APPENDIX 0: Aeroelasticity

For the analysis of the bridge structure it will be important to describe the aeroelastic effects that can occur and describe the possible model that can be used. A structure can undergo a motion induced by a wind flow. The structural motion can influence the flow around the structure. As a consequence the flow can affect the structural motions. It can be difficult to describe the interaction between the flow and the structure. For most bluff bodies that can be susceptible for self-excited behavior, wind tunnel testing is done. Phenomenon that reside to the self-excited behavior are, divergence, galloping, flutter and vortex induced vibrations. Another type of motion is the dynamic behavior due to external forces. Buffeting is an example of such a forced driven behavior.

Vortex induced oscillations
One of the possible oscillations can be due to vortexes of the wind. For a cylinder a Reynolds number between 30 and 5000 will cause Von Karman vortexes. This asymmetrical vortex trail can cause the object to oscillate. These oscillations can occur even when there is no turbulence, because the Von Karman vortex trail can occur in a constant flow. Along the length of the cylinder the vortex induced lift forces are not perfectly correlated. When the cylinder starts to oscillate, the forces along the cylinder become more correlated with each other if the frequency of the cylinder in transverse direction is close to the frequency of the vortex shedding. This gives that the amplitude of the oscillations will become larger. For bluff objects the boundary layer gets separated of the object, if very low Reynolds numbers are not considered. Reattachment of the boundary lower with the object stabilizes the object in the flow. Oscillations that want to occur due to the vortex shedding cannot occur because the flow holds the object into position by the reattachment of the flow. This effect of the reattachment is only applicable for objects that have sufficient length in the direction of the wind velocity.

The phenomenon of vortex induced oscillations can best be described on the basis of a cylinder in a fluid and is described by Allen (Allen, 1987):

“For example, consider a fluid particle in a inviscid uniform flow-field approaching a cylinder along a streamline. As the particle approaches the forward stagnation point, its velocity decreases while it experience an increase in pressure (the Bernoulli effect). After the forward stagnation point, the pressure accelerates the particle downstream around the cylinder until it reaches the widest section of the cylinder. In this region the velocity to a maximum while the pressure is at minimum. The increasing pressure along the back side of the cylinder decelerates the fluid particle until it reaches the rear stagnation point (180 degrees of the forward stagnation point). The fluid particle arrives at this point with the same velocity and pressure that it had at the forward stagnation point.”

“When the fluid is viscous, the frictional forces on the fluid particle in the boundary layer cause the particle to diffuse much of its momentum. When the fluid particle reaches the widest region of the cylinder, it has lost enough of its momentum such that it is unable to overcome the increasing pressure force along the back side of the cylinder surface.”

This means that fluid particles that are in contact with the freestream will have a much higher velocity than the particles that are not in contact with the freestream. This difference in velocity will cause the fluid to roll up, and vortexes are formed. At low Reynolds numbers (Re<40) the vortexes on both sides of the cylinder will be symmetric. At higher Reynolds numbers the vortexes becomes unbalanced by pressure fluctuations and other irregularities. When this happens the vortex at one side of the cylinder will draw fluid form the vortex at the other side of the cylinder. Eventually the vortex will shed and moves downstream. This vortex shedding will happen alternately. Every time a vortex is shed from the cylinder it will have influence on the forces that act on the cylinder. This is a result of the difference in pressure and shear stresses. The forces will cause a motion of the cylinder in transverse direction. When the frequency of the vortex shedding would be equal to the oscillations of the cylinder, the oscillation amplitude will grow larger. This is called Lock-on. The amplitude of the oscillations cannot grow infinitely large. At a certain amplitude the vortex forming is suppressed and therefore is the process limited.

In order to model the vortex induced vibrations, a mechanical model with single-degree-of-freedom system can be assumed. For the structure a simple mass-spring-damping system can be assumed. For the force that is generated by the Von Karman vortex trail, Scanlan developed an expression. The equation of motion can be expressed as following:

$$m\ddot{z} + c\dot{z} + kz = F_{vortex}(t)$$

Where m, c and k are respectively the lumped mass, the damping and the stiffness of the structure in the direction perpendicular to the wind velocity. The functions z, $\dot{z}$ and $\ddot{z}$ are respectively the displacement in the direction perpendicular to the wind velocity, and the first and second derivative with respect to time (t). This means that these functions are the velocity and the acceleration of the structure in the same direction as the displacement. Most of the times the equation of motion is rewritten.

$$\ddot{z} + 2\zeta\omega_n\dot{z} + {\omega_n}^2 z = \frac{F_{vortex}(t)}{m}$$

In this equation the ω_n is the natural frequency of the structure and can be written as $\omega_n = \sqrt{k/m}$. The $\zeta = c/2\sqrt{km}$ is the damping ratio. For the force introduced by the vortexes, Scanlan has written an expression for a cylinder in a uniform smooth flow.

$$F_{vortex}(t) = \frac{1}{2}\rho U^2 D \left[Y_1\left(\frac{\omega D}{U}\right)\left(1 - \varepsilon\frac{z^2}{D^2}\right)\right]\frac{\dot{z}}{U}$$

ρ is the specific mass of the air, U is the flow velocity, D is the diameter of the cylinder, Y_1 is a parameter that is dependent of the frequency ω, the diameter and the velocity of the flow. ε is also a parameter that should be obtained from empirical results. The flow velocity can be a constant value, because results of oscillations where observed in constant flow velocities. The vortex induced vibrations are important for objects with a short cross section, e.g. pipelines or chimneys. For concrete bridge sections, vortex induced vibrations are not common. The stay cables of a bridge are sensitive for such behavior.

Galloping

Galloping is a phenomenon where oscillations occur due to changes in the angle of attack of the wind flow velocity with respect to the object. Oscillations with a small amplitude can grow larger because the angle of attack changes when the object experience the oscillations. The forces that act on the structure are therefore self-exiting and can grow larger in time. The reason that the object will oscillate initially, can be explained by imperfections or a small perturbation. The forces that act on the structure will react on this motion. If the forces that act on the body will move the body even further in the displaced direction, the amplitude of the displacement becomes larger. When the frequency of the forces that act on the structure is comparable with the frequency of the structure, the system can become unstable. As a result a galloping motion can be observed.

The equation of motion for a single-degree-of-freedom mass-spring-damping system with a galloping force can be derived. The force that causes the motion of the structure in normal direction of the wind, is a composition of the drag and lift forces. The share and the value of the drag and lift force is dependent of the angle of attack.

$$m\ddot{z} + c\dot{z} + kz = F_{gallop}(\alpha)$$

$$F_{gallop}(\alpha) = F_L(\alpha)\cos(\alpha) + F_D(\alpha)\sin(\alpha)$$

The lift force and drag force are dependent of the wind velocity like is shown in equations below.

$$F_L(\alpha) = \frac{1}{2}\rho {U_\alpha}^2 BLC_L(\alpha)$$
$$F_D(\alpha) = \frac{1}{2}\rho {U_\alpha}^2 BLC_D(\alpha)$$

The wind velocity is in these equations the wind velocity under the angle of attack. For the wind velocity components with an angle of attack of zero, the relation can be expressed as following.

$$U = U_\alpha \cos(\alpha)$$

The galloping force can also be written as.

$$F_{gallop}(\alpha) = \frac{1}{2}\rho U^2 BLC_z(\alpha)$$

It follows that

$$C_z(\alpha) = \frac{C_L(\alpha) + C_D(\alpha)\tan(\alpha)}{\cos(\alpha)}$$

The expression of $C_z(\alpha)$ can be approximated for small angles of α.

$$C_z(\alpha) \approx \alpha\left(\frac{dC_z}{d\alpha}\right)_{\alpha=0}$$

The angle of attack can be approximated by dividing the velocity of the structure by the wind velocity

$$\alpha \approx -\frac{\dot{z}}{U}$$

The Taylor series can be used to get an approximation for the derivative of C_z to the angle of attack.

$$f(x)|_{x=a} \approx \sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n = f(a) + (x-a)\left.\frac{\partial f}{\partial x}\right|_{x=a} + \frac{(x-a)^2}{2}\left.\frac{\partial^2 f}{\partial x^2}\right|_{x=a} + \ \ldots$$

If only the first term of the Taylor series is taken into account the derivative of C_z can be approximated. With the general formulas for differentiation the first term can be written as:

$$\left.\frac{dC_z}{d\alpha}\right|_{\alpha=0} \approx \frac{\left(\left.\frac{dC_L}{d\alpha}\right|_{\alpha=0} + \left.\frac{dC_D}{d\alpha}\right|_{\alpha=0}\tan(0) + \mathrm{C_D}\sec^2(0)\right)\cos(0) + (C_L(0) + C_D(0)\tan(0))\sin(0)}{(\cos(0))^2}$$

The equation can now be simplified because most of the terms have the value of 0 or 1. There are only two terms that remain.

$$\left.\frac{dC_z}{d\alpha}\right|_{\alpha=0} \approx \left.\frac{dC_L}{d\alpha}\right|_{\alpha=0} + \mathrm{C_D}$$

The equation of motion with galloping force can be written down.

$$m\ddot{z} + c\dot{z} + kz = -\frac{1}{2}\rho UB\dot{z}\left(\frac{dC_z}{d\alpha}\right)_{\alpha=0}$$

$$m\ddot{z} + \left(c + \frac{1}{2}\rho UB\left(\frac{dC_z}{d\alpha}\right)_{\alpha=0}\right)\dot{z} + kz = 0$$

It means that the damping can be replaced by an effective damping. This damping can be negative since the derivative of C_z to the angle of attack can be negative. If the effective damping is negative than the amplitude of the oscillations will grow in time after a small initial displacement. That the effective damping is negative is a necessary criterion for galloping to occur, and is known as the Glauert-Den Hartog criterion. The system will become unstable and the amplitude of the oscillations will grow faster than resonance because the amplitude will increase exponentially. This would mean that the amplitude will grow infinitely large. This is however not the case, because the slope of C_z will become positive at a certain angle. As a consequence the amplitude of the oscillations will become constant. This effect can be represented in a nonlinear equation of motion.

$$m\ddot{z} + \left(c + \frac{1}{2}\rho UB\left(\frac{dC_z}{d\alpha}\right)_{\alpha=0} + \tilde{c}\dot{z}^2\right)\dot{z} + kz = 0$$

But can be represented with even more terms. A possibility is to make use of an abbreviated power series.

$$C_z = A_1\left(\frac{\dot{z}}{U}\right) + A_2\left(\frac{\dot{z}}{U}\right)^2\frac{\dot{z}}{|\dot{z}|} + A_3\left(\frac{\dot{z}}{U}\right)^3 + A_5\left(\frac{\dot{z}}{U}\right)^5 + A_7\left(\frac{\dot{z}}{U}\right)^7$$

The coefficients of this function can be found when the drag and lift force for the different angles are known.

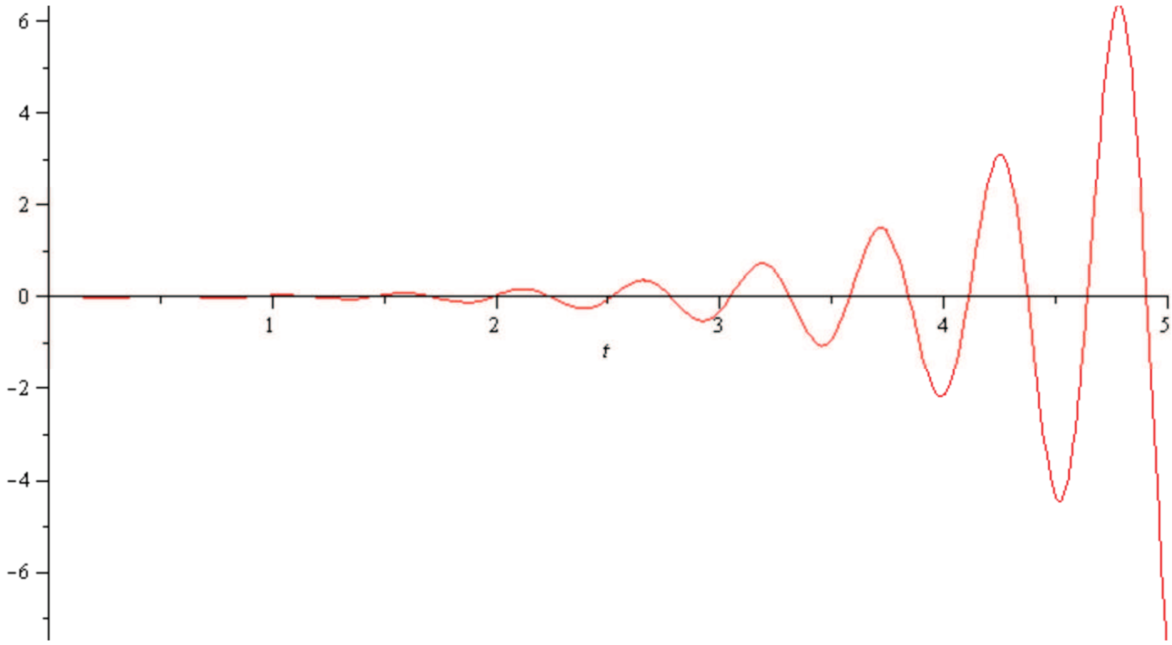


Figure 0.1 Amplitude of the displacement of a system that is sensitive for galloping. The system has a small initial displacement and the amplitude grows exponentially

It is difficult to find a solution for the nonlinear differential equation. It is however possible to show that the amplitude will not grow infinitely large. In order to show this, the equation is split up into two first order differential equations. For this it is necessary to introduce x and y.

$$x = z$$
$$y = \dot{z}$$

This gives that

$$\dot{x} = y = \dot{z}$$
$$\dot{y} = \ddot{z}$$

The nonlinear differential equation can be filled in with these functions. In vector notation the two equations can be written as

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{bmatrix} y \\ \frac{-(c_{eff} + \tilde{c}y^2)}{m} y - \frac{k}{m} x \end{bmatrix}$$

Where c_{eff} is the effective damping. It can be seen that the point (0,0) is the only critical point in the system. To evaluate the differential equation the linear and nonlinear part of the equation is separated.

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c_{eff}}{m} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{\tilde{c}y^3}{m} \end{bmatrix}$$

Near the origin the equation will behave as a linear equation. The eigenvalues of the linear part are:

$$\lambda_{1,2} = -\frac{c_{eff}}{2m} \pm \sqrt{\left(\frac{c_{eff}}{2m}\right)^2 - \frac{k}{m}}$$

When the system is sensitive for galloping the value of c_{eff} is negative. If the expression in the root has a negative value, then the node is an unstable spiral point. The nonlinear part of the equation can be investigated by using polar coordinates.

$$x = r\cos\theta$$
$$y = r\sin\theta$$

The first equation of the vector notation is multiplied by x and the second equation is multiplied with$y$. Both equations are summed up after this.

$$x\frac{dx}{dt} + y\frac{dy}{dt} = xy - \frac{\left(c_{eff} + \tilde{c}y^2\right)}{m}y^2 - \frac{k}{m}xy$$

x and y can be replaced by the polar coordinates.

$$\frac{dr}{dt} = \left(1 - \frac{k}{m}\right)r\cos\theta\sin\theta - \frac{c_{eff}}{m}r\sin^2\theta - \frac{\tilde{c}}{m}r^3\sin^4\theta$$

It can be seen that when r is small the first two terms dominate and the last term can be neglected. It follows that dr/dt has a value that can be negative as positive. However when the first two terms are plotted it can be seen that most part of dr/dt is larger than zero. It should be kept in mind that $c_{eff} < 0$. The circle will therefore grow. When the r grows larger the last term becomes more important. The parameter $\tilde{c}$ is always positive and sinus to the power four is positive as well. The last term decreases the value of dr/dt. When the first two terms are taken into account it can be seen that the r is rather stable. The trajectories do not have a perfect circle shape, but the circle is deformed.

Divergence

Torsional divergence occurs on a structure when there is a twisting moment acting on the structure. For flat shaped structures the twisting moment can result in a rotation of the structure. When this happens the twisting moment can increase. This means that the rotation can increase even further. Finally this can result in a situation where the condition of the structure is unstable. The unstable condition is reached when the wind flow reaches a critical velocity. Whether or not torsional divergence can occur is also dependent on the shape of the structure. Some structures are not susceptible for torsional divergence at all. For civil structures torsional divergence is in most cases not relevant because the critical velocity is too high to take into account in the design. For aerospace engineering torsional divergence is of much greater importance, because higher velocities of the flow.

To model torsional divergence the equation of motion can be written for a simple one-degree-of-freedom system.

$$J\ddot{\alpha} + c_\alpha\dot{\alpha} + k_\alpha\alpha = M_\varphi(\alpha)$$

J is here the mass moment of inertia, c_α is the rotational damping and the rotational stiffness is defined as k_α. The degree of freedom in this system is the rotation α.
The twisting moment is defined as;

$$M_\alpha(\alpha) = \frac{1}{2}\rho U^2 B^2 C_M(\alpha)$$

The aerodynamic moment coefficient $C_M(\alpha)$ is dependent of the angle of attack. The same problem arises as for galloping force. The angle of attack will change during the motion of the system. Because of this, the coefficients will change during the motion. It is however difficult to take this into account. For the aerodynamic moment coefficient an approximation for a small change of the angle of attack away from $\alpha = 0$ may be given by

$$C_M(\alpha) \approx C_M(0) + \left.\frac{dC_M}{d\alpha}\right|_{\alpha=0} \alpha$$

The evaluation of the twisting moment leads to the equation

$$J\ddot{\alpha} + c_\alpha\dot{\alpha} + k_\alpha\alpha = \frac{1}{2}\rho U^2 B^2 \left(C_M(0) + \left.\frac{dC_M}{d\alpha}\right|_{\alpha=0} \alpha\right)$$
$$J\ddot{\alpha} + c_\alpha\dot{\alpha} + \left(k_\alpha - \frac{1}{2}\rho U^2 B^2 \left.\frac{dC_M}{d\alpha}\right|_{\alpha=0}\right)\alpha = \frac{1}{2}\rho U^2 B^2 C_M(0)$$

The term within the brackets is denoted as the effective rotational stiffness. When this term is zero or smaller than zero the system has no stiffness and the rotation can grow larger in time. The critical velocity at which this can occur can be formulated as:

$$U_{crit} = \sqrt{\frac{2k_\alpha}{\rho B^2 \left.\frac{dC_M}{d\alpha}\right|_{\alpha=0}}}$$

Flutter
Another mechanism that can occur for structures in a flow is flutter. There are different types of flutter. The simplest of them is stall flutter. When the shear and pressure on a structure gives rise to an aerodynamic moment it is possible that the structure undergoes a rotation. Stall flutter has only one degree of freedom and this is the rotation of the structure. Classical flutter is another type of flutter. Here the rotation and the translation are coupled. For classical flutter the equations of motion can be written as

$$m\ddot{z} + c_z\dot{z} + k_z z = F_z$$
$$m\ddot{x} + c_x\dot{x} + k_x x = F_x$$
$$J\ddot{\alpha} + c_\alpha\dot{\alpha} + k_\alpha\alpha = M_\alpha$$

Where z, x and α are the vertical displacement, the horizontal displacement and the torsional rotation, respectively. F_z, F_x and M_α are respectively the lift force, the drag force and the aerodynamic moment acting on the structure. These forces are influenced by the displacements and velocities in both x and y direction and the rotation and rotation velocity. Forces that are proportional to the accelerations are negligible. In the equations below the forces are given

$$F_z = \frac{1}{2}\rho U^2 BL\left(C_1\frac{\dot{z}}{U} + C_2\frac{z}{U} + C_3\frac{\dot{x}}{U} + C_4 x + C_5 B\frac{\dot{\alpha}}{U} + C_6\alpha\right)$$
$$F_x = \frac{1}{2}\rho U^2 BL\left(C_7\frac{\dot{z}}{U} + C_8\frac{z}{U} + C_9\frac{\dot{x}}{U} + C_{10} x + C_{11} B\frac{\dot{\alpha}}{U} + C_{12}\alpha\right)$$
$$M_\alpha = \frac{1}{2}\rho U^2 B^2 L\left(C_{13}\frac{\dot{z}}{U} + C_{14}\frac{z}{U} + C_{15}\frac{\dot{x}}{U} + C_{16} x + C_{17} B\frac{\dot{\alpha}}{U} + C_{18}\alpha\right)$$

When these forces are substituted into the equations of motion the resulting equations of motion become

$$m\ddot{z} + \left(c_z - \frac{1}{2}\rho UBLC_1\right)\dot{z} + \left(k_z - \frac{1}{2}\rho UBLC_2\right)z - \frac{1}{2}\rho UBLC_3\dot{x} - \frac{1}{2}\rho UBLC_4 x - \frac{1}{2}\rho UB^2LC_5\dot{\alpha} - \frac{1}{2}\rho U^2BLC_6\alpha = 0$$
$$m\ddot{x} + \left(c_x - \frac{1}{2}\rho UBLC_9\right)\dot{x} + \left(k_x - \frac{1}{2}\rho UBLC_{10}\right)x - \frac{1}{2}\rho UBLC_7\dot{z} - \frac{1}{2}\rho UBLC_8 z - \frac{1}{2}\rho UB^2LC_{11}\dot{\alpha} - \frac{1}{2}\rho U^2BLC_{12}\alpha = 0$$
$$J\ddot{\alpha} + \left(c_\alpha - \frac{1}{2}\rho UB^3LC_{17}\right)\dot{\alpha} + \left(k_\alpha - \frac{1}{2}\rho U^2B^2LC_{18}\right)\alpha - \frac{1}{2}\rho UB^2LC_{13}\dot{z} - \frac{1}{2}\rho UB^2LC_{14} z - \frac{1}{2}\rho UB^2LC_{15}\dot{x} - \frac{1}{2}\rho UB^2LC_{16} x = 0$$

These equations can be written in matrix form

$$\boldsymbol{M}\underline{\ddot{u}} + \boldsymbol{C}\underline{\dot{u}} + \boldsymbol{K}\underline{u} = 0$$

Where

$$\boldsymbol{M} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & J \end{bmatrix}, \boldsymbol{C} = \begin{bmatrix} c_z - \frac{1}{2}\rho UBLC_1 & -\frac{1}{2}\rho UBLC_3 & -\frac{1}{2}\rho UB^2LC_5 \\ -\frac{1}{2}\rho UBLC_7 & c_x - \frac{1}{2}\rho UBLC_9 & -\frac{1}{2}\rho UB^2LC_{11} \\ -\frac{1}{2}\rho UB^2LC_{13} & -\frac{1}{2}\rho UB^2LC_{15} & c_\alpha - \frac{1}{2}\rho UB^3LC_{17} \end{bmatrix},$$
$$\boldsymbol{K} = \begin{bmatrix} k_z - \frac{1}{2}\rho UBLC_2 & -\frac{1}{2}\rho UBLC_4 & -\frac{1}{2}\rho U^2BLC_6 \\ -\frac{1}{2}\rho UBLC_8 & k_x - \frac{1}{2}\rho UBLC_{10} & -\frac{1}{2}\rho U^2BLC_{12} \\ -\frac{1}{2}\rho UB^2LC_{14} & -\frac{1}{2}\rho UB^2LC_{16} & k_\alpha - \frac{1}{2}\rho U^2B^2LC_{18} \end{bmatrix}, \underline{u} = [z \quad x \quad \alpha]^T$$

The characteristic equation can be obtained by taking the determinant of the dynamic stiffness matrix.

$$\det(\boldsymbol{M}{s_n}^2 + \boldsymbol{C}s_n + \boldsymbol{K}) = {s_n}^6 + a_1{s_n}^5 + a_2{s_n}^4 + a_3{s_n}^3 + a_4{s_n}^2 + a_5 s_n + a_6 = 0$$

The system is only stable if and only if the real values of s_n are all smaller or equal to zero. If the real part of one of the solutions is larger than zero, the system will be unstable. In figure 0.2 an example is plotted of the real part of the eigenvalues of the characteristic equation. It can be seen that the system becomes unstable when the velocity is larger than ≈ 2.5m/s.

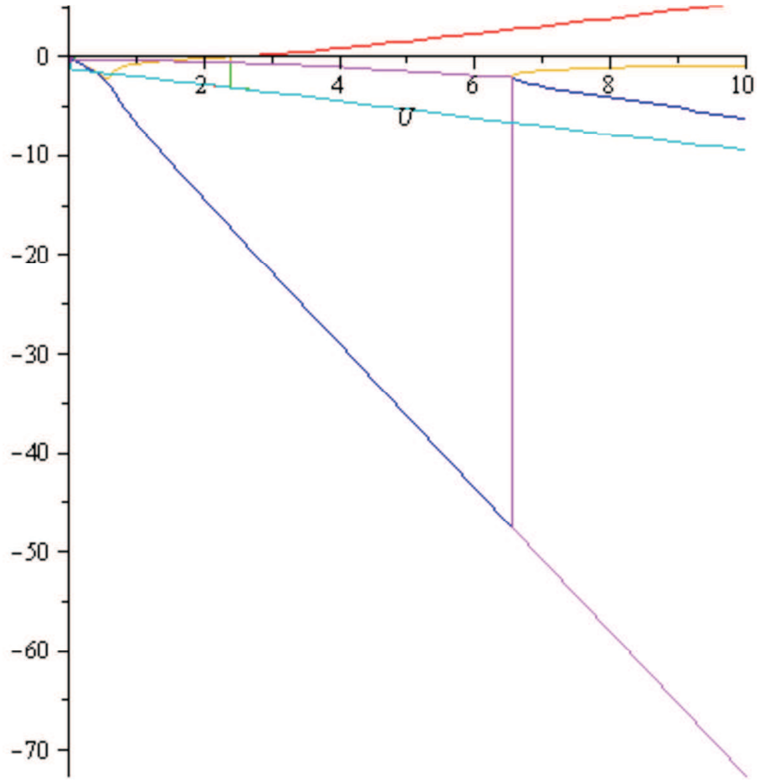


Figure 0.2 In this graph the real part of the eigenvalues of the characteristic equation are plotted. For m=10kg, J=20kgm^2, c_z=4Ns/m, c_x=24Ns/m, c_a =4Nms, k_z=90N/m, k_x=3240N/m, k_a=100Nm.

The aerodynamic vibrations of galloping, vortex induced oscillations and flutter, can according to Kubo, Hirata and Mikawa be closely related to flow patterns on the surface of the structure (Kubo, et al., 1992). In their article, published in 1992, the heaving and torsion of an H-shaped and rectangular shaped sections are investigated. The surface pressure on the structure causes the structure to rotate or vibrate. How the flow develops in the flow separation determines the pressure on the section. In figure 0.3 a figure form this article is presented to show the flow pattern and the behavior of the structure.

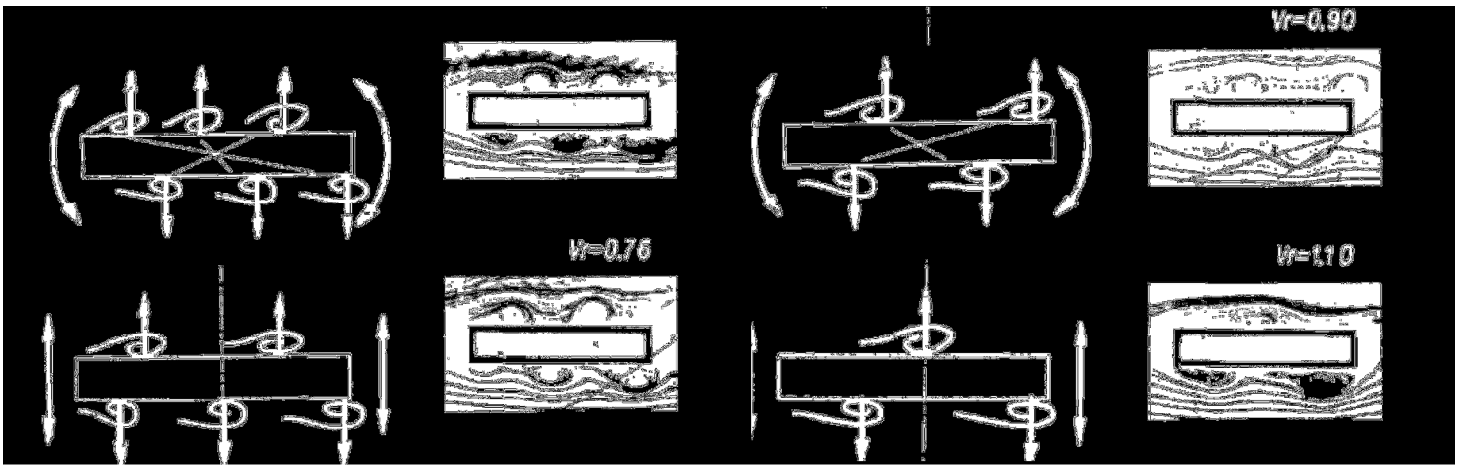


Figure 0.3 Explanation of the motion of a rectangular shaped cylinder based on the flow patterns according to Kubo et al.

APPENDIX I: Stationary Gaussian Process

A stationary process is a process where the mean is constant for all the points in time. When the process is also Gaussian the process is entirely determined by the mean and the auto-covariance function. To describe a process with the auto-covariance function is not very simple. Therefore a Fourier transform of the auto-covariance function can be used to describe the process. The wind velocity in time can be considered as a stationary Gaussian process. A basis element of a stationary Gaussian process can be given with the following formula:

$$x(t) = \hat{x}\sin(\omega t + \varphi)$$

Where the phase angle φ is stochastic and has a uniform distribution between 0 and 2π and determines the randomness of the formula. The process is stationary because the mean and standard deviation of this function are constant in time. When a summation of N sine functions with random phase angles is considered the following formula can be obtained.

$$x(t) = \sum_{n=1}^{N} \hat{x}_n \sin(\omega_n t + \varphi_n)$$

The mean of this function is zero because it is a summation of multiple sine functions. The standard deviation of this function is still constant in time.

$$\sigma(x)^2 = \sum_{n=1}^{N} \frac{1}{2} \hat{x}_n{}^2$$

From the trigonometric functions it can be seen that:

$$\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

Substituting this into the formula of $x(t)$gives:

$$x(t) = \sum_{n=1}^{N} \hat{x}_n \sin(\omega_n t)\cos(\varphi_n) + \hat{x}_n \cos(\omega_n t)\sin(\varphi_n)$$

When the function $x(t)$ is considered as a fourier series with $\hat{x}_n \cos(\varphi_n)$ as $A_n$and $\hat{x}_n \sin(\varphi_n)$ as B_n it can be written as:

$$x(t) = \sum_{n=1}^{N} A_n \sin(\omega_n t) + B_n \cos(\omega_n t)$$

When a period of $[0, T]$ is considered, the coefficients A_n and B_n of the Fourier series can be formulated as:

$$A_n = \frac{2}{T} \int_0^T x(t)\sin(\omega_n t)\, dt$$

$$B_n = \frac{2}{T}\int_0^T x(t)\cos(\omega_n t)\,dt$$

The frequency ω_n can be written as $2\pi n/T$. The amplitude $\hat{x}_n$ of the function $x(t)$can be expressed as:

$$\hat{x}_n{}^2 = A_n{}^2 + B_n{}^2$$

Substituting this into the equation of the standard deviation gives the following relation:

$$\sigma(x)^2 = \sum_{n=1}^{N} \frac{1}{2}A_n{}^2 + \frac{1}{2}B_n{}^2$$

This value approaches a constant value for very large N.

The variance spectrum can be introduced to give the relation between the frequency and the standard deviation.

$$S_{xx}(\omega)\Delta\omega = \sigma^2$$
$$S_{xx}(\omega)\Delta\omega = \frac{1}{2}\hat{x}_n{}^2$$

This gives that the amplitude of the sine function is equal to:

$$\hat{x}_n = \sqrt{2S_{xx}(\omega)\Delta\omega}$$

This would also mean that the variance spectrum can be written as:

$$S_{xx}(\omega) = \frac{1}{2\Delta\omega}(A_n{}^2 + B_n{}^2)$$

A relation between the variance spectrum and the Fourier series can be obtained. The Fourier transform of $x(t)$ is used:

$$S_x = \frac{1}{\pi}\int_{-T/2}^{T/2} x(t)(\cos(\omega t) - i\sin(\omega t))dt$$

With:

$$T \to \infty$$

Using the Fourier coefficients:

$$A(\omega) = \frac{1}{\pi}\int_{-T/2}^{T/2} x(t)\sin(\omega t)\,dt$$
$$B(\omega) = \frac{1}{\pi}\int_{-T/2}^{T/2} x(t)\cos(\omega t)\,dt$$

This gives:

$$S_x = B(\omega) - i\,A(\omega)$$
$$S_x{}^* = B(\omega) + i\,A(\omega)$$

Because $\pi = \Delta\omega T/2$ and $T \rightarrow \infty$, the relation between the coefficients can be written as:

$$A(\omega) = \frac{2}{\Delta\omega T}\int_{-T/2}^{T/2} x(t)\sin(\omega t)\,dt = \lim_{\Delta\omega\rightarrow 0}\frac{A_n}{\Delta\omega}$$
$$B(\omega) = \lim_{\Delta\omega\rightarrow 0}\frac{B_n}{\Delta\omega}$$

Substituting this into the equation of the variance spectrum gives:

$$S_{xx}(\omega) = \frac{\Delta\omega}{2}\left(A(\omega)^2 + B(\omega)^2\right)$$
$$S_{xx}(\omega) = \frac{\pi}{T} S_x S_x{}^*$$

Appendix II: Numerical Calculation for the Fluctuating Wind Velocity

The following numerical values are used to explain the process of modeling the wind velocity.

$h_0 = 10m$	Reference Height
$U(h_0) = 30m/s$	Mean wind velocity at reference height
$I = 0.2$	Turbulence intensity
$\sigma_v = 6\ m/s$	Standard deviation of the wind velocity
$z_1 = 10m$	Height for process 1
$z_2 = 20m$	Height for process 2
$\Delta\omega = 0.005\ rad/s$	Frequency step
$\omega_0 = 0.005\ rad/s$	Starting frequency
$N = 999$	Number of steps
$C_z = 10$	Coherence constant in z-direction

First the wind spectrum is plotted. For this analysis the reduced wind spectrum of Simiu is taken into account and can be given by:

$$\frac{fS_{vv}(f)}{{\sigma_v}^2} = \frac{2}{3}\frac{x}{(1+x)^{\frac{5}{3}}}$$

Where:

$S_{vv}(f)$	Variance spectrum of the wind velocity [m^2/s]
$x = fL/U(h_0)$	Dimensionless frequency
f	Frequency [Hz]
$L = 50z$	Characteristic length for reduced spectrum of Simiu, with z as the height [m]

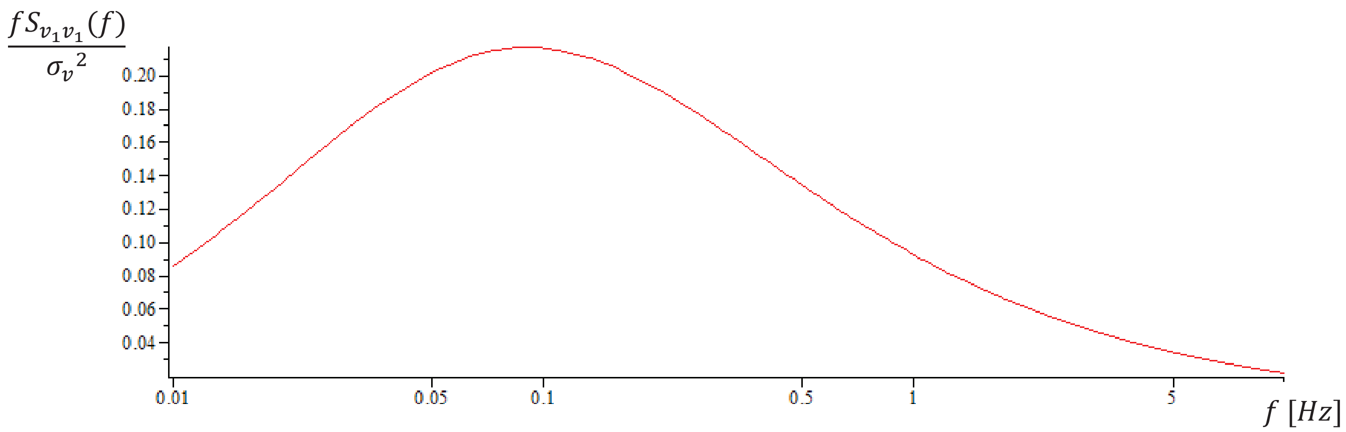


Figure II.1 Reduced spectrum of Simiu for the height z_1

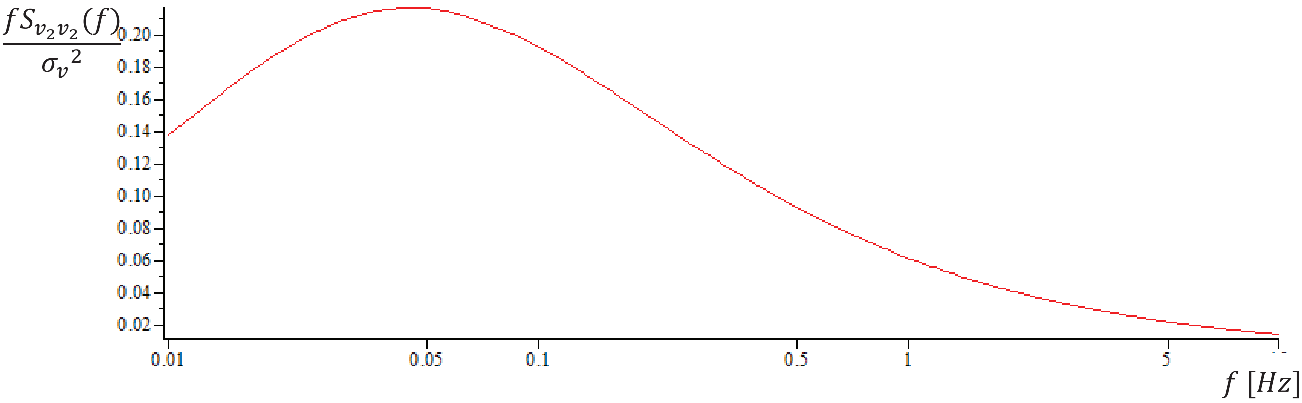


Figure II.2 Reduced spectrum of Simiu for the height z_2

The time dependent part of the wind velocity can be written as a summation of multiple sine functions with random phase angles.

$$U(t) = \sum_{k=1}^{N} a_k \sin(\omega_k t + \varphi_k)$$

The amplitude a_k is related to the variance spectrum as is shown in APPENDIX I.

$$a_k = \sqrt{2 S_{vv}(\omega_k)\Delta\omega}$$

The cross-spectrum can be given by:

$$S_{v_1 v_2}(f) = Coh^2{}_{v_1 v_2}(f) S_{v_1 v_1}(f) S_{v_2 v_2}(f)$$

The coherence function of the wind speed can be expressed with the following formula:

$$Coh_{v_1 v_2}(f) = \exp\left(-f \frac{\sqrt{C_z{}^2 (z_1 - z_2)^2}}{U(10)}\right)$$

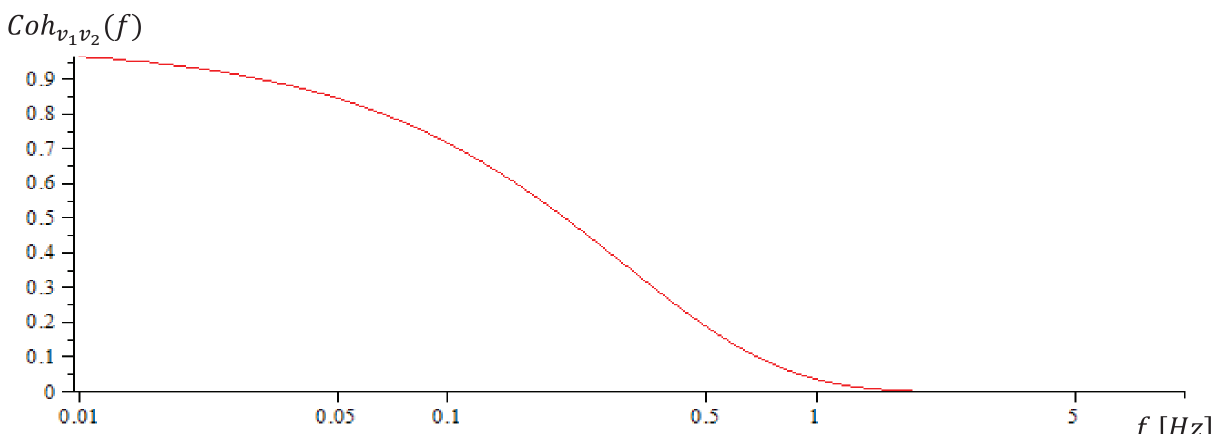


Figure II.3 coherence function

With the numerical values taken into account the fluctuating wind velocity can be calculated for height z_1 and z_2. The variance spectrum of the wind velocity at height z_1 is calculated as following:

$$S_{v_1v_1}(\omega_k) = \frac{50\omega_k}{9\pi\left(1+\frac{25\omega_k}{3\pi}\right)^{\frac{5}{3}}}$$

The frequency and the amplitude can be defined as:

$$\omega_k = 0.005 + 0.005k$$
$$a_{1,k} = \sqrt{0.01 S_{v_1v_1}(\omega_k)}$$

Finally, the fluctuating wind speed can be determined:

$$U_1(t) = \sum_{k=1}^{999} a_{1,k}\sin(\omega_k t + \varphi_k)$$

The variance spectrum at the height z_2 is given below:

$$S_{v_2v_2}(\omega_k) = \frac{100\omega_k}{9\pi\left(1+\frac{50\omega_k}{3\pi}\right)^{\frac{5}{3}}}$$

Based on both spectra and the coherence the cross-spectrum for the wind (v_1,v_2) can be determined.

$$S_{v_1v_2}(\omega_k) = \exp\left(-\omega\frac{5}{3\pi}\right)\sqrt{S_{v_1v_1}(\omega_k)S_{v_2v_2}(\omega_k)}$$

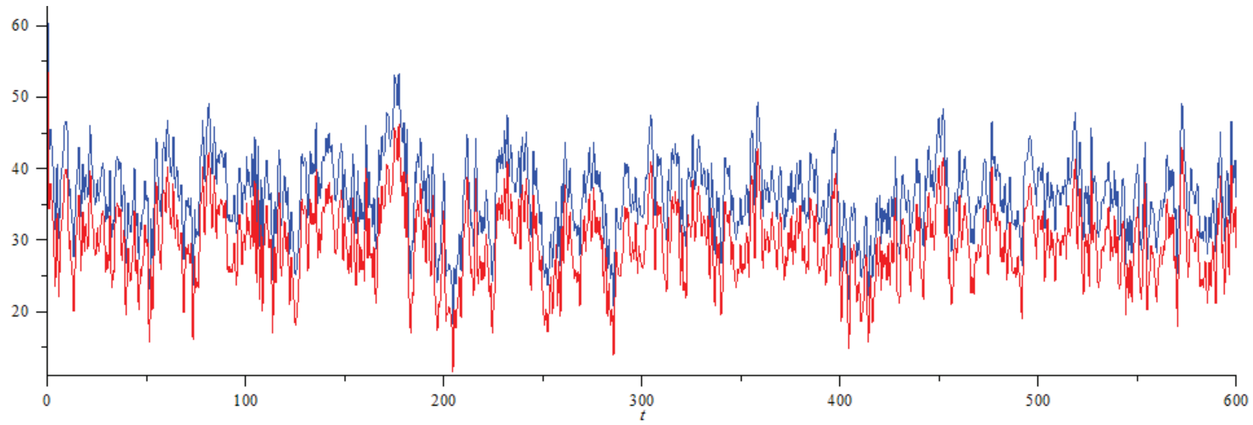


Figure II.4 Wind speed at different heights for two completely coupled processes

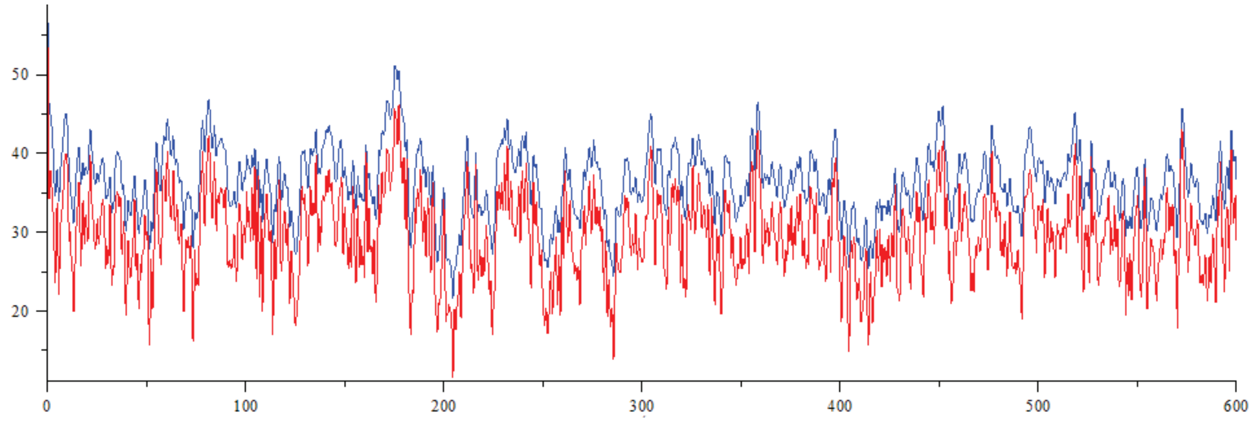


Figure II.5 Wind speed at different heights for two processes related with a cross-spectrum

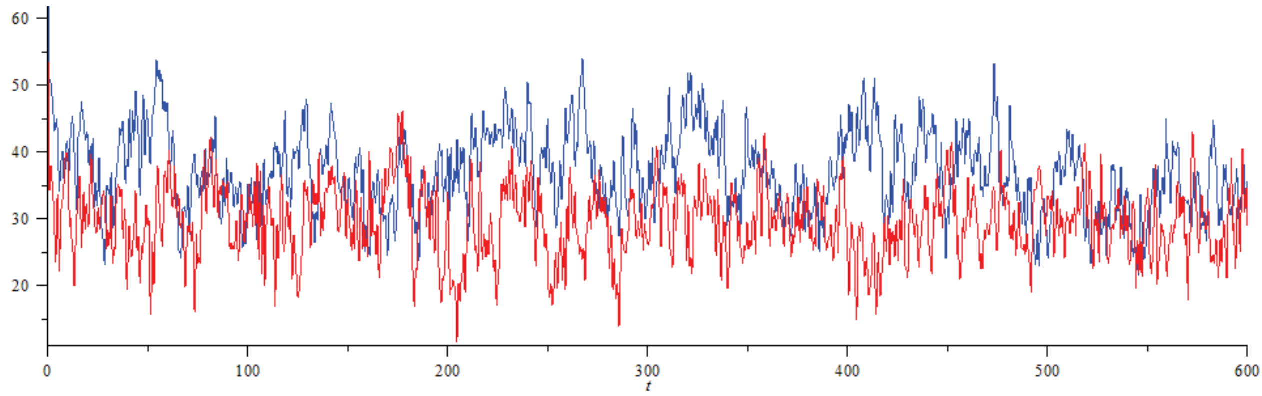


Figure II.6 Wind speed at different heights for two independent processes

Appendix III: Approach EuroCode

The EuroCode 1991-1-4 gives calculation methods for determining the loads due to wind. A part of the loads in the EuroCode are designed for the static approach of the wind load. The static force acting on a structure can be calculated with the following equation.

$$F_w = c_s c_d c_f q_p(z) A_{ref}$$

Where c_s is the reduction effect, c_d is a dynamic factor, c_f is the force coefficient, $q_p(z)$ is the extreme wind pressure and A_{ref} is the reference area of the structure. The extreme wind pressure can be calculated based on the average wind pressure multiplied with a factor for the turbulence.

$$q_p = (1 + 7I_v)\frac{1}{2}\rho {U_m}^2$$

The turbulence intensity multiplied with a factor 7 gives a extreme peak value for the wind pressure. The factor is based on the normalized variable that can be expressed as $\bar{g} \approx \sqrt{2\ln(n)}$. Where n is the number of peaks, which can be approximated with Tf_0. Where f_0 is the central frequency and has a value of about $0.13\ Hz$. This value can be regarded as the centre of gravity of the variance spectrum of the wind velocity. Using the central frequency the number of peaks in a certain period can be calculated. When one hour is taken into account for the period, the value $\sqrt{2\ln(3600 \cdot 0.13)} \approx 3.5$ can be obtained. The extreme wind pressure can be given by:

$$q_p = \mu + \bar{g}\sigma_p$$

Where μ is the mean of the wind pressure and σ_p is the standard deviation of the wind pressure. Filling in the mean and replacing the σ_p for $\rho U_m \sigma_v$ gives:

$$q_p = \frac{1}{2}\rho {U_m}^2 + \bar{g}\rho U_m \sigma_v$$

This expression is equal to the expression that can be found in the Eurocode.

$$q_p = (1 + 2\bar{g}I_v)\frac{1}{2}\rho {U_m}^2$$

The mean wind velocity can according to the EuroCode be expresses as:

$$U_m(z) = c_r(z)c_0(z)v_b$$

Where $c_r(z)$ is the roughness factor for the terrain, and can for heights between z_{min} and z_{max} be given by the following equation.

$$c_r(z) = 0.19\left(\frac{z_0}{z_{0,II}}\right)^{0.07} \cdot \ln\left(\frac{z}{z_0}\right)$$

The basic wind velocity is the wind velocity at a reference height of 10 meter and is based on the fundamental wind velocity for a certain wind area.

$$v_b = c_{dir} c_{season} v_{b,0}$$

In the equation for the static wind force a factor for the reduction effect is taken into account.

$$c_s = \frac{1 + 7I_v\sqrt{B^2}}{1 + 7I_v}$$

With I_v as the turbulence intensity and B^2 is the background-response factor. The dynamic factor can be described as:

$$c_d = \frac{1 + 2k_p I_v \sqrt{B^2 + R^2}}{1 + 7I_v\sqrt{B^2}}$$

The peakfactor is given by k_p, and R^2 is the resonance-response factor. It is stated that the combination of the reduction effect and the dynamic factor should not be less than 0.85. For bridges where no dynamic response calculation is necessary, the value for $c_s c_d$ is equal to one. The values for the resonance-response factor and the background-response factor are given in Appendix C of EuroCode 1991-1-4. The background-response factor gives a value to take into account the correlation for the wind pressure against the structure.

$$B^2 = \frac{1}{1 + \frac{3}{2}\sqrt{\left(\frac{b}{L(z)}\right)^2 + \left(\frac{h}{L(z)}\right)^2 + \left(\frac{bh}{L(z)^2}\right)^2}}$$

Where h and b are the height and width of the structure, respectively. $L(z)$ is the turbulence length:

$$L(z) = L_t \left(\frac{z}{z_t}\right)^{0.67 + 0.05\ln(z_0)}$$

$$R^2 = \frac{\pi^2}{2\delta} S_L(z, \omega_1) K_s(\omega_1)$$

In this equation ω_1 is the first Eigen frequency of the system. $K_s(\omega_1)$ is the dimension reduction function and can be approximated with:

$$K_s(\omega_1) = \frac{1}{1 + \sqrt{\left(\frac{4}{\pi}\frac{c_y b \omega_1}{U_m}\right)^2 + \left(\frac{1}{2}\frac{c_z h \omega_1}{U_m}\right)^2 + \left(\frac{4}{\pi}\frac{c_y c_z b h {\omega_1}^2}{{U_m}^2}\right)^2}}$$

The Eigen frequency of the structure can be estimated with the formulas given in appendix F of the Eurocode 1991-1-4. The reduced variance spectrum of the wind is in the Eurocode given by the formula.

$$S_L(z_e, \omega_1) = \frac{f S_{vv}(f)}{{\sigma_v}^2} = \frac{6.8x}{(1 + 10.2x)^{5/3}}$$

With $x = fL(z)/U_m(z)$.

The force coefficient c_f for bridges is different than the force coefficient used for buildings. The force coefficient is dependent of width-thickness ratio of the bridge and the angle of the wind velocity with the bridge deck. The force coefficient in horizontal direction is given in figure III.1.

$$c_{f,x} = c_{f,x0}$$

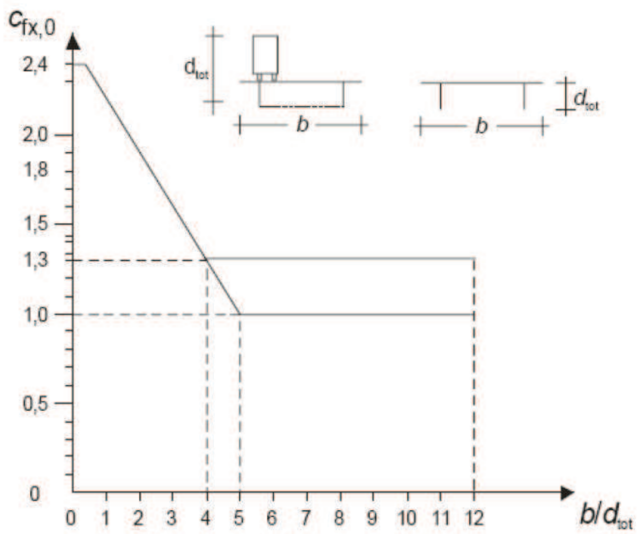


Figure III.1 pressure coefficient in horizontal direction according to EC1

For the force coefficient in vertical direction, the angle of attack and width-thickness ratio are important. The EuroCode does not recommend to use the vertical force coefficient for calculations with vertical vibrations.

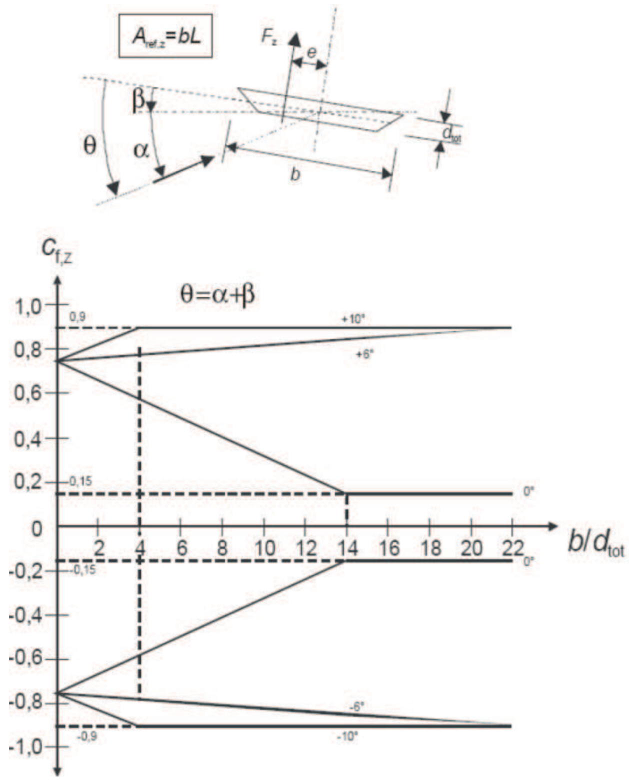


Figure III.2 Pressure coefficient in vertical direction according to EC1

The Eurocode 1991-1-4 gives a procedure to test the sensitivity of a structure for flutter and divergence. Three criteria are mentioned for a structure that could be sensible for divergence and flutter. When one of the criteria is not met, the structure should not be prone to flutter and divergence. The three criteria stated in the EuroCode 1991-1-4 are:

- The structure has a flat cross section with a width-thickness ratio smaller than 0.25.
- The torsion centre is positioned at a distance of at least d/4 of the edge at the leeward side.
- The vibration mode of the first eigenfrequency is a torsional mode. Or the first torsion eigenfrequency is smaller than two times the first bending eigenfrequency.

Appendix IV: Design Calculation Cable Stayed Bridge NSC

This design calculation is performed in order to determine the global dimensions of the bridge. Only the deck, the cross beams, the main girders and the stays are calculated in this appendix.

Loads

For the life load, the governing vehicle load is taken into account. The Eurocode 1991-2 gives the loading for bridges. The maximum possible number of lanes is 4 on both sides of the bridge.

Load model 1 (General loading due to lorries)

Lane number 1

$$Q_1 = 2 \cdot \alpha_{Q1} \cdot 300 = 2x300 \quad kN$$

$$p_1 = \alpha_{q1} \cdot 9.0 = 10.35 \quad kN/m^2$$

Lane number 2

$$Q_2 = 2 \cdot \alpha_{Q2} \cdot 200 = 2x200 \quad kN$$

$$p_2 = \alpha_{q2} \cdot 2.5 = 3.50 \quad kN/m^2$$

Lane number 3

$$Q_3 = 2 \cdot \alpha_{Q3} \cdot 100 = 2x100 \quad kN$$

$$p_2 = \alpha_{q2} \cdot 2.5 = 3.50 \quad kN/m^2$$

Lane number 4

$$p_2 = \alpha_{q2} \cdot 2.5 = 3.50 \quad kN/m^2$$

Remaining area

$$p_2 = \alpha_{q2} \cdot 2.5 = 3.50 \quad kN/m^2$$

Contact surface of each wheel is 400x400mm^2.

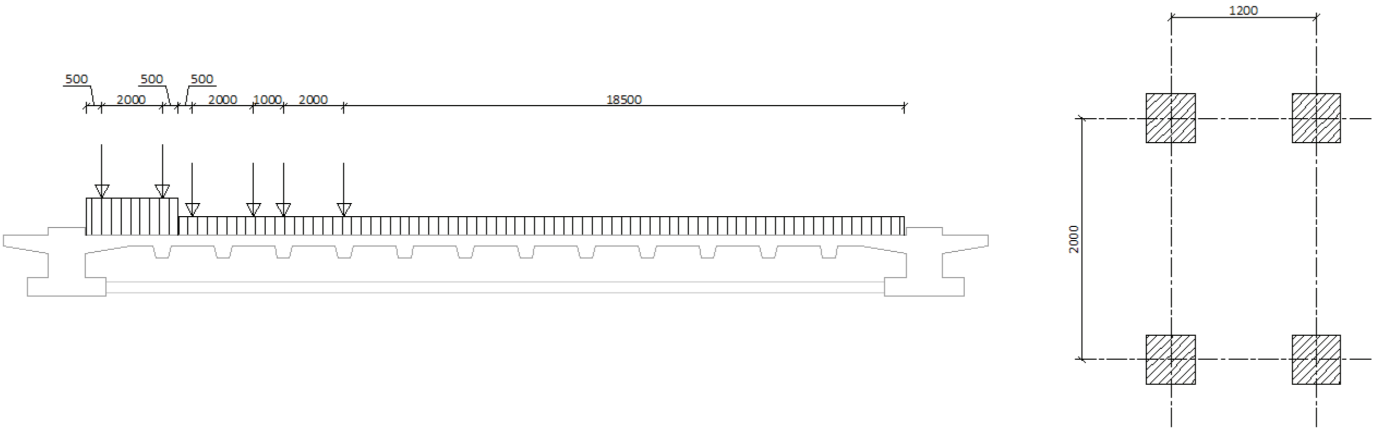


Figure I.1 Loads of load model 1

Load model 2 (Single axle)

$$Q_{ak} = 2 \cdot \beta_Q \cdot 400 = 2x400 \quad kN$$

Contact surface of each wheel is 600x350mm^2.

Load model 4 (Crowd)

$$p_4 = \quad = 5.00 \quad kN/m^2$$

Overview

The cable stayed bridge is constructed from the pylon. At both sides of the pylon the deck is built in sections. The pylon is temporary secured to the abutment. The falsework is attached to the casted bridge sections. When the casted concrete is sufficiently hardened the stay is installed and the post tensioning cables are tensioned. If the strength of the concrete is sufficient, the formwork and falsework is removed and installed for the next section. At both sides of the bridge the deck should be constructed at the same building speed, in order to make sure there is equilibrium during construction process.

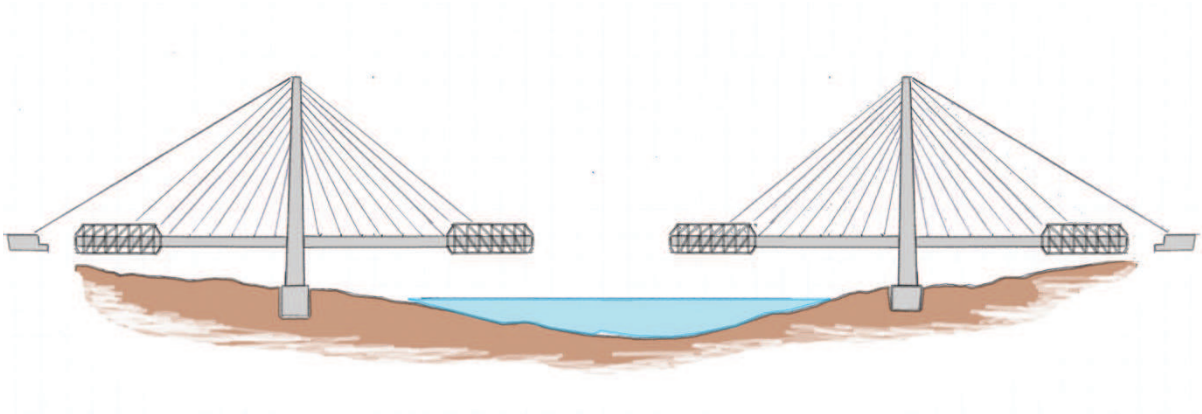


Figure IV.2 Construction of the cable stayed bridge

Material Properties

Concrete

Strength Class			$C35/45$	
Cement			$CEM\ 32.5\ N$	
Characteristic value of compressive strength	f_{ck}	=	35.00	N/mm^2
Design value of compressive strength	f_{cd}	=	23.33	N/mm^2
Mean value of axial tension strength	f_{ctm}	=	3.20	N/mm^2
Specific weight	ρ	=	2500	kg/m^3
Environmental class			$XD3$	
Young's modulus (short term)	$E_{cm(0)}$	=	34,000	N/mm^2
Strain for reaching maximum compressive strength	ε_c	=	$1.75 \cdot 10^{-3}$	
Maximum strain	ε_{cu}	=	$3.50 \cdot 10^{-3}$	

Post tensioning steel

Class			$Y1860S7-15$	
Number of wires in one strand			7	
Diameter of one strand			15.2	mm
Cross section one strand			150	mm^2
Characteristic tensile strength	f_{pk}	=	1860	N/mm^2
Characteristic 0.1% proof stress	$f_{p0.1k}$	=	1674	N/mm^2
Design value tensile strength	f_{pd}	=	1456	N/mm^2
Young's modulus	E_p	=	195,000	N/mm^2

Determination of the Global Dimensions Deck

For the bridge deck a ribbed floor is used to span between the cross beams. The ribs span over a distance of 4.4 meter, the c.t.c. distance of the cross beams. Due to the tension in the cables, a compressive normal force is activated in the bridge deck. The ribs gives some stiffness and strength to the bridge deck. The axial forces a relatively high and therefore it is necessary to create a deck that can transmit these forces to the cross girders. Based on a first indication of the global dimensions, the cross sectional properties are determined.

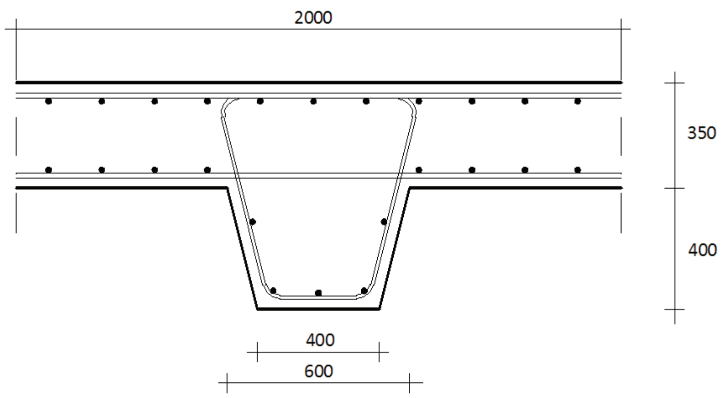


Figure IV.3 Dimensions of the ribbed floor

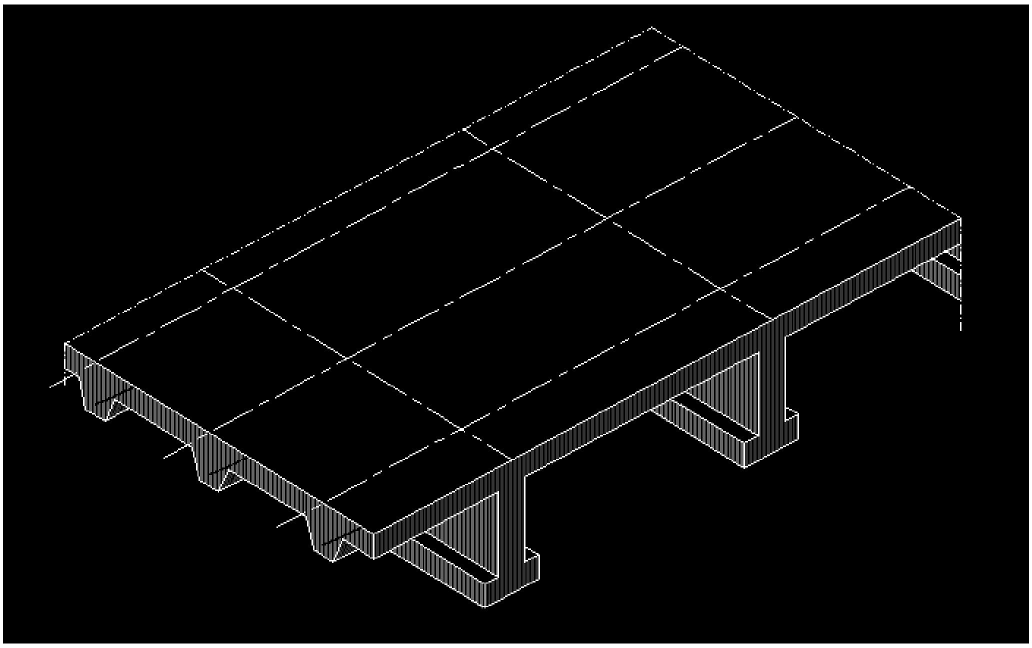


Figure IV.4 3D view of the ribs in the bridge deck plus the cross beams in transverse direction

The thickness of the deck:

$$t_d = 0.35\ m$$

Height of the rib:

$$h_r = 0.40\ m$$

Base width of the rib:

$$w_{br} = 0.40\ m$$

Top width of the rib:

$$w_{tr} = 0.60\ m$$

c.t.c. distance of the ribs:

$$c.t.c = 2.00\ m$$

Cross sectional properties of the ribbed bridge deck
The cross sectional properties of the deck can be calculated.
Cross sectional area:

$$A_c = 2.00 \cdot 0.35 + 0.40 \cdot 0.40 + 0.10 \cdot 0.40 = 0.900\ m^2$$

Distance of the bottom fibre to the normal center:

$$NC_{(z)} = \frac{2.00 \cdot 0.35 \cdot 0.575 + 0.40 \cdot 0.40 \cdot 0.20 + 0.10 \cdot 0.40 \cdot 2/3 \cdot 0.40}{A_c} = 0.4946\ m$$

Moment of inertia:

$$I_{zz} = \frac{1}{12} \cdot 2.00 \cdot 0.35^3 + 2.00 \cdot 0.35 \cdot (0.575 - 0.4946)^2$$
$$+ \frac{6 \cdot 0.40^2 + 6 \cdot 0.40 \cdot 0.10 + 0.10^2}{36 \cdot (2 \cdot 0.40 + 0.10)} \cdot 0.40^3 + 0.4 \cdot 0.4 \cdot (0.20 - 0.4946)^2$$
$$+0.1 \cdot 0.4 \cdot (2/3 \cdot 0.4 - 0.4946)^2 = 0.030025\ m^4$$

Section moduli:

$$W_{c,top} = \frac{I_{zz}}{0.75 - NC_{(z)}} = 0.11756\ m^3$$
$$W_{c,bottom} = \frac{I_{zz}}{NC_{(z)}} = 0.06071\ m^3$$

Determination of the Global Dimensions Cross Beams

A first assumption of the global dimensions of the cross section is done based on the rules of thumb. In order to reduce the number of post tensioning cables and the amount of concrete, there is chosen for an I-shaped cross section. Where the top flange is integrated into the deck floor. The cost for the formwork will increase due to the more complicated form of the cross section, when compared with a rectangular cross section. However the structural advantage of this type of cross section will reduce the costs for post tensioning cables and anchorage points.

Cross sectional properties
The effective width of the top flange can be determined according to:

$$b_{eff,i} = 0.2b_i + 0.1l_0 \leq 0.2l_0$$
$$b_{eff,i} = 0.2 \cdot 3.9 + 0.1 \cdot 28.3 = 3.61\ m \leq 5.66\ m$$

$$b_{eff} = b_{eff,i} + b_w \leq b$$
$$b_{eff} = 4.11\ m$$

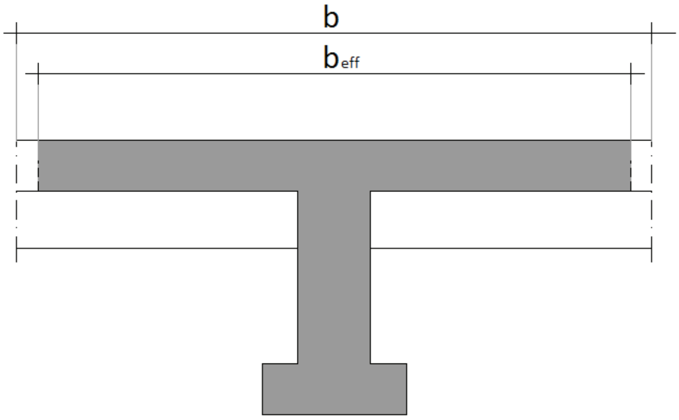


Figure IV.5 Effective width of the cross beam

Effective sectional area

$$A_c = 4.11 \cdot 0.35 + 1.30 \cdot 0.50 + 0.35 \cdot 1.00 = 2.438\ m^2$$

Position of the normal centre from the bottom fibre.

$$NC_{(z)} = \frac{4.11 \cdot 0.35 \cdot 1.825 + 1.30 \cdot 0.50 \cdot 1.05 + 0.35 \cdot 1.00 \cdot 0.175}{A_c} = 1.382\ m$$

Moment of Inertia

$$I_{zz} = \frac{1}{12} \cdot 4.11 \cdot 0.35^3 + 4.11 \cdot 0.35 \cdot (1.825 - 1.382)^2$$
$$+ \frac{1}{12} \cdot 0.50 \cdot 1.30^3 + 0.50 \cdot 1.30 \cdot (1.05 - 1.382)^2$$
$$+ \frac{1}{12} \cdot 1.00 \cdot 0.35^3 + 1.00 \cdot 0.35 \cdot (0.175 - 1.382)^2 = 0.974\ m^4$$

Section moduli

$$W_{c,top} = \frac{I_{zz}}{1.90 - NC_{(z)}} = 1.880\ m^3$$
$$W_{c,bottom} = \frac{I_{zz}}{NC_{(z)}} = 0.705\ m^3$$

Position of the post tensioning cable at midspan measured from the normal centre.

$$e_p = 1.172\ m$$

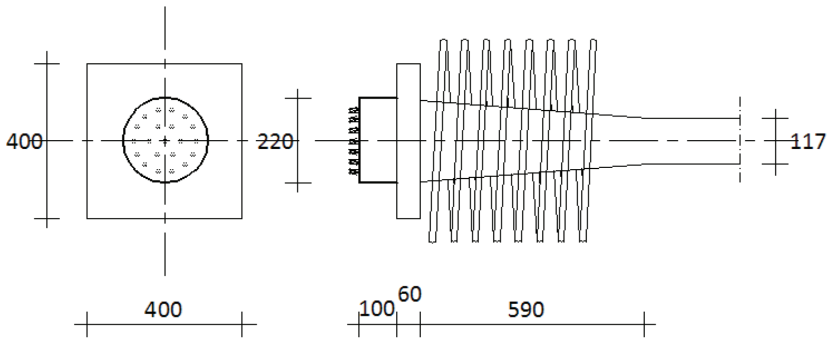


Figure I.6 Anchorage of the post-tensioning cable

In order to reduce the end moments in the girder one of the post tensioning cables has a certain curvature.

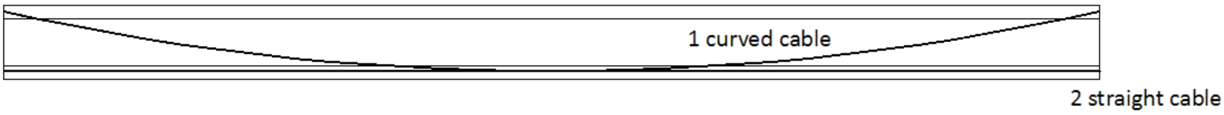


Figure IV.7 Side view of the cross beam

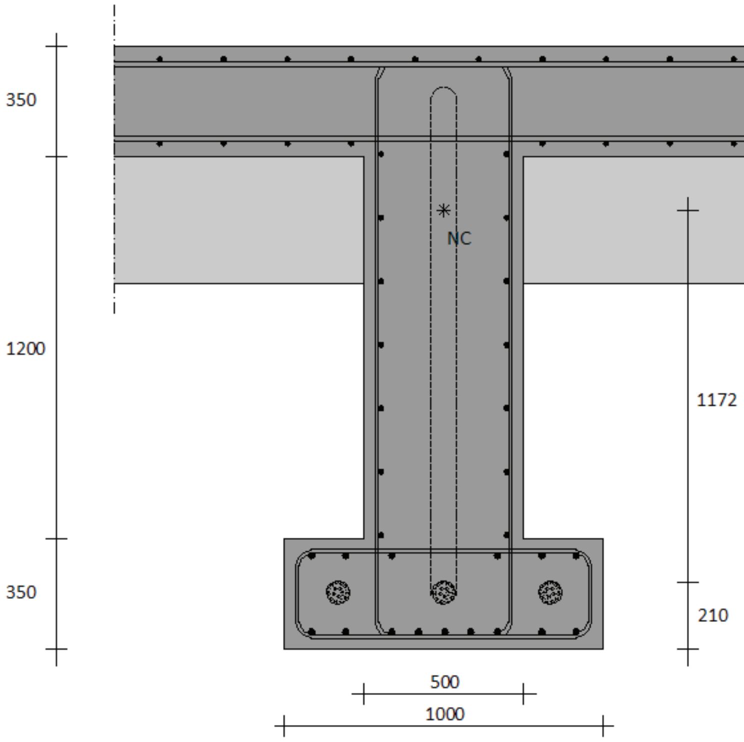


Figure IV.8 Cross section of the cross beam

Moments

The cross beam is modeled as a girder on hinged supports. The loading on the beam is based on the c.t.c. distance of the cross beams.

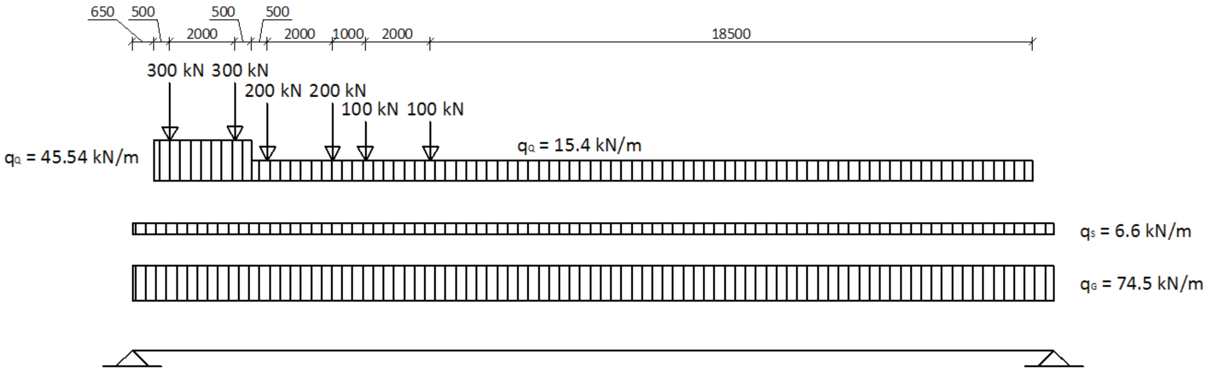


Figure IV.9 Loading of the cross beam due to the dead load, super dead load and the life load

Now the dimensions of the cross beam are known, the occurring moments can be calculated. These moments at midspan are calculated with a linear analysis in the program Matrixframe.

$$M_G = 7458\ kNm$$
$$M_S = 661\ kNm$$
$$M_Q = 4329\ kNm$$

Required post tensioning steel

The required value of the post tensioning force can be determined based on the minimum and maximum allowable stresses in the cross section.
The calculation of the stresses is performed for $t = 0$ and $t = \infty$ at midspan.

$t = 0$
Bottom fibre compression

$$\sigma_{c,bottom} = -\frac{P_{m,0}}{A_c} - \frac{P_{m,0} e_p}{W_{c,bottom}} + \frac{M_G}{W_{c,bottom}} \geq -0.6 f_{cd}$$

$$\sigma_{c,bottom} = -\frac{P_{m,0}}{2.438} - \frac{P_{m,0} \cdot 1.172}{0.705} + \frac{7458}{0.705} \geq -14000\ kN/m^2$$

$$P_{m,0} \leq 11859\ kN$$

$t = 0$
Top fibre tension

$$\sigma_{c,top} = -\frac{P_{m,0}}{A_c} + \frac{P_{m,0} e_p}{W_{c,top}} - \frac{M_G}{W_{c,top}} \leq 0.5 f_{ctm}$$

$$\sigma_{c,top} = -\frac{P_{m,0}}{2.438} + \frac{P_{m,0} \cdot 1.172}{1.880} - \frac{7458}{1.880} \leq 1600\ kN/m^2$$

$$P_{m,0} \leq 26108\ kN$$

$t = \infty$
Bottom fibre tension

$$\sigma_{c,bottom} = -\frac{P_{m,\infty}}{A_c} - \frac{P_{m,\infty} e_p}{W_{c,bottom}} + \frac{M_G + M_S + M_Q}{W_{c,bottom}} \leq 0.5 f_{ctm}$$

$$\sigma_{c,bottom} = -\frac{P_{m,\infty}}{2.438} - \frac{P_{m,\infty} \cdot 1.172}{0.705} + \frac{12448}{0.705} \leq 1600$$

$$P_{m,\infty} \geq 7747\ kN$$

If the prestressing losses are estimated on 20% the ratio between the prestressing force at $t = 0$ and $t = \infty$ is known.

$$P_{m,0} \geq \frac{7747}{0.80} = 9684\ kN$$

$$A_p = \frac{9684 \cdot 10^3}{1395} = 6942\ mm^2$$
$$\#strands = \frac{6942}{150} = 47\ strands$$

The maximum number of strands is (22x3) 66.
With 3 cables with 18 strands the prestressing force is:

$$P_{m,0} = 18 \cdot 3 \cdot 150 \cdot 1395 = 11300 \cdot 10^3\ N$$

Elastic losses

A certain deformation of the concrete, due to stressing of a cable, leads to a loss of stress in the cables that where already tensioned. The strain of the concrete can be calculated based on the linear stress-strain relation.

$$\Delta\varepsilon_c = \frac{\Delta\sigma_c}{E_{cm}} = \frac{1}{E_{cm}}\left(\frac{P_{m,0}}{A_c} + \frac{M_p e_p}{I_{zz}}\right)$$

The strain of the cable can also be determined.

$$\Delta\varepsilon_p = \frac{\Delta\sigma_p}{E_p} = \frac{\Delta P_{el}}{E_p A_p}$$

Compatibility requires:

$$\Delta\varepsilon_c = \Delta\varepsilon_p$$

$$\Delta P_{el} = \frac{E_p A_p}{E_{cm} A_c} P_{m,0}\left(1 + \frac{{e_p}^2 A_c}{I_{zz}}\right)$$

The number of post tensioning cables can be included in the formula. The first applied cable will experience the highest elastic losses.

$$\Delta P_{el} = \frac{n-1}{2n}\frac{E_p A_p}{E_{cm} A_c} P_{m,0}\left(1 + \frac{{e_p}^2 A_c}{I_{zz}}\right)$$

$$\Delta P_{el} = \frac{3-1}{2 \cdot 3}\frac{195{,}000 \cdot 8100}{34{,}000 \cdot 2.438 \cdot 10^6} \cdot 11300 \cdot 10^3 \cdot \left(1 + \frac{1{,}172^2 \cdot 2.438 \cdot 10^6}{0.974 \cdot 10^{12}}\right)$$

$$= 318.5 \cdot 10^3 N$$

This gives a mean loss of stress in the cables of:

$$\Delta\sigma_{el} = \frac{\Delta P_{el}}{A_p} = \frac{318.5 \cdot 10^3}{8100} = 39.3\ N/mm^2$$

The first cable has a loss of 78.6 N/mm^2 the second cable 39.3 N/mm^2 and the third cable has no elastic loss.

Losses due to friction

Because the beam consist of two types of cables, two straight cables and one curved cable, the friction losses for these cables are not the same.

$$\Delta P_\mu(x) = P_{max}(1 - e^{-\mu(\theta + kx)})$$

Where

$$\mu = 0.16$$
$$k = 0.010\ rad/m$$

Straight cables

$$\theta_1 = 0$$
$$\Delta P_\mu(x) = 3767 \cdot 10^3(1 - e^{-0.0016x})$$
$$\Delta P_\mu(28.3) = 3767 \cdot 10^3(1 - e^{-0.0016 \cdot 28.3}) = 166.8 \cdot 10^3 N$$

Curved cable

$$\theta_1 = 0.2098\ rad$$
$$\Delta P_\mu(x) = 3767 \cdot 10^3\left(1 - e^{-0.16(0.2098 + 0.010x)}\right)$$
$$\Delta P_\mu(28.3) = 3767 \cdot 10^3\left(1 - e^{-0.16(0.2098 + 0.010 \cdot 28.3)}\right) = 285.6 \cdot 10^3 N$$

The average friction loss as a change in stress per unit length is for the straight cable:

$$\Delta\sigma_{pm} = \frac{166.8 \cdot 10^3}{28{,}300 \cdot 2700} = 0.002183\ N/mm^2/mm$$

And for the curved cable:

$$\Delta\sigma_{pm} = \frac{285.6 \cdot 10^3}{28{,}300 \cdot 2700} = 0.003738\ N/mm^2/mm$$

Due to the settlement of the wedges a certain length of the cable experience a reduction in stress. The average friction loss is taken into account to calculate the length of the cable that experience the decrease of stress after the strands are released. It is assumed that the wedges will have on average a settlement of 6 mm. The length that is influenced by the settlement can be given by:

$$l_{set} = \sqrt{\frac{\Delta w_{set} E_p}{\Delta\sigma_{pm}}} = \sqrt{\frac{6 \cdot 195{,}000}{0.002183}} = 23151\ mm$$

$$l_{set} = \sqrt{\frac{\Delta w_{set} E_p}{\Delta\sigma_{pm}}} = \sqrt{\frac{6 \cdot 195{,}000}{0.003738}} = 17692\ mm$$

The loss due to wedge settlement is at the location of the anchorage point where the strands are tensioned is:

$$\Delta\sigma_{set} = 2\Delta\sigma_{pm} l_{set}$$

$$\Delta\sigma_{set} = 2 \cdot 0.002183 \cdot 23151 = 101\ N/mm^2$$

$$\Delta\sigma_{set} = 2 \cdot 0.003738 \cdot 17692 = 132\ N/mm^2$$

This means that the post tensioning force is lower at the side where the post tensioning cable is tensioned. At the side of the blind anchor the resulting stress in the cable is higher. This also means that tensioning of the cable at both anchorage points does not influence the stress in the cable positively.

It is possible to overstress the prestressing steel. The maximum allowable stress during prestressing is:

$$\sigma_{p,max} = 0.8 \cdot 1860 = 1488\ N/mm^2$$

First the two straight cables are tensioned. These cables will experience the elastic losses. The cables can be overstresses with 93 N/mm^2. This would mean that only a part of the loss of stress due to wedge settlement can be compensated. For the straight cable there is a loss of stress of:

$$\Delta\sigma = 101 + 78.6 - 93 = 86.6\ N/mm^2$$

For the curved cable there is a loss of:

$$\Delta\sigma = 132 - 93 = 39.0\ N/mm^2$$

The average stress in the cables at $t = 0$ is 1359 N/mm^2. With this stress taken into account the average prestressing force in the cables is 11008 kN.

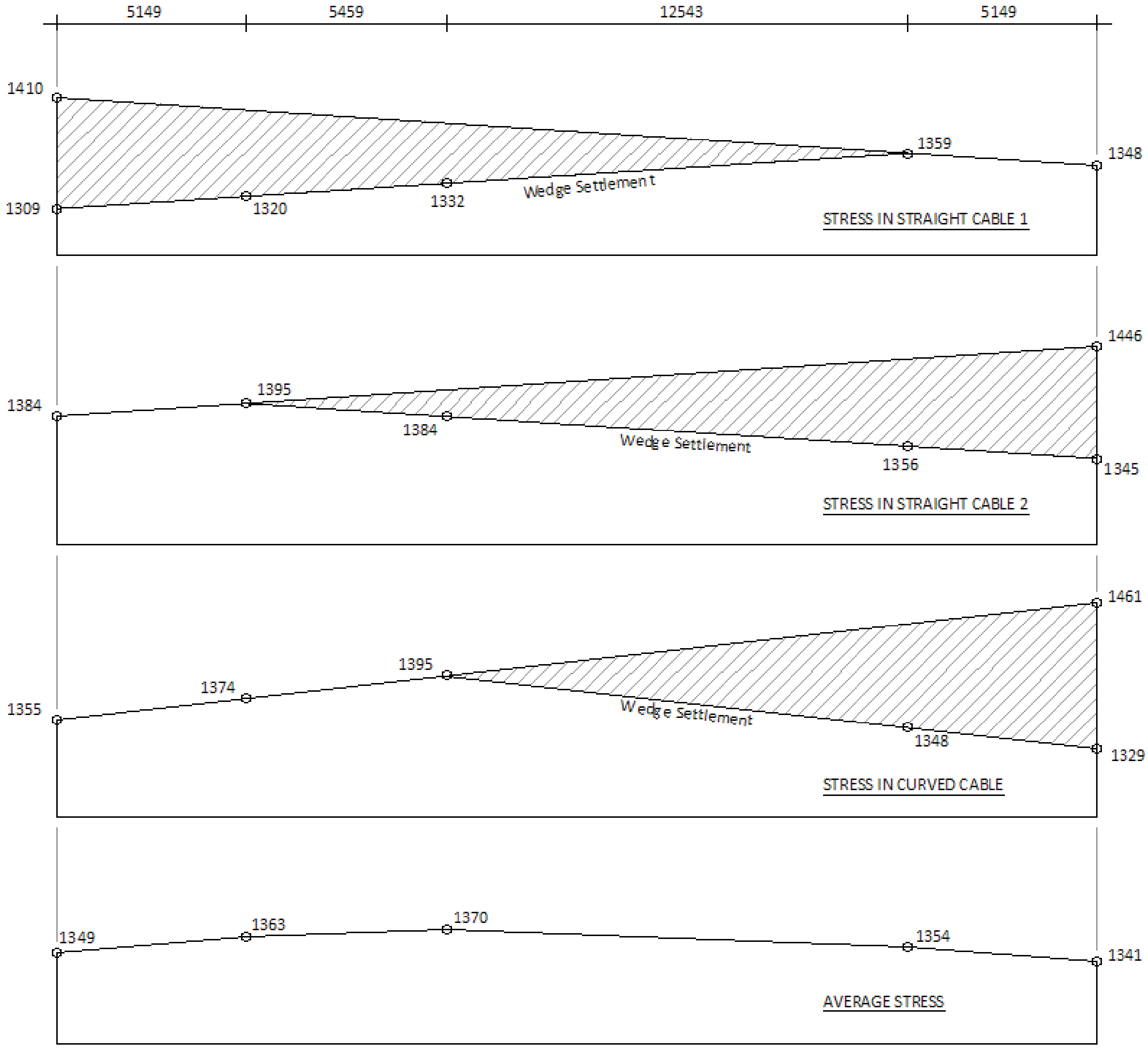


Figure IV.10 Stresses in cables with elastic losses, friction losses and wedge settlement included

Losses due to creep

For the prestressing losses due to creep Trost method is used.

$$\varepsilon_{c,cr}(t) = \frac{\sigma_c(0)}{E_{cm}{}'} + \frac{\Delta\sigma_c(t)}{E_{cm}{}''}$$

The initial stress at the location of the post tensioning cable is based on the post tensioning force at $t = 0$ and the moment due to the dead load.

$$\sigma_c(0) = -\frac{P_{m,0}}{A_c} - \frac{P_{m,0}e_p}{W_{c,bottom}} + \frac{M_G}{W_{c,bottom}}$$

$$\sigma_c(0) = -\frac{11008}{2.438} - \frac{11008 \cdot 1.172}{0.705} + \frac{7458}{0.705} = -12236\ kN/m^2$$

The additional stress is based on the super dead weight (asphalt layer) and the life load.

$$\Delta\sigma_c(t) = +\frac{M_S}{W_{c,bottom}} + \frac{\psi_2 M_Q}{W_{c,bottom}}$$

$$\Delta\sigma_c(t) = +\frac{661}{0.705} + \frac{0.4 \cdot 5236}{0.705} = +3908\ kN/m^2$$

The Young's modulus is adapted based on the creep coefficient.

$$E_{cm}{}' = \frac{E_{cm}}{1+\varphi} = \frac{34{,}000}{1+2.7} = 9189\ N/mm^2$$

$$E_{cm}{}'' = \frac{E_{cm}}{1+\chi\varphi} = \frac{34{,}000}{1+0.8\cdot 1.4} = 16038\ N/mm^2$$

The creep coefficient is determined based on the fictive thickness of the cross section h_0 and on the time of loading. The cables are post-tensioned after 3 days. It is assumed that the asphalt layer is applied after a 100 days. And that the life load is activated after 100 days as well.

$$h_0 = \frac{2A_c}{u} = \frac{2\cdot 2.438}{12.52} = 0.389\ m$$

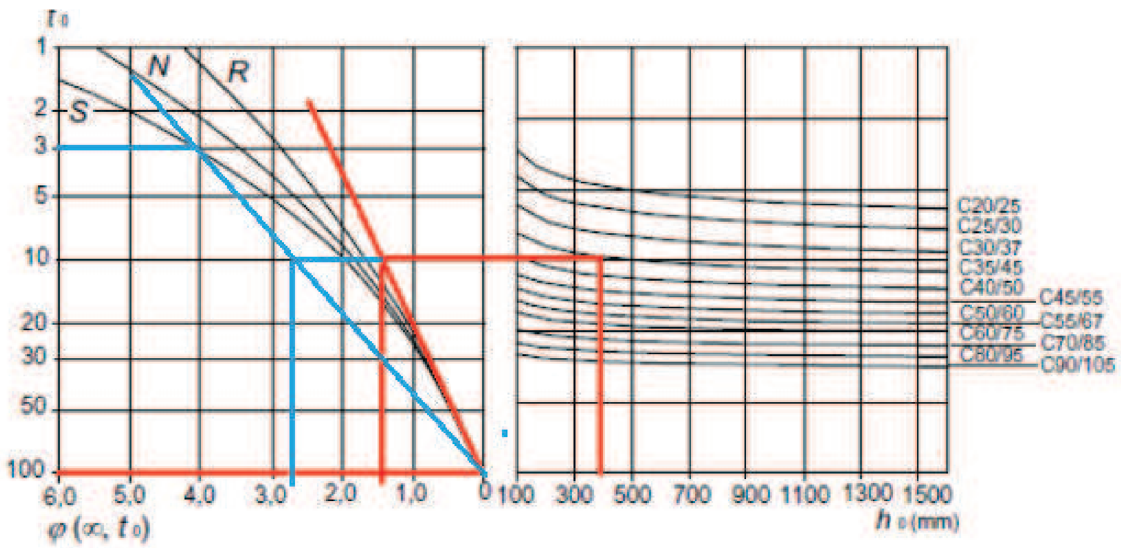


Figure IV.11 Determination of the creep coefficient

The strain due to creep can now be calculated. The initial stress at the location of the post tensioning cable has a negative value. The additional stress due to the super dead load and the life load has a positive value.

$$\varepsilon_{c,cr}(t) = \frac{-12.24}{9189} + \frac{3.90}{16038} = -1.089\cdot 10^{-3}$$

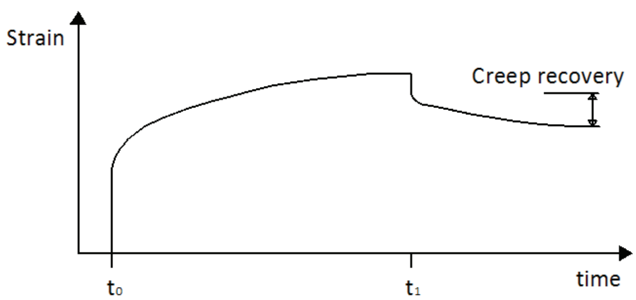


Figure IV.12 Schematic figure of the development of the creep in time

Losses due to shrinkage
According to the Eurocode 1992 art. 3.1.4, the shrinkage strain can be calculated as a combination of drying shrinkage and autogenous shrinkage.

$$\varepsilon_{c,sh} = \varepsilon_{cd,\infty} + \varepsilon_{ca,\infty}$$

The drying shrinkage ($\varepsilon_{cd,0}$) is based on Eurocode 1992 art. 3.1.4 table 3.2. Concrete class C35/45 combined with a relative humidity of 80%. k_h is based on table 3.3 with $h_0 = 388mm$.

$$\varepsilon_{cd,\infty} = k_h \varepsilon_{cd,0} = 0.728 \cdot 0.26 \cdot 10^{-3} = 0.189 \cdot 10^{-3}$$

For autogenous shrinkage at $t = \infty$ the EC2 gives:

$$\varepsilon_{ca,\infty} = 2.5(f_{ck} - 10) \cdot 10^{-6} = 0.0625 \cdot 10^{-3}$$

The amount of autogenous shrinkage is for this concrete class not very high. For higher concrete classes the autogenous shrinkage becomes more important due to the lower water-cement ratio. The total strain in the concrete that causes losses in the prestressing steel is the sum of the deformation due to creep and shrinkage.

$$\varepsilon_c(t) = \varepsilon_{c,cr}(t) + \varepsilon_{c,sh}(t) = -1.089 \cdot 10^{-3} - 0.189 \cdot 10^{-3} + 0.0625 \cdot 10^{-3} = 1.341 \cdot 10^{-3}$$

This results in a decrease of stress in the prestressing steel of:

$$\Delta\sigma_{pc} = \varepsilon_c(t) E_p = 1.341 \cdot 10^{-3} \cdot 195{,}000 = 261\ N/mm^2$$

Losses due to relaxation
For the loss of stress due to relaxation of the post tensioning steel the following formula is used.

$$\Delta\sigma_{pr} = \sigma_{pi} \cdot 0.66 \cdot \rho_{1000} e^{9.1\mu} \left(\frac{t}{1000}\right)^{0.75(1-\mu)} \cdot 10^{-5}$$

σ_{pi} is the initial peak stress value in the cables. After wedge settlement the peak stress in the cable is $1395\ N/mm^2$.
ρ_{1000} is the relaxation loss in percentage after a 1000 hours of stressing at a mean temperature of 20°C. Low relaxation steel is applied which results in $\rho_{1000} = 2.5\%$.
t is the time after tensioning in hours. For the total service life of the structure it can be said that $t = 500000\ hours$.
μ is the ratio between the initial peak stress and the yielding stress of the steel.

$$\mu = \frac{1395}{1860} = 0.75$$

$$\Delta\sigma_{pr} = 1395 \cdot 0.66 \cdot 2.5 \cdot e^{9.1 \cdot 0.75} \left(\frac{500000}{1000}\right)^{0.75(1-0.75)} \cdot 10^{-5} = 67.9\ N/mm^2$$

Because of the influences of creep and shrinkage, the value for relaxation will in practice be a bit lower. According to Eurocode 2 a reduction factor of 0.8 can be taken into account.

$$\Delta\sigma_{pr} = 0.8 \cdot 67.9 = 54.4\ N/mm^2$$

Actual stresses
Now the prestressing losses are known the actual stresses at midspan can now be calculated.

$$\sigma_{pi} = 1359\ N/mm^2$$
$$\Delta\sigma_{pc} = -261\ N/mm^2$$
$$\Delta\sigma_{pr} = -54.4\ N/mm^2$$
$$\sigma_{p\infty} = 1044\ N/mm^2$$

This results in a prestressing force at $t = \infty$ of

$$P_{m,\infty} = 1044 \cdot 18 \cdot 3 \cdot 150 \cdot 10^{-3} = 8456\ kN\ > 7747\ kN$$

Required amount of reinforcing steel
For the maximum occurring moment in the ULS it is allowable that the concrete cracks. Internal equilibrium should be reached by applying reinforcement steel.

$P_{m,\infty} = 8456\ kN$	Working prestressing force
$h = 1900\ mm$	Height of the girder
$d_p = h - 210 = 1790\ mm$	Distance from the top fibre to the prestressing cable
$d_{s1} = h - 65 = 1935\ mm$	Distance from the top fibre to the outer reinforcement
$d_{s2} = h - 285 = 1615\ mm$	Distance from the top fibre to the second layer of reinforcement
$E_s = 200{,}000\ N/mm^2$	Young's modulus of the reinforcement steel
$E_p = 195{,}000\ N/mm^2$	Young's modulus of the prestressing steel
$A_p = 8100\ mm^2$	Area of the prestressing steel
$f_{cd} = 23.33\ N/mm^2$	Design value compressive strength concrete
$\xi = 0.5$	Factor for the prestressing steel
x	Height compressive zone
ε_c	Maximum occurring concrete strain
$M_d = 1.32(7458 + 660.7) + 1.65 \cdot 4329 - 8456 \cdot 1.172 = 7949\ kNm$	Design value maximum moment
$A_{s1} = 3927\ mm^2$	Area outer reinforcement 8Ø25
$A_{s2} = 2945\ mm^2$	Area second layer of reinforcement 6Ø25

Assume that $x > 350mm$
There should be equilibrium of forces and moments

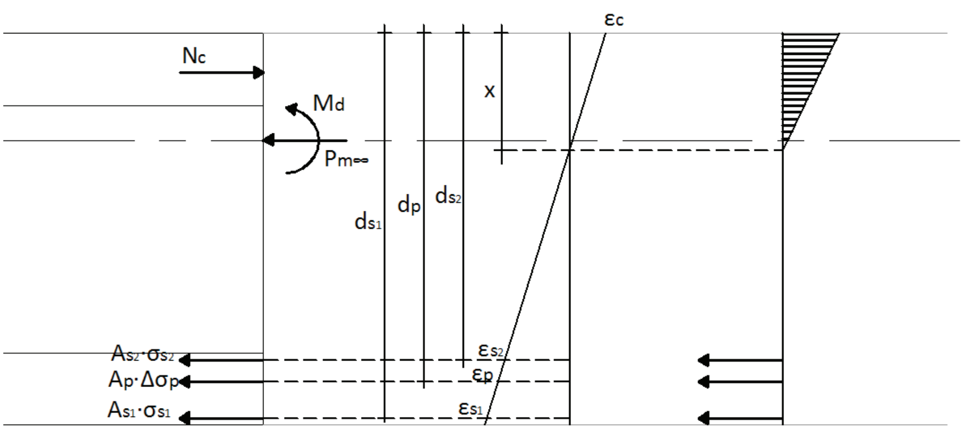


Figure IV.13 Internal forces in the midspan section

The internal force of the compression zone of the concrete can be expressed with the following formula:

$$N_c = \frac{1}{2} \cdot x b_f \left(\frac{\varepsilon_c}{1.75 \cdot 10^{-3}} f_{cd} \right) - \frac{1}{2} \cdot (b_f - b_w)(x - 350) \left(\frac{(x - 350)\varepsilon_c}{x \cdot 1.75 \cdot 10^{-3}} f_{cd} \right)$$

There should be horizontal equilibrium in the cross section.

$$\Sigma F_x = 0$$

$$\Sigma F_x = -N_c + P_{m,\infty} + A_{s1} \frac{(d_{s1} - x)\varepsilon_c}{x} E_s + A_{s2} \frac{(d_{s2} - x)\varepsilon_c}{x} E_s + \xi A_p \frac{(d_p - x)\varepsilon_c}{x} E_p = 0$$

A second condition is that there should be equilibrium of moments in the cross section.

$$\Sigma M = 0$$

$$\Sigma M = M_d - \frac{1}{2} \cdot x b_f \left(\frac{\varepsilon_c}{1.75 \cdot 10^{-3}} f_{cd}\right)\left(518 - \frac{1}{3}x\right) +$$

$$\frac{1}{2} \cdot (b_f - b_w)(x - 350)\left(\frac{(x-350)\varepsilon_c}{x \cdot 1.75 \cdot 10^{-3}} f_{cd}\right)\left(168 - \frac{1}{3}(x - 350)\right) -$$

$$A_{s1}\frac{(d_{s1} - 518)(d_{s1} - x)\varepsilon_c}{x} E_s - A_{s2}\frac{(d_{s2} - 518)(d_{s2} - x)\varepsilon_c}{x} E_s$$

$$- \xi A_p \frac{(d_p - 518)(d_p - x)\varepsilon_c}{x} E_p = 0$$

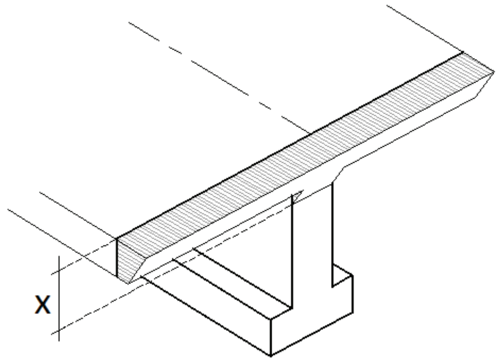


Figure IV.14 Compressive zone of the concrete

This results in:

$$x = 621\ mm$$
$$\varepsilon_c = 0.818 \cdot 10^{-3}$$

For this calculation some assumptions where made. These assumption should be checked afterwards in order to judge the correctness of the calculated values.

$$x > 350mm$$
$$x = 621mm$$

$$\varepsilon_c < 1.75 \cdot 10^{-3}$$
$$\varepsilon_c = 0.818 \cdot 10^{-3}$$

$$\Delta\sigma_p < 1395 - 1044 = 351\ N/mm^2$$
$$\Delta\sigma_p = \xi \frac{(d_p - x)\varepsilon_c}{x} E_p = 137\ N/mm^2$$

$$\sigma_{s1} = \frac{(d_{s1} - x)\varepsilon_c}{x} E_s = 320\ N/mm^2 < 435\ N/mm^2$$
$$\sigma_{s2} = \frac{(d_{s2} - x)\varepsilon_c}{x} E_s = 262\ N/mm^2 < 435\ N/mm^2$$

It can be seen that the cross section is able to carry the ULS design moment.

Determination of the Global Dimensions Stay Cables and Main girders

For the stay cables a Parallel Wire Strand (PWS) system is used. The maximum tensile strength of the cables is 1770 N/mm^2. The strands that are used have a diameter of 15.7mm. During the execution of the bridge deck, the stay cables should be able to carry the weight from two bridge section plus the formwork and falsework. When the load is equally divided per stay, the stay with the lowest angle will experience the highest force. The weight of each bridge section can be determined based on the dimensions of the deck and cross beams. For the weight of a main girder a value of 122.6 kN/m^1 is taken into account.

The weight per section of the bridge is divided into the dead weight, the super dead weight, the life load and the axle load. In the building phase it is necessary to take the weight of the false- and formwork into account. The load of the falsework is assumed to be 4.00 kN/m^2. For the formwork an additional temporary loading due to people and material a load of 2.00 kN/m^2 is taken into account.

$$W_G = 25 \cdot (1.00 \cdot 28.3 \cdot 2 + 0.45 \cdot 8.8 \cdot 24.1 + 2 \cdot 4.905 \cdot 8.8) = 5959kN$$
$$W_S = 1.50 \cdot 28.3 \cdot 8.8 = 374kN$$
$$W_Q = 8.8 \cdot (10.35 \cdot 3 + 25.3 \cdot 3.5) = 1052kN$$
$$W_D = 600 + 400 + 200 = 1200kN$$
$$W_B = (4.00 + 2.00) \cdot 28.3 \cdot 8.8 = 1494kN$$

For the building phase each stay should be able to carry the weight of two bridge sections. The load factors are for this phase equal to 1.0.

$$N_{s,i} = \frac{2 \cdot (W_G + W_S + W_B)}{2 \cdot \sin(\alpha_i)}$$

For the final phase (ULS) each stay will carry one bridge section and should be able to transfer the axle loading.

$$N_{s,i} = \frac{\gamma_G \kappa_{Fi}(W_G + W_S) + \gamma_Q \kappa_{Fi}(W_Q + W_D)}{2 \cdot \sin(\alpha_i)}$$

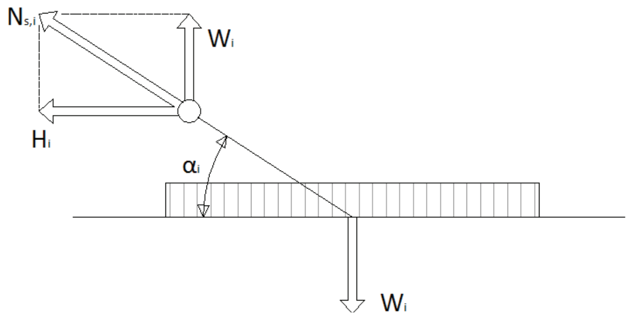


Figure IV.15 Determination of the cable force

The horizontal force is determined based on the final phase with the axle load positioned at the first stay. In table IV.1 the axial force is shown for the different stay cables. This table also shows the minimum and applied cross sectional area of the stays.

Table IV.1 Overview of the forces in the stay cables

Stay	Angle of the Stay Cable	Axial Force Stay Cable Building Phase $[kN]$	Axial Force Stay Cable Final Phase $[kN]$	Horizontal Force Final Phase $[kN]$	Minimum Required Area $[mm^2]$	Applied Stay Area $[mm^2]$
1	28.64°	16325	12594	11052	2x17976	2x22800
2	30.62°	15367	11855	9188	2x16593	2x21600
3	32.86°	14430	11132	8422	2x15210	2x20250
4	35.40°	13517	10428	7657	2x13827	2x18900
5	38.27°	12635	9747	6891	2x12445	2x17250
6	41.62°	11789	9095	6125	2x11062	2x16500
7	45.39°	10990	8478	5360	2x9679	2x15750
8	49.82°	10246	7904	4594	2x8296	2x14250
9	54.89°	9572	7384	3828	2x6914	2x13500
10	60.66°	8983	6930	3063	2x5531	2x12750
11	67.19°	8497	6555	2297	2x4148	2x12000
12	74.37°	8131	6273	1531	2x2765	2x12000

When the bending stiffness of the pylon is neglected. The back stay cables should be able to carry the resulting horizontal forces. The stays connected to the end support should be able to transfer the horizontal forces from the first, the second and the third cable. This results in an axial load of 34117kN for each of the stay cables. This gives a required area of 2x43500mm^2.

To insure that fatigue is not governing, the maximum stress in the stay should be limited to 45% of the ultimate stress.

$$\sigma_{st} = 0.45 f_{pu} = 797\ N/mm^2$$

This would mean that a minimum cable area of 17976mm^2 is needed for the cables with the lowest angle.

For the first analysis of the bridge the stay cables will be modeled as constrained supports for the dead load. For the life load the stays will be modeled as vertical springs. The spring stiffness is based on the vertical displacement of the stay under a unit load.

The vertical displacement of the stay cable can be expressed as:

$$v_{ver,i} = \sqrt{l_i^{\,2}\left(1 + \frac{N_{unit}}{EA_i}\right)^2 - b_i^{\,2}} - h$$

Where b_i is the horizontal distance from the pylon to the anchorage point of the considered stay cable. And h is the height of the pylon.

This gives a vertical spring stiffness of:

$$k_i = \frac{N_{unit}}{v_{ver,i}}$$

Table IV.2 Vertical spring stiffness for the different stay cables

Stay	Applied Stay Area $[mm^2]$	Modulus of Elasticity $[N/mm^2]$	Horizontal Distance from the Pylon to the Anchorage Point $[mm]$	Vertical Displacement under Unit Load $[mm]$	Vertical Spring Stiffness $[N/mm]$
1	2x22800	205000	114400	374	26704
2	2x21600	205000	105600	253	39513
3	2x20250	205000	96800	212	47182
4	2x18900	205000	88000	178	56315
5	2x17250	205000	79200	245	40756
6	2x16500	205000	70400	149	67293
7	2x15750	205000	61600	255	39235
8	2x14250	205000	52800	160	62346
9	2x13500	205000	44000	126	79093
10	2x12750	205000	35200	122	81846
11	2x12000	205000	26400	99	101415
12	2x12000	205000	17600	105	95214

Dead load

For the dead load the anchorage points of the stay cables are considered as fixed supports. The internal forces can be determined based on this assumption. The representative line load for the dead weight and the super dead weight is:

$$q_{G,rep} = 719.7\ kN/m$$

The moment distribution can be approached based on a continuous concrete beam over multiple fixed supports. The values for the field moment and the moment above the supports are estimated with:

$$M_{G,rep} = \pm \frac{1}{10} q_{G,rep} l^2$$

$$M_{G,rep} = \pm \frac{1}{10} \cdot 719.7 \cdot 17.6^2 = \pm 22292 kNm$$

The normal force in the bridge deck is dependent of the horizontal component from the stay cables. The maximum normal force can be found at the location of the pylon and has a value of 80252kN.

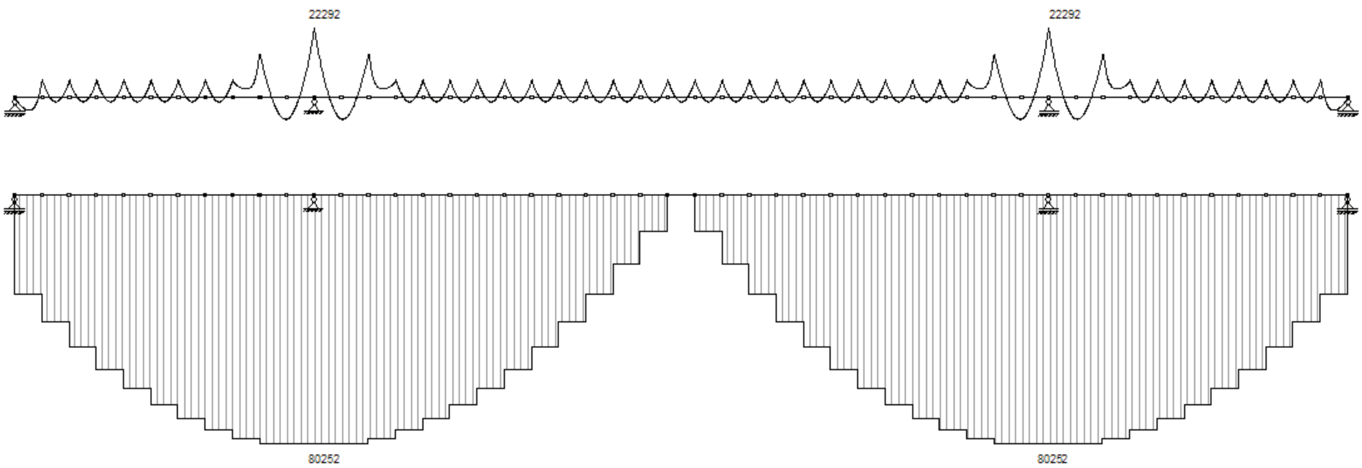


Figure IV.16 Global indication of the moment distribution and the normal force distribution in the bridge deck

Table IV.3 Horizontal forces due to the dead load and the life load

Stay	Angle of the Stay Cable	Horizontal Force Final Phase Dead Load $[kN]$	Horizontal Force Final Phase Life Load (distr.) $[kN]$	Horizontal Force Final Phase Life Load (axial) $[kN]$	Horizontal Force Final Phase Life Load (axial) $[kN]$
1	28.64°	11592	1342	-	236
2	30.62°	10700	1781	-	275
3	32.86°	9809	1854	-	226
4	35.40°	8917	1892	-	165
5	38.27°	8025	1149	-	63
6	41.62°	7133	1556	3	45
7	45.39°	6242	720	6	7
8	49.82°	5350	863	14	-
9	54.89°	4458	765	25	-
10	60.66°	3567	496	30	-
11	67.19°	2675	321	34	-
12	74.37°	1783	111	22	-
SUM		**80252**	**12851**	**135**	**1018**

Life load

For the life load the end supports and the supports at the pylons are modeled as fixed supports. The supports that represent the stay cables are modeled as vertical springs with a spring stiffness equal to the value calculated in table IV.2. The maximum moment for the distributed load can be found when the complete deck is loaded. The moment above the support without load factor is equal to -33153 kNm. The normal force has a value of -12851kN. For the axle load, two load cases can be considered. When the axle load is applied near the pylon, the occurring moment above the support will have the largest value (-3117kNm). In this case the normal force has a value of -135kN. Another possibility is when the axle load is positioned at midspan. In this case the normal force will have the largest value (-1018kN) and the moment above support has a value of 164kNm.

Cross sectional properties of the main girders and total bridge deck

The properties of the main girders can be determined. Based on the occurring moments and normal forces it should be checked whether or not the cross section complies with the requirements. The properties of the total cross section of the bridge are given.

Cross sectional area single main girder

$$A_{c,main} = 4.965\ m^2$$

Distance of the bottom fibre of the main girder to the normal centre

$$NC_{(z),main} = 1.168\ m$$

Moment of inertia of a single main girder

$$I_{zz,main} = 2.333\ m^4$$

Cross sectional area of the total bridge deck

$$A_{c,tot} = 2 \cdot 4.965 + 24.1 \cdot 0.45 = 20.775\ m^2$$

Distance of the bottom fibre of the main girders to the normal centre

$$NC_{(z)} = \frac{2 \cdot 4.965 \cdot 1.168 + 0.45 \cdot 24.1 \cdot 1.745}{A_{c,tot}} = 1.469\ m$$

Moment of inertia

$$I_{zz} = 2 \cdot 2.333 + 2 \cdot 4.965 \cdot (1.168 - 1.469)^2$$
$$+0.0150 \cdot 24.1 + 0.45 \cdot 24.1 \cdot (1.745 - 1.469)^2 = 6.753\ m^4$$

Section moduli

$$W_{c,top} = 8.647\ m^3$$
$$W_{c,bottom} = 4.597\ m^3$$

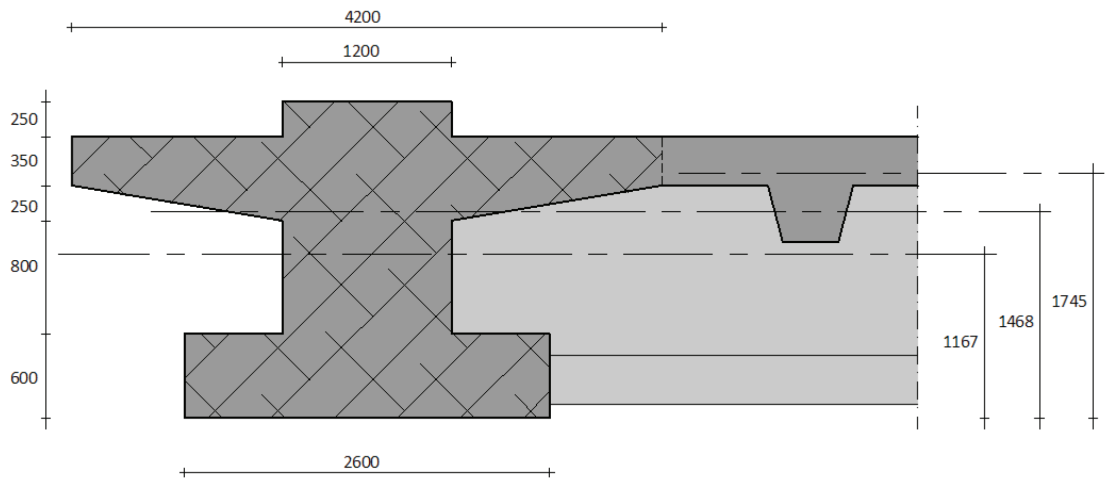


Figure IV.17 Cross section main girder

N-M-kappa diagram

A moment-kappa diagram can be determined with the occurring normal force at the support. For this N-M-kappa diagram the ultimate limit state is taken into account.
The design value for the normal force at the supports can be calculated based on the values found for the dead load and the life load.

$$N_d = 1.32 \cdot N_G + 1.65 \cdot (N_Q + N_D)$$
$$N_d = 1.32 \cdot 80252 + 1.65 \cdot (12851 + 135) = 127360\ kN$$

This gives a design moment with a value of:

$$M_d = 1.32 \cdot M_G + 1.65 \cdot (M_Q + M_D)$$
$$M_d = 1.32 \cdot 22292 + 1.65 \cdot (33153 + 3117) = 89271\ kNm$$

The normal force gives rise to a normal stress in the cross section.

$$\sigma_N = \frac{N_d}{A_c} = 6130\ kN/m^2$$

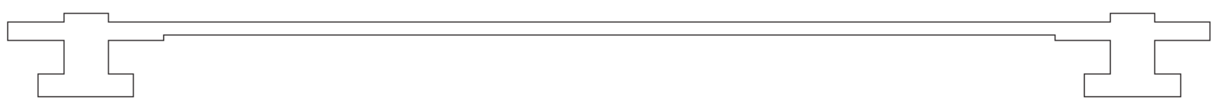


Figure IV.18 Simplification of the cross section of the bridge

The maximum additional compressive stress at the bottom fibre of the cross section has a value of $17202\ kN/m^2$. This means that there can be an additional moment applied on the cross section of $79125\ kNm$ before the bottom fibre reaches the design compressive strength of the concrete. The structure behaves as a linear elastic cross section when the moment is smaller than $79125\ kNm$. The curvature that belongs to this moment is:

$$\kappa_e = 0.879 \cdot 10^{-3}$$

If the moment is increased the stress at the bottom fibres will not increase any further. See also figure I.19. The stress at the top fibres can increase until the design tensile strength is reached. In order to calculate the value of the moment, horizontal equilibrium should be obtained. This gives due to the moment a compressive zone of:

$$x_r = 1.470m$$

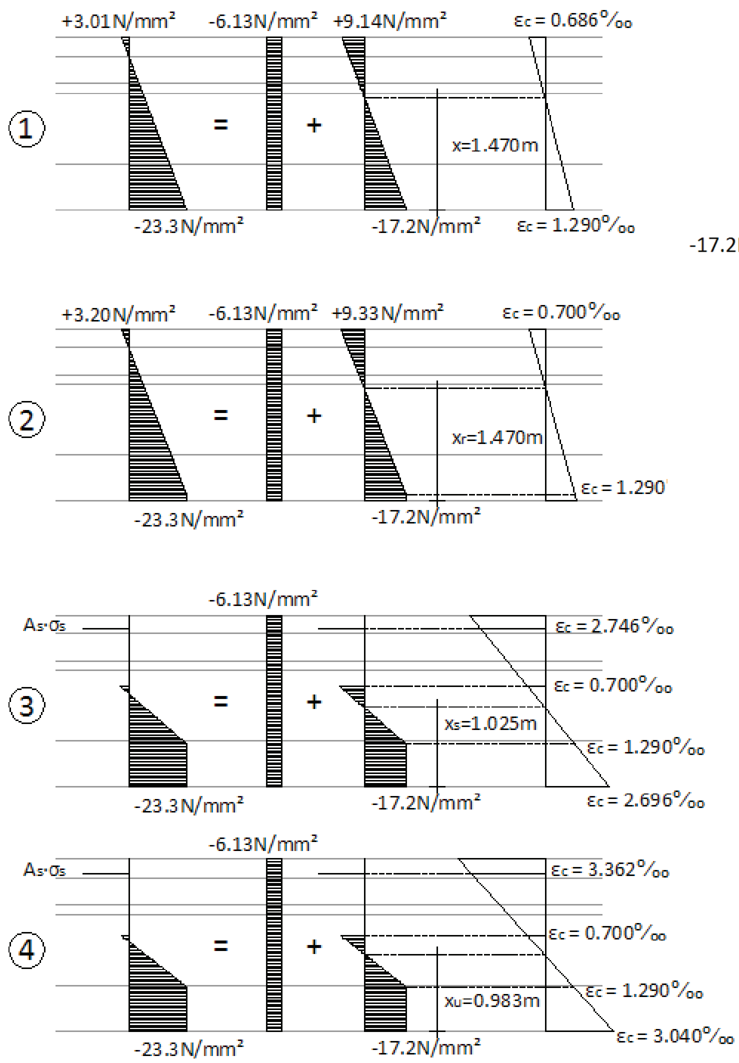
Where over a height of $0.080m$ the fibres has reached their maximum compressive strength. With these values the internal moment can have a value of:

$$M_r = 80285\ kNm$$

The curvature is based on the strains occurring at this moment.

$$\kappa_r = 0.897 \cdot 10^{-3}$$

The maximum strain of the concrete before reaching the design compressive strength is $1.75‰$. The normal force will cause a strain of $0.460‰$. For the moment remains a maximum resulting strain of $1.290‰$ before the concrete reaches the design compressive strength. This also means that the ultimate strain of the concrete caused by a moment is reduced with $0.460‰$. This gives that a maximum moment can be applied that causes a strain of $3.040‰$



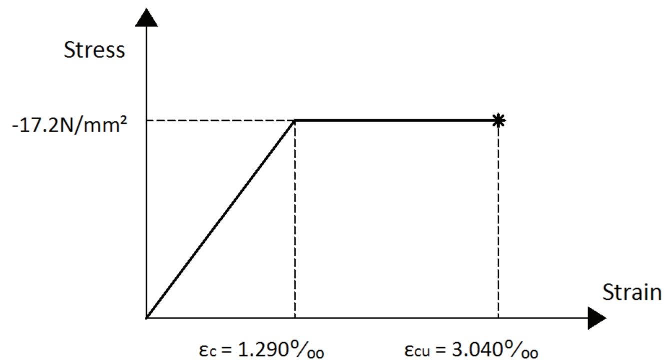


Figure IV.19 Assumed stress-strain relation

Figure IV.20 The four different phases in the N-M-kappa diagram

The third point in the N-M-kappa diagram is reached when the reinforcement starts to yield. The tension zone of the concrete is in this phase neglected. The compression zone of the concrete should make horizontal equilibrium with the force form the reinforcement. A reinforcement area of $0.14m^2$ is assumed in this calculation. This gives a compression zone of:

$$x_s = 1.025m$$

This gives a moment and curvature of

$$M_s = 104536\ kNm$$
$$\kappa_s = 2.629 \cdot 10^{-3}$$

The ultimate limit moment is based on the compression zone where the strain in the bottom fibre has reached its maximum value of 3.025‰.

$$M_u = 104689\ kNm$$
$$\kappa_u = 2.787 \cdot 10^{-3}$$

When these four points of the N-M-kappa diagram are taken into account figure IV.21 can be drawn. It can be seen that the design moment above the support has a value that occurs in the branch between the second and the third point. This means that the yield stress of the reinforcement is not reached, but most of the cracks are formed.

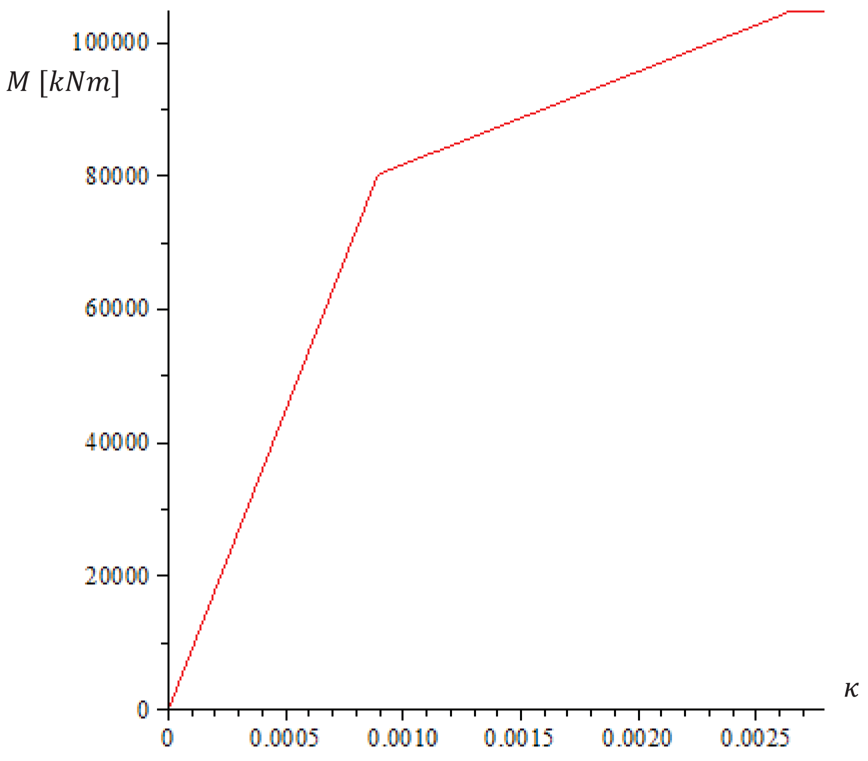


Figure IV.21 N-M-kappa diagram

FE-model

The hand calculation can be verified with a finite element analysis. The bridge is modeled with the FE-program Matrixframe. The stay cables are tensioned during the construction process. After all the bridge sections are in place, the cables are tensioned again. Therefore the dead weight of the bridge is not incorporated in the model. The internal forces and moments due to the dead load are added by using fixed supports at the location of the stays. In the FE-model the stay cables have a modulus of elasticity that is reduced. The stay cables experience a certain deflection due to the dead weight of the stay cable. This means that the deflection determines partly the equivalent modulus of elasticity. With the formula of Ernst the equivalent modulus of elasticity can be calculated for all the individual stay cables.

$$E_{eq} = \frac{E}{1 + \frac{\rho^2 L^2 E}{12 {\sigma_s}^3}}$$

Where ρ is the specific weigth of the stay cable, L is the length of the cable and σ_s is the axial stress in the cable. The length and the stress differs for the stay cables. Therefore it would be necessary to give each stay cable a different modulus of elasticity. Applying this in the model gives a redistribution of forces. As a result the stress in the cables, and the equivalent modulus of elasticity will change as well. Some iterations will be necessary to create the correct modulus of elasticity of the stay cables. In this analysis the modulus of elasticity is kept constant. Only one reference cable is taken into account in order to calculate a modified modulus of elasticity.

$$E_{eq} = \frac{205{,}000}{1 + \frac{(7.8 \cdot 10^{-5})^2 \cdot 87800^2 \cdot 205{,}000}{12 \cdot 250^3}} = 195{,}000 N/mm^2$$

The analysis is elaborated upon the dead load, the super dead load and the life load. For the life load four different load models are considered; the main span is fully loaded, the side spans are fully loaded, all spans are fully loaded and one side span and half of the main span fully loaded. The following properties are used in the FE-model.

Bridge deck

Concrete Strength Class			$C35/45$	
Area	A_c	=	20.775	m^2
Moment of Inertia	I_c	=	6.753	m^4
Young's modulus	E_c	=	34,000	N/mm^2

Stay cables

Young's modulus	E_c	=	195,000	N/mm^2

Pylons (assumed 2x2.0x2.5m^3)

Area	A_c	=	10.00	m^2
Moment of Inertia	I_c	=	5.208	m^4
Young's modulus	E_c	=	34,000	N/mm^2

Dead load

For the determination of the internal forces due to the dead load of the structure, the bridge deck is modeled as a beam over 46 supports. The reaction forces from this model are used to determine the normal force in the structure. The normal force in the bridge deck at the position of the pylon is $-76442\ kN$. The moment at this position is $-19968\ kNm$.

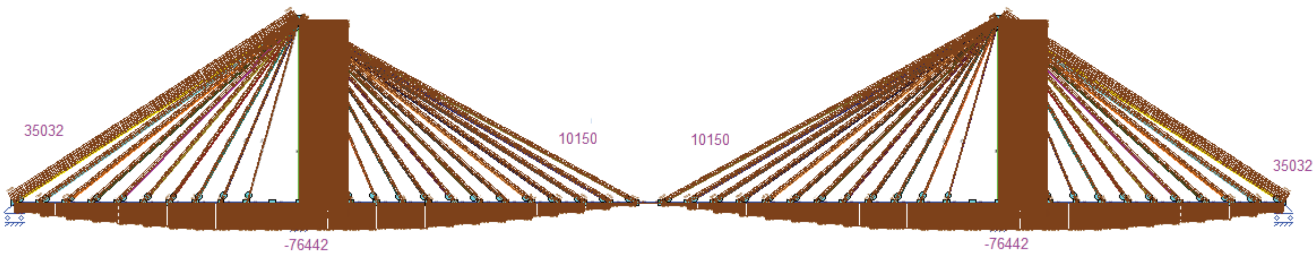


Figure IV.23 Normal force distribution due to dead load

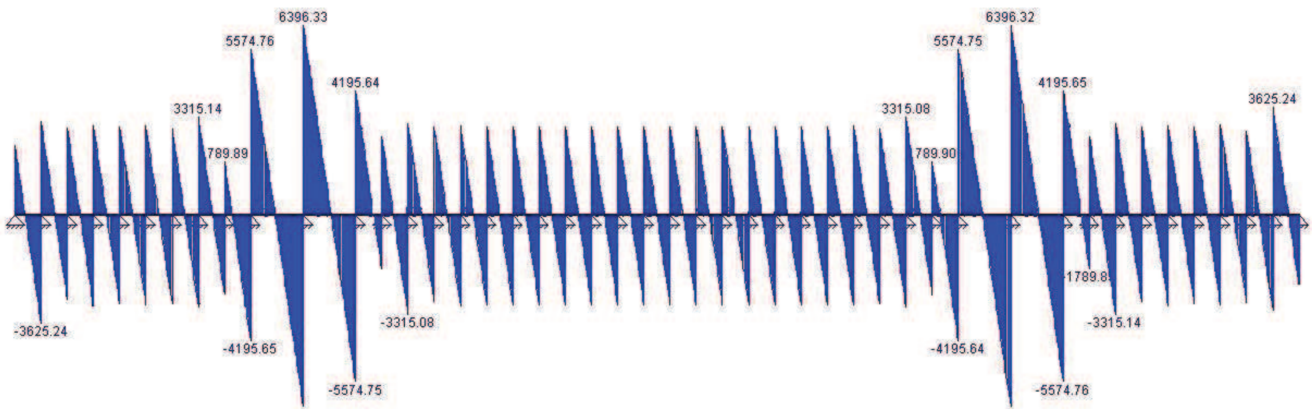


Figure IV.22 Shear force distribution of the bridge deck due to dead load

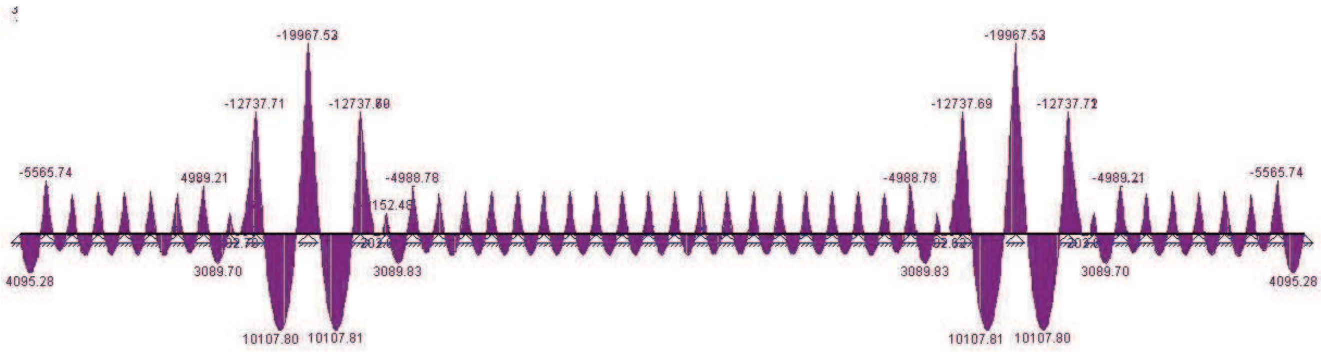


Figure IV.24 Moment distribution of the bridge deck due to dead load

Super dead load

For the determination of internal forces due to the super dead load and the life load a two-dimensional FE-model is used. The maximum normal force in the bridge deck is $-4541\ kN$. The maximum negative moment can be found in the bridge deck at the position of the pylon, ad has a value of $-9510\ kNm$.

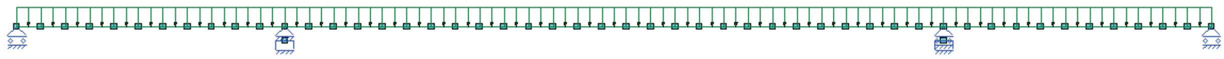


Figure IV.25 Loading super dead load

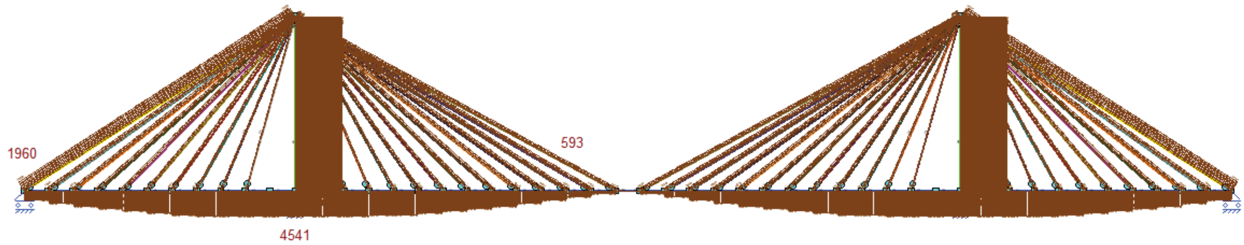


Figure IV.26 Normal force distribution due to the super dead load

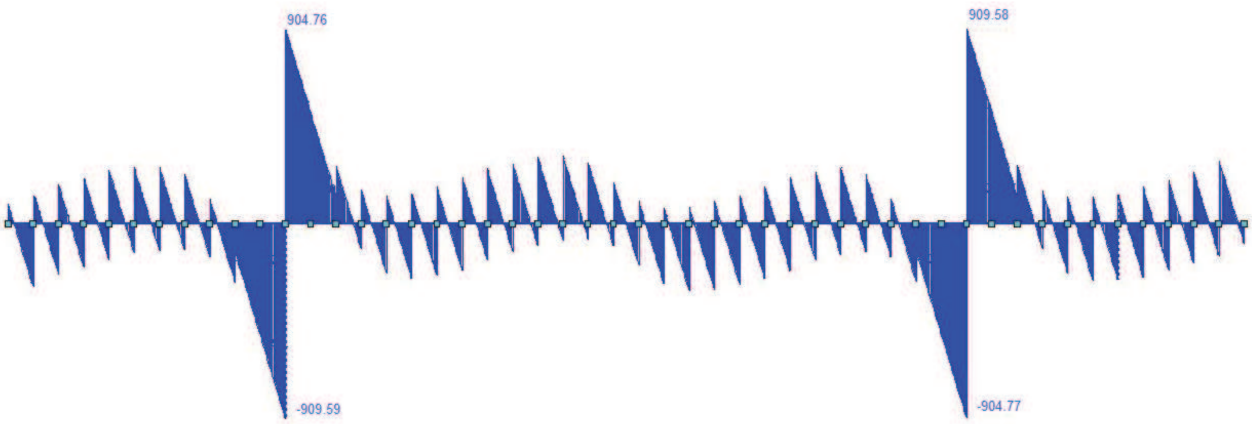


Figure IV.28 Shear force distribution of the bridge deck due to super dead load

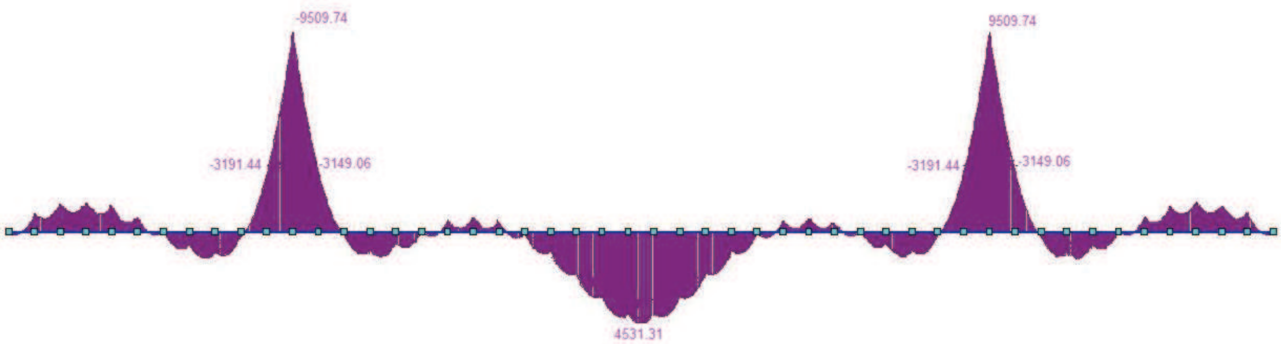
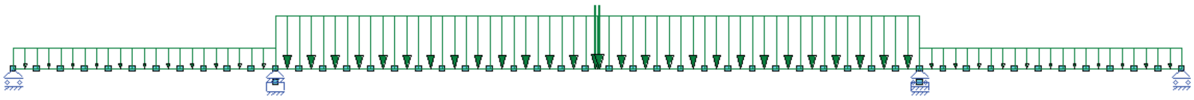


Figure IV.27 Moment distribution of the bridge deck due to super dead load

Life load: main span fully loaded

In this load case the main span of the bridge is fully loaded and the axle force is applied at midspan. The side spans of the bridge are partly loaded. This results in a maximum normal force of $-13826\ kN$ and a negative moment at the position of the pylon of $-19958\ kNm$.



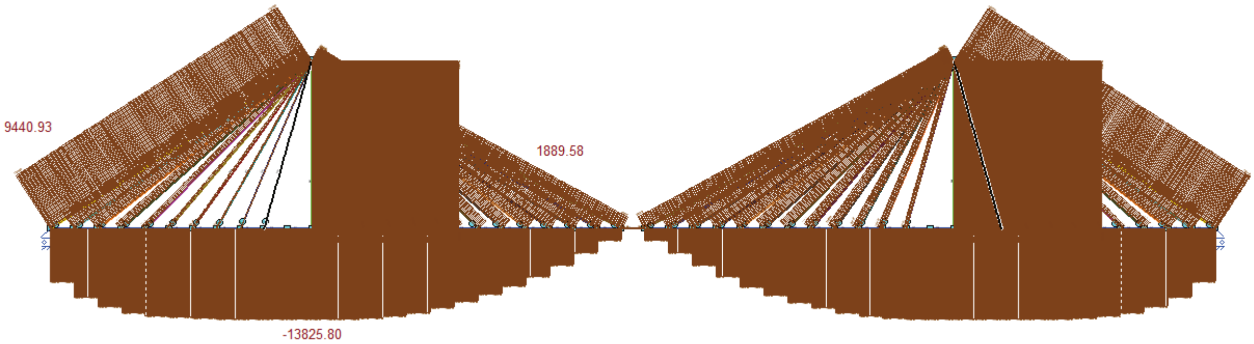


Figure IV.32 Normal force distribution due to the life load

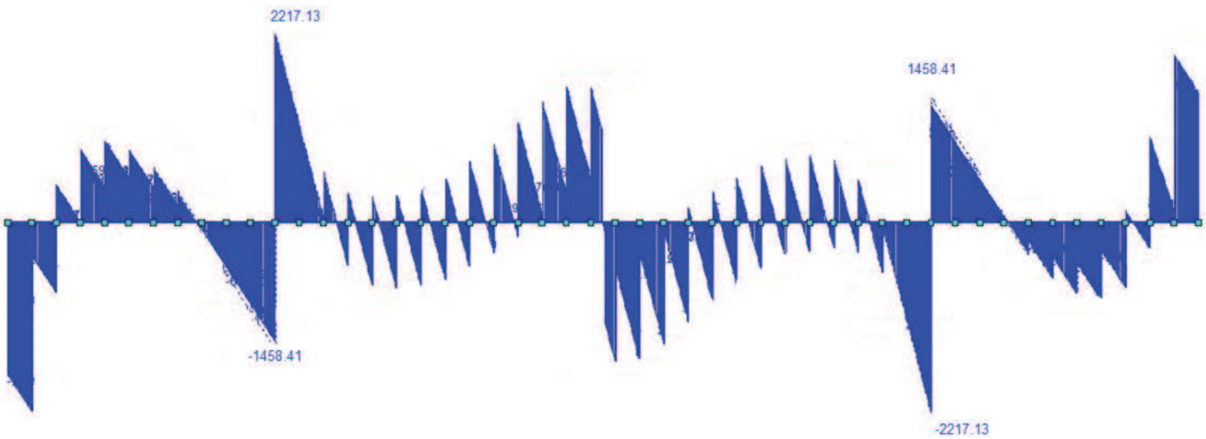


Figure IV.31 Shear force distribution due to the life load

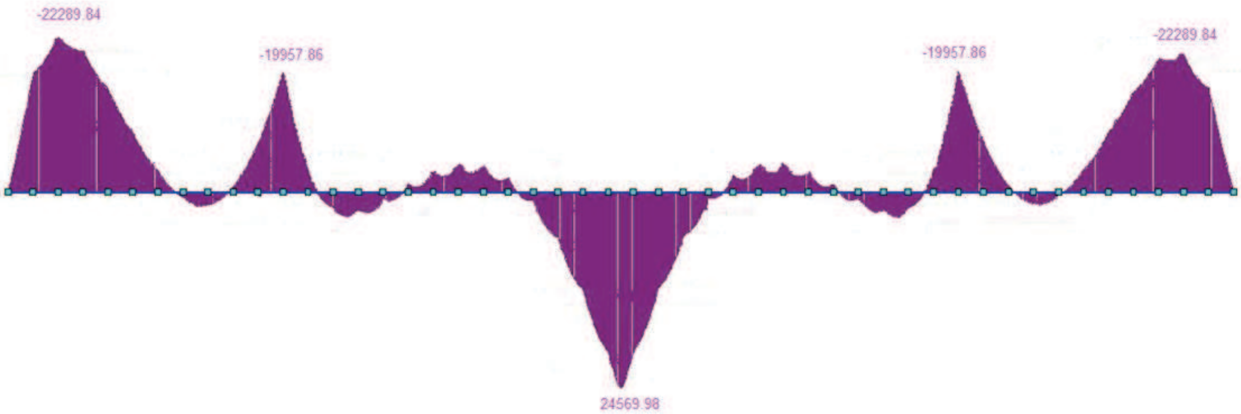


Figure IV.30 Moment distribution due to life load

Life load: side spans fully loaded

In this load case the side spans of the bridge are fully loaded and the main span has a reduced life load. The normal force has a value of $-5590kN$ and the moment has a value of $-19122\ kNm$ at the location of the pylon.

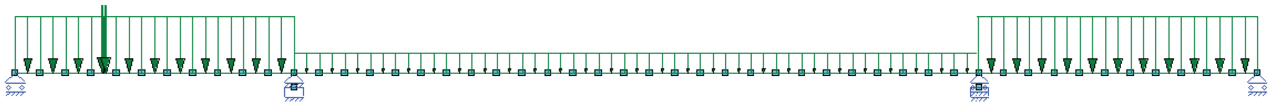


Figure IV.36 Loading life load side spans fully loaded

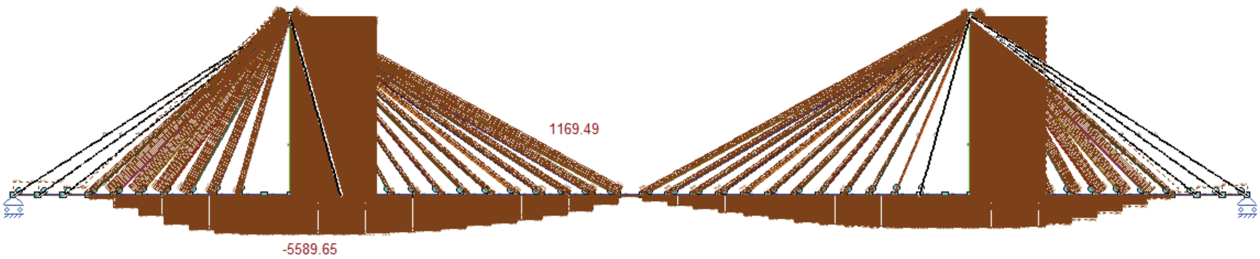


Figure IV.35 Normal force distribution due to the life load

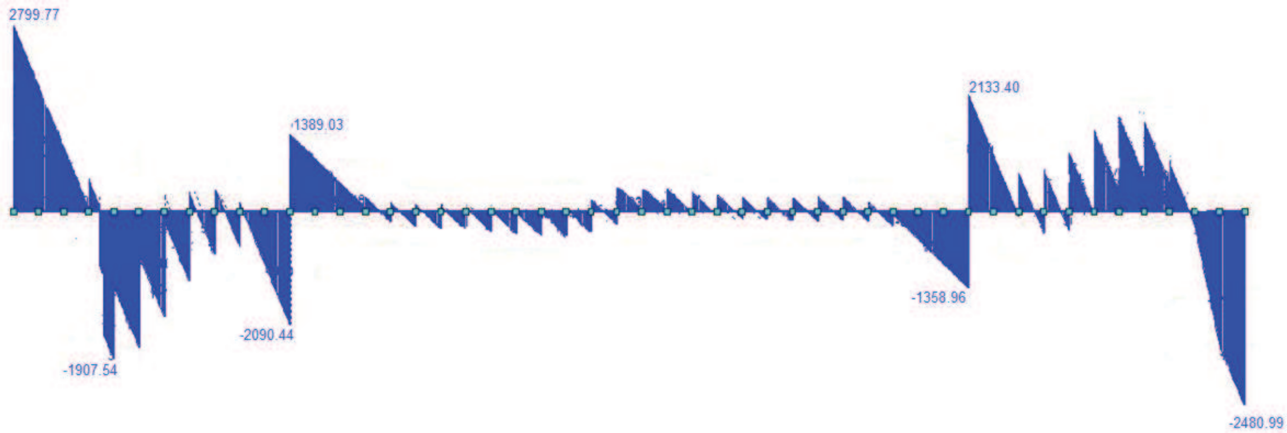


Figure IV.34 Shear force distribution due to the life load

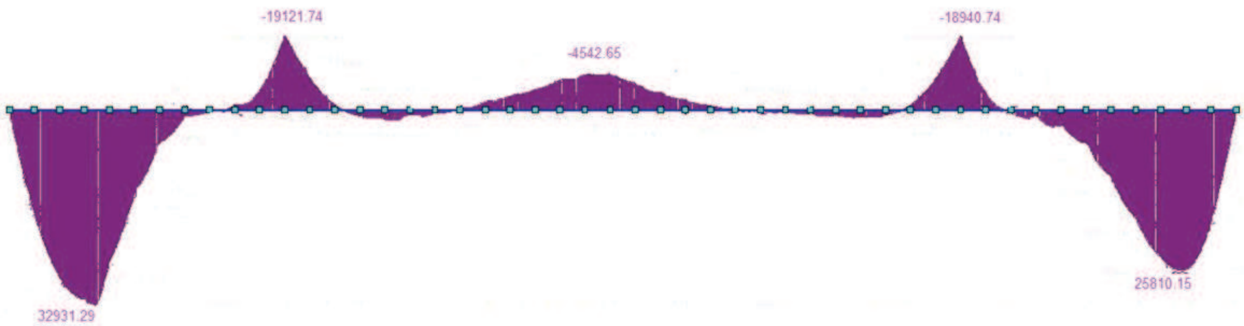


Figure IV.33 Moment distribution due to the life load

Life load: all spans fully loaded

In this load model the main span and the side spans are both fully loaded with the distributed life load and the axle load is applied at midspan. This results in a normal force of $-13779\ kN$ and a moment of $-26870\ kNm$ above the support.

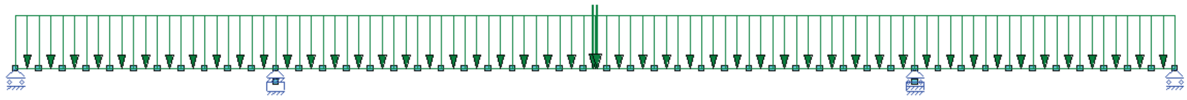


Figure IV.40 Loading life load all spans fully loaded

Figure IV.39 Normal force due to the life load

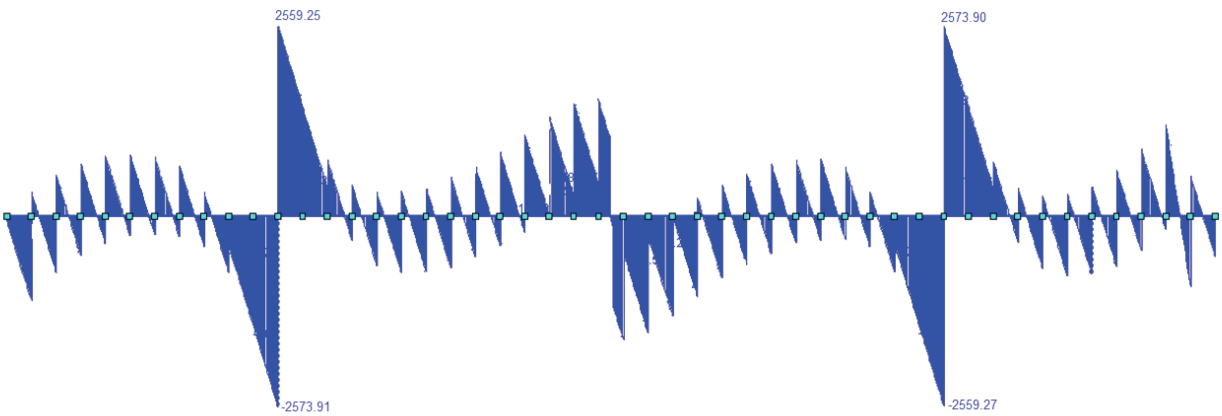


Figure IV.38 Shear force distribution due to the life load

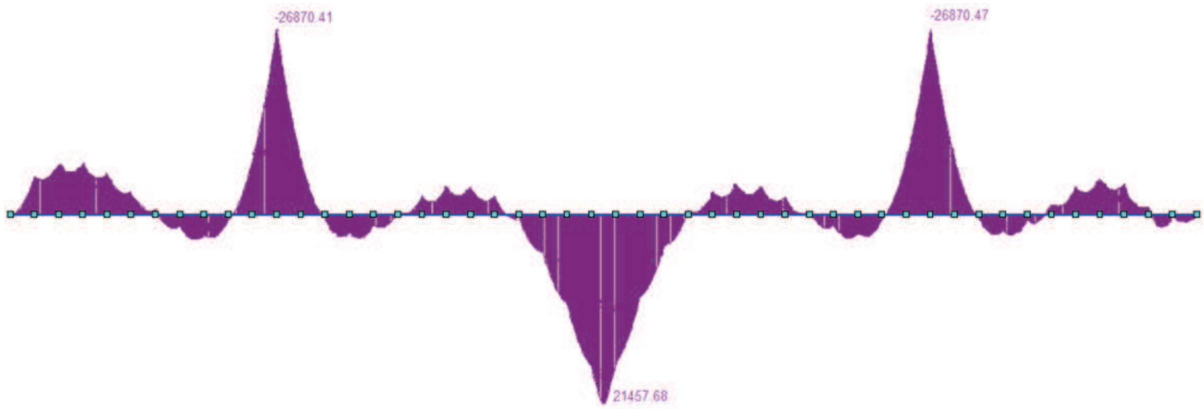


Figure IV.37 Moment distribution due to the life load

Life load: one side span + half main span fully loaded

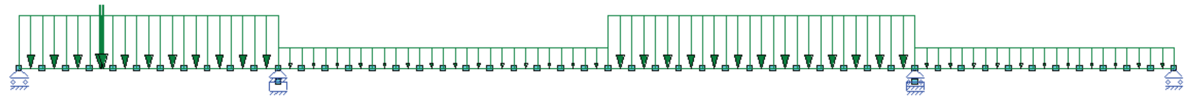


Figure IV.41 Loading life load one side span + half main span fully loaded

Figure IV.42 Normal force due to the life load

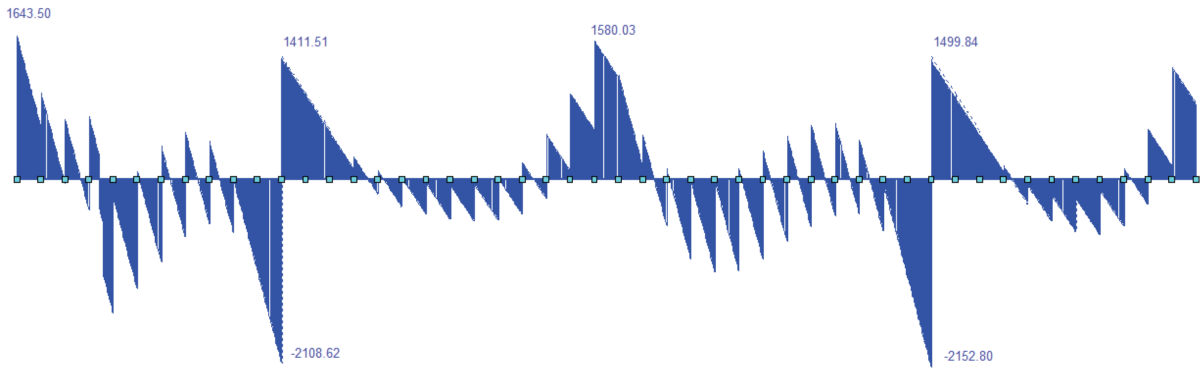


Figure IV.43 Shear force distribution due to the life load

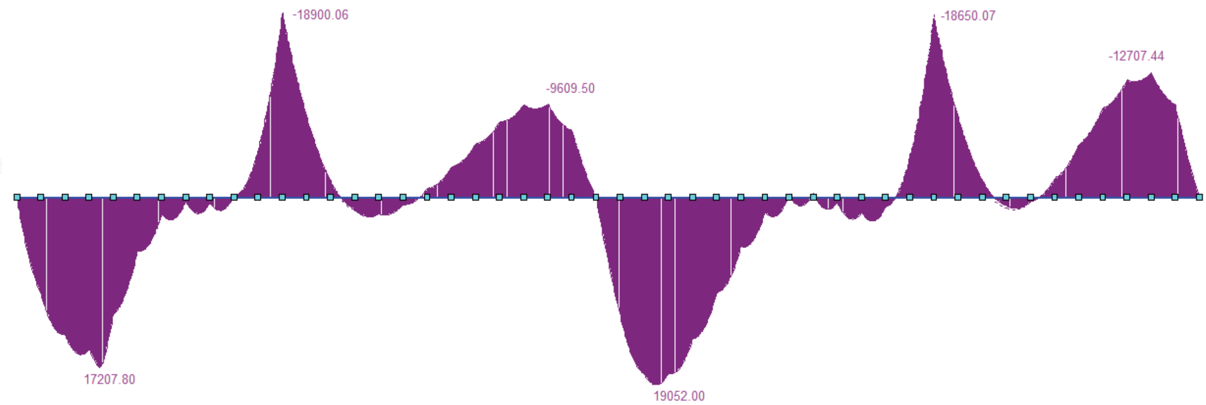


Figure IV.44 Moment distribution due to the life load

Conclusion

From the FE-analysis the maximum normal force, shear force and moment above the support at the location of the pylon can be determined.

Table IV.4 Normal force in the bridge deck at the location of the pylon for the different load cases

	Load factor	Dead load	Load factor	Super dead load	Load factor	Life load	Total
ULS 1	*1.35*	-76442 kN	*1.35*	-4541 kN	-	-	-109327 kN
ULS 2	*1.32*	-76442 kN	*1.32*	-4541 kN	*1.65*	-13826 kN	-129710 kN
ULS 3	*1.32*	-76442 kN	*1.32*	-4541 kN	*1.65*	-5590 kN	-116121 kN
ULS 4	*1.32*	-76442 kN	*1.32*	-4541 kN	*1.65*	-13779 kN	-129633 kN
ULS 5	*1.32*	-76442 kN	*1.32*	-4541 kN	*1.65*	-7610 kN	-119454 kN
SLS 1	*1.00*	-76442 kN	*1.00*	-4541 kN	*1.00*	-13826 kN	-94809 kN
SLS 2	*1.00*	-76442 kN	*1.00*	-4541 kN	*1.00*	-5590 kN	-86573 kN
SLS 3	*1.00*	-76442 kN	*1.00*	-4541 kN	*1.00*	-13779 kN	-94762 kN
SLS 4	*1.00*	-76442 kN	*1.00*	-4541 kN	*1.00*	-7610 kN	-88593 kN

Table IV.5 Shear force in the bridge deck at the location of the pylon for the different load cases

	Load factor	Dead load	Load factor	Super dead load	Load factor	Life load	Total
ULS 1	*1.35*	-6396 kN	*1.35*	-910 kN	-	-	-9863 kN
ULS 2	*1.32*	-6396 kN	*1.32*	-910 kN	*1.65*	-2217 kN	-13302 kN
ULS 3	*1.32*	-6396 kN	*1.32*	-910 kN	*1.65*	-2090 kN	-13092 kN
ULS 4	*1.32*	-6396 kN	*1.32*	-910 kN	*1.65*	-2574 kN	-13891 kN
ULS 5	*1.32*	-6396 kN	*1.32*	-910 kN	*1.65*	-2153 kN	-13196 kN
SLS 1	*1.00*	-6396 kN	*1.00*	-910 kN	*1.00*	-2217 kN	-9523 kN
SLS 2	*1.00*	-6396 kN	*1.00*	-910 kN	*1.00*	-2090 kN	-9396 kN
SLS 3	*1.00*	-6396 kN	*1.00*	-910 kN	*1.00*	-2574 kN	-9880 kN
SLS 4	*1.00*	-6396 kN	*1.00*	-910 kN	*1.00*	-2153 kN	-9459 kN

Table IV.6 Moments in the bridge deck at the location of the pylon for the different load cases

	Load factor	Dead load	Load factor	Super dead load	Load factor	Life load	Total
ULS 1	*1.35*	-19668 kNm	*1.35*	-9510 kNm	-	-	-39390 kNm
ULS 2	*1.32*	-19668 kNm	*1.32*	-9510 kNm	*1.65*	-19958 kNm	-71446 kNm
ULS 3	*1.32*	-19668 kNm	*1.32*	-9510 kNm	*1.65*	-19122 kNm	-70066 kNm
ULS 4	*1.32*	-19668 kNm	*1.32*	-9510 kNm	*1.65*	-26870 kNm	-82850 kNm
ULS 5	*1.32*	-19668 kNm	*1.32*	-9510 kNm	*1.65*	-18900 kNm	-69700 kNm
SLS 1	*1.00*	-19668 kNm	*1.00*	-9510 kNm	*1.00*	-19958 kNm	-49136 kNm
SLS 2	*1.00*	-19668 kNm	*1.00*	-9510 kNm	*1.00*	-19122 kNm	-48300 kNm
SLS 3	*1.00*	-19668 kNm	*1.00*	-9510 kNm	*1.00*	-26870 kNm	-56048 kNm
SLS 4	*1.00*	-19668 kNm	*1.00*	-9510 kNm	*1.00*	-18900 kNm	-48078 kNm

From table IV.4 it can be seen that the load model ULS 2 gives the highest normal force of $-129710\ kN$. The largest value for the moment above the support can be found with load model ULS 4 and this moment has a value of $-82850\ kNm$.

	Elastic beam on springs	**FE-model with stays**
Normal force	127360 kN	-129710 kN
Shear force	-	-13891 kN
Moment	89271 kN	-82850 kNm

The design value for the shear force should not be larger than the maximum shear force capacity of the cross section. The maximum design value of the shear force is equal to:

$$V_{Ed} = 13891\ kN$$

The maximum shear force capacity of the bridge deck can be calculated with:

$$V_{Rd,max} = \alpha_{cw} b_w z \nu_1 f_{cd} \cdot \cot(\theta)\,/(1 + \cot(\theta)^2)$$

Where:

$$\alpha_{cw} = 1.25$$
$$b_w = 2400mm$$
$$z = 2070mm$$
$$\nu_1 = 0.6$$
$$f_{cd} = 23.33N/mm^2$$

This gives that:

$$V_{Rd,max} = 86928 \cdot 10^3 \cdot \cot(\theta)\,/(1 + \cot(\theta)^2)$$

The angle θ of the compressive strut should have a value between:

$$0.380\ rad < \theta < 0.785\ rad$$

Filling in the minimum and maximum value for θ gives:

$$43464kN < V_{Rd,max} < 62084kN$$

Both values are higher than the occurring shear forces in the structure.

Alternative system

Another possible system that is applied in recent bridge decks is a combination of prefabricated box girders and in situ casted concrete main girders. The prefabricated box girders transfer the load to the main girders of the bridge. The main girders have a L-shaped cross section. The box girder have to be connected with the main girder to give the bridge deck sufficient robustness. To see whether or not this type of bridge deck can be applied for this particular bridge, some basic design calculations are made. The rule of thumb for the height of the box girder is:

$$h_{box} = \frac{1}{30} l_{span}$$

The span of the bridge is around 30 meter, this gives a required height of the box girder of 1.0 meter. There are many types of box girders of different companies. For this calculation the values of a type of box girder of *Spanbeton b.v.* are used. The dead weight of the girders is 18.9 kN/m. The c.t.c. distance of the girders is 1.5 meter. This means that the main girders have to carry a dead weight of $378kN/m$ of the box girders alone. The total dead weight of the bridge deck calculated in the first part of appendix I is 677.16 kN/m. The main girders can therefore have an additional weight of 299.16kN/m (677.16-378). This coincides with a cross section of the main girders of 11.97m. The available area that can be used for the normal force is therefore smaller compared with the value found in the first calculation.

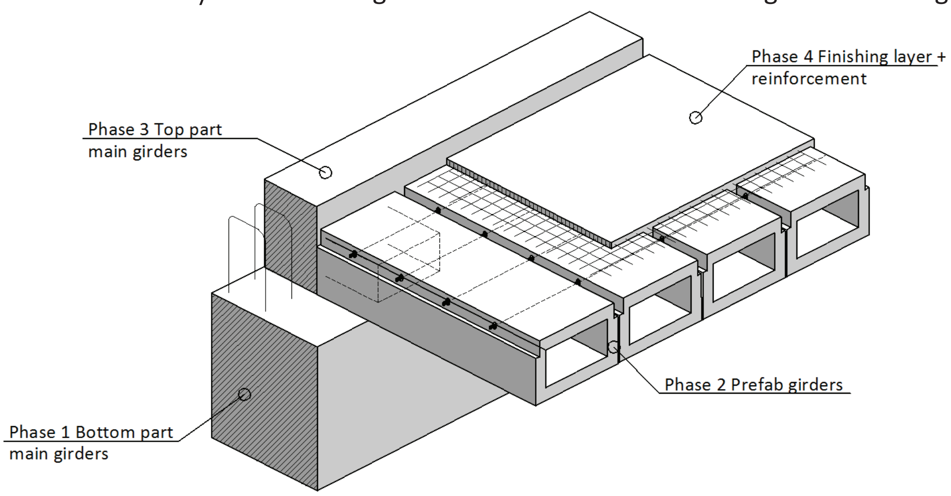


Figure IV.45 Alternative system with prefabricated box girders

Appendix V: Design Calculation Cable Stayed Bridge frUHSC

In this second design calculation the cable stayed bridge is designed with fibre reinforced ultra-high strength concrete (frUHSC). The concrete class C170/200 is used for this calculation. Research is still being done to the properties of the UHSC classes. The properties that are used for frUHSC are based on the information given at www.uhsb.nl.

The loads that where applied in APPENDIX IV, are also legitimate for this calculation.

Material Properties

Concrete

Strength Class			$C170/200$	
Cement			$CEM\ 52.5\ R$	
Characteristic value of compressive strength	f_{ck}	$=$	170.00	N/mm^2
Design value of compressive strength	f_{cd}	$=$	113.33	N/mm^2
Mean value of axial tension strength	f_{ctm}	$=$	6.22	N/mm^2
Specific weight	ρ	$=$	2900	kg/m^3
Environmental class			$XD3$	
Young's modulus (short term)	$E_{cm(0)}$	$=$	$52{,}185$	N/mm^2
Strain for reaching maximum compressive strength	ε_c	$=$	$2.3 \cdot 10^{-3}$	
Maximum strain	ε_{cu}	$=$	$2.6 \cdot 10^{-3}$	

Post tensioning steel

Class			$Y1860S7-15$	
Number of wires in one strand			7	
Diameter of one strand			15.2	mm
Cross section one strand			150	mm^2
Characteristic tensile strength	f_{pk}	$=$	1860	N/mm^2
Characteristic 0.1% proof stress	$f_{p0.1k}$	$=$	1674	N/mm^2
Design value tensile strength	f_{pd}	$=$	1456	N/mm^2
Young's modulus	E_p	$=$	$195{,}000$	N/mm^2

Determination of the Global Dimensions Bridge Deck

In comparison with the calculation in APPENDIX IV the cross section of the bridge deck can be reduced. Based on a first indication of the global dimensions the cross sectional properties are determined.

The thickness of the deck:

$$t_d = 0.18\ m$$

Height of the rib:

$$h_r = 0.32\ m$$

Base width of the rib:

$$w_{br} = 0.20\ m$$

Top width of the rib:

$$w_{tr} = 0.60\ m$$

c.t.c. distance of the ribs:

$$c.t.c = 2.00\ m$$

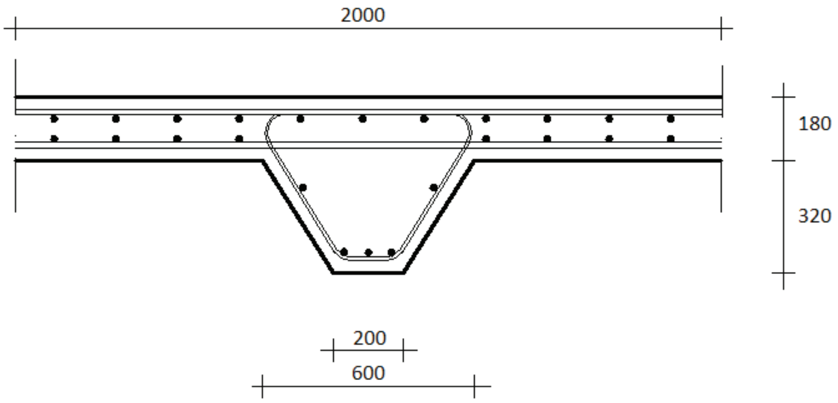


Figure V.1 Dimensions of the ribs in the deck floor

Cross sectional properties
The cross sectional properties can be calculated.
Cross sectional area:

$$A_c = 2.00 \cdot 0.18 + 0.20 \cdot 0.32 + 0.20 \cdot 0.32 = 0.488\, m^2$$

Distance of the bottom fibre to the normal center:

$$NC_{(z)} = \frac{2.00 \cdot 0.18 \cdot 0.41 + 0.20 \cdot 0.32 \cdot 0.16 + 0.20 \cdot 0.32 \cdot 2/3 \cdot 0.32}{A_c} = 0.3514\, m$$

Moment of inertia:

$$I_{zz} = \frac{1}{12} \cdot 2.00 \cdot 0.18^3 + 2.00 \cdot 0.18 \cdot (0.41 - 0.3514)^2$$
$$+ \frac{6 \cdot 0.20^2 + 6 \cdot 0.20 \cdot 0.20 + 0.20^2}{36 \cdot (2 \cdot 0.20 + 0.20)} \cdot 0.32^3 + 0.32 \cdot 0.20 \cdot (0.16 - 0.3514)^2$$
$$+ 0.20 \cdot 0.32 \cdot (2/3 \cdot 0.32 - 0.3514)^2 = 0.006562\, m^4$$

Section moduli:

$$W_{c,top} = \frac{I_{zz}}{0.50 - NC_{(z)}} = 0.04416\, m^3$$
$$W_{c,bottom} = \frac{I_{zz}}{NC_{(z)}} = 0.01867\, m^3$$

Determination of the Global Dimensions Cross Beams

The cross beams have a I-shaped cross section. The top flange is integrated into the bridge deck. In order to reduce the self-weight of the girders, the web has a relatively small width and increases near the flanges.

Cross sectional properties
The effective width of the top flange can be determined according to:

$$b_{eff,i} = 0.2 b_i + 0.1 l_0 \leq 0.2 l_0$$
$$b_{eff,i} = 0.2 \cdot 3.7 + 0.1 \cdot 28.3 = 3.57\, m \leq 5.66\, m$$

$$b_{eff} = b_{eff,i} + b_w \leq b$$
$$b_{eff} = 4.37\, m$$

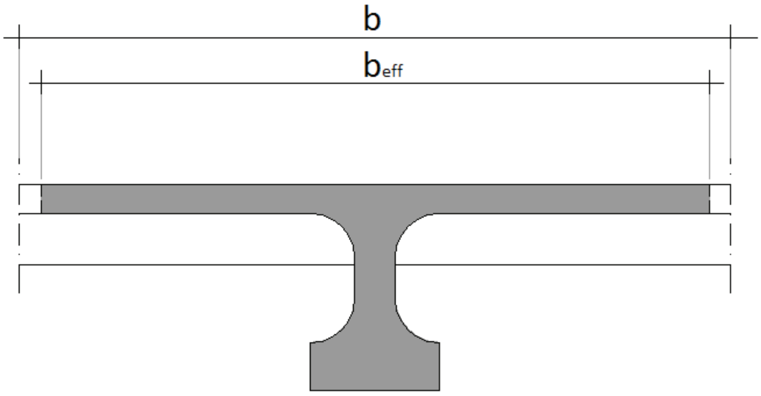


Figure V.2 Effective width of the cross beam

Effective sectional area

$$A_c = 4.37 \cdot 0.18 + 0.80 \cdot 0.25 + 0.30 \cdot 0.80 + (0.275 \cdot 2)^2 - 0.275^2\pi = 1.2915\, m^2$$

Position of the normal centre from the bottom fibre.

$$NC_{(z)} = \frac{4.37 \cdot 0.18 \cdot 1.19 + 0.80 \cdot 0.25 \cdot 0.70 + 0.30 \cdot 0.80 \cdot 0.15 + 0.03245 \cdot (0.3614 + 1.0386)}{A_c} = 0.896m$$

Moment of Inertia

$$I_{zz} = \frac{1}{12} \cdot 4.37 \cdot 0.18^3 + 4.37 \cdot 0.18 \cdot (1.19 - 0.896)^2 + \frac{1}{12} \cdot 0.25 \cdot 0.80^3 + 0.25 \cdot 0.80 \cdot (0.70 - 0.896)^2 + \frac{1}{12} \cdot 0.80 \cdot 0.30^3 + 0.80 \cdot 0.30 \cdot (0.15 - 0.896)^2 + \left(\frac{1}{3} - \frac{\pi}{16}\right) 0.275^4 \cdot 4 + 2 \cdot \left(1 - \frac{\pi}{4}\right) \cdot 0.275^2 \cdot (0.30 - 0.896)^2 + 2 \cdot \left(1 - \frac{\pi}{4}\right) \cdot 0.275^2 \cdot (1.10 - 0.896)^2 = 0.2398\, m^4$$

Section moduli

$$W_{c,top} = \frac{I_{zz}}{1.28 - NC_{(z)}} = 0.6246\, m^3$$

$$W_{c,bottom} = \frac{I_{zz}}{NC_{(z)}} = 0.2677\, m^3$$

Position of the post tensioning cable at midspan from the normal centre.

$$e_p = 0.716\, m$$

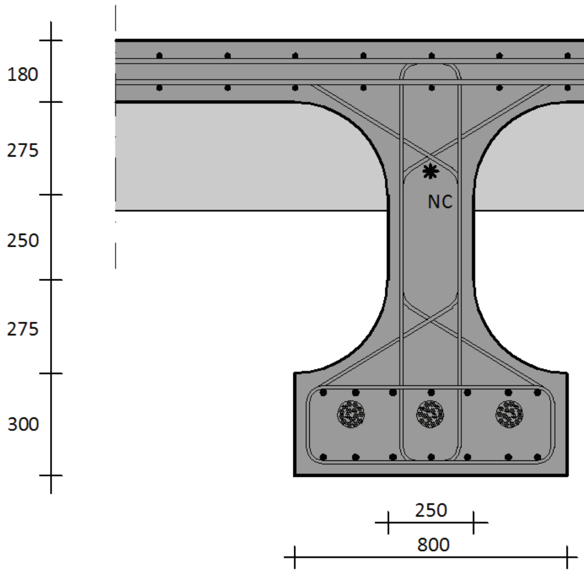


Figure V.3 Cross section of the cross beams

Moments

Now the dimensions of the cross beam are known, the occurring moments can be calculated. Only the moment due to the self-weight of the girder deviates from the moments found in APPENDIX IV.

$$M_G = 3951\ kNm$$
$$M_S = 661\ kNm$$
$$M_Q = 4329\ kNm$$

Required post tensioning steel

The required value of the post tensioning force can be determined based on the minimum and maximum allowable stresses in the cross section.

The calculation of the stresses is performed for $t = 0$ and $t = \infty$ at midspan.

$t = 0$

Bottom fibre compression

$$\sigma_{c,bottom} = -\frac{P_{m,0}}{A_c} - \frac{P_{m,0} e_p}{W_{c,bottom}} + \frac{M_G}{W_{c,bottom}} \geq -0.6 f_{cd}$$

$$\sigma_{c,bottom} = -\frac{P_{m,0}}{1.2915} - \frac{P_{m,0} \cdot 0.716}{0.2677} + \frac{3951}{0.2677} \geq -68000\ kN/m^2$$

$$P_{m,0} \leq 23996\ kN$$

$t = 0$

Top fibre tension

$$\sigma_{c,top} = -\frac{P_{m,0}}{A_c} + \frac{P_{m,0} e_p}{W_{c,top}} - \frac{M_G}{W_{c,top}} \leq 0.5 f_{ctm}$$

$$\sigma_{c,top} = -\frac{P_{m,0}}{1.2915} + \frac{P_{m,0} \cdot 0.716}{0.6246} - \frac{3951}{0.6246} \leq 3110\ kN/m^2$$

$$P_{m,0} \leq 25362\ kN$$

$t = \infty$
Bottom fibre tension

$$\sigma_{c,bottom} = -\frac{P_{m,\infty}}{A_c} - \frac{P_{m,\infty} e_p}{W_{c,bottom}} + \frac{M_G + M_S + M_Q}{W_{c,bottom}} \leq 0.5 f_{ctm}$$

$$\sigma_{c,bottom} = -\frac{P_{m,\infty}}{1.2915} - \frac{P_{m,\infty} \cdot 0.716}{0.2677} + \frac{8941}{0.2677} \leq 3110$$

$$P_{m,\infty} \geq 8724\ kN$$

If the prestressing losses are estimated on 20% the ratio between the prestressing force at $t = 0$ and $t = \infty$ is known.

$$P_{m,0} \geq \frac{8724}{0.80} = 10905\ kN$$

$$A_p = \frac{10905 \cdot 10^3}{1395} = 7817\ mm^2$$

$$\#strands = \frac{7817}{150} = 52\ strands$$

The maximum number of strands is (22x3) 66.
When in each of the cables 18 strands are applied the prestressing force is equal to:

$$P_{m,0} = 18 \cdot 3 \cdot 150 \cdot 1395 = 11300 \cdot 10^3\ N$$

Elastic losses
The strain of the concrete due to the post tensioning force reduces the stress in the post tensioning cable. The strain of the concrete at the location of the post tensioning cable can be determined.

$$\Delta\varepsilon_c = \frac{\Delta\sigma_c}{E_{cm}} = \frac{1}{E_{cm}}\left(\frac{P_{m,0}}{A_c} + \frac{M_p e_p}{I_{zz}}\right)$$

The strain of the cable is equal to

$$\Delta\varepsilon_p = \frac{\Delta\sigma_p}{E_p} = \frac{\Delta P_{el}}{E_p A_p}$$

Compatibility requires:

$$\Delta\varepsilon_c = \Delta\varepsilon_p$$

$$\Delta P_{el} = \frac{E_p A_p}{E_{cm} A_c} P_{m,0}\left(1 + \frac{{e_p}^2 A_c}{I_{zz}}\right)$$

The number of post tensioning cables can be included in the formula. The first applied cable will experience the highest elastic losses.

$$\Delta P_{el} = \frac{n-1}{2n}\frac{E_p A_p}{E_{cm} A_c} P_{m,0}\left(1 + \frac{{e_p}^2 A_c}{I_{zz}}\right)$$

$$\Delta P_{el} = \frac{3-1}{2 \cdot 3}\frac{195{,}000 \cdot 8100}{52{,}185 \cdot 1.2915 \cdot 10^6} \cdot 11300 \cdot 10^3 \cdot \left(1 + \frac{716^2 \cdot 1.2915 \cdot 10^6}{0.2398 \cdot 10^{12}}\right)$$

$$= 332 \cdot 10^3 N$$

This gives a mean loss of stress in the cables of:

$$\Delta\sigma_{el} = \frac{\Delta P_{el}}{A_p} = \frac{332 \cdot 10^3}{8100} = 41.0\ N/mm^2$$

The first cable will experience a loss of $82.0\ N/mm^2$ the second cable has a loss of $41.0\ N/mm^2$ and the third cable has no loss of stress.

Losses due to friction

The friction of the strands with the duct of the cable gives rise to certain stress losses. For these straight cables only the Wobble effect is taken into account.

$$\Delta P_\mu(x) = P_{max}(1 - e^{-\mu(\theta+kx)})$$

Where

$$\mu = 0.16$$
$$k = 0.010\ rad/m$$

Straight cables

$$\theta_1 = 0$$
$$\Delta P_\mu(x) = 3767 \cdot 10^3(1 - e^{-0.0016x})$$
$$\Delta P_\mu(28.3) = 3767 \cdot 10^3(1 - e^{-0.0016 \cdot 28.3}) = 166.8 \cdot 10^3 N$$

Assume wedge settlement of $6\ mm$.

The average friction loss as a change in stress per unit length is:

$$\Delta\sigma_{pm} = \frac{166.8 \cdot 10^3}{28{,}300 \cdot 2700} = 0.002182\ N/mm^2/mm$$

The length over which the settlement occurs is estimated with:

$$l_{set} = \sqrt{\frac{\Delta w_{set} E_p}{\Delta\sigma_{pm}}} = \sqrt{\frac{6 \cdot 195{,}000}{0.002182}} = 23156\ mm$$

The loss due to wedge settlement is :

$$\Delta\sigma_{set} = 2\Delta\sigma_{pm} l_{set}$$
$$\Delta\sigma_{set} = 2 \cdot 0.002182 \cdot 23156 = 101\ N/mm^2$$

It is possible to overstress the prestressing steel. The maximum allowable stress during prestressing is:

$$\sigma_{p,max} = 0.8 \cdot 1860 = 1488\ N/mm^2$$

The average stress in the cables at $t = 0$ is $1361\ N/mm^2$. The average prestressing force in the cables is equal to $11020\ kN$.

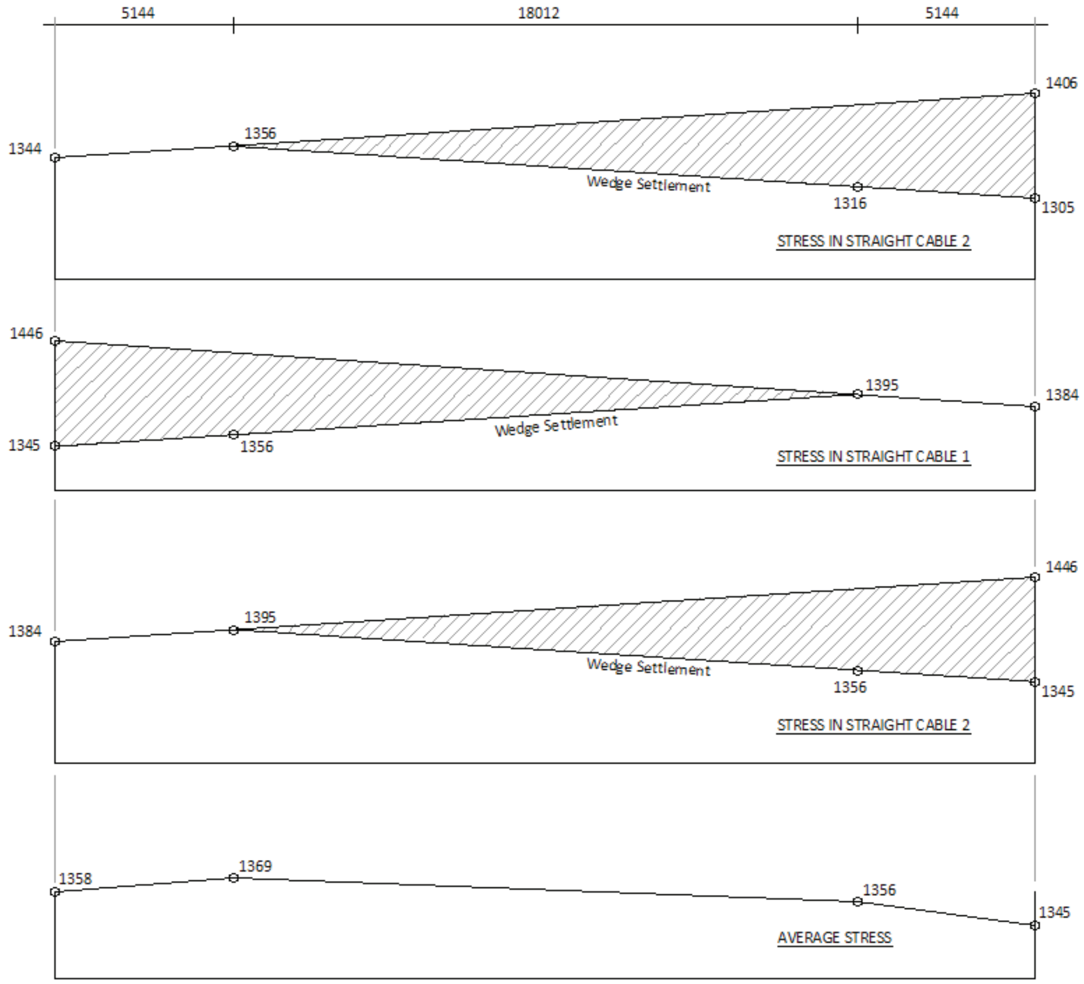


Figure V.1 Stresses in cables with elastic losses, friction losses and wedge settlement included

Losses due to creep

Trost method is used to determine the creep losses.

$$\varepsilon_{c,cr}(t) = \frac{\sigma_c(0)}{E_{cm}{}'} + \frac{\Delta\sigma_c(t)}{E_{cm}{}''}$$

The initial stress in the cross section at the location of the post tensioning cable is based on the initial post tensioning force and the moment due to the dead load.

$$\sigma_c(0) = -\frac{P_{m,0}}{A_c} - \frac{P_{m,0}{e_p}^2}{I_c} + \frac{M_G e_p}{I_c}$$

$$\sigma_c(0) = -\frac{11020}{1.2915} - \frac{11020 \cdot 0.716^2}{0.2398} + \frac{3951 \cdot 0.716}{0.2398} = -20295\ kN/m^2$$

The additional stress can be calculated by taking the moments due to the super dead load and the life load into account.

$$\Delta\sigma_c(t) = +\frac{M_S e_p}{I_c} + \frac{\psi_2 M_Q e_p}{I_c}$$

$$\Delta\sigma_c(t) = +\frac{661 \cdot 0.716}{0.2398} + \frac{0.4 \cdot 4329 \cdot 0.716}{0.2398} = +7144\ kN/m^2$$

The adapted Young's modulus is based on the creep coefficient. There is no definitive value available for the creep coefficient for this UHSC class. However from the information at www.uhsb.nl it seems that a creep coefficient of 0.8 is representable for this concrete class.

$$E_{cm}{}' = \frac{E_{cm}}{1+\varphi} = \frac{52{,}185}{1+0.8} = 28992\ N/mm^2$$

$$E_{cm}{}'' = \frac{E_{cm}}{1+\chi\varphi} = \frac{52{,}185}{1+0.8\cdot 0.8} = 31820\ N/mm^2$$

The strain due to the creep of the concrete can be calculated with these values.

$$\varepsilon_{c,cr}(t) = \frac{-20.3}{28992} + \frac{7.14}{31820} = -0.476\cdot 10^{-3}$$

Losses due to shrinkage

According to the Eurocode 1992 art. 3.1.4, the shrinkage strain can be calculated as a combination of drying shrinkage and autogenous shrinkage.

$$\varepsilon_{c,sh} = \varepsilon_{cd,\infty} + \varepsilon_{ca,\infty}$$

$$\varepsilon_{ca,\infty} = 2.5(f_{ck} - 10)\cdot 10^{-6} = 0.258\cdot 10^{-3}$$

Because there is no data of drying shrinkage available for frUHSC, concrete class C90/105 combined with a relative humidity of 80% is used to determine the drying strain. k_h is based on table 3.3 with $h_0 = 223mm$.

$$\varepsilon_{cd,\infty} = k_h\varepsilon_{cd,0} = 0.827\cdot 0.11\cdot 10^{-3} = 0.091\cdot 10^{-3}$$

The total strain in the concrete that causes losses in the prestressing steel is the sum of the deformation due to creep and shrinkage.

$$\varepsilon_c(t) = \varepsilon_{c,cr}(t) + \varepsilon_{c,sh}(t) = -0.476\cdot 10^{-3} - 0.091\cdot 10^{-3} + 0.258\cdot 10^{-3} = 0.825\cdot 10^{-3}$$

This results in a decrease of stress in the prestressing steel of:

$$\Delta\sigma_{pc} = \varepsilon_c(t)E_p = 0.825\cdot 10^{-3}\cdot 195{,}000 = 160.8\ N/mm^2$$

Losses due to relaxation

$$\Delta\sigma_{pr} = \sigma_{pi}\cdot 0.66\cdot \rho_{1000}e^{9.1\mu}\left(\frac{t}{1000}\right)^{0.75(1-\mu)}\cdot 10^{-5}$$

σ_{pi} is the initial peak stress value in the cables. After wedge settlement the peak stress in the cable is $1395\ N/mm^2$.

ρ_{1000} is the relaxation loss in percentage after a 1000 hours at a mean temperature of 20°C. Low relaxation steel is applied which results in $\rho_{1000} = 2.5\%$.

t is the time after tensioning in hours. For the total service life of the structure it can be said that $t = 500000\ hours$.

μ is the ratio between the initial peak stress and the yielding stress of the steel.

$$\mu = \frac{1395}{1860} = 0.75$$

$$\Delta\sigma_{pr} = 1395\cdot 0.66\cdot 2.5\cdot e^{9.1\cdot 0.75}\left(\frac{500000}{1000}\right)^{0.75(1-0.75)}\cdot 10^{-5} = 67.9\ N/mm^2$$

The relaxation losses can be reduced with 20% due to the influences of creep and shrinkage.

$$\Delta\sigma_{pr} = 0.8\cdot 67.9 = 54.4\ N/mm^2$$

Actual stresses
The actual stresses at midspan can now be calculated.

$\sigma_{pi} = 1361\ N/mm^2$
$\Delta\sigma_{pc} = -160.8\ N/mm^2$
$\Delta\sigma_{pr} = -54.4\ N/mm^2$
$\sigma_{p\infty} = 1146\ N/mm^2$

This results in a prestressing force at $t = \infty$ of

$P_{m,\infty} = 1146 \cdot 18 \cdot 3 \cdot 150 \cdot 10^{-3} = 9283\ kN > 8724\ kN$

Required amount of reinforcing steel
For the maximum occurring moment in the ULS it is allowable that the concrete cracks. Internal equilibrium should be reached by applying reinforcement steel.

$P_{m,\infty} = 9283\ kN$	Working prestressing force
$h = 1280\ mm$	Height of the girder
$d_p = h - 180 = 1100\ mm$	Distance from the top fibre to the prestressing cable
$d_{s1} = h - 65 = 1215\ mm$	Distance from the top fibre to the outer reinforcement
$d_{s2} = h - 235 = 1045\ mm$	Distance from the top fibre to the second layer of reinforcement
$E_s = 200{,}000\ N/mm^2$	Young's modulus of the reinforcement steel
$E_p = 195{,}000\ N/mm^2$	Young's modulus of the prestressing steel
$A_p = 8100\ mm^2$	Area of the prestressing steel
$f_{cd} = 113.3\ N/mm^2$	Design value compressive strength concrete
$\xi = 0.5$	Factor for the prestressing steel
x	Height compressive zone
ε_c	Maximum occurring concrete strain
$M_d = 1.32(3951 + 660.7) + 1.65 \cdot 4329 - 9283 \cdot 0.716 = 6584\ kNm$	Design value maximum moment
$A_{s1} = 2199\ mm^2$	Area outer reinforcement 7Ø20
$A_{s2} = 1571\ mm^2$	Area second layer of reinforcement 5Ø20

Assume that $x > 180mm$
There should be equilibrium of forces and moments

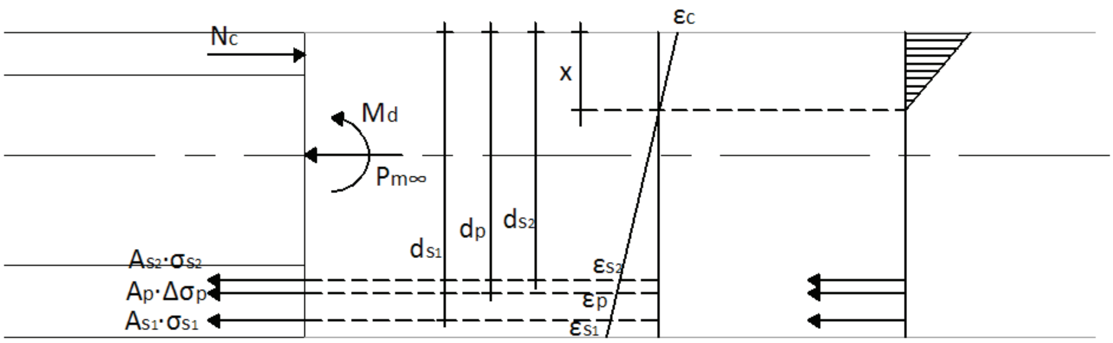


Figure V.5 Internal forces in the midspan section

Force equilibrium:

$$N_c = \frac{1}{2} \cdot x b_f \left(\frac{\varepsilon_c}{1.75 \cdot 10^{-3}} f_{cd}\right) - \frac{1}{2} \cdot (b_f - b_w)(x - 200)\left(\frac{(x-200)\varepsilon_c}{x \cdot 1.75 \cdot 10^{-3}} f_{cd}\right)$$

$$\Sigma F_x = 0$$

$$\Sigma F_x = -N_c + P_{m,\infty} + A_{s1}\frac{(d_{s1} - x)\varepsilon_c}{x} E_s + A_{s2}\frac{(d_{s2} - x)\varepsilon_c}{x} E_s + \xi A_p \frac{(d_p - x)\varepsilon_c}{x} E_p = 0$$

Moment equilibrium:

$$\Sigma M = 0$$

$$\Sigma M = M_d - \frac{1}{2} \cdot x b_f \left(\frac{\varepsilon_c}{1.75 \cdot 10^{-3}} f_{cd}\right)\left(443 - \frac{1}{3}x\right) + \frac{1}{2} \cdot (b_f - b_w)(x - 200)\left(\frac{(x-200)\varepsilon_c}{x \cdot 1.75 \cdot 10^{-3}} f_{cd}\right)\left(243 - \frac{1}{3}(x - 200)\right) - A_{s1}\frac{(d_{s1} - 443)(d_{s1} - x)\varepsilon_c}{x} E_s - A_{s2}\frac{(d_{s2} - 443)(d_{s2} - x)\varepsilon_c}{x} E_s - \xi A_p \frac{(d_p - 443)(d_p - x)\varepsilon_c}{x} E_p = 0$$

This results in:

$$x = 232\ mm$$
$$\varepsilon_c = 0.407 \cdot 10^{-3}$$

For this calculation some assumptions where made. These assumption should be checked afterwards in order to judge the correctness of the calculated values.

$$x > 180mm$$
$$x = 232mm$$

$$\varepsilon_c < 2.3 \cdot 10^{-3}$$
$$\varepsilon_c = 0.407 \cdot 10^{-3}$$

$$\Delta\sigma_p < 1395 - 1146 = 249\ N/mm^2$$
$$\Delta\sigma_p = \xi \frac{(d_p - x)\varepsilon_c}{x} E_p = 154\ N/mm^2$$

$$\sigma_{s1} = \frac{(d_{s1} - x)\varepsilon_c}{x} E_s = 345\ N/mm^2 < 435\ N/mm^2$$
$$\sigma_{s2} = \frac{(d_{s2} - x)\varepsilon_c}{x} E_s = 285\ N/mm^2 < 435\ N/mm^2$$

Determination of the Global Dimensions Stay Cables and Main girders

For the determination of the dimensions of the stays and the main girders the same approach as in APPENDIX IV is used. The weight of each bridge section can be determined based on the dimensions of the deck and cross beams. For the weight of the main girders a value of 67.7kN/m[1] is taken into account.

The weight per section of the bridge is divided into the dead weight, the super dead weight, the life load and the axle load. In the building phase it is necessary to take the weight of the false- and formwork into account. The load of the falsework is assumed to be 4.00 kN/m^2. For the formwork and additional temporary loading due to people and material a load of 2.00 kN/m^2 is taken into account.

$$W_G = 29 \cdot (0.505 \cdot 28.3 \cdot 2 + 0.244 \cdot 8.8 \cdot 28.3 + 2 \cdot 2.709 \cdot 8.8) = 3974kN$$
$$W_S = 1.50 \cdot 28.3 \cdot 8.8 = 374kN$$
$$W_Q = 8.8 \cdot (10.35 \cdot 3 + 25.3 \cdot 3.5) = 1052kN$$
$$W_D = 600 + 400 + 200 = 1200kN$$
$$W_B = (4.00 + 2.00) \cdot 28.3 \cdot 8.8 = 1494kN$$

For the building phase each stay should be able to carry the weight of two bridge sections. The load factors are for this phase equal to 1.0.

$$N_{s,i} = \frac{2 \cdot (W_G + W_S + W_B)}{2 \cdot \sin(\alpha_i)}$$

For the final phase each stay will carry a bridge section and should be able to transfer the axle loading.

$$N_{s,i} = \frac{\gamma_G \kappa_{Fi}(W_G + W_S) + \gamma_Q \kappa_{Fi}(W_Q + W_D)}{2 \cdot \sin(\alpha_i)}$$

The horizontal force is determined based on the final phase with the axle load positioned at the first stay.

Table V.1 Overview of the forces in the stay cables

Stay	Angle of the Stay Cable	Axial Force Stay Cable Building Phase $[kN]$	Axial Force Stay Cable Final Phase $[kN]$	Horizontal Force Final Phase $[kN]$	Minimum Required Area $[mm^2]$	Applied Stay Area $[mm^2]$
1	28.64°	12185	9861	8654	2x12373	2x14250
2	30.62°	11470	9283	6316	2x11647	2x13500
3	32.86°	10770	8716	5789	2x10937	2x12750
4	35.40°	10089	8165	5263	2x10245	2x12000
5	38.27°	9430	7632	4737	2x9576	2x11250
6	41.62°	8799	7122	4210	2x8935	2x10500
7	45.39°	8203	6638	3684	2x8329	2x9750
8	49.82°	7648	6189	3158	2x7766	2x9000
9	54.89°	7144	5782	2632	2x7255	2x9000
10	60.66°	6705	5426	2105	2x6808	2x9000
11	67.19°	6342	5132	1579	2x6440	2x9000
12	74.37°	6069	4912	1053	2x6163	2x9000

When the bending stiffness of the pylon is neglected, the back stay cables should be able to carry the resulting horizontal forces. The stays connected to the end support should be able to transfer the horizontal forces from the first, the second and the third cable. The force in these cables is equal to 20759kN. This gives a required area of 2x26046mm^2.

To insure that fatigue is not governing, the maximum stress in the stay should be limited to 45% of the ultimate stress.

$$\sigma_{st} = 0.45 f_{pu} = 797\ N/mm^2$$

This would mean that the minimum cable area of 12373mm^2 is needed.
For the moment distribution due to the dead load, the stay cables will be modeled as constrained supports. For the life load the stays will be modeled as vertical springs. The spring stiffness is based on the vertical displacement of the stay under a unit load.

The vertical displacement of the stay cable can be expressed as:

$$v_{ver,i} = \sqrt{{l_i}^2 \left(1 + \frac{N_{unit}}{EA_i}\right)^2 - {b_i}^2} - h$$

Where b_i is the horizontal distance from the pylon to the anchorage point of the considered stay cable. And h is the height of the pylon.

The vertical spring stiffness of the stays can be approximated with:

$$k_i = \frac{N_{unit}}{v_{ver,i}}$$

Table V.2 Vertical spring stiffness for the different stay cables

Stay	Applied Stay Area $[mm^2]$	Modulus of Elasticity $[N/mm^2]$	Horizontal Distance from the Pylon to the Anchorage Point $[mm]$	Vertical Displacement under Unit Load $[mm]$	Vertical Spring Stiffness $[N/mm]$
1	2x16500	205000	114400	485	20619
2	2x15750	205000	105600	354	28268
3	2x14250	205000	96800	319	31316
4	2x13500	205000	88000	274	36559
5	2x12750	205000	79200	326	30636
6	2x12000	205000	70400	227	44045
7	2x11250	205000	61600	331	30206
8	2x10500	205000	52800	226	44293
9	2x9750	205000	44000	191	52273
10	2x9000	205000	35200	188	53268
11	2x9000	205000	26400	148	67361
12	2x9000	205000	17600	151	66357

Dead load

For the dead load the anchorage points of the stay cables are considered as fixed supports. The internal forces can be determined based on this assumption. The representative line load for the dead weight and the super dead weight is equal to:

$$q_{G,rep} = 494.1\ kN/m$$

The moment distribution can be approached based on a continuous concrete beam over multiple fixed supports. The field moment and the moment above the supports is estimated with:

$$M_{G,rep} = \pm \frac{1}{10} q_{G,rep} l^2$$
$$M_{G,rep} = \pm \frac{1}{10} \cdot 494.1 \cdot 17.6^2 = \pm 15305 kNm$$

The normal force in the bridge deck is dependent of the horizontal component from the stay cables. The maximum normal force can be found at the location of the pylon and has a value of 54971kN.

Table V.3 Horizontal forces due to the dead load and the life load

Stay	Angle of the Stay Cable	Horizontal Force Final Phase Dead Load $[kN]$	Horizontal Force Final Phase Life Load (distr.) $[kN]$	Horizontal Force Final Phase Life Load (axial) $[kN]$	Horizontal Force Final Phase Life Load (axial) $[kN]$
1	28.64°	7940	1614	-	494
2	30.62°	7329	1894	-	400
3	32.86°	6719	1823	-	178
4	35.40°	6108	1569	-	41
5	38.27°	5497	1491	-	-
6	41.62°	4886	1036	-	-
7	45.39°	4276	1231	-	-
8	49.82°	3665	689	-	-
9	54.89°	3054	769	-	-
10	60.66°	2443	588	20	-
11	67.19°	1832	476	62	-
12	74.37°	1222	233	75	-
SUM		**54971**	**13415**	**156**	**1114**

Life load
In the analysis of the life load, the end supports and the supports at the pylons are modeled as fixed. The supports that represent the stay cables are modeled as vertical springs with a spring stiffness equal to the values from table V.2. The maximum moment due to the distributed load can be found when the deck is fully loaded. The moment above the support without load factor is equal to -10043 kNm. The normal force is equal to -13415kN. For the axle load, two load cases can be considered. When the axle load is applied near the pylon, the occurring moment above the support will have the largest value (-2097kNm). In this case the normal force has a value of -156kN. Another possibility is when the axle load is positioned at midspan. In this case the normal force will have the largest value (1114kN) and the moment above support has a value of -1.5kNm.

Cross sectional properties of the main girders
The properties of the main girders can be determined. Based on the occurring moments and normal forces it should be checked whether or not the cross section complies with the requirements. The properties of the total cross section of the bridge is taken into account.

Cross sectional area single main girder

$$A_{c,main} = 2.475 \, m^2$$

Distance of the bottom fibre of the main girder to the normal centre

$$NC_{(z),main} = 0.8308\ m$$

Moment of inertia of a single main girder

$$I_{zz,main} = 0.2148\ m^4$$

Cross sectional area of the total bridge deck

$$A_{c,tot} = 2 \cdot 2.475 + 24.1 \cdot 0.244 = 10.830\ m^2$$

Distance of the bottom fibre of the main girders to the normal centre

$$NC_{(z)} = \frac{2 \cdot 2.475 \cdot 0.8308 + 0.244 \cdot 24.1 \cdot 1.1514}{A_{c,tot}} = 1.0049\ m$$

Moment of inertia

$$I_{zz} = 2 \cdot 0.2148 + 2 \cdot 2.475 \cdot (1.0049 - 0.8308)^2$$
$$+0.00328 \cdot 24.1 + 0.244 \cdot 24.1 \cdot (1.0049 - 1.1514)^2 = 0.7849\ m^4$$

Section moduli

$$W_{c,top} = 2.6598\ m^3$$
$$W_{c,bottom} = 0.7811\ m^3$$

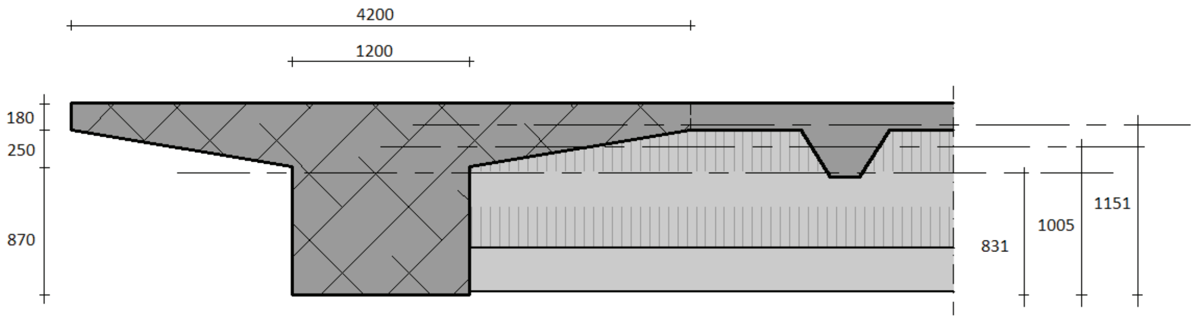


Figure V.6 Cross section main girder

N-M-kappa diagram

A moment-kappa diagram of the cross section, can be determined with the occurring normal force at the support. The design value for the normal force at the supports can be calculated based on the values found for the dead load and the life load.

$$N_d = 1.32 \cdot N_G + 1.65 \cdot (N_Q + N_D)$$
$$N_d = 1.32 \cdot 54971 + 1.65 \cdot (13415 + 156) = 94954\ kN$$

This gives a design moment with a value of:

$$M_d = 1.32 \cdot M_G + 1.65 \cdot (M_Q + M_D)$$
$$M_d = 1.32 \cdot 15305 + 1.65 \cdot (10043 + 2097) = 40234\ kNm$$

The normal stress in the cross section is equal to.

$$\sigma_N = \frac{N_d}{A_c} = 8768\ kN/m^2$$

At the bottom fibre an additional compressive stress can be formed before the bearing stress of the concrete is reached. The maximum additional tensile stress at the top fibre of the cross section has a value of $14988\ kN/m^2$. This means that there can be an additional moment applied on the cross section of $39864\ kNm$ before the top fibre reaches the mean tensile strength of the concrete. The structure behaves as a linear elastic cross section when the moment is smaller than $39864\ kNm$. The curvature that belongs to this moment is:

$$\kappa_{el} = 0.973 \cdot 10^{-3}$$

In reality the tensile strain can increase even further. This plastic part of the stress-strain diagram for the tensile strength is for simplicity not taken into account. This has the consequence that when the moment is increased it is assumed that the concrete cracks at the top and the reinforcement is supposed to take over the tensile stress. For the calculation it is assumed that 160 bars with a diameter of 32mm are applied in the top flange. The stress at the bottom fibres can increase until the design compressive strength is reached. It is assumed that the reinforcement yield first before the bottom fibres reach the design compressive strength. This gives result in a compressive zone of:

$$x_s = 0.535\ m$$

With these values the internal moment can have a value of:

$$M_s = 59715\ kNm$$

The curvature is based on the strains occurring at this moment.

$$\kappa_s = 3.312 \cdot 10^{-3}$$

The moment can be increased even further. This would mean that the concrete will reach the compressive design strength. Because the reinforcement steel already yields, the internal force equilibrium can only be reached when the height of the compressive zone is reduced. As a consequence the internal level arm will increase and the internal moment can increase as well. This gives a compressive zone of:

$$x_{st} = 0.446\ m$$

The moment and curvature can be calculated:

$$M_{st} = 61366\ kNm$$
$$\kappa_{st} = 4.757 \cdot 10^{-3}$$

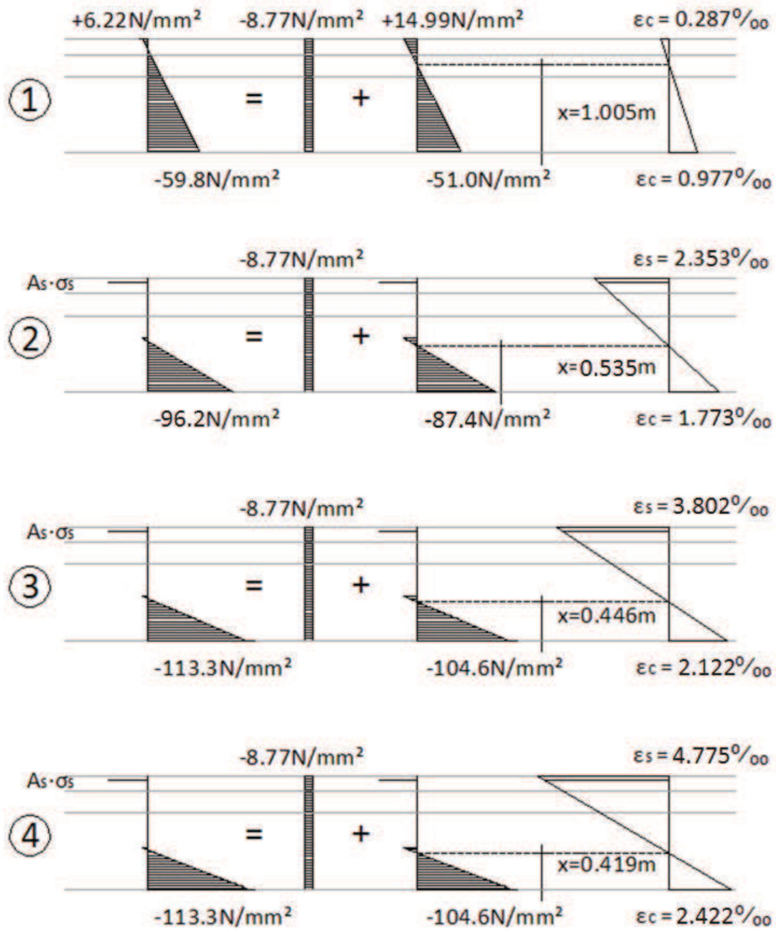
At this point the compressive strength of the bottom fibres cannot be increased. The ultimate limit moment is reached when the bottom fibres will reach the ultimate strain. This gives a compressive zone with a height of:

$$x_u = 0.419\ m$$

With a corresponding moment and curvature of:

$$M_u = 61768 kNm$$
$$\kappa_u = 5.783 \cdot 10^{-3}$$

When these values of the N-M-kappa diagram are taken into account figure V.9 can be drawn. After the ultimate limit moment the figure will experience a material softening curve. This means that when the curvature is increased the moment capacity of the cross section will decrease. It can be seen that the design moment is lower than the moment that is necessary to reach yield stress of the steel. Therefore it seems that the cross section of the main girders meet the requirements. Locally, where the moments are larger than the elastic moment, cracks appear in the concrete. These cracks will reduce the Young's modulus of the concrete. This means that the stiffness of the bridge deck will be reduced as well.



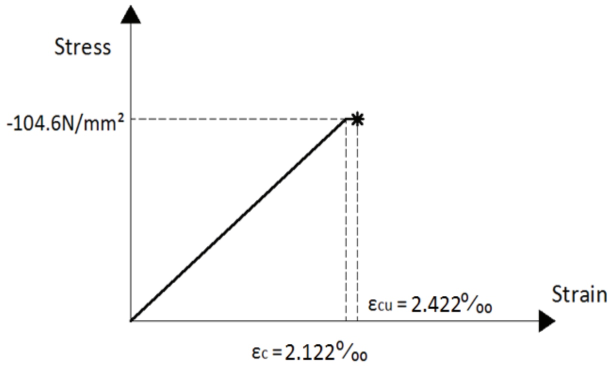


Figure V.7 Stress-strain relation

Figure V.8 The four different phases in the N-M-kappa diagram

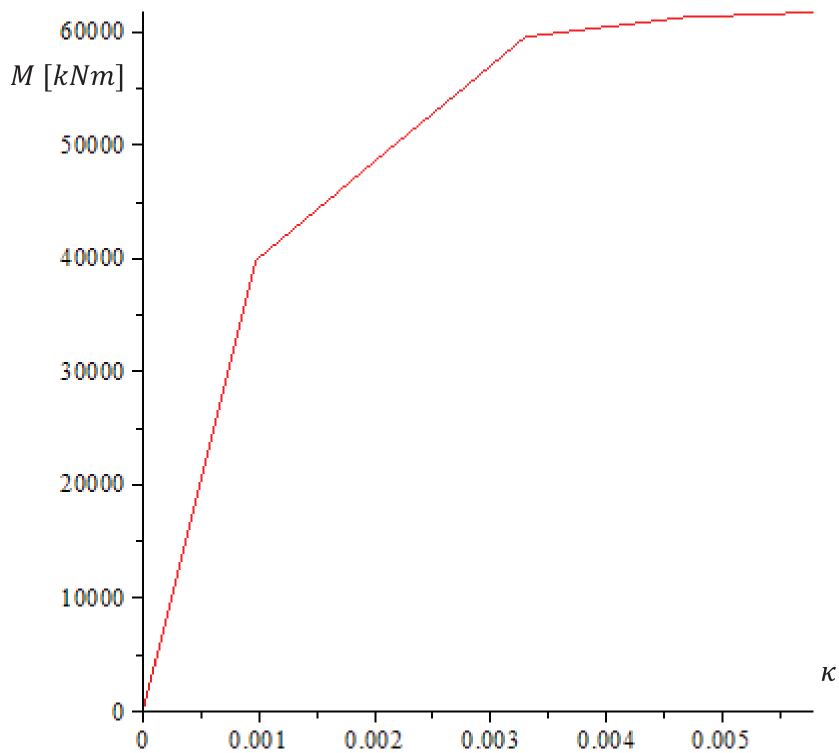


Figure V.9 N-M-kappa diagram

FE-model

For the FE-model the same boundary conditions as for the NSC model are taken into account. The dead load, super dead load and the life loads are split up in different load cases.

Dead load

The analysis of the self-weight of the bridge is based on a beam with the properties of the bridge deck over multiple stiff supports.

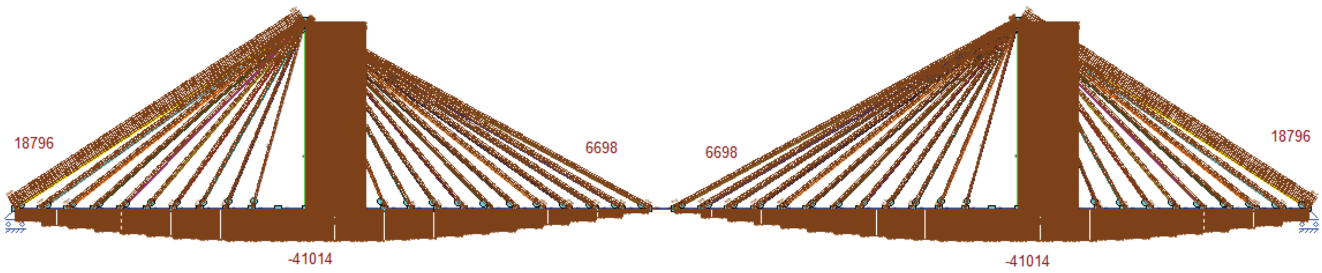


Figure V.10 Normal force distribution due to the dead load

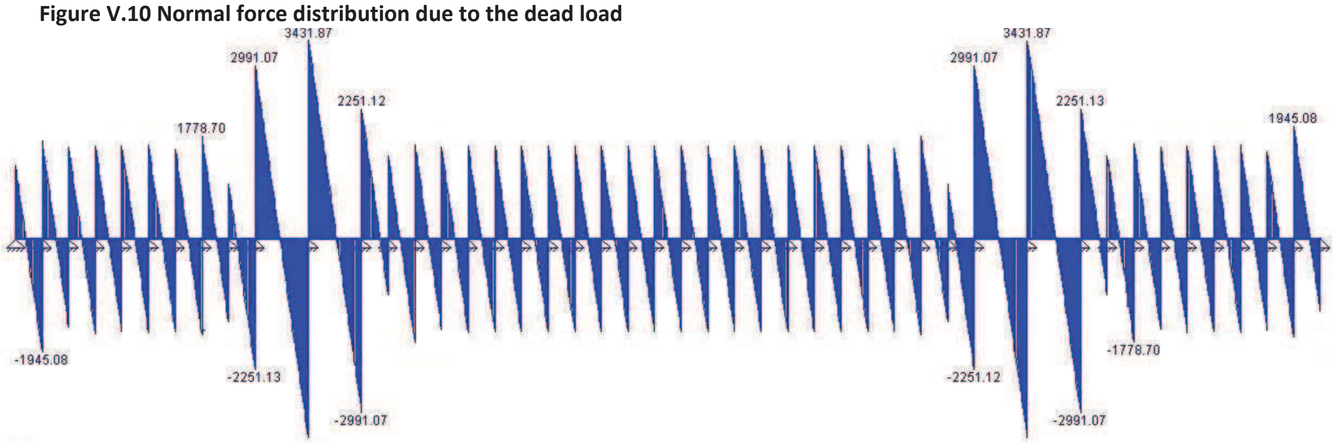


Figure V.11 Shear force distribution due to the dead load

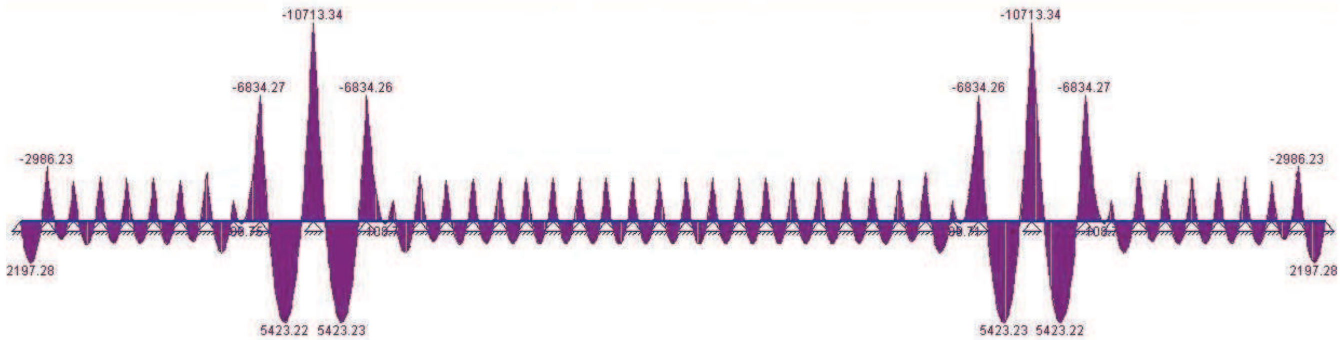


Figure V.12 Moment distribution due to the dead load

Super dead load

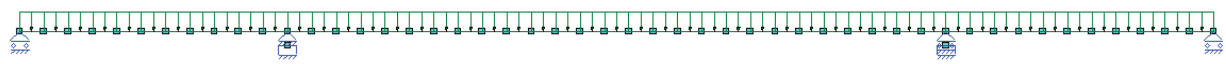


Figure V.13 Loading super dead load

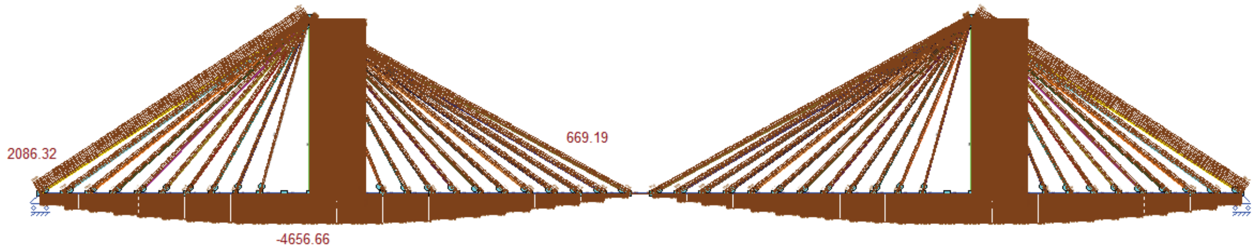


Figure V.14 Normal force distribution due to the super dead load

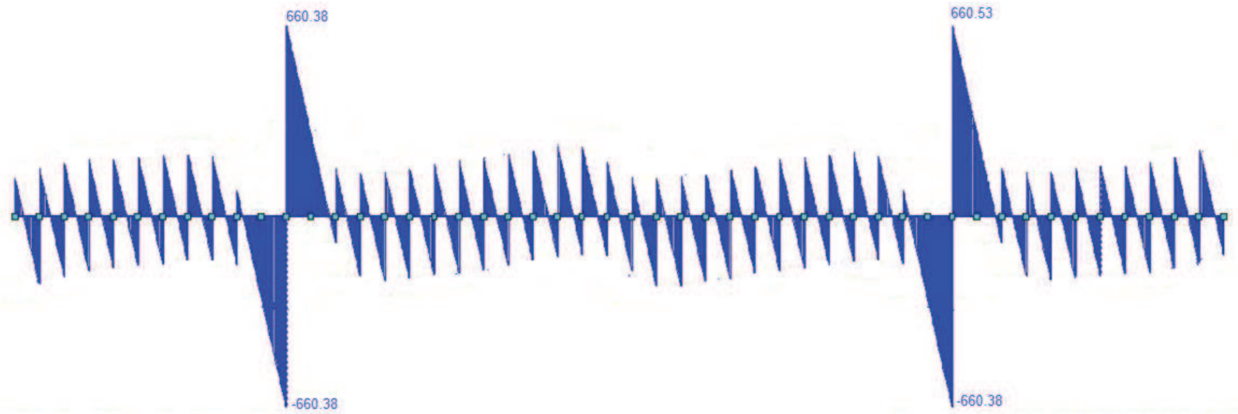


Figure V.15 Shear force distribution due to the super dead load

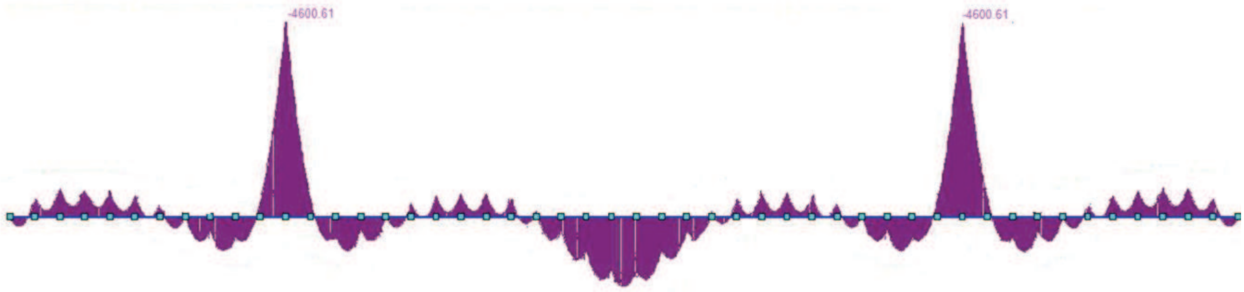


Figure V.16 Moment distribution due to super dead load

Life load: main span fully loaded

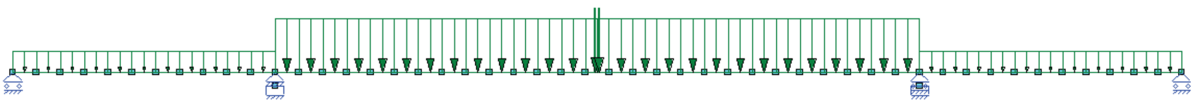


Figure V.17 loading life load

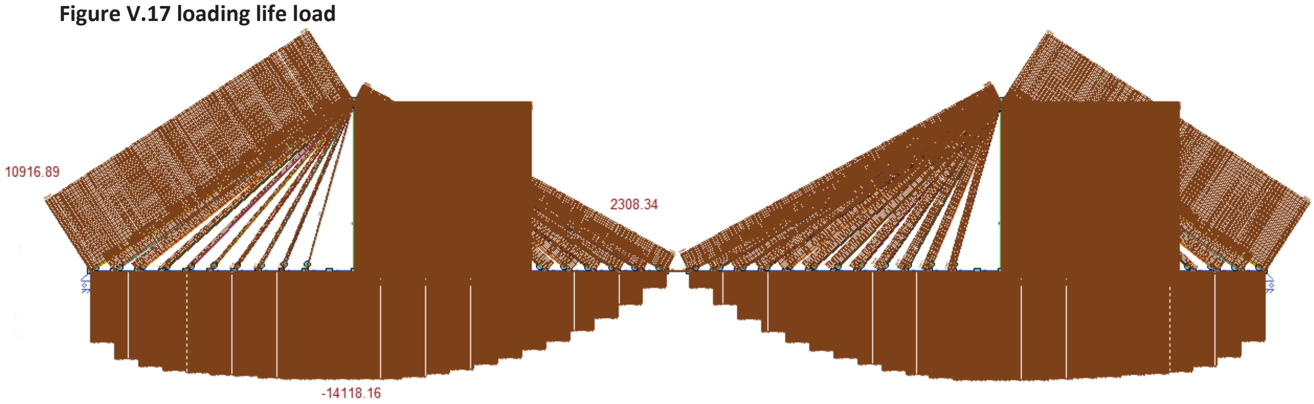


Figure V.18 Normal force distribution due to the life load

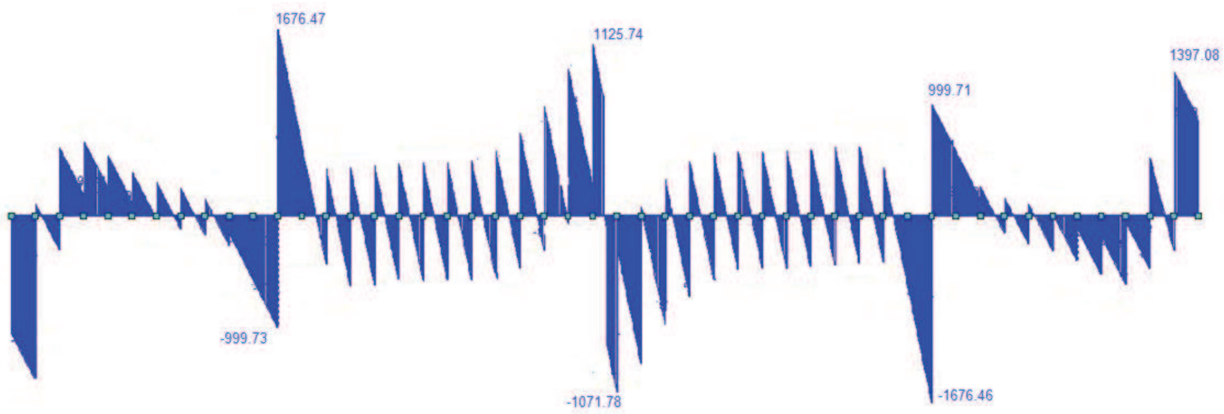


Figure V.19 Shear force distribution due to the life load

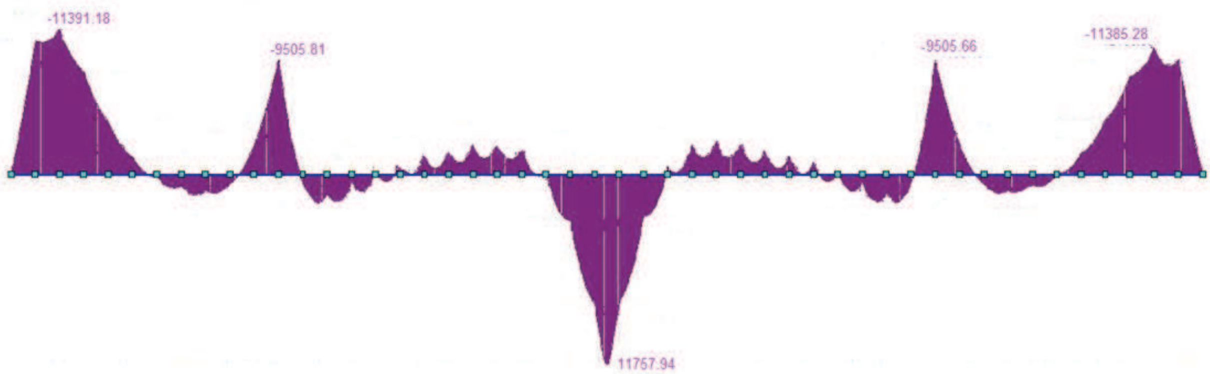


Figure V.20 Moment distribution due to the life load

Life load: side spans fully loaded

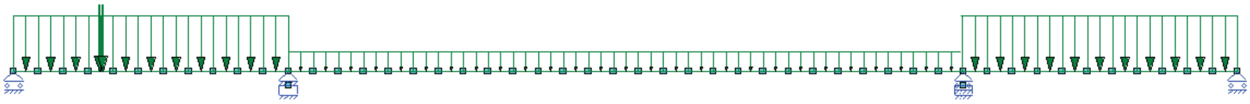


Figure V.21 Loading life load

Figure V.22 Normal force distribution due to life load

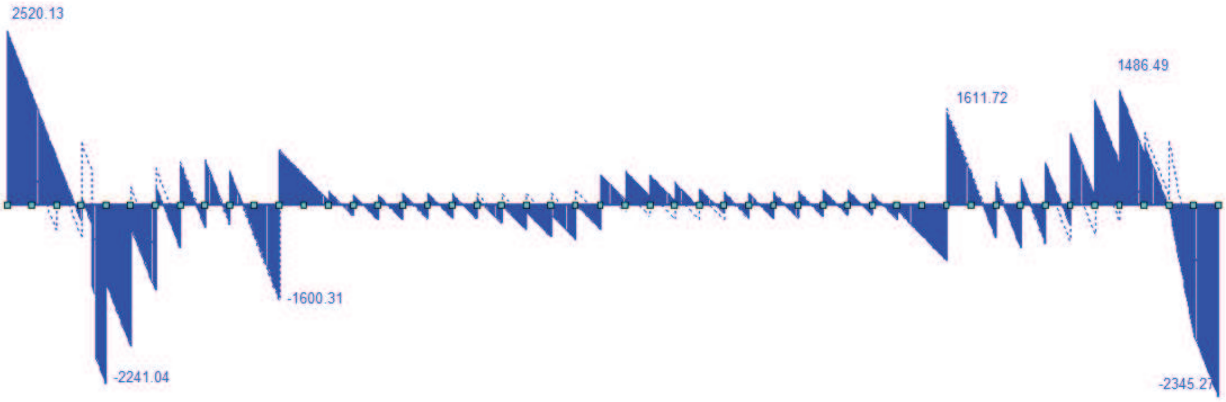


Figure V.23 Shear force distribution due to life load

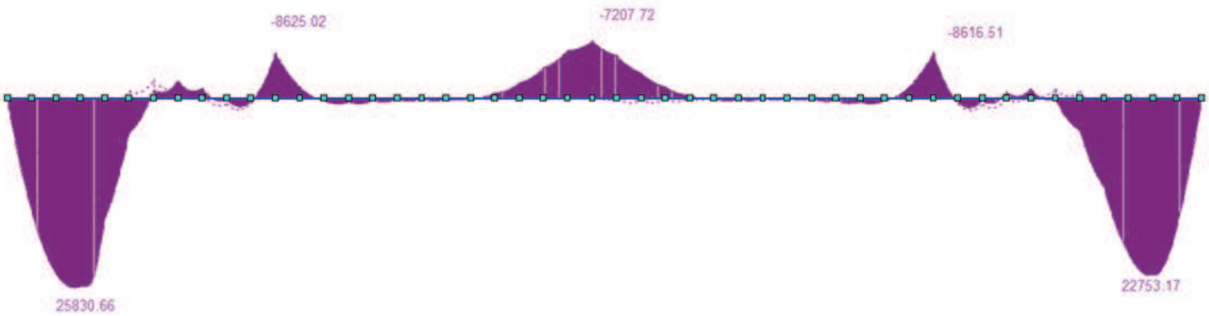


Figure V.24 Moment distribution due to the life load

Life load: all spans fully loaded

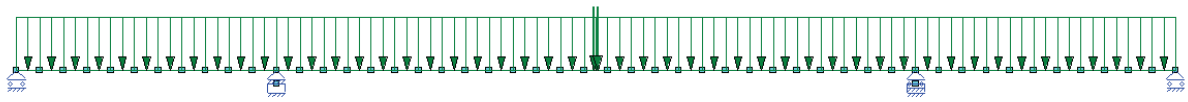


Figure V.25 Loading life load

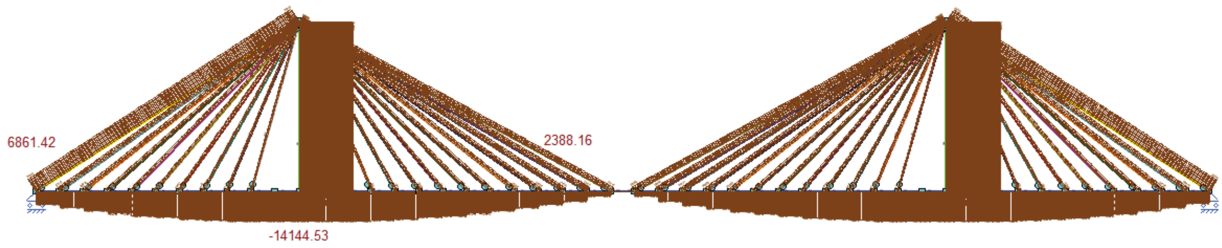


Figure V.26 Normal force distribution due to life load

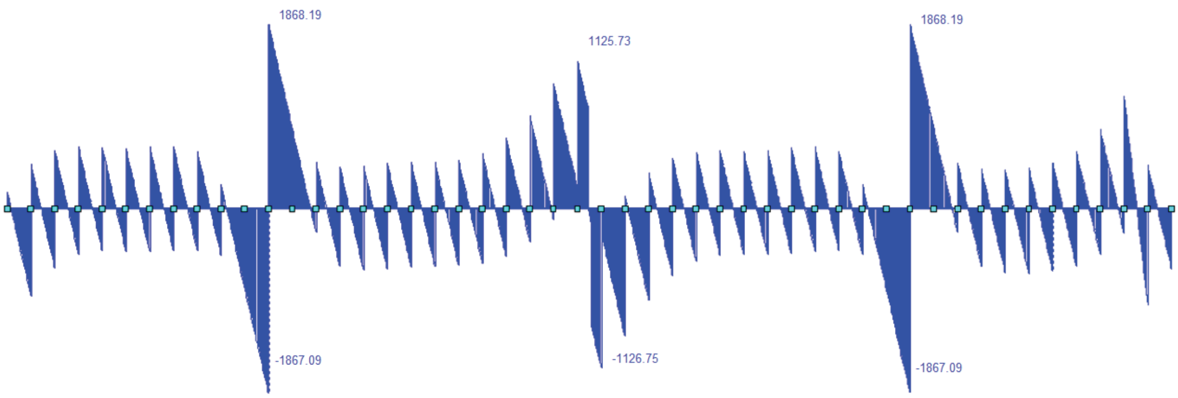


Figure V.27 Shear force distribution due to life load

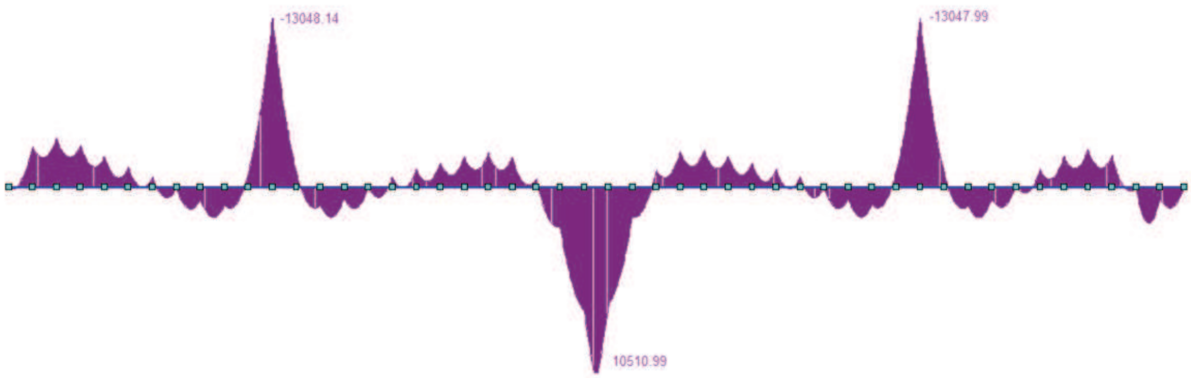


Figure V.28 Moment distribution due to the life load

Life load: one side span + half main span fully loaded

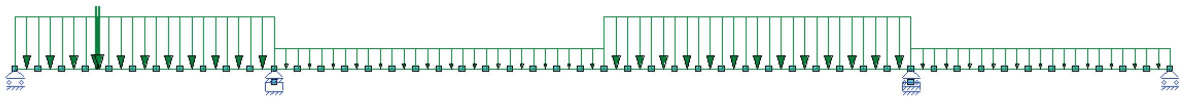


Figure V.29 Loading life load

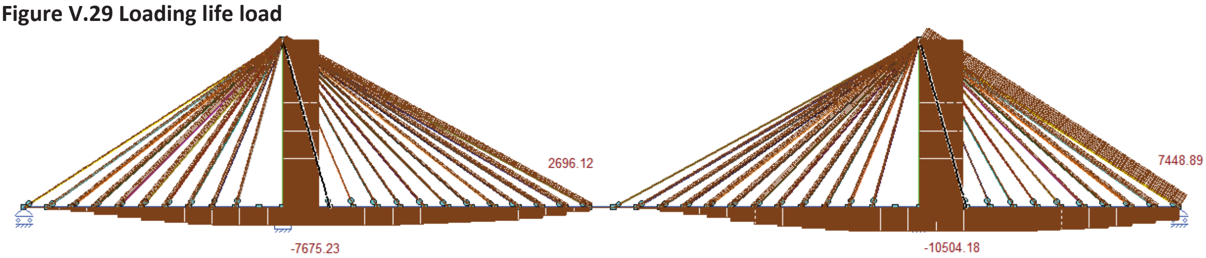


Figure V.30 Normal force distribution due to life load

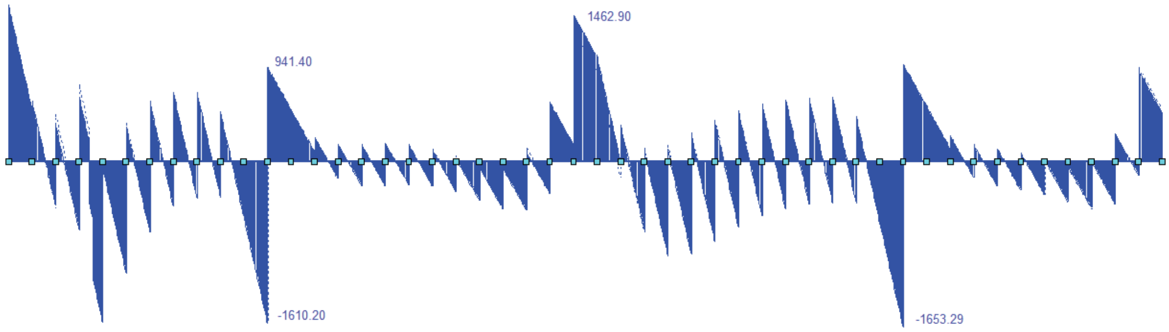


Figure V.31 Shear force distribution due to life load

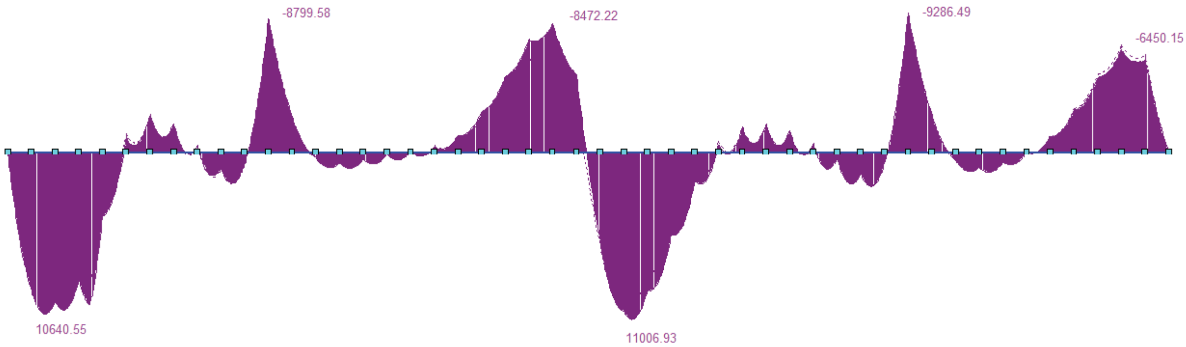


Figure V.32 Moment force distribution due to life load

Conclusion

The internal forces calculated with the FE-model for the cross section of the bridge deck at the position of the pylon are listed in the tables V.4, V.5 and V.6. It can be seen that for the Ultimate Limit State (ULS) the normal forces calculated in the FE-model with stays are lower compared with the values found in the model of the beam on elastic supports. The moments are for both calculation methods about the same.

Table V.4 Normal force in the bridge deck at the location of the pylon for the different load cases

	Load factor	Dead load	Load factor	Super dead load	Load factor	Life load	Total
ULS 1	*1.35*	-41014 kN	*1.35*	-4655 kN	-	-	-61653 kN
ULS 2	*1.32*	-41014 kN	*1.32*	-4655 kN	*1.65*	-14102 kN	-83551 kN
ULS 3	*1.32*	-41014 kN	*1.32*	-4655 kN	*1.65*	-6130 kN	-70398 kN
ULS 4	*1.32*	-41014 kN	*1.32*	-4655 kN	*1.65*	-14135 kN	-83606 kN
ULS 5	*1.32*	-41014 kN	*1.32*	-4655 kN	*1.65*	-7620 kN	-72856 kN
SLS 1	*1.00*	-41014 kN	*1.00*	-4655 kN	*1.00*	-14102 kN	-59771 kN
SLS 2	*1.00*	-41014 kN	*1.00*	-4655 kN	*1.00*	-6130 kN	-51799 kN
SLS 3	*1.00*	-41014 kN	*1.00*	-4655 kN	*1.00*	-14135 kN	-59804 kN
SLS 4	*1.00*	-41014 kN	*1.00*	-4655 kN	*1.00*	-7620 kN	-53289 kN

Table V.5 Shear force in the bridge deck at the location of the pylon for the different load cases

	Load factor	Dead load	Load factor	Super dead load	Load factor	Life load	Total
ULS 1	*1.35*	-3431 kN	*1.35*	-655 kN	-	-	-5516 kN
ULS 2	*1.32*	-3431 kN	*1.32*	-655 kN	*1.65*	-1663 kN	-8137 kN
ULS 3	*1.32*	-3431 kN	*1.32*	-655 kN	*1.65*	-1581 kN	-8002 kN
ULS 4	*1.32*	-3431 kN	*1.32*	-655 kN	*1.65*	-1851 kN	-8448 kN
ULS 5	*1.32*	-3431 kN	*1.32*	-655 kN	*1.65*	-1631 kN	-8085 kN
SLS 1	*1.00*	-3431 kN	*1.00*	-655 kN	*1.00*	-1663 kN	-5749 kN
SLS 2	*1.00*	-3431 kN	*1.00*	-655 kN	*1.00*	-1581 kN	-5667 kN
SLS 3	*1.00*	-3431 kN	*1.00*	-655 kN	*1.00*	-1851 kN	-5937 kN
SLS 4	*1.00*	-3431 kN	*1.00*	-655 kN	*1.00*	-1631 kN	-5717 kN

Table V.6 Moments in the bridge deck at the location of the pylon for the different load cases

	Load factor	Dead load	Load factor	Super dead load	Load factor	Life load	Total
ULS 1	*1.35*	-10713 kNm	*1.35*	-4560 kNm	-	-	-20619 kNm
ULS 2	*1.32*	-10713 kNm	*1.32*	-4560 kNm	*1.65*	-9429 kNm	-35718 kNm
ULS 3	*1.32*	-10713 kNm	*1.32*	-4560 kNm	*1.65*	-8546 kNm	-34261 kNm
ULS 4	*1.32*	-10713 kNm	*1.32*	-4560 kNm	*1.65*	-12934 kNm	-41501 kNm
ULS 5	*1.32*	-10713 kNm	*1.32*	-4560 kNm	*1.65*	-8709 kNm	-34530 kNm
SLS 1	*1.00*	-10713 kNm	*1.00*	-4560 kNm	*1.00*	-9429 kNm	-24702 kNm
SLS 2	*1.00*	-10713 kNm	*1.00*	-4560 kNm	*1.00*	-8546 kNm	-23819 kNm
SLS 3	*1.00*	-10713 kNm	*1.00*	-4560 kNm	*1.00*	-12934 kNm	-28207 kNm
SLS 4	*1.00*	-10713 kNm	*1.00*	-4560 kNm	*1.00*	-8709 kNm	-23982 kNm

	Elastic beam on springs	**FE-model with stays**
Normal force	94954 kN	-83606 kN
Shear force	-	-8448 kN
Moment	40234 kNm	-41501 kNm

The design value for the shear force should not be larger than the maximum shear force capacity of the structure. The maximum design value of the shear force is equal to:

$$V_{Ed} = 8482\ kN$$

The maximum shear force capacity of the structure can be calculated with:

$$V_{Rd,max} = \alpha_{cw} b_w z v_1 f_{cd} \cdot \cot(\theta)\,/(1 + \cot(\theta)^2)$$

Where:

$$\alpha_{cw} = 1.07$$
$$b_w = 2400mm$$
$$z = 1245mm$$
$$v_1 = 0.5$$
$$f_{cd} = 113.33N/mm^2$$

This gives that:

$$V_{Rd,max} = 180850 \cdot 10^3 \cdot \cot(\theta)\,/(1 + \cot(\theta)^2)$$

The angle θ of the compressive strut should have a value between:

$$0.380\ rad < \theta < 0.785\ rad$$

Filling in the minimum and maximum value for θ gives:

$$90430kN < V_{Rd,max} < 129220kN$$

Both values are higher than the occurring shear forces in the structure.

From this design calculation it seems that the assumed dimensions are sufficient for the design of the frUHSC bridge.

APPENDIX VI: Matlab Code wind.m

```
%wind.m
%Simulation of the turbulent wind velocity in longitudinal and vertical
%direction. The Von Karman-Harris spectrum is used for the longitudinal
%wind velocity and the Bush and Panofsky spectrum is used for the
%vertical turbulent wind velocity. In this numerical analysis 6000 time
%steps of 0,05 seconds are used. The turbulent wind velocity is
%simulated with a Fourier series.

Lu=200;                    %(Turbulence length longitudinal[m])
Lw=18;                     %(Turbulence length vertical[m])
Iu=0.11;                   %(Turbulence intensity longitudinal)
Iw=0.06;                   %(Turbulence intensity vertical)
v50=36.62;                 %(Mean wind velocity[m/s])
sigmau=Iu*v50;             %(Standard deviation longitudinal[m/s])
sigmaw=Iw*v50;             %(Standard deviation vertical[m/s])
CL=0.9;                    %(Lift coefficient)
dCL=1;                     %(Derivative lift coefficient)
N=12000;                   %(Number of frequency steps)
deltat=0.05;               %(Timestep[s])
deltaf=0.001;              %(Frequencystep[1/s])
numberf=[1:N]';
f=deltaf*numberf;
Suu=zeros(N,1);
Sww=zeros(N,1);
au=zeros(N,1);
aw=zeros(N,1);
phiu=zeros(N,1);
phiw=zeros(N,1);
u=zeros(N,6000);
w=zeros(N,6000);

for i=1:N;                 %(Calculation of longitudinal wind velocity)
    Suu(i)=4*(f(i)*Lu/v50)/((1+70.4*(f(i)*Lu/(v50))^2)^(5/6))*
    sigmau^2/(f(i));
    au(i)=sqrt(2*Suu(i)*deltaf);
    phiu=rand(N,1)*2*pi;
   for t=1:6000;
       u(i,t)=au(i)*sin(f(i)*t*deltat+phiu(i));
   end
    utot=sum(u);
end

for i=1:N;                 %(Calculation of vertical wind velocity)
    Sww(i)=4*(f(i)*Lw/v50)/((1+70.4*(f(i)*Lw/(v50))^2)^(5/6))
    *sigmaw^2/(f(i));
    aw(i)=sqrt(2*Sww(i)*deltaf);
    phiw=rand(N,1)*2*pi;
   for t=1:6000;
       w(i,t)=aw(i)*sin(f(i)*t*deltat+phiw(i));
   end
    wtot=sum(w);
end

figure(1);                 %(See also figure 5.1)
```

```
    plot(utot);
    figure(2);
    plot(wtot);
```

APPENDIX VII: Matlab Code bridge.m

```
%bridge.m
%Matlab code for linear elastic FE-model of the cable stayed bridge. In
%this code the cross sectional properties of the bridge are given per
%element. The coordinates of all the individual nodes are introduced. The
%nodes are connected with Euler-Bernoulli beam elements. There are three
%degrees of freedom per node, horizontal displacement, vertical
%displacement and rotation. Subsequently the empty matrix/vectors of the
%stiffness matrix, displacement vector and right-hand-side vector are
%given. The BC are given and the total stiffness matrix is formed. Based
%on the stiffness matrix and the rhs the displacements can be obtained.

%Cross sectional properties
L=8800;
a=pi()/12;
E1=52185;
E2=34000;
E3=195000;
A1=10.83*10^6;
A2=2*2000*2500;
A3=2*14250;
A4=2*13500;
A5=2*12750;
A6=2*12000;
A7=2*11250;
A8=2*10500;
A9=2*9750;
A10=2*9000;
A11=2*9000;
A12=2*9000;
A13=2*9000;
A14=2*39000;
I1=0.7849*10^12;
I2=5.205*10^12;
GAs=0;

%Allocation of the properties for each element
EI(1)=E1*I1;
EA(1)=E1*A1;
EI(2)=E1*I1;
EA(2)=E1*A1;
EI(3)=E1*I1;
EA(3)=E1*A1;
EI(4)=E1*I1;
EA(4)=E1*A1;
EI(5)=E1*I1;
EA(5)=E1*A1;
EI(6)=E1*I1;
EA(6)=E1*A1;
EI(7)=E1*I1;
EA(7)=E1*A1;
EI(8)=E1*I1;
EA(8)=E1*A1;
EI(9)=E1*I1;
EA(9)=E1*A1;
```

```
EI(10)=E1*I1;
EA(10)=E1*A1;
EI(11)=E1*I1;
EA(11)=E1*A1;
EI(12)=E1*I1;
EA(12)=E1*A1;
EI(13)=E1*I1;
EA(13)=E1*A1;
EI(14)=E1*I1;
EA(14)=E1*A1;
EI(15)=E1*I1;
EA(15)=E1*A1;
EI(16)=E1*I1;
EA(16)=E1*A1;
EI(17)=E1*I1;
EA(17)=E1*A1;
EI(18)=E1*I1;
EA(18)=E1*A1;
EI(19)=E1*I1;
EA(19)=E1*A1;
EI(20)=E1*I1;
EA(20)=E1*A1;
EI(21)=E1*I1;
EA(21)=E1*A1;
EI(22)=E1*I1;
EA(22)=E1*A1;
EI(23)=E1*I1;
EA(23)=E1*A1;
EI(24)=E1*I1;
EA(24)=E1*A1;
EI(25)=E1*I1;
EA(25)=E1*A1;
EI(26)=E1*I1;
EA(26)=E1*A1;
EI(27)=E1*I1;
EA(27)=E1*A1;
EI(28)=E1*I1;
EA(28)=E1*A1;
EI(29)=E1*I1;
EA(29)=E1*A1;
EI(30)=E1*I1;
EA(30)=E1*A1;
EI(31)=E1*I1;
EA(31)=E1*A1;
EI(32)=E1*I1;
EA(32)=E1*A1;
EI(33)=E1*I1;
EA(33)=E1*A1;
EI(34)=E1*I1;
EA(34)=E1*A1;
EI(35)=E1*I1;
EA(35)=E1*A1;
EI(36)=E1*I1;
EA(36)=E1*A1;
EI(37)=E1*I1;
EA(37)=E1*A1;
EI(38)=E1*I1;
EA(38)=E1*A1;
EI(39)=E1*I1;
EA(39)=E1*A1;
```

```
EI(40)=E1*I1;
EA(40)=E1*A1;
EI(41)=E1*I1;
EA(41)=E1*A1;
EI(42)=E1*I1;
EA(42)=E1*A1;
EI(43)=E1*I1;
EA(43)=E1*A1;
EI(44)=E1*I1;
EA(44)=E1*A1;
EI(45)=E1*I1;
EA(45)=E1*A1;
EI(46)=E1*I1;
EA(46)=E1*A1;
EI(47)=E1*I1;
EA(47)=E1*A1;
EI(48)=E1*I1;
EA(48)=E1*A1;
EI(49)=E1*I1;
EA(49)=E1*A1;
EI(50)=E2*I2;
EA(50)=E2*A2;
EI(51)=E2*I2;
EA(51)=E2*A2;
EI(52)=0;
EA(52)=E3*A14;
EI(53)=0;
EA(53)=E3*A6;
EI(54)=0;
EA(54)=E3*A7;
EI(55)=0;
EA(55)=E3*A8;
EI(56)=0;
EA(56)=E3*A9;
EI(57)=0;
EA(57)=E3*A10;
EI(58)=0;
EA(58)=E3*A11;
EI(59)=0;
EA(59)=E3*A12;
EI(60)=0;
EA(60)=E3*A13;
EI(61)=0;
EA(61)=E3*A13;
EI(62)=0;
EA(62)=E3*A13;
EI(63)=0;
EA(63)=E3*A13;
EI(64)=0;
EA(64)=E3*A12;
EI(65)=0;
EA(65)=E3*A11;
EI(66)=0;
EA(66)=E3*A10;
EI(67)=0;
EA(67)=E3*A9;
EI(68)=0;
EA(68)=E3*A8;
EI(69)=0;
EA(69)=E3*A7;
```

```
EI(70)=0;
EA(70)=E3*A6;
EI(71)=0;
EA(71)=E3*A5;
EI(72)=0;
EA(72)=E3*A4;
EI(73)=0;
EA(73)=E3*A3;
EI(74)=0;
EA(74)=E3*A3;
EI(75)=0;
EA(75)=E3*A4;
EI(76)=0;
EA(76)=E3*A5;
EI(77)=0;
EA(77)=E3*A6;
EI(78)=0;
EA(78)=E3*A7;
EI(79)=0;
EA(79)=E3*A8;
EI(80)=0;
EA(80)=E3*A9;
EI(81)=0;
EA(81)=E3*A10;
EI(82)=0;
EA(82)=E3*A11;
EI(83)=0;
EA(83)=E3*A12;
EI(84)=0;
EA(84)=1E3*A13;
EI(85)=0;
EA(85)=E3*A13;
EI(86)=0;
EA(86)=E3*A13;
EI(87)=0;
EA(87)=E3*A13;
EI(88)=0;
EA(88)=E3*A12;
EI(89)=0;
EA(89)=E3*A11;
EI(90)=0;
EA(90)=E3*A10;
EI(91)=0;
EA(91)=E3*A9;
EI(92)=0;
EA(92)=E3*A8;
EI(93)=0;
EA(93)=E3*A7;
EI(94)=0;
EA(94)=E3*A6;
EI(95)=0;
EA(95)=E3*A5;

%Loads
gamg=1.32;
gamq=1.65;
psi=0.4;
qg=314.07;
qq=0;
q1=qq;
```

```
q2=qq;
q3=qq;
F=880000;

%Coordinates on the x-y axis for all the nodes
coord=[0 0; 8800 0; 17600 0; 26400 0; 35200 0; 44000 0; 52800 0; 61600 0;
70400 0; 79200 0; 88000 0; 96800 0; 105600 0; 114400 0; 123200 0; 132000
0; 140800 0; 149600 0; 158400 0; 167200 0; 176000 0; 184800 0; 193600 0;
202400 0; 211200 0; 220000 0; 228800 0; 237600 0; 246400 0; 255200 0;
264000 0; 272800 0; 281600 0; 290400 0; 299200 0; 308000 0; 316800 0;
325600 0; 334400 0; 343200 0; 352000 0; 360800 0; 369600 0; 378400 0;
387200 0; 396000 0; 404800 0; 413600 0; 422400 0; 431200 0; 96800 -5000;
96800 62500; 334400 -5000; 334400 62500]

%Forming of the beam elements between the nodes
conn=[1 2; 2 3; 3 4; 4 5; 5 6; 6 7; 7 8; 8 9; 9 10; 10 11; 11 12; 12 13;
13 14; 14 15; 15 16; 16 17; 17 18; 18 19; 19 20; 20 21; 21 22; 22 23; 23
24; 24 25; 25 26; 26 27; 27 28; 28 29; 29 30; 30 31; 31 32; 32 33; 33 34;
34 35; 35 36; 36 37; 37 38; 38 39; 39 40; 40 41; 41 42; 42 43; 43 44; 44
45; 45 46; 46 47; 47 48; 48 49; 49 50; 51 52; 53 54; 1 52; 2 52; 3 52; 4
52; 5 52; 6 52; 7 52; 8 52; 9 52; 10 52; 14 52; 15 52; 16 52; 17 52; 18
52; 19 52; 20 52; 21 52; 22 52; 23 52; 24 52; 25 52; 26 54; 27 54; 28 54;
29 54; 30 54; 31 54; 32 54; 33 54; 34 54; 35 54; 36 54; 37 54; 41 54; 42
54; 43 54; 44 54; 45 54; 46 54; 47 54; 48 54; 49 54; 50 54];

%Number of degrees of freedom, nodes and elements
dofNode=3;
nNodes=size(coord,1);
nElements=size(conn,1);

%Allocate arrays in ka=f
nDofs=dofNode*nNodes;   %(Total number of DOFs)
k=zeros(nDofs,nDofs);   %(Stiffness matrix)
a=zeros(nDofs,1);       %(Displacement vector)
f=zeros(nDofs,1);       %(Right-hand-side)

%Define boundary conditions:
%Constrained DOFs and applied load
constrainedDofs=[2 34 35 116 149 151 152 153 157 158 159];
f(5)=-q1*L;
f(8)=-q1*L;
f(11)=-q1*L;
f(14)=-q1*L;
f(17)=-q1*L;
f(20)=-q1*L;
f(23)=-q1*L;
f(26)=-q1*L;
f(29)=-q1*L;
f(32)=-q1*L;
f(38)=-q2*L;
f(41)=-q2*L;
f(44)=-q2*L;
f(47)=-q2*L;
f(50)=-q2*L;
f(53)=-q2*L;
f(56)=-q2*L;
f(59)=-q2*L;
f(62)=-q2*L;
f(65)=-q2*L;
f(68)=-q2*L;
```

```
f(71)=-q2*L;
f(74)=-q2*L;
f(74)=-F;
f(77)=-q2*L;
f(80)=-q2*L;
f(83)=-q2*L;
f(86)=-q2*L;
f(89)=-q2*L;
f(92)=-q2*L;
f(95)=-q2*L;
f(98)=-q2*L;
f(101)=-q2*L;
f(104)=-q2*L;
f(107)=-q2*L;
f(110)=-q2*L;
f(113)=-q2*L;
f(119)=-q3*L;
f(122)=-q3*L;
f(125)=-q3*L;
f(128)=-q3*L;
f(131)=-q3*L;
f(134)=-q3*L;
f(137)=-q3*L;
f(140)=-q3*L;
f(143)=-q3*L;
f(146)=-q3*L;

%Assemble stiffness matrix
%First bar
for e=1:nElements;

  %element connectivity table
  eleConn=conn(e,:);

  %Element coordinates and length
  x1=coord(eleConn(1),1);
  x2=coord(eleConn(2),1);
  y1=coord(eleConn(1),2);
  y2=coord(eleConn(2),2);
  len=sqrt((x2-x1)*(x2-x1)+(y2-y1)*(y2-y1));

  %c and s are cosine and sine of the
  %angle between local and global axes
  c=(x2-x1)/len;
  s=(y2-y1)/len;

  %Phi is a constant depending of the shear modulus
  phi=0;      %phi=0 gives Euler-Bernoulli beam elements

  %Element stiffness matrix (local system)
  ke_loc=[EA(e)/len 0 0 -EA(e)/len 0 0;
          0 12*EI(e)/(len^3*(1+phi)) 6*EI(e)/(len^2*(1+phi)) 0 -
12*EI(e)/(len^3*(1+phi)) 6*EI(e)/(len^2*(1+phi));
          0 6*EI(e)/(len^2*(1+phi)) (4+phi)*EI(e)/(len*(1+phi)) 0 -
6*EI(e)/(len^2*(1+phi)) (2-phi)*EI(e)/(len*(1+phi));
          -EA(e)/len 0 0 EA(e)/len 0 0;
          0 -12*EI(e)/(len^3*(1+phi)) -6*EI(e)/(len^2*(1+phi)) 0
12*EI(e)/(len^3*(1+phi)) -6*EI(e)/(len^2*(1+phi));
          0 6*EI(e)/(len^2*(1+phi)) (2-phi)*EI(e)/(len*(1+phi)) 0 -
```

```
6*EI(e)/(len^2*(1+phi)) (4+phi)*EI(e)/(len*(1+phi))]

  %Rotation and transformation matrix
  R=[c s 0;
    -s c 0;
    0 0 1];

  Zero=[0 0 0;
        0 0 0;
        0 0 0];

  T=[R Zero;
     Zero R];

  %Element stiffness matrix (global system)
  ke=T'*ke_loc*T

  %Compute system dofs associated with each element
  %(node 1 has dofs 1, 2 and 3, node 2 has dofs 4, 5 and 6...)
  index(1)=dofNode*eleConn(1)-2;
  index(2)=dofNode*eleConn(1)-1;
  index(3)=dofNode*eleConn(1);
  index(4)=dofNode*eleConn(2)-2;
  index(5)=dofNode*eleConn(2)-1;
  index(6)=dofNode*eleConn(2);

  %Assemble ke into K
  edof = length(index);
  for i=1:edof
    ii=index(i);
    for j=1:edof
      jj=index(j);
      k(ii,jj)=k(ii,jj)+ke(i,j);
    end
  end

end

%Apply boundary conditions by zeroing out
%rows and colums and putting ones on the diagonal
k1=k
k1(constrainedDofs,:)=0;
k1(:,constrainedDofs)=0;
k1(constrainedDofs,constrainedDofs)=eye(length(constrainedDofs));

%Solve ka=f for a
kinv=inv(k1)
a=kinv*f
f=k*a

%Determination of the reaction forces
Re=f(constrainedDofs,:)+[q1*L/2; 0; q1*L/2+q2*L/2; q2*L/2+q3*L/2; q3*L/2;
0; 0; 0; 0; 0; 0]

%Plot results
mag=500;                    %(Scale factor for plot)
clf
hold on
```

```
for e=1:nElements          %(See also figure 5.2)
  x=coord(conn(e,:),1);
  y=coord(conn(e,:),2);
  u=a(3*conn(e,:)-2);
  v=a(3*conn(e,:)-1);
  title('Deformed plot')
  axis equal
  plot(x,y,'r--o')
  plot(x+mag*u,y+mag*v,'k-o')
end
```

APPENDIX VIII: Matlab Code stiffness.m

```
%stiffness.m
%Determination of the mass, damping and stiffness matrix. The inverse
%stiffness matrix is determined based on the node displacements of the FE-
%model (See also APPENDIX VII). A lumped mass matrix is used in this
%model. The damping matrix is a proportional Rayleigh damping matrix.

L=237.6;
Lpart=8.8;

%Properties NSC bridge
rhoc=2500;
Ac=20.775;
mass=Lpart*Ac*rhoc+4145*Lpart+Lpart*5606;
zeta1=0.0072;
zeta2=0.0070;

%Properties frUHSC bridge
rhocHSC=2900;
AcHSC=10.83;
massHSC=Lpart*AcHSC*rhocHSC+4145*Lpart+5606*Lpart;
zeta1HSC=0.0055;
zeta2HSC=0.0054;

%Stiffness matrix
 Ki=-1/880000000*[-1.143964862  -1.579556779   -1.469717091   -
1.200800754    -0.961519478   -0.816170483   -0.764456955   -
0.779323296    -0.826532683   -0.873665293   -0.894154726   -
0.870238569    -0.796566857   -0.68505146    -0.563425702   -
0.452014382    -0.360112632   -0.289312115   -0.236655117   -
0.197216867    -0.165901924   -0.138517034   -0.112203387   -
0.085466894    -0.057840826   -0.029202492;
-1.579556779    -2.908948282   -3.132647521   -2.744194834   -
2.258704599    -1.897722764   -1.712652883   -1.673034557   -
1.717240746    -1.779053832   -1.800944464   -1.742678792   -
1.589817935    -1.363929963   -1.119578067   -0.896809941   -
0.713707296    -0.573076758   -0.468760875   -0.390782225   -
0.328907252    -0.27476017    -0.222657982   -0.169649494   -
0.114830199    -0.057978237;
-1.469717091    -3.132647521   -4.320290637   -4.386899869   -
3.887310774    -3.345122911   -2.968698199   -2.788277356   -
2.747406564    -2.758832802   -2.736194706   -2.614072308   -
2.364817394    -2.015974068   -1.646302322   -1.313366849   -
1.042244102    -0.835676717   -0.683522174   -0.570366133   -
0.480740213    -0.402160364   -0.326261709   -0.248773686   -
0.168455433    -0.085065951;
-1.200800754    -2.744194834   -4.386899869   -5.564525048   -
5.641064287    -5.15918379    -4.629319703   -4.248410736   -
4.030266692    -3.900862216   -3.759668417   -3.519587272   -3.13795255
-2.644795778    -2.139648835   -1.694165245   -1.337328152   -
1.069400038    -0.87460422    -0.731136303   -0.617891069   -
0.518244074    -0.421304643   -0.321687431   -0.217992487   -
0.110109884;
-0.961519478    -2.258704599   -3.887310774   -5.641064287   -
6.936425852    -7.106919311   -6.674553823   -6.140594794   -
```

```
5.690561164    -5.327931955    -4.973789384    -4.535939967    -
3.961945949    -3.283896723    -2.619354286    -2.050124559    -
1.604944028    -1.27796244     -1.045017064    -0.876106232    -
0.743522209    -0.626167809    -0.510675458    -0.390760991    -
0.265108276    -0.133962691;
-0.816170483    -1.897722764    -3.345122911    -5.15918379    -
7.106919311    -8.565117167    -8.833437233    -8.415372972    -
7.796679294    -7.156031724    -6.500781216    -5.771267427    -
4.923946217    -3.999776016    -3.134533494    -2.4171412    -
1.871753144    -1.482004596    -1.21162403     -1.019689633    -0.87020648
-0.73679739    -0.603412596    -0.463000316    -0.314588981    -
0.159048698;
-0.764456955    -1.712652883    -2.968698199    -4.629319703    -
6.674553823    -8.833437233    -10.43965366    -10.75747276    -
10.26061931    -9.424798043    -8.43919548     -7.341646846    -
6.136235871    -4.891929866    -3.768954746    -2.863637455    -
2.192939882    -1.7261327      -1.410920562    -1.192175312    -
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0.375123936    -0.189752202;
-0.779323296    -1.673034557    -2.788277356    -4.248410736    -
6.140594794    -8.415372972    -10.75747276    -12.45179996    -
12.71498196    -12.00026892    -10.79511641    -9.327495743    -
7.712358508    -6.082954252    -4.639919887    -3.494076522    -
2.657449853    -2.084107602    -1.703305145    -1.442832672    -
1.242831432    -1.061636493    -0.875367819    -0.674673093    -
0.459513121    -0.23251229;
-0.826532683    -1.717240746    -2.747406564    -4.030266692    -
5.690561164    -7.796679294    -10.26061931    -12.71498196    -
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0.295309368;
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5.327931955    -7.156031724    -9.424798043    -12.00026892    -
14.45837969    -15.98574545    -15.76090701    -14.275445    -
12.13780391    -9.79512469     -7.631070638    -5.854756522    -
4.516573714    -3.568981857    -2.91762563     -2.458749501    -
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-12.35094319    -9.942023822    -7.84004972     -6.170731429    -
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-1.95061533    -1.490511208    -1.010463161    -0.510466771;
-0.870238569    -1.742678792    -2.614072308    -3.519587272    -
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11.70701657    -14.275445      -16.56014479    -17.74252537    -
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1.674729306    -0.843748909;
```

```
-0.68505146     -1.363929963    -2.015974068    -2.644795778    -
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12.13780391    -14.275445    -15.76090701    -15.98574545    -
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7.796679294    -5.690561164    -4.030266692    -2.747406564    -
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0.873665293	-0.826532683	-0.779323296	-0.764456955	-
0.816170483	-0.961519478	-1.200800754	-1.469717091	-
1.579556779	-1.143964862]


%Inverse stiffness matrix frUHSC
KiHSC=-1/880000000*[-3.788998302	-4.020546349	-2.538935718	-
1.322875732	-0.752893725	-0.658645919	-0.806328501	-
1.034705796	-1.255191393	-1.418990273	-1.490818196	-
1.441463156	-1.261028861	-0.99064166 -0.729675819	-0.532842482
-0.405970999	-0.331772217	-0.287844726	-0.256231461	-
0.226307038	-0.193705509	-0.157801292	-0.119439285	-0.07959067
-0.039299403;
-4.020546349	-6.913149345	-5.839616968	-3.671156445	-
2.233240686	-1.70042737	-1.765442076	-2.110712368	-
2.518387589	-2.850678837	-3.010063226	-2.923750384	-
2.566502879	-2.021392743	-1.491968934	-1.091106189	-
0.831834107	-0.679664999	-0.589312282	-0.524281024	-
0.462879682	-0.396131283	-0.322692753	-0.244248304	-
0.162766171	-0.080371694;
-2.538935718	-5.839616968	-8.276360256	-7.161789534	-
5.034506992	-3.631506727	-3.149621876	-3.298282978	-
3.734544339	-4.181285075	-4.428943833	-4.327708314	-3.81911426
-3.020380211	-2.236652624	-1.639533824	-1.251196882	-
1.021995716	-0.885273318	-0.786848752	-0.694286064	-
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0.120528132;
-1.322875732	-3.671156445	-7.161789534	-9.979989254	-
9.096308758	-6.992411333	-5.51656324	-4.975223928	-
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0.658645919    -0.752893725    -1.322875732    -2.538935718    -
4.020546349    -3.788998302]

%Stiffness matrix NSC
K=inv(Ki);

%Stiffness matrix frUHSC
KHSC=inv(KiHSC);

%Mass matrix NSC
M=zeros(26,26);
for n=1:1:26;
    M(n,n)=mass;
end

%Mass matrix frUHSC
MHSC=zeros(26,26);
for n=1:1:26;
    MHSC(n,n)=massHSC;
end

%Eigenfrequency NSC
[E,omegakw]=eig(K,M);

omega1=omegakw(1,1);
omega2=omegakw(2,2);
```

```
Kstar=E'*K*E;
Mstar=E'*M*E;

E1=zeros(26,1);
for n=1:26;
    E1(n,1)=E(n,1);
    E2(n,2)=E(n,2);
    E3(n,3)=E(n,3);
end

figure(1);
plot(E1);
figure(2);
plot(E2);
figure(3);
plot(E3);

%Eigenfrequency frUHSC
[EHSC,omegakwHSC]=eig(KHSC,MHSC);

omega1HSC=omegakwHSC(2,2);
omega2HSC=omegakwHSC(3,3);

KstarHSC=EHSC'*KHSC*EHSC;
MstarHSC=EHSC'*MHSC*EHSC;

E1HSC=zeros(26,1);
for n=1:26;
    E1HSC(n,1)=EHSC(n,2);
    E2HSC(n,2)=EHSC(n,3);
    E3HSC(n,3)=EHSC(n,4);
end

figure(4);
plot(E1HSC);
figure(5);
plot(E2HSC);
figure(6);
plot(E3HSC);

%Damping matrix NSC (Rayleigh)
a0=2*omega1*omega2*(zeta1*omega2-zeta2*omega1)/(omega2^2-omega1^2);
a1=2*(zeta2*omega2-zeta1*omega1)/(omega2^2-omega1^2);
Cs=zeros(26,26);
Cs=a0*M+a1*K;

%Damping matrix frUHSC (Rayleigh)
a0HSC=2*omega1HSC*omega2HSC*(zeta1HSC*omega2HSC-
zeta2HSC*omega1HSC)/(omega2HSC^2-omega1HSC^2);
a1HSC=2*(zeta2HSC*omega2HSC-zeta1HSC*omega1HSC)/(omega2HSC^2-omega1HSC^2);
CsHSC=zeros(26,26);
CsHSC=a0HSC*M+a1HSC*K;
```

APPENDIX IX: Matlab Code transfer.m

```
%transfer.m
%Uses data from stiffness.m
%Transfer function displacement (Hx13) and acceleration (Ha13) of the node
%at midspan for the NSC bridge

for n=1:1:10000;                          %(Number of frequency steps)
    omega(n)=n/100;
    Suf=K-M*omega(n)^2+i*omega(n)*Cs;
    Hu=Suf^-1;
    Hx=abs(Hu);
    Hx13(n)=sum(Hx(13,:));
    Ha13(n)=sum(Hx(13,:))*omega(n)^2;
end

figure(7);
plot(omega,Hx13);

figure(9);
plot(omega,Ha13);

%Transfer function displacement (Hx13) and acceleration (Ha13) of the node
at midspan for the frUHSC bridge

for n=1:1:10000;
    omega(n)=n/100;
    AHSC=KHSC-MHSC*omega(n)^2+i*omega(n)*CsHSC;
    HuHSC=AHSC^-1;
    HxHSC=abs(HuHSC);
    HxHSC13(n)=sum(HxHSC(13,:));
    HaHSC13(n)=sum(HxHSC(13,:))*omega(n)^2;
end


figure(8);
plot(omega,HxHSC13);

figure(10);
plot(omega,HaHSC13);
```

APPENDIX X: Matlab Code forcespectrum.m

```
%forcespectrum.m
%Lift force spectrum based on the Von Karman-Harris spectrum for the
%longitudinal wind velocity and the Bush and Panofsky spectrum for the
%vertical turbulent wind velocity.

Lu=200;                    %(Turbulence length longitudinal[m])
Lw=18;                     %(Turbulence length vertical[m])
Iu=0.11;                   %(Turbulence intensity longitudinal)
Iw=0.06;                   %(Turbulence intensity vertical)
v50=36.62;                 %(Mean wind velocity[m/s])
rho=1.25;                  %(Density of air[kg/m^3])
sigmau=Iu*v50;             %(Standard deviation longitudinal[m/s])
sigmaw=Iw*v50;             %(Standard deviation vertical[m/s])
CL=0.9;                    %(Lift coefficient)
dCL=1;                     %(Derivative lift coefficient)
N=10000;                   %(Number of frequency steps)
B=32.5;                    %(width of the bridge deck[m])
Cy=7;                      %(Factor for coherence in y-direction)

%Matrix with mutual distance between nodes
ypoint=8.8:8.8:228.8;
yall=ypoint'*ones(1,26);
ydif=zeros(26,26);
for m=1:26
    for n=m+1:26
        ydif(m,n)=abs(yall(m)-yall(n));
        ydif(n,m)=ydiff(m,n);
    end
end

omega=zeros(N,1);
Suu=zeros(N,1);
Sww=zeros(N,1);

%Assembly of the full force matrix with force spectra for all the different
%nodes and cross spectra
for n=1:10000;
    omega(n)=n/100;
Suu(n)=(4*omega(n)*Lu/(v50*pi*2))/(1+70.4*(omega(n)*Lu/(v50*pi*2))^2)^(5/6)
*sigmau^2/(omega(n)/(2*pi));
Sww(n)=(2.15*omega(n)*Lw/(v50*pi*2))/(1+11.16*(omega(n)*Lw/(v50*pi*2))^2)^(
5/3)*sigmaw^2/(omega(n)/(2*pi));
    Chisquare(n)=((1+(1.1*omega(n)/pi)^(4/3))^-1)^2;
    cohy=exp(-1*Cy*omega(n)/(2*pi*v50)*ydif);
    SLL(n)=Lpart^2*rho^2*v50^2*B^2*(CL^2*Suu(n)+1/4*dCL^2*Sww(n))
    *Chisquare(n);
end

figure(11);                 %(See also figure 5.5)
plot(omega,SLL);
```

APPENDIX XI: Matlab Code response.m

```
%response.m
%Uses transfer.m and forcespectrum.m
%Assembly of the response spectrum of the midnode (13) for the
%displacements and the accelerations. In this response spectrum it is
%assumed that all the forces on the nodes are completely coupled.

for n=1:10000;
    for j=1:26;
        for k=1:26;
            Sxx(n)=Hx13(n)*conj(Hx13(n))*SLL(n);
            SxxHSC(n)=HxHSC13(n)*conj(HxHSC13(n))*SLL(n);
            Saa(n)=Ha13(n)*conj(Ha13(n))*SLL(n);
            SaaHSC(n)=HaHSC13(n)*conj(HaHSC13(n))*SLL(n);
        end
    end
end

figure(12);                     %(See also figure 5.6)
plot(omega,Sxx);

figure(13);
plot(omega,SxxHSC);

figure(14);                     %(See also figure 5.8)
plot(omega,Saa);

figure(15);
plot(omega,SaaHSC);

%Determination of the peakvalues during a certain time period.
for n=1:10000;
    variance=sum(Sxx(1,:))*0.01;
    varianceHSC=sum(SxxHSC(1,:))*0.01;
    standev=sqrt(variance);
    standevHSC=sqrt(varianceHSC);
    variancea=sum(Saa(1,:))*0.01;
    varianceHSCa=sum(SaaHSC(1,:))*0.01;
    standeva=sqrt(variancea);
    standevHSCa=sqrt(varianceHSCa);
end

Peakx=standev*sqrt(2*log(0.532*600));
PeakxHSC=standevHSC*sqrt(2*log(0.532*600));
Peaka=standeva*sqrt(2*log(0.525*600));
PeakaHSC=standevHSCa*sqrt(2*log(0.525*600));
```

APPENDIX XII: Matlab Code coherence.m

```
%coherence.m
%This file is used to obtain the response spectra when the force spectra
%for the different nodes are related with cross spectra.

Lu=200;                    %(Turbulence length longitudinal[m])
Lw=18;                     %(Turbulence length vertical[m])
Iu=0.11;                   %(Turbulence intensity longitudinal)
Iw=0.06;                   %(Turbulence intensity vertical)
v50=36.62;                 %(Mean wind velocity[m/s])
rho=1.25;                  %(Density of air[kg/m^3])
sigmau=Iu*v50;             %(Standard deviation longitudinal[m/s])
sigmaw=Iw*v50;             %(Standard deviation vertical[m/s])
CL=0.9;                    %(Lift coefficient)
dCL=1;                     %(Derivative lift coefficient)
N=10000;                   %(Number of frequency steps)
B=32.5;                    %(width of the bridge deck[m])
Cy=7;                      %(Factor for coherence in y-direction)

%Matrix with mutual distance between nodes
ypoint=8.8:8.8:228.8;
yall=ypoint'*ones(1,26);
ydif=zeros(26,26);
for m=1:26
    for n=m+1:26
        ydif(m,n)=abs(yall(m)-yall(n));
        ydif(n,m)=ydif(m,n);
    end
end

%Assembly of the response spectra of the midnode for the displacement and
%acceleration. With a full force matrix with force spectra for all the
%different nodes and cross spectra.
for n=1:1:10000;
    omega(n)=n/100;
    Suf=K-M*omega(n)^2+i*omega(n)*Cs;
    Hu=Suf^-1;
    Hx=abs(Hu);
Suu(n)=(4*omega(n)*Lu/(v50*pi*2))/(1+70.4*(omega(n)*Lu/(v50*pi*2))^2)^(5/6)
*sigmau^2/(omega(n)/(2*pi));
Sww(n)=(2.15*omega(n)*Lw/(v50*pi*2))/(1+11.16*(omega(n)*Lw/(v50*pi*2))^2)^(
5/3)*sigmaw^2/(omega(n)/(2*pi));
    for j=1:26;
        for k=1:26;
            Hx13F(j)=Hx(j,13);
            Ha13F(j)=Hx(j,13)*omega(n)^2;
            Hx13FT(k)=Hx(13,k);
            Ha13FT(k)=Hx(13,k)*omega(n)^2;
            HxF(j,k)=Hx13F(j)'*conj(Hx13FT(k));
            HaF(j,k)=Ha13F(j)'*conj(Ha13FT(k));
            cohy(j,k)=exp(-1*Cy*omega(n)/(2*pi*v50)*ydif(j,k));
            SFF(j,k)=Lpart^2*rho^2*v50^2*B^2*(CL^2*Suu(n)+1/4*dCL^2*Sww(n))
            * cohy(j,k);
            Szz(j,k)=HxF(j,k)*SFF(j,k);
            Szz13(n)=sum(sum(Szz,1),2);
```

```
            Saa(j,k)=HaF(j,k)*SFF(j,k);
            Saa13(n)=sum(sum(Saa,1),2);
        end
    end
end

for n=1:1:10000;
    omega(n)=n/100;
    SufHSC=KHSC-MHSC*omega(n)^2+i*omega(n)*CsHSC;
    HuHSC=SufHSC^-1;
    HxHSC=abs(HuHSC);
Suu(n)=(4*omega(n)*Lu/(v50*pi*2))/(1+70.4*(omega(n)*Lu/(v50*pi*2))^2)^(5/6)
*sigmau^2/(omega(n)/(2*pi));
Sww(n)=(2.15*omega(n)*Lw/(v50*pi*2))/(1+11.16*(omega(n)*Lw/(v50*pi*2))^2)^(
5/3)*sigmaw^2/(omega(n)/(2*pi));
    for j=1:26;
        for k=1:26;
            Hx13FHSC(j)=HxHSC(j,13);
            Ha13FHSC(j)=HxHSC(j,13)*omega(n)^2;
            Hx13FTHSC(k)=HxHSC(13,k);
            Ha13FTHSC(k)=HxHSC(13,k)*omega(n)^2;
            HxFHSC(j,k)=Hx13FHSC(j)'*conj(Hx13FTHSC(k));
            HaFHSC(j,k)=Ha13FHSC(j)'*conj(Ha13FTHSC(k));
            cohy(j,k)=exp(-1*Cy*omega(n)/(2*pi*v50)*ydif(j,k));
            SFFHSC(j,k)=Lpart^2*rho^2*v50^2*B^2*
            (CL^2*Suu(n)+1/4*dCL^2*Sww(n))* cohy(j,k);
            SzzHSC(j,k)=HxFHSC(j,k)*SFFHSC(j,k);
            Szz13HSC(n)=sum(sum(SzzHSC,1),2);
            SaaHSC(j,k)=HaFHSC(j,k)*SFFHSC(j,k);
            Saa13HSC(n)=sum(sum(SaaHSC,1),2);
        end
    end
end

figure(10);                   %(See also figure 5.7)
plot(omega,Szz13);

figure(11);                   %(See also figure 5.7)
plot(omega,Szz13HSC);

figure(12);                   %(See also figure 5.9)
plot(omega,Saa13);

figure(13);                   %(See also figure 5.9)
plot(omega,Saa13HSC);

%Determination of the peakvalues during a certain time period.
for n=1:10000;
    variance=sum(Szz13(1,:))*0.01;
    varianceHSC=sum(Szz13HSC(1,:))*0.01;
    standev=sqrt(variance);
    standevHSC=sqrt(varianceHSC);
    variancea=sum(Saa13(1,:))*0.01;
    varianceHSCa=sum(Saa13HSC(1,:))*0.01;
    standeva=sqrt(variancea);
    standevHSCa=sqrt(varianceHSCa);
end
```