominator. The same denominator for all expressions of e_1 will be made.

$$e_1 \frac{3\pi q L_{bm} L_{cln}{}^3 EI_{bm}}{16 L_{cln}{}^2 \left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)} + e_1 \frac{16\pi^2 EI_{cln} \left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)}{16 L_{cln}{}^2 \left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)}$$
$$-e_1 \frac{8 q L_{bm} L_{cln}{}^2 \left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)}{16 L_{cln}{}^2 \left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)} = \frac{3q^2 L_{bm}{}^4 L_{cln}{}^3}{384 L_{cln} \left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)} + \frac{q L_{bm} e_0}{2} + \frac{q L_{bm}{}^3 EI_{cln}}{8\left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)}$$

Make one denominator for all expressions of e_1. Combine some expressions.

$$e_1 \frac{3\pi q L_{bm} L_{cln}{}^3 EI_{bm} + \left(16\pi^2 EI_{cln} - 8 q L_{bm} L_{cln}{}^2\right)\left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)}{16 L_{cln}{}^2 \left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)}$$
$$= \frac{3q^2 L_{bm}{}^4 L_{cln}{}^3}{384 L_{cln} \left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)} + \frac{q L_{bm} e_0}{2} + \frac{q L_{bm}{}^3 EI_{cln}}{8\left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)}$$

Make everywhere the same denominator.

$$e_1 \frac{24\left[3 q \pi L_{bm} L_{cln}{}^3 EI_{bm} + \left(16\pi^2 EI_{cln} - 8 q L_{bm} L_{cln}{}^2\right)\left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)\right]}{384 L_{cln}{}^2 \left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)}$$
$$= \frac{3q^2 L_{bm}{}^4 L_{cln}{}^4 + 192 q L_{bm} L_{cln}{}^2 e_0 \left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right) + 48 q L_{bm}{}^3 L_{cln}{}^2 EI_{cln}}{384 L_{cln}{}^2 \left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)}$$

This results in a formula of e_1.

$$e_1 = \frac{q L_{bm} L_{cln}{}^2 \left(q L_{bm}{}^3 L_{cln}{}^2 + 64 e_0 \left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right) + 16 L_{bm}{}^2 EI_{cln}\right)}{8\left[3\pi q L_{bm} L_{cln}{}^3 EI_{bm} + \left(16\pi^2 EI_{cln} - 8 q L_{bm} L_{cln}{}^2\right)\left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)\right]}$$

A formula for the additional deflection is found. This formula can be used to calculate the additional rotation and the additional bending moments. The ultimate load can be calculated by these values. As summary of this Appendix the formulas will be repeated.

$$e_1 = \frac{q L_{bm} L_{cln}{}^2 \left(q L_{bm}{}^3 L_{cln}{}^2 + 64 e_0 \left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right) + 16 L_{bm}{}^2 EI_{cln}\right)}{8\left[3\pi q L_{bm} L_{cln}{}^3 EI_{bm} + \left(16\pi^2 EI_{cln} - 8 q L_{bm} L_{cln}{}^2\right)\left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)\right]}$$

$$e_{1,total} = \frac{q L_{bm}{}^3 L_{cln}{}^2}{64\left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)} + e_0 - \frac{\varphi_{extra,x=0} L_{cln}{}^2 EI_{bm}}{8 L_{bm} EI_{cln}} + e_1$$

$$\varphi_{extra,x=0} = \frac{3\pi L_{bm} e_1 EI_{cln}}{L_{cln} \left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)}$$

$$F = \frac{qL_{bm}{}^{3}EI_{cln}}{4L_{cln}\left(3L_{bm}EI_{cln} + 2L_{cln}EI_{bm}\right)}$$

The force F is the linear reaction force. The second order deflection has influence on the rotation of the beam and on the reaction forces. The second order reaction force is smaller than the first order reaction force. The difference between these forces is called ΔF and can be calculated by the following formula:

$$\Delta F = \frac{3\varphi_{extra,x=0}EI_{cln}}{L_{cln}{}^{2}}$$

The bending moments in the column can be calculated.

$$M_{top} = \left(F - \Delta F\right)L_{cln}$$

$$M_{middle} = 0.5\left(F - \Delta F\right)L_{cln} + 0.5qL_{bm}e_{total}$$

The critical load can be calculated by a well known formula.

$$-f_y = -\frac{N}{A} \pm \frac{M}{Z}$$

The formulas become more clear at the calculation example in Appendix P.

Appendix O Residual stress in non-linear analysis

Appendix N was about the non linear analysis of a portal frame. Residual stress was not taken into account. This Appendix is about the influence of residual stress. The Appendix is split in three parts. The order of this Appendix is the order of loading. The first part is about the second load case. The second load case starts if the right flange starts to yield. The second part is about the third load case. The third load case starts if the left flange starts to yield too. The third part is the conclusion of this Appendix. It is possible that the third or even the second load case does not occur. There are two possibilities that results in failure before the third load case starts. First: the stiffness has reduced too much. Second: the left flange does not yield before the right flange fully yields. The analysis in this Appendix is the same analysis as in Appendix N but more expended and more complex. The load-deflection graphic of the analysis can be found in Figure O.1.

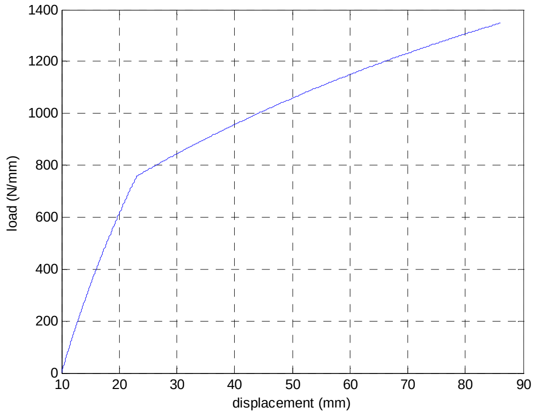


Figure O.1: Load-deflection Graphic

Column: Length 10m, HE 360A
Beam: Length 5 m, HE 900A

O.1 Analysis if one part yields

The portal frame is loaded by a load q_1. By this load the column starts to yield. The effective cross-section and the effective stiffness decrease. A part of the right flange is not effective anymore. This results in an asymmetric section. The centre of gravity has been shift. This results in extra bending moments and in an extra deflection. This influence is also taken into account at the analysis. The braced portal frame is a symmetric structure. Both columns of the portal frame yield. The stiffness of both columns decreases.

There are five deflection parts.

1. Deflection due to moment (linear analysis)
2. Starting deflection
3. Deflection due to the rotation spring
4. Additional deflection
5. Deflection due to the shift of point of gravity.

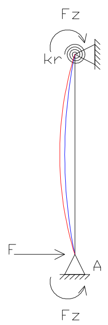


Figure O.2: Column

The moment distribution is a little bit difference than discussed in Appendix N. The section is not symmetric. There is an eccentric moment. This eccentric moment results in an extra deflection (Fig. O.2).

The equilibrium of the displacement of point A is the follow formula:

$$\frac{q_2 L_{bm}^{\ 3} L_{cln}}{24EI_{bm}} + \frac{N_2 z_2 L_{cln}^{\ 2}}{2EI_{cln,2}} - \frac{M_2 L_{cln}}{k_r} - \frac{F_2 L_{cln}^{\ 3}}{3EI_{cln,2}} = 0$$

Know is the vertical load $\left(N_2 = \frac{1}{2} q_2 L_{bm}\right)$.

$$\frac{q_2 L_{bm}{}^3 L_{cln}}{24EI_{bm}} + \frac{q_2 L_{bm} L_{cln}{}^2 z_2}{4EI_{cln,2}} - \frac{F_2 L_{bm} L_{cln}{}^2}{2EI_{bm}} - \frac{F_2 L_{cln}{}^3}{3EI_{cln,2}} = 0$$

The expressions of F_2 has been separate from the other expressions.

$$\frac{F_2 L_{bm} L_{cln}{}^2}{2EI_{bm}} + \frac{F_2 L_{cln}{}^3}{3EI_{cln,2}} = \frac{q_2 L_{bm}{}^3 L_{cln}}{24EI_{bm}} + \frac{q_2 L_{bm} L_{cln}{}^2 z_2}{4EI_{cln,2}}$$

There can be made one denominator for all expressions.

$$\frac{12F_2 L_{bm} L_{cln}{}^2 EI_{cln,2} + 8F_2 L_{cln}{}^3 EI_{bm}}{24EI_{cln,2} EI_{bm}} = \frac{q_2 L_{bm}{}^3 L_{cln} EI_{cln,2} + 6q_2 L_{bm} L_{cln}{}^2 z_2 EI_{bm}}{24EI_{cln,2} EI_{bm}}$$

The same denominator can be neglected.

$$4F_2 L_{cln}{}^2 \left(3L_{bm} EI_{cln,2} + 2L_{cln} EI_{bm}\right) = q_2 L_{bm} L_{cln} \left(L_{bm}{}^2 EI_{cln,2} + 6L_{cln} z_2 EI_{bm}\right)$$

The formula of F_2 can be found.

$$F_2 = \frac{q_2 L_{bm} \left(L_{bm}{}^2 EI_{cln,2} + 6L_{cln} z_2 EI_{bm}\right)}{4L_{cln} \left(3L_{bm} EI_{cln,2} + 2L_{cln} EI_{bm}\right)}$$

Known from Appendix N is the formula of F_1.

$$F_1 = \frac{q_1 L_{bm}{}^3 EI_{cln,1}}{4L_{cln} \left(3L_{bm} EI_{cln,1} + 2L_{cln} EI_{bm}\right)}$$

With these formulas the deflections can be calculated.

Deflection due to moment

$$M_{1,2} = F_2 \left(L_{cln} - x\right)$$

$$\kappa_{1,2} = \frac{-F_2 \left(L_{cln} - x\right)}{EI_{cln,2}}$$

$$\varphi_{1,2} = \frac{-F_2 \left(2L_{cln} x - x^2\right)}{2EI_{cln,2}} + C_1$$

$$y_{1,2} = \frac{-F_2 \left(3L_{cln} x^2 - x^3\right)}{6EI_{cln,2}} + C_1 x + C_2$$

Use the following boundary conditions to find the integral constants.

$y_{1,2,x=0} = 0$ → $C_2 = 0$

$y_{1,2,x=L_{cln}} = 0$ → $C_1 = \frac{2F_2 L_{cln}{}^2}{6EI_{cln,2}}$

This results in the follow formula.

$$y_{1,2}=\frac{F_2\left(2L_{cln}{}^2x-3L_{cln}x^2+x^3\right)}{6EI_{cln,2}}$$

Use the formula of F_2

$$y_{1,2}=\frac{q_2L_{bm}\left(L_{bm}{}^2EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)}{4L_{cln}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}\frac{\left(2L_{cln}{}^2x-3L_{cln}x^2+x^3\right)}{6EI_{cln,2}}$$

$$y_{1,2}=\frac{q_2L_{bm}\left(L_{bm}{}^2EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)\left(2L_{cln}{}^2x-3L_{cln}x^2+x^3\right)}{24L_{cln}EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}$$

Starting deflection

$$y_2=e_0\sin\left(\frac{\pi x}{L_{cln}}\right)$$

This deflection does not change at another load case.

Deflection due to rotation spring:

$$M_{3,2}=-\varphi_2k_r$$

$$M_{3,2,x=0}=-\varphi_{extra,2,x=0}k_r$$

$$M_{3,2}=\frac{-\varphi_{extra,2,x=0}k_r\left(L_{cln}-x\right)}{L_{cln}}$$

$$\kappa_{3,2}=\frac{\varphi_{extra,2,x=0}k_r\left(L_{cln}-x\right)}{L_{cln}EI_{cln,2}}$$

$$\varphi_{3,2}=\frac{\varphi_{extra,2,x=0}k_r\left(2L_{cln}x-x^2\right)}{2L_{cln}EI_{cln,2}}+C_1$$

$$y_{3,2}=\frac{\varphi_{extra,2,x=0}k_r\left(3L_{cln}x^2-x^3\right)}{6L_{cln}EI_{cln,2}}+C_1x+C_2$$

Use the boundary conditions.

$y_{3,2,x=0}=0$ → $C_2 = 0$

$y_{3,2,x=L_{cln}}=0$ → $C_1=\frac{-2\varphi_{extra,2,x=0}k_rL_{cln}{}^2}{6L_{cln}EI_{cln,2}}$

This results in the following formula.

$$y_{3,2} = \frac{\varphi_{extra,2,x=0} k_r \left(-2L_{cln}{}^2 x + 3L_{cln} x^2 - x^3\right)}{6L_{cln} EI_{cln,2}}$$

Use the formula of k_r in this formula.

$$y_{3,2} = \frac{\varphi_{extra,2,x=0} \dfrac{2EI_{bm}}{L_{bm}} \left(-2L_{cln}{}^2 x + 3L_{cln} x^2 - x^3\right)}{6L_{cln} EI_{cln,2}}$$

$$y_{3,2} = \frac{\varphi_{extra,2,x=0} EI_{bm} \left(-2L_{cln}{}^2 x + 3L_{cln} x^2 - x^3\right)}{3L_{bm} L_{cln} EI_{cln,2}}$$

Additional deflection

$$y_{4,2} = e_2 \sin\left(\frac{\pi x}{L_{cln}}\right)$$

Deflection due to shift of the centre of gravity.

$$M_{5,2} = N_2 z_2$$

Use the formula of N_2.

$$M_{5,2} = \tfrac{1}{2} q_2 L_{bm} z_2$$

$$\kappa_{5,2} = \frac{-q_2 L_{bm} z_2}{2EI_{cln,2}}$$

$$\varphi_{5,2} = \frac{-q_2 L_{bm} z_2 x}{2EI_{cln,2}} + C_1$$

$$y_{5,2} = \frac{-q_2 L_{bm} z_2 x^2}{4EI_{cln,2}} + C_1 x + C_2$$

Use the boundary conditions to find the integral constants.

$y_{5,2,x=0} = 0$ → $C_2 = 0$

$y_{5,2,x=L_{cln}} = 0$ → $C_1 = \dfrac{q_2 L_{bm} z_2 L_{cln}}{4EI_{cln,2}}$

This results in the following formula.

$$y_{5,2} = \frac{q_2 L_{bm} z_2 \left(L_{cln} x - x^2\right)}{4EI_{cln,2}}$$

There follows a list of all deflection formulas. The formulas are found in this Appendix and in Appendix N.

$$y_{1,2}=\frac{q_2 L_{bm}\left(L_{bm}{}^2 EI_{cln,2}+6L_{cln}z_2 EI_{bm}\right)\left(2L_{cln}{}^2 x-3L_{cln}x^2+x^3\right)}{24L_{cln}EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}$$

$$y_{1,1}=\frac{q_1 L_{bm}{}^3\left(2L_{cln}{}^2 x-3L_{cln}x^2+x^3\right)}{24L_{cln}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)}$$

$$y_2=e_0\sin\left(\frac{\pi x}{L_{cln}}\right)$$

$$y_{3,2}=\frac{\varphi_{extra,2,x=0}EI_{bm}\left(-2L_{cln}{}^2 x+3L_{cln}x^2-x^3\right)}{3L_{bm}L_{cln}EI_{cln,2}}$$

$$y_{3,1}=\frac{\varphi_{extra,1,x=0}EI_{bm}\left(-2L_{cln}{}^2 x+3L_{cln}x^2-x^3\right)}{3L_{bm}L_{cln}EI_{cln}}$$

$$y_{4,2}=e_2\sin\left(\frac{\pi x}{L_{cln}}\right)$$

$$y_{4,1}=e_1\sin\left(\frac{\pi x}{L_{cln}}\right)$$

$$y_{5,2}=\frac{q_2 L_{bm}z_2\left(L_{cln}x-x^2\right)}{4EI_{cln,2}}$$

$$y_{total,2}=\frac{q_1 L_{bm}{}^3 L_{cln}{}^2}{64\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)}+\frac{q_2 L_{bm}L_{cln}{}^2\left(L_{bm}{}^2 EI_{cln,2}+6L_{cln}z_2 EI_{bm}\right)}{64EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}+e_0-\frac{\varphi_{extra,1,x=0}L_{cln}{}^2 EI_{bm}}{8L_{bm}EI_{cln}}$$
$$-\frac{\varphi_{extra,2,x=0}L_{cln}{}^2 EI_{bm}}{8L_{bm}EI_{cln,2}}+e_1+e_2+\frac{q_2 L_{bm}L_{cln}{}^2 z_2}{16EI_{cln,2}}$$

To calculate the additional rotation only deflection $y_{3,2}$ and $y_{4,2}$ are important. The influence of $y_{5,2}$ is already taken into account at the calculation of $y_{4,2}$. The follow derivatives are important.

$$y_{3,2}=\frac{\varphi_{extra,2,x=0}EI_{bm}\left(-2L_{cln}{}^2 x+3L_{cln}x^2-x^3\right)}{3L_{bm}L_{cln}EI_{cln,2}}$$

$$\varphi_{3,2}=\frac{\varphi_{extra,2,x=0}EI_{bm}\left(-2L_{cln}{}^2+6L_{cln}x-3x^2\right)}{3L_{bm}L_{cln}EI_{cln,2}}$$

$$\kappa_{3,2}=\frac{\varphi_{extra,2,x=0}EI_{bm}\left(6L_{cln}-6x\right)}{3L_{bm}L_{cln}EI_{cln,2}}$$

$$y_{4,2}=e_2\sin\left(\frac{\pi x}{L_{cln}}\right)$$

$$\varphi_{4,2}=\frac{\pi}{L_{cln}}e_2\cos\left(\frac{\pi x}{L_{cln}}\right)$$

$$\kappa_{4,2}=\frac{-\pi^2}{L_{cln}{}^2}e_2\sin\left(\frac{\pi x}{L_{cln}}\right)$$

With these formulas the additional rotation can be calculated as function of the additional deflection.

$$\varphi_{extra,2,x=0} = \varphi_{3,2,x=0} + \varphi_{4,2,x=0}$$

$$\varphi_{extra,2,x=0} = \frac{\varphi_{extra,2,x=0} EI_{bm}\left(-2L_{cln}{}^{2} + 6L_{cln} * 0 - 3 * 0^{2}\right)}{3L_{bm}L_{cln}EI_{cln,2}} + \frac{\pi}{L_{cln}} e_{2} \cos\left(\frac{\pi * 0}{L_{cln}}\right)$$

Neglect some expressions.

$$\varphi_{extra,2,x=0} = \frac{-2\varphi_{extra,2,x=0} L_{cln} EI_{bm}}{3L_{bm}EI_{cln,2}} + \frac{\pi}{L_{cln}} e_{2}$$

Separate the expressions of $\varphi_{extra,2,x=0}$.

$$\varphi_{extra,2,x=0} + \frac{2\varphi_{extra,2,x=0} L_{cln} EI_{bm}}{3L_{bm}EI_{cln,2}} = \frac{\pi}{L_{cln}} e_{2}$$

Make everywhere the same denominator.

$$\varphi_{extra,2,x=0}\left(\frac{3L_{bm}L_{cln}EI_{cln,2} + 2L_{cln}{}^{2}EI_{bm}}{3L_{bm}L_{cln}EI_{cln,2}}\right) = \frac{3\pi L_{bm} e_{2} EI_{cln,2}}{3L_{cln}L_{bm}EI_{cln,2}}$$

Find the formula of $\varphi_{extra,2,x=0}$.

$$\varphi_{extra,2,x=0} = \frac{3\pi L_{bm} e_{2} EI_{cln,2}}{L_{cln}\left(3L_{bm}EI_{cln,2} + 2L_{cln}EI_{bm}\right)}$$

This formula can be used for the deflection $y_{3,2}$.

$$y_{3,2} = \frac{3\pi L_{bm} e_{2} EI_{cln,2}}{L_{cln}\left(3L_{bm}EI_{cln,2} + 2L_{cln}EI_{bm}\right)} \frac{EI_{bm}\left(-2L_{cln}{}^{2}x + 3L_{cln}x^{2} - x^{3}\right)}{3L_{bm}L_{cln}EI_{cln,2}}$$

$$y_{3,2} = \frac{\pi e_{2} EI_{bm}\left(-2L_{cln}{}^{2}x + 3L_{cln}x^{2} - x^{3}\right)}{L_{cln}{}^{2}\left(3L_{bm}EI_{cln,2} + 2L_{cln}EI_{bm}\right)}$$

$$\varphi_{3,2} = \frac{\pi e_{2} EI_{bm}\left(-2L_{cln}{}^{2} + 6L_{cln}x - 3x^{2}\right)}{L_{cln}{}^{2}\left(3L_{bm}EI_{cln,2} + 2L_{cln}EI_{bm}\right)}$$

$$\kappa_{3,2} = \frac{6\pi e_{2} EI_{bm}\left(L_{cln} - x\right)}{L_{cln}{}^{2}\left(3L_{bm}EI_{cln,2} + 2L_{cln}EI_{bm}\right)}$$

Known from earlier analysis was the following formula:

$$y_{3,1} = \frac{\pi e_{1} EI_{bm}\left(-2L_{cln}{}^{2}x + 3L_{cln}x^{2} - x^{3}\right)}{L_{cln}{}^{2}\left(3L_{bm}EI_{cln,1} + 2L_{cln}EI_{bm}\right)}$$

All necessary formulas are known. The differential equation can start. The differential equation is based on the equilibrium between the internal and the external moments.

$$\Delta M_{intern} = \Delta M_{extern}$$

$$-\Delta\kappa_2 = N_1 y_2 + N_2 y_{total,2} + \frac{M_{B,2}x}{L_{cln}} - \frac{k_r \varphi_{extra,2,x=0} x}{L_{cln}}$$

$$-\left(\kappa_{3,2} + \kappa_{4,2}\right) = N_1\left(y_{1,2} + y_{3,2} + y_{4,2} + y_{5,2}\right) + N_2\left(y_{1,1} + y_{1,2} + y_2 + y_{3,1} + y_{3,2} + y_{4,1} + y_{4,2} + y_{5,2}\right)$$

All known formulas can be used in this formula.

$$-\left(\frac{6\pi e_2 EI_{bm}\left(L_{cln} - x\right)}{L_{cln}{}^2\left(3L_{bm}EI_{cln,2} + 2L_{cln}EI_{bm}\right)} + \frac{-\pi^2}{L_{cln}{}^2} e_2 \sin\left(\frac{\pi x}{L_{cln}}\right)\right) EI_{cln,2}$$

$$= N_1\left(\begin{array}{l} \frac{q_2 L_{bm}\left(L_{bm}{}^2 EI_{cln,2} + 6L_{cln}z_2 EI_{bm}\right)\left(2L_{cln}{}^2 x - 3L_{cln}x^2 + x^3\right)}{24L_{cln}EI_{cln,2}\left(3L_{bm}EI_{cln,2} + 2L_{cln}EI_{bm}\right)} + \frac{\pi e_2 EI_{bm}\left(-2L_{cln}{}^2 x + 3L_{cln}x^2 - x^3\right)}{L_{cln}{}^2\left(3L_{bm}EI_{cln,2} + 2L_{cln}EI_{bm}\right)} \\ + e_2 \sin\left(\frac{\pi x}{L_{cln}}\right) + \frac{q_2 L_{bm} z_2\left(L_{cln}x - x^2\right)}{4EI_{cln,2}} \end{array}\right)$$

$$+ N_2\left(\begin{array}{l} \frac{q_1 L_{bm}{}^3\left(2L_{cln}{}^2 x - 3L_{cln}x^2 + x^3\right)}{24L_{cln}\left(3L_{bm}EI_{cln,1} + 2L_{cln}EI_{bm}\right)} + \frac{q_2 L_{bm}\left(L_{bm}{}^2 EI_{cln,2} + 6L_{cln}z_2 EI_{bm}\right)\left(2L_{cln}{}^2 x - 3L_{cln}x^2 + x^3\right)}{24L_{cln}EI_{cln,2}\left(3L_{bm}EI_{cln,2} + 2L_{cln}EI_{bm}\right)} \\ + e_0 \sin\left(\frac{\pi x}{L_{cln}}\right) + \frac{\pi e_1 EI_{bm}\left(-2L_{cln}{}^2 x + 3L_{cln}x^2 - x^3\right)}{L_{cln}{}^2\left(3L_{bm}EI_{cln,1} + 2L_{cln}EI_{bm}\right)} + \frac{\pi e_2 EI_{bm}\left(-2L_{cln}{}^2 x + 3L_{cln}x^2 - x^3\right)}{L_{cln}{}^2\left(3L_{bm}EI_{cln,2} + 2L_{cln}EI_{bm}\right)} \\ + e_1 \sin\left(\frac{\pi x}{L_{cln}}\right) + e_2 \sin\left(\frac{\pi x}{L_{cln}}\right) + \frac{q_2 L_{bm} z_2\left(L_{cln}x - x^2\right)}{4EI_{cln,2}} \end{array}\right)$$

$$+ \frac{q_2 L_{bm}\left(L_{bm}{}^2 EI_{cln,2} + 6L_{cln}z_2 EI_{bm}\right)}{4L_{cln}\left(3L_{bm}EI_{cln,2} + 2L_{cln}EI_{bm}\right)} x - \frac{6\pi e_2 EI_{bm}EI_{cln,2}}{L_{cln}{}^2\left(3L_{bm}EI_{cln,2} + 2L_{cln}EI_{bm}\right)} x$$

Write out some expressions to neglect the brackets.

$$\frac{-6\pi e_2 EI_{bm} EI_{cln,2}\left(L_{cln}-x\right)}{L_{cln}{}^2\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}+\frac{\pi^2 EI_{cln,2}}{L_{cln}{}^2}e_2\sin\left(\frac{\pi x}{L_{cln}}\right)$$

$$=\frac{q_2N_1L_{bm}\left(L_{bm}{}^2EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)\left(2L_{cln}{}^2x-3L_{cln}x^2+x^3\right)}{24L_{cln}EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}+\frac{\pi N_1e_2EI_{bm}\left(-2L_{cln}{}^2x+3L_{cln}x^2-x^3\right)}{L_{cln}{}^2\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}$$

$$+N_1e_2\sin\left(\frac{\pi x}{L_{cln}}\right)+\frac{q_2N_1L_{bm}z_2\left(L_{cln}x-x^2\right)}{4EI_{cln,2}}$$

$$+\frac{q_1N_2L_{bm}{}^3\left(2L_{cln}{}^2x-3L_{cln}x^2+x^3\right)}{24L_{cln}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)}+\frac{q_2N_2L_{bm}\left(L_{bm}{}^2EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)\left(2L_{cln}{}^2x-3L_{cln}x^2+x^3\right)}{24L_{cln}EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}$$

$$+N_2e_0\sin\left(\frac{\pi x}{L_{cln}}\right)+\frac{\pi N_2e_1EI_{bm}\left(-2L_{cln}{}^2x+3L_{cln}x^2-x^3\right)}{L_{cln}{}^2\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)}+\frac{\pi N_2e_2EI_{bm}\left(-2L_{cln}{}^2x+3L_{cln}x^2-x^3\right)}{L_{cln}{}^2\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}$$

$$+N_2e_1\sin\left(\frac{\pi x}{L_{cln}}\right)+N_2e_2\sin\left(\frac{\pi x}{L_{cln}}\right)+\frac{q_2N_2L_{bm}z_2\left(L_{cln}x-x^2\right)}{4EI_{cln,2}}$$

$$+\frac{q_2L_{bm}\left(L_{bm}{}^2EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)x}{4L_{cln}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}-\frac{6\pi e_2EI_{bm}EI_{cln,2}x}{L_{cln}{}^2\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}$$

The portal frame and the loads on the portal frame are symmetric. The values of N_1 and N_2 can be used ($N_1 = \frac{1}{2}q_1L_{bm}$; $N_2 = \frac{1}{2}q_2L_{bm}$). This results in the following formula.

$$\frac{-6\pi e_2 EI_{bm} EI_{cln,2}\left(L_{cln}-x\right)}{L_{cln}{}^2\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}+\frac{\pi^2 EI_{cln,2}}{L_{cln}{}^2}e_2\sin\left(\frac{\pi x}{L_{cln}}\right)$$

$$=\frac{q_1q_2L_{bm}{}^2\left(L_{bm}{}^2EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)\left(2L_{cln}{}^2x-3L_{cln}x^2+x^3\right)}{48L_{cln}EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}+\frac{\pi q_1L_{bm}e_2EI_{bm}\left(-2L_{cln}{}^2x+3L_{cln}x^2-x^3\right)}{2L_{cln}{}^2\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}$$

$$+\frac{q_1L_{bm}e_2\sin\left(\frac{\pi x}{L_{cln}}\right)}{2}+\frac{q_1q_2L_{bm}{}^2z_2\left(L_{cln}x-x^2\right)}{8EI_{cln,2}}$$

$$+\frac{q_1q_2L_{bm}{}^4\left(2L_{cln}{}^2x-3L_{cln}x^2+x^3\right)}{48L_{cln}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)}+\frac{q_2q_2L_{bm}{}^2\left(L_{bm}{}^2EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)\left(2L_{cln}{}^2x-3L_{cln}x^2+x^3\right)}{48L_{cln}EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}$$

$$+\frac{q_2L_{bm}e_0\sin\left(\frac{\pi x}{L_{cln}}\right)}{2}+\frac{\pi q_2L_{bm}e_1EI_{bm}\left(-2L_{cln}{}^2x+3L_{cln}x^2-x^3\right)}{2L_{cln}{}^2\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)}+\frac{\pi q_2L_{bm}e_2EI_{bm}\left(-2L_{cln}{}^2x+3L_{cln}x^2-x^3\right)}{2L_{cln}{}^2\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}$$

$$+\frac{q_2L_{bm}e_1\sin\left(\frac{\pi x}{L_{cln}}\right)}{2}+\frac{q_2L_{bm}e_2\sin\left(\frac{\pi x}{L_{cln}}\right)}{2}+\frac{q_2{}^2L_{bm}{}^2z_2\left(L_{cln}x-x^2\right)}{8EI_{cln,2}}$$

$$+\frac{q_2L_{bm}\left(L_{bm}{}^2EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)x}{4L_{cln}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}-\frac{6\pi e_2EI_{bm}EI_{cln,2}x}{L_{cln}{}^2\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}$$

As same as in Appendix N the deflection must be calculated at the half of the column. Deflection at $x=0.5L_{cln}$.

$$\frac{-3\pi e_2 EI_{bm} EI_{cln,2}}{L_{cln}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}+\frac{\pi^2 e_2 EI_{cln,2}}{L_{cln}^{2}}=\frac{q_1q_2L_{bm}^{2}L_{cln}^{2}\left(L_{bm}^{2}EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)}{128EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}$$
$$-\frac{3\pi q_1L_{bm}L_{cln}e_2EI_{bm}}{16\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}+\frac{q_1L_{bm}e_2}{2}+\frac{q_1q_2L_{bm}^{2}L_{cln}^{2}z_2}{32EI_{cln,2}}+\frac{q_1q_2L_{bm}^{4}L_{cln}^{2}}{128\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)}$$
$$+\frac{q_2^{2}L_{bm}^{2}\left(L_{cln}^{2}L_{bm}^{2}EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)}{128EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}+\frac{q_2L_{bm}e_0}{2}-\frac{3\pi q_2L_{bm}L_{cln}e_1EI_{bm}}{16\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)}$$
$$-\frac{3\pi q_2L_{bm}L_{cln}e_2EI_{bm}}{16\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}+\frac{q_2L_{bm}e_1}{2}+\frac{q_2L_{bm}e_2}{2}+\frac{q_2^{2}L_{bm}^{2}L_{cln}^{2}z_2}{32EI_{cln,2}}$$
$$+\frac{q_2L_{bm}\left(L_{bm}^{2}EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)}{8\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}-\frac{3\pi e_2EI_{bm}EI_{cln,2}}{L_{cln}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}$$

On expression is the same at both sides of the equation. These expressions can be neglected.

$$\frac{\pi^2 e_2 EI_{cln,2}}{L_{cln}^{2}}=\frac{q_1q_2L_{bm}^{2}L_{cln}^{2}\left(L_{bm}^{2}EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)}{128EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}$$
$$-\frac{3\pi q_1L_{bm}L_{cln}e_2EI_{bm}}{16\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}+\frac{q_1L_{bm}e_2}{2}+\frac{q_1q_2L_{bm}^{2}L_{cln}^{2}z_2}{32EI_{cln,2}}+\frac{q_1q_2L_{bm}^{4}L_{cln}^{2}}{128\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)}$$
$$+\frac{q_2^{2}L_{bm}^{2}\left(L_{cln}^{2}L_{bm}^{2}EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)}{128EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}+\frac{q_2L_{bm}e_0}{2}-\frac{3\pi q_2L_{bm}L_{cln}e_1EI_{bm}}{16\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)}$$
$$-\frac{3\pi q_2L_{bm}L_{cln}e_2EI_{bm}}{16\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}+\frac{q_2L_{bm}e_1}{2}+\frac{q_2L_{bm}e_2}{2}+\frac{q_2^{2}L_{bm}^{2}L_{cln}^{2}z_2}{32EI_{cln,2}}$$
$$+\frac{q_2L_{bm}\left(L_{bm}^{2}EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)}{8\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}$$

In this formula e_2 is the only unknown. To find the unknown value all expressions of e_2 will be placed on one side of the equation.

$$\frac{3\pi q_1L_{bm}L_{cln}e_2EI_{bm}}{16\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}+\frac{3\pi q_2L_{bm}L_{cln}e_2EI_{bm}}{16\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}+\frac{\pi^2 e_2EI_{cln,2}}{L_{cln}^{2}}$$
$$-\frac{q_1L_{bm}e_2}{2}-\frac{q_2L_{bm}e_2}{2}=\frac{q_1q_2L_{bm}^{2}L_{cln}^{2}\left(L_{bm}^{2}EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)}{128EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}+\frac{q_1q_2L_{bm}^{2}L_{cln}^{2}z_2}{32EI_{cln,2}}$$
$$+\frac{q_1q_2L_{bm}^{4}L_{cln}^{2}}{128\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)}+\frac{q_2^{2}L_{bm}^{2}L_{cln}^{2}\left(L_{bm}^{2}EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)}{128EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}+\frac{q_2L_{bm}e_0}{2}$$
$$-\frac{3\pi q_2L_{bm}L_{cln}e_1EI_{bm}}{16\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)}+\frac{q_2L_{bm}e_1}{2}+\frac{q_2^{2}L_{bm}^{2}L_{cln}^{2}z_2}{32EI_{cln,2}}+\frac{q_2L_{bm}\left(L_{bm}^{2}EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)}{8\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}$$

All expressions must have the same denominator. To start with the expressions of e_2.

$$e_2\left(\begin{array}{l}\frac{3\pi q_1 L_{bm} L_{cln}{}^3 EI_{bm}}{16L_{cln}{}^2\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}+\frac{3\pi q_2 L_{bm} L_{cln}{}^3 EI_{bm}}{16L_{cln}{}^2\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}\\ +\frac{16\pi^2 EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}{16L_{cln}{}^2\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}-\frac{8q_1 L_{bm} L_{cln}{}^2\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}{16L_{cln}{}^2\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}\\ -\frac{8q_2 L_{bm} L_{cln}{}^2\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}{16L_{cln}{}^2\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}\end{array}\right)$$

$$=\frac{q_1q_2L_{bm}{}^2L_{cln}{}^2\left(L_{bm}{}^2EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)}{128EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}+\frac{q_1q_2L_{bm}{}^2L_{cln}{}^2z_2}{32EI_{cln,2}}$$
$$+\frac{q_1q_2L_{bm}{}^4L_{cln}{}^2}{128\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)}+\frac{q_2{}^2L_{bm}{}^2L_{cln}{}^2\left(L_{bm}{}^2EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)}{128EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}+\frac{q_2L_{bm}e_0}{2}$$
$$-\frac{3\pi q_2L_{bm}L_{cln}e_1EI_{bm}}{16\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)}+\frac{q_2L_{bm}e_1}{2}+\frac{q_2{}^2L_{bm}{}^2L_{cln}{}^2z_2}{32EI_{cln,2}}+\frac{q_2L_{bm}\left(L_{bm}{}^2EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)}{8\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}$$

Make one denominator for all expressions of e_2.

$$e_2\frac{\left(\begin{array}{l}3\pi q_1L_{bm}L_{cln}{}^3EI_{bm}+3\pi q_2L_{bm}L_{cln}{}^3EI_{bm}+16\pi^2EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\\ -8q_1L_{bm}L_{cln}{}^2\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)-8q_2L_{bm}L_{cln}{}^2\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\end{array}\right)}{16L_{cln}{}^2\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}$$

$$=\frac{q_1q_2L_{bm}{}^2L_{cln}{}^2\left(L_{bm}{}^2EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)}{128EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}+\frac{q_1q_2L_{bm}{}^2L_{cln}{}^2z_2}{32EI_{cln,2}}$$
$$+\frac{q_1q_2L_{bm}{}^4L_{cln}{}^2}{128\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)}+\frac{q_2{}^2L_{bm}{}^2L_{cln}{}^2\left(L_{bm}{}^2EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)}{128EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}+\frac{q_2L_{bm}e_0}{2}$$
$$-\frac{3\pi q_2L_{bm}L_{cln}e_1EI_{bm}}{16\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)}+\frac{q_2L_{bm}e_1}{2}+\frac{q_2{}^2L_{bm}{}^2L_{cln}{}^2z_2}{32EI_{cln,2}}+\frac{q_2L_{bm}\left(L_{bm}{}^2EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)}{8\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}$$

Simplify the numerator

$$e_2\frac{3\pi\left(q_1+q_2\right)L_{bm}L_{cln}{}^3EI_{bm}+8\left(2\pi^2EI_{cln,2}-\left(q_1+q_2\right)L_{bm}L_{cln}{}^2\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}{16L_{cln}{}^2\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}$$

$$=\frac{q_1q_2L_{bm}{}^2L_{cln}{}^2\left(L_{bm}{}^2EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)}{128EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}+\frac{q_1q_2L_{bm}{}^2L_{cln}{}^2z_2}{32EI_{cln,2}}$$
$$+\frac{q_1q_2L_{bm}{}^4L_{cln}{}^2}{128\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)}+\frac{q_2{}^2L_{bm}{}^2L_{cln}{}^2\left(L_{bm}{}^2EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)}{128EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}+\frac{q_2L_{bm}e_0}{2}$$
$$-\frac{3\pi q_2L_{bm}L_{cln}e_1EI_{bm}}{16\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)}+\frac{q_2L_{bm}e_1}{2}+\frac{q_2{}^2L_{bm}{}^2L_{cln}{}^2z_2}{32EI_{cln,2}}+\frac{q_2L_{bm}\left(L_{bm}{}^2EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)}{8\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}$$

It is a complex situation to find everywhere the same denominator. The stiffness of the column is not equally in every denominator. This results in a complex formula. In the following formula every expression has the same denominator.

$$e_2 \frac{8EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\begin{pmatrix}3\pi\left(q_1+q_2\right)L_{bm}L_{cln}{}^3EI_{bm}\\ +8\left(2\pi^2EI_{cln,2}-\left(q_1+q_2\right)L_{bm}L_{cln}{}^2\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\end{pmatrix}}{128L_{cln}{}^2EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}$$

$$=\frac{q_1q_2L_{bm}{}^2L_{cln}{}^4\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(L_{bm}{}^2EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)}{128L_{cln}{}^2EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}$$

$$+\frac{4q_1q_2L_{bm}{}^2L_{cln}{}^4z_2\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}{128L_{cln}{}^2EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}$$

$$+\frac{q_1q_2L_{bm}{}^4L_{cln}{}^4EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}{128L_{cln}{}^2EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}$$

$$+\frac{q_2{}^2L_{bm}{}^2L_{cln}{}^4\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(L_{bm}{}^2EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)}{128L_{cln}{}^2EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}$$

$$+\frac{64q_2L_{bm}L_{cln}{}^2e_0EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}{128L_{cln}{}^2EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}$$

$$-\frac{24\pi q_2L_{bm}L_{cln}{}^3e_1EI_{bm}EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}{128L_{cln}{}^2EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}$$

$$+\frac{64q_2L_{bm}L_{cln}{}^2e_1EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}{128L_{cln}{}^2EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}$$

$$+\frac{4q_2{}^2L_{bm}{}^2L_{cln}{}^4z_2\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}{128L_{cln}{}^2EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}$$

$$+\frac{16q_2L_{bm}L_{cln}{}^2EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(L_{bm}{}^2EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)}{128L_{cln}{}^2EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}$$

The same denominator can be neglected. This has been done in the following formula.

$$e_2 8EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\begin{pmatrix}3\pi\left(q_1+q_2\right)L_{bm}{L_{cln}}^3EI_{bm}\\+8\left(2\pi^2EI_{cln,2}-\left(q_1+q_2\right)L_{bm}{L_{cln}}^2\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\end{pmatrix}$$

$$\begin{aligned}
&= q_1q_2{L_{bm}}^2{L_{cln}}^4\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left({L_{bm}}^2EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)\\
&+4q_1q_2{L_{bm}}^2{L_{cln}}^4z_2\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\\
&+q_1q_2{L_{bm}}^4{L_{cln}}^4EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\\
&+{q_2}^2{L_{bm}}^2{L_{cln}}^4\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left({L_{bm}}^2EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)\\
&+64q_2L_{bm}{L_{cln}}^2e_0EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\\
&-24\pi q_2L_{bm}{L_{cln}}^3e_1EI_{bm}EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\\
&+64q_2L_{bm}{L_{cln}}^2e_1EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\\
&+4{q_2}^2{L_{bm}}^2{L_{cln}}^4z_2\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\\
&+16q_2L_{bm}{L_{cln}}^2EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left({L_{bm}}^2EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)
\end{aligned}$$

This formula can result in a formula of e_2.

$$e_2=\frac{\begin{pmatrix}q_1q_2{L_{bm}}^2{L_{cln}}^4\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left({L_{bm}}^2EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)\\+4q_1q_2{L_{bm}}^2{L_{cln}}^4z_2\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\\+q_1q_2{L_{bm}}^4{L_{cln}}^4EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\\+{q_2}^2{L_{bm}}^2{L_{cln}}^4\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left({L_{bm}}^2EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)\\+64q_2L_{bm}{L_{cln}}^2e_0EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\\-24\pi q_2L_{bm}{L_{cln}}^3e_1EI_{bm}EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\\+64q_2L_{bm}{L_{cln}}^2e_1EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\\+4{q_2}^2{L_{bm}}^2{L_{cln}}^4z_2\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\\+16q_2L_{bm}{L_{cln}}^2EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left({L_{bm}}^2EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)\end{pmatrix}}{8EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\begin{pmatrix}3\pi\left(q_1+q_2\right)L_{bm}{L_{cln}}^3EI_{bm}\\+8\left(2\pi^2EI_{cln,2}-\left(q_1+q_2\right)L_{bm}{L_{cln}}^2\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\end{pmatrix}}$$

This formula is quite difficult. This formula can be simplified by combine the same expressions. The following formula is the result.

$$e_2 = \frac{q_2 L_{bm} L_{cln}^2 \begin{pmatrix} \left((q_1+q_2) L_{bm} L_{cln}^2 + 16 EI_{cln,2}\right)\left(3L_{bm} EI_{cln,1} + 2L_{cln} EI_{bm}\right)\left(L_{bm}^2 EI_{cln,2} + 6L_{cln} z_2 EI_{bm}\right) \\ +\left(3L_{bm} EI_{cln,2} + 2L_{cln} EI_{bm}\right)\begin{pmatrix} 4(q_1+q_2) L_{bm} L_{cln}^2 z_2 \left(3L_{bm} EI_{cln,1} + 2L_{cln} EI_{bm}\right) \\ +L_{cln} EI_{cln,2}\left(q_1 L_{bm}^3 L_{cln} - 24\pi e_1 EI_{bm}\right) \\ +64 EI_{cln,2}\left(3L_{bm} EI_{cln,1} + 2L_{cln} EI_{bm}\right)(e_0 + e_1) \end{pmatrix} \end{pmatrix}}{8 EI_{cln,2}\left(3L_{bm} EI_{cln,1} + 2L_{cln} EI_{bm}\right)\begin{pmatrix} 3\pi(q_1+q_2) L_{bm} L_{cln}^3 EI_{bm} \\ +8\left(2\pi^2 EI_{cln,2} - (q_1+q_2) L_{bm} L_{cln}^2\right)\left(3L_{bm} EI_{cln,2} + 2L_{cln} EI_{bm}\right) \end{pmatrix}}$$

To avoid mistakes in the formula, the formula has been checked on several aspects. The first check is the check on the dimensions.

$$m = \frac{Nm^{-1} mm^2 \begin{pmatrix} \left((Nm^{-1} + Nm^{-1}) mm^2 + Nm^2\right)\left(mNm^2 + mNm^2\right)\left(m^2 Nm^2 + mmNm^2\right) \\ +\left(mNm^2 + mNm^2\right)\begin{pmatrix} \left(Nm^{-1} + Nm^{-1}\right) mm^2 m \left(mNm^2 + mNm^2\right) \\ +mNm^2\left(Nm^{-1} m^3 m - mNm^2\right) \\ +Nm^2\left(mNm^2 + mNm^2\right)(m + m) \end{pmatrix} \end{pmatrix}}{Nm^2\left(mNm^2 + mNm^2\right)\begin{pmatrix} \left(Nm^{-1} + Nm^{-1}\right) mm^3 Nm^2 \\ +\left(Nm^2 - \left(Nm^{-1} + Nm^{-1}\right) mm^2\right)\left(mNm^2 + mNm^2\right) \end{pmatrix}}$$

$$m = \frac{Nm^2\left(\left(Nm^2\right)\left(Nm^3\right)\left(Nm^3\right) + \left(Nm^3\right)\left(\left(Nm^{-1}\right) m^4 \left(Nm^3\right) + Nm^3\left(Nm^3\right) + Nm^2\left(Nm^3\right)(m)\right)\right)}{Nm^2\left(Nm^3\right)\left(\left(Nm^{-1}\right) Nm^6 + \left(Nm^2\right)\left(Nm^3\right)\right)}$$

$$m = \frac{N^4 m^{11}}{N^4 m^{10}}$$

The dimensions are correct.

The second check is to find the original formula (App. N). There are a few differences between the starting position of this formula and the starting position in Appendix N. The first difference is the shift of the centre of gravity. This is neglected from the formula.

$$e_2 = \frac{q_2 L_{bm} L_{cln}^2 \begin{pmatrix} \left((q_1+q_2) L_{bm} L_{cln}^2 + 16 EI_{cln,2}\right)\left(3L_{bm} EI_{cln,1} + 2L_{cln} EI_{bm}\right)\left(L_{bm}^2 EI_{cln,2}\right) \\ +\left(3L_{bm} EI_{cln,2} + 2L_{cln} EI_{bm}\right)\begin{pmatrix} +L_{cln} EI_{cln,2}\left(q_1 L_{bm}^3 L_{cln} - 24\pi e_1 EI_{bm}\right) \\ +64 EI_{cln,2}\left(3L_{bm} EI_{cln,1} + 2L_{cln} EI_{bm}\right)(e_0 + e_1) \end{pmatrix} \end{pmatrix}}{8 EI_{cln,2}\left(3L_{bm} EI_{cln,1} + 2L_{cln} EI_{bm}\right)\begin{pmatrix} 3\pi(q_1+q_2) L_{bm} L_{cln}^3 EI_{bm} \\ +8\left(2\pi^2 EI_{cln,2} - (q_1+q_2) L_{bm} L_{cln}^2\right)\left(3L_{bm} EI_{cln,2} + 2L_{cln} EI_{bm}\right) \end{pmatrix}}$$

The second difference is the original load. In the formula of Appendix N, there is no original load. The original load must be neglected. q_1 = 0. This also results in e_1 = 0.

$$e_2 = \frac{q_2 L_{bm} L_{cln}^2 \begin{pmatrix} \left(q_2 L_{bm} L_{cln}^2 + 16 EI_{cln,2}\right)\left(3L_{bm} EI_{cln,1} + 2L_{cln} EI_{bm}\right)\left(L_{bm}^2 EI_{cln,2}\right) \\ +\left(3L_{bm} EI_{cln,2} + 2L_{cln} EI_{bm}\right)\left(64 e_0 EI_{cln,2}\left(3L_{bm} EI_{cln,1} + 2L_{cln} EI_{bm}\right)\right) \end{pmatrix}}{8 EI_{cln,2}\left(3L_{bm} EI_{cln,1} + 2L_{cln} EI_{bm}\right) \begin{pmatrix} 3 q_2 \pi L_{bm} L_{cln}^3 EI_{bm} \\ +8\left(2\pi^2 EI_{cln,2} - q_2 L_{bm} L_{cln}^2\right)\left(3L_{bm} EI_{cln,2} + 2L_{cln} EI_{bm}\right) \end{pmatrix}}$$

Again some expressions can be neglected.

$$e_2 = \frac{q_2 L_{bm} L_{cln}^2 \left(\left(q_2 L_{bm}^3 L_{cln}^2 + 16 EI_{cln,2}\right) + 64 e_0 \left(3L_{bm} EI_{cln,2} + 2L_{cln} EI_{bm}\right)\right)}{8\left(3 q_2 \pi L_{bm} L_{cln}^3 EI_{bm} + 8\left(2\pi^2 EI_{cln,2} - q_2 L_{bm} L_{cln}^2\right)\left(3L_{bm} EI_{cln,2} + 2L_{cln} EI_{bm}\right)\right)}$$

The formula what is found in Appendix N is the following one:

$$e_1 = \frac{q_1 L_{bm} L_{cln}^2 \left(q_1 L_{bm}^3 L_{cln}^2 + 64 e_0 \left(3L_{bm} EI_{cln,1} + 2L_{cln} EI_{bm}\right) + 16 L_{bm}^2 EI_{cln,1}\right)}{8\left[3\pi q_1 L_{bm} L_{cln}^3 EI_{bm} + \left(16\pi^2 EI_{cln,1} - 8 q_1 L_{bm} L_{cln}^2\right)\left(3L_{bm} EI_{cln,1} + 2L_{cln} EI_{bm}\right)\right]}$$

The only difference between these two formulas is the stiffness and the load. If q_2 will be replaced by q_1, $EI_{cln,2}$ will be replaced by $EI_{cln,1}$ and if e_2 will be replaced by e_1 the original formula is found. It is proven that the formula does not have big mistakes.

The formula of e_2 has been found. If e_2 has been found all reaction forces and internal stresses can be calculated. This could be done if the formulas from the begin of this Appendix are used. How to use these formulas become clear at the calculation example in Appendix P.

O.2 Analysis if two parts yield

If both the right flange as the left flange partial yield and the stiffness of the reduced section is large enough to resist more loads, the third load case starts. The portal frame is loaded by load q_1 and by load q_2. Load q_1 results in yield in the right flange. Load q_2 results in yield in the left flange. Due to q_2 the stress in the right flange increase, but does not reaches the second critical stress. In the analysis of Appendix O.1 a shift of the centre of gravity has taken into account. In the third load case, both the right flange as well as the left flange partial yield. In the middle of the column the effective section is double symmetric again. There is no shift in the centre of gravity anymore. The stress in the midsection has been generalized whole in the structure. There will be no extra deflection due to the shift of the centre of gravity. The deflection of the second load case must be taken into account at the calculation of the external moment. Because of this the shift of the point of gravity has a small influence on the total deflection in the third load case.

Again the analysis starts with the calculation of the horizontal reaction force in support A.

$$\frac{q_3 L_{bm}{}^3 L_{cln}}{24EI_{bm}} - \frac{M_3 L_{cln}}{k_r} - \frac{F_3 L_{cln}{}^3}{3EI_{cln,3}} = 0$$

Use the formula of k_r.

$$\frac{q_3 L_{bm}{}^3 L_{cln}}{24EI_{bm}} - \frac{F_3 L_{bm} L_{cln}{}^2}{2EI_{bm}} - \frac{F_3 L_{cln}{}^3}{3EI_{cln,3}} = 0$$

All expressions of F_3 has been separate from the rest of the formula.

$$\frac{F_3 L_{bm} L_{cln}{}^2}{2EI_{bm}} + \frac{F_3 L_{cln}{}^3}{3EI_{cln,3}} = \frac{q_3 L_{bm}{}^3 L_{cln}}{24EI_{bm}}$$

Make the same denominator.

$$\frac{12F_3 L_{bm} L_{cln}{}^2 EI_{cln,3} + 8F_3 L_{cln}{}^3 EI_{bm}}{24EI_{cln,3} EI_{bm}} = \frac{q_3 L_{bm}{}^3 L_{cln} EI_{cln,3}}{24EI_{cln,3} EI_{bm}}$$

Neglect the denominator.

$$F_3\left(12L_{bm} L_{cln}{}^2 EI_{cln,3} + 8L_{cln}{}^3 EI_{bm}\right) = q_3 L_{bm}{}^3 L_{cln} EI_{cln,3}$$

Find the formula of F_3

$$F_3 = \frac{q_3 L_{bm}{}^3 EI_{cln,3}}{4L_{cln}\left(3L_{bm} EI_{cln,3} + 2L_{cln} EI_{bm}\right)}$$

This formula looks like the formula found in Appendix N. The formulas found in Appendix N and in Appendix O.1 are:

$$F_1 = \frac{q_1 L_{bm}{}^3 EI_{cln,1}}{4L_{cln}\left(3L_{bm} EI_{cln,1} + 2L_{cln} EI_{bm}\right)} \qquad F_2 = \frac{q_2 L_{bm}\left(L_{bm}{}^2 EI_{cln,2} + 6L_{cln} z_2 EI_{bm}\right)}{4L_{cln}\left(3L_{bm} EI_{cln,2} + 2L_{cln} EI_{bm}\right)}$$

The deflections are the follow.

Deflection due to moment

$$M_{1,3} = F_3\left(L_{cln} - x\right)$$

$$\kappa_{1,3} = \frac{-F_3\left(L_{cln} - x\right)}{EI_{cln,3}}$$

$$\varphi_{1,3} = \frac{-F_3\left(2L_{cln} x - x^2\right)}{2EI_{cln,3}} + C_1$$

$$y_{1,3} = \frac{-F_3\left(3L_{cln}x^2 - x^3\right)}{6EI_{cln,3}} + C_1x + C_2$$

Find the integration constants.

$y_{1,3,x=0} = 0$ → $C_2 = 0$

$y_{1,3,x=L_{cln}} = 0$ → $C_1 = \frac{2F_3L_{cln}{}^2}{6EI_{cln,3}}$

$$y_{1,3} = \frac{F_3\left(2L_{cln}{}^2x - 3L_{cln}x^2 + x^3\right)}{6EI_{cln,3}}$$

Use the formula of F_3.

$$y_{1,3} = \frac{q_3L_{bm}{}^3EI_{cln,3}}{4L_{cln}\left(3L_{bm}EI_{cln,3} + 2L_{cln}EI_{bm}\right)}\frac{\left(2L_{cln}{}^2x - 3L_{cln}x^2 + x^3\right)}{6EI_{cln,3}}$$

$$y_{1,3} = \frac{q_3L_{bm}{}^3\left(2L_{cln}{}^2x - 3L_{cln}x^2 + x^3\right)}{24L_{cln}\left(3L_{bm}EI_{cln,3} + 2L_{cln}EI_{bm}\right)}$$

Starting deflection

$$y_2 = e_0\sin\left(\frac{\pi x}{L_{cln}}\right)$$

This deflection does not change.

Deflection due to rotation spring:

$$M_{3,3} = -\varphi_3k_r$$

$$M_{3,3,x=0} = -\varphi_{extra,3,x=0}k_r$$

$$M_{3,3} = \frac{-\varphi_{extra,3,x=0}k_r\left(L_{cln} - x\right)}{L_{cln}}$$

$$\kappa_{3,3} = \frac{\varphi_{extra,3,x=0}k_r\left(L_{cln} - x\right)}{L_{cln}EI_{cln,3}}$$

$$\varphi_{3,3} = \frac{\varphi_{extra,3,x=0}k_r\left(2L_{cln}x - x^2\right)}{2L_{cln}EI_{cln,3}} + C_1$$

$$y_{3,3}=\frac{\varphi_{extra,3,x=0}k_r\left(3L_{cln}x^2-x^3\right)}{6L_{cln}EI_{cln,3}}+C_1x+C_2$$

Find the integration constants.

$y_{3,3,x=0}=0$ → $C_2 = 0$

$y_{3,3,x=L_{cln}}=0$ → $C_1=\frac{-2\varphi_{extra,3,x=0}k_rL_{cln}^2}{6L_{cln}EI_{cln,3}}$

$$y_{3,3}=\frac{\varphi_{extra,3,x=0}k_r\left(-2L_{cln}^2x+3L_{cln}x^2-x^3\right)}{6L_{cln}EI_{cln,3}}$$

Use the formula of k_r.

$$y_{3,3}=\frac{\varphi_{extra,3,x=0}\frac{2EI_{bm}}{L_{bm}}\left(-2L_{cln}^2x+3L_{cln}x^2-x^3\right)}{6L_{cln}EI_{cln,3}}$$

$$y_{3,3}=\frac{\varphi_{extra,3,x=0}EI_{bm}\left(-2L_{cln}^2x+3L_{cln}x^2-x^3\right)}{3L_{bm}L_{cln}EI_{cln,3}}$$

Additional deflection

$$y_{4,3}=e_3\sin\left(\frac{\pi x}{L_{cln}}\right)$$

Here follows a list of all deflection formulas.

$$y_{1,3}=\frac{q_3L_{bm}^3\left(2L_{cln}^2x-3L_{cln}x^2+x^3\right)}{24L_{cln}\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}$$

$$y_{1,1}=\frac{q_1L_{bm}^3\left(2L_{cln}^2x-3L_{cln}x^2+x^3\right)}{24L_{cln}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)}$$

$$y_{1,2}=\frac{q_2L_{bm}\left(L_{bm}^2EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)\left(2L_{cln}^2x-3L_{cln}x^2+x^3\right)}{24L_{cln}EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}$$

$$y_2=e_0\sin\left(\frac{\pi x}{L_{cln}}\right)$$

$$y_{3,3}=\frac{\varphi_{extra,3,x=0}EI_{bm}\left(-2L_{cln}^2x+3L_{cln}x^2-x^3\right)}{3L_{bm}L_{cln}EI_{cln,3}}$$

$$y_{3,1}=\frac{\varphi_{extra,1,x=0}EI_{bm}\left(-2L_{cln}^2x+3L_{cln}x^2-x^3\right)}{3L_{bm}L_{cln}EI_{cln}}$$

$$y_{3,2}=\frac{\varphi_{extra,2,x=0}EI_{bm}\left(-2L_{cln}^2x+3L_{cln}x^2-x^3\right)}{3L_{bm}L_{cln}EI_{cln,2}}$$

$$y_{4,3} = e_3 \sin\left(\frac{\pi x}{L_{cln}}\right)$$

$$y_{4,1} = e_1 \sin\left(\frac{\pi x}{L_{cln}}\right)$$

$$y_{4,2} = e_2 \sin\left(\frac{\pi x}{L_{cln}}\right)$$

$$y_{5,2} = \frac{q_2 L_{bm} z_2 \left(L_{cln} x - x^2\right)}{4EI_{cln,2}}$$

$$y_{total,3} = \frac{q_1 L_{bm}{}^3 L_{cln}{}^2}{64\left(3L_{bm}EI_{cln,1} + 2L_{cln}EI_{bm}\right)} + \frac{q_2 L_{bm} L_{cln}{}^2 \left(L_{bm}{}^2 EI_{cln,2} + 6L_{cln} z_2 EI_{bm}\right)}{64EI_{cln,2}\left(3L_{bm}EI_{cln,2} + 2L_{cln}EI_{bm}\right)} + \frac{q_3 L_{bm}{}^3 L_{cln}{}^2}{64\left(3L_{bm}EI_{cln,3} + 2L_{cln}EI_{bm}\right)} + e_0$$
$$- \frac{\varphi_{extra,1,x=0} L_{cln}{}^2 EI_{bm}}{8L_{bm}EI_{cln}} - \frac{\varphi_{extra,2,x=0} L_{cln}{}^2 EI_{bm}}{8L_{bm}EI_{cln,2}} - \frac{\varphi_{extra,3,x=0} L_{cln}{}^2 EI_{bm}}{8L_{bm}EI_{cln,3}} + e_1 + e_2 + e_3 + \frac{q_2 L_{bm} L_{cln}{}^2 z_2}{16EI_{cln,2}}$$

Deflection $y_{3,3}$ and deflection $y_{3,4}$ are the additional deflections. These deflections are important to calculate the additional rotation. The additional rotation must be calculated at x = 0.

$$\varphi_{extra,3,x=0} = \frac{\varphi_{extra,3,x=0} EI_{bm}\left(-2L_{cln}{}^2 + 6L_{cln} * 0 - 3 * 0^2\right)}{3L_{bm}L_{cln}EI_{cln,3}} + \frac{\pi}{L_{cln}} e_3 \cos\left(\frac{\pi * 0}{L_{cln}}\right)$$

Neglect some expressions.

$$\varphi_{extra,3,x=0} = \frac{-2\varphi_{extra,3,x=0} L_{cln} EI_{bm}}{3L_{bm}EI_{cln,3}} + \frac{\pi}{L_{cln}} e_3$$

Separate $\varphi_{extra,3,x=0}$ from e_3.

$$\varphi_{extra,3,x=0} + \frac{2\varphi_{extra,3,x=0} L_{cln} EI_{bm}}{3L_{bm}EI_{cln,3}} = \frac{\pi e_3}{L_{cln}}$$

Make the same denominator for all expressions.

$$\varphi_{extra,3,x=0}\left(\frac{L_{cln}\left(3L_{bm}EI_{cln,3} + 2L_{cln}EI_{bm}\right)}{3L_{bm}L_{cln}EI_{cln,3}}\right) = \frac{3\pi L_{bm} e_3 EI_{cln,3}}{3L_{bm}L_{cln}EI_{cln,3}}$$

This results in the following formula.

$$\varphi_{extra,3,x=0} = \frac{3\pi L_{bm} e_3 EI_{cln,3}}{L_{cln}\left(3L_{bm}EI_{cln,3} + 2L_{cln}EI_{bm}\right)}$$

This formula can be used for the deflection $y_{3,3}$.

$$y_{3,3} = \frac{3\pi L_{bm} e_3 EI_{cln,3}}{L_{cln}\left(3L_{bm}EI_{cln,3} + 2L_{cln}EI_{bm}\right)} \frac{EI_{bm}\left(-2L_{cln}{}^2 x + 3L_{cln} x^2 - x^3\right)}{3L_{bm}L_{cln}EI_{cln,3}}$$

$$y_{3,3} = \frac{\pi e_3 EI_{bm}\left(-2L_{cln}{}^2 x + 3L_{cln}x^2 - x^3\right)}{L_{cln}{}^2\left(3L_{bm}EI_{cln,3} + 2L_{cln}EI_{bm}\right)}$$

$$\varphi_{3,3} = \frac{\pi e_3 EI_{bm}\left(-2L_{cln}{}^2 + 6L_{cln}x - 3x^2\right)}{L_{cln}{}^2\left(3L_{bm}EI_{cln,3} + 2L_{cln}EI_{bm}\right)}$$

$$\kappa_{3,3} = \frac{6\pi e_3 EI_{bm}\left(L_{cln} - x\right)}{L_{cln}{}^2\left(3L_{bm}EI_{cln,3} + 2L_{cln}EI_{bm}\right)}$$

Already known are the following formulas:

$$y_{3,2} = \frac{\pi e_2 EI_{bm}\left(-2L_{cln}{}^2 x + 3L_{cln}x^2 - x^3\right)}{L_{cln}{}^2\left(3L_{bm}EI_{cln,2} + 2L_{cln}EI_{bm}\right)} \qquad y_{3,1} = \frac{\pi e_1 EI_{bm}\left(-2L_{cln}{}^2 x + 3L_{cln}x^2 - x^3\right)}{L_{cln}{}^2\left(3L_{bm}EI_{cln,1} + 2L_{cln}EI_{bm}\right)}$$

All necessary formulas are known. The differential equation can start. The differential equation is based on the equilibrium between the internal and the external moments.

$$\Delta M_{intern} = \Delta M_{extern}$$

$$-\Delta\kappa_3 = \left(N_1 + N_2\right)y_3 + N_3 y_{total,3} + \frac{M_{B,3}x}{L_{cln}} - \frac{k_r \varphi_{extra,3,x=0}x}{L_{cln}}$$

$$-\left(\kappa_{3,3} + \kappa_{4,3}\right)EI_{cln,3} = \left(N_1 + N_2\right)\left(y_{1,3} + y_{3,3} + y_{4,3}\right)$$
$$+N_3\left(y_{1,1} + y_{1,2} + y_{1,3} + y_2 + y_{3,1} + y_{3,2} + y_{3,3} + y_{4,1} + y_{4,2} + y_{4,3} + y_{5,2}\right)$$
$$+\frac{M_{B,3}x}{L_{cln}} - \frac{k_r \varphi_{extra,3,x=0}x}{L_{cln}}$$

All known formulas can be used in this formula.

$$-\left(\frac{6\pi e_3 EI_{bm}\left(L_{cln}-x\right)}{L_{cln}{}^2\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}+\frac{-\pi^2}{L_{cln}{}^2}e_3\sin\left(\frac{\pi x}{L_{cln}}\right)\right)EI_{cln,3}$$

$$=\left(N_1+N_2\right)\left(\frac{q_3L_{bm}{}^3\left(2L_{cln}{}^2x-3L_{cln}x^2+x^3\right)}{24L_{cln}\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}+\frac{\pi e_3EI_{bm}\left(-2L_{cln}{}^2x+3L_{cln}x^2-x^3\right)}{L_{cln}{}^2\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}+e_3\sin\left(\frac{\pi x}{L_{cln}}\right)\right)$$

$$+N_3\left(\begin{aligned}&\frac{q_1L_{bm}{}^3\left(2L_{cln}{}^2x-3L_{cln}x^2+x^3\right)}{24L_{cln}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)}+\frac{q_2L_{bm}\left(L_{bm}{}^2EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)\left(2L_{cln}{}^2x-3L_{cln}x^2+x^3\right)}{24L_{cln}EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}\\&+\frac{q_3L_{bm}{}^3\left(2L_{cln}{}^2x-3L_{cln}x^2+x^3\right)}{24L_{cln}\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}+e_0\sin\left(\frac{\pi x}{L_{cln}}\right)+\frac{\pi e_1EI_{bm}\left(-2L_{cln}{}^2x+3L_{cln}x^2-x^3\right)}{L_{cln}{}^2\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)}\\&+\frac{\pi e_2EI_{bm}\left(-2L_{cln}{}^2x+3L_{cln}x^2-x^3\right)}{L_{cln}{}^2\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}+\frac{\pi e_3EI_{bm}\left(-2L_{cln}{}^2x+3L_{cln}x^2-x^3\right)}{L_{cln}{}^2\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}+e_1\sin\left(\frac{\pi x}{L_{cln}}\right)\\&+e_2\sin\left(\frac{\pi x}{L_{cln}}\right)+e_3\sin\left(\frac{\pi x}{L_{cln}}\right)+\frac{q_2L_{bm}z_2\left(L_{cln}x-x^2\right)}{4EI_{cln,2}}\end{aligned}\right)$$

$$+\frac{q_3L_{bm}{}^3EI_{cln,3}x}{4L_{cln}\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}-\frac{6\pi e_3EI_{bm}EI_{cln,3}x}{L_{cln}{}^2\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}$$

Write out some expressions to neglect the brackets.

$$\frac{-6\pi e_3EI_{bm}EI_{cln,3}\left(L_{cln}-x\right)}{L_{cln}{}^2\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}+\frac{\pi^2EI_{cln,3}}{L_{cln}{}^2}e_3\sin\left(\frac{\pi x}{L_{cln}}\right)$$

$$=\frac{q_3\left(N_1+N_2\right)L_{bm}{}^3\left(2L_{cln}{}^2x-3L_{cln}x^2+x^3\right)}{24L_{cln}\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}+\frac{\pi\left(N_1+N_2\right)e_3EI_{bm}\left(-2L_{cln}{}^2x+3L_{cln}x^2-x^3\right)}{L_{cln}{}^2\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}$$

$$+\left(N_1+N_2\right)e_3\sin\left(\frac{\pi x}{L_{cln}}\right)+N_3e_2\sin\left(\frac{\pi x}{L_{cln}}\right)+N_3e_3\sin\left(\frac{\pi x}{L_{cln}}\right)+\frac{q_2N_3L_{bm}z_2\left(L_{cln}x-x^2\right)}{4EI_{cln,2}}$$

$$+\frac{q_1N_3L_{bm}{}^3\left(2L_{cln}{}^2x-3L_{cln}x^2+x^3\right)}{24L_{cln}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)}+\frac{q_2N_3L_{bm}\left(L_{bm}{}^2EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)\left(2L_{cln}{}^2x-3L_{cln}x^2+x^3\right)}{24L_{cln}EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}$$

$$+\frac{q_3N_3L_{bm}{}^3\left(2L_{cln}{}^2x-3L_{cln}x^2+x^3\right)}{24L_{cln}\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}+N_3e_0\sin\left(\frac{\pi x}{L_{cln}}\right)+\frac{\pi N_3e_1EI_{bm}\left(-2L_{cln}{}^2x+3L_{cln}x^2-x^3\right)}{L_{cln}{}^2\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)}$$

$$+\frac{\pi N_3e_2EI_{bm}\left(-2L_{cln}{}^2x+3L_{cln}x^2-x^3\right)}{L_{cln}{}^2\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}+\frac{\pi N_3e_3EI_{bm}\left(-2L_{cln}{}^2x+3L_{cln}x^2-x^3\right)}{L_{cln}{}^2\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}+N_3e_1\sin\left(\frac{\pi x}{L_{cln}}\right)$$

$$+\frac{q_3L_{bm}{}^3EI_{cln,3}x}{4L_{cln}\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}-\frac{6\pi e_3EI_{bm}EI_{cln,3}x}{L_{cln}{}^2\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}$$

The loads are known. The values of N_1, N_2 and N_3 can be used $\left(N_1=\tfrac{1}{2}q_1L_{bm};N_2=\tfrac{1}{2}q_2L_{bm};N_3=\tfrac{1}{2}q_3L_{bm}\right)$. This results in the following formula.

$$\frac{-6\pi e_3 EI_{bm} EI_{cln,3}\left(L_{cln}-x\right)}{L_{cln}^{\ 2}\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}+\frac{\pi^2 EI_{cln,3}}{L_{cln}^{\ 2}}e_3\sin\left(\frac{\pi x}{L_{cln}}\right)$$

$$=\frac{q_3\left(q_1+q_2\right)L_{bm}^{\ 4}\left(2L_{cln}^{\ 2}x-3L_{cln}x^2+x^3\right)}{48L_{cln}\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}+\frac{\pi\left(q_1+q_2\right)L_{bm}e_3EI_{bm}\left(-2L_{cln}^{\ 2}x+3L_{cln}x^2-x^3\right)}{2L_{cln}^{\ 2}\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}$$

$$+\frac{\left(q_1+q_2\right)L_{bm}e_3\sin\left(\frac{\pi x}{L_{cln}}\right)}{2}+\frac{\Delta q_3L_{bm}e_2\sin\left(\frac{\pi x}{L_{cln}}\right)}{2}+\frac{q_3L_{bm}e_3\sin\left(\frac{\pi x}{L_{cln}}\right)}{2}+\frac{q_2q_3L_{bm}^{\ 2}z_2\left(L_{cln}x-x^2\right)}{8EI_{cln,2}}$$

$$+\frac{q_1q_3L_{bm}^{\ 4}\left(2L_{cln}^{\ 2}x-3L_{cln}x^2+x^3\right)}{48L_{cln}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)}+\frac{q_2q_3L_{bm}^{\ 2}\left(L_{bm}^{\ 2}EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)\left(2L_{cln}^{\ 2}x-3L_{cln}x^2+x^3\right)}{48L_{cln}EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}$$

$$+\frac{q_3^{\ 2}L_{bm}^{\ 4}\left(2L_{cln}^{\ 2}x-3L_{cln}x^2+x^3\right)}{48L_{cln}\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}+\frac{q_3L_{bm}e_0\sin\left(\frac{\pi x}{L_{cln}}\right)}{2}+\frac{\pi q_3L_{bm}e_1EI_{bm}\left(-2L_{cln}^{\ 2}x+3L_{cln}x^2-x^3\right)}{2L_{cln}^{\ 2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)}$$

$$+\frac{\pi q_3L_{bm}e_2EI_{bm}\left(-2L_{cln}^{\ 2}x+3L_{cln}x^2-x^3\right)}{2L_{cln}^{\ 2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}+\frac{\pi q_3L_{bm}e_3EI_{bm}\left(-2L_{cln}^{\ 2}x+3L_{cln}x^2-x^3\right)}{2L_{cln}^{\ 2}\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}+\frac{q_3L_{bm}e_1\sin\left(\frac{\pi x}{L_{cln}}\right)}{2}$$

$$+\frac{q_3L_{bm}^{\ 3}EI_{cln,3}x}{4L_{cln}\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}-\frac{6\pi e_3EI_{bm}EI_{cln,3}x}{L_{cln}^{\ 2}\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}$$

The most important deflection is the deflection at half the column length. Calculate the deflection at $x=0.5L_{cln}$.

$$\frac{-3\pi e_3EI_{bm}EI_{cln,3}}{L_{cln}\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}+\frac{\pi^2e_3EI_{cln,3}}{L_{cln}^{\ 2}}=\frac{q_3\left(q_1+q_2\right)L_{bm}^{\ 4}L_{cln}^{\ 2}}{128\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}-\frac{3\pi\left(q_1+q_2\right)L_{bm}L_{cln}e_3EI_{bm}}{16\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}$$

$$+\frac{\left(q_1+q_2\right)L_{bm}e_3}{2}+\frac{q_1q_3L_{bm}^{\ 4}L_{cln}^{\ 2}}{128\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)}+\frac{q_2q_3L_{bm}^{\ 2}L_{cln}^{\ 2}\left(L_{bm}^{\ 2}EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)}{128EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}$$

$$+\frac{q_3^{\ 2}L_{bm}^{\ 4}L_{cln}^{\ 2}}{128\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}+\frac{q_3L_{bm}e_0}{2}-\frac{3\pi q_3L_{bm}L_{cln}e_1EI_{bm}}{16\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)}-\frac{3\pi q_3L_{bm}L_{cln}e_2EI_{bm}}{16\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}$$

$$-\frac{3\pi q_3L_{bm}L_{cln}e_3EI_{bm}}{16\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}+\frac{q_3L_{bm}e_1}{2}+\frac{q_3L_{bm}e_2}{2}+\frac{q_3L_{bm}e_3}{2}+\frac{q_2q_3L_{bm}^{\ 2}L_{cln}^{\ 2}z_2}{32EI_{cln,2}}$$

$$+\frac{q_3L_{bm}^{\ 3}EI_{cln,3}}{8\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}-\frac{3\pi e_3EI_{bm}EI_{cln,3}}{L_{cln}\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}$$

On expression is the same at both sides of the equation. These expressions can be neglected.

$$\frac{\pi^2 e_3 EI_{cln,3}}{L_{cln}{}^2} = \frac{q_3(q_1+q_2)L_{bm}{}^4 L_{cln}{}^2}{128(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm})} - \frac{3\pi(q_1+q_2)L_{bm}L_{cln}e_3EI_{bm}}{16(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm})}$$

$$+\frac{(q_1+q_2)L_{bm}e_3}{2} + \frac{q_1q_3L_{bm}{}^4L_{cln}{}^2}{128(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm})} + \frac{q_2q_3L_{bm}{}^2L_{cln}{}^2(L_{bm}{}^2EI_{cln,2}+6L_{cln}z_2EI_{bm})}{128EI_{cln,2}(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm})}$$

$$+\frac{q_3{}^2L_{bm}{}^4L_{cln}{}^2}{128(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm})} + \frac{q_3L_{bm}e_0}{2} - \frac{3\pi q_3L_{bm}L_{cln}e_1EI_{bm}}{16(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm})} - \frac{3\pi q_3L_{bm}L_{cln}e_2EI_{bm}}{16(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm})}$$

$$-\frac{3\pi q_3L_{bm}L_{cln}e_3EI_{bm}}{16(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm})} + \frac{q_3L_{bm}e_1}{2} + \frac{q_3L_{bm}e_2}{2} + \frac{q_3L_{bm}e_3}{2} + \frac{q_2q_3L_{bm}{}^2L_{cln}{}^2z_2}{32EI_{cln,2}}$$

$$+\frac{q_3L_{bm}{}^3EI_{cln,3}}{8(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm})}$$

In this formula e_3 is the only unknown. To find the unknown value all expressions of e_3 must be placed on one side of the equation.

$$\frac{3\pi(q_1+q_2)L_{bm}L_{cln}e_3EI_{bm}}{16(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm})} + \frac{3\pi q_3L_{bm}L_{cln}e_3EI_{bm}}{16(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm})} - \frac{(q_1+q_2)L_{bm}e_3}{2} - \frac{q_3L_{bm}e_3}{2} + \frac{\pi^2e_3EI_{cln,3}}{L_{cln}{}^2}$$

$$= \frac{q_3(q_1+q_2)L_{bm}{}^4L_{cln}{}^2}{128(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm})} + \frac{q_1q_3L_{bm}{}^4L_{cln}{}^2}{128(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm})} + \frac{q_2q_3L_{bm}{}^2L_{cln}{}^2(L_{bm}{}^2EI_{cln,2}+6L_{cln}z_2EI_{bm})}{128EI_{cln,2}(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm})}$$

$$+\frac{q_3{}^2L_{bm}{}^4L_{cln}{}^2}{128(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm})} + \frac{q_3L_{bm}e_0}{2} - \frac{3\pi q_3L_{bm}L_{cln}e_1EI_{bm}}{16(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm})} - \frac{3\pi q_3L_{bm}L_{cln}e_2EI_{bm}}{16(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm})}$$

$$+\frac{q_3L_{bm}e_1}{2} + \frac{q_3L_{bm}e_2}{2} + \frac{q_2q_3L_{bm}{}^2L_{cln}{}^2z_2}{32EI_{cln,2}} + \frac{q_3L_{bm}{}^3EI_{cln,3}}{8(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm})}$$

All expressions must have the same denominator. To make the same denominator starts with the expressions of e_3.

$$e_3\left(\begin{array}{l}\frac{3\pi(q_1+q_2)L_{bm}L_{cln}^3EI_{bm}}{16L_{cln}^2(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm})}+\frac{3\pi q_3L_{bm}L_{cln}^3EI_{bm}}{16L_{cln}^2(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm})}\\-\frac{8(q_1+q_2)L_{bm}L_{cln}^2(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm})}{16L_{cln}^2(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm})}-\frac{8q_3L_{bm}L_{cln}^2(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm})}{16L_{cln}^2(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm})}\\+\frac{16\pi^2EI_{cln,3}(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm})}{16L_{cln}^2(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm})}\end{array}\right)$$

$$=\frac{q_3(q_1+q_2)L_{bm}^4L_{cln}^2}{128(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm})}+\frac{q_1q_3L_{bm}^4L_{cln}^2}{128(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm})}+\frac{q_2q_3L_{bm}^2L_{cln}^2(L_{bm}^2EI_{cln,2}+6L_{cln}z_2EI_{bm})}{128EI_{cln,2}(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm})}$$
$$+\frac{q_3^2L_{bm}^4L_{cln}^2}{128(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm})}+\frac{q_3L_{bm}e_0}{2}-\frac{3\pi q_3L_{bm}L_{cln}e_1EI_{bm}}{16(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm})}-\frac{3\pi q_3L_{bm}L_{cln}e_2EI_{bm}}{16(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm})}$$
$$+\frac{q_3L_{bm}e_1}{2}+\frac{q_3L_{bm}e_2}{2}+\frac{q_2q_3L_{bm}^2L_{cln}^2z_2}{32EI_{cln,2}}+\frac{q_3L_{bm}^3EI_{cln,3}}{8(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm})}$$

Make one denominator for all expressions of e_3.

$$e_3\frac{\left(\begin{array}{l}3\pi(q_1+q_2)L_{bm}L_{cln}^3EI_{bm}+3\pi q_3L_{bm}L_{cln}^3EI_{bm}-8(q_1+q_2)L_{bm}L_{cln}^2(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm})\\-8q_3L_{bm}L_{cln}^2(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm})+16\pi^2EI_{cln,3}(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm})\end{array}\right)}{16L_{cln}^2(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm})}$$

$$=\frac{q_3(q_1+q_2)L_{bm}^4L_{cln}^2}{128(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm})}+\frac{q_1q_3L_{bm}^4L_{cln}^2}{128(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm})}+\frac{q_2q_3L_{bm}^2L_{cln}^2(L_{bm}^2EI_{cln,2}+6L_{cln}z_2EI_{bm})}{128EI_{cln,2}(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm})}$$
$$+\frac{q_3^2L_{bm}^4L_{cln}^2}{128(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm})}+\frac{q_3L_{bm}e_0}{2}-\frac{3\pi q_3L_{bm}L_{cln}e_1EI_{bm}}{16(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm})}-\frac{3\pi q_3L_{bm}L_{cln}e_2EI_{bm}}{16(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm})}$$
$$+\frac{q_3L_{bm}e_1}{2}+\frac{q_3L_{bm}e_2}{2}+\frac{q_2q_3L_{bm}^2L_{cln}^2z_2}{32EI_{cln,2}}+\frac{q_3L_{bm}^3EI_{cln,3}}{8(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm})}$$

Simplify the numerator

$$e_3 \frac{\begin{pmatrix} 3\pi\left(q_1+q_2+q_3\right)L_{bm}{L_{cln}}^3 EI_{bm} \\ +8\left(2\pi^2 EI_{cln,3}-\left(q_1+q_2+q_3\right)L_{bm}{L_{cln}}^2\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)\end{pmatrix}}{16{L_{cln}}^2\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}$$

$$=\frac{q_3\left(q_1+q_2\right){L_{bm}}^4{L_{cln}}^2}{128\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}+\frac{q_1 q_3 {L_{bm}}^4{L_{cln}}^2}{128\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)}+\frac{q_2 q_3 {L_{bm}}^2{L_{cln}}^2\left({L_{bm}}^2EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)}{128EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}$$

$$+\frac{{q_3}^2{L_{bm}}^4{L_{cln}}^2}{128\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}+\frac{q_3L_{bm}e_0}{2}-\frac{3\pi q_3 L_{bm}L_{cln}e_1EI_{bm}}{16\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)}-\frac{3\pi q_3 L_{bm}L_{cln}e_2EI_{bm}}{16\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}$$

$$+\frac{q_3L_{bm}e_1}{2}+\frac{q_3L_{bm}e_2}{2}+\frac{q_2q_3{L_{bm}}^2{L_{cln}}^2z_2}{32EI_{cln,2}}+\frac{q_3{L_{bm}}^3EI_{cln,3}}{8\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}$$

It is complex to find everywhere the same denominator. In Appendix O.1 there were two different stiffness. In this Appendix there are three different stiffness. This results in a very complex formula. In the following formula every expression has the same denominator.

$$
e_3\frac{8EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\begin{pmatrix}3\pi\left(q_1+q_2+q_3\right)L_{bm}L_{cln}{}^{3}EI_{bm}\\+8\left(2\pi^2EI_{cln,3}-\left(q_1+q_2+q_3\right)L_{bm}L_{cln}{}^{2}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)\end{pmatrix}}{128L_{cln}{}^{2}EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}
$$

$$
=\frac{q_3\left(q_1+q_2\right)L_{bm}{}^{4}L_{cln}{}^{4}EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}{128L_{cln}{}^{2}EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}
$$

$$
+\frac{q_1q_3L_{bm}{}^{4}L_{cln}{}^{4}EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}{128L_{cln}{}^{2}EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}
$$

$$
+\frac{q_2q_3L_{bm}{}^{2}L_{cln}{}^{4}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(L_{bm}{}^{2}EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}{128L_{cln}{}^{2}EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}
$$

$$
+\frac{q_3{}^{2}L_{bm}{}^{4}L_{cln}{}^{4}EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}{128L_{cln}{}^{2}EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}
$$

$$
+\frac{64q_3L_{bm}L_{cln}{}^{2}e_0EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}{128L_{cln}{}^{2}EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}
$$

$$
-\frac{24\pi q_3L_{bm}L_{cln}{}^{3}e_1EI_{bm}EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}{128L_{cln}{}^{2}EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}
$$

$$
-\frac{24\pi q_3L_{bm}L_{cln}{}^{3}e_2EI_{bm}EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}{128L_{cln}{}^{2}EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}
$$

$$
+\frac{64q_3L_{bm}L_{cln}{}^{2}e_1EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}{128L_{cln}{}^{2}EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}
$$

$$
+\frac{64q_3L_{bm}L_{cln}{}^{2}e_2EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}{128L_{cln}{}^{2}EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}
$$

$$
+\frac{4q_2q_3L_{bm}{}^{2}L_{cln}{}^{4}z_2\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}{128L_{cln}{}^{2}EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}
$$

$$
+\frac{16q_3L_{bm}{}^{3}L_{cln}{}^{2}EI_{cln,2}EI_{cln,3}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}{128L_{cln}{}^{2}EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}
$$

Neglect the denominator.

$$
\begin{aligned}
&8EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\\
&\begin{pmatrix}3\pi\left(q_1+q_2+q_3\right)L_{bm}L_{cln}{}^3EI_{bm}\\+8\left(2\pi^2EI_{cln,3}-\left(q_1+q_2+q_3\right)L_{bm}L_{cln}{}^2\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)\end{pmatrix}\\
&=q_3\left(q_1+q_2\right)L_{bm}{}^4L_{cln}{}^4EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\\
&+q_1q_3L_{bm}{}^4L_{cln}{}^4EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)\\
&+q_2q_3L_{bm}{}^2L_{cln}{}^4\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(L_{bm}{}^2EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)\\
&+q_3{}^2L_{bm}{}^4L_{cln}{}^4EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\\
&+64q_3L_{bm}L_{cln}{}^2e_0EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)\\
&-24\pi q_3L_{bm}L_{cln}{}^3e_1EI_{bm}EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)\\
&-24\pi q_3L_{bm}L_{cln}{}^3e_2EI_{bm}EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)\\
&+64q_3L_{bm}L_{cln}{}^2e_1EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)\\
&+64q_3L_{bm}L_{cln}{}^2e_2EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)\\
&+4q_2q_3L_{bm}{}^2L_{cln}{}^4z_2\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)\\
&+16q_3L_{bm}{}^3L_{cln}{}^2EI_{cln,2}EI_{cln,3}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)
\end{aligned}
$$

Find the formula of e_3.

$$
e_3=\frac{\begin{bmatrix}q_3\left(q_1+q_2\right)L_{bm}{}^4L_{cln}{}^4EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\\
+q_1q_3L_{bm}{}^4L_{cln}{}^4EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)\\
+q_2q_3L_{bm}{}^2L_{cln}{}^4\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(L_{bm}{}^2EI_{cln,2}+6L_{cln}z_2EI_{bm}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)\\
+q_3{}^2L_{bm}{}^4L_{cln}{}^4EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\\
+64q_3L_{bm}L_{cln}{}^2e_0EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)\\
-24\pi q_3L_{bm}L_{cln}{}^3e_1EI_{bm}EI_{cln,2}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)\\
-24\pi q_3L_{bm}L_{cln}{}^3e_2EI_{bm}EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)\\
+64q_3L_{bm}L_{cln}{}^2e_1EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)\\
+64q_3L_{bm}L_{cln}{}^2e_2EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)\\
+4q_2q_3L_{bm}{}^2L_{cln}{}^4z_2\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)\\
+16q_3L_{bm}{}^3L_{cln}{}^2EI_{cln,2}EI_{cln,3}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\end{bmatrix}}{\begin{matrix}8EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\\
\begin{pmatrix}3\pi\left(q_1+q_2+q_3\right)L_{bm}L_{cln}{}^3EI_{bm}\\+8\left(2\pi^2EI_{cln,3}-\left(q_1+q_2+q_3\right)L_{bm}L_{cln}{}^2\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)\end{pmatrix}\end{matrix}}
$$

Simplify the numerator.

$$e_3 = \frac{\begin{array}{l} q_3 L_{bm}{}^3 L_{cln}{}^2 EI_{cln,2}\left(\left(q_1+q_2+q_3\right)L_{bm}L_{cln}{}^2+16EI_{cln,3}\right)\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right) \\ +\left(\begin{array}{l} q_3 L_{bm} L_{cln}{}^3\left(\begin{array}{l} q_2 L_{bm} L_{cln}\left(L_{bm}{}^2 EI_{cln,2}+6L_{cln}z_2 EI_{bm}\right) \\ -24\pi e_2 EI_{bm}EI_{cln,2}\end{array}\right)\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right) \\ +q_3 L_{bm} L_{cln}{}^2\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right) \\ \left(\begin{array}{l} L_{cln}EI_{cln,2}\left(q_1 L_{bm}{}^3 L_{cln}-24\pi e_1 EI_{bm}\right) \\ +\left(64EI_{cln,2}\left(e_0+e_1+e_2\right)+4q_2 L_{bm}L_{cln}{}^2 z_2\right)\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\end{array}\right)\end{array}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)\end{array}}{\begin{array}{l} 8EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right) \\ \left(\begin{array}{l} 3\pi\left(q_1+q_2+q_3\right)L_{bm}L_{cln}{}^3 EI_{bm} \\ +8\left(2\pi^2 EI_{cln,3}-\left(q_1+q_2+q_3\right)L_{bm}L_{cln}{}^2\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)\end{array}\right)\end{array}}$$

To check this formula on errors, there are made some checks. The first check is the check on the dimensions.

$$m = \frac{\begin{array}{l} Nm^{-1}m^3m^2Nm^2\left(\left(Nm^{-1}+Nm^{-1}+Nm^{-1}\right)mm^2+Nm^2\right)\left(mNm^2+mNm^2\right)\left(mNm^2+mNm^2\right) \\ +\left(\begin{array}{l} Nm^{-1}mm^3\left(\begin{array}{l} Nm^{-1}mm\left(m^2Nm^2+mmNm^2\right) \\ -mNm^2Nm^2\end{array}\right)\left(mNm^2+mNm^2\right) \\ +\left(Nm^{-1}mm^2\left(mNm^2+mNm^2\right)\right) \\ \left(\begin{array}{l} +mNm^2\left(Nm^{-1}m^3m-mNm^2\right) \\ +\left(Nm^2\left(m+m+m\right)+Nm^{-1}mm^2m\right)\left(mNm^2+mNm^2\right)\end{array}\right)\end{array}\right)\left(mNm^2+mNm^2\right)\end{array}}{Nm^2\left(mNm^2+mNm^2\right)\left(mNm^2+mNm^2\right)\left(\begin{array}{l} \left(Nm^2+\left(Nm^{-1}+Nm^{-1}+Nm^{-1}\right)mm^3Nm^2\right) \\ +\left(Nm^2-\left(Nm^{-1}+Nm^{-1}+Nm^{-1}\right)mm^2\right)\left(mNm^2+mNm^2\right)\end{array}\right)}$$

$$m = \frac{\left(\begin{array}{l} N^2m^6\left(Nm^2\right)\left(Nm^3\right)\left(Nm^3\right) \\ +\left(Nm^3\left(Nm\left(Nm^4\right)-N^2m^5\right)\left(Nm^3\right)+Nm^2\left(+Nm^3\left(Nm^3\right)+\left(Nm^2\left(m\right)\right)\left(Nm^3\right)\right)\left(Nm^3\right)\right)\left(Nm^3\right)\end{array}\right)}{Nm^2\left(Nm^3\right)\left(Nm^3\right)\left(Nm^3\left(Nm^2\right)+\left(Nm^2-\left(Nm^{-1}\right)m^3\right)\left(Nm^3\right)\right)}$$

$$m = \frac{N^5m^{14}}{N^5m^{13}}$$

The dimensions are correct.

The second check is to find the known formula by neglecting some expressions. First q_2 will be taken zero. As result of this e_2 is also taken zero.

$$e_3 = \frac{q_3 L_{bm}{}^3 L_{cln}{}^2 EI_{cln,2}\left(\left(q_1+q_3\right)L_{bm}L_{cln}{}^2+16EI_{cln,3}\right)\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right) + \left(q_3 L_{bm}L_{cln}{}^2\begin{pmatrix}3L_{bm}EI_{cln,2}\\+2L_{cln}EI_{bm}\end{pmatrix}\begin{pmatrix}L_{cln}EI_{cln,2}\left(q_1 L_{bm}{}^3 L_{cln}-24\pi e_1 EI_{bm}\right)\\+\left(64EI_{cln,2}\left(e_0+e_1\right)\right)\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\end{pmatrix}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}{8EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\begin{pmatrix}3\pi\left(q_1+q_2+q_3\right)L_{bm}L_{cln}{}^3EI_{bm}\\+8\left(2\pi^2EI_{cln,3}-\left(q_1+q_3\right)L_{bm}L_{cln}{}^2\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)\end{pmatrix}}$$

Some expressions are in the numerator as well in the denominator. These expressions can be neglected.

$$e_3 = \frac{q_3 L_{bm}{}^3 L_{cln}{}^2\left(\left(q_1+q_3\right)L_{bm}L_{cln}{}^2+16EI_{cln,3}\right)\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right) + \left(q_3 L_{bm}L_{cln}{}^2\begin{pmatrix}L_{cln}\left(q_1 L_{bm}{}^3 L_{cln}-24\pi e_1 EI_{bm}\right)\\+\left(64\left(e_0+e_1\right)\right)\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\end{pmatrix}\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)}{8\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\begin{pmatrix}3\pi\left(q_1+q_2+q_3\right)L_{bm}L_{cln}{}^3EI_{bm}\\+8\left(2\pi^2EI_{cln,3}-\left(q_1+q_3\right)L_{bm}L_{cln}{}^2\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)\end{pmatrix}}$$

Combine some expressions.

$$e_3 = \frac{q_3 L_{bm}L_{cln}{}^2\begin{pmatrix}L_{bm}{}^2\left(\left(q_1+q_3\right)L_{bm}L_{cln}{}^2+16EI_{cln,3}\right)\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\\+\begin{pmatrix}L_{cln}\left(q_1 L_{bm}{}^3 L_{cln}-24\pi e_1 EI_{bm}\right)\\+\left(64\left(e_0+e_1\right)\right)\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\end{pmatrix}\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)\end{pmatrix}}{8\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\begin{pmatrix}3\pi\left(q_1+q_2+q_3\right)L_{bm}L_{cln}{}^3EI_{bm}\\+8\left(2\pi^2EI_{cln,3}-\left(q_1+q_3\right)L_{bm}L_{cln}{}^2\right)\left(3L_{bm}EI_{cln,3}+2L_{cln}EI_{bm}\right)\end{pmatrix}}$$

In Appendix O.1 a formula is found. A shift in the centre point of gravity is taken into account at this formula. If this shift is neglected the following formula has been analysed.

$$e_2 = \frac{q_2 L_{bm}L_{cln}{}^2\begin{pmatrix}L_{bm}{}^2\left(\left(q_1+q_2\right)L_{bm}L_{cln}{}^2+16EI_{cln,2}\right)\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\\+\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\begin{pmatrix}L_{cln}\left(q_1 L_{bm}{}^3 L_{cln}-24\pi e_1 EI_{bm}\right)\\+64\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(e_0+e_1\right)\end{pmatrix}\end{pmatrix}}{8\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\begin{pmatrix}3\pi\left(q_1+q_2\right)L_{bm}L_{cln}{}^3EI_{bm}\\+8\left(2\pi^2EI_{cln,2}-\left(q_1+q_2\right)L_{bm}L_{cln}{}^2\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\end{pmatrix}}$$

There are just a few differences between these formulas. This is the stiffness and the load. e_3 becomes e_2 and $EI_{cln,3}$ becomes $EI_{cln,2}$. In other words the formula has no big mistakes.

O.3 Conclusion

The additional deflection has been analysed in Appendix O.1 and in Appendix O.2. Appendix O.1 was about the second load case. The second load case starts if the right flange starts to yield. The additional deflection in this load case is called e_2. Appendix O.2 was about the third load case. The third load case starts if both the right and the left flange is partial yielded. The additional deflection in the third load case is called e_3. In many situations the third load case does not occur. The follow formulas have been derived.

$$e_2 = \frac{q_2 L_{bm} L_{cln}^2 \begin{pmatrix} \left((q_1+q_2) L_{bm} L_{cln}^2 + 16 EI_{cln,2}\right)\left(3L_{bm} EI_{cln,1} + 2L_{cln} EI_{bm}\right)\left(L_{bm}^2 EI_{cln,2} + 6L_{cln} z_2 EI_{bm}\right) \\ +\left(3L_{bm} EI_{cln,2} + 2L_{cln} EI_{bm}\right)\begin{pmatrix} 4(q_1+q_2) L_{bm} L_{cln}^2 z_2 \left(3L_{bm} EI_{cln,1} + 2L_{cln} EI_{bm}\right) \\ +L_{cln} EI_{cln,2}\left(q_1 L_{bm}^3 L_{cln} - 24\pi e_1 EI_{bm}\right) \\ +64 EI_{cln,2}\left(3L_{bm} EI_{cln,1} + 2L_{cln} EI_{bm}\right)(e_0 + e_1) \end{pmatrix} \end{pmatrix}}{8EI_{cln,2}\left(3L_{bm} EI_{cln,1} + 2L_{cln} EI_{bm}\right)\begin{pmatrix} 3\pi(q_1+q_2) L_{bm} L_{cln}^3 EI_{bm} \\ +8\left(2\pi^2 EI_{cln,2} - (q_1+q_2) L_{bm} L_{cln}^2\right)\left(3L_{bm} EI_{cln,2} + 2L_{cln} EI_{bm}\right) \end{pmatrix}}$$

$$e_3 = \frac{\begin{matrix} q_3 L_{bm}^3 L_{cln}^2 EI_{cln,2}\left((q_1+q_2+q_3) L_{bm} L_{cln}^2 + 16EI_{cln,3}\right)\left(3L_{bm} EI_{cln,1} + 2L_{cln} EI_{bm}\right)\left(3L_{bm} EI_{cln,2} + 2L_{cln} EI_{bm}\right) \\ + \begin{pmatrix} q_3 L_{bm} L_{cln}^3 \begin{pmatrix} q_2 L_{bm} L_{cln}\left(L_{bm}^2 EI_{cln,2} + 6L_{cln} z_2 EI_{bm}\right) \\ -24\pi e_2 EI_{bm} EI_{cln,2} \end{pmatrix}\left(3L_{bm} EI_{cln,1} + 2L_{cln} EI_{bm}\right) \\ +q_3 L_{bm} L_{cln}^2\left(3L_{bm} EI_{cln,2} + 2L_{cln} EI_{bm}\right) \\ \begin{pmatrix} L_{cln} EI_{cln,2}\left(q_1 L_{bm}^3 L_{cln} - 24\pi e_1 EI_{bm}\right) \\ +\left(64EI_{cln,2}(e_0 + e_1 + e_2) + 4q_2 L_{bm} L_{cln}^2 z_2\right)\left(3L_{bm} EI_{cln,1} + 2L_{cln} EI_{bm}\right) \end{pmatrix} \end{pmatrix}\left(3L_{bm} EI_{cln,3} + 2L_{cln} EI_{bm}\right) \end{matrix}}{\begin{matrix} 8EI_{cln,2}\left(3L_{bm} EI_{cln,1} + 2L_{cln} EI_{bm}\right)\left(3L_{bm} EI_{cln,2} + 2L_{cln} EI_{bm}\right) \\ \begin{pmatrix} 3\pi(q_1+q_2+q_3) L_{bm} L_{cln}^3 EI_{bm} \\ +8\left(2\pi^2 EI_{cln,3} - (q_1+q_2+q_3) L_{bm} L_{cln}^2\right)\left(3L_{bm} EI_{cln,3} + 2L_{cln} EI_{bm}\right) \end{pmatrix} \end{matrix}}$$

$$y_{total,2} = \frac{q_1 L_{bm}^3 L_{cln}^2}{64\left(3L_{bm} EI_{cln,1} + 2L_{cln} EI_{bm}\right)} + \frac{q_2 L_{bm} L_{cln}^2\left(L_{bm}^2 EI_{cln,2} + 6L_{cln} z_2 EI_{bm}\right)}{64EI_{cln,2}\left(3L_{bm} EI_{cln,2} + 2L_{cln} EI_{bm}\right)} + e_0 - \frac{\varphi_{extra,1,x=0} L_{cln}^2 EI_{bm}}{8L_{bm} EI_{cln}}$$
$$-\frac{\varphi_{extra,2,x=0} L_{cln}^2 EI_{bm}}{8L_{bm} EI_{cln,2}} + e_1 + e_2 + \frac{q_2 L_{bm} L_{cln}^2 z_2}{16EI_{cln,2}}$$

$$y_{total,3} = \frac{q_1 L_{bm}^3 L_{cln}^2}{64\left(3L_{bm} EI_{cln,1} + 2L_{cln} EI_{bm}\right)} + \frac{q_2 L_{bm} L_{cln}^2\left(L_{bm}^2 EI_{cln,2} + 6L_{cln} z_2 EI_{bm}\right)}{64EI_{cln,2}\left(3L_{bm} EI_{cln,2} + 2L_{cln} EI_{bm}\right)} + \frac{q_3 L_{bm}^3 L_{cln}^2}{64\left(3L_{bm} EI_{cln,3} + 2L_{cln} EI_{bm}\right)} + e_0$$
$$-\frac{\varphi_{extra,1,x=0} L_{cln}^2 EI_{bm}}{8L_{bm} EI_{cln}} - \frac{\varphi_{extra,2,x=0} L_{cln}^2 EI_{bm}}{8L_{bm} EI_{cln,2}} - \frac{\varphi_{extra,3,x=0} L_{cln}^2 EI_{bm}}{8L_{bm} EI_{cln,3}} + e_1 + e_2 + e_3 + \frac{q_2 L_{bm} L_{cln}^2 z_2}{16EI_{cln,2}}$$

These are formulas to calculate the additional deflection or the total deflection. These deflections results in additional rotations and in additional bending moments. The follow formulas must be used to calculate the additional rotations.

$$\varphi_{extra,2,x=0} = \frac{3\pi L_{bm} e_2 EI_{cln,2}}{L_{cln}\left(3L_{bm}EI_{cln,2} + 2L_{cln}EI_{bm}\right)}$$

$$\varphi_{extra,3,x=0} = \frac{3\pi L_{bm} e_3 EI_{cln,3}}{L_{cln}\left(3L_{bm}EI_{cln,3} + 2L_{cln}EI_{bm}\right)}$$

The horizontal reaction forces:

$$F_2 = \frac{q_2 L_{bm}\left(L_{bm}{}^2 EI_{cln,2} + 6L_{cln} z_2 EI_{bm}\right)}{4L_{cln}\left(3L_{bm}EI_{cln,2} + 2L_{cln}EI_{bm}\right)}$$

$$\Delta F_2 = \frac{3\varphi_{extra,2,x=0} EI_{cln,2}}{L_{cln}{}^2}$$

$$F_3 = \frac{q_3 L_{bm}{}^3 EI_{cln,3}}{4L_{cln}\left(3L_{bm}EI_{cln,3} + 2L_{cln}EI_{bm}\right)}$$

$$\Delta F_3 = \frac{3\varphi_{extra,3,x=0} EI_{cln,3}}{L_{cln}{}^2}$$

The vertical reaction force:

$$N_2 = 0.5 q_2 L_{bm}$$

$$N_3 = 0.5 q_3 L_{bm}$$

The additional moments can be calculated by the follow formulas.

$$M_{top,2} = \left(F_2 - \Delta F_2\right) L_{cln}$$

$$M_{middle,2} = 0.5\left(F_2 - \Delta F_2\right) L_{cln} + 0.5 q_1 L_{bm}\left(e_{total,2} - e_{total,1}\right) + 0.5 q_2 L_{bm} e_{total,2}$$

$$M_{top,3} = \left(F_3 - \Delta F_3\right) L_{cln}$$

$$M_{middle,3} = 0.5\left(F_3 - \Delta F_3\right) L_{cln} + 0.5\left(q_1 + q_2\right) L_{bm}\left(e_{total,3} - e_{total,2}\right) + 0.5 q_3 L_{bm} e_{total,3}$$

The formulas for the bending moments and the formulas for the normal forces can be used to calculate the stresses. The critical loads (for the different load cases) can be calculated by the following formula.

$$-f_y = -\frac{N}{A} \pm \frac{M}{Z}$$

The formulas become clearer at the calculations in Appendix P.

Appendix P Calculations example of a braced portal frame

In Appendix N and in Appendix O some formulas have been derived. These formulas are based on a non-linear analysis. The linear analysis of Appendix M has been made as initial situation for the non-linear analysis. The linear analysis is also made to understand the non-linear analysis better. This Appendix is about the how to use of the formulas and to calculate the ultimate load. It is assumed that the columns are critical and that the portal frame fails if the columns fail. The beam is strong enough to resist all failure mechanisms. This assumption must be checked afterwards. The column can fail on many locations. Two of these locations are most logical and will be calculated. These locations are: at the end of the column and in the middle of the column. The failure location depends on the moment distribution (Fig. P.1).

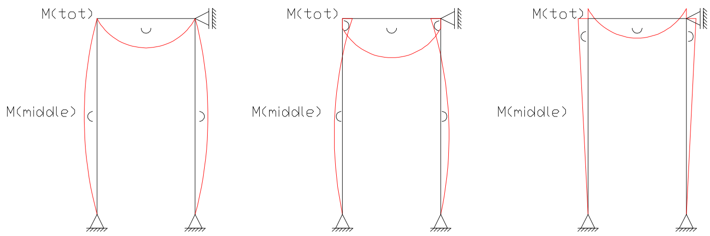


Figure P.1:
Moment distributions

The moment distribution depends on the relative slenderness of the column and the relative slenderness of the beam. If the calculation starts it is not clear where the column fails. Both possibilities must be calculated. The lowest failure load is the ultimate load. The formulas are too complex to make hand-calculations. A computer program is used to make the iterations. The calculation procedure will be found in Appendix P.1. The calculation file for this computer program can be found in Appendix P.2. The formulas which are used for the calculations have been analysed in Appendix N and in Appendix O. The calculation according to the Dutch code will be discussed in Appendix P.3. In Appendix P.4 the same problem is solved by another computer program based on the finite element method (in shortly FEM).

P.1 Calculation example

This Appendix is about the manual calculation of a braced portal frame. The first load must be calculated by the formulas of Appendix N. The formulas of Appendix O will be used if the right flange starts to yield.

The second load case needs some extra attention. The second load case starts if the right flange partial yields and ends if the left flange partial yields or (in this calculation example) if the right flange fully yields. In the second load case, the stress in the web can reaches the first critical stress. If the web partial yields, the influence on the effective stiffness is negligible but the influence on the effective cross-section must take into account. The second load case has been split in two parts. One part with and one part without a partial yielded web. This becomes clear at the calculation.

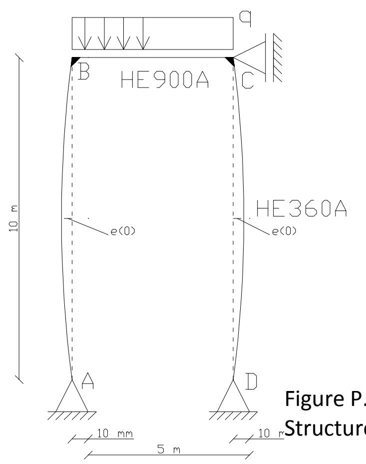


Figure P.2:
Structure

The calculation example in this Appendix is about a beam section HE 900A and a column section HE 360A (Fig. P.2). According to the calculation the middle of the column is the critical location. The moments in the end of the column will be checked. The structure fails at the second load case. The whole right flange yields before the left flange starts to yield. The formulas of the third load case will not be used.

The section properties are:

L_{cln} = 10000 mm
L_{bm} = 5000 mm
e_0 = 10 mm
$EI_{cln,1}$ = 6.959*10^13 Nmm^2
$EI_{cln,2}$ = 5.263*10^13 Nmm^2
$EI_{cln,3}$ = 3.573*10^13 Nmm^2
$Z_{cln,1}$ = 1.891*10^6 mm^3
$Z_{cln,2}$ = 1.432*10^6 mm^3
$Z_{cln,3}$ = 9.721*10^5 mm^3
EI_{bm} = 8.864*10^14 Nmm^2
$f_{c,1}$ = 177.5 N/mm^2 (first critical stress)
$f_{c,2}$ = 532.5 N/mm^2 (second critical stress)
$A_{cln,1}$ = 14280 mm^2
$A_{cln,2a}$ = 11655 mm^2
$A_{cln,2b}$ = 9905 mm^2 ($A_{cln,2a} - 0.5ht_w$)
$A_{cln,3}$ = 7280 mm^2
z_2 = 39.6 mm

The first critical load is:

y_1 = 755 N/mm (kN/m)

The additional deflection can be calculated.

$$e_1 = \frac{q_1 L_{bm} L_{cln}{}^2 \left(q_1 L_{bm}{}^3 L_{cln}{}^2 + 64 e_0 \left(3L_{bm} EI_{cln,1} + 2L_{cln} EI_{bm}\right)\right)}{8\left[3\pi L_{cln} EI_{bm}\left(-16 EI_{cln,1} + q_1 L_{bm} L_{cln}{}^2\right) + \left(16\pi^2 EI_{cln,1} - 8 q_1 L_{bm} L_{cln}{}^2\right)\left(3L_{bm} EI_{cln,1} + 2L_{cln} EI_{bm}\right)\right]}$$

$$e_1 = \frac{q_1 L_{bm} L_{cln}{}^2 \left(q_1 L_{bm}{}^3 L_{cln}{}^2 + 64 e_0 \left(3L_{bm} EI_{cln,1} + 2L_{cln} EI_{bm}\right) + 16 L_{bm}{}^2 EI_{cln,1}\right)}{8\left[3\pi q_1 L_{bm} L_{cln}{}^3 EI_{bm} + \left(16\pi^2 EI_{cln,1} - 8 q_1 L_{bm} L_{cln}{}^2\right)\left(3L_{bm} EI_{cln,1} + 2L_{cln} EI_{bm}\right)\right]}$$

$$e_1 = \frac{755 * 5000 * 10000^2 \begin{pmatrix} 755 * 5000^3 * 10000^2 + 64 * 10\left(3 * 5000 * 6.959 * 10^{13} + 2 * 10000 * 8.864 * 10^{14}\right) \\ +16 * 5000^2 * 6.959 * 10^{13} \end{pmatrix}}{8\begin{bmatrix} 3\pi * 755 * 5000 * 10000^3 * 8.864 * 10^{14} \\ +\left(16\pi^2 6.959 * 10^{13} - 8 * 755 * 5000 * 10000^2\right)\left(3 * 5000 * 6.959 * 10^{13} + 2 * 10000 * 8.864 * 10^{14}\right) \end{bmatrix}}$$

$$e_1 = \frac{1.859 * 10^{37}}{1.553 * 10^{36}}$$

$e_1 = 12.8mm$

The additional deflection can be used to calculate the additional rotation.

$$\varphi_{extra,1,x=0} = \frac{3\pi L_{bm} e_1 EI_{cln,1}}{L_{cln}\left(3L_{bm}EI_{cln,1} + 2L_{cln}EI_{bm}\right)}$$

$$\varphi_{extra,1,x=0} = \frac{3\pi * 5000 * 12.8 * 6.959 * 10^{13}}{10000\left(3 * 5000 * 6.959 * 10^{13} + 2 * 10000 * 8.864 * 10^{14}\right)}$$

$$\varphi_{extra,1,x=0} = \frac{4.208 * 10^{19}}{1.877 * 10^{23}}$$

$$\varphi_{extra,1,x=0} = 2.242 * 10^{-4} rad$$

The total deflection follows from the following formula:

$$y_{total,1} = \frac{q_1 L_{bm}{}^3 L_{cln}{}^2}{64\left(3L_{bm}EI_{cln} + 2L_{cln}EI_{bm}\right)} + e_0 - \frac{\varphi_{extra1,x=0} L_{cln}{}^2 EI_{bm}}{8L_{bm}EI_{cln}} + e_1$$

$$y_{total,1} = \frac{755 * 5000^3 * 10000^2}{64\left(3 * 5000 * 6.959 * 10^{13} + 2 * 10000 * 8.864 * 10^{14}\right)} + 10 - \frac{2.242 * 10^{-4} * 10000^2 * 8.864 * 10^{14}}{8 * 5000 * 6.959 * 10^{13}} + 12.8$$

$$y_{total,1} = 23.6mm$$

The reaction forces are:

$$F_1 = \frac{q_1 L_{bm}{}^3 EI_{cln}}{4L_{cln}\left(3L_{bm}EI_{cln} + 2L_{cln}EI_{bm}\right)}$$

$$F_1 = \frac{755 * 5000^3 * 6.959 * 10^{13}}{4 * 10000\left(3 * 5000 * 6.959 * 10^{13} + 2 * 10000 * 8.865 * 10^{14}\right)}$$

$$F_1 = 8267N$$

$$\Delta F_1 = \frac{3\varphi_{extra,1,x=0} EI_{cln,1}}{L_{cln}{}^2}$$

$$\Delta F_1 = \frac{3 * 2.242 * 10^{-4} * 6.959 * 10^{13}}{10000^2}$$

$$\Delta F_1 = 467N$$

$$N_1 = 0.5 q_1 L_{bm}$$

$$N_1 = 0.5 * 755 * 5000$$

$$N_1 = 1891.5 * 10^3 N$$

Bending moment in the middle of the column.

$$M_{middle,1} = 0.5\left(F_1 - \Delta F_1\right) L_{cln} + 0.5 q_1 L_{bm} e_{total}$$

$$M_{middle,1} = 0.5(8767 - 467)10000 + 0.5 * 755 * 5000 * 23.6$$

$$M_{middle,1} = 85.8 * 10^6 Nmm$$

Calculate the bending moment at the end of the column (as check).

$$M_{end,1} = \left(F_1 - \Delta F_1\right) L_{cln}$$

$$M_{end,1} = \left(8767 - 467\right) * 10000$$

$$M_{end,1} = 83.0 * 10^6 \, Nmm$$

$$M_{end,1} < M_{middle,1}$$

Stresses:

$$\sigma_{right,1} = -\frac{N_1}{A_{cln,1}} - \frac{M_1}{Z_{cln,1}}$$

$$\sigma_{right,1} = -\frac{1891.5 * 10^3}{14280} - \frac{85.8 * 10^6}{1.891 * 10^6}$$

$$\sigma_{right,1} = -177.6 \, N / mm^2$$ first critical stress

$$\sigma_{left,1} = -\frac{N_1}{A_{cln,1}} + \frac{M_1}{Z_{cln,1}}$$

$$\sigma_{left,1} = -\frac{1891.5 * 10^3}{14280} + \frac{85.8 * 10^6}{1.891 * 10^6}$$

$$\sigma_{left,1} = -86.8 \, N / mm^2$$

$$\sigma_{centre,1} = -\frac{N_1}{A_{cln,1}}$$

$$\sigma_{centre,1} = -\frac{1891.5 * 10^3}{14280}$$

$$\sigma_{centre,1} = -132.2 \, N / mm^2$$

The second critical load is:

y_2 = 563 N/mm (kN/m)

This has been split in:

y_{2a} = 212 N/mm (kN/m)

y_{2b} = 351 N/mm (kN/m)

The web yields at y_{2a}.

The additional deflection can be calculated.

$$e_2 = \frac{q_2 L_{bm} L_{cln}^2 \begin{pmatrix} \left(\left(q_1+q_2\right)L_{bm}L_{cln}^2 + 16EI_{cln,2}\right)\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(L_{bm}^2 EI_{cln,2}+6L_{cln}z_2EI_{bm}\right) \\ +\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\begin{pmatrix}4\left(q_1+q_2\right)L_{bm}L_{cln}^2 z_2\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right) \\ +L_{cln}EI_{cln,2}\left(q_1L_{bm}^3L_{cln}-24\pi e_1 EI_{bm}\right) \\ +64EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\left(e_0+e_1\right)\end{pmatrix}\end{pmatrix}}{8EI_{cln,2}\left(3L_{bm}EI_{cln,1}+2L_{cln}EI_{bm}\right)\begin{pmatrix}3\pi\left(q_1+q_2\right)L_{bm}L_{cln}^3EI_{bm} \\ +8\left(2\pi^2EI_{cln,2}-\left(q_1+q_2\right)L_{bm}L_{cln}^2\right)\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)\end{pmatrix}}$$

$$e_2 = \frac{\begin{pmatrix} \left(1318*5000*10000^2+16*5.263*10^{13}\right)\begin{pmatrix}3*5000*6.959*10^{13} \\ +2*10000*8.864*10^{14}\end{pmatrix}\begin{pmatrix}5000^2*5.263*10^{13} \\ +6*10000*39.6*8.864*10^{14}\end{pmatrix} \\ +\begin{pmatrix}3*5000*5.263*10^{13} \\ +2*10000*8.864*10^{14}\end{pmatrix}\begin{pmatrix}4*591*1348*5000^2*10000^4*39.6*\begin{pmatrix}3*5000*6.959*10^{13} \\ +2*10000*8.864*10^{14}\end{pmatrix} \\ +591*5000*10000^3*5.263*10^{13}\begin{pmatrix}757*5000^3*10000 \\ -24\pi*11.6*8.864*10^{14}\end{pmatrix} \\ +64*591*5000*10000^2*5.263*10^{13}\begin{pmatrix}3*5000*6.959*10^{13} \\ +2*10000*8.864*10^{14}\end{pmatrix}*21.6\end{pmatrix}\end{pmatrix}}{8*5.263*10^{13}\begin{pmatrix}3*5000*6.959*10^{13} \\ +2*10000*8.864*10^{14}\end{pmatrix}\begin{pmatrix}3\pi*1318*5000*10000^3*8.864*10^{14} \\ +8\begin{pmatrix}2\pi^2 5.263*10^{13} \\ -1348*5000*10000^2\end{pmatrix}\begin{pmatrix}3*5000*5.263*10^{13} \\ +2*10000*8.864*10^{14}\end{pmatrix}\end{pmatrix}}$$

$$e_2 = \frac{4.509*10^{70}}{8.789*10^{68}}$$

$$e_2 = 51.3mm$$

The additional deflection can be used to calculate the additional rotation.

$$\varphi_{extra,2,x=0} = \frac{3\pi L_{bm} e_2 EI_{cln,2}}{L_{cln}\left(3L_{bm}EI_{cln,2}+2L_{cln}EI_{bm}\right)}$$

$$\varphi_{extra,2,x=0} = \frac{3\pi*5000*51.3*5.263*10^{13}}{10000\left(3*5000*5.263*10^{13}+2*10000*8.864*10^{14}\right)}$$

$$\varphi_{extra,2,x=0} = \frac{1.272*10^{20}}{1.852*10^{23}}$$

$$\varphi_{extra,2,x=0} = 6.868*10^{-4} rad$$

The total deflection follows from the following formula:

$$y_{total,2} = \frac{q_1 L_{bm}{}^3 L_{cln}{}^2}{64\left(3L_{bm}EI_{cln,1} + 2L_{cln}EI_{bm}\right)} + \frac{q_2 L_{bm} L_{cln}{}^2\left(L_{bm}{}^2 EI_{cln,2} + 6L_{cln}z_2 EI_{bm}\right)}{64EI_{cln,2}\left(3L_{bm}EI_{cln,2} + 2L_{cln}EI_{bm}\right)} + e_0 + e_1 + e_2$$

$$-\frac{\varphi_{extra,1,x=0} L_{cln}{}^2 EI_{bm}}{8L_{bm}EI_{cln,1}} - \frac{\varphi_{extra,2,x=0} L_{cln}{}^2 EI_{bm}}{8L_{bm}EI_{cln,2}} + \frac{q_2 L_{bm} L_{cln}{}^2 z_2}{16EI_{cln,2}}$$

$$y_{total,2} = \frac{755*5000^3*10000^2}{64\left(3*5000*6.595*10^{13} + 2*10000*8.864*10^{14}\right)}$$

$$+\frac{563*5000*10000^2\left(5000^2*5.263*10^{13} + 6*10000*39.6*8.864*10^{14}\right)}{64*5.263*10^{13}\left(3*5000*5.263*10^{13} + 2*10000*8.864*10^{14}\right)} + 10 + 12.8 + 51.3$$

$$-\frac{2.017*10^{-4}*10000^2*8.864*10^{14}}{8*5000*6.959*10^{13}} - \frac{6.868*10^{-4}*10000^2*8.864*10^{14}}{8*5000*5.263*10^{13}} + \frac{563*5000*10000^2*39.6}{16*5.263*10^{13}}$$

$$y_{total,2} = 74.6mm$$

The reaction forces are:

$$F_2 = \frac{q_2 L_{bm}\left(L_{bm}{}^2 EI_{cln,2} + 6L_{cln}z_2 EI_{bm}\right)}{4L_{cln}\left(3L_{bm}EI_{cln,2} + 2L_{cln}EI_{bm}\right)}$$

$$F_2 = \frac{563*5000\left(5000^2*5.263*10^{13} + 6*10000*39.6*8.864*10^{14}\right)}{4*10000\left(3*5000*5.263*10^{13} + 2*10000*8.864*10^{14}\right)}$$

$$F_2 = 12999N$$

$$\Delta F_2 = \frac{3\varphi_{extra,2,x=0} EI_{cln,2}}{L_{cln}{}^2}$$

$$\Delta F_2 = \frac{3*8.686*10^{-4}*5.263*10^{13}}{10000^2}$$

$$\Delta F_2 = 1084N$$

$$N_{2a} = 0.5q_{2a}L_{bm}$$

$$N_{2a} = 0.5*212*5000$$

$$N_{2a} = 526*10^3 N$$

$$N_{2b} = 0.5q_{2b}L_{bm}$$

$$N_{2b} = 0.5*351*5000$$

$$N_{2b} = 877.5*10^3 N$$

Bending moment in the middle of the column.

$$M_{middle,2} = 0.5\left(F_2 - \Delta F_2\right)L_{cln} + 0.5q_1 L_{bm}\left(e_{total,2} - e_{total,1}\right) + 0.5q_2 L_{bm} e_{total,2}$$

$$M_{middle,2} = 0.5(12999 - 1084)10000 + 0.5*755*5000(74.6 - 23.6) + 0.5*563*5000*74.6$$

$$M_{middle,2} = 316.7*10^6 Nmm$$

Calculate the bending moment at the end of the column (as check).

$$M_{end,2} = (F_2 - \Delta F_2) L_{cln}$$

$$M_{end,2} = (12999 - 1084) * 10000$$

$$M_{end,2} = 119.2 * 10^6 \, Nmm$$

$$M_{end,2} < M_{middle,2}$$

Stresses:

$$\sigma_{right,2} = \sigma_{right,1} - \frac{N_{2a}}{A_{cln,2a}} - \frac{N_{2b}}{A_{cln,2b}} - \frac{M_2}{Z_{cln,2}}$$

$$\sigma_{right,2} = -177.6 - \frac{526 * 10^3}{11655} - \frac{877.5 * 10^3}{9905} - \frac{316.7 * 10^6}{1.432 * 10^6}$$

$$\sigma_{right,2} = -532.8 N/mm^2$$ second critical stress

$$\sigma_{left,2} = \sigma_{left,1} - \frac{N_{2a}}{A_{cln,2a}} - \frac{N_{2b}}{A_{cln,2b}} + \frac{M_2}{Z_{cln,2}}$$

$$\sigma_{left,2} = -86.8 - \frac{526 * 10^3}{11655} - \frac{877.5 * 10^3}{9905} + \frac{316.7 * 10^6}{1.432 * 10^6}$$

$$\sigma_{left,2} = 0.3 N/mm^2$$

$$\sigma_{centre,2} = \sigma_{centre,1} - \frac{N_{2a}}{A_{cln,2a}}$$

$$\sigma_{centre,2} = -132.5 - \frac{526 * 10^3}{11655}$$

$$\sigma_{centre,2} = -177.5 N/mm^2$$ first critical stress

$$\sigma_{centre,2} = \sigma_{centre,1} - \frac{N_{2a}}{A_{cln,2a}} - \frac{N_{2b}}{A_{cln,2b}}$$

$$\sigma_{centre,2} = -132.5 - \frac{526 * 10^3}{11655} - \frac{877.5 * 10^3}{9905}$$

$$\sigma_{centre,2} = -266.2 N/mm^2$$

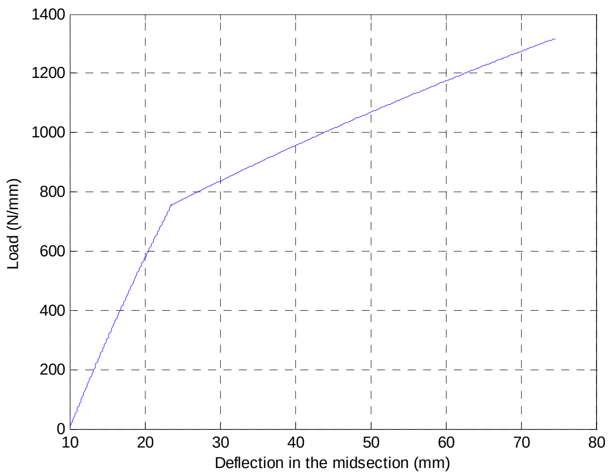


Figure P.3:
Load -deflection
Graphic

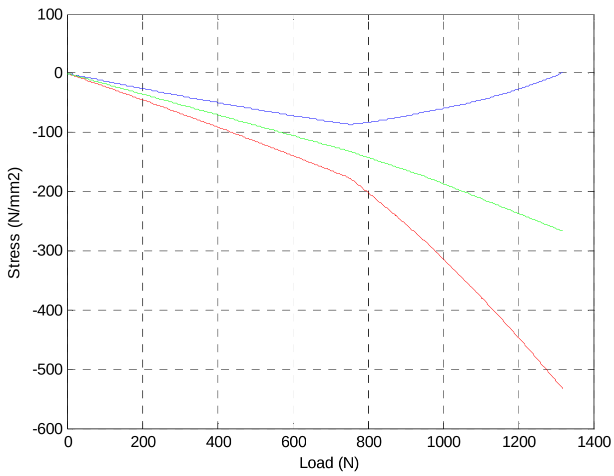


Figure P.4:
Stress load
Graphic

The deflection of the column can be found in Figure P.3.

The calculation has some interesting aspects. These aspects will be discussed.

- The bending moments in the middle of the column and at the end of the column are very different. In the first load case these moments are close together. In the second load case the moment at the end of the beam decrease while the moment in the middle in the middle of the column increases.
- The stresses in the failure situation. The right flange fully yields. The web partial yield and the left flange is nearly free of stresses. Figure P.4 is a load-stress graphic.
- The load case. The stress in the right flange has increased to the second critical stress. The right flange fully yields. If one whole flange yields, the column fails. In other words the fails at the second load case. The third load case is not taken into account.

The column fails at a load of 1318 kN/m.

It is assumed that the beam will not fail. An extra calculation is needed to check this assumption.

$$f_y \leq \frac{\frac{1}{8}(q_1+q_2)L_{bm}^{\;2}}{Z_{bm}} - \frac{M_{top,2}}{Z_{bm}}$$

$$f_y \leq \frac{\frac{1}{8}*1318*5000^2}{9.485*10^6} - \frac{119.2*10^6}{9.485*10^6}$$

$f_y \leq 422.3 N/mm^2$ Steel grade S460 is needed.

P.2 Computer calculation file

The formulas are very complex. It is nearly impossible to make calculations by hand. It is necessary to make iterations. The computer is a useful instrument to make iterations. The computer program MatLab is used to calculate the ultimate load. The calculation file (called m-file) is placed in this Appendix.

```
clear; clf; clc; close;
E=210000;
fy=355;
qq=1;%belastingsstap
delta=0.001;
Lbm=5000;
Lcln=10000;
e0=delta*Lcln;

%input profiles
HEA=[2.124E+03  2.534E+03   3.142E+03   3.877E+03   4.525E+03   5.383E+03
6.434E+03   7.684E+03   8.682E+03   9.726E+03   1.125E+04   1.244E+04
1.335E+04   1.428E+04   1.590E+04   1.780E+04   1.975E+04   2.118E+04
2.265E+04   2.416E+04   2.605E+04   2.858E+04   3.205E+04   3.468E+04;%A
(cross-section)

3.492E+06   6.062E+06   1.033E+07   1.673E+07   2.510E+07   3.692E+07
5.410E+07   7.763E+07   1.046E+08   1.367E+08   1.826E+08   2.293E+08
```

```
2.769E+08   3.309E+08   4.507E+08   6.372E+08   8.698E+08   1.119E+09
1.412E+09   1.752E+09   2.153E+09   3.034E+09   4.221E+09   5.538E+09;%I
(moment of inertia)

8.301E+04   1.195E+05   1.735E+05   2.451E+05   3.249E+05   4.295E+05
5.685E+05   7.446E+05   9.198E+05   1.112E+06   1.383E+06   1.628E+06
1.850E+06   2.088E+06   2.562E+06   3.216E+06   3.949E+06   4.622E+06
5.350E+06   6.136E+06   7.032E+06   8.699E+06   1.081E+07   1.282E+07;%Z
plastic (Section modulus)

7.276E+04   1.063E+05   1.554E+05   2.201E+05   2.936E+05   3.886E+05
5.152E+05   6.751E+05   8.364E+05   1.013E+06   1.260E+06   1.479E+06
1.678E+06   1.891E+06   2.311E+06   2.896E+06   3.550E+06   4.146E+06
4.787E+06   5.474E+06   6.241E+06   7.682E+06   9.485E+06   1.119E+07;%Z
elastic (Section modulus)

4.055E+01   4.891E+01   5.734E+01   6.569E+01   7.448E+01   8.282E+01
9.170E+01   1.005E+02   1.097E+02   1.186E+02   1.274E+02   1.358E+02
1.440E+02   1.522E+02   1.684E+02   1.892E+02   2.099E+02   2.299E+02
2.497E+02   2.693E+02   2.875E+02   3.258E+02   3.629E+02   3.996E+02;%i
(Gyration radius)

96 114 133 152 171 190 210 230 250 270 290 310 330 350 390 440 490 540 590
640 690 790 890 990;%h height

100 120 140 160 180 200 220 240 260 280 300 300 300 300 300 300 300 300 300
300 300 300 300 300;%b width

8 8 8.5 9 9.5 10 11 12 12.5 13 14 15.5 16.5 17.5 19 21 23 24 25 26 27 28 30
31;%tf thickness flange

5 5 5.5 6 6 6.5 7 7.5 7.5 8 8.5 9 9.5 10 11 11.5 12 12.5 13 13.5 14.5 15 16
16.5];%tw thickness web

cln=14;
    Acln(1,1)=HEA(1,cln);
    Icln(1,1)=HEA(2,cln);
    Zcln(1,1)=HEA(4,cln);
    hcln=HEA(6,cln);
    bcln=HEA(7,cln);
    tfcln=HEA(8,cln);
    twcln=HEA(9,cln);

bm=23;
    Abm(1,1)=HEA(1,bm);
    Ibm(1,1)=HEA(2,bm);
    Zbm(1,1)=HEA(4,bm);

    Npcln=Acln(1,1)*fy;
    Mpcln=Zcln(1,1)*fy;

     if cln<=14 ;
         S=0.5;
     else
         S=0.3;
     end% if
    yield1=-(1-S)*fy;
    yield2=-(1+S)*fy;
```

```
Acln(1,2)=Acln(1,1)-2*(0.25*bcln)*tfcln;
Acln(1,3)=Acln(1,2)-2*(0.25*bcln)*tfcln-twcln*hcln*0.5;
Acln(1,4)=Acln(1,2)-twcln*hcln*0.5;
Icln(1,2)=Icln(1,1)-2*(0.25*bcln)*tfcln*(0.5*hcln)^2;
Icln(1,3)=Icln(1,2)-2*(0.25*bcln)*tfcln*(0.5*hcln)^2;
Zcln(1,2)=2*Icln(1,2)/hcln;
Zcln(1,3)=2*Icln(1,3)/hcln;

z(1,1)=0;
z(1,2)=(0.5*bcln*tfcln*0.5*tfcln+(hcln-
2*tfcln)*twcln*0.5*hcln+bcln*tfcln*(hcln-0.5*tfcln))/(0.5*bcln*tfcln+(hcln-
2*tfcln)*twcln+bcln*tfcln)-0.5*hcln;
z(1,3)=0;

hulp(1,1)=3*Lbm*E*Icln(1,1)+2*Lcln*E*Ibm;
hulp(1,2)=3*Lbm*E*Icln(1,2)+2*Lcln*E*Ibm;
hulp(1,3)=3*Lbm*E*Icln(1,3)+2*Lcln*E*Ibm;

%end column
q1a=0;
sigmatopa=0;
sigmabottoma=0;
sigmacentrea=0;
while sigmatopa>yield1 & sigmabottoma>yield1;
    q1a=q1a+1;
    Q1a(q1a)=q1a*qq;
    Qtota(q1a)=Q1a(q1a);
    N1a(q1a)=0.5*Q1a(q1a)*Lbm;
    Ntota(q1a)=N1a(q1a);

    hulp11(q1a)=Q1a(q1a)*Lbm*Lcln^2*(Q1a(q1a)*Lbm^3*Lcln^2 +
64*e0*hulp(1,1) + 16*Lbm^2*E*Icln(1,1));
    hulp12(q1a)=8*(3*pi*Q1a(q1a)*Lbm*Lcln^3*E*Ibm + (16*pi^2*E*Icln(1,1)-
8*Q1a(q1a)*Lbm*Lcln^2)*hulp(1,1));
    e1a(q1a)=hulp11(q1a)/hulp12(q1a);
    phi1a(q1a)=3*pi*Lbm*e1a(q1a)*E*Icln(1,1)/(Lcln*hulp(1,1));
    etota(q1a)=Q1a(q1a)*Lbm^3*Lcln^2/(64*hulp(1,1)) + e0 -
phi1a(q1a)*Lcln^2*E*Ibm/(8*Lbm*E*Icln(1,1)) + e1a(q1a);

    F1a(q1a)=Q1a(q1a)*Lbm^3*E*Icln(1,1)/(4*Lcln*hulp(1,1)) -
3*phi1a(q1a)*E*Icln(1,1)/(Lcln^2);
    Ftota(q1a)=F1a(q1a);
    M1a(q1a)=F1a(q1a)*Lcln;
    Mtota(q1a)=M1a(q1a);

    sigmatopa=-N1a(q1a)/Acln(1,1) - M1a(q1a)/Zcln(1,1);
    sigmatoptota(q1a)=sigmatopa;
    sigmabottoma=-N1a(q1a)/Acln(1,1) + M1a(q1a)/Zcln(1,1);
    sigmabottomtota(q1a)=sigmabottoma;
    sigmacentrea=-N1a(q1a)/Acln(1,1);
    sigmacentretota(q1a)=sigmacentrea;
end% while q1a

q2a=0;
if (3*pi*Lcln*E*Ibm*((Qtota(q1a)+2*qq)*Lbm*Lcln^2) + 8*(2*pi^2*E*Icln(1,2)-
(Qtota(q1a)+2*qq)*Lbm*Lcln^2)*hulp(1,2))>0;

    while (sigmabottoma>yield1 | sigmatopa>yield1) & (sigmabottoma>yield2)
& (sigmatopa>yield2);
    q2a=q2a+1;
```

```
    Q2a(q2a)=q2a*qq;
        if sigmacentrea>yield1
            Q2aa(q2a)=q2a*qq;
        else
            Q2aa(q2a)=Q2aa(q2a-1);
        end%if

    Qtota(q1a+q2a)=Qtota(q1a)+Q2a(q2a);
    N2a(q2a)=0.5*Q2a(q2a)*Lbm;
    N2aa(q2a)=0.5*Q2aa(q2a)*Lbm;
    Ntota(q1a+q2a)=Ntota(q1a)+N2a(q2a);


hulp21(q2a)=(Qtota(q1a+q2a)*Lbm*Lcln^2+16*E*Icln(1,2))*hulp(1,1)*(Lbm^2*E*I
cln(1,2)+6*Lcln*z(1,2)*E*Ibm);
    hulp22(q2a)=hulp(1,2)*(4*Qtota(q1a+q2a)*Lbm*Lcln^2*z(1,2)*hulp(1,1) +
Lcln*E*Icln(1,2)*(Qtota(q1a)*Lbm^3*Lcln-24*pi*e1a(q1a)*E*Ibm) +
64*E*Icln(1,2)*hulp(1,1)*(e0+e1a(q1a)));

hulp23(q2a)=8*E*Icln(1,2)*hulp(1,1)*(3*pi*(Qtota(q1a+q2a)*Lbm*Lcln^3*E*Ibm)
+ 8*(2*pi^2*E*Icln(1,2)-Qtota(q1a+q2a)*Lbm*Lcln^2)*hulp(1,2));
    e2a(q2a)=Q2a(q2a)*Lbm*Lcln^2*(hulp21(q2a)+hulp22(q2a))/hulp23(q2a);
    phi2a(q2a)=3*pi*Lbm*e2a(q2a)*E*Icln(1,2)/(Lcln*hulp(1,2));
    etota(q1a+q2a)=Q1a(q1a)*Lbm^3*Lcln^2/(64*hulp(1,1)) +
Q2a(q2a)*Lbm*Lcln^2*(Lbm^2*E*Icln(1,2)+6*Lcln*z(1,2)*E*Ibm)/(64*E*Icln(1,2)
*hulp(1,2)) + e0 - phi1a(q1a)*Lcln^2*E*Ibm/(8*Lbm*E*Icln(1,1)) -
phi2a(q2a)*Lcln^2*E*Ibm/(8*Lbm*E*Icln(1,2)) + e1a(q1a) + e2a(q2a) +
Q2a(q2a)*Lbm*Lcln^2*z(1,2)/(16*E*Icln(1,2));


F2a(q2a)=Q2a(q2a)*Lbm*(Lbm^2*E*Icln(1,2)+6*Lcln*z(1,2)*E*Ibm)/(4*Lcln*hulp(
1,2)) - 3*phi2a(q2a)*E*Icln(1,2)/(Lcln^2);
    Ftota(q1a+q2a)=Ftota(q1a)+F2a(q2a);
    M2a(q2a)=F2a(q2a)*Lcln + N2a(q2a)*z(1,2);
    Mtota(q1a+q2a)=Mtota(q1a)+M2a(q2a);

    sigmatopa=sigmatoptota(q1a) - N2aa(q2a)/Acln(1,2) - (N2a(q2a)-
N2aa(q2a))/Acln(1,4) - M2a(q2a)/Zcln(1,2);
    sigmatoptota(q1a+q2a)=sigmatopa;
    sigmabottoma=sigmabottomtota(q1a) - N2aa(q2a)/Acln(1,2) - (N2a(q2a)-
N2aa(q2a))/Acln(1,4) + M2a(q2a)/Zcln(1,2);
    sigmabottomtota(q1a+q2a)=sigmabottoma;
    sigmacentrea=sigmacentretota(q1a) - N2aa(q2a)/Acln(1,2) - (N2a(q2a)-
N2aa(q2a))/Acln(1,4) ;
    sigmacentretota(q1a+q2a)=sigmacentrea;
end% while q2a

else
    q2a=q2a;
end% if

q3a=0;
if (3*pi*Lcln*E*Ibm*((Qtota(q1a+q2a)+2*qq)*Lbm*Lcln^2) +
8*(2*pi*E*Icln(1,3)-(Qtota(q1a+q2a)+2*qq)*Lbm*Lcln^2)*hulp(1,3))>0;

while sigmatopa>sigmabottoma>yield2 & sigmatopa>yield2;
    q3a=q3a+1;
    Q3a(q3a)=q3a*qq;
    Qtota(q1a+q2a+q3a)=Qtota(q1a+q2a)+Q3a(q3a);
    N3a(q3a)=0.5*Q3a(q3a)*Lbm;
```

```
    Ntota(q1a+q2a+q3a)=Ntota(q1a+q3a)+N3a(q3a);


hulp31(q3a)=Q3a(q3a)*Lbm^3*Lcln^2*E*Icln(1,2)*(Qtota(q1a+q2a+q3a)*Lbm*Lcln^
2+16*E*Icln(1,3))*hulp(1,1)*hulp(1,2);

hulp32(q3a)=Q3a(q3a)*Lbm*Lcln^3*(Q2a(q2a)*Lbm*Lcln*(Lbm^2*E*Icln(1,2)+6*Lcl
n*z(1,2)*E*Ibm)-24*pi*e2a(q2a)*E*Ibm*E*Icln(1,2))*hulp(1,1);

hulp33(q3a)=(Q3a(q3a)*Lbm*Lcln^2*hulp(1,2))*(Lcln*E*Icln(1,2)*(Q1a(q1a)*Lbm
^3*Lcln-24*pi*e1a(q1a)*E*Ibm) +
(64*E*Icln(1,2)*(e0+e1a(q1a)+e2a(q2a))+4*Q2a(q2a)*Lbm*Lcln^2*z(1,2))*hulp(1
,1));

hulp34(q3a)=8*E*Icln(1,2)*hulp(1,1)*hulp(1,2)*(3*pi*Qtota(q1a+q2a+q3a)*Lbm*
Lcln^3*E*Ibm + 8*(2*pi*E*Icln(1,3)-
Qtota(q1a+q2a+q3a)*Lbm*Lcln^2)*hulp(1,3));
    e3a(q3a)=(hulp31(q3a)+(hulp32(q3a)+hulp33(q3a))*hulp(1,3))/hulp34(q3a);
    phi3a(q3a)=3*pi*Lbm*e3a(q3a)*E*Icln(1,3)/(Lcln*hulp(1,3));
    etota(q1a+q2a+q3a)=Q1a(q1a)*Lbm^3*Lcln^2/(64*hulp(1,1)) +
Q2a(q2a)*Lbm*Lcln^2*(Lbm^2*E*Icln(1,2)+6*Lcln*z(1,2)*E*Ibm)/(64*E*Icln(1,2)
*hulp(1,2)) + Q3a(q3a)*Lbm^3*Lcln^2/(64*hulp(1,3)) + e0 -
phi1a(q1a)*Lcln^2*E*Ibm/(8*Lbm*E*Icln(1,1)) -
phi2a(q2a)*Lcln^2*E*Ibm/(8*Lbm*E*Icln(1,2)) -
phi3a(q3a)*Lcln^2*E*Ibm/(8*Lbm*E*Icln(1,3)) + e1a(q1a) + e2a(q2a) +
e3a(q3a) + Q2a(q2a)*Lbm*Lcln^2*z(1,2)/(16*E*Icln(1,2));

    F3a(q3a)=Q3a(q3a)*Lbm^3*E*Icln(1,3)/(4*Lcln*hulp(1,3)) -
3*phi3a(q3a)*E*Icln(1,3)/(Lcln^2);
    Ftota(q1a+q2a+q3a)=Ftota(q1a+q2a)+F3a(q3a);
    M3a(q3a)=F3a(q3a)*Lcln;
    Mtota(q1a+q2a+q3a)=Mtota(q1a+q2a)+M3a(q3a);

%     sigmatopa=0.01*q3a*yield2;
    sigmatopa=sigmatoptota(q1a+q2a) - N3a(q3a)/Acln(1,3) -
M3a(q3a)/Zcln(1,3);
    sigmatoptota(q1a+q2a+q3a)=sigmatopa;
    sigmabottoma=sigmabottomtota(q1a+q2a) - N1a(q3a)/Acln(1,3) +
M3a(q3a)/Zcln(1,3);
    sigmabottomtota(q1a+q2a+q3a)=sigmabottoma;
    sigmacentrea=sigmacentretota(q1a+q2a) - N3a(q3a)/Acln(1,3);
    sigmacentretota(q1a+q2a+q3a)=sigmacentrea;

end%ehile q3a

else
    q3a=q3a;
end% if
Qmaxa=Qtota(q1a+q2a+q3a);

clear F1a;clear F2a;clear F3a;
clear M1a;clear M2a;clear M3a;
clear N1a;clear N2a;clear N3a; clear N2aa;
clear Q1a;clear Q2a;clear Q3a;clear Q2aa;
clear e1a;clear e2a;clear e3a;
clear phi1a;clear phi2a;clear phi3a;
clear sigmatopa; clear sigmabottoma; clear sigmacentrea;
clear hulp11;clear hulp12;
clear hulp21;clear hulp22;clear hulp23;
clear hulp31;clear hulp32;clear hulp33;
```

```
%middle column
q1b=0;
sigmatopb=0;
sigmabottomb=0;
sigmacentreb=0;
while sigmatopb>yield1 & sigmabottomb>yield1;
    q1b=q1b+1;
    Q1b(q1b)=q1b*qq;
    Qtotb(q1b)=Q1b(q1b);
    N1b(q1b)=0.5*Q1b(q1b)*Lbm;
    Ntotb(q1b)=N1b(q1b);

    hulp11(q1b)=Q1b(q1b)*Lbm*Lcln^2*(Q1b(q1b)*Lbm^3*Lcln^2 +
64*e0*hulp(1,1) + 16*Lbm^2*E*Icln(1,1));
    hulp12(q1b)=8*(3*pi*Q1b(q1b)*Lbm*Lcln^3*E*Ibm + (16*pi^2*E*Icln(1,1)-
8*Q1b(q1b)*Lbm*Lcln^2)*hulp(1,1));
    e1b(q1b)=hulp11(q1b)/hulp12(q1b);
    phi1b(q1b)=3*pi*Lbm*e1b(q1b)*E*Icln(1,1)/(Lcln*hulp(1,1));
    etotb(q1b)=Q1b(q1b)*Lbm^3*Lcln^2/(64*hulp(1,1)) + e0 -
phi1b(q1b)*Lcln^2*E*Ibm/(8*Lbm*E*Icln(1,1)) + e1b(q1b);

    F1b(q1b)=Q1b(q1b)*Lbm^3*E*Icln(1,1)/(4*Lcln*hulp(1,1)) -
3*phi1b(q1b)*E*Icln(1,1)/(Lcln^2);
    Ftotb(q1b)=F1b(q1b);
    M1b(q1b)=0.5*F1b(q1b)*Lcln + N1b(q1b)*etotb(q1b);
    Mtotb(q1b)=M1b(q1b);

    sigmatopb=-N1b(q1b)/Acln(1,1) - M1b(q1b)/Zcln(1,1);
    sigmatoptotb(q1b)=sigmatopb;
    sigmabottomb=-N1b(q1b)/Acln(1,1) + M1b(q1b)/Zcln(1,1);
    sigmabottomtotb(q1b)=sigmabottomb;
    sigmacentreb=-N1b(q1b)/Acln(1,1);
    sigmacentretotb(q1b)=sigmacentreb;
end% while q1b

q2b=0;
if (3*pi*Lcln*E*Ibm*((Qtotb(q1b)+2*qq)*Lbm*Lcln^2) + 8*(2*pi^2*E*Icln(1,2)-
(Qtotb(q1b)+2*qq)*Lbm*Lcln^2)*hulp(1,2))>0;

    while (sigmabottomb>yield1 | sigmatopb>yield1) & (sigmabottomb>yield2)
& (sigmatopb>yield2);
    q2b=q2b+1;
    Q2b(q2b)=q2b*qq;
        if sigmacentreb>yield1
            Q2bb(q2b)=q2b*qq;
        else
            Q2bb(q2b)=Q2bb(q2b-1);
        end%if

    Qtotb(q1b+q2b)=Qtotb(q1b)+Q2b(q2b);
    N2b(q2b)=0.5*Q2b(q2b)*Lbm;
    N2bb(q2b)=0.5*Q2bb(q2b)*Lbm;
    Ntotb(q1b+q2b)=Ntotb(q1b)+N2b(q2b);
```

```
hulp21(q2b)=(Qtotb(q1b+q2b)*Lbm*Lcln^2+16*E*Icln(1,2))*hulp(1,1)*(Lbm^2*E*I
cln(1,2)+6*Lcln*z(1,2)*E*Ibm);
    hulp22(q2b)=hulp(1,2)*(4*Qtotb(q1b+q2b)*Lbm*Lcln^2*z(1,2)*hulp(1,1) +
Lcln*E*Icln(1,2)*(Qtotb(q1b)*Lbm^3*Lcln-24*pi*e1b(q1b)*E*Ibm) +
64*E*Icln(1,2)*hulp(1,1)*(e0+e1b(q1b)));

hulp23(q2b)=8*E*Icln(1,2)*hulp(1,1)*(3*pi*(Qtotb(q1b+q2b)*Lbm*Lcln^3*E*Ibm)
+ 8*(2*pi^2*E*Icln(1,2)-Qtotb(q1b+q2b)*Lbm*Lcln^2)*hulp(1,2));
    e2b(q2b)=Q2b(q2b)*Lbm*Lcln^2*(hulp21(q2b)+hulp22(q2b))/hulp23(q2b);
    phi2b(q2b)=3*pi*Lbm*e2b(q2b)*E*Icln(1,2)/(Lcln*hulp(1,2));
    etotb(q1b+q2b)=Q1b(q1b)*Lbm^3*Lcln^2/(64*hulp(1,1)) +
Q2b(q2b)*Lbm*Lcln^2*(Lbm^2*E*Icln(1,2)+6*Lcln*z(1,2)*E*Ibm)/(64*E*Icln(1,2)
*hulp(1,2)) + e0 - phi1b(q1b)*Lcln^2*E*Ibm/(8*Lbm*E*Icln(1,1)) -
phi2b(q2b)*Lcln^2*E*Ibm/(8*Lbm*E*Icln(1,2)) + e1b(q1b) + e2b(q2b) +
Q2b(q2b)*Lbm*Lcln^2*z(1,2)/(16*E*Icln(1,2));


F2b(q2b)=Q2b(q2b)*Lbm*(Lbm^2*E*Icln(1,2)+6*Lcln*z(1,2)*E*Ibm)/(4*Lcln*hulp(
1,2)) - 3*phi2b(q2b)*E*Icln(1,2)/(Lcln^2);
    Ftotb(q1b+q2b)=Ftotb(q1b)+F2b(q2b);
    M2b(q2b)=0.5*F2b(q2b)*Lcln + N2b(q2b)*etotb(q1b+q2b) +
N1b(q1b)*(etotb(q1b+q2b)-etotb(q1b)) + N2b(q2b)*z(1,2);
    Mtotb(q1b+q2b)=Mtotb(q1b)+M2b(q2b);

    sigmatopb=sigmatoptotb(q1b) - N2bb(q2b)/Acln(1,2) - (N2b(q2b)-
N2bb(q2b))/Acln(1,4) - M2b(q2b)/Zcln(1,2);
    sigmatoptotb(q1b+q2b)=sigmatopb;
    sigmabottomb=sigmabottomtotb(q1b) - N2bb(q2b)/Acln(1,2) - (N2b(q2b)-
N2bb(q2b))/Acln(1,4) + M2b(q2b)/Zcln(1,2);
    sigmabottomtotb(q1b+q2b)=sigmabottomb;
    sigmacentreb=sigmacentretotb(q1b) - N2bb(q2b)/Acln(1,2) - (N2b(q2b)-
N2bb(q2b))/Acln(1,4) ;
    sigmacentretotb(q1b+q2b)=sigmacentreb;
end% while q2b

else
    q2b=q2b;
end% if

q3b=0;
if (3*pi*Lcln*E*Ibm*((Qtotb(q1b+q2b)+2*qq)*Lbm*Lcln^2) +
8*(2*pi*E*Icln(1,3)-(Qtotb(q1b+q2b)+2*qq)*Lbm*Lcln^2)*hulp(1,3))>0;

while sigmatopb>sigmabottomb>yield2 & sigmatopb>yield2;
    q3b=q3b+1;
    Q3b(q3b)=q3b*qq;
    Qtotb(q1b+q2b+q3b)=Qtotb(q1b+q2b)+Q3b(q3b);
    N3b(q3b)=0.5*Q3b(q3b)*Lbm;
    Ntotb(q1b+q2b+q3b)=Ntotb(q1b+q3b)+N3b(q3b);


hulp31(q3b)=Q3b(q3b)*Lbm^3*Lcln^2*E*Icln(1,2)*(Qtotb(q1b+q2b+q3b)*Lbm*Lcln^
2+16*E*Icln(1,3))*hulp(1,1)*hulp(1,2);

hulp32(q3b)=Q3b(q3b)*Lbm*Lcln^3*(Q2b(q2b)*Lbm*Lcln*(Lbm^2*E*Icln(1,2)+6*Lcl
n*z(1,2)*E*Ibm)-24*pi*e2b(q2b)*E*Ibm*E*Icln(1,2))*hulp(1,1);

hulp33(q3b)=(Q3b(q3b)*Lbm*Lcln^2*hulp(1,2))*(Lcln*E*Icln(1,2)*(Q1b(q1b)*Lbm
^3*Lcln-24*pi*e1b(q1b)*E*Ibm) +
```

```
(64*E*Icln(1,2)*(e0+e1b(q1b)+e2b(q2b))+4*Q2b(q2b)*Lbm*Lcln^2*z(1,2))*hulp(1
,1));

hulp34(q3b)=8*E*Icln(1,2)*hulp(1,1)*hulp(1,2)*(3*pi*Qtotb(q1b+q2b+q3b)*Lbm*
Lcln^3*E*Ibm + 8*(2*pi*E*Icln(1,3)-
Qtotb(q1b+q2b+q3b)*Lbm*Lcln^2)*hulp(1,3));
    e3b(q3b)=(hulp31(q3b)+(hulp32(q3b)+hulp33(q3b))*hulp(1,3))/hulp34(q3b);
    phi3b(q3b)=3*pi*Lbm*e3b(q3b)*E*Icln(1,3)/(Lcln*hulp(1,3));
    etotb(q1b+q2b+q3b)=Q1b(q1b)*Lbm^3*Lcln^2/(64*hulp(1,1)) +
Q2b(q2b)*Lbm*Lcln^2*(Lbm^2*E*Icln(1,2)+6*Lcln*z(1,2)*E*Ibm)/(64*E*Icln(1,2)
*hulp(1,2)) + Q3b(q3b)*Lbm^3*Lcln^2/(64*hulp(1,3)) + e0 -
phi1b(q1b)*Lcln^2*E*Ibm/(8*Lbm*E*Icln(1,1)) -
phi2b(q2b)*Lcln^2*E*Ibm/(8*Lbm*E*Icln(1,2)) -
phi3b(q3b)*Lcln^2*E*Ibm/(8*Lbm*E*Icln(1,3)) + e1b(q1b) + e2b(q2b) +
e3b(q3b) + Q2b(q2b)*Lbm*Lcln^2*z(1,2)/(16*E*Icln(1,2));

    F3b(q3b)=Q3b(q3b)*Lbm^3*E*Icln(1,3)/(4*Lcln*hulp(1,3)) -
3*phi3b(q3b)*E*Icln(1,3)/(Lcln^2);
    Ftotb(q1b+q2b+q3b)=Ftotb(q1b+q2b)+F3b(q3b);
    M3b(q3b)=0.5*F3b(q3b)*Lcln + N3b(q3b)*etotb(q1b+q2b+q3b) +
(N1b(q1b)+N2b(q2b))*(etotb(q1b+q2b+q3b)-etotb(q1a+q2b));
    Mtotb(q1b+q2b+q3b)=Mtotb(q1b+q2b)+M3b(q3b);

    sigmatopb=sigmatoptotb(q1b+q2b) - N3b(q3b)/Acln(1,3) -
M3b(q3b)/Zcln(1,3);
    sigmatoptotb(q1b+q2b+q3b)=sigmatopb;
    sigmabottomb=sigmabottomtotb(q1b+q2b) - N1b(q3b)/Acln(1,3) +
M3b(q3b)/Zcln(1,3);
    sigmabottomtotb(q1b+q2b+q3b)=sigmabottomb;
    sigmacentreb=sigmacentretotb(q1b+q2b) - N3b(q3b)/Acln(1,3);
    sigmacentretotb(q1b+q2b+q3b)=sigmacentreb;

end%ehile q3a

else
    q3b=q3b;
end% if
Qmaxb=Qtotb(q1b+q2b+q3b);

clear F1b;clear F2b;clear F3b;
clear M1b;clear M2b;clear M3b;
clear N1b;clear N2b;clear N3b; clear N2bb;
clear Q1b;clear Q2b;clear Q3b;clear Q2bb;
clear e1b;clear e2b;clear e3b;
clear phi1b;clear phi2b;clear phi3;
clear sigmatopb; clear sigmabottomb; clear sigmacentreb;
clear hulp11;clear hulp12;
clear hulp21;clear hulp22;clear hulp23;
clear hulp31;clear hulp32;clear hulp33;

if Qmaxa<Qmaxb;
    Qmax=Qmaxa;
    Nmax=Ntota;
    Mtot=Mtota;
    Qtot=Qtota;
    etot=etota;
    q1=q1a;
    q2=q2a;
    q3=q3a;
    sigmatop=sigmatoptota;
```

```
    sigmabottom=sigmabottomtota;
    sigmacentre=sigmacentretota;
    AAA='end column';

else
    Qmax=Qmaxb;
    Nmax=Ntotb;
    Mtot=Mtotb;
    Qtot=Qtotb;
    etot=etotb;
    q1=q1b;
    q2=q2b;
    q3=q3b;
    sigmatop=sigmatoptotb;
    sigmabottom=sigmabottomtotb;
    sigmacentre=sigmacentretotb;
    AAA='middle column';


end%if Qmax

clear Mtota;clear Mtotb;clear Ftota;clear Ftotb;clear Qtota;clear Qtotb;
clear Qmaxa;clear Qmaxb;clear etota;clear etotb;
clear q1a;clear q2a;clear q3a;%clear q1b;clear q2b;clear q3b;
clear sigmatoptota;clear sigmatoptotb;clear sigmabottomtota;clear
sigmabottomtotb;clear sigmacentretota;clear sigmacentretotb;
clear delta;clear qq;clear hulp;

Qmax
%plot(sigmatop,'r');hold
on;plot(sigmabottom);plot(sigmacentre,'g');xlabel('Deflection in the
midsection (mm)');ylabel('Load (N/mm)');title('Length is 10 meter');grid
%plot(etot,Qtot);xlabel('Deflection in the midsection (mm)');ylabel('Load
(N/mm)');title('Length is 10 meter');grid
```

P.3 Calculation according to the Dutch code

According to the Dutch code, the calculation of a braced portal frame is almost the same as the calculation of an unbraced portal frame. The calculation of an unbraced portal frame is discussed in Appendix G. The main difference is the buckling length ratio. Another graphic must be used to find the buckling length (Fig. P.5). The value of C_A and the value of C_B can be calculated by the following formula:

$$C = \frac{\sum \frac{I_{cln}}{L_{cln}}}{\sum \mu \frac{I_{bm}}{L_{bm}}} \quad \text{(NEN 6770 art. 12.1.1.3)}$$

In which μ is a correction factor. This correction factor depends on the connections of the beam. In the case of a portal frame μ=3.

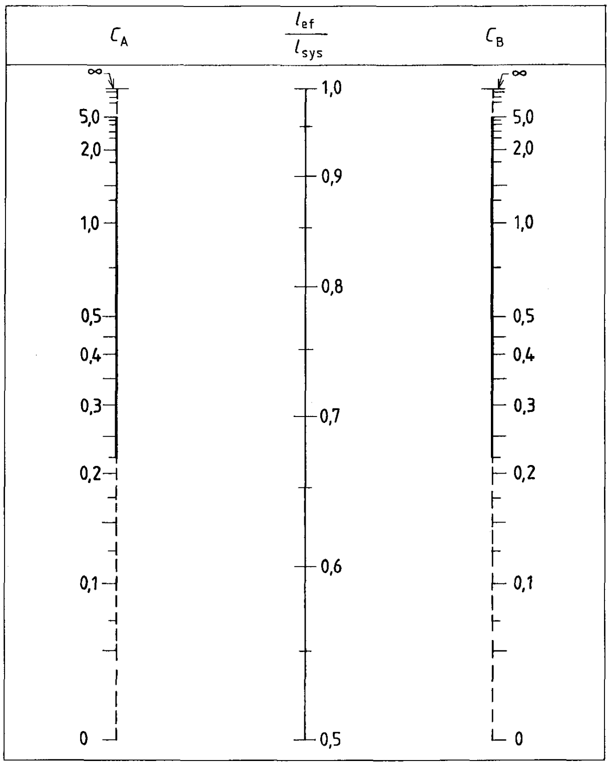


Figure P.5:
Buckling length ratio
(NEN 6770 art. 12.1.1.3)

To calculate the ultimate load according to the Dutch code, the force must be split to a bending moment and a normal force. This can be done by a linear analysis. If the bending moments (as a function of the normal force) and the normal forces
are known the ultimate load can be calculated by the following formula:

$$\frac{N_{c;s;d}}{N_{c;u;d}} + \frac{n_y}{n_y - 1} \frac{M_{y;equ;s;d} + F_{y;tot;s;d} e_y^*}{M_{y;u;d}} \leq 1.0 \quad \text{(NEN 6771 art. 12.3.1)}$$

The following expressions must be used in the formula.

$$n_y = \frac{F_E}{F_{y;tot;s;d}}$$

$$e_y^* = \alpha_k \left(\lambda_{y;rel} - \lambda_0 \right) \frac{M_{y;u;d}}{N_{c;u;d}}$$

$M_{y;equ;s;d}$ depends on the different moments on the structure and the type of structure. The following formulas must be used.

$$M_{y;equ;s;d} = \max \begin{pmatrix} \left| 0.1 \left(M_{y;2;s;d} - M_{y;1;s;d} \right) + M_{y;mid;s;d} \right| \\ \left| 0.6 M_{y;2;s;d} \right| \end{pmatrix}$$

By these formulas the ultimate load can be calculated.

P.4 Calculation according to Matrix Frame

The same portal frame can be calculated by a finite element method. A computer program Matrix Frame is such a program. The results of these calculations can be found in this Appendix.

# Appendix Q	Loads of an extended Frame

The loads on the frame are calculated according to the Dutch code (NEN 6702). The NEN 6702 is about loading rules and safety rules. The rules in this code are used to find the ultimate load.

If a beam deflects, the connected elements deflects too. The largest deflection of the beam results in the largest deflection of the column. For the non linear analysis the largest deflection is most interesting. The largest deflection occurs if not all elements are loaded (Fig. Q.1).

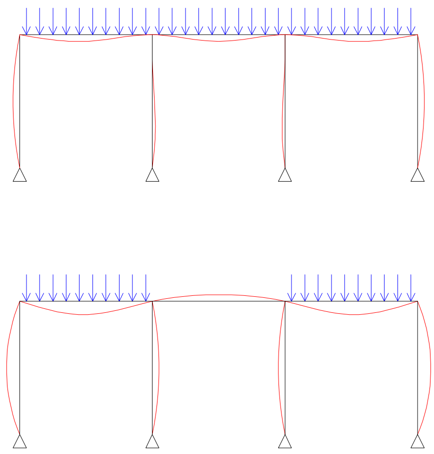


Figure Q.1: Deflection example

The discussed frame is a two-dimensional structure. The loads are based on three-dimensional values.

The range of the frame is taken five meters.

Q.1 Different loads

The NEN 6702 describes three load types: the permanent load, the variable load and the special loads. The special loads are about fire and explosions. These loads are not taken into account in this study.

All permanent loads must be taken into account. The permanent load of the frame is the dead load. Other permanent loads like water pressure and prestressing are not relevant and are neglected. The frame cannot be used without a floor. The dead load of this floor must also be taken into account. The weight of the floor is estimated on 1.5 kN/m^2. This load corresponds with a 100 mm sandwich panel. Also the weight of a partition wall must take into account. The NEN 6702 has estimated this load on 0.5 kN/m^2.

The total dead load is:

Beam:	2.5 kN/m	
Column:	1.1 kN/m	
Floor:	1.5 kN/m^2	= 7.5 kN/m
Wall:	0.5 kN/m^2	= 2.5 kN/m

According to the NEN 6702 wind load must be taken into account. Wind load have a positive influence on the ultimate load calculation (Fig. Q.2). The ultimate load increases if the wind load is taken into account. The wind load is neglected in the analysis.

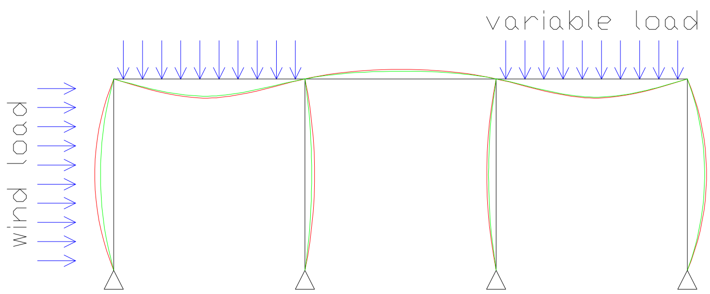


Figure Q.2: Influence of wind

Another load that must be taken into account is the snow load. The snow load on a flat roof is 0.56 kN/m^2 (2.8 kN/m). This load is summed up at the dead load of the roof.

Q.2 Safety rules

The NEN 6702 describes many safety classes. The heaviest safety class is used for the calculations. The maximum load is 1.2 times the permanent load and 1.5 times the variable load ($q_{design} = 1.2q_{dead} + 1.5q_{variable}$).

The design loads are:

On the roof: $q_{design} = 1.2(2.5+7.5) + 1.5(2.8)$

$q_{design} = 16\ kN/m^1$

On the floor: $q_{design} = 1.2(2.5+7.5+2.5)$

$q_{design} = 15\ kN/m^1$

Column: $F_{design} = 1.2*1.1*5$

$F_{design} = 5.5\ kN$

Q.3 Loading

The frame is symmetric. The real load distribution depends on the deflection of the structure. It is assumed that the design loads from Appendix Q.2 are linear distributed. The distributed loads results in point loads on the columns. Due to symmetry tere are no bending moments. The difference between this assumption and the real distribution is very small compare with the real load distribution. The differences are negligible.

The structure is loaded by several point loads. See Figure Q.3 for the different loads. The loads are:

$F_1 = 0.5q_{design,roof}L_{bm} + F_{design,column}$

$F_1 = 0.5*16*5 +5.5$

$F_1 = 45.5\ kN$

$F_2 = q_{design,roof}L_{bm} + F_{design,column}$

$F_2 = 16*5 +5.5$

$F_2 = 85.5\ kN$

$F_3 = 0.5q_{design,floor}L_{bm} + F_{design,column}$

$F_3 = 0.5*15*5 +5.5$

$F_3 = 43.0\ kN$

$F_4 = q_{design,floor}L_{bm} + F_{design,column}$

$F_4 = 15*5 +5.5$

$F_4 = 80.5\ kN$

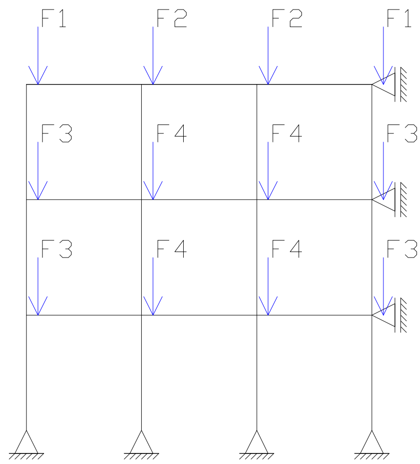


Figure Q.3: Different loads

The loads discussed in this Appendix are the required loads. The ultimate load is the limit of the extra variable loads.

Appendix R Linear analysis extended frame

The framework that will be analyzed is a frame with three floors and three bays (Fig. R.1). The loads F_1 till F_4 are required loads. These loads have been discussed in Appendix Q. The variable load is located on beam JK.

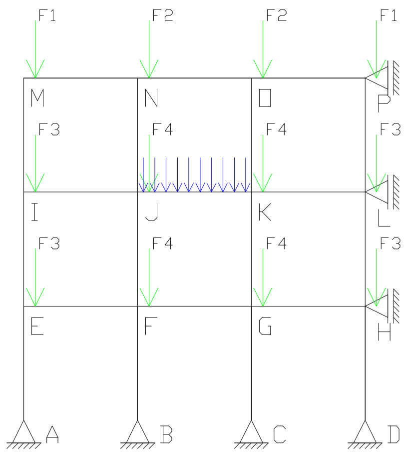


Figure R.1: Structure

All columns have the same length (L_{cln}) and the same section properties. Also the length and the section properties of all beams are the equal. The section properties of the beam and the length of the beam can be different from the section properties and the length of the column. The frame is symmetrical. The ultimate load results in a limit of the variable load. It is assumed that the variable load is equally distributed over the columns. The vertical loads on the columns are:

Column FJ: $0.5q_{variable}L_{bm} + F_2 + F_4$

Column GK: $0.5q_{variable}L_{bm} + F_2 + F_4$

The maximum column deflection is the deflection of the columns FJ and GK. These columns are taken as critical construction elements. It is assumed that the beams will not yield. It is also assumed that all other columns do not yield. These assumptions must be checked afterwards. Due to the symmetry of the frame, the columns FJ and GK are loaded equal. The analyzed column is column FJ.

Column FJ has been schematized as a column with an initial deflection, an initial rotation and a two rotation springs (Fig. R.2). The initial deflection is the starting deflection (imperfections). The initial rotation is the rotation of the beam due to the uniformly distributed variable load. The stiffness of the rotational springs depends on the stiffness of the construction elements. These rotational springs (one on both sides of the column) are not equal.

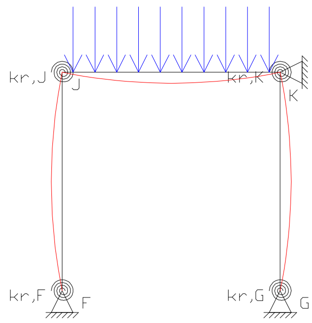


Figure R.2: Structure

For an analysis it is nearly impossible to take the bending of all elements into account. It is assumed that there are virtual hinges at the points E, H, M and P. The influence of the rotation of these points on the moment distribution of column FJ is very small. It becomes clearer at the analysis.

Point I

$$\varphi_{I,top} = \frac{-M_{I,top}L_{cln}}{3EI_{cln}} \rightarrow M_{I,top} = \frac{-3\varphi_{I,top}EI_{cln}}{L_{cln}}$$

$$\varphi_{I,bottom} = \frac{-M_{I,bottom}L_{cln}}{3EI_{cln}} \rightarrow M_{I,bottom} = \frac{-3\varphi_{I,bottom}EI_{cln}}{L_{cln}}$$

Moment equilibrium

$$M_{I,right} = M_{I,top} + M_{I,bottom}$$

$$M_{I,right} = \frac{-3\varphi_{I,bottom} EI_{cln}}{L_{cln}} + \frac{-3\varphi_{I,top} EI_{cln}}{L_{cln}}$$

Rotation equilibrium

$$\varphi_{I,bottom} = \varphi_{I,top} = \varphi_I$$

$$M_{I,right} = \left(\frac{-3EI_{cln}}{L_{cln}} + \frac{-3EI_{cln}}{L_{cln}} \right) \varphi_I$$

$$M_{I,right} = \left(\frac{-6EI_{cln}}{L_{cln}} \right) \varphi_I$$

Point N

$$\varphi_{N,right} = \frac{-M_{N,right} L_{bm}}{2EI_{bm}} \rightarrow M_{N,right} = \frac{-2\varphi_{N,right} EI_{bm}}{L_{bm}}$$ (Symmetric with point O)

$$\varphi_{N,left} = \frac{M_{N,left} L_{bm}}{3EI_{bm}} - \frac{2NL_{cln}}{L_{bm} EA_{cln}} \rightarrow M_{N,left} = \frac{6NL_{cln} EI_{bm}}{L_{bm}{}^2 EA_{cln}} + \frac{3\varphi_{N,left} EI_{bm}}{L_{bm}}$$

Moment equilibrium

$$M_{N,bottom} = M_{N,left} - M_{N,right}$$

$$M_{N,bottom} = \left(\frac{6NL_{cln} EI_{bm}}{L_{bm}{}^2 EA_{cln}} + \frac{3\varphi_{N,left} EI_{bm}}{L_{bm}} \right) - \left(\frac{-2\varphi_{N,right} EI_{bm}}{L_{bm}} \right)$$

Rotation equilibrium

$$\varphi_{N,bottom} = \varphi_{N,top} = \varphi_N$$

$$M_{N,bottom} = \frac{6NL_{cln} EI_{bm}}{L_{bm}{}^2 EA_{cln}} + \left(\frac{3EI_{bm}}{L_{bm}} + \frac{2EI_{bm}}{L_{bm}} \right) \varphi_N$$

$$M_{N,bottom} = \frac{6NL_{cln} EI_{bm}}{L_{bm}{}^2 EA_{cln}} + \left(\frac{5EI_{bm}}{L_{bm}} \right) \varphi_N$$

Point F

$$\varphi_{F,right} = \frac{-M_{F,right} L_{bm}}{2EI_{bm}} \rightarrow M_{F,right} = \frac{-2\varphi_{F,right} EI_{bm}}{L_{bm}}$$ (Symmetric with point G)

$$\varphi_{F,left} = \frac{M_{F,left} L_{bm}}{3EI_{bm}} - \frac{NL_{cln}}{L_{bm} EA_{cln}} \rightarrow M_{F,left} = \frac{3NL_{cln} EI_{bm}}{L_{bm}{}^2 EA_{cln}} + \frac{3\varphi_{F,left} EI_{bm}}{L_{bm}}$$

$$\varphi_{F,bottom} = \frac{-M_{F,bottom} L_{cln}}{3EI_{cln}} \rightarrow M_{F,bottom} = \frac{-3\varphi_{F,bottom} EI_{cln}}{L_{cln}}$$

Moment equilibrium

$$M_{F,top} = M_{F,left} - M_{F,bottom} - M_{F,right}$$

$$M_{F,top} = \left(\frac{3NL_{cln}EI_{bm}}{L_{bm}{}^2 EA_{cln}} + \frac{3\varphi_{F,left}EI_{bm}}{L_{bm}} \right) - \left(\frac{-3\varphi_{F,bottom}EI_{cln}}{L_{cln}} \right) - \left(\frac{-2\varphi_{F,right}EI_{bm}}{L_{bm}} \right)$$

Rotation equilibrium

$$\varphi_{F,bottom} = \varphi_{F,left} = \varphi_{F,right} = \varphi_F$$

$$M_{F,top} = \frac{3NL_{cln}EI_{bm}}{L_{bm}{}^2 EA_{cln}} + \left(\frac{3EI_{bm}}{L_{bm}} + \frac{3EI_{cln}}{L_{cln}} + \frac{2EI_{bm}}{L_{bm}} \right)\varphi_F$$

$$M_{F,top} = \frac{3NL_{cln}EI_{bm}}{L_{bm}{}^2 EA_{cln}} + \left(\frac{3L_{bm}EI_{cln} + 5L_{cln}EI_{bm}}{L_{bm}L_{cln}} \right)\varphi_F$$

The moments which are found are:

$$M_{I,right} = \left(\frac{-6EI_{cln}}{L_{cln}} \right)\varphi_I$$

$$M_{N,bottom} = \frac{6NL_{cln}EI_{bm}}{L_{bm}{}^2 EA_{cln}} + \left(\frac{5EI_{bm}}{L_{bm}} \right)\varphi_N$$

$$M_{F,top} = \frac{3NL_{cln}EI_{bm}}{L_{bm}{}^2 EA_{cln}} + \left(\frac{3L_{bm}EI_{cln} + 5L_{cln}EI_{bm}}{L_{bm}L_{cln}} \right)\varphi_F$$

To find the bending moments distribution in point J and the rotation in point J it is necessarily to know the influence of the connected elements (see figure R.3). The known formulas can be used to find the rotation in point J.

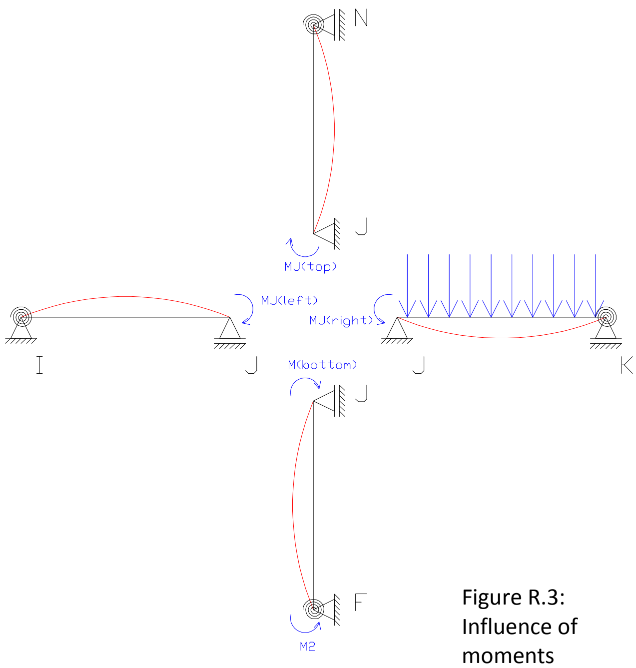


Figure R.3: Influence of moments

Column JN.

$$\varphi_N = \frac{M_{J,top}L_{cln}}{6EI_{cln}} - \frac{M_{N,bottom}L_{cln}}{3EI_{cln}}$$

Use the known formula of $M_{N,bottom}$.

$$\varphi_N = \frac{M_{J,top}L_{cln}}{6EI_{cln}} - \frac{\left(\frac{6NL_{cln}EI_{bm}}{L_{bm}{}^2 EA_{cln}} + \left(\frac{5EI_{bm}}{L_{bm}} \right)\varphi_N \right)L_{cln}}{3EI_{cln}}$$

Write out the formulas.

$$\varphi_N = \frac{M_{J,top}L_{cln}}{6EI_{cln}} - \frac{2NL_{cln}{}^2 EI_{bm}}{L_{bm}{}^2 EA_{cln}EI_{cln}} - \frac{5\varphi_N L_{cln}EI_{bm}}{3L_{bm}EI_{cln}}$$

All expressions of φ_N are separated from the rest of the formula.

$$\varphi_N + \frac{5\varphi_N L_{cln}EI_{bm}}{3L_{bm}EI_{cln}} = \frac{M_{J,top}L_{cln}}{6EI_{cln}} - \frac{2NL_{cln}{}^2 EI_{bm}}{L_{bm}{}^2 EA_{cln}EI_{cln}}$$

Make everywhere the same denominator.

$$\varphi_N\left(\frac{2L_{bm}EA_{cln}\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}{6L_{bm}^{2}EA_{cln}EI_{cln}}\right)=\frac{M_{J,top}L_{bm}^{2}L_{cln}EA_{cln}}{6L_{bm}^{2}EA_{cln}EI_{cln}}-\frac{12NL_{cln}^{2}EI_{bm}}{6L_{bm}^{2}EA_{cln}EI_{cln}}$$

Find the formula of φ_N.

$$\varphi_N=\frac{M_{J,top}L_{bm}^{2}L_{cln}EA_{cln}-12NL_{cln}^{2}EI_{bm}}{2L_{bm}EA_{cln}\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}$$

The rotation of point N has been found. This rotation can be used. The formula of the rotation in point J is:

$$\varphi_{J,top}=-\frac{M_{J,top}L_{cln}}{3EI_{cln}}+\frac{M_{N,bottom}L_{cln}}{6EI_{cln}}$$

The formula of $M_{N,bottom}$ can be used.

$$\varphi_{J,top}=-\frac{M_{J,top}L_{cln}}{3EI_{cln}}+\frac{\left(\frac{6NL_{cln}EI_{bm}}{L_{bm}^{2}EA_{cln}}+\left(\frac{5EI_{bm}}{L_{bm}}\right)\varphi_N\right)L_{cln}}{6EI_{cln}}$$

The rotation of point N has been found en can be used.

$$\varphi_{J,top}=-\frac{M_{J,top}L_{cln}}{3EI_{cln}}+\frac{\left(\frac{6NL_{cln}EI_{bm}}{L_{bm}^{2}EA_{cln}}+\left(\frac{5EI_{bm}}{L_{bm}}\right)\left(\frac{M_{J,top}L_{bm}^{2}L_{cln}EA_{cln}-12NL_{cln}^{2}EI_{bm}}{2L_{bm}EA_{cln}\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}\right)\right)L_{cln}}{6EI_{cln}}$$

Writing out the expression results in:

$$\varphi_{J,top}=-\frac{M_{J,top}L_{cln}}{3EI_{cln}}+\frac{NL_{cln}^{2}EI_{bm}}{L_{bm}^{2}EA_{cln}EI_{cln}}+\frac{5M_{J,top}L_{bm}^{2}L_{cln}^{2}EA_{cln}EI_{bm}}{12L_{bm}^{2}EA_{cln}EI_{cln}\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}$$
$$-\frac{60NL_{cln}^{3}\left(EI_{bm}\right)^{2}}{12L_{bm}^{2}EA_{cln}EI_{cln}\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}$$

Separate $M_{J,top}$ from the rest of the formula.

$$\frac{M_{J,top}L_{cln}}{3EI_{cln}}-\frac{5M_{J,top}L_{bm}^{2}L_{cln}^{2}EA_{cln}EI_{bm}}{12L_{bm}^{2}EA_{cln}EI_{cln}\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}=\frac{NL_{cln}^{2}EI_{bm}}{L_{bm}^{2}EA_{cln}EI_{cln}}$$
$$-\frac{60NL_{cln}^{3}\left(EI_{bm}\right)^{2}}{12L_{bm}^{2}EA_{cln}EI_{cln}\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}-\varphi_{J,top}$$

Make the same denominator.

$$\frac{4M_{J,top}L_{bm}{}^{2}L_{cln}EA_{cln}\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}{12L_{bm}{}^{2}EI_{cln}EA_{cln}\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}-\frac{5M_{J,top}L_{bm}{}^{2}L_{cln}{}^{2}EA_{cln}EI_{bm}}{12L_{bm}{}^{2}EA_{cln}EI_{cln}\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}$$

$$=\frac{12NL_{cln}{}^{2}EI_{bm}\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}{12L_{bm}{}^{2}EA_{cln}EI_{cln}\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}-\frac{60NL_{cln}{}^{3}\left(EI_{bm}\right)^{2}}{12L_{bm}{}^{2}EA_{cln}EI_{cln}\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}$$

$$-\frac{12\varphi_{J,top}L_{bm}{}^{2}EA_{cln}EI_{cln}\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}{12L_{bm}{}^{2}EA_{cln}EI_{cln}\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}$$

Neglect the denominator.

$$4M_{J,top}L_{bm}{}^{2}L_{cln}EA_{cln}\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)-5M_{J,top}L_{bm}{}^{2}L_{cln}{}^{2}EA_{cln}EI_{bm}$$

$$=12NL_{cln}{}^{2}EI_{bm}\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)-60NL_{cln}{}^{3}\left(EI_{bm}\right)^{2}-12\varphi_{J,top}L_{bm}{}^{2}EA_{cln}EI_{cln}\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)$$

Combine the same functions.

$$3M_{J,top}L_{bm}{}^{2}L_{cln}EA_{cln}\left(4L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)$$

$$=12L_{bm}EI_{cln}\left(3NL_{cln}{}^{2}EI_{bm}-\varphi_{J,top}L_{bm}EA_{cln}\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\right)$$

Find the formula of $M_{J,top}$.

$$M_{J,top}=\frac{4EI_{cln}\left(3NL_{cln}{}^{2}EI_{bm}-\varphi_{J,top}L_{bm}EA_{cln}\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\right)}{L_{bm}L_{cln}EA_{cln}\left(4L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}$$

Column FJ.

$$\varphi_{F}=\frac{M_{J,bottom}L_{cln}}{6EI_{cln}}-\frac{M_{F,top}L_{cln}}{3EI_{cln}}$$

Use the formula of $M_{F,top}$.

$$\varphi_{F}=\frac{M_{J,bottom}L_{cln}}{6EI_{cln}}-\frac{\left(\frac{3NL_{cln}EI_{bm}}{L_{bm}{}^{2}EA_{cln}}+\left(\frac{3L_{bm}EI_{cln}+5L_{cln}EI_{bm}}{L_{bm}L_{cln}}\right)\varphi_{F}\right)L_{cln}}{3EI_{cln}}$$

Write out the expression.

$$\varphi_{F}=\frac{M_{J,bottom}L_{cln}}{6EI_{cln}}-\frac{NL_{cln}{}^{2}EI_{bm}}{L_{bm}{}^{2}EA_{cln}EI_{cln}}-\frac{\varphi_{F}L_{cln}\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}{3L_{bm}L_{cln}EI_{cln}}$$

Separate the expressions of φ_{F}.

$$\varphi_{F}+\frac{\varphi_{F}\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}{3L_{bm}EI_{cln}}=\frac{M_{J,bottom}L_{cln}}{6EI_{cln}}-\frac{NL_{cln}{}^{2}EI_{bm}}{L_{bm}{}^{2}EA_{cln}EI_{cln}}$$

Make everywhere the same denominator.

$$\varphi_F\left(\frac{2L_{bm}EA_{cln}\left(3L_{bm}EI_{cln}+\left(5L_{cln}EI_{bm}+3L_{bm}EI_{cln}\right)\right)}{6L_{bm}{}^2EA_{cln}EI_{cln}}\right)=\frac{M_{J,bottom}L_{bm}{}^2L_{cln}EA_{cln}}{6L_{bm}{}^2EA_{cln}EI_{cln}}-\frac{6NL_{cln}{}^2EI_{bm}}{6L_{bm}{}^2EA_{cln}EI_{cln}}$$

Find the formula of φ_F .

$$\varphi_F=\frac{M_{J,bottom}L_{bm}{}^2L_{cln}EA_{cln}-6NL_{cln}{}^2EI_{bm}}{2L_{bm}EA_{cln}\left(6L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}$$

The rotation of point F has been found. This rotation can be used. The formula of the rotation in point J is:

$$\varphi_{J,bottom}=-\frac{M_{J,bottom}L_{cln}}{3EI_{cln}}+\frac{M_{F,top}L_{cln}}{6EI_{cln}}$$

The formula of $M_{F,top}$ is known. This formula can be used.

$$\varphi_{J,bottom}=-\frac{M_{J,bottom}L_{cln}}{3EI_{cln}}+\frac{\left(\frac{3NL_{cln}EI_{bm}}{L_{bm}{}^2EA_{cln}}+\left(\frac{3L_{bm}EI_{cln}+5L_{cln}EI_{bm}}{L_{bm}L_{cln}}\right)\varphi_F\right)L_{cln}}{6EI_{cln}}$$

The formula of φ_F can be used.

$$\varphi_{J,bottom}=-\frac{M_{J,bottom}L_{cln}}{3EI_{cln}}+\frac{\left(\frac{3NL_{cln}EI_{bm}}{L_{bm}{}^2EA_{cln}}+\left(\frac{3L_{bm}EI_{cln}+5L_{cln}EI_{bm}}{L_{bm}L_{cln}}\right)\left(\frac{M_{J,bottom}L_{bm}{}^2L_{cln}EA_{cln}-6NL_{cln}{}^2EI_{bm}}{2L_{bm}EA_{cln}\left(6L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}\right)\right)L_{cln}}{6EI_{cln}}$$

Write out the expression.

$$\varphi_{J,bottom}=-\frac{M_{J,bottom}L_{cln}}{3EI_{cln}}+\frac{NL_{cln}{}^2EI_{bm}}{2L_{bm}{}^2EA_{cln}EI_{cln}}+\frac{M_{J,bottom}L_{bm}{}^2L_{cln}EA_{cln}\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}{12L_{bm}{}^2EA_{cln}EI_{cln}\left(6L_{bm}EI_{cln}5L_{cln}EI_{bm}+\right)}$$
$$-\frac{6NL_{cln}{}^2EI_{bm}\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}{12L_{bm}{}^2EA_{cln}EI_{cln}\left(6L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}$$

Separate $M_{J,bottom}$ from the rest of the formula.

$$\frac{M_{J,bottom}L_{cln}}{3EI_{cln}}-\frac{M_{J,bottom}L_{bm}{}^2L_{cln}EA_{cln}\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}{12L_{bm}{}^2EA_{cln}EI_{cln}\left(6L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}$$
$$=\frac{NL_{cln}{}^2EI_{bm}}{2L_{bm}{}^2EA_{cln}EI_{cln}}-\frac{6NL_{cln}{}^2EI_{bm}\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}{12L_{bm}{}^2EA_{cln}EI_{cln}\left(6L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}-\varphi_{J,bottom}$$

Make the same denominator.

$$\frac{4M_{J,bottom}L_{bm}{}^2L_{cln}EA_{cln}\left(6L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}{12L_{bm}{}^2EA_{cln}EI_{cln}\left(6L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}-\frac{M_{J,bottom}L_{bm}{}^2L_{cln}EA_{cln}\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}{12L_{bm}{}^2EA_{cln}EI_{cln}\left(6L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}$$

$$=\frac{6NL_{cln}{}^2EI_{bm}\left(6L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}{12L_{bm}{}^2EA_{cln}EI_{cln}\left(6L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}-\frac{6NL_{cln}{}^2EI_{bm}\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}{12L_{bm}{}^2EA_{cln}EI_{cln}\left(6L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}$$

$$-\frac{12\varphi_{J,bottom}L_{bm}{}^2EA_{cln}EI_{cln}\left(6L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}{12L_{bm}{}^2EA_{cln}EI_{cln}\left(6L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}$$

Neglect the denominator.

$$4M_{J,bottom}L_{bm}{}^2L_{cln}EA_{cln}\left(6L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)-M_{J,bottom}L_{bm}{}^2L_{cln}EA_{cln}\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)$$

$$=6NL_{cln}{}^2EI_{bm}\left(6L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)-6NL_{cln}{}^2EI_{bm}\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)$$

$$-12\varphi_{J,bottom}L_{bm}{}^2EA_{cln}EI_{cln}\left(6L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)$$

Combine the same functions.

$$3M_{J,bottom}L_{bm}{}^2L_{cln}EA_{cln}\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)$$

$$=18NL_{bm}L_{cln}{}^2EI_{bm}EI_{cln}-12\varphi_{J,bottom}L_{bm}{}^2EA_{cln}EI_{cln}\left(6L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)$$

Find the formula of $M_{J,bottom}$.

$$M_{J,bottom}=\frac{2EI_{cln}\left(3NL_{cln}{}^2EI_{bm}-2\varphi_{J,bottom}L_{bm}EA_{cln}\left(6L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\right)}{L_{bm}L_{cln}EA_{cln}\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}$$

Beam IJ.

For the influence of beam IJ the elongation of the columns must be taken into account. The length of the columns BF and FJ will decrease. The length of the columns AE and EI remain constant.

$$\varphi_I=\frac{M_{J,left}L_{bm}}{6EI_{bm}}+\frac{M_{I,right}L_{bm}}{3EI_{bm}}-\frac{2NL_{cln}}{L_{bm}EA_{cln}}$$

Use the formula of $M_{I,right}$.

$$\varphi_I=\frac{M_{J,left}L_{bm}}{6EI_{bm}}+\frac{\left(\frac{-6EI_{cln}}{L_{cln}}\right)\varphi_IL_{bm}}{3EI_{bm}}-\frac{2NL_{cln}}{L_{bm}EA_{cln}}$$

Write out the expression.

$$\varphi_I=\frac{M_{J,left}L_{bm}}{6EI_{bm}}-\frac{6\varphi_IL_{bm}EI_{cln}}{3L_{cln}EI_{bm}}-\frac{2NL_{cln}}{L_{bm}EA_{cln}}$$

Separate the expressions of φ_I.

$$\varphi_I + \frac{6\varphi_I L_{bm} EI_{cln}}{3L_{cln} EI_{bm}} = \frac{M_{J,left} L_{bm}}{6EI_{bm}} - \frac{2NL_{cln}}{L_{bm} EA_{cln}}$$

Make everywhere the same denominator.

$$\varphi_I \left(\frac{6L_{bm} EA_{cln} \left(2L_{bm} EI_{cln} + L_{cln} EI_{bm} \right)}{6L_{bm} L_{cln} EA_{cln} EI_{bm}} \right) = \frac{M_{J,left} L_{bm}{}^2 L_{cln} EA_{cln}}{6L_{bm} L_{cln} EA_{cln} EI_{bm}} - \frac{12NL_{cln}{}^2 EI_{bm}}{6L_{bm} L_{cln} EA_{cln} EI_{bm}}$$

Find the formula of φ_I.

$$\varphi_I = \frac{M_{J,left} L_{bm}{}^2 L_{cln} EA_{cln} - 12NL_{cln}{}^2 EI_{bm}}{6L_{bm} EA_{cln} \left(2L_{bm} EI_{cln} + L_{cln} EI_{bm} \right)}$$

The rotation of point I have been found. This rotation can be used. The formula of the rotation in point J is:

$$\varphi_{J,left} = -\frac{M_{J,left} L_{bm}}{3EI_{bm}} - \frac{M_{I,right} L_{bm}}{6EI_{bm}} - \frac{2NL_{cln}}{L_{bm} EA_{cln}}$$

Use the formula of $M_{I,right}$.

$$\varphi_{J,left} = -\frac{M_{J,left} L_{bm}}{3EI_{bm}} - \frac{\left(\left(\frac{-6EI_{cln}}{L_{cln}} \right) \varphi_I \right) L_{bm}}{6EI_{bm}} - \frac{2NL_{cln}}{L_{bm} EA_{cln}}$$

Use the formula of φ_I.

$$\varphi_{J,left} = -\frac{M_{J,left} L_{bm}}{3EI_{bm}} - \frac{\left(\left(\frac{-6EI_{cln}}{L_{cln}} \right) \left(\frac{M_{J,left} L_{bm}{}^2 L_{cln} EA_{cln} - 12NL_{cln}{}^2 EI_{bm}}{6L_{bm} EA_{cln} \left(2L_{bm} EI_{cln} + L_{cln} EI_{bm} \right)} \right) \right) L_{bm}}{6EI_{bm}} - \frac{2NL_{cln}}{L_{bm} EA_{cln}}$$

Write out the expression.

$$\varphi_{J,left} = -\frac{M_{J,left} L_{bm}}{3EI_{bm}} + \frac{M_{J,left} L_{bm}{}^2 L_{cln} EA_{cln} EI_{cln}}{6L_{cln} EA_{cln} EI_{bm} \left(2L_{bm} EI_{cln} + L_{cln} EI_{bm} \right)} - \frac{12NL_{cln}{}^2 EI_{bm} EI_{cln}}{6L_{cln} EA_{cln} EI_{bm} \left(2L_{bm} EI_{cln} + L_{cln} EI_{bm} \right)} - \frac{2NL_{cln}}{L_{bm} EA_{cln}}$$

Separate $M_{J,left}$ from the rest of the formula.

$$-\frac{M_{J,left} L_{bm}}{3EI_{bm}} + \frac{M_{J,left} L_{bm}{}^2 L_{cln} EA_{cln} EI_{cln}}{6L_{cln} EA_{cln} EI_{bm} \left(2L_{bm} EI_{cln} + L_{cln} EI_{bm} \right)}$$

$$= \varphi_{J,left} + \frac{12NL_{cln}{}^2 EI_{bm} EI_{cln}}{6L_{cln} EA_{cln} EI_{bm} \left(2L_{bm} EI_{cln} + L_{cln} EI_{bm} \right)} + \frac{2NL_{cln}}{L_{bm} EA_{cln}}$$

Make the same denominator.

$$-\frac{2M_{J,left}L_{bm}^{2}L_{cln}EA_{cln}\left(2L_{bm}EI_{cln}+L_{cln}EI_{bm}\right)}{6L_{bm}L_{cln}EA_{cln}EI_{bm}\left(2L_{bm}EI_{cln}+L_{cln}EI_{bm}\right)}+\frac{M_{J,left}L_{bm}^{3}L_{cln}EA_{cln}EI_{cln}}{6L_{bm}L_{cln}EA_{cln}EI_{bm}\left(2L_{bm}EI_{cln}+L_{cln}EI_{bm}\right)}$$
$$=\frac{6\varphi_{J,left}L_{bm}L_{cln}EA_{cln}EI_{bm}\left(2L_{bm}EI_{cln}+L_{cln}EI_{bm}\right)}{6L_{bm}L_{cln}EA_{cln}EI_{bm}\left(2L_{bm}EI_{cln}+L_{cln}EI_{bm}\right)}+\frac{12NL_{bm}L_{cln}^{2}EI_{bm}EI_{cln}}{6L_{bm}L_{cln}EA_{cln}EI_{bm}\left(2L_{bm}EI_{cln}+L_{cln}EI_{bm}\right)}$$
$$+\frac{12NL_{cln}^{2}EI_{bm}\left(2L_{bm}EI_{cln}+L_{cln}EI_{bm}\right)}{6L_{bm}L_{cln}EA_{cln}EI_{bm}\left(2L_{bm}EI_{cln}+L_{cln}EI_{bm}\right)}$$

Neglect the denominator.

$$-2M_{J,left}L_{bm}^{2}L_{cln}EA_{cln}\left(2L_{bm}EI_{cln}+L_{cln}EI_{bm}\right)+M_{J,left}L_{bm}^{3}L_{cln}EA_{cln}EI_{cln}$$
$$=6\varphi_{J,left}L_{bm}L_{cln}EA_{cln}EI_{bm}\left(2L_{bm}EI_{cln}+L_{cln}EI_{bm}\right)+12NL_{bm}L_{cln}^{2}EI_{bm}EI_{cln}+12NL_{cln}^{2}EI_{bm}\left(2L_{bm}EI_{cln}+L_{cln}EI_{bm}\right)$$

Combine the same functions.

$$M_{J,left}L_{bm}^{2}L_{cln}EA_{cln}\left(-3L_{bm}EI_{cln}-2L_{cln}EI_{bm}\right)$$
$$=6L_{cln}EI_{bm}\left(\varphi_{J,left}L_{bm}EA_{cln}\left(2L_{bm}EI_{cln}+L_{cln}EI_{bm}\right)+2NL_{cln}\left(3L_{bm}EI_{cln}+L_{cln}EI_{bm}\right)\right)$$

Find the formula of $M_{J,left}$.

$$M_{J,left}=\frac{-6EI_{bm}\left(\varphi_{J,left}L_{bm}EA_{cln}\left(2L_{bm}EI_{cln}+L_{cln}EI_{bm}\right)+2NL_{cln}\left(3L_{bm}EI_{cln}+L_{cln}EI_{bm}\right)\right)}{L_{bm}^{2}EA_{cln}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)}$$

The bending moments which are found are:

$$M_{J,top}=\frac{4EI_{cln}\left(3NL_{cln}^{2}EI_{bm}-\varphi_{J,top}L_{bm}EA_{cln}\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\right)}{L_{bm}L_{cln}EA_{cln}\left(4L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}$$
$$M_{J,bottom}=\frac{2EI_{cln}\left(3NL_{cln}^{2}EI_{bm}-2\varphi_{J,bottom}L_{bm}EA_{cln}\left(6L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\right)}{L_{bm}L_{cln}EA_{cln}\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}$$
$$M_{J,left}=\frac{-6EI_{bm}\left(\varphi_{J,left}L_{bm}EA_{cln}\left(2L_{bm}EI_{cln}+L_{cln}EI_{bm}\right)+2NL_{cln}\left(3L_{bm}EI_{cln}+L_{cln}EI_{bm}\right)\right)}{L_{bm}^{2}EA_{cln}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)}$$

Point J has been split in four parts. All of these parts have their own bending moment. The formulas for three of these moments are found as function of the rotation. The fourth formula can be found using moment equilibrium. The rotation in point J is the same for all parts. The following formulas can be used to find the bending moments.

$$\varphi_{J,right}=\frac{M_{right}L_{bm}}{2EI_{bm}}-\frac{qL_{bm}^{3}}{24EI_{bm}}$$
$$M_{J,right}=M_{J,top}+M_{J,bottom}+M_{J,left}$$
$$\varphi_{J,right}=\varphi_{J,top}=\varphi_{J,bottom}=\varphi_{J,left}=\varphi_{J}$$

The rotation of $\varphi_{J,right}$ is:

$$\varphi_{J,right} = \frac{M_{right} L_{bm}}{2EI_{bm}} - \frac{qL_{bm}{}^3}{24EI_{bm}}$$

Use moments equilibrium.

$$\varphi_{J,right} = \frac{\left(M_{J,top} + M_{J,bottom} + M_{J,left}\right) L_{bm}}{2EI_{bm}} - \frac{qL_{bm}{}^3}{24EI_{bm}}$$

Use the formulas for the different bending moment.

$$\varphi_{J,right} = \frac{\left(\begin{array}{l} \dfrac{4EI_{cln}\left(3NL_{cln}{}^2 EI_{bm} - \varphi_{J,top} L_{bm} EA_{cln}\left(3L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)\right)}{L_{bm}L_{cln}EA_{cln}\left(4L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)} \\ + \dfrac{2EI_{cln}\left(3NL_{cln}{}^2 EI_{bm} - 2\varphi_{J,bottom} L_{bm} EA_{cln}\left(6L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)\right)}{L_{bm}L_{cln}EA_{cln}\left(7L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)} \\ - \dfrac{6EI_{bm}\left(\varphi_{J,left} L_{bm} EA_{cln}\left(2L_{bm}EI_{cln} + L_{cln}EI_{bm}\right) + 2NL_{cln}\left(3L_{bm}EI_{cln} + L_{cln}EI_{bm}\right)\right)}{L_{bm}{}^2 EA_{cln}\left(3L_{bm}EI_{cln} + 2L_{cln}EI_{bm}\right)} \end{array} \right) L_{bm}}{2EI_{bm}} - \frac{qL_{bm}{}^3}{24EI_{bm}}$$

Write out the expressions.

$$\begin{aligned} \varphi_J = &\frac{2EI_{cln}\left(3NL_{cln}{}^2 EI_{bm} - \varphi_J L_{bm} EA_{cln}\left(3L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)\right)}{L_{cln}EA_{cln}EI_{bm}\left(4L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)} \\ &+ \frac{EI_{cln}\left(3NL_{cln}{}^2 EI_{bm} - 2\varphi_{J,bottom} L_{bm} EA_{cln}\left(6L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)\right)}{L_{cln}EA_{cln}EI_{bm}\left(7L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)} \\ &- \frac{3\left(\varphi_J L_{bm} EA_{cln}\left(2L_{bm}EI_{cln} + L_{cln}EI_{bm}\right) + 2NL_{cln}\left(3L_{bm}EI_{cln} + L_{cln}EI_{bm}\right)\right)}{L_{bm}EA_{cln}\left(3L_{bm}EI_{cln} + 2L_{cln}EI_{bm}\right)} - \frac{qL_{bm}{}^3}{24EI_{bm}} \end{aligned}$$

Some expressions depend on both φ_J and on N. These expressions are written out.

$$\begin{aligned} \varphi_J = &\frac{6NL_{cln}{}^2 EI_{bm} EI_{cln}}{L_{cln}EA_{cln}EI_{bm}\left(4L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)} - \frac{2\varphi_J L_{bm} EA_{cln} EI_{cln}\left(3L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)}{L_{cln}EA_{cln}EI_{bm}\left(4L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)} \\ &+ \frac{3NL_{cln}{}^2 EI_{bm} EI_{cln}}{L_{cln}EA_{cln}EI_{bm}\left(7L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)} - \frac{2\varphi_{J,bottom} L_{bm} EA_{cln} EI_{cln}\left(6L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)}{L_{cln}EA_{cln}EI_{bm}\left(7L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)} \\ &- \frac{3\varphi_J L_{bm} EA_{cln}\left(2L_{bm}EI_{cln} + L_{cln}EI_{bm}\right)}{L_{bm}EA_{cln}\left(3L_{bm}EI_{cln} + 2L_{cln}EI_{bm}\right)} - \frac{6NL_{cln}\left(3L_{bm}EI_{cln} + L_{cln}EI_{bm}\right)}{L_{bm}EA_{cln}\left(3L_{bm}EI_{cln} + 2L_{cln}EI_{bm}\right)} - \frac{qL_{bm}{}^3}{24EI_{bm}} \end{aligned}$$

Separate the expressions of φ_J .

$$\varphi_J + \frac{2\varphi_J L_{bm} EI_{cln} \left(6L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)}{L_{cln} EI_{bm} \left(7L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)} + \frac{3\varphi_J L_{bm} EA_{cln} \left(2L_{bm} EI_{cln} + L_{cln} EI_{bm}\right)}{L_{bm} EA_{cln} \left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)}$$

$$+ \frac{2\varphi_J L_{bm} EA_{cln} EI_{cln} \left(3L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)}{L_{cln} EA_{cln} EI_{bm} \left(4L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)} = \frac{6NL_{cln}{}^2 EI_{bm} EI_{cln}}{L_{cln} EA_{cln} EI_{bm} \left(4L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)}$$

$$+ \frac{3NL_{cln}{}^2 EI_{bm} EI_{cln}}{L_{cln} EA_{cln} EI_{bm} \left(7L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)} - \frac{6NL_{cln} \left(3L_{bm} EI_{cln} + L_{cln} EI_{bm}\right)}{L_{bm} EA_{cln} \left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)} - \frac{qL_{bm}{}^3}{24EI_{bm}}$$

Make everywhere the same denominator.

$$\frac{24\varphi_J L_{bm} L_{cln} EA_{cln} EI_{bm} \left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)\left(4L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)\left(7L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)}{24L_{bm} L_{cln} EA_{cln} EI_{bm} \left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)\left(4L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)\left(7L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)}$$

$$+ \frac{48\varphi_J L_{bm}{}^2 EA_{cln} EI_{cln} \left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)\left(4L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)\left(6L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)}{24L_{bm} L_{cln} EA_{cln} EI_{bm} \left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)\left(4L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)\left(7L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)}$$

$$+ \frac{72\varphi_J L_{bm} L_{cln} EA_{cln} EI_{bm} \left(2L_{bm} EI_{cln} + L_{cln} EI_{bm}\right)\left(4L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)\left(7L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)}{24L_{bm} L_{cln} EA_{cln} EI_{bm} \left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)\left(4L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)\left(7L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)}$$

$$+ \frac{48\varphi_J L_{bm}{}^2 EA_{cln} EI_{cln} \left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)\left(3L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)\left(7L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)}{24L_{bm} L_{cln} EA_{cln} EI_{bm} \left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)\left(4L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)\left(7L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)}$$

$$= \frac{144NL_{bm} L_{cln}{}^2 EI_{bm} EI_{cln} \left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)\left(7L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)}{24L_{bm} L_{cln} EA_{cln} EI_{bm} \left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)\left(4L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)\left(7L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)}$$

$$+ \frac{72NL_{bm} L_{cln}{}^2 EI_{bm} EI_{cln} \left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)\left(4L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)}{24L_{bm} L_{cln} EA_{cln} EI_{bm} \left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)\left(4L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)\left(7L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)}$$

$$- \frac{144NL_{cln}{}^2 EI_{bm} \left(3L_{bm} EI_{cln} + L_{cln} EI_{bm}\right)\left(4L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)\left(7L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)}{24L_{bm} L_{cln} EA_{cln} EI_{bm} \left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)\left(4L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)\left(7L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)}$$

$$- \frac{qL_{bm}{}^4 L_{cln} EA_{cln} \left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)\left(4L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)\left(7L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)}{24L_{bm} L_{cln} EA_{cln} EI_{bm} \left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)\left(4L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)\left(7L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)}$$

Neglect the denominator.

$$24\varphi_J L_{bm} L_{cln} EA_{cln} EI_{bm} \left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)\left(4L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)\left(7L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)$$

$$+48\varphi_J L_{bm}{}^2 EA_{cln} EI_{cln} \left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)\left(4L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)\left(6L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)$$

$$+72\varphi_J L_{bm} L_{cln} EA_{cln} EI_{bm} \left(2L_{bm} EI_{cln} + L_{cln} EI_{bm}\right)\left(4L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)\left(7L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)$$

$$+48\varphi_J L_{bm}{}^2 EA_{cln} EI_{cln} \left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)\left(3L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)\left(7L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)$$

$$=144NL_{bm} L_{cln}{}^2 EI_{bm} EI_{cln} \left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)\left(7L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)$$

$$+72NL_{bm} L_{cln}{}^2 EI_{bm} EI_{cln} \left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)\left(4L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)$$

$$-144NL_{cln}{}^2 EI_{bm} \left(3L_{bm} EI_{cln} + L_{cln} EI_{bm}\right)\left(4L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)\left(7L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)$$

$$-qL_{bm}{}^4 L_{cln} EA_{cln} \left(3L_{bm} EI_{cln} + 2L_{cln} EI_{bm}\right)\left(4L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)\left(7L_{bm} EI_{cln} + 5L_{cln} EI_{bm}\right)$$

The normal force N depends on the variable load and on the required loads. It is assumed column FJ and column GK carries half of the variable load. The required loads very small compare with the variable loads. The influence of the rotation is neglected. The required loads are taken into account at the calculation of the stresses. $\left(N=0.5qL_{bm}\right)$

$$
\begin{aligned}
&24\varphi_J L_{bm}L_{cln}EA_{cln}EI_{bm}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)\left(4L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\\
&+48\varphi_J L_{bm}{}^2EA_{cln}EI_{cln}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)\left(4L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\left(6L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\\
&+72\varphi_J L_{bm}L_{cln}EA_{cln}EI_{bm}\left(2L_{bm}EI_{cln}+L_{cln}EI_{bm}\right)\left(4L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\\
&+48\varphi_J L_{bm}{}^2EA_{cln}EI_{cln}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\\
&=72qL_{bm}{}^2L_{cln}{}^2EI_{bm}EI_{cln}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\\
&+36qL_{bm}{}^2L_{cln}{}^2EI_{bm}EI_{cln}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)\left(4L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\\
&-72qL_{bm}L_{cln}{}^2EI_{bm}\left(3L_{bm}EI_{cln}+L_{cln}EI_{bm}\right)\left(4L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\\
&-qL_{bm}{}^4L_{cln}EA_{cln}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)\left(4L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)
\end{aligned}
$$

Combine the same expressions.

$$
24\varphi_J L_{bm}EA_{cln}\begin{pmatrix}
L_{cln}EI_{bm}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)\left(4L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\\
+2L_{bm}EI_{cln}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)\left(4L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\left(6L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\\
+3L_{cln}EI_{bm}\left(2L_{bm}EI_{cln}+L_{cln}EI_{bm}\right)\left(4L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\\
+2L_{bm}EI_{cln}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)
\end{pmatrix}
$$

$$
=qL_{bm}L_{cln}\begin{pmatrix}
72L_{bm}L_{cln}EI_{bm}EI_{cln}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\\
+36L_{bm}L_{cln}EI_{bm}EI_{cln}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)\left(4L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\\
-72L_{cln}EI_{bm}\left(3L_{bm}EI_{cln}+L_{cln}EI_{bm}\right)\left(4L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\\
-L_{bm}{}^3EA_{cln}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)\left(4L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)
\end{pmatrix}
$$

The formula of φ_J can be found.

$$
\varphi_J=\frac{qL_{bm}L_{cln}\begin{pmatrix}
72L_{bm}L_{cln}EI_{bm}EI_{cln}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\\
+36L_{bm}L_{cln}EI_{bm}EI_{cln}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)\left(4L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\\
-72L_{cln}EI_{bm}\left(3L_{bm}EI_{cln}+L_{cln}EI_{bm}\right)\left(4L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\\
-L_{bm}{}^3EA_{cln}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)\left(4L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)
\end{pmatrix}}{24EA_{cln}\begin{pmatrix}
L_{cln}EI_{bm}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)\left(4L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\\
+2L_{bm}EI_{cln}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)\left(4L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\left(6L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\\
+3L_{cln}EI_{bm}\left(2L_{bm}EI_{cln}+L_{cln}EI_{bm}\right)\left(4L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\\
+2L_{bm}EI_{cln}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)
\end{pmatrix}}
$$

The formula what is found is not clear. The expressions must be written out to find a clearer formula. To write out the formula, the formula is split is parts. First the numerator has been split. The last (fourth) part of the numerator is not written out.

The first part:

$$72L_{bm}L_{cln}EI_{bm}EI_{cln}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)$$

$$=\left(216L_{bm}{}^{2}L_{cln}EI_{bm}\left(EI_{cln}\right)^{2}+144L_{bm}L_{cln}{}^{2}\left(EI_{bm}\right)^{2}EI_{cln}\right)\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)$$

$$=1512L_{bm}{}^{3}L_{cln}EI_{bm}\left(EI_{cln}\right)^{3}+1008L_{bm}{}^{2}L_{cln}{}^{2}\left(EI_{bm}\right)^{2}\left(EI_{cln}\right)^{2}$$
$$+1080L_{bm}{}^{2}L_{cln}{}^{2}\left(EI_{bm}\right)^{2}\left(EI_{cln}\right)^{2}+720L_{bm}L_{cln}{}^{3}\left(EI_{bm}\right)^{3}EI_{cln}$$

$$=720L_{bm}L_{cln}{}^{3}\left(EI_{bm}\right)^{3}EI_{cln}+2088L_{bm}{}^{2}L_{cln}{}^{2}\left(EI_{bm}\right)^{2}\left(EI_{cln}\right)^{2}+1512L_{bm}{}^{3}L_{cln}EI_{bm}\left(EI_{cln}\right)^{3}$$

The second part:

$$36L_{bm}L_{cln}EI_{bm}EI_{cln}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)\left(4L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)$$

$$=\left(108L_{bm}{}^{2}L_{cln}EI_{bm}\left(EI_{cln}\right)^{2}+72L_{bm}L_{cln}{}^{2}\left(EI_{bm}\right)^{2}EI_{cln}\right)\left(4L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)$$

$$=432L_{bm}{}^{3}L_{cln}EI_{bm}\left(EI_{cln}\right)^{3}+288L_{bm}{}^{2}L_{cln}{}^{2}\left(EI_{bm}\right)^{2}\left(EI_{cln}\right)^{2}$$
$$+540L_{bm}{}^{2}L_{cln}{}^{2}\left(EI_{bm}\right)^{2}\left(EI_{cln}\right)^{2}+360L_{bm}L_{cln}{}^{3}\left(EI_{bm}\right)^{3}EI_{cln}$$

$$=360L_{bm}L_{cln}{}^{3}\left(EI_{bm}\right)^{3}EI_{cln}+828L_{bm}{}^{2}L_{cln}{}^{2}\left(EI_{bm}\right)^{2}\left(EI_{cln}\right)^{2}+432L_{bm}{}^{3}L_{cln}EI_{bm}\left(EI_{cln}\right)^{3}$$

The third part:

$$-72L_{cln}EI_{bm}\left(3L_{bm}EI_{cln}+L_{cln}EI_{bm}\right)\left(4L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)$$

$$=\left(-216L_{bm}L_{cln}EI_{bm}EI_{cln}-72L_{cln}{}^{2}\left(EI_{bm}\right)^{2}\right)\left(4L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)$$

$$=\left(-864L_{bm}{}^{2}L_{cln}EI_{bm}\left(EI_{cln}\right)^{2}-288L_{bm}L_{cln}{}^{2}\left(EI_{bm}\right)^{2}EI_{cln}\right)\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)$$
$$+\left(-1080L_{bm}L_{cln}{}^{2}\left(EI_{bm}\right)^{2}EI_{cln}-360L_{cln}{}^{3}\left(EI_{bm}\right)^{3}\right)\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)$$

$$=-6048L_{bm}{}^{3}L_{cln}EI_{bm}\left(EI_{cln}\right)^{3}-2016L_{bm}{}^{2}L_{cln}{}^{2}\left(EI_{bm}\right)^{2}\left(EI_{cln}\right)^{2}$$
$$-4320L_{bm}{}^{2}L_{cln}{}^{2}\left(EI_{bm}\right)^{2}\left(EI_{cln}\right)^{2}-1440L_{bm}L_{cln}{}^{3}\left(EI_{bm}\right)^{3}EI_{cln}$$
$$-7560L_{bm}{}^{2}L_{cln}{}^{2}\left(EI_{bm}\right)^{2}\left(EI_{cln}\right)^{2}-2520L_{bm}L_{cln}{}^{3}\left(EI_{bm}\right)^{3}EI_{cln}$$
$$-5400L_{bm}L_{cln}{}^{3}\left(EI_{bm}\right)^{3}EI_{cln}-1800L_{cln}{}^{4}\left(EI_{bm}\right)^{4}$$

$$=-1800L_{cln}{}^{4}\left(EI_{bm}\right)^{4}-9360L_{bm}L_{cln}{}^{3}\left(EI_{bm}\right)^{3}EI_{cln}-13896L_{bm}{}^{2}L_{cln}{}^{2}\left(EI_{bm}\right)^{2}\left(EI_{cln}\right)^{2}-6048L_{bm}{}^{3}L_{cln}EI_{bm}\left(EI_{cln}\right)^{3}$$

The expressions in the denominator have been split in four parts.
The first part:

$$L_{cln}EI_{bm}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)\left(4L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)$$

$$=\left(3L_{bm}L_{cln}EI_{bm}EI_{cln}+2L_{cln}{}^{2}\left(EI_{bm}\right)^{2}\right)\left(4L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)$$

$$=\left(12L_{bm}{}^{2}L_{cln}EI_{bm}\left(EI_{cln}\right)^{2}+8L_{bm}L_{cln}{}^{2}\left(EI_{bm}\right)^{2}EI_{cln}\right)\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)$$
$$+\left(15L_{bm}L_{cln}{}^{2}\left(EI_{bm}\right)^{2}EI_{cln}+10L_{cln}{}^{3}\left(EI_{bm}\right)^{3}\right)\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)$$

$$=84L_{bm}{}^{3}L_{cln}EI_{bm}\left(EI_{cln}\right)^{3}+56L_{bm}{}^{2}L_{cln}{}^{2}\left(EI_{bm}\right)^{2}\left(EI_{cln}\right)^{2}+60L_{bm}{}^{2}L_{cln}{}^{2}\left(EI_{bm}\right)^{2}\left(EI_{cln}\right)^{2}$$
$$+40L_{bm}L_{cln}{}^{3}\left(EI_{bm}\right)^{3}EI_{cln}+105L_{bm}{}^{2}L_{cln}{}^{2}\left(EI_{bm}\right)^{2}\left(EI_{cln}\right)^{2}+70L_{bm}L_{cln}{}^{3}\left(EI_{bm}\right)^{3}EI_{cln}$$
$$+75L_{bm}L_{cln}{}^{3}\left(EI_{bm}\right)^{3}EI_{cln}+50L_{cln}{}^{4}\left(EI_{bm}\right)^{4}$$

$$=50L_{cln}{}^{4}\left(EI_{bm}\right)^{4}+185L_{bm}L_{cln}{}^{3}\left(EI_{bm}\right)^{3}EI_{cln}+221L_{bm}{}^{2}L_{cln}{}^{2}\left(EI_{bm}\right)^{2}\left(EI_{cln}\right)^{2}+84L_{bm}{}^{3}L_{cln}EI_{bm}\left(EI_{cln}\right)^{3}$$

The second part:

$$2L_{bm}EI_{cln}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)\left(4L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\left(6L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)$$

$$=\left(6L_{bm}{}^{2}\left(EI_{cln}\right)^{2}+4L_{bm}L_{cln}EI_{bm}EI_{cln}\right)\left(4L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\left(6L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)$$

$$=\left(24L_{bm}{}^{3}\left(EI_{cln}\right)^{3}+16L_{bm}{}^{2}L_{cln}EI_{bm}\left(EI_{cln}\right)^{2}\right)\left(6L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)$$
$$+\left(30L_{bm}{}^{2}L_{cln}EI_{bm}\left(EI_{cln}\right)^{2}+20L_{bm}L_{cln}{}^{2}\left(EI_{bm}\right)^{2}EI_{cln}\right)\left(6L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)$$

$$=144L_{bm}{}^{4}\left(EI_{cln}\right)^{4}+96L_{bm}{}^{3}L_{cln}EI_{bm}\left(EI_{cln}\right)^{3}+120L_{bm}{}^{3}L_{cln}EI_{bm}\left(EI_{cln}\right)^{3}$$
$$+80L_{bm}{}^{2}L_{cln}{}^{2}\left(EI_{bm}\right)^{2}\left(EI_{cln}\right)^{2}+180L_{bm}{}^{3}L_{cln}EI_{bm}\left(EI_{cln}\right)^{3}+120L_{bm}{}^{2}L_{cln}{}^{2}\left(EI_{bm}\right)^{2}\left(EI_{cln}\right)^{2}$$
$$+150L_{bm}{}^{2}L_{cln}{}^{2}\left(EI_{bm}\right)^{2}\left(EI_{cln}\right)^{2}+100L_{bm}L_{cln}{}^{3}\left(EI_{bm}\right)^{3}EI_{cln}$$

$$=100L_{bm}L_{cln}{}^{3}\left(EI_{bm}\right)^{3}EI_{cln}+350L_{bm}{}^{2}L_{cln}{}^{2}\left(EI_{bm}\right)^{2}\left(EI_{cln}\right)^{2}+396L_{bm}{}^{3}L_{cln}EI_{bm}\left(EI_{cln}\right)^{3}+144L_{bm}{}^{4}\left(EI_{cln}\right)^{4}$$

The third part:

$$3L_{cln}EI_{bm}\left(2L_{bm}EI_{cln}+L_{cln}EI_{bm}\right)\left(4L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)$$

$$=\left(6L_{bm}L_{cln}EI_{bm}EI_{cln}+3L_{cln}{}^{2}\left(EI_{bm}\right)^{2}\right)\left(4L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)$$

$$=\left(24L_{bm}{}^{2}L_{cln}EI_{bm}\left(EI_{cln}\right)^{2}+12L_{bm}L_{cln}{}^{2}\left(EI_{bm}\right)^{2}EI_{cln}\right)\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)$$
$$+\left(30L_{bm}L_{cln}{}^{2}\left(EI_{bm}\right)^{2}EI_{cln}+15L_{cln}{}^{3}\left(EI_{bm}\right)^{3}\right)\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)$$

$$=168L_{bm}{}^{3}L_{cln}EI_{bm}\left(EI_{cln}\right)^{3}+84L_{bm}{}^{2}L_{cln}{}^{2}\left(EI_{bm}\right)^{2}\left(EI_{cln}\right)^{2}+120L_{bm}{}^{2}L_{cln}{}^{2}\left(EI_{bm}\right)^{2}\left(EI_{cln}\right)^{2}$$
$$+60L_{bm}L_{cln}{}^{3}\left(EI_{bm}\right)^{3}EI_{cln}+210L_{bm}{}^{2}L_{cln}{}^{2}\left(EI_{bm}\right)^{2}\left(EI_{cln}\right)^{2}+105L_{bm}L_{cln}{}^{3}\left(EI_{bm}\right)^{3}EI_{cln}$$
$$+150L_{bm}L_{cln}{}^{3}\left(EI_{bm}\right)^{3}EI_{cln}+75L_{cln}{}^{4}\left(EI_{bm}\right)^{4}$$

$$=75L_{cln}{}^{4}\left(EI_{bm}\right)^{4}+315L_{bm}L_{cln}{}^{3}\left(EI_{bm}\right)^{3}EI_{cln}+414L_{bm}{}^{2}L_{cln}{}^{2}\left(EI_{bm}\right)^{2}\left(EI_{cln}\right)^{2}+168L_{bm}{}^{3}L_{cln}EI_{bm}\left(EI_{cln}\right)^{3}$$

The fourth part:

$$2L_{bm}EI_{cln}\left(3L_{bm}EI_{cln}+2L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)$$

$$=\left(6L_{bm}{}^{2}\left(EI_{cln}\right)^{2}+4L_{bm}L_{cln}EI_{bm}EI_{cln}\right)\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)$$

$$=\left(18L_{bm}{}^{3}\left(EI_{cln}\right)^{3}+12L_{bm}{}^{2}L_{cln}EI_{bm}\left(EI_{cln}\right)^{2}\right)\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)$$
$$+\left(30L_{bm}{}^{2}L_{cln}EI_{bm}\left(EI_{cln}\right)^{2}+20L_{bm}L_{cln}{}^{2}\left(EI_{bm}\right)^{2}EI_{cln}\right)\left(7L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)$$

$$=126L_{bm}{}^{4}\left(EI_{cln}\right)^{4}+84L_{bm}{}^{3}L_{cln}EI_{bm}\left(EI_{cln}\right)^{3}+90L_{bm}{}^{3}L_{cln}EI_{bm}\left(EI_{cln}\right)^{3}$$
$$+60L_{bm}{}^{2}L_{cln}{}^{2}\left(EI_{bm}\right)^{2}\left(EI_{cln}\right)^{2}+210L_{bm}{}^{3}L_{cln}EI_{bm}\left(EI_{cln}\right)^{3}+140L_{bm}{}^{2}L_{cln}{}^{2}\left(EI_{bm}\right)^{2}\left(EI_{cln}\right)^{2}$$
$$+150L_{bm}{}^{2}L_{cln}{}^{2}\left(EI_{bm}\right)^{2}\left(EI_{cln}\right)^{2}+100L_{bm}L_{cln}{}^{3}\left(EI_{bm}\right)^{3}EI_{cln}$$

$$=100L_{bm}L_{cln}{}^{3}\left(EI_{bm}\right)^{3}EI_{cln}+350L_{bm}{}^{2}L_{cln}{}^{2}\left(EI_{bm}\right)^{2}\left(EI_{cln}\right)^{2}+384L_{bm}{}^{3}L_{cln}EI_{bm}\left(EI_{cln}\right)^{3}+126L_{bm}{}^{4}\left(EI_{cln}\right)^{4}$$

All these parts are used in the formula.

$$\varphi_J = \frac{qL_{bm}L_{cln}\begin{pmatrix} 720L_{bm}L_{cln}^{3}(EI_{bm})^{3}EI_{cln} + 2088L_{bm}^{2}L_{cln}^{2}(EI_{bm})^{2}(EI_{cln})^{2} + 1512L_{bm}^{3}L_{cln}EI_{bm}(EI_{cln})^{3} \\ +360L_{bm}L_{cln}^{3}(EI_{bm})^{3}EI_{cln} + 828L_{bm}^{2}L_{cln}^{2}(EI_{bm})^{2}(EI_{cln})^{2} + 432L_{bm}^{3}L_{cln}EI_{bm}(EI_{cln})^{3} \\ -1800L_{cln}^{4}(EI_{bm})^{4} - 9360L_{bm}L_{cln}^{3}(EI_{bm})^{3}EI_{cln} - 13896L_{bm}^{2}L_{cln}^{2}(EI_{bm})^{2}(EI_{cln})^{2} \\ -6048L_{bm}^{3}L_{cln}EI_{bm}(EI_{cln})^{3} \\ -L_{bm}^{3}EA_{cln}(3L_{bm}EI_{cln} + 2L_{cln}EI_{bm})(4L_{bm}EI_{cln} + 5L_{cln}EI_{bm})(7L_{bm}EI_{cln} + 5L_{cln}EI_{bm}) \end{pmatrix}}{24EA_{cln}\begin{pmatrix} 50L_{cln}^{4}(EI_{bm})^{4} + 185L_{bm}L_{cln}^{3}(EI_{bm})^{3}EI_{cln} + 221L_{bm}^{2}L_{cln}^{2}(EI_{bm})^{2}(EI_{cln})^{2} \\ +84L_{bm}^{3}L_{cln}EI_{bm}(EI_{cln})^{3} + 100L_{bm}L_{cln}^{3}(EI_{bm})^{3}EI_{cln} + 350L_{bm}^{2}L_{cln}^{2}(EI_{bm})^{2}(EI_{cln})^{2} \\ +396L_{bm}^{3}L_{cln}EI_{bm}(EI_{cln})^{3} + 144L_{bm}^{4}(EI_{cln})^{4} + 75L_{cln}^{4}(EI_{bm})^{4} + 60L_{bm}L_{cln}^{3}(EI_{bm})^{3}EI_{cln} \\ +414L_{bm}^{2}L_{cln}^{2}(EI_{bm})^{2}(EI_{cln})^{2} + 423L_{bm}^{3}L_{cln}EI_{bm}(EI_{cln})^{3} + 100L_{bm}L_{cln}^{3}(EI_{bm})^{3}EI_{cln} \\ +350L_{bm}^{2}L_{cln}^{2}(EI_{bm})^{2}(EI_{cln})^{2} + 384L_{bm}^{3}L_{cln}EI_{bm}(EI_{cln})^{3} + 126L_{bm}^{4}(EI_{cln})^{4} \end{pmatrix}}$$

Combine the same expressions

$$\varphi_J = \frac{qL_{bm}L_{cln}\begin{pmatrix} -1800L_{cln}^{4}(EI_{bm})^{4} - 8280L_{bm}L_{cln}^{3}(EI_{bm})^{3}EI_{cln} - 10980L_{bm}^{2}L_{cln}^{2}(EI_{bm})^{2}(EI_{cln})^{2} \\ -4104L_{bm}^{3}L_{cln}EI_{bm}(EI_{cln})^{3} \\ -L_{bm}^{3}EA_{cln}(3L_{bm}EI_{cln} + 2L_{cln}EI_{bm})(4L_{bm}EI_{cln} + 5L_{cln}EI_{bm})(7L_{bm}EI_{cln} + 5L_{cln}EI_{bm}) \end{pmatrix}}{24EA_{cln}\begin{pmatrix} 125L_{cln}^{4}(EI_{bm})^{4} + 445L_{bm}L_{cln}^{3}(EI_{bm})^{3}EI_{cln} + 1335L_{bm}^{2}L_{cln}^{2}(EI_{bm})^{2}(EI_{cln})^{2} \\ +1287L_{bm}^{3}L_{cln}EI_{bm}(EI_{cln})^{3} + 270L_{bm}^{4}(EI_{cln})^{4} \end{pmatrix}}$$

The bending moments can be calculated by the numerical value of φ_J. The bending moments are:

$$M_{J,bottom} = \frac{2EI_{cln}\left(3NL_{cln}^{2}EI_{bm} - 2\varphi_{J,bottom}L_{bm}EA_{cln}(6L_{bm}EI_{cln} + 5L_{cln}EI_{bm})\right)}{L_{bm}L_{cln}EA_{cln}(7L_{bm}EI_{cln} + 5L_{cln}EI_{bm})}$$

$$M_{F,top} = \frac{3NL_{cln}EI_{bm}}{L_{bm}^{2}EA_{cln}} + \left(\frac{3L_{bm}EI_{cln} + 5L_{cln}EI_{bm}}{L_{bm}L_{cln}}\right)\varphi_F$$

$$\varphi_F = \frac{M_{J,bottom}L_{bm}^{2}L_{cln}EA_{cln} - 6NL_{cln}^{2}EI_{bm}}{2L_{bm}EA_{cln}(6L_{bm}EI_{cln} + 5L_{cln}EI_{bm})}$$

The deflection of column FJ can be calculated by the following formula:

$$e_1 = \frac{M_{J,bottom}L_{cln}^{2}}{16EI_{cln}} - \frac{M_{I,top}L_{cln}^{2}}{16EI_{cln}}$$

$$e_1 = \frac{(M_{J,bottom} - M_{I,top})L_{cln}^{2}}{16EI_{cln}}$$

Appendix S Non-linear analyses extended frame

This Appendix is about the non-linear analysis of an extended frame. The structure can be found in Figure S.1. The linear analysis of this frame can be found in Appendix R. This non-linear analysis is based on the equilibrium between the internal and the external bending moments of column FJ. Column FJ is taken as critical element of the structure. Column GK is loaded equally to column FJ (because of the symmetry of the frame). It is assumed that only these columns fail. All other construction elements are free of yielding.

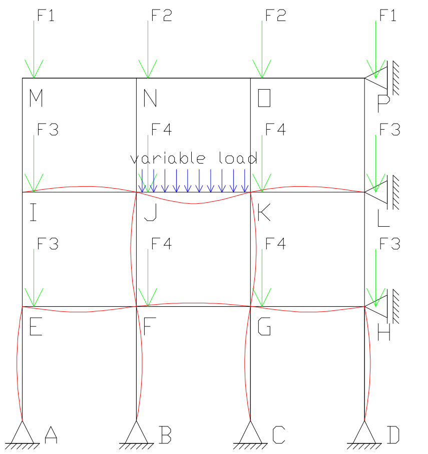


Figure S.1: Structure

If column FJ deflects, the rest of the frame resists. This resist of the deflection has been schematized as a rotational spring at both ends of the column. The non-linear analysis starts to calculate the spring stiffness in point F and J. For the calculation of the spring stiffness only the bending of the direct connected construction elements are taken into account. It is possible that rotation spring will be utilized by other construction elements. A safe assumption is to use just one quarter of the spring stiffness as the effective spring stiffness. The reduction of the spring stiffness become later on in the analysis.

Figure S.2 is the load-deflection graphic of the analyzed structure.

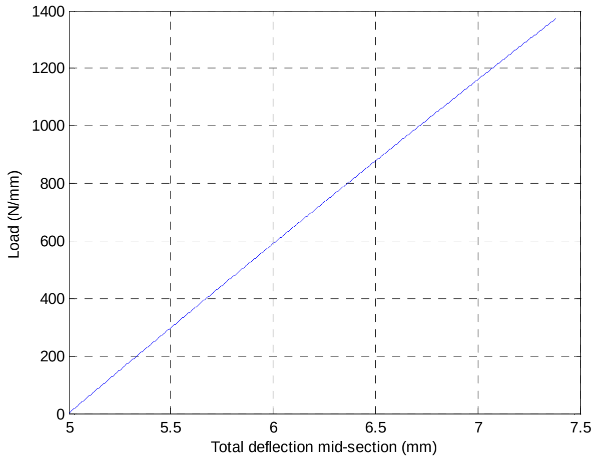


Figure S.2: Load-deflection Graphic

Column: Length 5 m
HE 360A
Beam: Length 5m
HE 900A

If column FJ deflects, also the column JN and the beams IJ and JK will deflect. To calculate these deflections, a bending moment is introduced.

$$\varphi_{J,right} = \frac{M_{J,right} L_{bm}}{2EI_{bm}} \rightarrow M_{J,right} = \frac{2\varphi_{J,right} EI_{bm}}{L_{bm}}$$

$$\varphi_{J,top} = \frac{M_{J,top} L_{cln}}{3EI_{cln}} \rightarrow M_{J,top} = \frac{3\varphi_{J,top} EI_{cln}}{L_{cln}}$$

$$\varphi_{J,left} = \frac{M_{J,left} L_{bm}}{3EI_{bm}} \rightarrow M_{J,left} = \frac{3\varphi_{J,left} EI_{bm}}{L_{bm}}$$

The rotation of point J is the same in all directions.

$$\varphi_{J,right} = \varphi_{J,top} = \varphi_{J,left} = \varphi_J$$

All bending moments in point J are in equilibrium. The bending moment $M_{J,bottom}$ is the sum of the other moments in point J.

$$M_{J,bottom} = M_{J,right} + M_{J,top} + M_{J,left}$$

The formulas of the bending moment can be used.

$$M_{J,bottom} = \frac{2\varphi_{J,right} EI_{bm}}{L_{bm}} + \frac{3\varphi_{J,top} EI_{cln}}{L_{cln}} + \frac{3\varphi_{J,left} EI_{bm}}{L_{bm}}$$

Separate φ_J .

$$M_{J,bottom} = \left(\frac{2EI_{bm}}{L_{bm}} + \frac{3EI_{cln}}{L_{cln}} + \frac{3EI_{bm}}{L_{bm}} \right) \varphi_J$$

Combine the expressions.

$$M_{J,bottom} = \left(\frac{3L_{bm} EI_{cln} + 5L_{cln} EI_{bm}}{L_{bm} L_{cln}} \right) \varphi_J$$

Find the rotational stiffness.

$$k_{r,J} = \frac{3L_{bm} EI_{cln} + 5L_{cln} EI_{bm}}{L_{bm} L_{cln}}$$

Only one quarter of the spring stiffness is taken into account.

$$k_{r,J} = \frac{3L_{bm} EI_{cln} + 5L_{cln} EI_{bm}}{4L_{bm} L_{cln}}$$

Only the connected elements are taken into account at the calculation of the spring stiffness. The spring stiffness of point F is the same as the spring stiffness of column J.

$$k_{r,F} = \frac{3L_{bm} EI_{cln} + 5L_{cln} EI_{bm}}{4L_{bm} L_{cln}}$$

As same as the non-linear analysis of the braced portal frame (App. N), the non-linear analysis of the extended frame is based on the equilibrium between internal en external moments. The deflection of the columns has been split in four parts. These parts are:

1. Starting deflection
2. Deflection due to the linear analysis
3. Additional deflection
4. Deflection due to the rotational springs

The starting deflection

$$y_1 = e_0 \sin\left(\frac{\pi x}{L_{cln}} \right)$$

$$\varphi_1 = e_0 \frac{\pi}{L_{cln}} \cos\left(\frac{\pi x}{L_{cln}} \right)$$

$$\kappa_1 = -e_0 \frac{\pi^2}{L_{cln}{}^2} \sin\left(\frac{\pi x}{L_{cln}} \right)$$

Deflection due to the linear analysis

The deflection due to the linear analysis dependa on the linear bending moments at both column ends. The following formula is the bending in the column.

$$M_2 = \frac{\left(M_{J,bottom} + M_{F,top}\right)\left(L_{cln} - x\right)}{L_{cln}} - M_{F,top}$$

The formula can be simplified at the following formula.

$$M_2 = \frac{M_{J,bottom}\left(L_{cln} - x\right) - M_{F,top}x}{L_{cln}}$$

$$\kappa_2 = -\frac{M_{J,bottom}\left(L_{cln} - x\right) - M_{F,top}x}{L_{cln}EI_{cln}}$$

$$\kappa_2 = \frac{M_{J,bottom}\left(-L_{cln} + x\right) + M_{F,top}x}{L_{cln}EI_{cln}}$$

$$\varphi_2 = \frac{M_{J,bottom}\left(-L_{cln}x + \tfrac{1}{2}x^2\right) + M_{F,top}\tfrac{1}{2}x^2}{L_{cln}EI_{cln}} + C_1$$

$$y_2 = \frac{M_{J,bottom}\left(-\tfrac{1}{2}L_{cln}x^2 + \tfrac{1}{6}x^3\right) + M_{F,top}\tfrac{1}{6}x^3}{L_{cln}EI_{cln}} + C_1x + C_2$$

Use the boundary conditions to find the integral constants.

$y_{2,x=0} = 0$ → $C_2 = 0$

$y_{2,x=L_{cln}} = 0$ → $C_1 = \frac{\tfrac{1}{3}M_{J,bottom}L_{cln} - \tfrac{1}{6}M_{F,top}L_{cln}}{EI_{cln}}$

Use these integral constants in the formula.

$$y_2 = \frac{M_{J,bottom}\left(-\tfrac{1}{2}L_{cln}x^2 + \tfrac{1}{6}x^3\right) + M_{F,top}\tfrac{1}{6}x^3}{L_{cln}EI_{cln}} + \frac{\tfrac{1}{3}M_{J,bottom}L_{cln} - \tfrac{1}{6}M_{F,top}L_{cln}}{EI_{cln}}x$$

Combine the same expressions.

$$y_2 = \frac{M_{J,bottom}\left(\tfrac{1}{3}L_{cln}{}^2x - \tfrac{1}{2}L_{cln}x^2 + \tfrac{1}{6}x^3\right) + M_{F,top}\left(-\tfrac{1}{6}L_{cln}{}^2x + \tfrac{1}{6}x^3\right)}{L_{cln}EI_{cln}}$$

Neglect the fractions.

$$y_2 = \frac{M_{J,bottom}\left(2L_{cln}{}^2x - 3L_{cln}x^2 + x^3\right) + M_{F,top}\left(-L_{cln}{}^2x + x^3\right)}{6L_{cln}EI_{cln}}$$

$$\varphi_2 = \frac{M_{J,bottom}\left(2L_{cln}{}^2 - 6L_{cln}x + 3x^2\right) + M_{F,top}\left(-L_{cln}{}^2 + 3x^2\right)}{6L_{cln}EI_{cln}}$$

$$\kappa_2 = \frac{M_{J,bottom}\left(-6L_{cln} + 6x\right) + M_{F,top}\left(6x\right)}{6L_{cln}EI_{cln}}$$

Additional deflection

$$y_3 = e_1 \sin\left(\frac{\pi x}{L_{cln}}\right)$$

$$\varphi_3 = e_1 \frac{\pi}{L_{cln}} \cos\left(\frac{\pi x}{L_{cln}}\right)$$

$$\kappa_3 = -e_1 \frac{\pi^2}{L_{cln}{}^2} \sin\left(\frac{\pi x}{L_{cln}}\right)$$

Deflection due to the rotational spring

The stiffness of both rotational springs is equal. The bending moment due to the rotational springs is constant.

$$M_4 = -\varphi_{extra,x=0} k_r$$

$$\kappa_4 = \frac{\varphi_{extra,x=0} k_r}{EI_{cln}}$$

$$\varphi_4 = \frac{\varphi_{extra,x=0} k_r x}{EI_{cln}} + C_1$$

$$y_4 = \frac{\varphi_{extra,x=0} k_r x^2}{2EI_{cln}} + C_1 x + C_2$$

Use the boundary conditions to find the integral constants.

$y_{4,x=0} = 0$ → $C_2 = 0$

$y_{4,x=L_{cln}} = 0$ → $C_1 = -\frac{\varphi_{extra,x=0} k_r L_{cln}}{2EI_{cln}}$

Use these constants in the formula.

$$y_4 = \frac{\varphi_{extra,x=0} k_r x^2}{2EI_{cln}} + -\frac{\varphi_{extra,x=0} k_r L_{cln}}{2EI_{cln}} x$$

Combine the same expressions.

$$y_4 = \frac{\varphi_{extra,x=0} k_r \left(-L_{cln} x + x^2\right)}{2EI_{cln}}$$

$$\varphi_4 = \frac{\varphi_{extra,x=0} k_r \left(-L_{cln} + 2x\right)}{2EI_{cln}}$$

$$\kappa_4 = \frac{\varphi_{extra,x=0} k_r}{EI_{cln}}$$

The additional rotation depends on φ_3 and on φ_4. The formulas of φ_3 and φ_4 can be used.

$$\varphi_{extra} = e_1 \frac{\pi}{L_{cln}} \cos\left(\frac{\pi x}{L_{cln}}\right) + \frac{\varphi_{extra,x=0} k_r \left(-L_{cln} + 2x\right)}{2EI_{cln}}$$

The additional rotation at the x=0 is taken.

$$\varphi_{extra,x=0} = e_1 \frac{\pi}{L_{cln}} - \frac{\varphi_{extra,x=0} L_{cln} k_r}{2EI_{cln}}$$

The expressions of $\varphi_{extra,x=0}$ is separated from the expression on e_1.

$$\varphi_{extra,x=0} + \frac{\varphi_{extra,x=0} L_{cln} k_r}{2EI_{cln}} = e_1 \frac{\pi}{L_{cln}}$$

Use the spring stiffness $\left(k_r = \frac{3L_{bm}EI_{cln} + 5L_{cln}EI_{bm}}{4L_{bm}L_{cln}} \right)$ in this formula.

$$\varphi_{extra,x=0} + \frac{\varphi_{extra,x=0} L_{cln} \left(\frac{3L_{bm}EI_{cln} + 5L_{cln}EI_{bm}}{4L_{bm}L_{cln}} \right)}{2EI_{cln}} = e_1 \frac{\pi}{L_{cln}}$$

Write out the expression.

$$\varphi_{extra,x=0} + \frac{\varphi_{extra,x=0} \left(3L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)}{8L_{bm}EI_{cln}} = e_1 \frac{\pi}{L_{cln}}$$

Same denominator

$$\varphi_{extra,x=0} \left(\frac{L_{cln}\left(8L_{bm}EI_{cln} + \left(3L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)\right)}{8L_{bm}L_{cln}EI_{cln}} \right) = \frac{8\pi L_{bm} e_1 EI_{cln}}{8L_{bm}L_{cln}EI_{cln}}$$

Find the formula of $\varphi_{extra,x=0}$:

$$\varphi_{extra,x=0} = \frac{8\pi L_{bm} e_1 EI_{cln}}{L_{cln}\left(11L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)}$$

The additional rotation must be used in the formula of y_4.

$$y_4 = \frac{\varphi_{extra,x=0} k_r \left(-L_{cln}x + x^2\right)}{2EI_{cln}}$$

The additional rotation and the spring stiffness can be filled in.

$$y_4 = \left(\frac{8\pi L_{bm} e_1 EI_{cln}}{L_{cln}\left(11L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)} \right) \left(\frac{3L_{bm}EI_{cln} + 5L_{cln}EI_{bm}}{4L_{bm}L_{cln}} \right) \frac{\left(-L_{cln}x + x^2\right)}{2EI_{cln}}$$

This formula can be simplified.

$$y_4 = \frac{\pi e_1 \left(3L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)\left(-L_{cln}x + x^2\right)}{L_{cln}^{\ 2}\left(11L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)}$$

$$\varphi_4 = \frac{\pi e_1 \left(3L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)\left(-L_{cln} + 2x\right)}{L_{cln}^{\ 2}\left(11L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)}$$

$$\kappa_4 = \frac{2\pi e_1 \left(3L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)}{L_{cln}{}^2\left(11L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)}$$

With the formulas above the differential equation can be solved. The differential equation is the equilibrium between the internal and the external moments.

$$M_{intern} = M_{extern}$$

$$-\kappa EI_{cln} = Ny_{total}$$

$$-\left(\kappa_3 + \kappa_4\right)EI_{cln} = N\left(y_1 + y_2 + y_3 + y_4\right) + \frac{M_{J,bottom}x}{L_{cln}} - \frac{k_r\varphi_{extra,x=0}x}{L_{cln}} + \frac{M_{F,top}\left(L_{cln} - x\right)}{L_{cln}} - \frac{k_r\varphi_{extra,x=0}\left(L_{cln} - x\right)}{L_{cln}}$$

Combine the same expressions

$$-\left(\kappa_3 + \kappa_4\right)EI_{cln} = N\left(y_1 + y_2 + y_3 + y_4\right) + \frac{M_{J,bottom}x}{L_{cln}} - k_r\varphi_{extra,x=0}L_{cln} + \frac{M_{F,top}\left(L_{cln} - x\right)}{L_{cln}}$$

Use the known formulas to solve this equation.

$$-\left(-e_1\frac{\pi^2}{L_{cln}{}^2}\sin\left(\frac{\pi x}{L_{cln}}\right) + \frac{2\pi e_1\left(3L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)}{L_{cln}{}^2\left(11L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)}\right)EI_{cln}$$

$$= N\left(\begin{array}{l} e_0\sin\left(\frac{\pi x}{L_{cln}}\right) + \frac{M_{J,bottom}\left(2L_{cln}{}^2x - 3L_{cln}x^2 + x^3\right) + M_{F,top}\left(-L_{cln}{}^2x + x^3\right)}{6L_{cln}EI_{cln}} + e_1\sin\left(\frac{\pi x}{L_{cln}}\right) \\ + \frac{\pi e_1\left(3L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)\left(-L_{cln}x + x^2\right)}{L_{cln}{}^2\left(11L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)} \end{array}\right)$$

$$+ \frac{M_{J,bottom}x}{L_{cln}} - \left(\frac{3L_{bm}EI_{cln} + 5L_{cln}EI_{bm}}{4L_{bm}L_{cln}}\right)\left(\frac{8\pi L_{bm}e_1EI_{cln}}{L_{cln}\left(11L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)}\right) + \frac{M_{F,top}\left(L_{cln} - x\right)}{L_{cln}}$$

Write out the expressions to neglect the brackets.

$$e_1\frac{\pi^2 EI_{cln}}{L_{cln}{}^2}\sin\left(\frac{\pi x}{L_{cln}}\right) - \frac{2\pi e_1 EI_{cln}\left(3L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)}{L_{cln}{}^2\left(11L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)}$$

$$= Ne_0\sin\left(\frac{\pi x}{L_{cln}}\right) + \frac{NM_{J,bottom}\left(2L_{cln}{}^2x - 3L_{cln}x^2 + x^3\right) + NM_{F,top}\left(-L_{cln}{}^2x + x^3\right)}{6L_{cln}EI_{cln}} + Ne_1\sin\left(\frac{\pi x}{L_{cln}}\right)$$

$$+ \frac{\pi Ne_1\left(3L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)\left(-L_{cln}x + x^2\right)}{L_{cln}{}^2\left(11L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)}$$

$$+ \frac{M_{J,bottom}x}{L_{cln}} - \frac{2\pi e_1 EI_{cln}\left(3L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)}{L_{cln}{}^2\left(11L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)} + \frac{M_{F,top}\left(L_{cln} - x\right)}{L_{cln}}$$

The load N exists in two parts. One part is the variable load and the second part is the required loads. The required loads are small compare with the variable loads and are neglected. $\left(N = 0.5qL_{bm}\right)$. The load can be used in the formula.

$$e_1 \frac{\pi^2 EI_{cln}}{L_{cln}^2} \sin\left(\frac{\pi x}{L_{cln}}\right) - \frac{2\pi e_1 EI_{cln}\left(3L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)}{L_{cln}^2\left(11L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)}$$

$$= \frac{qL_{bm}e_0 \sin\left(\frac{\pi x}{L_{cln}}\right)}{2} + \frac{qL_{bm}M_{J,bottom}\left(2L_{cln}^2 x - 3L_{cln}x^2 + x^3\right) + qL_{bm}M_{F,top}\left(-L_{cln}^2 x + x^3\right)}{12L_{cln}EI_{cln}} + \frac{qL_{bm}e_1 \sin\left(\frac{\pi x}{L_{cln}}\right)}{2}$$

$$+ \frac{\pi q L_{bm} e_1\left(3L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)\left(-L_{cln}x + x^2\right)}{2L_{cln}^2\left(11L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)}$$

$$+ \frac{M_{J,bottom}x}{L_{cln}} - \frac{2\pi e_1 EI_{cln}\left(3L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)}{L_{cln}^2\left(11L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)} + \frac{M_{F,top}\left(L_{cln} - x\right)}{L_{cln}}$$

The value e_1 must be calculated in the middle of the column. This value must be calculated. $\left(x = 0.5L_{cln}\right)$

$$e_1 \frac{\pi^2 EI_{cln}}{L_{cln}^2} - \frac{2\pi e_1 EI_{cln}\left(3L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)}{L_{cln}^2\left(11L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)}$$

$$= \frac{qL_{bm}e_0}{2} + \frac{qL_{bm}L_{cln}^2 M_{J,bottom}}{48EI_{cln}} - \frac{qL_{bm}L_{cln}^2 M_{F,top}}{48EI_{cln}} + \frac{qL_{bm}e_1}{2} - \frac{\pi q L_{bm} e_1\left(3L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)}{8\left(11L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)}$$

$$+ \frac{M_{J,bottom}}{2} - \frac{2\pi e_1 EI_{cln}\left(3L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)}{L_{cln}^2\left(11L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)} + \frac{M_{F,top}}{2}$$

One expression can be found at both sides of the equation. This expression can be neglected.

$$e_1 \frac{\pi^2 EI_{cln}}{L_{cln}^2} = \frac{qL_{bm}e_0}{2} + \frac{qL_{bm}L_{cln}^2 M_{J,bottom}}{48EI_{cln}} - \frac{qL_{bm}L_{cln}^2 M_{F,top}}{48EI_{cln}} + \frac{qL_{bm}e_1}{2}$$

$$- \frac{\pi q L_{bm} e_1\left(3L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)}{8\left(11L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)} + \frac{M_{J,bottom}}{2} + \frac{M_{F,top}}{2}$$

The only unknown value is the value of e_1. To calculate this value, all expressions of e_1 must be separate from the rest of the formula. All expressions of e_1 are placed on one side of the equation.

$$e_1 \frac{\pi^2 EI_{cln}}{L_{cln}^2} - \frac{qL_{bm}e_1}{2} + \frac{\pi q L_{bm} e_1\left(3L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)}{8\left(11L_{bm}EI_{cln} + 5L_{cln}EI_{bm}\right)}$$

$$= \frac{qL_{bm}e_0}{2} + \frac{qL_{bm}L_{cln}^2 M_{J,bottom}}{48EI_{cln}} - \frac{qL_{bm}L_{cln}^2 M_{F,top}}{48EI_{cln}} + \frac{M_{J,bottom}}{2} + \frac{M_{F,top}}{2}$$

To calculate the value e_1 it is necessary to have one denominator.

$$e_1\frac{48\pi^2\left(EI_{cln}\right)^2\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}{48L_{cln}{}^2EI_{cln}\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}-\frac{24qL_{bm}L_{cln}{}^2e_1EI_{cln}\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}{48L_{cln}{}^2EI_{cln}\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}$$

$$+\frac{6\pi qL_{bm}L_{cln}{}^2e_1EI_{cln}\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}{48L_{cln}{}^2EI_{cln}\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}=\frac{24qL_{bm}L_{cln}{}^2e_0EI_{cln}\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}{48L_{cln}{}^2EI_{cln}\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}$$

$$+\frac{qL_{bm}L_{cln}{}^4M_{J,bottom}\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}{48L_{cln}{}^2EI_{cln}\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}-\frac{qL_{bm}L_{cln}{}^4M_{F,top}\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}{48L_{cln}{}^2EI_{cln}\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}$$

$$+\frac{24M_{J,bottom}L_{cln}{}^2EI_{cln}\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}{48L_{cln}{}^2EI_{cln}\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}+\frac{24M_{F,top}L_{cln}{}^2EI_{cln}\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}{48L_{cln}{}^2EI_{cln}\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}$$

Neglect the denominator and separate e_1.

$$e_1\begin{pmatrix}48\pi^2\left(EI_{cln}\right)^2\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)-24qL_{bm}L_{cln}{}^2EI_{cln}\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\\+6\pi qL_{bm}L_{cln}{}^2EI_{cln}\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\end{pmatrix}$$

$$=24qL_{bm}L_{cln}{}^2e_0EI_{cln}\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)+qL_{bm}L_{cln}{}^4M_{J,bottom}\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)$$

$$-qL_{bm}L_{cln}{}^4M_{F,top}\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)$$

$$+24M_{J,bottom}L_{cln}{}^2EI_{cln}\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)+24M_{F,top}L_{cln}{}^2EI_{cln}\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)$$

Find the formula of e_1.

$$e_1=\frac{\begin{pmatrix}24qL_{bm}L_{cln}{}^2e_0EI_{cln}\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)+qL_{bm}L_{cln}{}^4M_{J,bottom}\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\\-qL_{bm}L_{cln}{}^4M_{F,top}\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)+24M_{J,bottom}L_{cln}{}^2EI_{cln}\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\\+24M_{F,top}L_{cln}{}^2EI_{cln}\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\end{pmatrix}}{\begin{pmatrix}48\pi^2\left(EI_{cln}\right)^2\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)-24qL_{bm}L_{cln}{}^2EI_{cln}\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\\+6\pi qL_{bm}L_{cln}{}^2EI_{cln}\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\end{pmatrix}}$$

Combine the same expressions.

$$e_1=\frac{\begin{matrix}qL_{bm}L_{cln}{}^2\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\left(24e_0EI_{cln}+L_{cln}{}^2\left(M_{J,bottom}-M_{F,top}\right)\right)\\+24\left(M_{J,bottom}+M_{F,top}\right)L_{cln}{}^2EI_{cln}\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\end{matrix}}{6EI_{cln}\begin{pmatrix}8\pi^2EI_{cln}\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\\+qL_{bm}L_{cln}{}^2\left(\left(3\pi-44\right)L_{bm}EI_{cln}+\left(5\pi-20\right)L_{cln}EI_{bm}\right)\end{pmatrix}}$$

The formula of e_1 is found. e_1 is the additional defection due to the second order analysis.
The formula of e_1 is checked on the dimensions:

$$m=\frac{Nm^{-1}mm^2\left(mNm^2+mNm^2\right)\left(mNm^2+m^2\left(Nm-Nm\right)\right)+\left(Nm+Nm\right)m^2Nm^2\left(mNm^2+mNm^2\right)}{Nm^2\left(Nm^2\left(mNm^2+mNm^2\right)+Nm^{-1}mm^2\left(mNm^2+mNm^2\right)\right)}$$

$$m=\frac{Nm^2\left(Nm^3\right)\left(Nm^3\right)+\left(Nm\right)Nm^4\left(Nm^3\right)}{Nm^2\left(Nm^2\left(Nm^3\right)+Nm^2\left(Nm^3\right)\right)}$$

$$m=\frac{N^3m^8}{N^3m^7}$$

The dimensions of the formula are correct

The formula of the additional deflection e_1 is found. The additional deflection e_1 can be used to calculate the total deflection, the change in bending moments and the internal stress distribution. The formulas are:

$$y_1=e_0$$

$$y_2=\frac{M_{J,bottom}{L_{cln}}^2-M_{F,top}{L_{cln}}^2}{16EI_{cln}}$$

$$y_3=e_1$$

$$y_4=\frac{-\pi e_1\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}{4\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}$$

$$e_1=\frac{\begin{array}{l}qL_{bm}{L_{cln}}^2\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\left(24e_0EI_{cln}+{L_{cln}}^2\left(M_{J,bottom}-M_{F,top}\right)\right)\\+24\left(M_{J,bottom}+M_{F,top}\right){L_{cln}}^2EI_{cln}\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\end{array}}{6EI_{cln}\left(\begin{array}{l}8\pi^2EI_{cln}\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)\\+qL_{bm}{L_{cln}}^2\left(\left(3\pi-44\right)L_{bm}EI_{cln}+\left(5\pi-20\right)L_{cln}EI_{bm}\right)\end{array}\right)}$$

$$\varphi_{extra,x=0}=\frac{8\pi L_{bm}e_1EI_{cln}}{L_{cln}\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}$$

$$k_r=\frac{3L_{bm}EI_{cln}+5L_{cln}EI_{bm}}{4L_{bm}L_{cln}}$$

Appendix T Residual stress in the non-linear analysis

This Appendix is about the influence of residual stress on the non-linear analysis of an extended frame (Fig. T.1). The residual stress distribution is the same as in the analysis of the column (App. D.1) and of the analysis of the portal frames (App. K and App. O).

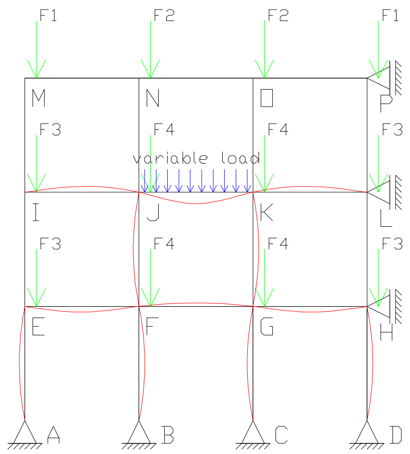


Figure T.1: Structure

The analyses in this Appendix are an extension of the analysis in Appendix S. Appendix S is about the non-linear analysis. The analysis in Appendix S is the first load case. The second and the third load case will be discussed in this Appendix. The second load case starts if the stress in one of the flanges (right one) has reached the yield stress. The third load case starts if both flanges partial yield.

Due to partial yielding, the effective section properties decrease. This results in extra deflections. The total load-deflection is graphic displayed in Fig. T.2.

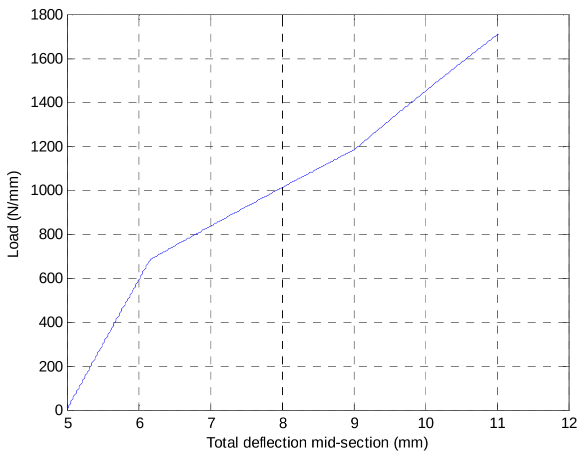


Figure T.2: Load-deflection Graphic

Column: Length 5 m, HE 360A
Beam: Length 5m, HE 900A

It is assumed that only column FJ and column GK fail. The effective section properties of these columns decrease. All other construction elements are free of yield stresses. The section properties of these elements do not decrease. The spring stiffness at the ends of the columns (determined in App. S) remains constant. This spring stiffness is.

$$k_r = \frac{3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}}{4L_{bm}L_{cln}}$$

T.1 Analysis if one part yields

The total deflection has been split in five parts. These parts are:

1. Starting deflection
2. Deflection due to the linear analysis
3. Additional deflection
4. Deflection due to the rotational springs
5. Deflection due to the shift of the centre of gravity

The starting deflection is the initial deflection due to the imperfections. This deflection does not change.

$$y_1 = e_0 \sin\left(\frac{\pi x}{L_{cln}}\right)$$

$$\varphi_1 = e_0 \frac{\pi}{L_{cln}} \cos\left(\frac{\pi x}{L_{cln}}\right)$$

$$\kappa_1 = -e_0 \frac{\pi^2}{L_{cln}{}^2} \sin\left(\frac{\pi x}{L_{cln}}\right)$$

The deflection due to the linear analysis depends on the linear bending moments at both ends of the column. The difference between this deflection and the deflection in Appendix S is the stiffness of the column.

$$M_{2,2} = \frac{\left(M_{J,bottom,2} + M_{F,top,2}\right)\left(L_{cln} - x\right)}{L_{cln}} - M_{F,top,2}$$

$$M_{2,2} = \frac{M_{J,bottom,2}\left(L_{cln} - x\right) - M_{F,top,2}x}{L_{cln}}$$

$$\kappa_{2,2} = -\frac{M_{J,bottom,2}\left(L_{cln} - x\right) - M_{F,top,2}x}{L_{cln}EI_{cln,2}}$$

$$\kappa_{2,2} = \frac{M_{J,bottom,2}\left(-L_{cln} + x\right) + M_{F,top,2}x}{L_{cln}EI_{cln,2}}$$

$$\varphi_{2,2} = \frac{M_{J,bottom,2}\left(-L_{cln}x + \tfrac{1}{2}x^2\right) + M_{F,top,2}\tfrac{1}{2}x^2}{L_{cln}EI_{cln,2}} + C_1$$

$$y_{2,2} = \frac{M_{J,bottom,2}\left(-\tfrac{1}{2}L_{cln}x^2 + \tfrac{1}{6}x^3\right) + M_{F,top,2}\tfrac{1}{6}x^3}{L_{cln}EI_{cln,2}} + C_1x + C_2$$

Use the boundary conditions to find the integral constants.

$y_{2,2,x=0} = 0$ → $C_2 = 0$

$y_{2,2,x=L_{cln}} = 0$ → $C_1 = \dfrac{\tfrac{1}{3}M_{J,bottom,2}L_{cln} - \tfrac{1}{6}M_{F,top,2}L_{cln}}{EI_{cln,2}}$

Use these integral constants in the formula.

$$y_{2,2} = \frac{M_{J,bottom,2}\left(-\tfrac{1}{2}L_{cln}x^2 + \tfrac{1}{6}x^3\right) + M_{F,top,2}\tfrac{1}{6}x^3}{L_{cln}EI_{cln,2}} + \frac{\tfrac{1}{3}M_{J,bottom,2}L_{cln} - \tfrac{1}{6}M_{F,top,2}L_{cln}}{EI_{cln,2}}x$$

Combine the same expressions.

$$y_{2,2} = \frac{M_{J,bottom,2}\left(\tfrac{1}{3}L_{cln}{}^2x - \tfrac{1}{2}L_{cln}x^2 + \tfrac{1}{6}x^3\right) + M_{F,top,2}\left(-\tfrac{1}{6}L_{cln}{}^2x + \tfrac{1}{6}x^3\right)}{L_{cln}EI_{cln,2}}$$

Make the formula more clear.

$$y_{2,2} = \frac{M_{J,bottom,2}\left(2L_{cln}{}^2x - 3L_{cln}x^2 + x^3\right) + M_{F,top,2}\left(-L_{cln}{}^2x + x^3\right)}{6L_{cln}EI_{cln,2}}$$

$$\varphi_{2,2} = \frac{M_{J,bottom,2}\left(2L_{cln}{}^2 - 6L_{cln}x + 3x^2\right) + M_{F,top,2}\left(-L_{cln}{}^2 + 3x^2\right)}{6L_{cln}EI_{cln,2}}$$

$$\kappa_{2,2} = \frac{M_{J,bottom,2}\left(-6L_{cln} + 6x\right) + M_{F,top,2}\left(6x\right)}{6L_{cln}EI_{cln,2}}$$

The additional deflection

$$y_{3,2} = e_2 \sin\left(\frac{\pi x}{L_{cln}}\right)$$

$$\varphi_{3,2} = e_2 \frac{\pi}{L_{cln}} \cos\left(\frac{\pi x}{L_{cln}}\right)$$

$$\kappa_{3,2} = -e_2 \frac{\pi^2}{L_{cln}{}^2} \sin\left(\frac{\pi x}{L_{cln}}\right)$$

The rotational spring stiffness remains constant. The stiffness of the analyzed column decreased. This has results on the deflection due to the rotational spring

$$M_{4,2} = -\varphi_{extra,2,x=0} k_r$$

$$\kappa_{4,2} = \frac{\varphi_{extra,2,x=0} k_r}{EI_{cln,2}}$$

$$\varphi_{4,2} = \frac{\varphi_{extra,2,x=0} k_r x}{EI_{cln,2}} + C_1$$

$$y_{4,2} = \frac{\varphi_{extra,2,x=0} k_r x^2}{2EI_{cln,2}} + C_1 x + C_2$$

Use the boundary conditions to find the integral constants.

$y_{4,2,x=0} = 0$ → $C_2 = 0$

$y_{4,2,,x=L_{cln}} = 0$ → $C_1 = -\frac{\varphi_{extra,2,x=0} k_r L_{cln}}{2EI_{cln,2}}$

Use these constants in the formula.

$$y_{4,2} = \frac{\varphi_{extra,2,x=0} k_r x^2}{2EI_{cln,2}} + -\frac{\varphi_{extra,2,x=0} k_r L_{cln}}{2EI_{cln,2}} x$$

Combine the same expressions.

$$y_{4,2} = \frac{\varphi_{extra,2,x=0} k_r \left(-L_{cln} x + x^2\right)}{2EI_{cln,2}}$$

$$\varphi_{4,2} = \frac{\varphi_{extra,2,x=0} k_r \left(-L_{cln} + 2x\right)}{2EI_{cln,2}}$$

$$\kappa_{4,2} = \frac{\varphi_{extra,2,x=0} k_r}{EI_{cln,2}}$$

The deflection due to the shift of the centre of gravity

$$M_{5,2} = N_2 z_2$$

$$\kappa_{5,2} = \frac{-N_2 z_2}{EI_{cln,2}}$$

$$\varphi_{5,2} = \frac{-N_2 z_2 x}{EI_{cln,2}} + C_1$$

$$y_{5,2} = \frac{-N_2 z_2 x^2}{2EI_{cln,2}} + C_1 x + C_2$$

Use the boundary conditions to find the integral constants.

$y_{5,2,x=0} = 0$ → $C_2 = 0$

$y_{5,2,,x=L_{cln}} = 0$ → $C_1 = \frac{N_2 z_2 L_{cln}}{2EI_{cln,2}}$

Use these constants in the formula.

$$y_{5,2} = \frac{-N_2 z_2 x^2}{2EI_{cln,2}} + \frac{N_2 z_2 L_{cln}}{2EI_{cln,2}} x$$

Combine the same expressions.

$$y_{5,2} = \frac{N_2 z_2 \left(L_{cln} x - x^2\right)}{2EI_{cln,2}}$$

$$\varphi_{5,2} = \frac{N_2 z_2 \left(L_{cln} - 2x\right)}{2EI_{cln,2}}$$

$$\kappa_{5,2} = \frac{-N_2 z_2}{EI_{cln,2}}$$

The additional rotation depends on φ_3, φ_4 and φ_5.

$$\varphi_{extra,2} = e_2 \frac{\pi}{L_{cln}} \cos\left(\frac{\pi x}{L_{cln}}\right) + \frac{\varphi_{extra,2,x=0} k_r \left(-L_{cln} + 2x\right)}{2EI_{cln,2}} + \frac{N_2 z_2 \left(L_{cln} - 2x\right)}{2EI_{cln,2}}$$

Calculate the additional rotation at x=0.

$$\varphi_{extra,2,x=0} = e_2 \frac{\pi}{L_{cln}} - \frac{\varphi_{extra,2,x=0} k_r L_{cln}}{2EI_{cln,2}} + \frac{N_2 L_{cln} z_2}{2EI_{cln,2}}$$

Put all the expressions of $\varphi_{extra,2,x=0}$ to one side of the equation.

$$\varphi_{extra,2,x=0} + \frac{\varphi_{extra,2,x=0} k_r L_{cln}}{2EI_{cln,2}} = e_2 \frac{\pi}{L_{cln}} + \frac{N_2 L_{cln} z_2}{2EI_{cln,2}}$$

Combine the expressions of $\varphi_{extra,2,x=0}$.

$$\varphi_{extra,2,x=0} \left(\frac{L_{cln}\left(2EI_{cln,2} + k_r L_{cln}\right)}{2L_{cln} EI_{cln,2}}\right) = \frac{2\pi e_2 EI_{cln,2} + N_2 L_{cln}{}^2 z_2}{2L_{cln} EI_{cln,2}}$$

Use the spring stiffness $k_r = \frac{3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}}{4L_{bm}L_{cln}}$

$$\varphi_{extra,2,x=0}\left(\frac{L_{cln}\left(2EI_{cln,2} + \left(\frac{3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}}{4L_{bm}L_{cln}}\right)L_{cln}\right)}{2L_{cln}EI_{cln,2}}\right) = \frac{2\pi e_2 EI_{cln,2} + N_2 L_{cln}{}^2 z_2}{2L_{cln}EI_{cln,2}}$$

Write out one expression.

$$\varphi_{extra,2,x=0}\left(\frac{L_{cln}\left(8L_{bm}EI_{cln,2} + 3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)}{8L_{bm}L_{cln}EI_{cln,2}}\right) = \frac{2\pi e_2 EI_{cln,2} + N_2 L_{cln}{}^2 z_2}{2L_{cln}EI_{cln,2}}$$

Everywhere the same denominator.

$$\varphi_{extra,2,x=0}\left(\frac{L_{cln}\left(8L_{bm}EI_{cln,2} + 3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)}{8L_{bm}L_{cln}EI_{cln,2}}\right) = \frac{4L_{bm}\left(2\pi e_2 EI_{cln,2} + N_2 L_{cln}{}^2 z_2\right)}{8L_{bm}L_{cln}EI_{cln,2}}$$

Find the formula of $\varphi_{extra,2,x=0}$ as a function of e_2.

$$\varphi_{extra,2,x=0} = \frac{4L_{bm}\left(2\pi e_2 EI_{cln,2} + N_2 L_{cln}{}^2 z_2\right)}{L_{cln}\left(8L_{bm}EI_{cln,2} + 3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)}$$

The additional rotation can be used in the formula of $y_{4,2}$.

$$y_{4,2} = \frac{\varphi_{extra,2,x=0}k_r\left(-L_{cln}x + x^2\right)}{2EI_{cln,2}}$$

The additional rotation and the spring stiffness can be filled in.

$$y_{4,2} = \left(\frac{4L_{bm}\left(2\pi e_2 EI_{cln,2} + N_2 L_{cln}{}^2 z_2\right)}{L_{cln}\left(8L_{bm}EI_{cln,2} + 3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)}\right)\left(\frac{3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}}{4L_{bm}L_{cln}}\right)\frac{\left(-L_{cln}x + x^2\right)}{2EI_{cln,2}}$$

Neglect and combine some parameters.

$$y_{4,2} = \frac{\left(2\pi e_2 EI_{cln,2} + N_2 L_{cln}{}^2 z_2\right)\left(3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)\left(-L_{cln}x + x^2\right)}{2L_{cln}{}^2 EI_{cln,2}\left(8L_{bm}EI_{cln,2} + 3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)}$$

$$\varphi_{4,2} = \frac{\left(2\pi e_2 EI_{cln,2} + N_2 L_{cln}{}^2 z_2\right)\left(3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)\left(-L_{cln} + 2x\right)}{2L_{cln}{}^2 EI_{cln,2}\left(8L_{bm}EI_{cln,2} + 3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)}$$

$$\kappa_{4,2} = \frac{\left(2\pi e_2 EI_{cln,2} + N_2 L_{cln}{}^2 z_2\right)\left(3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)}{L_{cln}{}^2 EI_{cln,2}\left(8L_{bm}EI_{cln,2} + 3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)}$$

With the formulas above the equilibrium between the internal and the external bending moments can be analyzed.

$$\Delta M_{intern} = \Delta M_{extern}$$

$$-\Delta\kappa EI_{cln,2} = N_1 y_2 + N_2 y_{total,2} + N_2 z_2 + \frac{M_{J,bottom,2}x}{L_{cln}} - \frac{k_r \varphi_{extra,2,x=0} x}{L_{cln}} + \frac{M_{F,top,2}\left(L_{cln} - x\right)}{L_{cln}} - \frac{k_r \varphi_{extra,2,x=0}\left(L_{cln} - x\right)}{L_{cln}}$$

$$-\left(\kappa_{3,2} + \kappa_{4,2} + \kappa_{5,2}\right) EI_{cln,2}$$
$$= N_1\left(y_{2,2} + y_{3,2} + y_{4,2} + y_{5,2}\right) + N_2\left(y_1 + y_{2,1} + y_{2,2} + y_{3,1} + y_{3,2} + y_{4,1} + y_{4,2} + y_{5,2} + z_2\right)$$
$$+\frac{M_{J,bottom,2}x}{L_{cln}} - \frac{k_r \varphi_{extra,2,x=0} x}{L_{cln}} + \frac{M_{F,top,2}\left(L_{cln} - x\right)}{L_{cln}} - \frac{k_r \varphi_{extra,2,x=0}\left(L_{cln} - x\right)}{L_{cln}}$$

Combine the same expressions.

$$-\left(\kappa_{3,2} + \kappa_{4,2} + \kappa_{5,2}\right) EI_{cln,2}$$
$$= N_1\left(y_{2,2} + y_{3,2} + y_{4,2} + y_{5,2}\right) + N_2\left(y_1 + y_{2,1} + y_{2,2} + y_{3,1} + y_{3,2} + y_{4,1} + y_{4,2} + y_{5,2} + z_2\right)$$
$$+\frac{M_{J,bottom,2}x}{L_{cln}} - k_r \varphi_{extra,2,x=0} + \frac{M_{F,top,2}\left(L_{cln} - x\right)}{L_{cln}}$$

Use the known formulas to solve this equation.

$$-\left(-\frac{e_2\pi^2}{L_{cln}^{\ 2}}\sin\left(\frac{\pi x}{L_{cln}}\right) + \frac{\left(2\pi e_2 EI_{cln,2} + N_2 L_{cln}^{\ 2} z_2\right)\left(3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)}{L_{cln}^{\ 2}EI_{cln,2}\left(8L_{bm}EI_{cln,2} + 3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)} + \frac{-N_2 z_2}{EI_{cln,2}}\right) EI_{cln,2}$$

$$= N_1\left(\begin{aligned}&\frac{M_{J,bottom,2}\left(2L_{cln}^{\ 2}x - 3L_{cln}x^2 + x^3\right) + M_{F,top,2}\left(-L_{cln}^{\ 2}x + x^3\right)}{6L_{cln}EI_{cln,2}} + e_2\sin\left(\frac{\pi x}{L_{cln}}\right)\\ &+\frac{\left(2\pi e_2 EI_{cln,2} + N_2 L_{cln}^{\ 2} z_2\right)\left(3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)\left(-L_{cln}x + x^2\right)}{2L_{cln}^{\ 2}EI_{cln,2}\left(8L_{bm}EI_{cln,2} + 3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)} + \frac{N_2 z_2\left(L_{cln}x - x^2\right)}{2EI_{cln,2}}\end{aligned}\right)$$

$$+N_2\left(\begin{aligned}&e_0\sin\left(\frac{\pi x}{L_{cln}}\right) + \frac{M_{J,bottom,1}\left(2L_{cln}^{\ 2}x - 3L_{cln}x^2 + x^3\right) + M_{F,top,1}\left(-L_{cln}^{\ 2}x + x^3\right)}{6L_{cln}EI_{cln,1}}\\ &+\frac{M_{J,bottom,2}\left(2L_{cln}^{\ 2}x - 3L_{cln}x^2 + x^3\right) + M_{F,top,2}\left(-L_{cln}^{\ 2}x + x^3\right)}{6L_{cln}EI_{cln,2}} + e_1\sin\left(\frac{\pi x}{L_{cln}}\right) + e_2\sin\left(\frac{\pi x}{L_{cln}}\right)\\ &+\frac{\pi e_1\left(3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)\left(-L_{cln}x + x^2\right)}{L_{cln}^{\ 2}\left(11L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)}\\ &+\frac{\left(2\pi e_2 EI_{cln,2} + N_2 L_{cln}^{\ 2} z_2\right)\left(3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)\left(-L_{cln}x + x^2\right)}{2L_{cln}^{\ 2}EI_{cln,2}\left(8L_{bm}EI_{cln,2} + 3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)} + \frac{N_2 z_2\left(L_{cln}x - x^2\right)}{2EI_{cln,2}} + z_2\end{aligned}\right)$$

$$+\frac{M_{J,bottom,2}x}{L_{cln}} - \left(\frac{3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}}{4L_{bm}L_{cln}}\right)\left(\frac{4L_{bm}\left(2\pi e_2 EI_{cln,2} + N_2 L_{cln}^{\ 2} z_2\right)}{L_{cln}\left(8L_{bm}EI_{cln,2} + 3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)}\right) + \frac{M_{F,top,2}\left(L_{cln} - x\right)}{L_{cln}}$$

Write out the formula to neglect the brackets.

$$\frac{e_2\pi^2 EI_{cln,2}}{L_{cln}{}^2}\sin\left(\frac{\pi x}{L_{cln}}\right)-\frac{2\pi e_2 EI_{cln,2}\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{L_{cln}{}^2\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}-\frac{N_2 L_{cln}{}^2 z_2\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{L_{cln}{}^2\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$+N_2 z_2=\frac{N_1 M_{J,bottom,2}\left(2L_{cln}{}^2 x-3L_{cln}x^2+x^3\right)}{6L_{cln}EI_{cln,2}}+\frac{N_1 M_{F,top,2}\left(-L_{cln}{}^2 x+x^3\right)}{6L_{cln}EI_{cln,2}}+N_1 e_2\sin\left(\frac{\pi x}{L_{cln}}\right)$$

$$+\frac{\pi N_1 e_2 EI_{cln,2}\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(-L_{cln}x+x^2\right)}{L_{cln}{}^2 EI_{cln,2}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}+\frac{N_1 N_2 L_{cln}{}^2 z_2\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(-L_{cln}x+x^2\right)}{2L_{cln}{}^2 EI_{cln,2}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$+\frac{N_1 N_2 z_2\left(L_{cln}x-x^2\right)}{2EI_{cln,2}}+N_2 e_0\sin\left(\frac{\pi x}{L_{cln}}\right)+\frac{N_2 M_{J,bottom,1}\left(2L_{cln}{}^2 x-3L_{cln}x^2+x^3\right)}{6L_{cln}EI_{cln,1}}+\frac{N_2 M_{F,top,1}\left(-L_{cln}{}^2 x+x^3\right)}{6L_{cln}EI_{cln,1}}$$

$$+\frac{N_2 M_{J,bottom,2}\left(2L_{cln}{}^2 x-3L_{cln}x^2+x^3\right)}{6L_{cln}EI_{cln,2}}+\frac{N_2 M_{F,top,2}\left(-L_{cln}{}^2 x+x^3\right)}{6L_{cln}EI_{cln,2}}+N_2 e_1\sin\left(\frac{\pi x}{L_{cln}}\right)+N_2 e_2\sin\left(\frac{\pi x}{L_{cln}}\right)$$

$$+\frac{N_2\pi e_1\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(-L_{cln}x+x^2\right)}{L_{cln}{}^2\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}+\frac{\pi N_2 e_2 EI_{cln,2}\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(-L_{cln}x+x^2\right)}{L_{cln}{}^2 EI_{cln,2}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$+\frac{N_2{}^2 L_{cln}{}^2 z_2\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(-L_{cln}x+x^2\right)}{2L_{cln}{}^2 EI_{cln,2}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}+\frac{N_2{}^2 z_2\left(L_{cln}x-x^2\right)}{2EI_{cln,2}}+N_2 z_2+\frac{M_{J,bottom,2}x}{L_{cln}}$$

$$-\frac{2\pi e_2 EI_{cln,2}\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{L_{cln}{}^2\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}-\frac{N_2 L_{cln}{}^2 z_2\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{L_{cln}{}^2\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}+\frac{M_{F,top,2}\left(L_{cln}-x\right)}{L_{cln}}$$

Use the formulas of the normal force in this formula $\left(N_1=0.5q_1L_{bm};N_2=0.5q_2L_{bm}\right)$. Neglect the influence of the required forces.

$$\frac{e_2\pi^2 EI_{cln,2}}{L_{cln}^{\ 2}}\sin\left(\frac{\pi x}{L_{cln}}\right)-\frac{2\pi e_2 EI_{cln,2}\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{L_{cln}^{\ 2}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$-\frac{q_2 L_{bm}L_{cln}^{\ 2}z_2\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{2L_{cln}^{\ 2}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}=\frac{q_1 M_{J,bottom,2}L_{bm}\left(2L_{cln}^{\ 2}x-3L_{cln}x^2+x^3\right)}{12L_{cln}EI_{cln,2}}$$

$$+\frac{q_1 M_{F,top,2}L_{bm}\left(-L_{cln}^{\ 2}x+x^3\right)}{12L_{cln}EI_{cln,2}}+\frac{q_1 L_{bm}e_2\sin\left(\frac{\pi x}{L_{cln}}\right)}{2}+\frac{\pi q_1 L_{bm}e_2 EI_{cln,2}\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(-L_{cln}x+x^2\right)}{2L_{cln}^{\ 2}EI_{cln,2}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$+\frac{q_1 q_2 L_{bm}^{\ 2}L_{cln}^{\ 2}z_2\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(-L_{cln}x+x^2\right)}{8L_{cln}^{\ 2}EI_{cln,2}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}+\frac{q_1 q_2 L_{bm}^{\ 2}z_2\left(L_{cln}x-x^2\right)}{8EI_{cln,2}}+\frac{q_2 L_{bm}e_0\sin\left(\frac{\pi x}{L_{cln}}\right)}{2}$$

$$+\frac{q_2 M_{J,bottom,1}L_{bm}\left(2L_{cln}^{\ 2}x-3L_{cln}x^2+x^3\right)}{12L_{cln}EI_{cln,1}}+\frac{q_2 M_{F,top,1}L_{bm}\left(-L_{cln}^{\ 2}x+x^3\right)}{12L_{cln}EI_{cln,1}}$$

$$+\frac{q_2 M_{J,bottom,2}L_{bm}\left(2L_{cln}^{\ 2}x-3L_{cln}x^2+x^3\right)}{12L_{cln}EI_{cln,2}}+\frac{q_2 M_{F,top,2}L_{bm}\left(-L_{cln}^{\ 2}x+x^3\right)}{12L_{cln}EI_{cln,2}}+\frac{q_2 L_{bm}e_1\sin\left(\frac{\pi x}{L_{cln}}\right)}{2}$$

$$+\frac{q_2 L_{bm}e_2\sin\left(\frac{\pi x}{L_{cln}}\right)}{2}+\frac{q_2\pi L_{bm}e_1\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(-L_{cln}x+x^2\right)}{2L_{cln}^{\ 2}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$+\frac{\pi q_2 L_{bm}e_2 EI_{cln,2}\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(-L_{cln}x+x^2\right)}{2L_{cln}^{\ 2}EI_{cln,2}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}+\frac{q_2^{\ 2}L_{bm}^{\ 2}L_{cln}^{\ 2}z_2\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(-L_{cln}x+x^2\right)}{8L_{cln}^{\ 2}EI_{cln,2}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$+\frac{q_2^{\ 2}L_{bm}^{\ 2}z_2\left(L_{cln}x-x^2\right)}{8EI_{cln,2}}+\frac{M_{J,bottom,2}x}{L_{cln}}-\frac{2\pi e_2 EI_{cln,2}\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{L_{cln}^{\ 2}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$-\frac{q_2 L_{bm}L_{cln}^{\ 2}z_2\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{2L_{cln}^{\ 2}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}+\frac{M_{F,top,2}\left(L_{cln}-x\right)}{L_{cln}}$$

Calculate the deflection in the middle of the column $\left(x=0.5L_{cln}\right)$.

$$\frac{e_2\pi^2 EI_{cln,2}}{L_{cln}{}^2}-\frac{2\pi e_2 EI_{cln,2}\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{L_{cln}{}^2\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}-\frac{q_2L_{bm}L_{cln}{}^2z_2\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{2L_{cln}{}^2\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$=\frac{q_1M_{J,bottom,2}L_{bm}L_{cln}{}^2}{48EI_{cln,2}}-\frac{q_1M_{F,top,2}L_{bm}L_{cln}{}^2}{48EI_{cln,2}}+\frac{q_1L_{bm}e_2}{2}-\frac{\pi q_1L_{bm}e_2EI_{cln,2}\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{8EI_{cln,2}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$-\frac{q_1q_2L_{bm}{}^2L_{cln}{}^2z_2\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{32EI_{cln,2}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}+\frac{q_1q_2L_{bm}{}^2z_2L_{cln}{}^2}{32EI_{cln,2}}+\frac{q_2L_{bm}e_0}{2}+\frac{q_2M_{J,bottom,1}L_{bm}L_{cln}{}^2}{48EI_{cln,1}}$$

$$-\frac{q_2M_{F,top,1}L_{bm}L_{cln}{}^2}{48EI_{cln,1}}+\frac{q_2M_{J,bottom,2}L_{bm}L_{cln}{}^2}{48EI_{cln,2}}-\frac{q_2M_{F,top,2}L_{bm}L_{cln}{}^2}{48EI_{cln,2}}+\frac{q_2L_{bm}e_1}{2}+\frac{q_2L_{bm}e_2}{2}$$

$$-\frac{q_2\pi L_{bm}e_1\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{8\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}-\frac{\pi q_2L_{bm}e_2EI_{cln,2}\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{8EI_{cln,2}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$-\frac{q_2{}^2L_{bm}{}^2L_{cln}{}^2z_2\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{32EI_{cln,2}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}+\frac{q_2{}^2L_{bm}{}^2L_{cln}{}^2z_2}{32EI_{cln,2}}+\frac{M_{J,bottom,2}}{2}$$

$$-\frac{2\pi e_2EI_{cln,2}\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{L_{cln}{}^2\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}-\frac{q_2L_{bm}L_{cln}{}^2z_2\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{2L_{cln}{}^2\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}+\frac{M_{F,top,2}}{2}$$

Some expressions can be found at both sides of the equation. These expressions can be neglected.

$$\frac{e_2\pi^2 EI_{cln,2}}{L_{cln}{}^2}=\frac{q_1M_{J,bottom,2}L_{bm}L_{cln}{}^2}{48EI_{cln,2}}-\frac{q_1M_{F,top,2}L_{bm}L_{cln}{}^2}{48EI_{cln,2}}+\frac{q_1L_{bm}e_2}{2}$$

$$-\frac{\pi q_1L_{bm}e_2EI_{cln,2}\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{8EI_{cln,2}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}-\frac{q_1q_2L_{bm}{}^2L_{cln}{}^2z_2\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{32EI_{cln,2}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$+\frac{q_1q_2L_{bm}{}^2z_2L_{cln}{}^2}{32EI_{cln,2}}+\frac{q_2L_{bm}e_0}{2}+\frac{q_2M_{J,bottom,1}L_{bm}L_{cln}{}^2}{48EI_{cln,1}}-\frac{q_2M_{F,top,1}L_{bm}L_{cln}{}^2}{48EI_{cln,1}}+\frac{q_2M_{J,bottom,2}L_{bm}L_{cln}{}^2}{48EI_{cln,2}}$$

$$-\frac{q_2M_{F,top,2}L_{bm}L_{cln}{}^2}{48EI_{cln,2}}+\frac{q_2L_{bm}e_1}{2}+\frac{q_2L_{bm}e_2}{2}-\frac{q_2\pi L_{bm}e_1\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{8\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$-\frac{\pi q_2L_{bm}e_2EI_{cln,2}\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{8EI_{cln,2}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}-\frac{q_2{}^2L_{bm}{}^2L_{cln}{}^2z_2\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{32EI_{cln,2}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$+\frac{q_2{}^2L_{bm}{}^2L_{cln}{}^2z_2}{32EI_{cln,2}}+\frac{M_{J,bottom,2}}{2}+\frac{M_{F,top,2}}{2}$$

In this formula e_2 is the only unknown. To find the unknown value all expressions of e_2 are located to one side of the equation.

$$\frac{e_2\pi^2 EI_{cln,2}}{L_{cln}{}^2}-\frac{q_1L_{bm}e_2}{2}+\frac{\pi q_1L_{bm}e_2EI_{cln,2}\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{8EI_{cln,2}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}-\frac{q_2L_{bm}e_2}{2}$$

$$+\frac{\pi q_2L_{bm}e_2EI_{cln,2}\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{8EI_{cln,2}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}=\frac{q_1M_{J,bottom,2}L_{bm}L_{cln}{}^2}{48EI_{cln,2}}-\frac{q_1M_{F,top,2}L_{bm}L_{cln}{}^2}{48EI_{cln,2}}$$

$$-\frac{q_1q_2L_{bm}{}^2L_{cln}{}^2z_2\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{32EI_{cln,2}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}+\frac{q_1q_2L_{bm}{}^2L_{cln}{}^2z_2}{32EI_{cln,2}}+\frac{q_2L_{bm}e_0}{2}+\frac{q_2M_{J,bottom,1}L_{bm}L_{cln}{}^2}{48EI_{cln,1}}$$

$$-\frac{q_2M_{F,top,1}L_{bm}L_{cln}{}^2}{48EI_{cln,1}}+\frac{q_2M_{J,bottom,2}L_{bm}L_{cln}{}^2}{48EI_{cln,2}}-\frac{q_2M_{F,top,2}L_{bm}L_{cln}{}^2}{48EI_{cln,2}}+\frac{q_2L_{bm}e_1}{2}-\frac{q_2\pi L_{bm}e_1\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{8\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$-\frac{q_2{}^2L_{bm}{}^2L_{cln}{}^2z_2\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{32EI_{cln,2}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}+\frac{q_2{}^2L_{bm}{}^2L_{cln}{}^2z_2}{32EI_{cln,2}}+\frac{M_{J,bottom,2}}{2}+\frac{M_{F,top,2}}{2}$$

Combine the same expressions. Separate e_2.

$$e_2\left(\frac{\pi^2EI_{cln,2}}{L_{cln}{}^2}-\frac{(q_1+q_2)L_{bm}}{2}+\frac{\pi(q_1+q_2)L_{bm}EI_{cln,2}\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{8EI_{cln,2}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}\right)$$

$$=\frac{(q_1+q_2)\left(M_{J,bottom,2}-M_{F,top,2}\right)L_{bm}L_{cln}{}^2}{48EI_{cln,2}}-\frac{q_2(q_1+q_2)L_{bm}{}^2L_{cln}{}^2z_2\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{32EI_{cln,2}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$+\frac{q_2(q_1+q_2)L_{bm}{}^2L_{cln}{}^2z_2}{32EI_{cln,2}}+\frac{q_2L_{bm}(e_0+e_1)}{2}+\frac{q_2\left(M_{J,bottom,1}-M_{F,top,1}\right)L_{bm}L_{cln}{}^2}{48EI_{cln,1}}$$

$$-\frac{q_2\pi L_{bm}e_1\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{8\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}+\frac{M_{J,bottom,2}}{2}+\frac{M_{F,top,2}}{2}$$

Everywhere the same denominator.

$$e_2\left(\begin{array}{l}\frac{96\pi^2 EI_{cln,1}\left(EI_{cln,2}\right)^2\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{96L_{cln}{}^2EI_{cln,1}EI_{cln,2}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}\\-\frac{48\left(q_1+q_2\right)L_{bm}L_{cln}{}^2EI_{cln,1}EI_{cln,2}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{96L_{cln}{}^2EI_{cln,1}EI_{cln,2}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}\\+\frac{12\pi\left(q_1+q_2\right)L_{bm}L_{cln}{}^2EI_{cln,1}EI_{cln,2}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{96L_{cln}{}^2EI_{cln,1}EI_{cln,2}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}\end{array}\right)$$

$$=\frac{2\left(q_1+q_2\right)\left(M_{J,bottom,2}-M_{F,top,2}\right)L_{bm}L_{cln}{}^4EI_{cln,1}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{96L_{cln}{}^2EI_{cln,1}EI_{cln,2}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$-\frac{3q_2\left(q_1+q_2\right)L_{bm}{}^2L_{cln}{}^4z_2EI_{cln,1}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{96L_{cln}{}^2EI_{cln,1}EI_{cln,2}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$+\frac{3q_2\left(q_1+q_2\right)L_{bm}{}^2L_{cln}{}^4z_2EI_{cln,1}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{96L_{cln}{}^2EI_{cln,1}EI_{cln,2}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$+\frac{48q_2L_{bm}L_{cln}{}^2\left(e_0+e_1\right)EI_{cln,1}EI_{cln,2}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{96L_{cln}{}^2EI_{cln,1}EI_{cln,2}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$+\frac{2q_2\left(M_{J,bottom,1}-M_{F,top,1}\right)L_{bm}L_{cln}{}^4EI_{cln,2}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{96L_{cln}{}^2EI_{cln,1}EI_{cln,2}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$-\frac{12q_2\pi L_{bm}L_{cln}{}^2e_1EI_{cln,1}EI_{cln,2}\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{96L_{cln}{}^2EI_{cln,1}EI_{cln,2}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$+\frac{48M_{J,bottom,2}L_{cln}{}^2EI_{cln,1}EI_{cln,2}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{96L_{cln}{}^2EI_{cln,1}EI_{cln,2}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$+\frac{48M_{F,top,2}L_{cln}{}^2EI_{cln,1}EI_{cln,2}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{96L_{cln}{}^2EI_{cln,1}EI_{cln,2}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

Neglect the denominator

$$
e_2\begin{pmatrix} 96\pi^2 EI_{cln,1}\left(EI_{cln,2}\right)^2\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right) \\ -48\left(q_1+q_2\right)L_{bm}L_{cln}{}^2 EI_{cln,1}EI_{cln,2}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right) \\ +12\pi\left(q_1+q_2\right)L_{bm}L_{cln}{}^2 EI_{cln,1}EI_{cln,2}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right) \end{pmatrix}
$$

$$
\begin{aligned}
&=2\left(q_1+q_2\right)\left(M_{J,bottom,2}-M_{F,top,2}\right)L_{bm}L_{cln}{}^4 EI_{cln,1}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right) \\
&-3q_2\left(q_1+q_2\right)L_{bm}{}^2 L_{cln}{}^4 z_2 EI_{cln,1}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right) \\
&+3q_2\left(q_1+q_2\right)L_{bm}{}^2 L_{cln}{}^4 z_2 EI_{cln,1}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right) \\
&+48q_2L_{bm}L_{cln}{}^2\left(e_0+e_1\right)EI_{cln,1}EI_{cln,2}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right) \\
&+2q_2\left(M_{J,bottom,1}-M_{F,top,1}\right)L_{bm}L_{cln}{}^4 EI_{cln,2}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right) \\
&-12q_2\pi L_{bm}L_{cln}{}^2 e_1 EI_{cln,1}EI_{cln,2}\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right) \\
&+48M_{J,bottom,2}L_{cln}{}^2 EI_{cln,1}EI_{cln,2}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right) \\
&+48M_{F,top,2}L_{cln}{}^2 EI_{cln,1}EI_{cln,2}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)
\end{aligned}
$$

Combine the same expressions

$$
12e_2 EI_{cln,1}EI_{cln,2}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\begin{pmatrix} 4\left(2\pi^2 EI_{cln,2}-\left(q_1+q_2\right)L_{bm}L_{cln}{}^2\right)\begin{pmatrix} 8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1} \\ +5L_{cln}EI_{bm} \end{pmatrix} \\ +\pi\left(q_1+q_2\right)L_{bm}L_{cln}{}^2\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right) \end{pmatrix}
$$

$$
\begin{aligned}
&=2L_{bm}L_{cln}{}^4\begin{pmatrix} \left(q_1+q_2\right)\left(M_{J,bottom,2}-M_{F,top,2}\right)EI_{cln,1} \\ +q_2\left(M_{J,bottom,1}-M_{F,top,1}\right)EI_{cln,2} \end{pmatrix}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\begin{pmatrix} 8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1} \\ +5L_{cln}EI_{bm} \end{pmatrix} \\
&+24q_2\left(q_1+q_2\right)L_{bm}{}^3 L_{cln}{}^4 z_2 EI_{cln,1}EI_{cln,2}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right) \\
&+12q_2L_{bm}L_{cln}{}^2 EI_{cln,1}EI_{cln,2}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\begin{pmatrix} 4e_0\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right) \\ +e_1\begin{pmatrix} \left(44-3\pi\right)L_{bm}EI_{cln,1} \\ +\left(20-5\pi\right)L_{cln}EI_{bm} \end{pmatrix} \end{pmatrix} \\
&+48\left(M_{J,bottom,2}+M_{F,top,2}\right)L_{cln}{}^2 EI_{cln,1}EI_{cln,2}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)
\end{aligned}
$$

The formula of e_2 can be found.

$$e_2=\frac{\begin{pmatrix}2L_{bm}L_{cln}{}^4\begin{pmatrix}(q_1+q_2)\left(M_{J,bottom,2}-M_{F,top,2}\right)EI_{cln,1}\\+q_2\left(M_{J,bottom,1}-M_{F,top,1}\right)EI_{cln,2}\end{pmatrix}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\begin{pmatrix}8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}\\+5L_{cln}EI_{bm}\end{pmatrix}\\+24q_2\left(q_1+q_2\right)L_{bm}{}^3L_{cln}{}^4z_2EI_{cln,1}EI_{cln,2}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\\+12q_2L_{bm}L_{cln}{}^2EI_{cln,1}EI_{cln,2}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\begin{pmatrix}4e_0\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\\+e_1\begin{pmatrix}(44-3\pi)L_{bm}EI_{cln,1}\\+(20-5\pi)L_{cln}EI_{bm}\end{pmatrix}\end{pmatrix}\\+48\left(M_{J,bottom,2}+M_{F,top,2}\right)L_{cln}{}^2EI_{cln,1}EI_{cln,2}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\end{pmatrix}}{12EI_{cln,1}EI_{cln,2}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\begin{pmatrix}4\left(2\pi^2EI_{cln,2}-(q_1+q_2)L_{bm}L_{cln}{}^2\right)\begin{pmatrix}8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}\\+5L_{cln}EI_{bm}\end{pmatrix}\\+\pi(q_1+q_2)L_{bm}L_{cln}{}^2\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\end{pmatrix}}$$

This formula is checked on the correctness of the dimensions.

$$m=\frac{\begin{pmatrix}mm^4\begin{pmatrix}\left(Nm^{-1}+Nm^{-1}\right)\left(Nm-Nm\right)Nm^2\\+Nm^{-1}\left(Nm-Nm\right)Nm^2\end{pmatrix}\left(mNm^2+mNm^2\right)\left(mNm^2+mNm^2+mNm^2\right)\\+Nm^{-1}\left(Nm^{-1}+Nm^{-1}\right)m^3m^4mNm^2Nm^2\left(mNm^2+mNm^2\right)\\+Nm^{-1}mm^2Nm^2Nm^2\left(mNm^2+mNm^2+mNm^2\right)\left(m\left(mNm^2+mNm^2\right)+m\left(mNm^2+mNm^2\right)\right)\\+\left(Nm+Nm\right)m^2Nm^2Nm^2\left(mNm^2+mNm^2\right)\left(mNm^2+mNm^2+mNm^2\right)\end{pmatrix}}{Nm^2Nm^2\left(mNm^2+mNm^2\right)\begin{pmatrix}\left(Nm^2-\left(Nm^{-1}+Nm^{-1}\right)mm^2\right)\left(mNm^2+mNm^2+mNm^2\right)\\+\left(Nm^{-1}+Nm^{-1}\right)Nm^{-1}mm^2\left(mNm^2+mNm^2\right)\end{pmatrix}}$$

$$m=\frac{\begin{pmatrix}m^5\left(\left(Nm^{-1}\right)\left(Nm\right)Nm^2\right)\left(Nm^3\right)\left(Nm^3\right)+Nm^{-1}\left(Nm^{-1}\right)N^2m^{12}\left(Nm^3\right)\\+N^3m^6\left(Nm^3\right)\left(m\left(Nm^3\right)+m\left(Nm^3\right)\right)+\left(Nm\right)N^2m^6\left(Nm^3\right)\left(Nm^3\right)\end{pmatrix}}{N^2m^4\left(Nm^3\right)\left(\left(Nm^2-\left(Nm^{-1}\right)m^3\right)\left(Nm^3\right)+\left(Nm^{-1}\right)Nm^2\left(Nm^3\right)\right)}$$

$$m=\frac{N^5m^{13}}{N^5m^{12}}$$ Dimensions are correct

A second check is made. If the boundary conditions will changed in the same boundary conditions as in the analysis without residual stress is the same formula as found in Appendix S must be found.

$$e_2 = \frac{\begin{pmatrix} 2L_{bm}L_{cln}^{\ 4}\begin{pmatrix}(q_1+q_2)(M_{J,bottom,2}-M_{F,top,2})EI_{cln,1} \\ +q_2(M_{J,bottom,1}-M_{F,top,1})EI_{cln,2}\end{pmatrix}(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm})\begin{pmatrix}8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1} \\ +5L_{cln}EI_{bm}\end{pmatrix} \\ +24q_2(q_1+q_2)L_{bm}^{\ 3}L_{cln}^{\ 4}z_2EI_{cln,1}EI_{cln,2}(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}) \\ +12q_2L_{bm}L_{cln}^{\ 2}EI_{cln,1}EI_{cln,2}(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm})\begin{pmatrix}4e_0(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}) \\ +e_1\begin{pmatrix}(44-3\pi)L_{bm}EI_{cln,1} \\ +(20-5\pi)L_{cln}EI_{bm}\end{pmatrix}\end{pmatrix} \\ +48(M_{J,bottom,2}+M_{F,top,2})L_{cln}^{\ 2}EI_{cln,1}EI_{cln,2}(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm})(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm})\end{pmatrix}}{12EI_{cln,1}EI_{cln,2}(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm})\begin{pmatrix}4(2\pi^2EI_{cln,2}-(q_1+q_2)L_{bm}L_{cln}^{\ 2})\begin{pmatrix}8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1} \\ +5L_{cln}EI_{bm}\end{pmatrix} \\ +\pi(q_1+q_2)L_{bm}L_{cln}^{\ 2}(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm})\end{pmatrix}}$$

Take z=0;

$$e_2 = \frac{\begin{pmatrix} 2L_{bm}L_{cln}^{\ 4}\begin{pmatrix}(q_1+q_2)(M_{J,bottom,2}-M_{F,top,2})EI_{cln,1} \\ +q_2(M_{J,bottom,1}-M_{F,top,1})EI_{cln,2}\end{pmatrix}(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm})\begin{pmatrix}8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1} \\ +5L_{cln}EI_{bm}\end{pmatrix} \\ +12q_2L_{bm}L_{cln}^{\ 2}EI_{cln,1}EI_{cln,2}(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm})\begin{pmatrix}4e_0(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}) \\ +e_1\begin{pmatrix}(44-3\pi)L_{bm}EI_{cln,1} \\ +(20-5\pi)L_{cln}EI_{bm}\end{pmatrix}\end{pmatrix} \\ +48(M_{J,bottom,2}+M_{F,top,2})L_{cln}^{\ 2}EI_{cln,1}EI_{cln,2}(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm})(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm})\end{pmatrix}}{12EI_{cln,1}EI_{cln,2}(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm})\begin{pmatrix}4(2\pi^2EI_{cln,2}-(q_1+q_2)L_{bm}L_{cln}^{\ 2})\begin{pmatrix}8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1} \\ +5L_{cln}EI_{bm}\end{pmatrix} \\ +\pi(q_1+q_2)L_{bm}L_{cln}^{\ 2}(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm})\end{pmatrix}}$$

Take all stiffness the same $\left(EI_{cln,1}=EI_{cln,2}=EI_{cln}\right)$.

$$e_2 = \frac{\begin{pmatrix} 2L_{bm}L_{cln}{}^4 \begin{pmatrix} (q_1+q_2)(M_{J,bottom,2}-M_{F,top,2})EI_{cln} \\ +q_2(M_{J,bottom,1}-M_{F,top,1})EI_{cln} \end{pmatrix} (11L_{bm}EI_{cln}+5L_{cln}EI_{bm}) \begin{pmatrix} 8L_{bm}EI_{cln}+3L_{bm}EI_{cln} \\ +5L_{cln}EI_{bm} \end{pmatrix} \\ +12q_2L_{bm}L_{cln}{}^2EI_{cln}EI_{cln}(8L_{bm}EI_{cln}+3L_{bm}EI_{cln}+5L_{cln}EI_{bm}) \begin{pmatrix} 4e_0(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}) \\ +e_1\begin{pmatrix} (44-3\pi)L_{bm}EI_{cln} \\ +(20-5\pi)L_{cln}EI_{bm} \end{pmatrix} \end{pmatrix} \\ +48(M_{J,bottom,2}+M_{F,top,2})L_{cln}{}^2EI_{cln}EI_{cln}(11L_{bm}EI_{cln}+5L_{cln}EI_{bm})(8L_{bm}EI_{cln}+3L_{bm}EI_{cln}+5L_{cln}EI_{bm}) \end{pmatrix}}{12EI_{cln}EI_{cln}(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}) \begin{pmatrix} 4\left(2\pi^2EI_{cln}-(q_1+q_2)L_{bm}L_{cln}{}^2\right)\begin{pmatrix} 8L_{bm}EI_{cln}+3L_{bm}EI_{cln} \\ +5L_{cln}EI_{bm} \end{pmatrix} \\ +\pi(q_1+q_2)L_{bm}L_{cln}{}^2(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}) \end{pmatrix}}$$

Neglect the same expressions

$$e_2 = \frac{\begin{pmatrix} L_{bm}L_{cln}{}^4\left((q_1+q_2)(M_{J,bottom,2}-M_{F,top,2})+q_2(M_{J,bottom,1}-M_{F,top,1})\right)(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}) \\ +6q_2L_{bm}L_{cln}{}^2EI_{cln}\begin{pmatrix} 4e_0(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}) \\ +e_1\left((44-3\pi)L_{bm}EI_{cln}+(20-5\pi)L_{cln}EI_{bm}\right) \end{pmatrix} \\ +24(M_{J,bottom,2}+M_{F,top,2})L_{cln}{}^2EI_{cln}(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}) \end{pmatrix}}{6EI_{cln}\begin{pmatrix} 4\left(2\pi^2EI_{cln}-(q_1+q_2)L_{bm}L_{cln}{}^2\right)(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}) \\ +\pi(q_1+q_2)L_{bm}L_{cln}{}^2(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}) \end{pmatrix}}$$

Neglect the original loads $\left(q_1=0; e_1=0; M_{J,bottom,1}=0; M_{F,top,1}=0\right)$. Combine the same expressions in the numerator.

$$e_2 = \frac{\begin{pmatrix} q_2L_{bm}L_{cln}{}^4(M_{J,bottom,2}-M_{F,top,2})(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}) \\ +24q_2L_{bm}L_{cln}{}^2e_0EI_{cln}(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}) \\ +24(M_{J,bottom,2}+M_{F,top,2})L_{cln}{}^2EI_{cln}(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}) \end{pmatrix}}{6EI_{cln}\begin{pmatrix} 4\left(2\pi^2EI_{cln}-q_2L_{bm}L_{cln}{}^2\right)(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}) \\ +\pi q_2L_{bm}L_{cln}{}^2(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}) \end{pmatrix}}$$

Combine the expressions in the numerator.

$$e_2 = \frac{\begin{pmatrix} q_2L_{bm}L_{cln}{}^2\left(24e_0EI_{cln}+L_{cln}{}^2(M_{J,bottom,2}-M_{F,top,2})\right)(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}) \\ +24(M_{J,bottom,2}+M_{F,top,2})L_{cln}{}^2EI_{cln}(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}) \end{pmatrix}}{6EI_{cln}\begin{pmatrix} 4\left(2\pi^2EI_{cln}-q_2L_{bm}L_{cln}{}^2\right)(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}) \\ +\pi q_2L_{bm}L_{cln}{}^2(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}) \end{pmatrix}}$$

Combine the expressions in the denominator.

$$e_2 = \frac{\begin{pmatrix} q_2 L_{bm} L_{cln}{}^2 \left(24 e_0 EI_{cln} + L_{cln}{}^2 \left(M_{J,bottom,2} - M_{F,top,2}\right)\right)\left(11 L_{bm} EI_{cln} + 5 L_{cln} EI_{bm}\right) \\ +24\left(M_{J,bottom,2} + M_{F,top,2}\right) L_{cln}{}^2 EI_{cln} \left(11 L_{bm} EI_{cln} + 5 L_{cln} EI_{bm}\right) \end{pmatrix}}{6 EI_{cln} \begin{pmatrix} 8\pi^2 EI_{cln} \left(11 L_{bm} EI_{cln} + 5 L_{cln} EI_{bm}\right) \\ + q L_{bm} L_{cln}{}^2 \left((3\pi - 44) L_{bm} EI_{cln} + (5\pi - 20) L_{cln} EI_{bm}\right) \end{pmatrix}}$$

The original formula was:

$$e_1 = \frac{\begin{matrix} q L_{bm} L_{cln}{}^2 \left(11 L_{bm} EI_{cln} + 5 L_{cln} EI_{bm}\right)\left(24 e_0 EI_{cln} + L_{cln}{}^2 \left(M_{J,bottom} - M_{F,top}\right)\right) \\ +24\left(M_{J,bottom} + M_{F,top}\right) L_{cln}{}^2 EI_{cln} \left(11 L_{bm} EI_{cln} + 5 L_{cln} EI_{bm}\right) \end{matrix}}{6 EI_{cln} \begin{pmatrix} 8\pi^2 EI_{cln} \left(11 L_{bm} EI_{cln} + 5 L_{cln} EI_{bm}\right) \\ + q L_{bm} L_{cln}{}^2 \left((3\pi - 44) L_{bm} EI_{cln} + (5\pi - 20) L_{cln} EI_{bm}\right) \end{pmatrix}}$$

These formulas are the same.

T.2 Analysis if two parts yield

The third load case starts if both flanges partial yield. The additional (third) deflection will be analyzed in this Appendix.

The rotation spring remain constant.

$$k_r = \frac{3 L_{bm} EI_{cln} + 5 L_{cln} EI_{bm}}{4 L_{bm} L_{cln}}$$

In Appendix T.1 the shift of the centre of gravity is a part of the additional deflection. In this load case the effective cross-section is symmetric. The total deflection is split in four parts.

The four deflections are:

1. Starting deflection
2. Deflection due to the linear analysis
3. Additional deflection
4. Deflection due to the rotational springs

The starting deflection

$$y_1 = e_0 \sin\left(\frac{\pi x}{L_{cln}}\right)$$

$$\varphi_1 = e_0 \frac{\pi}{L_{cln}} \cos\left(\frac{\pi x}{L_{cln}}\right)$$

$$\kappa_1 = -e_0 \frac{\pi^2}{L_{cln}{}^2} \sin\left(\frac{\pi x}{L_{cln}}\right)$$

Deflection due to the linear analysis

$$M_{2,3} = \frac{\left(M_{J,bottom,3} + M_{F,top,3}\right)\left(L_{cln} - x\right)}{L_{cln}} - M_{F,top,3}$$

$$M_{2,3} = \frac{M_{J,bottom,3}\left(L_{cln} - x\right) - M_{F,top,3}x}{L_{cln}}$$

$$\kappa_{2,3} = -\frac{M_{J,bottom,3}\left(L_{cln} - x\right) - M_{F,top,3}x}{L_{cln}EI_{cln,3}}$$

$$\kappa_{2,3} = \frac{M_{J,bottom,3}\left(-L_{cln} + x\right) + M_{F,top,3}x}{L_{cln}EI_{cln,3}}$$

$$\varphi_{2,3} = \frac{M_{J,bottom,3}\left(-L_{cln}x + \tfrac{1}{2}x^2\right) + M_{F,top,3}\tfrac{1}{2}x^2}{L_{cln}EI_{cln,3}} + C_1$$

$$y_{2,3} = \frac{M_{J,bottom3}\left(-\tfrac{1}{2}L_{cln}x^2 + \tfrac{1}{6}x^3\right) + M_{F,top,3}\tfrac{1}{6}x^3}{L_{cln}EI_{cln,3}} + C_1x + C_2$$

Use the boundary conditions to find the integral constants.

$y_{2,3,x=0} = 0$ → $C_2 = 0$

$y_{2,3,x=L_{cln}} = 0$ → $C_1 = \frac{\tfrac{1}{3}M_{J,bottom,3}L_{cln} - \tfrac{1}{6}M_{F,top,3}L_{cln}}{EI_{cln,3}}$

Use these integral constants in the formula.

$$y_{2,3} = \frac{M_{J,bottom,3}\left(-\tfrac{1}{2}L_{cln}x^2 + \tfrac{1}{6}x^3\right) + M_{F,top,3}\tfrac{1}{6}x^3}{L_{cln}EI_{cln,3}} + \frac{\tfrac{1}{3}M_{J,bottom,3}L_{cln} - \tfrac{1}{6}M_{F,top,3}L_{cln}}{EI_{cln,3}}x$$

Combine the same expressions.

$$y_{2,3} = \frac{M_{J,bottom,3}\left(\tfrac{1}{3}L_{cln}{}^2x - \tfrac{1}{2}L_{cln}x^2 + \tfrac{1}{6}x^3\right) + M_{F,top,3}\left(-\tfrac{1}{6}L_{cln}{}^2x + \tfrac{1}{6}x^3\right)}{L_{cln}EI_{cln,3}}$$

Make the formula clearer.

$$y_{2,3} = \frac{M_{J,bottom,3}\left(2L_{cln}{}^2x - 3L_{cln}x^2 + x^3\right) + M_{F,top,3}\left(-L_{cln}{}^2x + x^3\right)}{6L_{cln}EI_{cln,3}}$$

$$\varphi_{2,3} = \frac{M_{J,bottom,3}\left(2L_{cln}{}^2 - 6L_{cln}x + 3x^2\right) + M_{F,top,3}\left(-L_{cln}{}^2 + 3x^2\right)}{6L_{cln}EI_{cln,3}}$$

$$\kappa_{2,3} = \frac{M_{J,bottom,3}\left(-6L_{cln} + 6x\right) + M_{F,top,3}\left(6x\right)}{6L_{cln}EI_{cln,3}}$$

Additional deflection.

$$y_{3,3} = e_3\sin\left(\frac{\pi x}{L_{cln}}\right)$$

$$\varphi_{3,3} = e_3 \frac{\pi}{L_{cln}} \cos\left(\frac{\pi x}{L_{cln}}\right)$$

$$\kappa_{3,3} = -e_3 \frac{\pi^2}{L_{cln}{}^2} \sin\left(\frac{\pi x}{L_{cln}}\right)$$

Deflection due to the rotational spring

Because of symmetry, the bending moment in the column due to the spring stiffness is constant.

$$M_{4,3} = -\varphi_{extra,3,x=0} k_r$$

$$\kappa_{4,3} = \frac{\varphi_{extra,3,x=0} k_r}{EI_{cln,3}}$$

$$\varphi_{4,3} = \frac{\varphi_{extra,3,x=0} k_r x}{EI_{cln,3}} + C_1$$

$$y_{4,3} = \frac{\varphi_{extra,3,x=0} k_r x^2}{2EI_{cln,3}} + C_1 x + C_2$$

Use the boundary conditions to find the integral constants.

$y_{4,3,x=0} = 0$ → $C_2 = 0$

$y_{4,3,,x=L_{cln}} = 0$ → $C_1 = -\frac{\varphi_{extra,3,x=0} k_r L_{cln}}{2EI_{cln,3}}$

Use these constants in the formula.

$$y_{4,3} = \frac{\varphi_{extra,3,x=0} k_r x^2}{2EI_{cln,3}} + -\frac{\varphi_{extra,3,x=0} k_r L_{cln}}{2EI_{cln,3}} x$$

Combine the same expressions.

$$y_{4,3} = \frac{\varphi_{extra,3,x=0} k_r \left(-L_{cln} x + x^2\right)}{2EI_{cln,3}}$$

$$\varphi_{4,3} = \frac{\varphi_{extra,3,x=0} k_r \left(-L_{cln} + 2x\right)}{2EI_{cln,3}}$$

$$\kappa_{4,3} = \frac{\varphi_{extra,3,x=0} k_r}{EI_{cln,3}}$$

The additional rotation depends on $\varphi_{3,3}$ and $\varphi_{4,3}$.

$$\varphi_{extra,3} = e_3 \frac{\pi}{L_{cln}} \cos\left(\frac{\pi x}{L_{cln}}\right) + \frac{\varphi_{extra,3,x=0} k_r \left(-L_{cln} + 2x\right)}{2EI_{cln,3}}$$

Calculate the additional rotation at x=0.

$$\varphi_{extra,3,x=0} = e_3 \frac{\pi}{L_{cln}} - \frac{\varphi_{extra,3,x=0} k_r L_{cln}}{2EI_{cln,3}}$$

Put all expressions of $\varphi_{extra,3,x=0}$ on one side of the equation.

$$\varphi_{extra,3,x=0} + \frac{\varphi_{extra,3,x=0} k_r L_{cln}}{2EI_{cln,3}} = e_3 \frac{\pi}{L_{cln}}$$

Use the spring stiffness $\left(k_r = \frac{3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}}{4L_{bm}L_{cln}} \right)$

$$\varphi_{extra,3,x=0} + \frac{\varphi_{extra,3,x=0} \left(\frac{3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}}{4L_{bm}L_{cln}} \right) L_{cln}}{2EI_{cln,3}} = e_3 \frac{\pi}{L_{cln}}$$

Write out one expression.

$$\varphi_{extra,3,x=0} + \frac{\varphi_{extra,3,x=0} \left(3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm} \right)}{8L_{bm}EI_{cln,3}} = e_3 \frac{\pi}{L_{cln}}$$

Same denominator

$$\varphi_{extra,3,x=0} \left(\frac{L_{cln} \left(8L_{bm}EI_{cln,3} + \left(3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm} \right) \right)}{8L_{bm}L_{cln}EI_{cln,3}} \right) = \frac{8\pi L_{bm} e_3 EI_{cln,3}}{8L_{bm}L_{cln}EI_{cln,3}}$$

Find the formula of $\varphi_{extra,3,x=0}$:

$$\varphi_{extra,3,x=0} = \frac{8\pi L_{bm} e_3 EI_{cln,3}}{L_{cln} \left(8L_{bm}EI_{cln,3} + 3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm} \right)}$$

The additional rotation can be used in the formula of $y_{4,3}$.

$$y_{4,3} = \frac{\varphi_{extra,3,x=0} k_r \left(-L_{cln} x + x^2 \right)}{2EI_{cln,3}}$$

Use the additional rotation and the spring stiffness in this formula.

$$y_{4,3} = \left(\frac{8\pi L_{bm} e_3 EI_{cln,3}}{L_{cln} \left(8L_{bm}EI_{cln,3} + 3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm} \right)} \right) \left(\frac{3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}}{4L_{bm}L_{cln}} \right) \frac{\left(-L_{cln} x + x^2 \right)}{2EI_{cln,3}}$$

Combine and neglect the same parameters.

$$y_{4,3} = \frac{\pi e_3 \left(3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm} \right) \left(-L_{cln} x + x^2 \right)}{L_{cln}^{\ 2} \left(8L_{bm}EI_{cln,3} + 3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm} \right)}$$

$$\varphi_{4,3} = \frac{\pi e_3 \left(3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm} \right) \left(-L_{cln} + 2x \right)}{L_{cln}^{\ 2} \left(8L_{bm}EI_{cln,3} + 3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm} \right)}$$

$$\kappa_{4,3} = \frac{2\pi e_3 \left(3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm} \right)}{L_{cln}^{\ 2} \left(8L_{bm}EI_{cln,3} + 3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm} \right)}$$

A list of all deflection formulas.

$$y_1 = e_0 \sin\left(\frac{\pi x}{L_{cln}}\right)$$

$$y_{2,1} = \frac{M_{J,bottom,1}\left(2L_{cln}{}^2 x - 3L_{cln}x^2 + x^3\right) + M_{F,top,1}\left(-L_{cln}{}^2 x + x^3\right)}{6L_{cln}EI_{cln,1}}$$

$$y_{2,2} = \frac{M_{J,bottom,2}\left(2L_{cln}{}^2 x - 3L_{cln}x^2 + x^3\right) + M_{F,top,2}\left(-L_{cln}{}^2 x + x^3\right)}{6L_{cln}EI_{cln,2}}$$

$$y_{2,3} = \frac{M_{J,bottom,3}\left(2L_{cln}{}^2 x - 3L_{cln}x^2 + x^3\right) + M_{F,top,3}\left(-L_{cln}{}^2 x + x^3\right)}{6L_{cln}EI_{cln,3}}$$

$$y_{3,1} = e_1 \sin\left(\frac{\pi x}{L_{cln}}\right)$$

$$y_{3,2} = e_2 \sin\left(\frac{\pi x}{L_{cln}}\right)$$

$$y_{3,3} = e_3 \sin\left(\frac{\pi x}{L_{cln}}\right)$$

$$\kappa_{3,3} = -e_3 \frac{\pi^2}{L_{cln}{}^2} \sin\left(\frac{\pi x}{L_{cln}}\right)$$

$$y_{4,1} = \frac{\pi e_1\left(3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)\left(-L_{cln}x + x^2\right)}{L_{cln}{}^2\left(11L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)}$$

$$y_{4,2} = \frac{\left(2\pi e_2 EI_{cln,2} + N_2 L_{cln}{}^2 z_2\right)\left(3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)\left(-L_{cln}x + x^2\right)}{2L_{cln}{}^2 EI_{cln,2}\left(8L_{bm}EI_{cln,2} + 3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)}$$

$$y_{4,3} = \frac{\pi e_3\left(3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)\left(-L_{cln}x + x^2\right)}{L_{cln}{}^2\left(8L_{bm}EI_{cln,3} + 3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)}$$

$$\kappa_{4,3} = \frac{2\pi e_3\left(3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)}{L_{cln}{}^2\left(8L_{bm}EI_{cln,3} + 3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)}$$

With the formulas above the equilibrium between the internal and the external moments can be analyzed.

$$\Delta M_{intern} = \Delta M_{extern}$$

$$-\Delta\kappa EI_{cln,3} = \left(N_1 + N_2\right)y_3 + N_3 y_{total,3} + \frac{M_{J,bottom,3}x}{L_{cln}} - \frac{k_r \varphi_{extra,3,x=0}x}{L_{cln}} + \frac{M_{F,top,3}\left(L_{cln} - x\right)}{L_{cln}} - \frac{k_r \varphi_{extra,3,x=0}\left(L_{cln} - x\right)}{L_{cln}}$$

$$-\left(\kappa_{3,3}+\kappa_{4,3}\right)EI_{cln,3}$$
$$=\left(N_1+N_2\right)\left(y_{2,3}+y_{3,3}+y_{4,3}\right)+N_3\left(y_1+y_{2,1}+y_{2,2}+y_{2,3}+y_{3,1}+y_{3,2}+y_{3,3}+y_{4,1}+y_{4,2}+y_{4,3}\right)$$
$$+\frac{M_{J,bottom,3}x}{L_{cln}}-\frac{k_r\varphi_{extra,3,x=0}x}{L_{cln}}+\frac{M_{F,top,3}\left(L_{cln}-x\right)}{L_{cln}}-\frac{k_r\varphi_{extra,3,x=0}\left(L_{cln}-x\right)}{L_{cln}}$$

Combine the same expressions.

$$-\left(\kappa_{3,3}+\kappa_{4,3}\right)EI_{cln,3}$$
$$=\left(N_1+N_2\right)\left(y_{2,3}+y_{3,3}+y_{4,3}\right)+N_3\left(y_1+y_{2,1}+y_{2,2}+y_{2,3}+y_{3,1}+y_{3,2}+y_{3,3}+y_{4,1}+y_{4,2}+y_{4,3}\right)$$
$$+\frac{M_{J,bottom,3}x}{L_{cln}}-k_r\varphi_{extra,3,x=0}+\frac{M_{F,top,3}\left(L_{cln}-x\right)}{L_{cln}}$$

Use the known formulas in this formula.

$$-\left(-e_3\frac{\pi^2}{L_{cln}{}^2}\sin\left(\frac{\pi x}{L_{cln}}\right)+\frac{2\pi e_3\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{L_{cln}{}^2\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}\right)EI_{cln,3}$$
$$=\left(N_1+N_2\right)\left(\begin{array}{l}\frac{M_{J,bottom,3}\left(2L_{cln}{}^2x-3L_{cln}x^2+x^3\right)+M_{F,top,3}\left(-L_{cln}{}^2x+x^3\right)}{6L_{cln}EI_{cln,3}}+e_3\sin\left(\frac{\pi x}{L_{cln}}\right)\\+\frac{\pi e_3\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(-L_{cln}x+x^2\right)}{L_{cln}{}^2\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}\end{array}\right)$$
$$+N_3\left(\begin{array}{l}e_0\sin\left(\frac{\pi x}{L_{cln}}\right)+\frac{M_{J,bottom,1}\left(2L_{cln}{}^2x-3L_{cln}x^2+x^3\right)+M_{F,top,1}\left(-L_{cln}{}^2x+x^3\right)}{6L_{cln}EI_{cln,1}}\\+\frac{M_{J,bottom,2}\left(2L_{cln}{}^2x-3L_{cln}x^2+x^3\right)+M_{F,top,2}\left(-L_{cln}{}^2x+x^3\right)}{6L_{cln}EI_{cln,2}}\\+\frac{M_{J,bottom,3}\left(2L_{cln}{}^2x-3L_{cln}x^2+x^3\right)+M_{F,top,3}\left(-L_{cln}{}^2x+x^3\right)}{6L_{cln}EI_{cln,3}}+e_1\sin\left(\frac{\pi x}{L_{cln}}\right)+e_2\sin\left(\frac{\pi x}{L_{cln}}\right)+e_3\sin\left(\frac{\pi x}{L_{cln}}\right)\\+\frac{\pi e_1\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(-L_{cln}x+x^2\right)}{L_{cln}{}^2\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}\\+\frac{\left(2\pi e_2EI_{cln,2}+N_2L_{cln}{}^2z_2\right)\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(-L_{cln}x+x^2\right)}{2L_{cln}{}^2EI_{cln,2}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}\\+\frac{\pi e_3\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(-L_{cln}x+x^2\right)}{L_{cln}{}^2\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}\end{array}\right)$$
$$+\frac{M_{J,bottom,3}x}{L_{cln}}-\left(\frac{3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}}{4L_{bm}L_{cln}}\right)\left(\frac{8\pi L_{bm}e_3EI_{cln,3}}{L_{cln}\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}\right)+\frac{M_{F,top,3}\left(L_{cln}-x\right)}{L_{cln}}$$

Write out the expressions to neglect the brackets.

$$\frac{e_3\pi^2 EI_{cln,3}}{L_{cln}{}^2}\sin\left(\frac{\pi x}{L_{cln}}\right)-\frac{2\pi e_3 EI_{cln,3}\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{L_{cln}{}^2\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$=\frac{\left(N_1+N_2\right)M_{J,bottom,3}\left(2L_{cln}{}^2x-3L_{cln}x^2+x^3\right)}{6L_{cln}EI_{cln,3}}+\frac{\left(N_1+N_2\right)M_{F,top,3}\left(-L_{cln}{}^2x+x^3\right)}{6L_{cln}EI_{cln,3}}$$

$$+\left(N_1+N_2\right)e_3\sin\left(\frac{\pi x}{L_{cln}}\right)+\frac{\pi\left(N_1+N_2\right)e_3\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(-L_{cln}x+x^2\right)}{L_{cln}{}^2\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$+N_3e_0\sin\left(\frac{\pi x}{L_{cln}}\right)+\frac{N_3M_{J,bottom,1}\left(2L_{cln}{}^2x-3L_{cln}x^2+x^3\right)}{6L_{cln}EI_{cln,1}}+\frac{N_3M_{F,top,1}\left(-L_{cln}{}^2x+x^3\right)}{6L_{cln}EI_{cln,1}}$$

$$+\frac{N_3M_{J,bottom,2}\left(2L_{cln}{}^2x-3L_{cln}x^2+x^3\right)}{6L_{cln}EI_{cln,2}}+\frac{N_3M_{F,top,2}\left(-L_{cln}{}^2x+x^3\right)}{6L_{cln}EI_{cln,2}}$$

$$+\frac{N_3M_{J,bottom,3}\left(2L_{cln}{}^2x-3L_{cln}x^2+x^3\right)}{6L_{cln}EI_{cln,3}}+\frac{N_3M_{F,top,3}\left(-L_{cln}{}^2x+x^3\right)}{6L_{cln}EI_{cln,3}}+N_3e_1\sin\left(\frac{\pi x}{L_{cln}}\right)$$

$$+N_3e_2\sin\left(\frac{\pi x}{L_{cln}}\right)+N_3e_3\sin\left(\frac{\pi x}{L_{cln}}\right)+\frac{N_3\pi e_1\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(-L_{cln}x+x^2\right)}{L_{cln}{}^2\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$+\frac{N_3\left(2\pi e_2EI_{cln,2}+N_2L_{cln}{}^2z_2\right)\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(-L_{cln}x+x^2\right)}{2L_{cln}{}^2EI_{cln,2}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$+\frac{\pi N_3e_3\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(-L_{cln}x+x^2\right)}{L_{cln}{}^2\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$+\frac{M_{J,bottom,3}x}{L_{cln}}-\frac{2\pi e_3EI_{cln,3}\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{L_{cln}{}^2\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}+\frac{M_{F,top,3}\left(L_{cln}-x\right)}{L_{cln}}$$

Known are the loads N_1, N_2 and N_3 $\left(N_1=0.5q_1L_{bm};N_2=0.5q_2L_{bm};N_3=0.5q_3L_{bm}\right)$. The loads can be used in the formula. Neglect the influence of the required loads.

$$
\begin{aligned}
&e_3 \frac{\pi^2 EI_{cln,3}}{L_{cln}{}^2}\sin\left(\frac{\pi x}{L_{cln}}\right) - \frac{2\pi e_3 EI_{cln,3}\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{L_{cln}{}^2\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)} \\
&= \frac{(q_1+q_2)M_{J,bottom,3}L_{bm}\left(2L_{cln}{}^2x-3L_{cln}x^2+x^3\right)}{12L_{cln}EI_{cln,3}} + \frac{(q_1+q_2)M_{F,top,3}L_{bm}\left(-L_{cln}{}^2x+x^3\right)}{12L_{cln}EI_{cln,3}} \\
&+ \frac{(q_1+q_2)L_{bm}e_3\sin\left(\frac{\pi x}{L_{cln}}\right)}{2} + \frac{\pi(q_1+q_2)L_{bm}e_3\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(-L_{cln}x+x^2\right)}{2L_{cln}{}^2\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)} \\
&+ \frac{q_3L_{bm}e_0\sin\left(\frac{\pi x}{L_{cln}}\right)}{2} + \frac{q_3M_{J,bottom,1}L_{bm}\left(2L_{cln}{}^2x-3L_{cln}x^2+x^3\right)}{12L_{cln}EI_{cln,1}} + \frac{q_3M_{F,top,1}L_{bm}\left(-L_{cln}{}^2x+x^3\right)}{12L_{cln}EI_{cln,1}} \\
&+ \frac{q_3M_{J,bottom,2}L_{bm}\left(2L_{cln}{}^2x-3L_{cln}x^2+x^3\right)}{12L_{cln}EI_{cln,2}} + \frac{q_3M_{F,top,2}L_{bm}\left(-L_{cln}{}^2x+x^3\right)}{12L_{cln}EI_{cln,2}} \\
&+ \frac{q_3M_{J,bottom,3}L_{bm}\left(2L_{cln}{}^2x-3L_{cln}x^2+x^3\right)}{12L_{cln}EI_{cln,3}} + \frac{q_3M_{F,top,3}L_{bm}\left(-L_{cln}{}^2x+x^3\right)}{12L_{cln}EI_{cln,3}} + \frac{q_3L_{bm}e_1\sin\left(\frac{\pi x}{L_{cln}}\right)}{2} \\
&+ \frac{q_3L_{bm}e_2\sin\left(\frac{\pi x}{L_{cln}}\right)}{2} + \frac{q_3L_{bm}e_3\sin\left(\frac{\pi x}{L_{cln}}\right)}{2} + \frac{q_3\pi L_{bm}e_1\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(-L_{cln}x+x^2\right)}{2L_{cln}{}^2\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)} \\
&+ \frac{q_3L_{bm}\left(4\pi e_2EI_{cln,2}+q_2L_{bm}L_{cln}{}^2z_2\right)\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(-L_{cln}x+x^2\right)}{8L_{cln}{}^2EI_{cln,2}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)} \\
&+ \frac{\pi q_3L_{bm}e_3\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(-L_{cln}x+x^2\right)}{2L_{cln}{}^2\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)} \\
&+ \frac{M_{J,bottom,3}x}{L_{cln}} - \frac{2\pi e_3EI_{cln,3}\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{L_{cln}{}^2\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)} + \frac{M_{F,top,3}\left(L_{cln}-x\right)}{L_{cln}}
\end{aligned}
$$

The value e_3 must be calculated in the middle of the column. This value must be calculated. $\left(x = 0.5L_{cln}\right)$

$$e_3\frac{\pi^2 EI_{cln,3}}{L_{cln}{}^2}-\frac{2\pi e_3 EI_{cln,3}\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{L_{cln}{}^2\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$=\frac{(q_1+q_2)M_{J,bottom,3}L_{bm}L_{cln}{}^2}{48EI_{cln,3}}-\frac{(q_1+q_2)M_{F,top,3}L_{bm}L_{cln}{}^2}{48EI_{cln,3}}+\frac{(q_1+q_2)L_{bm}e_3}{2}$$

$$-\frac{\pi(q_1+q_2)L_{bm}e_3\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{8\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}+\frac{q_3L_{bm}e_0}{2}+\frac{q_3M_{J,bottom,1}L_{bm}L_{cln}{}^2}{48EI_{cln,1}}-\frac{q_3M_{F,top,1}L_{bm}L_{cln}{}^2}{48EI_{cln,1}}$$

$$+\frac{q_3M_{J,bottom,2}L_{bm}L_{cln}{}^2}{48EI_{cln,2}}-\frac{q_3M_{F,top,2}L_{bm}L_{cln}{}^2}{48EI_{cln,2}}+\frac{q_3M_{J,bottom,3}L_{bm}L_{cln}{}^2}{48EI_{cln,3}}-\frac{q_3M_{F,top,3}L_{bm}L_{cln}{}^2}{48EI_{cln,3}}+\frac{q_3L_{bm}e_1}{2}$$

$$+\frac{q_3L_{bm}e_2}{2}+\frac{q_3L_{bm}e_3}{2}-\frac{q_3\pi L_{bm}e_1\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{8\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$-\frac{q_3L_{bm}\left(4\pi e_2EI_{cln,2}+q_2L_{bm}L_{cln}{}^2z_2\right)\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{32EI_{cln,2}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}-\frac{\pi q_3L_{bm}e_3\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{8\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$+\frac{M_{J,bottom,3}}{2}-\frac{2\pi e_3EI_{cln,3}\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{L_{cln}{}^2\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}+\frac{M_{F,top,3}}{2}$$

Some expressions can be found at both sides of the equation. These expressions can be neglected.

$$e_3\frac{\pi^2 EI_{cln,3}}{L_{cln}{}^2}=\frac{(q_1+q_2)M_{J,bottom,3}L_{bm}L_{cln}{}^2}{48EI_{cln,3}}-\frac{(q_1+q_2)M_{F,top,3}L_{bm}L_{cln}{}^2}{48EI_{cln,3}}+\frac{(q_1+q_2)L_{bm}e_3}{2}$$

$$-\frac{\pi(q_1+q_2)L_{bm}e_3\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{8\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}+\frac{q_3L_{bm}e_0}{2}+\frac{q_3M_{J,bottom,1}L_{bm}L_{cln}{}^2}{48EI_{cln,1}}-\frac{q_3M_{F,top,1}L_{bm}L_{cln}{}^2}{48EI_{cln,1}}$$

$$+\frac{q_3M_{J,bottom,2}L_{bm}L_{cln}{}^2}{48EI_{cln,2}}-\frac{q_3M_{F,top,2}L_{bm}L_{cln}{}^2}{48EI_{cln,2}}+\frac{q_3M_{J,bottom,3}L_{bm}L_{cln}{}^2}{48EI_{cln,3}}-\frac{q_3M_{F,top,3}L_{bm}L_{cln}{}^2}{48EI_{cln,3}}+\frac{q_3L_{bm}e_1}{2}$$

$$+\frac{q_3L_{bm}e_2}{2}+\frac{q_3L_{bm}e_3}{2}-\frac{q_3\pi L_{bm}e_1\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{8\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$-\frac{q_3L_{bm}\left(4\pi e_2EI_{cln,2}+q_2L_{bm}L_{cln}{}^2z_2\right)\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{32EI_{cln,2}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}-\frac{\pi q_3L_{bm}e_3\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{8\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$+\frac{M_{J,bottom,3}}{2}+\frac{M_{F,top,3}}{2}$$

In this formula e_3 is the only unknown. To find the unknown value all expressions of e_3 are put on one side of the equation.

$$e_3 \frac{\pi^2 EI_{cln,3}}{L_{cln}{}^2} - \frac{(q_1+q_2)L_{bm}e_3}{2} + \frac{\pi(q_1+q_2)L_{bm}e_3\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{8\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)} - \frac{q_3L_{bm}e_3}{2}$$

$$+\frac{\pi q_3L_{bm}e_3\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{8\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)} = \frac{(q_1+q_2)M_{J,bottom,3}L_{bm}L_{cln}{}^2}{48EI_{cln,3}} - \frac{(q_1+q_2)M_{F,top,3}L_{bm}L_{cln}{}^2}{48EI_{cln,3}} + \frac{q_3L_{bm}e_0}{2}$$

$$+\frac{q_3M_{J,bottom,1}L_{bm}L_{cln}{}^2}{48EI_{cln,1}} - \frac{q_3M_{F,top,1}L_{bm}L_{cln}{}^2}{48EI_{cln,1}} + \frac{q_3M_{J,bottom,2}L_{bm}L_{cln}{}^2}{48EI_{cln,2}} - \frac{q_3M_{F,top,2}L_{bm}L_{cln}{}^2}{48EI_{cln,2}} + \frac{q_3M_{J,bottom,3}L_{bm}L_{cln}{}^2}{48EI_{cln,3}}$$

$$-\frac{q_3M_{F,top,3}L_{bm}L_{cln}{}^2}{48EI_{cln,3}} + \frac{q_3L_{bm}e_1}{2} + \frac{q_3L_{bm}e_2}{2} - \frac{q_3\pi L_{bm}e_1\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{8\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$-\frac{q_3L_{bm}\left(4\pi e_2EI_{cln,2}+q_2L_{bm}L_{cln}{}^2z_2\right)\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{32EI_{cln,2}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)} + \frac{M_{J,bottom,3}}{2} + \frac{M_{F,top,3}}{2}$$

Combine the same expressions. Separate e_3.

$$e_3\left(\frac{\pi^2 EI_{cln,3}}{L_{cln}{}^2} - \frac{(q_1+q_2+q_3)L_{bm}}{2} + \frac{\pi(q_1+q_2+q_3)L_{bm}\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{8\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}\right)$$

$$=\frac{(q_1+q_2)\left(M_{J,bottom,3}-M_{F,top,3}\right)L_{bm}L_{cln}{}^2}{48EI_{cln,3}} + \frac{q_3\left(M_{J,bottom,1}-M_{F,top,1}\right)L_{bm}L_{cln}{}^2}{48EI_{cln,1}}$$

$$+\frac{q_3\left(M_{J,bottom,2}-M_{F,top,2}\right)L_{bm}L_{cln}{}^2}{48EI_{cln,2}} + \frac{q_3\left(M_{J,bottom,3}-M_{F,top,3}\right)L_{bm}L_{cln}{}^2}{48EI_{cln,3}}$$

$$+\frac{q_3L_{bm}(e_0+e_1+e_2)}{2} - \frac{q_3\pi L_{bm}e_1\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{8\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$-\frac{q_3L_{bm}\left(4\pi e_2EI_{cln,2}+q_2L_{bm}L_{cln}{}^2z_2\right)\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{32EI_{cln,2}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)} + \frac{M_{J,bottom,3}}{2} + \frac{M_{F,top,3}}{2}$$

Everywhere the same denominator.

$$e_3\left(\begin{array}{l}\frac{96\pi^2 EI_{cln,1}EI_{cln,2}\left(EI_{cln,3}\right)^2\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{96L_{cln}{}^2EI_{cln,1}EI_{cln,2}EI_{cln,3}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}\\ -\frac{48\left(q_1+q_2+q_3\right)L_{bm}L_{cln}{}^2EI_{cln,1}EI_{cln,2}EI_{cln,3}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{96L_{cln}{}^2EI_{cln,1}EI_{cln,2}EI_{cln,3}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}\\ +\frac{12\pi\left(q_1+q_2+q_3\right)L_{bm}L_{cln}{}^2EI_{cln,1}EI_{cln,2}EI_{cln,3}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{96L_{cln}{}^2EI_{cln,1}EI_{cln,2}EI_{cln,3}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}\end{array}\right)$$

$$=\frac{2\left(q_1+q_2\right)\left(M_{J,bottom,3}-M_{F,top,3}\right)L_{bm}L_{cln}{}^4EI_{cln,1}EI_{cln,2}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{96L_{cln}{}^2EI_{cln,1}EI_{cln,2}EI_{cln,3}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$+\frac{2q_3\left(M_{J,bottom,1}-M_{F,top,1}\right)L_{bm}L_{cln}{}^4EI_{cln,2}EI_{cln,3}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{96L_{cln}{}^2EI_{cln,1}EI_{cln,2}EI_{cln,3}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$+\frac{2q_3\left(M_{J,bottom,2}-M_{F,top,2}\right)L_{bm}L_{cln}{}^4EI_{cln,1}EI_{cln,3}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{96L_{cln}{}^2EI_{cln,1}EI_{cln,2}EI_{cln,3}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$+\frac{2q_3\left(M_{J,bottom,3}-M_{F,top,3}\right)L_{bm}L_{cln}{}^4EI_{cln,1}EI_{cln,2}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{96L_{cln}{}^2EI_{cln,1}EI_{cln,2}EI_{cln,3}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$+\frac{48q_3L_{bm}L_{cln}{}^2\left(e_0+e_1+e_2\right)EI_{cln,1}EI_{cln,2}EI_{cln,3}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{96L_{cln}{}^2EI_{cln,1}EI_{cln,2}EI_{cln,3}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$-\frac{12q_3\pi L_{bm}L_{cln}{}^2e_1\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)EI_{cln,1}EI_{cln,2}EI_{cln,3}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{96L_{cln}{}^2EI_{cln,1}EI_{cln,2}EI_{cln,3}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$-\frac{3q_3L_{bm}L_{cln}{}^2EI_{cln,1}EI_{cln,3}\left(4\pi e_2EI_{cln,2}+q_2L_{bm}L_{cln}{}^2z_2\right)\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{96L_{cln}{}^2EI_{cln,1}EI_{cln,2}EI_{cln,3}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$+\frac{48M_{J,bottom,3}L_{cln}{}^2EI_{cln,1}EI_{cln,2}EI_{cln,3}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{96L_{cln}{}^2EI_{cln,1}EI_{cln,2}EI_{cln,3}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$+\frac{48M_{F,top,3}L_{cln}{}^2EI_{cln,1}EI_{cln,2}EI_{cln,3}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{96L_{cln}{}^2EI_{cln,1}EI_{cln,2}EI_{cln,3}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

Neglect the denominator

$$e_3\left(\begin{array}{l}96\pi^2EI_{cln,1}EI_{cln,2}\left(EI_{cln,3}\right)^2\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\\ -48\left(q_1+q_2+q_3\right)L_{bm}L_{cln}{}^2EI_{cln,1}EI_{cln,2}EI_{cln,3}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\\ +12\pi\left(q_1+q_2+q_3\right)L_{bm}L_{cln}{}^2EI_{cln,1}EI_{cln,2}EI_{cln,3}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\end{array}\right)$$

$$=2\left(q_1+q_2\right)\left(M_{J,bottom,3}-M_{F,top,3}\right)L_{bm}L_{cln}{}^4EI_{cln,1}EI_{cln,2}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)$$

$$+2q_3\left(M_{J,bottom,1}-M_{F,top,1}\right)L_{bm}L_{cln}{}^4EI_{cln,2}EI_{cln,3}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)$$

$$+2q_3\left(M_{J,bottom,2}-M_{F,top,2}\right)L_{bm}L_{cln}{}^4EI_{cln,1}EI_{cln,3}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)$$

$$+2q_3\left(M_{J,bottom,3}-M_{F,top,3}\right)L_{bm}L_{cln}{}^4EI_{cln,1}EI_{cln,2}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)$$

$$+48q_3L_{bm}L_{cln}{}^2\left(e_0+e_1+e_2\right)EI_{cln,1}EI_{cln,2}EI_{cln,3}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)$$

$$-12q_3\pi L_{bm}L_{cln}{}^2e_1\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)EI_{cln,1}EI_{cln,2}EI_{cln,3}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)$$

$$-3q_3L_{bm}L_{cln}{}^2EI_{cln,1}EI_{cln,3}\left(4\pi e_2EI_{cln,2}+q_2L_{bm}L_{cln}{}^2z_2\right)\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)$$

$$+48M_{J,bottom,3}L_{cln}{}^2EI_{cln,1}EI_{cln,2}EI_{cln,3}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)$$

$$+48M_{F,top,3}L_{cln}{}^2EI_{cln,1}EI_{cln,2}EI_{cln,3}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)$$

Combine the same expressions

$$12e_3EI_{cln,1}EI_{cln,2}EI_{cln,3}\begin{pmatrix}11L_{bm}EI_{cln,1}\\+5L_{cln}EI_{bm}\end{pmatrix}\begin{pmatrix}8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}\\+5L_{cln}EI_{bm}\end{pmatrix}\begin{pmatrix}\begin{pmatrix}8\pi^2EI_{cln,3}\\-4\left(q_1+q_2+q_3\right)L_{bm}L_{cln}{}^2\end{pmatrix}\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\\+\pi\left(q_1+q_2+q_3\right)L_{bm}L_{cln}{}^2\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\end{pmatrix}$$

$$=2L_{bm}L_{cln}{}^4\begin{pmatrix}\left(q_1+q_2+q_3\right)\left(M_{J,bottom,3}-M_{F,top,3}\right)EI_{cln,1}EI_{cln,2}\\+q_3\left(M_{J,bottom,1}-M_{F,top,1}\right)EI_{cln,2}EI_{cln,3}\\+q_3\left(M_{J,bottom,2}-M_{F,top,2}\right)EI_{cln,1}EI_{cln,3}\end{pmatrix}\begin{pmatrix}11L_{bm}EI_{cln,1}\\+5L_{cln}EI_{bm}\end{pmatrix}\begin{pmatrix}8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}\\+5L_{cln}EI_{bm}\end{pmatrix}\begin{pmatrix}8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}\\+5L_{cln}EI_{bm}\end{pmatrix}$$

$$+3q_3L_{bm}L_{cln}{}^2EI_{cln,1}EI_{cln,3}\begin{pmatrix}4EI_{cln,2}\begin{pmatrix}4\left(e_0+e_1+e_2\right)\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\\-\pi e_1\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\end{pmatrix}\begin{pmatrix}8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}\\+5L_{cln}EI_{bm}\end{pmatrix}\\-\left(4\pi e_2EI_{cln,2}+q_2L_{bm}L_{cln}{}^2z_2\right)\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\end{pmatrix}\begin{pmatrix}8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}\\+5L_{cln}EI_{bm}\end{pmatrix}$$

$$+48\left(M_{J,bottom,3}+M_{F,top,3}\right)L_{cln}{}^2EI_{cln,1}EI_{cln,2}EI_{cln,3}\begin{pmatrix}11L_{bm}EI_{cln,1}\\+5L_{cln}EI_{bm}\end{pmatrix}\begin{pmatrix}8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}\\+5L_{cln}EI_{bm}\end{pmatrix}\begin{pmatrix}8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}\\+5L_{cln}EI_{bm}\end{pmatrix}$$

The formula of e_3 can be found.

$$e_3=\frac{\left[\begin{matrix}2L_{bm}L_{cln}{}^4\begin{pmatrix}\left(q_1+q_2+q_3\right)\left(M_{J,bottom,3}-M_{F,top,3}\right)EI_{cln,1}EI_{cln,2}\\+q_3\left(M_{J,bottom,1}-M_{F,top,1}\right)EI_{cln,2}EI_{cln,3}\\+q_3\left(M_{J,bottom,2}-M_{F,top,2}\right)EI_{cln,1}EI_{cln,3}\end{pmatrix}\begin{pmatrix}11L_{bm}EI_{cln,1}\\+5L_{cln}EI_{bm}\end{pmatrix}\begin{pmatrix}8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}\\+5L_{cln}EI_{bm}\end{pmatrix}\begin{pmatrix}8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}\\+5L_{cln}EI_{bm}\end{pmatrix}\\+3q_3L_{bm}L_{cln}{}^2EI_{cln,1}EI_{cln,3}\begin{pmatrix}4EI_{cln,2}\begin{pmatrix}4\left(e_0+e_1+e_2\right)\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\\-\pi e_1\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\end{pmatrix}\begin{pmatrix}8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}\\+5L_{cln}EI_{bm}\end{pmatrix}\\-\left(4\pi e_2EI_{cln,2}+q_2L_{bm}L_{cln}{}^2z_2\right)\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\end{pmatrix}\begin{pmatrix}8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}\\+5L_{cln}EI_{bm}\end{pmatrix}\\+48\left(M_{J,bottom,3}+M_{F,top,3}\right)L_{cln}{}^2EI_{cln,1}EI_{cln,2}EI_{cln,3}\begin{pmatrix}11L_{bm}EI_{cln,1}\\+5L_{cln}EI_{bm}\end{pmatrix}\begin{pmatrix}8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}\\+5L_{cln}EI_{bm}\end{pmatrix}\begin{pmatrix}8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}\\+5L_{cln}EI_{bm}\end{pmatrix}\end{matrix}\right]}{12EI_{cln,1}EI_{cln,2}EI_{cln,3}\begin{pmatrix}11L_{bm}EI_{cln,1}\\+5L_{cln}EI_{bm}\end{pmatrix}\begin{pmatrix}8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}\\+5L_{cln}EI_{bm}\end{pmatrix}\begin{pmatrix}\begin{pmatrix}8\pi^2EI_{cln,3}\\-4\left(q_1+q_2+q_3\right)L_{bm}L_{cln}{}^2\end{pmatrix}\left(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\\+\pi\left(q_1+q_2+q_3\right)L_{bm}L_{cln}{}^2\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\end{pmatrix}}$$

Check the dimensions.

$$m=\frac{\left[\begin{array}{l} mm^4\begin{pmatrix}\left(Nm^{-1}+Nm^{-1}+Nm^{-1}\right)\left(Nm-Nm\right)Nm^2Nm^2\\ +Nm^{-1}\left(Nm-Nm\right)Nm^2Nm^2\\ +Nm^{-1}\left(Nm-Nm\right)Nm^2Nm^2\end{pmatrix}\begin{pmatrix}mNm^2\\ +mNm^2\end{pmatrix}\begin{pmatrix}mNm^2+mNm^2\\ +mNm^2\end{pmatrix}\begin{pmatrix}mNm^2+mNm^2\\ +mNm^2\end{pmatrix}\\ +Nm^{-1}mm^2Nm^2Nm^2\begin{pmatrix}Nm^2\begin{pmatrix}\left(m+m+m\right)\left(mNm^2+mNm^2\right)\\ -m\left(mNm^2+mNm^2\right)\end{pmatrix}\begin{pmatrix}mNm^2+mNm^2\\ +mNm^2\end{pmatrix}\\ -\left(mNm^2+Nm^{-1}mm^2m\right)\left(mNm^2+mNm^2\right)\left(mNm^2+mNm^2\right)\end{pmatrix}\begin{pmatrix}mNm^2+mNm^2\\ +mNm^2\end{pmatrix}\\ +\left(Nm+Nm\right)m^2Nm^2Nm^2Nm^2\begin{pmatrix}mNm^2\\ +mNm^2\end{pmatrix}\begin{pmatrix}mNm^2+mNm^2\\ +mNm^2\end{pmatrix}\begin{pmatrix}mNm^2+mNm^2\\ +mNm^2\end{pmatrix}\end{array}\right]}{Nm^2Nm^2Nm^2\begin{pmatrix}mNm^2\\ +mNm^2\end{pmatrix}\begin{pmatrix}mNm^2+mNm^2\\ +mNm^2\end{pmatrix}\begin{pmatrix}\begin{pmatrix}Nm^2\\ -\left(Nm^{-1}+Nm^{-1}+Nm^{-1}\right)mm^2\end{pmatrix}\left(mNm^2+mNm^2+mNm^2\right)\\ +\left(Nm^{-1}+Nm^{-1}+Nm^{-1}\right)mm^2\left(mNm^2+mNm^2\right)\end{pmatrix}}$$

$$m=\frac{\left[\begin{array}{l} m^5\left(\left(Nm^{-1}\right)\left(Nm\right)N^2m^4\right)\left(Nm^3\right)\left(Nm^3\right)\left(Nm^3\right)\\ +N^3m^6\left(Nm^2\left(\left(m\right)\left(Nm^3\right)\right)\left(Nm^3\right)\right)\left(Nm^3\right)\\ +\left(Nm\right)N^3m^8\left(Nm^3\right)\left(Nm^3\right)\left(Nm^3\right)\end{array}\right]}{N^3m^6\left(Nm^3\right)\left(Nm^3\right)\left(\left(Nm^2\right)\left(Nm^3\right)\right)}$$

$$m=\frac{N^7m^{18}+N^7m^{18}+N^7m^{18}}{N^7m^{17}}$$

The dimensions are correct.

T.3 Conclusion

The conclusions of the non-linear analysis in Appendix T are some formulas to calculate the additional deflection. The most important formulas are repeated in this part of the Appendix. In Appendix U a calculation example is made. The use of the formulas becomes clear at Appendix U. The formulas are:

$$y_1=e_0$$

$$y_{2,2}=\frac{L_{cln}{}^2\left(M_{J,bottom,2}-M_{F,top,2}\right)}{16EI_{cln,2}}$$

$$y_{3,2} = e_2$$

$$y_{4,2} = -\frac{\left(2\pi e_2 EI_{cln,2} + N_2 L_{cln}{}^2 z_2\right)\left(3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)}{8EI_{cln,2}\left(8L_{bm}EI_{cln,2} + 3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)}$$

$$y_{5,2} = \frac{N_2 L_{cln}{}^2 z_2}{8EI_{cln,2}}$$

$$\varphi_{extra,2,x=0} = \frac{4L_{bm}\left(2\pi e_2 EI_{cln,2} + N_2 L_{cln}{}^2 z_2\right)}{L_{cln}\left(8L_{bm}EI_{cln,2} + 3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)}$$

$$e_2 = \frac{\begin{pmatrix} 2L_{bm}L_{cln}{}^4\begin{pmatrix}\left(q_1+q_2\right)\left(M_{J,bottom,2} - M_{F,top,2}\right)EI_{cln,1} \\ +q_2\left(M_{J,bottom,1} - M_{F,top,1}\right)EI_{cln,2}\end{pmatrix}\left(11L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)\begin{pmatrix}8L_{bm}EI_{cln,2} + 3L_{bm}EI_{cln,1} \\ +5L_{cln}EI_{bm}\end{pmatrix} \\ +24q_2\left(q_1+q_2\right)L_{bm}{}^3L_{cln}{}^4 z_2 EI_{cln,1}EI_{cln,2}\left(11L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right) \\ +12q_2L_{bm}L_{cln}{}^2EI_{cln,1}EI_{cln,2}\left(8L_{bm}EI_{cln,2} + 3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)\begin{pmatrix}4e_0\left(11L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right) \\ +e_1\begin{pmatrix}\left(44-3\pi\right)L_{bm}EI_{cln,1} \\ +\left(20-5\pi\right)L_{cln}EI_{bm}\end{pmatrix}\end{pmatrix} \\ +48\left(M_{J,bottom,2} + M_{F,top,2}\right)L_{cln}{}^2EI_{cln,1}EI_{cln,2}\left(11L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2} + 3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)\end{pmatrix}}{12EI_{cln,1}EI_{cln,2}\left(11L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)\begin{pmatrix}4\left(2\pi^2EI_{cln,2} - \left(q_1+q_2\right)L_{bm}L_{cln}{}^2\right)\begin{pmatrix}8L_{bm}EI_{cln,2} + 3L_{bm}EI_{cln,1} \\ +5L_{cln}EI_{bm}\end{pmatrix} \\ +\pi\left(q_1+q_2\right)L_{bm}L_{cln}{}^2\left(3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)\end{pmatrix}}$$

$$y_{2,3} = \frac{L_{cln}{}^2\left(M_{J,bottom,3} - M_{F,top,3}\right)}{16EI_{cln,3}}$$

$$y_{3,3} = e_3$$

$$y_{4,3} = -\frac{\pi e_3\left(3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)}{4\left(8L_{bm}EI_{cln,3} + 3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)}$$

$$\varphi_{extra,3,x=0} = \frac{8\pi L_{bm}e_3EI_{cln,3}}{L_{cln}\left(8L_{bm}EI_{cln,3} + 3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)}$$

$$e_3 = \frac{\left[\begin{array}{l} 2L_{bm}L_{cln}^{\ 4}\begin{pmatrix} (q_1+q_2+q_3)(M_{J,bottom,3}-M_{F,top,3})EI_{cln,1}EI_{cln,2} \\ +q_3(M_{J,bottom,1}-M_{F,top,1})EI_{cln,2}EI_{cln,3} \\ +q_3(M_{J,bottom,2}-M_{F,top,2})EI_{cln,1}EI_{cln,3} \end{pmatrix}\begin{pmatrix} 11L_{bm}EI_{cln,1} \\ +5L_{cln}EI_{bm} \end{pmatrix}\begin{pmatrix} 8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1} \\ +5L_{cln}EI_{bm} \end{pmatrix}\begin{pmatrix} 8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1} \\ +5L_{cln}EI_{bm} \end{pmatrix} \\ +3q_3L_{bm}L_{cln}^{\ 2}EI_{cln,1}EI_{cln,3}\begin{pmatrix} 4EI_{cln,2}\begin{pmatrix} 4(e_0+e_1+e_2)(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}) \\ -\pi e_1(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}) \end{pmatrix}\begin{pmatrix} 8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1} \\ +5L_{cln}EI_{bm} \end{pmatrix} \\ -(4\pi e_2EI_{cln,2}+q_2L_{bm}L_{cln}^{\ 2}z_2)(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm})(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}) \end{pmatrix}\begin{pmatrix} 8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1} \\ +5L_{cln}EI_{bm} \end{pmatrix} \\ +48(M_{J,bottom,3}+M_{F,top,3})L_{cln}^{\ 2}EI_{cln,1}EI_{cln,2}EI_{cln,3}\begin{pmatrix} 11L_{bm}EI_{cln,1} \\ +5L_{cln}EI_{bm} \end{pmatrix}\begin{pmatrix} 8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1} \\ +5L_{cln}EI_{bm} \end{pmatrix}\begin{pmatrix} 8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1} \\ +5L_{cln}EI_{bm} \end{pmatrix} \end{array}\right]}{12EI_{cln,1}EI_{cln,2}EI_{cln,3}\begin{pmatrix} 11L_{bm}EI_{cln,1} \\ +5L_{cln}EI_{bm} \end{pmatrix}\begin{pmatrix} 8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1} \\ +5L_{cln}EI_{bm} \end{pmatrix}\begin{pmatrix} \begin{pmatrix} 8\pi^2EI_{cln,3} \\ -4(q_1+q_2+q_3)L_{bm}L_{cln}^{\ 2} \end{pmatrix}(8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}) \\ +\pi(q_1+q_2+q_3)L_{bm}L_{cln}^{\ 2}(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}) \end{pmatrix}}$$

Appendix U Calculations

This Appendix is about the calculations of an extended frame. In Appendix S and in Appendix T some formulas are derived. These formulas will be used to calculate the ultimate load. Appendix U.1 is about the manual calculation of the frame (according to the analyzed formulas). Appendix U.2 exists in the MatLab file. Appendix U.3 is the calculation according to the Dutch code. Appendix U.4 is the calculation file of Matrix Frame.

U.1 Manual calculations.

This Appendix is about the calculation of the ultimate load according to the formulas derived in Appendices S and T. The structure and the loads on the structure can be found in Figure U.1. The formulas are too complex to make the calculations by hand. The computer program MatLab has been used to calculate the numerical values. The MatLab file can be found in Appendix U.2.

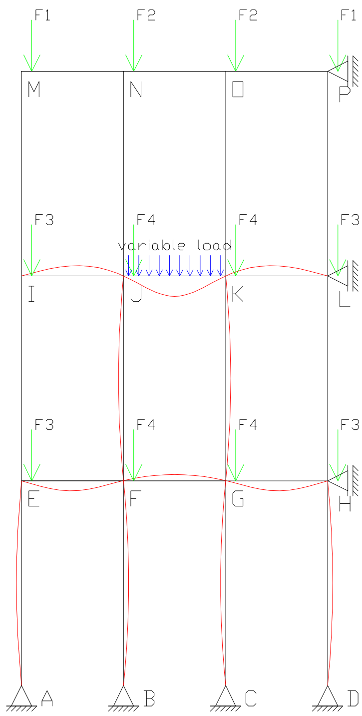


Figure U.1: Structure

The section properties are:

L_{cln} = 10000 mm
L_{bm} = 5000 mm
e_0 = 10 mm
$EI_{cln,1}$ = 6.959*10^{13} Nmm^2
$EI_{cln,2}$ = 5.263*10^{13} Nmm^2
$EI_{cln,3}$ = 3.573*10^{13} Nmm^2
$Z_{cln,1}$ = 1.891*10^{6} mm^3
$Z_{cln,2}$ = 1.432*10^{6} mm^3
$Z_{cln,3}$ = 9.721*10^{5} mm^3
EI_{bm} = 8.864*10^{14} Nmm^2
$f_{c,1}$ = 177.5 N/mm^2 (first critical stress)
$f_{c,2}$ = 532.5 N/mm^2 (second critical stress)
$A_{cln,1}$ = 14280 mm^2
$A_{cln,2a}$ = 11655 mm^2
$A_{cln,b}$ = 9905 mm^2
$A_{cln,3}$ = 7280 mm^2
$EA_{cln,1}$ = 2.999*10^{9} N
$EA_{cln,2}$ = 2.448*10^{9} N
$EA_{cln,3}$ = 1.529*10^{9} N
z_2 = 39.6 mm

The required loads are:

F_2 = 85.5*10^{3} N
F_4 = 80.5*10^{3} N

The total load on column FJ is:
$0.5q_{variable}L_{bm} + 166*10^3$ N

$$k_r = \frac{3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}}{4L_{bm}L_{cln}}$$

$$k_r = \frac{3*5000*6.959*10^{13} + 5*10000*8.864*10^{14}}{4*5000*10000}$$

$$k_r = 2.268*10^{11}\,Nm/rad$$

The first load case:

q_1 = 836 N/mm

N_1 = $0.5q_1L_{bm}$
N_1 = 0.5*777*5000
N_1 = $1942.5*10^3$ N

Calculate the linear rotation.

$$\varphi_{J,1} = \frac{q_1L_{bm}L_{cln}\begin{pmatrix} -1800L_{cln}^{\ 4}\left(EI_{bm}\right)^4 - 8280L_{bm}L_{cln}^{\ 3}\left(EI_{bm}\right)^3EI_{cln,1} - 10980L_{bm}^{\ 2}L_{cln}^{\ 2}\left(EI_{bm}\right)^2\left(EI_{cln,1}\right)^2 \\ -4104L_{bm}^{\ 3}L_{cln}EI_{bm}\left(EI_{cln,1}\right)^3 \\ -L_{bm}^{\ 3}EA_{cln}\left(3L_{bm}EI_{cln,1} + 2L_{cln}EI_{bm}\right)\left(4L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)\left(7L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right) \end{pmatrix}}{24EA_{cln,1}\begin{pmatrix} 125L_{cln}^{\ 4}\left(EI_{bm}\right)^4 + 445L_{bm}L_{cln}^{\ 3}\left(EI_{bm}\right)^3EI_{cln,1} + 1335L_{bm}^{\ 2}L_{cln}^{\ 2}\left(EI_{bm}\right)^2\left(EI_{cln,1}\right)^2 \\ +1287L_{bm}^{\ 3}L_{cln}EI_{bm}\left(EI_{cln,1}\right)^3 + 270L_{bm}^{\ 4}\left(EI_{cln,1}\right)^4 \end{pmatrix}}$$

$$\varphi_{J,1} = \frac{777*5000*10000\begin{pmatrix} -1800*10000^4*\left(8.864*10^{14}\right)^4 - 8280*5000*10000^3*\left(8.864*10^{14}\right)^3*6.959*10^{13} \\ -10980*5000^2*10000^2*\left(8.864*10^{14}\right)^2\left(6.959*10^{13}\right)^2 \\ -4104*5000^3*10000*8.864*10^{14}\left(6.959*10^{13}\right)^3 \\ -5000^3*2.999*10^9\begin{pmatrix} 3*5000*6.959*10^{13} \\ +2*10000*8.864*10^{14}\end{pmatrix}\begin{pmatrix} 4*5000*6.959*10^{13} \\ +5*10000*8.864*10^{14}\end{pmatrix}\begin{pmatrix} 7*50000*6.959*10^{13} \\ +5*10000*8.864*10^{14}\end{pmatrix} \end{pmatrix}}{24*2.999*10^9\begin{pmatrix} 125*10000^4*\left(8.864*10^{14}\right)^4 + 445*5000*10000^3*\left(8.864*10^{14}\right)^3*6.959*10^{13} \\ +1335*5000^2*10000^2*\left(8.864*10^{14}\right)^2*\left(6.959*10^{13}\right)^2 \\ +1287*5000^3*10000*8.864*10^{14}*\left(6.959*10^{13}\right)^3 + 270*5000^4*\left(6.959*10^{13}\right)^4 \end{pmatrix}}$$

$$\varphi_{J,1} = -0.0034rad$$

$$M_{J,bottom,1} = \frac{EI_{cln,1}\left(3q_1L_{cln}^{\ 2}EI_{bm} - 4\varphi_{J,1}EA_{cln,1}\left(6L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)\right)}{L_{cln}EA_{cln,1}\left(7L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)}$$

$$M_{J,bottom,1}=\frac{6.959*10^{13}\begin{pmatrix}3*777*10000^2*8.864*10^{14}\\ -4*-0.0034*2.999*10^9*\begin{pmatrix}6*5000*6.959*10^{13}\\ +5*10000*8.864*10^{14}\end{pmatrix}\end{pmatrix}}{10000*2.999*10^9\left(7*5000*6.959*10^{13}+5*10000*8.864*10^{14}\right)}$$

$$M_{J,bottom,1}=104.54*10^6\,Nmm$$

$$\varphi_{F,1}=\frac{M_{J,bottom,1}L_{bm}L_{cln}EA_{cln,1}-3q_1L_{cln}{}^2EI_{bm}}{2EA_{cln,1}\left(6L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$\varphi_{F,1}=\frac{104.54*10^6*5000*10000*2.999*10^9-3*777*10000^2*8.864*10^{14}}{2*2.999*10^9\left(6*5000*6.959*10^{13}+5*10000*8.864*10^{14}\right)}$$

$$\varphi_{F,1}=-6.861*10^{-4}\,rad$$

$$M_{F,top,1}=\frac{1.5q_1L_{cln}EI_{bm}}{L_{bm}EA_{cln,1}}+\left(\frac{3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}}{L_{bm}L_{cln}}\right)\varphi_F$$

$$M_{F,top,1}=\frac{1.5*777*10000*8.864*10^{14}}{5000*2.999*10^9}+\left(\frac{3*5000*6.959*10^{13}+5*10000*8.864*10^{14}}{5000*10000}\right)*-6.861*10^{-4}$$

$$M_{F,top,1}=66.57*10^6\,Nm$$

$$y_{2,1}=\frac{\left(M_{J,bottom,1}-M_{F,top,1}\right)L_{cln}{}^2}{16EI_{cln,1}}$$

$$y_{2,1}=\frac{\left(104.54*10^6-66.57*10^6\right)*10000^2}{16*6.959*10^{13}}$$

$$y_{2,1}=3.4mm$$

$$e_1=\frac{\begin{matrix}q_1L_{bm}L_{cln}{}^2\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(24e_0EI_{cln,1}+L_{cln}{}^2\left(M_{J,bottom,1}-M_{F,top,1}\right)\right)\\ +24\left(M_{J,bottom,1}+M_{F,top,1}\right)L_{cln}{}^2EI_{cln,1}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\end{matrix}}{6EI_{cln}\begin{pmatrix}8\pi^2EI_{cln}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\\ +q_1L_{bm}L_{cln}{}^2\left(\left(3\pi-44\right)L_{bm}EI_{cln,1}+\left(5\pi-20\right)L_{cln}EI_{bm}\right)\end{pmatrix}}$$

$$e_1 = \frac{777*5000*10000^2*\begin{pmatrix}11*5000*6.959*10^{13}\\+5*10000*8.864*10^{14}\end{pmatrix}\left(24*10*6.959*10^{13}+10000^2*\begin{pmatrix}105.54*10^6\\-66.57*10^6\end{pmatrix}\right) +24*\left(105.54*10^6+66.57*10^6\right)*10000^2*6.959*10^{13}*\begin{pmatrix}11*5000*6.959*10^{13}\\+5*10000*8.864*10^{14}\end{pmatrix}}{6*6.959*10^{13}\begin{pmatrix}8\pi^2*6.959*10^{13}*\left(11*5000*6.959*10^{13}+5*10000*8.864*10^{14}\right)\\+777*5000*10000^2*\begin{pmatrix}\left(3\pi-44\right)*5000*6.959*10^{13}\\+\left(5\pi-20\right)*10000*8.864*10^{14}\end{pmatrix}\end{pmatrix}}$$

$$e_1 = 17.2mm$$

$$y_{4,1} = \frac{-\pi e_1\left(3L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}{4\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}$$
$$y_{4,1} = \frac{-\pi*17.2*\left(3*5000*6.959*10^{13}+5*10000*8.864*10^{14}\right)}{4*\left(11*5000*6.959*10^{13}+5*10000*8.864*10^{14}\right)}$$
$$y_{4,1} = -12.7mm$$

$$y_{total,1} = e_0 + y_{2,1} + e_1 + y_{4,1}$$
$$y_{total,1} = 10+3.4+17.2-12.7$$
$$y_{total,1} = 17.9mm$$

$$\varphi_{extra,x=0} = \frac{8\pi L_{bm}e_1EI_{cln}}{L_{cln}\left(11L_{bm}EI_{cln}+5L_{cln}EI_{bm}\right)}$$
$$\varphi_{extra,x=0} = \frac{8\pi*5000*17.2*6.959*10^{13}}{10000*\left(11*5000*6.959*10^{13}+5*10000*8.864*10^{14}\right)}$$
$$\varphi_{extra,1,x=0} = 3.124*10^{-4}rad$$

$$M_{J,bottom,tot,1} = M_{J,bottom,1} - \varphi_{extra,1,x=0}k_r$$
$$M_{J,bottom,tot,1} = 104.54*10^6 - 3.124*10^{-4}*2.268*10^{11}$$
$$M_{J,bottom,tot,1} = 33.70*10^6\,Nmm$$

$$M_{F,top,tot,1} = M_{F,top,1} - \varphi_{extra,1,x=0}k_r$$
$$M_{F,top,tot,1} = 66.57*10^6 - 3.124*10^{-4}*2.268*10^{11}$$
$$M_{F,top,tot,1} = -4.27*10^6\,Nmm$$

$$\sigma_{right,1} = -\frac{\tfrac{1}{2}\left(M_{J,bottom,tot,1} - M_{F,top,tot,1}\right)}{Z_{cln,1}} - \frac{\left(N_1 + N_{required}\right)}{A_{cln,1}} - \frac{\left(N_1 + N_{required}\right)y_{total,1}}{Z_{cln,1}}$$

$$\sigma_{right,1} = -\frac{\tfrac{1}{2}\left(33.70*10^6 + 4.27*10^6\right)}{1.891*10^6} - \frac{\left(1942.5*10^3 + 166*10^3\right)}{14280} - \frac{\left(1942.5*10^3 + 166*10^3\right)*17.9}{1.891*10^6}$$

$\sigma_{right,1} = -177.6$ N/mm^2 (first critical stress)

$$\sigma_{left,1} = \frac{\tfrac{1}{2}\left(M_{J,bottom,tot,1} - M_{F,top,tot,1}\right)}{Z_{cln,1}} - \frac{\left(N_1 + N_{required}\right)}{A_{cln,1}} + \frac{\left(N_1 + N_{required}\right)y_{total,1}}{Z_{cln,1}}$$

$$\sigma_{left,1} = \frac{\tfrac{1}{2}\left(33.70*10^6 + 4.27*10^6\right)}{1.891*10^6} - \frac{\left(1942.5*10^3 + 166*10^3\right)}{14280} + \frac{\left(1942.5*10^3 + 166*10^3\right)*17.9}{1.891*10^6}$$

$\sigma_{left,1} = -117.7$ N/mm^2

$$\sigma_{centre,1} = -\frac{\left(N_1 + N_{required}\right)}{A_{cln,1}}$$

$$\sigma_{centre,1} = -\frac{\left(1942.5*10^3 + 166*10^3\right)}{14280}$$

$\sigma_{centre,1} = -147.7$ N/mm^2

The second load case:

q_2 = 416 N/mm

q_2 has been split in two parts:

q_{2a} = 140 N/mm

q_{2b} = 276 N/mm

$N_2 = 0.5q_2L_{bm}$	$N_{2a} = 0.5q_{2a}L_{bm}$	$N_{2ba} = 0.5q_{2ba}L_{bm}$
N_2 = 0.5*416*5000	N_{2a} = 0.5*140*5000	N_{2ba} = 0.5*276*5000
$N_2 = 1040*10^3$ N	$N_{2a} = 350*10^3$ N	$N_{2ba} = 690*10^3$ N

The calculation of the linear rotation is the same as in the first load case. Only the numerical values are given.

$$\varphi_{J,2} = -0.0018 rad$$

$$M_{J,bottom,2} = 55.97*10^6 Nm$$

$$\varphi_{F,2} = -3.673*10^{-4} rad$$

$$M_{F,top,2} = 35.64*10^6 Nm$$

$$y_{2,2} = 2.4 mm$$

$$e_2 = \frac{\begin{pmatrix} 2L_{bm}L_{cln}^{4}\begin{pmatrix}(q_1+q_2)\left(M_{J,bottom,2}-M_{F,top,2}\right)EI_{cln,1} \\ +q_2\left(M_{J,bottom,1}-M_{F,top,1}\right)EI_{cln,2}\end{pmatrix}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\begin{pmatrix}8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1} \\ +5L_{cln}EI_{bm}\end{pmatrix} \\ +24q_2(q_1+q_2)L_{bm}^{3}L_{cln}^{4}z_2EI_{cln,1}EI_{cln,2}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right) \\ +12q_2L_{bm}L_{cln}^{2}EI_{cln,1}EI_{cln,2}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\begin{pmatrix}4e_0\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right) \\ +e_1\begin{pmatrix}(44-3\pi)L_{bm}EI_{cln,1} \\ +(20-5\pi)L_{cln}EI_{bm}\end{pmatrix}\end{pmatrix} \\ +48\left(M_{J,bottom,2}+M_{F,top,2}\right)L_{cln}^{2}EI_{cln,1}EI_{cln,2}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\end{pmatrix}}{12EI_{cln,1}EI_{cln,2}\left(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\begin{pmatrix}4\left(2\pi^2EI_{cln,2}-(q_1+q_2)L_{bm}L_{cln}^{2}\right)\begin{pmatrix}8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1} \\ +5L_{cln}EI_{bm}\end{pmatrix} \\ +\pi(q_1+q_2)L_{bm}L_{cln}^{2}\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)\end{pmatrix}}$$

$$e_2 = 15.6mm$$

$$y_{4,2} = -\frac{\left(4\pi e_2EI_{cln,2}+q_2L_{bm}L_{cln}^{2}z_2\right)\left(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}{16EI_{cln,2}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$y_{4,2} = -\frac{\left(4\pi*15.6*5.263*10^{13}+416*5000*10000^2*39.6\right)\left(3*5000*6.959*10^{13}+5*10000*8.864*10^{14}\right)}{16*5.263*10^{13}\left(8*5000*5.263*10^{13}+3*5000*6.959*10^{13}+5*10000*8.864*10^{14}\right)}$$

$$y_{4,2} = -21.0mm$$

$$y_{5,2} = \frac{q_2L_{bm}L_{cln}^{2}z_2}{16EI_{cln,2}}$$

$$y_{5,2} = \frac{416*5000*10000^2*39.6}{16*5.263*10^{13}}$$

$$y_{5,2} = 9.8mm$$

$$y_{total,2} = y_{total,1}+y_{2,2}+e_2+y_{4,2}+y_{5,2}$$

$$y_{total,2} = 17.9+2.4+15.6-21.0+9.8$$

$$y_{total,2} = 24.6mm$$

$$\varphi_{extra,2,x=0} = \frac{2L_{bm}\left(4\pi e_2EI_{cln,2}+q_2L_{bm}L_{cln}^{2}z_2\right)}{L_{cln}\left(8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}\right)}$$

$$\varphi_{extra,2,x=0} = \frac{2*5000\left(4\pi*15.6*5.263*10^{13}+416*5000*10000^2*39.6\right)}{10000*\left(8*5000*5.263*10^{13}+3*5000*6.959*10^{13}+5*10000*8.864*10^{14}\right)}$$

$$\varphi_{extra,2,x=0} = 3.905*10^{-4}rad$$

$$M_{J,bottom,tot,2} = M_{J,bottom,2} - \varphi_{extra,2,x=0} k_r$$
$$M_{J,bottom,tot,2} = 55.97*10^6 - 3.905*10^{-4} * 2.268*10^{11}$$
$$M_{J,bottom,tot,2} = -32.60*10^6 \, Nmm$$

$$M_{F,top,tot,2} = M_{F,top,2} - \varphi_{extra,2,x=0} k_r$$
$$M_{F,top,tot,2} = 35.64*10^6 - 3.905*10^{-4} * 2.268*10^{11}$$
$$M_{F,top,tot,2} = -57.20*10^6 \, Nmm$$

$$\sigma_{right,2} = \sigma_{right,1} - \frac{\frac{1}{2}\left(M_{J,bottom,tot,2} - M_{F,top,tot,2}\right)}{Z_{cln,2}} - \frac{N_{2a}}{A_{cln,2a}} - \frac{N_{2b}}{A_{cln,2b}} - \frac{N_2 y_{total,2}}{Z_{cln,2}}$$
$$- \frac{\left(N_1 + N_{required} + N_2\right)\left(y_{total,2} - y_{total,1}\right)}{Z_{cln,2}}$$
$$\sigma_{right,2} = -177.6 - \frac{\frac{1}{2}\left(-32.60*10^6 + 57.20*10^6\right)}{1.432*10^6} - \frac{350*10^3}{11655} - \frac{690*10^3}{9905} - \frac{1040*10^3 * 24.6}{1.432*10^6}$$
$$- \frac{\left(1942.5*10^3 + 166*10^3 + 1040*10^3\right)\left(24.6 - 17.9\right)}{1.432*10^6}$$
$\sigma_{right,2} = -317.1$ N/mm²

$$\sigma_{left,2} = \sigma_{left,1} + \frac{\frac{1}{2}\left(M_{J,bottom,tot,2} - M_{F,top,tot,2}\right)}{Z_{cln,2}} - \frac{N_{2a}}{A_{cln,2a}} - \frac{N_{2b}}{A_{cln,2b}} + \frac{N_2 y_{total,2}}{Z_{cln,2}}$$
$$+ \frac{\left(N_1 + N_{required} + N_2\right)\left(y_{total,2} - y_{total,1}\right)}{Z_{cln,2}}$$
$$\sigma_{left,2} = -117.7 + \frac{\frac{1}{2}\left(-32.60*10^6 + 57.20*10^6\right)}{1.432*10^6} - \frac{350*10^3}{11655} - \frac{690*10^3}{9905} + \frac{1040*10^3 * 24.6}{1.432*10^6}$$
$$+ \frac{\left(1942.5*10^3 + 166*10^3 + 1040*10^3\right)\left(24.6 - 17.9\right)}{1.432*10^6}$$
$\sigma_{left,2} = -177.5$ N/mm² (first critical stress)

$$\sigma_{centre,2a} = \sigma_{centre,1} - \frac{N_{2a}}{A_{cln,2a}}$$
$$\sigma_{centre,2a} = -147.7 - \frac{340*10^3}{11655}$$
$\sigma_{centre,2a} = -177.7$ N/mm² (first critical stress)

$$\sigma_{centre,2} = \sigma_{centre,1} - \frac{N_{2a}}{A_{cln,2a}} - \frac{N_{2b}}{A_{cln,2b}}$$
$$\sigma_{centre,2} = -147.7 - \frac{350*10^3}{11655} - \frac{960*10^3}{9905}$$

$\sigma_{centre,2a} = -247.3$ N/mm^2

The third load case:

q_3 = 390 N/mm

N_3 = $0.5q_3L_{bm}$

N_3 = 0.5*390*5000

N_3 = $975*10^3$ N

The calculation of the linear rotation is the same as in the first load case. Only the numerical values are given.

$\varphi_{J,3} = -0.0017 rad$

$M_{J,bottom,3} = 52.47*10^6 Nm$

$\varphi_{F,3} = -3.444*10^{-4} rad$

$M_{F,top,3} = 33.42*10^6 Nm$

$y_{2,3} = 3.3mm$

$$e_3 = \frac{\left[\begin{array}{l} 2L_{bm}L_{cln}^4 \begin{pmatrix} (q_1+q_2+q_3)(M_{J,bottom,3}-M_{F,top,3})EI_{cln,1}EI_{cln,2} \\ +q_3(M_{J,bottom,1}-M_{F,top,1})EI_{cln,2}EI_{cln,3} \\ +q_3(M_{J,bottom,2}-M_{F,top,2})EI_{cln,1}EI_{cln,3} \end{pmatrix} \begin{pmatrix} 11L_{bm}EI_{cln,1} \\ +5L_{cln}EI_{bm} \end{pmatrix} \begin{pmatrix} 8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1} \\ +5L_{cln}EI_{bm} \end{pmatrix} \begin{pmatrix} 8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1} \\ +5L_{cln}EI_{bm} \end{pmatrix} \\ +3q_3L_{bm}L_{cln}^2EI_{cln,1}EI_{cln,3} \begin{pmatrix} 4EI_{cln,2} \begin{pmatrix} 4(e_0+e_1+e_2)(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}) \\ -\pi e_1(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}) \end{pmatrix} \begin{pmatrix} 8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1} \\ +5L_{cln}EI_{bm} \end{pmatrix} \\ -(4\pi e_2EI_{cln,2}+q_2L_{bm}L_{cln}^2z_2)(11L_{bm}EI_{cln,1}+5L_{cln}EI_{bm})(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}) \end{pmatrix} \begin{pmatrix} 8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1} \\ +5L_{cln}EI_{bm} \end{pmatrix} \\ +48(M_{J,bottom,3}+M_{F,top,3})L_{cln}^2EI_{cln,1}EI_{cln,2}EI_{cln,3} \begin{pmatrix} 11L_{bm}EI_{cln,1} \\ +5L_{cln}EI_{bm} \end{pmatrix} \begin{pmatrix} 8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1} \\ +5L_{cln}EI_{bm} \end{pmatrix} \begin{pmatrix} 8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1} \\ +5L_{cln}EI_{bm} \end{pmatrix} \end{array}\right]}{12EI_{cln,1}EI_{cln,2}EI_{cln,3} \begin{pmatrix} 11L_{bm}EI_{cln,1} \\ +5L_{cln}EI_{bm} \end{pmatrix} \begin{pmatrix} 8L_{bm}EI_{cln,2}+3L_{bm}EI_{cln,1} \\ +5L_{cln}EI_{bm} \end{pmatrix} \begin{pmatrix} \begin{pmatrix} 8\pi^2EI_{cln,3} \\ -4(q_1+q_2+q_3)L_{bm}L_{cln}^2 \end{pmatrix} (8L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}) \\ +\pi(q_1+q_2+q_3)L_{bm}L_{cln}^2(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm}) \end{pmatrix}}$$

$e_3 = 23.8mm$

$$y_{4,3} = -\frac{\pi e_3(3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm})}{4(2L_{bm}EI_{cln,3}+3L_{bm}EI_{cln,1}+5L_{cln}EI_{bm})}$$

$$y_{4,3} = -\frac{\pi*23.8(3*5000*6.959*10^{13}+5*10000*10^{14})}{4(2*5000*3.572*10^{13}+3*5000*6.959*10^{13}+5*10000*8.864*10^{14})}$$

$y_{4,3} = -18.1mm$

$y_{total,3} = y_{total,2} + y_{2,3} + e_3 + y_{4,3}$
$y_{total,3} = 24.6 + 3.3 + 23.8 - 18.1$
$y_{total,3} = 33.6mm$

$$\varphi_{extra,3,x=0} = \frac{2\pi L_{bm} e_3 EI_{cln,3}}{L_{cln}\left(2L_{bm}EI_{cln,3} + \left(3L_{bm}EI_{cln,1} + 5L_{cln}EI_{bm}\right)\right)}$$
$$\varphi_{extra,3,x=0} = \frac{2\pi * 5000 * 23.8 * 3.573 * 10^{13}}{10000\left(2 * 5000 * 3.573 * 10^{13} + 3 * 5000 * 6.959 * 10^{13} + 5 * 10000 * 8.864 * 10^{14}\right)}$$
$$\varphi_{extra,3,x=0} = 2.285 * 10^{-4} rad$$

$M_{J,bottom,tot,3} = M_{J,bottom,3} - \varphi_{extra,3,x=0} k_r$
$M_{J,bottom,tot,2} = 52.47 * 10^6 - 2.285 * 10^{-4} * 2.268 * 10^{11}$
$M_{J,bottom,tot,2} = 0.64 * 10^6 \, Nmm$

$M_{F,top,tot,2} = M_{F,top,2} - \varphi_{extra,2,x=0} k_r$
$M_{F,top,tot,2} = 33.42 * 10^6 - 2.285 * 10^{-4} * 2.268 * 10^{11}$
$M_{F,top,tot,1} = -18.42 * 10^6 \, Nmm$

$$\sigma_{right,3} = \sigma_{right,2} - \frac{½\left(M_{J,bottom,tot,3} - M_{F,top,tot,3}\right)}{Z_{cln,3}} - \frac{N_3}{A_{cln,3}} - \frac{N_3 y_{total,3}}{Z_{cln,3}}$$
$$- \frac{\left(N_1 + N_{required} + N_2 + N_3\right)\left(y_{total,3} - y_{total,2}\right)}{Z_{cln,3}}$$
$$\sigma_{right,3} = -317.1 - \frac{½\left(0.64 * 10^6 + 18.42 * 10^6\right)}{9.721 * 10^6} - \frac{975 * 10^3}{7280} - \frac{975 * 10^3 * 33.6}{9.721 * 10^6}$$
$$- \frac{\left(1942.5 * 10^3 + 166 * 10^3 + 1040 * 10^3 + 975 * 10^3\right)\left(33.6 - 24.6\right)}{9.721 * 10^6}$$
$\sigma_{right,3} = -532.9$ N/mm^2 (second critical stress)

$$\sigma_{left,3} = \sigma_{left,2} + \frac{½\left(M_{J,bottom,tot,3} - M_{F,top,tot,3}\right)}{Z_{cln,3}} - \frac{N_3}{A_{cln,3}} + \frac{N_3 y_{total,3}}{Z_{cln,3}}$$
$$+\frac{\left(N_1 + N_{required} + N_2 + N_3\right)\left(y_{total,3} - y_{total,2}\right)}{Z_{cln,3}}$$

$$\sigma_{left,3} = -177.7 + \frac{½\left(0.64*10^6 + 18.42*10^6\right)}{9.721*10^6} - \frac{975*10^3}{7280} + \frac{975*10^3*33.6}{9.721*10^6}$$
$$+\frac{\left(1942.5*10^3 + 166*10^3 + 1040*10^3 + 975*10^3\right)\left(33.6 - 24.6\right)}{9.721*10^6}$$
$$\sigma_{left,3} = -229.7 \text{ N/mm}^2$$

$$\sigma_{centre,3} = \sigma_{centre,2} - \frac{N_3}{A_{cln,23}}$$
$$\sigma_{centre,3} = -247.3 - \frac{975*10^3}{7280}$$
$$\sigma_{centre,2a} = -381.3 \text{ N/mm}^2$$

After the third load case, the right flange fully yields and the column fails. The bearing capacity is 1583 N/mm. The load on column FJ is $2145*10^3$ N.

The total deflection of column FJ is 33.6 mm.

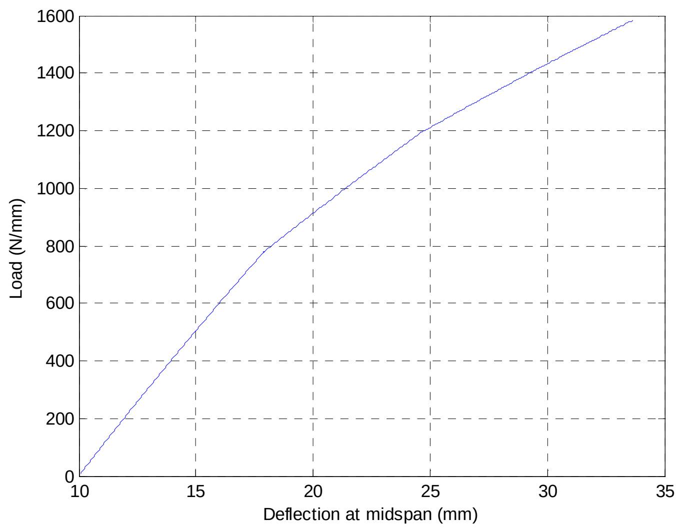


Figure U.2: Load-deflection graphic

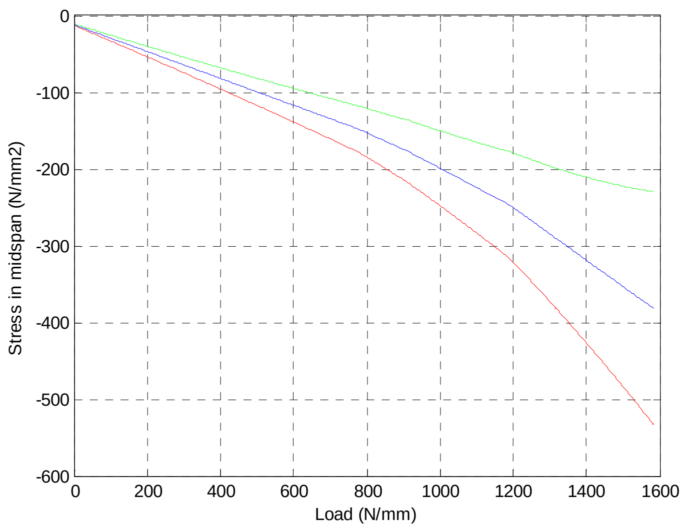


Figure U.3: Stresses

Figure U.2 is the load-deflection graphic of the analyzed structure.

Figure U.3 is the stress distribution in the mid-section. In Figure U.3 is

- The red line with the stress in the right flange
- The blue line is the stress in the centre of the web
- The green line is the stress in the left flange.

U.2 Calculation file.

The computer program MatLab has been used to make the calculations. In this Appendix the calculation file of the extended frame can be found.

```
clear; clf; clc; close;
%NON-Linear analysis extended frame, residual stress

E=210000;
fy=355;
qq=1;%belastingsstap
delta=0.001;
Lbm=5000;
Lcln=10000;
e0=delta*Lcln;
Freq=166e3;

%input profiles
HEA=[2.124E+03  2.534E+03   3.142E+03   3.877E+03   4.525E+03   5.383E+03
6.434E+03   7.684E+03   8.682E+03   9.726E+03   1.125E+04   1.244E+04
1.335E+04   1.428E+04   1.590E+04   1.780E+04   1.975E+04   2.118E+04
2.265E+04   2.416E+04   2.605E+04   2.858E+04   3.205E+04   3.468E+04;%A
(cross-section)

3.492E+06   6.062E+06   1.033E+07   1.673E+07   2.510E+07   3.692E+07
5.410E+07   7.763E+07   1.046E+08   1.367E+08   1.826E+08   2.293E+08
2.769E+08   3.309E+08   4.507E+08   6.372E+08   8.698E+08   1.119E+09
1.412E+09   1.752E+09   2.153E+09   3.034E+09   4.221E+09   5.538E+09;%I
(moment of inertia)

8.301E+04   1.195E+05   1.735E+05   2.451E+05   3.249E+05   4.295E+05
5.685E+05   7.446E+05   9.198E+05   1.112E+06   1.383E+06   1.628E+06
1.850E+06   2.088E+06   2.562E+06   3.216E+06   3.949E+06   4.622E+06
5.350E+06   6.136E+06   7.032E+06   8.699E+06   1.081E+07   1.282E+07;%Z
plastic (Section modulus)

7.276E+04   1.063E+05   1.554E+05   2.201E+05   2.936E+05   3.886E+05
5.152E+05   6.751E+05   8.364E+05   1.013E+06   1.260E+06   1.479E+06
1.678E+06   1.891E+06   2.311E+06   2.896E+06   3.550E+06   4.146E+06
4.787E+06   5.474E+06   6.241E+06   7.682E+06   9.485E+06   1.119E+07;%Z
elastic (Section modulus)

4.055E+01   4.891E+01   5.734E+01   6.569E+01   7.448E+01   8.282E+01
9.170E+01   1.005E+02   1.097E+02   1.186E+02   1.274E+02   1.358E+02
1.440E+02   1.522E+02   1.684E+02   1.892E+02   2.099E+02   2.299E+02
2.497E+02   2.693E+02   2.875E+02   3.258E+02   3.629E+02   3.996E+02;%i
(Gyration radius)

96 114 133 152 171 190 210 230 250 270 290 310 330 350 390 440 490 540 590
640 690 790 890 990;%h height

100 120 140 160 180 200 220 240 260 280 300 300 300 300 300 300 300 300 300
300 300 300 300 300;%b width

8 8 8.5 9 9.5 10 11 12 12.5 13 14 15.5 16.5 17.5 19 21 23 24 25 26 27 28 30
31;%tf thickness flange
```

```
5 5 5.5 6 6 6.5 7 7.5 7.5 8 8.5 9 9.5 10 11 11.5 12 12.5 13 13.5 14.5 15 16
16.5];%tw thickness web

cln=14;
    Acln(1,1)=HEA(1,cln);
    Icln(1,1)=HEA(2,cln);
    Zcln(1,1)=HEA(4,cln);
    hcln=HEA(6,cln);
    bcln=HEA(7,cln);
    tfcln=HEA(8,cln);
    twcln=HEA(9,cln);

bm=23;
    Abm(1,1)=HEA(1,bm);
    Ibm(1,1)=HEA(2,bm);
    Zbm(1,1)=HEA(4,bm);

    Npcln=Acln(1,1)*fy;
    Mpcln=Zcln(1,1)*fy;

     if cln<=14 ;
         S=0.5;
     else
         S=0.3;
     end% if
    yield1=-(1-S)*fy;
    yield2=-(1+S)*fy;

Acln(1,2)=Acln(1,1)-2*(0.25*bcln)*tfcln;
Acln(1,3)=Acln(1,2)-2*(0.25*bcln)*tfcln-twcln*hcln*0.5;
Acln(1,4)=Acln(1,2)-twcln*hcln*0.5;
Icln(1,2)=Icln(1,1)-2*(0.25*bcln)*tfcln*(0.5*hcln)^2;
Icln(1,3)=Icln(1,2)-2*(0.25*bcln)*tfcln*(0.5*hcln)^2;
Zcln(1,2)=2*Icln(1,2)/hcln;
Zcln(1,3)=2*Icln(1,3)/hcln;

z(1,1)=0;
z(1,2)=1*((0.5*bcln*tfcln*0.5*tfcln+(hcln-
2*tfcln)*twcln*0.5*hcln+bcln*tfcln*(hcln-0.5*tfcln))/(0.5*bcln*tfcln+(hcln-
2*tfcln)*twcln+bcln*tfcln)-0.5*hcln);
z(1,3)=0;

kr=(3*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm)/(4*Lbm*Lcln);

hulpa=-1800*Lcln^4*(E*Ibm)^4 - 8280*Lbm*Lcln^3*(E*Ibm)^3*E*Icln(1,1) -
10980*Lbm^2*Lcln^2*(E*Ibm)^2*(E*Icln(1,1))^2 -
4104*Lbm^3*Lcln*E*Ibm*(E*Icln(1,1))^3 -
Lbm^3*E*Acln(1,1)*(3*Lbm*E*Icln(1,1)+2*Lcln*E*Ibm)*(4*Lbm*E*Icln(1,1)+5*Lcl
n*E*Ibm)*(7*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm);
hulpb=125*Lcln^4*(E*Ibm)^4 + 445*Lbm*Lcln^3*(E*Ibm)^3*E*Icln(1,1) +
1335*Lbm^2*Lcln^2*(E*Ibm)^2*(E*Icln(1,1))^2 +
1287*Lbm^3*Lcln*E*Ibm*(E*Icln(1,1))^3 + 270*Lbm^4*(E*Icln(1,1))^4;

q1a=0;
sigmatopa=0;
sigmabottoma=0;
while sigmatopa>yield1 & sigmabottoma>yield1;
    q1a=q1a+1;
```

```
    Q1a(q1a)=q1a*qq;
    N1a(q1a)=0.5*Q1a(q1a)*Lbm + Freq;
    Qtota(q1a)=Q1a(q1a);
    Ntota(q1a)=N1a(q1a);

    phiJ1a(q1a)=Q1a(q1a)*Lbm*Lcln*hulpa/(24*Lbm*E*Acln(1,1)*hulpb);%linear
    MJbottom1a(q1a)=2*E*Icln(1,1)*(1.5*Q1a(q1a)*Lcln^2*E*Ibm -
2*phiJ1a(q1a)*E*Acln(1,1)*(6*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm))/(Lcln*E*Acln(1,
1)*(7*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm));%linear
    phiF1a(q1a)=(MJbottom1a(q1a)*Lbm*Lcln*E*Acln(1,1)-
3*Q1a(q1a)*Lcln^2*E*Ibm)/(2*E*Acln(1,1)*(6*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm));%
linear
    MFtop1a(q1a)=1.5*Q1a(q1a)*Lcln*E*Ibm/(Lbm*E*Acln(1,1)) +
(3*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm)*phiF1a(q1a)/(Lbm*Lcln);%linear


hulp11a(q1a)=Q1a(q1a)*Lbm*Lcln^2*(11*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm)*(24*e0*E
*Icln(1,1)+Lcln^2*(MJbottom1a(q1a)-MFtop1a(q1a))) +
24*(MJbottom1a(q1a)+MFtop1a(q1a))*Lcln^2*E*Icln(1,1)*(11*Lbm*E*Icln(1,1)+5*
Lcln*E*Ibm);

hulp12a(q1a)=6*E*Icln(1,1)*(8*pi^2*E*Icln(1,1)*(11*Lbm*E*Icln(1,1)+5*Lcln*E
*Ibm) + Q1a(q1a)*Lbm*Lcln^2*((3*pi-44)*Lbm*E*Icln(1,1)+(5*pi-
20)*Lcln*E*Ibm));
    e1a(q1a)=hulp11a(q1a)/hulp12a(q1a);%non-linear
    e1aa(q1a)=(MJbottom1a(q1a)-MFtop1a(q1a))*Lcln^2/(16*E*Icln(1,1));

phiextra1a(q1a)=8*pi*Lbm*e1a(q1a)*E*Icln(1,1)/(Lcln*(11*Lbm*E*Icln(1,1)+5*L
cln*E*Ibm));
    e1ab(q1a)=phiextra1a(q1a)*kr*Lcln^2/(8*E*Icln(1,1));

    etota(q1a)=e0 + e1a(q1a) + e1aa(q1a) - e1ab(q1a);

    MJbottomtot1a(q1a)=MJbottom1a(q1a) - phiextra1a(q1a)*kr;
    MJbottomtota(q1a)=MJbottomtot1a(q1a);
    MFtoptot1a(q1a)=MFtop1a(q1a) - phiextra1a(q1a)*kr;
    MFtoptota(q1a)=MFtoptot1a(q1a);
    phiFtota(q1a)=phiF1a(q1a) - phiextra1a(q1a);

    sigmatopa=-0.5*(MJbottomtot1a(q1a)-MFtoptot1a(q1a))/Zcln(1,1) -
Ntota(q1a)/Acln(1,1) - Ntota(q1a)*etota(q1a)/Zcln(1,1);% - Freq/Acln(1,1);
    sigmatoptota(q1a)=sigmatopa;
    sigmabottoma=0.5*(MJbottomtot1a(q1a)-MFtoptot1a(q1a))/Zcln(1,1) -
Ntota(q1a)/Acln(1,1) + Ntota(q1a)*etota(q1a)/Zcln(1,1);% - Freq/Acln(1,1);
    sigmabottomtota(q1a)=sigmabottoma;
    sigmacentrea= - Ntota(q1a)/Acln(1,1);% - Freq/Acln(1,1);
    sigmacentretota(q1a)=sigmacentrea;

end% while q1

q2a=0;

if (4*(2*pi^2*E*Icln(1,2)-
(Qtota(q1a)+2*qq)*Lbm*Lcln^2)*(8*Lbm*E*Icln(1,2)+3*Lbm*E*Icln(1,1)+5*Lcln*E
*Ibm) +
pi*(Qtota(q1a)+2*qq)*Lbm*Lcln^2*(3*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm))>0;

while (sigmatopa>yield1 | sigmabottoma>yield1) & sigmatopa>yield2 &
sigmabottoma<-yield1;
    q2a=q2a+1;
```

```
    Q2a(q2a)=q2a*qq;
        if sigmacentrea>yield1
            Q2aa(q2a)=q2a*qq;
        else
            Q2aa(q2a)=Q2aa(q2a-1);
        end%if

    Qtota(q1a+q2a)=Qtota(q1a)+Q2a(q2a);

    N2a(q2a)=0.5*Q2a(q2a)*Lbm;
    N2aa(q2a)=0.5*Q2aa(q2a)*Lbm;
    Qtota(q1a+q2a)=Qtota(q1a)+Q2a(q2a);
    Ntota(q1a+q2a)=Ntota(q1a)+N2a(q2a);

    phiJ2a(q2a)=Q2a(q2a)*Lbm*Lcln*hulpa/(24*Lbm*E*Acln(1,1)*hulpb);%linear
    MJbottom2a(q2a)=2*E*Icln(1,1)*(1.5*Q2a(q2a)*Lcln^2*E*Ibm - 
2*phiJ2a(q2a)*E*Acln(1,1)*(6*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm))/(Lcln*E*Acln(1,
1)*(7*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm));%linear
    phiF2a(q2a)=(MJbottom2a(q2a)*Lbm*Lcln*E*Acln(1,1)-
3*Q2a(q2a)*Lcln^2*E*Ibm)/(2*E*Acln(1,1)*(6*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm));%
linear
    MFtop2a(q2a)=1.5*Q2a(q2a)*Lcln*E*Ibm/(Lbm*E*Acln(1,1)) + 
(3*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm)*phiF2a(q2a)/(Lbm*Lcln);%linear

    hulp21a(q2a)=2*Lbm*Lcln^4*(Qtota(q1a+q2a)*(MJbottom2a(q2a)-
MFtop2a(q2a))*E*Icln(1,1)+Q2a(q2a)*(MJbottom1a(q1a)-
MFtop1a(q1a))*E*Icln(1,2))*(11*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm)*(8*Lbm*E*Icln(
1,2)+3*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm);

hulp22a(q2a)=24*Q2a(q2a)*Qtota(q1a+q2a)*Lbm^3*Lcln^4*z(1,2)*E*Icln(1,1)*E*I
cln(1,2)*(11*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm);

hulp23a(q2a)=12*Q2a(q2a)*Lbm*Lcln^2*E*Icln(1,1)*E*Icln(1,2)*(8*Lbm*E*Icln(1
,2)+3*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm)*(4*e0*(11*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm)
+e1a(q1a)*((44-3*pi)*Lbm*E*Icln(1,1)+(20-5*pi)*Lcln*E*Ibm));

hulp24a(q2a)=48*(MJbottom2a(q2a)+MFtop2a(q2a))*Lcln^2*E*Icln(1,1)*E*Icln(1,
2)*(11*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm)*(8*Lbm*E*Icln(1,2)+3*Lbm*E*Icln(1,1)+5
*Lcln*E*Ibm);

hulp25a(q2a)=12*E*Icln(1,1)*E*Icln(1,2)*(11*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm)*(
4*(2*pi^2*E*Icln(1,2)-
Qtota(q1a+q2a)*Lbm*Lcln^2)*(8*Lbm*E*Icln(1,2)+3*Lbm*E*Icln(1,1)+5*Lcln*E*Ib
m) + pi*Qtota(q1a+q2a)*Lbm*Lcln^2*(3*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm));

e2a(q2a)=(hulp21a(q2a)+hulp22a(q2a)+hulp23a(q2a)+hulp24a(q2a))/hulp25a(q2a)
;%non-linear

    e2aa(q2a)=(MJbottom2a(q2a)-MFtop2a(q2a))*Lcln^2/(16*E*Icln(1,2));

phiextra2a(q2a)=4*Lbm*(2*pi*e2a(q2a)*E*Icln(1,2)+N2a(q2a)*Lcln^2*z(1,2))/(L
cln*(8*Lbm*E*Icln(1,2)+3*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm));
    e2ab(q2a)=phiextra2a(q2a)*kr*Lcln^2/(8*E*Icln(1,2));
    e2ac(q2a)=N2a(q2a)*z(1,2)*Lcln^2/(8*E*Icln(1,2));

    etota(q1a+q2a)=e0 + e1a(q1a) + e2a(q2a) + e1aa(q1a) + e2aa(q2a) - 
e1ab(q1a) - e2ab(q2a) + e2ac(q2a);

    MJbottomtot2a(q2a)=MJbottom2a(q2a) - phiextra2a(q2a)*kr;
    MJbottomtota(q1a+q2a)=MJbottomtota(q1a)+MJbottomtot2a(q2a);
```

```
    MFtoptot2a(q2a)=MFtop2a(q2a) - phiextra2a(q2a)*kr;
    MFtoptota(q1a+q2a)=MFtoptota(q1a)+MFtoptot2a(q2a);
    phiFtota(q1a+q2a)=phiFtota(q1a) + phiF2a(q2a) - phiextra2a(q2a);

    sigmatopa=sigmatoptota(q1a) - 0.5*(MJbottomtot2a(q2a)-
MFtoptot2a(q2a))/Zcln(1,2) - N2aa(q2a)/Acln(1,2) - (N2a(q2a)-
N2aa(q2a))/Acln(1,4)  - Ntota(q1a+q2a)*(etota(q1a+q2a)-
etota(q1a))/Zcln(1,2) - N2a(q2a)*etota(q1a+q2a)/Zcln(1,2);
    sigmatoptota(q1a+q2a)=sigmatopa;
    sigmabottoma=sigmabottomtota(q1a) + 0.5*(MJbottomtot2a(q2a)-
MFtoptot2a(q2a))/Zcln(1,2) - N2aa(q2a)/Acln(1,2) - (N2a(q2a)-
N2aa(q2a))/Acln(1,4)  + Ntota(q1a+q2a)*(etota(q1a+q2a)-
etota(q1a))/Zcln(1,2) + N2a(q2a)*etota(q1a+q2a)/Zcln(1,2);
    sigmabottomtota(q1a+q2a)=sigmabottoma;
    sigmacentrea=sigmacentretota(q1a) - N2aa(q2a)/Acln(1,2) - (N2a(q2a)-
N2aa(q2a))/Acln(1,4);
    sigmacentretota(q1a+q2a)=sigmacentrea;

end% while q2

else
    Qtota(q1a+q2a)=Qtota(q1a+q2a);
end% if

q3a=0;
if ((8*pi^2*E*Icln(1,3) -
4*(Qtota(q1a+q2a)+2*qq)*Lbm*Lcln^2)*(8*Lbm*E*Icln(1,3)+3*Lbm*E*Icln(1,1)+5*
Lcln*E*Ibm) +
pi*(Qtota(q1a+q2a)+2*qq)*Lbm*Lcln^2*(3*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm))>0;

while sigmatopa>yield2;
    q3a=q3a+1;
    Q3a(q3a)=q3a*qq;
    Qtota(q1a+q2a+q3a)=Qtota(q1a+q2a)+Q3a(q3a);

    N3a(q3a)=0.5*Q3a(q3a)*Lbm;
    Qtota(q1a+q2a+q3a)=Qtota(q1a+q2a)+Q3a(q3a);
    Ntota(q1a+q2a+q3a)=Ntota(q1a+q2a)+N3a(q3a);

    phiJ3a(q3a)=Q3a(q3a)*Lbm*Lcln*hulpa/(24*Lbm*E*Acln(1,1)*hulpb);%linear
    MJbottom3a(q3a)=2*E*Icln(1,1)*(1.5*Q3a(q3a)*Lcln^2*E*Ibm -
2*phiJ3a(q3a)*E*Acln(1,1)*(6*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm))/(Lcln*E*Acln(1,
1)*(7*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm));%linear
    phiF3a(q3a)=(MJbottom3a(q3a)*Lbm*Lcln*E*Acln(1,1)-
3*Q3a(q3a)*Lcln^2*E*Ibm)/(2*E*Acln(1,1)*(6*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm));%
linear
    MFtop3a(q3a)=1.5*Q3a(q3a)*Lcln*E*Ibm/(Lbm*E*Acln(1,1)) +
(3*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm)*phiF3a(q3a)/(Lbm*Lcln);%linear

    hulp31a(q3a)=2*Lbm*Lcln^4*(Qtota(q1a+q2a+q3a)*(MJbottom3a(q3a)-
MFtop3a(q3a))*E*Icln(1,1)*E*Icln(1,3) + Q3a(q3a)*(MJbottom1a(q1a)-
MFtop1a(q1a))*E*Icln(1,2)*E*Icln(1,3) + Q3a(q3a)*(MJbottom2a(q2a)-
MFtop2a(q2a))*E*Icln(1,1)*E*Icln(1,3)) * (11*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm)
* (8*Lbm*E*Icln(1,2)+3*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm) *
(8*Lbm*E*Icln(1,3)+3*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm);
    hulp32a(q3a)=3*Q3a(q3a)*Lbm*Lcln^2*E*Icln(1,1)*E*Icln(1,3) *
(4*E*Icln(1,2)*(4*(e0+e1a(q1a)+e2a(q2a))*(11*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm)
- pi*e1a(q1a)*(3*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm)) *
(8*Lbm*E*Icln(1,2)+3*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm) -
(4*pi*e2a(q2a)*E*Icln(1,2)+Q2a(q2a)*Lbm*Lcln^2*z(1,2)) *
```

```
(11*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm) *
(3*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm))*(8*Lbm*E*Icln(1,3)+3*Lbm*E*Icln(1,1)+5*Lc
ln*E*Ibm);

hulp33a(q3a)=48*(MJbottom3a(q3a)+MFtop3a(q3a))*Lcln^2*E*Icln(1,1)*E*Icln(1,
2)*E*Icln(1,3) * (11*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm) *
(8*Lbm*E*Icln(1,2)+3*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm) *
(8*Lbm*E*Icln(1,3)+3*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm);
    hulp34a(q3a)=12*E*Icln(1,1)*E*Icln(1,2)*E*Icln(1,3) *
(11*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm) *
(8*Lbm*E*Icln(1,2)+3*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm) * ((8*pi^2*E*Icln(1,3) -
4*Qtota(q1a+q2a+q3a)*Lbm*Lcln^2)*(8*Lbm*E*Icln(1,3)+3*Lbm*E*Icln(1,1)+5*Lcl
n*E*Ibm) +
pi*Qtota(q1a+q2a+q3a)*Lbm*Lcln^2*(3*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm));
    e3a(q3a)=(hulp31a(q3a)+hulp32a(q3a)+hulp33a(q3a))/hulp34a(q3a);%non-
linear

    e3aa(q3a)=(MJbottom3a(q3a)-MFtop3a(q3a))*Lcln^2/(16*E*Icln(1,3));

phiextra3a(q3a)=8*pi*Lbm*e3a(q3a)*E*Icln(1,3)/(Lcln*(8*Lbm*E*Icln(1,3)+3*Lb
m*E*Icln(1,1)+5*Lcln*E*Ibm));
    e3ab(q3a)=phiextra3a(q3a)*kr*Lcln^2/(8*E*Icln(1,3));

    etota(q1a+q2a+q3a)=e0 + e1a(q1a) + e2a(q2a) + e3a(q3a) + e1aa(q1a) +
e2aa(q2a) + e3aa(q3a) - e1ab(q1a) - e2ab(q2a) - e3ab(q3a) + e2ac(q2a);

    MJbottomtot3a(q3a)=MJbottom3a(q3a) - phiextra3a(q3a)*kr;
    MJbottomtota(q1a+q2a+q3a)=MJbottomtota(q1a+q2a)+MJbottomtot3a(q3a);
    MFtoptot3a(q3a)=MFtop3a(q3a) - phiextra3a(q3a)*kr;
    MFtoptota(q1a+q2a+q3a)=MFtoptota(q1a+q2a)+MFtoptot3a(q3a);
    phiFtota(q1a+q2a+q3a)=phiFtota(q1a+q2a) + phiF3a(q3a) -
phiextra3a(q3a);

    sigmatopa=sigmatoptota(q1a+q2a) - 0.5*(MJbottomtot3a(q3a)-
MFtoptot3a(q3a))/Zcln(1,3) - N3a(q3a)/Acln(1,3) -
Ntota(q1a+q2a+q3a)*(etota(q1a+q2a+q3a)-etota(q1a+q2a))/Zcln(1,3) -
N3a(q3a)*etota(q1a+q2a+q3a)/Zcln(1,3);
    sigmatoptota(q1a+q2a+q3a)=sigmatopa;
    sigmabottoma=sigmabottomtota(q1a+q2a) + 0.5*(MJbottomtot3a(q3a)-
MFtoptot3a(q3a))/Zcln(1,3) - N3a(q3a)/Acln(1,3) +
Ntota(q1a+q2a+q3a)*(etota(q1a+q2a+q3a)-etota(q1a+q2a))/Zcln(1,3) +
N3a(q3a)*etota(q1a+q2a+q3a)/Zcln(1,3);
    sigmabottomtota(q1a+q2a+q3a)=sigmabottoma;
    sigmacentrea=sigmacentretota(q1a+q2a) - N3a(q3a)/Acln(1,3);
    sigmacentretota(q1a+q2a+q3a)=sigmacentrea;

end% while q3

else
    Qtota(q1a+q2a+q3a)=Qtota(q1a+q2a+q3a);
end% if

Qmaxa=Qtota(q1a+q2a+q3a);

clear MFtop1a;clear MFtop2a;clear MFtop3a;
clear MFtoptot1a;clear MFtoptot2a;clear MFtoptot3a;
clear MJbottom1a;clear MJbottom2a;clear MJbottom3a;
clear MJbottomtot1a;clear MJbottomtot2a;clear MJbottomtot3a;
clear N1a;clear N2a;clear N3a;clear N2aa;
clear Q1a;clear Q2a;clear Q3a;clear Q2aa;
```

```
clear e1a;clear e2a;clear e3a;
clear e1aa;clear e1ab;clear e2aa;clear e2ab;clear e2ac;clear e3aa;clear
e3ab;
clear hulp11a;clear hulp12a;
clear hulp21a;clear hulp22a;clear hulp23a;clear hulp24a;clear hulp25a;
clear hulp31a;clear hulp32a;clear hulp33a;clear hulp34a
clear phiF1a;clear phiF2a;clear phiF3a;
clear phiJ1a;clear phiJ2a;clear phiJ3a;
clear phiextra1a;clear phiextra2a;clear phiextra3a;

q1b=0;
sigmatopb=0;
sigmabottomb=0;
while sigmatopb>yield1 & sigmabottomb>yield1;
    q1b=q1b+1;
    Q1b(q1b)=q1b*qq;
    N1b(q1b)=0.5*Q1b(q1b)*Lbm + Freq;
    Qtotb(q1b)=Q1b(q1b);
    Ntotb(q1b)=N1b(q1b);

    phiJ1b(q1b)=Q1b(q1b)*Lbm*Lcln*hulpa/(24*Lbm*E*Acln(1,1)*hulpb);%linear
    MJbottom1b(q1b)=2*E*Icln(1,1)*(1.5*Q1b(q1b)*Lcln^2*E*Ibm -
2*phiJ1b(q1b)*E*Acln(1,1)*(6*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm))/(Lcln*E*Acln(1,
1)*(7*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm));%linear
    phiF1b(q1b)=(MJbottom1b(q1b)*Lbm*Lcln*E*Acln(1,1)-
3*Q1b(q1b)*Lcln^2*E*Ibm)/(2*E*Acln(1,1)*(6*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm));%
linear
    MFtop1b(q1b)=1.5*Q1b(q1b)*Lcln*E*Ibm/(Lbm*E*Acln(1,1)) +
(3*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm)*phiF1b(q1b)/(Lbm*Lcln);%linear


hulp11b(q1b)=Q1b(q1b)*Lbm*Lcln^2*(11*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm)*(24*e0*E
*Icln(1,1)+Lcln^2*(MJbottom1b(q1b)-MFtop1b(q1b))) +
24*(MJbottom1b(q1b)+MFtop1b(q1b))*Lcln^2*E*Icln(1,1)*(11*Lbm*E*Icln(1,1)+5*
Lcln*E*Ibm);

hulp12b(q1b)=6*E*Icln(1,1)*(8*pi^2*E*Icln(1,1)*(11*Lbm*E*Icln(1,1)+5*Lcln*E
*Ibm) + Q1b(q1b)*Lbm*Lcln^2*((3*pi-44)*Lbm*E*Icln(1,1)+(5*pi-
20)*Lcln*E*Ibm));
    e1b(q1b)=hulp11b(q1b)/hulp12b(q1b);%non-linear
    e1ba(q1b)=(MJbottom1b(q1b)-MFtop1b(q1b))*Lcln^2/(16*E*Icln(1,1));

phiextra1b(q1b)=8*pi*Lbm*e1b(q1b)*E*Icln(1,1)/(Lcln*(11*Lbm*E*Icln(1,1)+5*L
cln*E*Ibm));
    e1bb(q1b)=phiextra1b(q1b)*kr*Lcln^2/(8*E*Icln(1,1));

    etotb(q1b)=e0 + e1b(q1b) + e1ba(q1b) - e1bb(q1b);

    MJbottomtot1b(q1b)=MJbottom1b(q1b) - phiextra1b(q1b)*kr;
    MJbottomtotb(q1b)=MJbottomtot1b(q1b);
    MFtoptot1b(q1b)=MFtop1b(q1b) - phiextra1b(q1b)*kr;
    MFtoptotb(q1b)=MFtoptot1b(q1b);

    sigmatopb=-MJbottomtot1b(q1b)/Zcln(1,1) - Ntotb(q1b)/Acln(1,1);
    sigmatoptotb(q1b)=sigmatopb;
    sigmabottomb=MJbottomtot1b(q1b)/Zcln(1,1) - Ntotb(q1b)/Acln(1,1);
    sigmabottomtotb(q1b)=sigmabottomb;
    sigmacentreb= - Ntotb(q1b)/Acln(1,1);
    sigmacentretotb(q1b)=sigmacentreb;
```

```
end% while q1

q2b=0;

if (4*(2*pi^2*E*Icln(1,2)-
(Qtotb(q1b)+2*qq)*Lbm*Lcln^2)*(8*Lbm*E*Icln(1,2)+3*Lbm*E*Icln(1,1)+5*Lcln*E
*Ibm) +
pi*(Qtotb(q1b)+2*qq)*Lbm*Lcln^2*(3*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm))>0;

while (sigmatopb>yield1 | sigmabottomb>yield1) & sigmatopb>yield2 &
sigmabottomb<-yield1;
    q2b=q2b+1;
    Q2b(q2b)=q2b*qq;
        if sigmacentreb>yield1
            Q2ba(q2b)=q2b*qq;
        else
            Q2ba(q2b)=Q2ba(q2b-1);
        end%if

    Qtotb(q1b+q2b)=Qtotb(q1b)+Q2b(q2b);

    N2b(q2b)=0.5*Q2b(q2b)*Lbm;
    N2ba(q2b)=0.5*Q2ba(q2b)*Lbm;
    Qtotb(q1b+q2b)=Qtotb(q1b)+Q2b(q2b);
    Ntotb(q1b+q2b)=Ntotb(q1b)+N2b(q2b);

    phiJ2b(q2b)=Q2b(q2b)*Lbm*Lcln*hulpa/(24*Lbm*E*Acln(1,1)*hulpb);%linear
    MJbottom2b(q2b)=2*E*Icln(1,1)*(1.5*Q2b(q2b)*Lcln^2*E*Ibm -
2*phiJ2b(q2b)*E*Acln(1,1)*(6*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm))/(Lcln*E*Acln(1,
1)*(7*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm));%linear
    phiF2b(q2b)=(MJbottom2b(q2b)*Lbm*Lcln*E*Acln(1,1)-
3*Q2b(q2b)*Lcln^2*E*Ibm)/(2*E*Acln(1,1)*(6*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm));%
linear
    MFtop2b(q2b)=1.5*Q2b(q2b)*Lcln*E*Ibm/(Lbm*E*Acln(1,1)) +
(3*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm)*phiF2b(q2b)/(Lbm*Lcln);%linear

    hulp21b(q2b)=2*Lbm*Lcln^4*(Qtotb(q1b+q2b)*(MJbottom2b(q2b)-
MFtop2b(q2b))*E*Icln(1,1)+Q2b(q2b)*(MJbottom1b(q1b)-
MFtop1b(q1b))*E*Icln(1,2))*(11*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm)*(8*Lbm*E*Icln(
1,2)+3*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm);

hulp22b(q2b)=24*Q2b(q2b)*Qtotb(q1b+q2b)*Lbm^3*Lcln^4*z(1,2)*E*Icln(1,1)*E*I
cln(1,2)*(11*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm);

hulp23b(q2b)=12*Q2b(q2b)*Lbm*Lcln^2*E*Icln(1,1)*E*Icln(1,2)*(8*Lbm*E*Icln(1
,2)+3*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm)*(4*e0*(11*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm)
+e1b(q1b)*((44-3*pi)*Lbm*E*Icln(1,1)+(20-5*pi)*Lcln*E*Ibm));

hulp24b(q2b)=48*(MJbottom2b(q2b)+MFtop2b(q2b))*Lcln^2*E*Icln(1,1)*E*Icln(1,
2)*(11*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm)*(8*Lbm*E*Icln(1,2)+3*Lbm*E*Icln(1,1)+5
*Lcln*E*Ibm);

hulp25b(q2b)=12*E*Icln(1,1)*E*Icln(1,2)*(11*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm)*(
4*(2*pi^2*E*Icln(1,2)-
Qtotb(q1b+q2b)*Lbm*Lcln^2)*(8*Lbm*E*Icln(1,2)+3*Lbm*E*Icln(1,1)+5*Lcln*E*Ib
m) + pi*Qtotb(q1b+q2b)*Lbm*Lcln^2*(3*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm));

e2b(q2b)=(hulp21b(q2b)+hulp22b(q2b)+hulp23b(q2b)+hulp24b(q2b))/hulp25b(q2b)
;%non-linear
```

```
    e2ba(q2b)=(MJbottom2b(q2b)-MFtop2b(q2b))*Lcln^2/(16*E*Icln(1,2));

phiextra2b(q2b)=4*Lbm*(2*pi*e2b(q2b)*E*Icln(1,2)+N2b(q2b)*Lcln^2*z(1,2))/(L
cln*(8*Lbm*E*Icln(1,2)+3*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm));
    e2bb(q2b)=phiextra2b(q2b)*kr*Lcln^2/(8*E*Icln(1,2));
    e2bc(q2b)=N2b(q2b)*z(1,2)*Lcln^2/(8*E*Icln(1,2));

    etotb(q1b+q2b)=e0 + e1b(q1b) + e2b(q2b) + e1ba(q1b) + e2ba(q2b) -
e1bb(q1b) - e2bb(q2b) + e2bc(q2b);

    MJbottomtot2b(q2b)=MJbottom2b(q2b) - phiextra2b(q2b)*kr;
    MJbottomtotb(q1b+q2b)=MJbottomtotb(q1b)+MJbottomtot2b(q2b);
    MFtoptot2b(q2b)=MFtop2b(q2b) - phiextra2b(q2b)*kr;
    MFtoptotb(q1b+q2b)=MFtoptotb(q1b)+MFtoptot2b(q2b);

    sigmatopb=sigmatoptotb(q1b) -MJbottomtot2b(q2b)/Zcln(1,2) -
N2ba(q2b)/Acln(1,2) - (N2b(q2b)-N2ba(q2b))/Acln(1,4);
    sigmatoptotb(q1b+q2b)=sigmatopb;
    sigmabottomb=sigmabottomtotb(q1b) + MJbottomtot2b(q2b)/Zcln(1,2) -
N2ba(q2b)/Acln(1,2) - (N2b(q2b)-N2ba(q2b))/Acln(1,4);
    sigmabottomtotb(q1b+q2b)=sigmabottomb;
    sigmacentreb=sigmacentretotb(q1b) - N2ba(q2b)/Acln(1,2) - (N2b(q2b)-
N2ba(q2b))/Acln(1,4);
    sigmacentretotb(q1b+q2b)=sigmacentreb;

end% while q2

else
    Qtotb(q1b+q2b)=Qtotb(q1b+q2b);
end% if

q3b=0;
if ((8*pi^2*E*Icln(1,3) -
4*(Qtotb(q1b+q2b)+2*qq)*Lbm*Lcln^2)*(8*Lbm*E*Icln(1,3)+3*Lbm*E*Icln(1,1)+5*
Lcln*E*Ibm) +
pi*(Qtotb(q1b+q2b)+2*qq)*Lbm*Lcln^2*(3*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm))>0;

while sigmatopb>yield2;
    q3b=q3b+1;
    Q3b(q3b)=q3b*qq;
    Qtotb(q1b+q2b+q3b)=Qtotb(q1b+q2b)+Q3b(q3b);

    N3b(q3b)=0.5*Q3b(q3b)*Lbm;
    Qtotb(q1b+q2b+q3b)=Qtotb(q1b+q2b)+Q3b(q3b);
    Ntotb(q1b+q2b+q3b)=Ntotb(q1b+q2b)+N3b(q3b);

    phiJ3b(q3b)=Q3b(q3b)*Lbm*Lcln*hulpa/(24*Lbm*E*Acln(1,1)*hulpb);%linear
    MJbottom3b(q3b)=2*E*Icln(1,1)*(1.5*Q3b(q3b)*Lcln^2*E*Ibm -
2*phiJ3b(q3b)*E*Acln(1,1)*(6*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm))/(Lcln*E*Acln(1,
1)*(7*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm));%linear
    phiF3b(q3b)=(MJbottom3b(q3b)*Lbm*Lcln*E*Acln(1,1)-
3*Q3b(q3b)*Lcln^2*E*Ibm)/(2*E*Acln(1,1)*(6*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm));%
linear
    MFtop3b(q3b)=1.5*Q3b(q3b)*Lcln*E*Ibm/(Lbm*E*Acln(1,1)) +
(3*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm)*phiF3b(q3b)/(Lbm*Lcln);%linear

    hulp31b(q3b)=2*Lbm*Lcln^4*(Qtotb(q1b+q2b+q3b)*(MJbottom3b(q3b)-
MFtop3b(q3b))*E*Icln(1,1)*E*Icln(1,3) + Q3b(q3b)*(MJbottom1b(q1b)-
```

```
MFtop1b(q1b))*E*Icln(1,2)*E*Icln(1,3) + Q3b(q3b)*(MJbottom2b(q2b)-
MFtop2b(q2b))*E*Icln(1,1)*E*Icln(1,3)) * (11*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm)
* (8*Lbm*E*Icln(1,2)+3*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm) *
(8*Lbm*E*Icln(1,3)+3*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm);
    hulp32b(q3b)=3*Q3b(q3b)*Lbm*Lcln^2*E*Icln(1,1)*E*Icln(1,3) *
(4*E*Icln(1,2)*(4*(e0+e1b(q1b)+e2b(q2b))*(11*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm)
- pi*e1b(q1b)*(3*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm)) *
(8*Lbm*E*Icln(1,2)+3*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm) -
(4*pi*e2b(q2b)*E*Icln(1,2)+Q2b(q2b)*Lbm*Lcln^2*z(1,2)) *
(11*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm) *
(3*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm))*(8*Lbm*E*Icln(1,3)+3*Lbm*E*Icln(1,1)+5*Lc
ln*E*Ibm);

hulp33b(q3b)=48*(MJbottom3b(q3b)+MFtop3b(q3b))*Lcln^2*E*Icln(1,1)*E*Icln(1,
2)*E*Icln(1,3) * (11*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm) *
(8*Lbm*E*Icln(1,2)+3*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm) *
(8*Lbm*E*Icln(1,3)+3*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm);
    hulp34b(q3b)=12*E*Icln(1,1)*E*Icln(1,2)*E*Icln(1,3) *
(11*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm) *
(8*Lbm*E*Icln(1,2)+3*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm) * ((8*pi^2*E*Icln(1,3) -
4*Qtotb(q1b+q2b+q3b)*Lbm*Lcln^2)*(8*Lbm*E*Icln(1,3)+3*Lbm*E*Icln(1,1)+5*Lcl
n*E*Ibm) +
pi*Qtotb(q1b+q2b+q3b)*Lbm*Lcln^2*(3*Lbm*E*Icln(1,1)+5*Lcln*E*Ibm));
    e3b(q3b)=(hulp31b(q3b)+hulp32b(q3b)+hulp33b(q3b))/hulp34b(q3b);%non-
linear

    e3ba(q3b)=(MJbottom3b(q3b)-MFtop3b(q3b))*Lcln^2/(16*E*Icln(1,3));

phiextra3b(q3b)=8*pi*Lbm*e3b(q3b)*E*Icln(1,3)/(Lcln*(8*Lbm*E*Icln(1,3)+3*Lb
m*E*Icln(1,1)+5*Lcln*E*Ibm));
    e3bb(q3b)=phiextra3b(q3b)*kr*Lcln^2/(8*E*Icln(1,3));

    etotb(q1b+q2b+q3b)=e0 + e1b(q1b) + e2b(q2b) + e3b(q3b) + e1ba(q1b) +
e2ba(q2b) + e3ba(q3b) - e1bb(q1b) - e2bb(q2b) - e3bb(q3b) + e2bc(q2b);

    MJbottomtot3b(q3b)=MJbottom3b(q3b) - phiextra3b(q3b)*kr;
    MJbottomtotb(q1b+q2b+q3b)=MJbottomtotb(q1b+q2b)+MJbottomtot3b(q3b);
    MFtoptot3b(q3b)=MFtop3b(q3b) - phiextra3b(q3b)*kr;
    MFtoptotb(q1b+q2b+q3b)=MFtoptotb(q1b+q2b)+MFtoptot3b(q3b);

    sigmatopb=sigmatoptotb(q1b+q2b) - MJbottomtot3b(q3b)/Zcln(1,3) -
N3b(q3b)/Acln(1,3);
    sigmatoptotb(q1b+q2b+q3b)=sigmatopb;
    sigmabottomb=sigmabottomtotb(q1b+q2b) + MJbottomtot3b(q3b)/Zcln(1,3) -
N3b(q3b)/Acln(1,3);
    sigmabottomtotb(q1b+q2b+q3b)=sigmabottomb;
    sigmacentreb=sigmacentretotb(q1b+q2b) - N3b(q3b)/Acln(1,3);
    sigmacentretotb(q1b+q2b+q3b)=sigmacentreb;

end% while q3

else
    Qtotb(q1b+q2b+q3b)=Qtotb(q1b+q2b+q3b);
end% if

Qmaxb=Qtotb(q1b+q2b+q3b);

clear MFtop1b;clear MFtop2b;clear MFtop3b
clear MFtoptot1b;clear MFtoptot2b;clear MFtoptot3b;
clear MJbottom1b;clear MJbottom2b;clear MJbottom3b;
```

```
clear MJbottomtot1b;clear MJbottomtot2b;clear MJbottomtot3b;
clear N1b;clear N2b;clear N3b;clear N2ba;
clear Q1b;clear Q2b;clear Q3b;clear Q2ba;
clear e0;clear e1b;clear e2b;clear e3b;
clear e1ba;clear e1bb;clear e2ba;clear e2bb;clear e2bc;clear e3ba;clear
e3bb;
clear hulp11b;clear hulp12b;
clear hulp21b;clear hulp22b;clear hulp23b;clear hulp24b;clear hulp25b;
clear hulp31b;clear hulp32b;clear hulp33b;clear hulp34b
clear hulpa;clear hulpb;
clear phiF1b;clear phiF2b;clear phiF3b;
clear phiJ1b;clear phiJ2b;clear phiJ3b;
clear phiextra1b;clear phiextra2b;clear phiextra3b;
clear delta;

if Qmaxa<Qmaxb;
    Qmax=Qmaxa;
    MFtoptot=MFtoptota;
    MJbottomtot=MJbottomtota;
    Ntot=Ntota;
    Qtot=Qtota;
    etot=etota;
    q1=q1a;
    q2=q2a;
    q3=q3a;
    sigmatop=sigmatopa;
    sigmabottom=sigmabottoma;
    sigmacentre=sigmacentrea;
    sigmatoptot=sigmatoptota;
    sigmabottomtot=sigmabottomtota;
    sigmacentretot=sigmacentretota;
%    AAA='end column';

else
    Qmax=Qmaxb;
    MFtoptot=MFtoptotb;
    MJbottomtot=MJbottomtotb;
    Ntot=Ntotb;
    Qtot=Qtotb;
    etot=etotb;
    q1=q1b;
    q2=q2b;
    q3=q3b;
    sigmatop=sigmatopb;
    sigmabottom=sigmabottomb;
    sigmacentre=sigmacentreb;
    sigmatoptot=sigmatoptotb;
    sigmabottomtot=sigmabottomtotb;
    sigmacentretot=sigmacentretotb;
%    AAA='middle column';


end%if Qmax
clear MFtoptota;clear MFtoptotb;clear MJbottomtota;clear MJbottomtotb;
clear Ntota;clear Ntotb;
clear Qmaxa;clear Qmaxb;clear Qtota;clear Qtotb;clear etota;clear etotb;
clear q1a;clear q2a;clear q3a;clear q1b;clear q2b;clear q3b;
clear sigmatoptota;clear sigmatoptotb;clear sigmabottomtota;clear
sigmabottomtotb;clear sigmacentretota;clear sigmacentretotb;
clear sigmatop;clear sigmatopb;clear sigmabottoma;clear sigmabottomb;clear
sigmacentrea;clear sigmacentreb;
```

```
clear delta;clear qq;clear hulp;

Qmax

%plot(sigmatoptot,'r');hold
on;plot(sigmabottomtot,'g');plot(sigmacentretot);ylabel('Stress in the mid-
section (N/mm2)');xlabel('Load (N/mm)');grid

%plot(etot,Qtot);xlabel('Deflection at midspan (mm)');ylabel('Load
(N/mm)');grid
```

U.3 Calculation according to the Dutch code

The same construction is calculated according to the Dutch code. The ultimate load is calculated as function of the required loads ($F_{required}$) and the variable uniform distributed load (q). The result of the calculations is the ultimate load q.

$$N_{c;u;d} = A_{cln} f_y$$
$$N_{c;u;d} = 14280 * 355$$
$$N_{c;u;d} = 5069 * 10^3 N$$

$$M_{y;u;d} = Z_{cln} f_y$$
$$M_{y;u;d} = 1.891 * 10^6 * 355$$
$$M_{y;u;d} = 671 * 10^6 Nmm$$

$$L_{buc} = \omega_{buc} L_{sys}$$ $\omega_{buc} = 0.67$ (NEN 6770 art. 12.1.1.3)
$$L_{buc} = 0.67 * 5000$$
$$L_{buc} = 3350 mm$$

$$F_E = \frac{\pi^2 EI}{L_{buc}{}^2}$$
$$F_E = \frac{\pi^2 * 6.959 * 10^{13}}{3350^2}$$
$$F_E = 61112 * 10^3 N$$

$$\lambda_{y;rel} = \frac{L_{buc}}{\pi \sqrt{\frac{EI}{Af_y}}}$$
$$\lambda_{y;rel} = \frac{3350}{\pi \sqrt{\frac{6.959 * 10^{13}}{14280 * 355}}}$$
$$\lambda_{y;rel} = 0.29$$

$$\lambda_0 = 0.2$$
$$\alpha_k = 0.34$$

$$e_y^* = \alpha_k \left(\lambda_{y;rel} - \lambda_0\right)\frac{M_{y;u;d}}{N_{c;u;d}}$$
$$e_y^* = 0.34\left(0.29 - 0.20\right)\frac{M_{y;u;d}}{N_{c;u;d}}$$
$$e_y^* = 0.031\frac{M_{y;u;d}}{N_{c;u;d}}$$
$$M_{y;equ;s;d} = \max\begin{pmatrix} \left|0.1\left(M_{y;2;s;d} - M_{y;1;s;d}\right) + M_{y;mid;s;d}\right| \\ \left|0.6M_{y;2;s;d}\right| \end{pmatrix}$$
$$M_{y;equ;s;d} = \max\begin{pmatrix} \left|0.1\left(178*10^3q + 111*10^3q\right) + 41*10^3q\right| \\ \left|0.6*178*10^3q\right| \end{pmatrix}$$
$$M_{y;equ;s;d} = 107*10^3q$$

$$N_{c;s;d} = 0.5qL_{bm} + F_{required}$$
$$N_{c;s;d} = 0.5q*5000 + 166*10^3$$
$$N_{c;s;d} = 2500q + 166*10^3$$

$$e_y^* = 0.031\frac{107*10^3q}{2500q + 166*10^3}$$
$$e_y^* = \frac{3317q}{2500q + 166*10^3}$$

$$\frac{N_{c;s;d}}{N_{c;u;d}} + \frac{n_y}{n_y - 1}\frac{M_{y;equ;s;d} + F_{y;tot;s;d}e_y^*}{M_{y;u;d}} \le 1.0$$

$$\frac{2500q + 166*10^3}{5069*10^3} + \frac{61112*10^3}{61112*10^3 - 2500q - 166*10^3}\frac{107*10^3q + \left(2500q + 166*10^3\right)\left(\frac{3317q}{2500q + 166*10^3}\right)}{671*10^6} \le 1.0$$

$q \le 1448$ N/mm

U.4 Matrix Frame calculations.

Two Matrix Frame calculations are made. Both calculations have the same structure but are different loaded (Fig. U.4 and Fig. U.5).

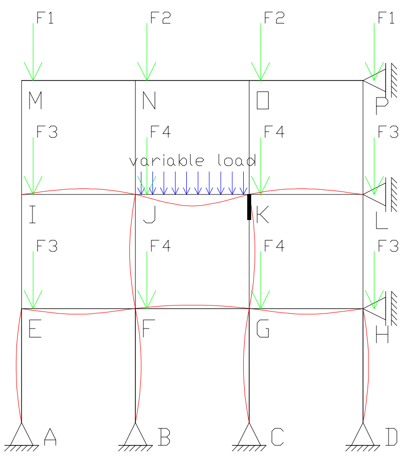


Figure U.4:
First load situation

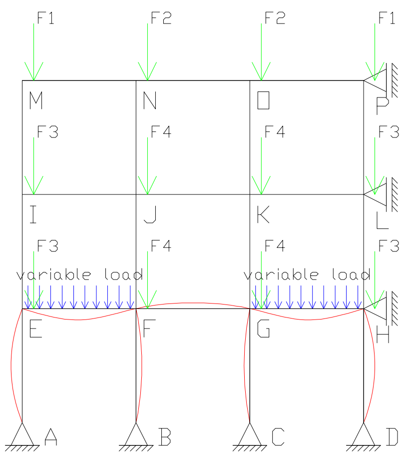


Figure U.5:
Second load situation

The loads F_1 till F_4 are required loads. These required loads are discussed in Appendix Q. The numerical value of these loads are:

F_1 = 45.5 kN
F_2 = 85.5 kN
F_3 = 43.0 kN
F_4 = 80.5 kN

The calculation file can be found is inserted at the end of this Appendix. The imperfections are inserted by the same way as did in the single column (App.) and the braced portal frame (App.). The residual stress is also taken into account. This is done by making two calculatations and sunmmate the results (clearly described in Chapter 2.9). Matrix Frame has calculated the ultimate load for the first and the second situation. The ultimate loads are:

1533 N/mm First situation (Fig. U.4)
1565 N/mm Second situation (Fig. U.5)

There is only a small difference (2.0%) between the ultimate load in the first situation and he ultimate load in the second situation. The differences between the calculations can differ considerably by other column lengths (results in Chapter 5.7).

The calculation file of Matrix frame follows on the next pages.