Crack width in reinforced steel fibre concrete

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Graduation committee

Prof.dr.ir. J.C. Walraven
Dr.ir. C.B.M. Blom
Dr.Ir. P.C.J. Hoogenboom
Ir. L.J.M. Houben

Technische Universiteit Delft

Gemeente Rotterdam
Gemeentewerken

Preface

This report is the graduation thesis for obtaining my master's degree in Civil Engineering at Delft University of Technology. The project was carried out at the engineering office of Gemeentewerken Rotterdam (Public Works Rotterdam).

The following members took part in the graduation committee:

- Prof.dr.ir. J.C. Walraven — Delft University of Technology
- Dr.ir. C.B.M. Blom — Gemeentewerken Rotterdam / Delft University of Technology
- Dr.ir. P.C.J. Hoogenboom — Delft University of Technology
- Ir. L.J.M. Houben — Delft University of Technology

I would like to thank the engineering office of Gemeentewerken Rotterdam for giving me the opportunity to do the research within their company. The seven months were a very instructive period. My gratitude goes to my colleagues at the office for the pleasant ambience during my graduation time. I would also like to thank my graduation committee for their useful support and advices.

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Rik Cederhout
Rotterdam, February 2012

Samenvatting

In de huidige tijd waarin duurzaamheid een steeds grotere rol gaat spelen rondom de bouw van constructies, is scheurwijdte een belangrijk onderdeel in de duurzaamheid. De toevoeging van staalvezels aan het betonmengsel is niet onbekend en uit onderzoeken is gebleken dat de scheurwijdte verminderd kan worden wanneer er staalvezels worden toegevoegd. Staalvezels zijn al in diverse projecten aan het beton toegevoegd, echter ging dit voornamelijk om vloeren en onderwaterbeton. Voor bredere inzet van staalvezels zijn rekenregels nodig. De vraag van deze afstudeerscriptie is om de invloed van staalvezels op de scheurwijdte te onderzoeken.

Uit het onderzoek naar de rekenregels van de scheurwijdte in staalvezelbeton kwam naar voren dat de staalvezels invloed hebben op zowel de overdrachtslengte als op de staalspanning. De scheurwijdte-formule is op twee manieren afgeleid: op de semi-empirische manier en door middel van een differentiaal vergelijking. De gevonden formule is dezelfde formule als die gebruikt wordt in de Eurocode voor gewapend beton zonder staalvezels, maar in de formule is een extra variabele toegevoegd, welke de toevoeging van staalvezels meerekent.

De gevonden formule voor de scheurwijdte is gebaseerd op een axiale trekstaaf. In balken welke op buiging belast worden, kan een deel van de doorsnede rondom de wapening gezien worden als een trekstaaf. De afmetingen van deze trekstaaf zijn afhankelijk van de effectieve hoogte. Voor staalvezelbeton is dezelfde effectieve hoogte gevonden als voor gewapend beton.

Met de gevonden scheurwijdte-formule is de invloed van staalvezels bepaald op zowel de scheurwijdte als op de hoeveelheid wapening. Dit is gebeurd door middel van twee verschillende case studies. Een case studie is een fiets-/voetgangersbrug welke op buiging wordt belast en een case studie waarin de vloer van een parkeergarage wordt beschouwd. Deze vloer is belast op zowel opgelegde vervormingen als op moment.

Uit beide case studies blijkt dat de scheurwijdte significant verminderd kan worden. De vermindering van de scheurwijdte is afhankelijk van de nascheursterkte van het beton. Met een nascheursterkte van 2.00 N/mm² kan de scheurwijdte met een 75% verminderd worden als de constructie wordt belast door externe belastingen. Bij opgelegde vervormingen kan de scheurwijdte tot wel 90% worden verminderd bij een nascheursterkte van 2.00 N/mm². Deze nascheursterkte van 2.00 N/mm² is relatief aan de hoge kant. In de praktijk meestal wordt een hoeveelheid van 25-35 kg vezels per m³ gebruikt. Dit geeft een nascheursterkte van ongeveer 0.75-1.00 N/mm². Met deze nascheursterkte komt de vermindering van de scheurwijdte uit op ongeveer 30-55%.

Naast de scheurwijdte kan ook de hoeveelheid wapening verminderd worden, als staalvezels worden toegepast en er wordt gerekend met dezelfde scheurwijdte. De hoeveelheid wapening die verminderd kan worden, bij een nascheursterkte van 2.00 N/mm² is ongeveer 50% van de wapening voor gewapend beton. Bij opgelegde vervormingen is er een vermindering van de wapening tot wel 70% mogelijk, bij een nascheursterkte van 2.00 N/mm². Voor een hoeveelheid van 25-35 kg/m³ staalvezels kan een vermindering van de wapening worden gevonden van 20 tot 35%.

De nauwkeurigheid van de scheurwijdte controleberekeningen tussen de constante spanning over de trekzone en het nascheurgedrag met een afnemende nascheurspanning is gecontroleerd. Voor een afnemende nascheurspanning wordt de scheurwijdte in de meeste gevallen onderschat. Om deze onderschatting te compenseren of te verminderen wordt de constante spanning over de trekzone verminderd. Uit beide case studies blijkt dat een vermindering van 10% van de spanning een kleine en acceptabel aangenomen onderschatting van de scheurwijdte geeft.

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Summary of the study

Nowadays durability plays a bigger and bigger role in structures. The so-called crack width is a very important issue in the durability. The addition of steel fibres to the concrete matrix is not unheard of and tests concluded that the crack width can be reduced. Currently steel fibres are added to the concrete matrix in different projects, however these projects were mainly floors. For more general use of steel fibres, calculation rules are needed. The main question of this graduation thesis was to study the influence of steel fibres on the crack width.

The study of the crack width in steel fibre concrete shows that the steel fibres influence the transmission length and the steel stress. A crack width expression has been derived in two different ways: a semi-empirical method and by the use of differential equations. The expression which is found is the same expression as found in reinforced concrete without steel fibres. In this expression an extra variable has been added, the variable that contains the influence of the steel fibres.

The expression that has been found for the crack width is based on an axial tensile bar. In beams which are loaded in bending, a part of the cross section around the reinforcement can be seen as an axial tensile bar. The height of this tensile bar is the effective height. For steel fibre reinforced concrete the same effective height is found as for reinforced concrete.

By using the expression for the crack width, the influence of the steel fibres can be found of the crack width and the amount of reinforcement. Two different case studies have been carried out: one case study is a bicycle and pedestrian bridge that is loaded in bending. The second case is a case study where the floor of the parking garage is loaded in bending and imposed deformations.

From both cases it is concluded that the crack width will be significantly reduced by the use of steel fibres. This reduction is mostly dependent of the post-cracking strength of the concrete. With a post-cracking strength of 2.00 N/mm² the crack width can be reduced by 75% in the case of bending moments. In case of an imposed deformation the crack width can be reduced by 90% with a post-cracking strength of 2.00 N/mm².
In practical application for e.g. an amount of 25-35 kg fibres per m³ is used. This results in a post-cracking strength of 0.75-1.00 N/mm². By the use of this post-cracking strength, the reduction of the crack width is in the range of 30-55%.

Not only the crack width can be reduced, but also the reinforcement can be reduced if steel fibres are added and the calculation is made for the same crack width. The amount of reinforcement which can be reduced is at a post-cracking strength of 2.00 N/mm², about 50% of the reinforcement. With an imposed deformation a reduction of 70% is possible by a post-cracking strength of 2.00 N/mm³. The amount of steel fibres in the range of 25-35 kg/m³ can reduce the reinforcement by 20-35%.

The accuracy of the calculations of the maximum crack width is checked between a constant stress over the tensile zone and the post-cracking behaviour with a reducing post-cracking stress. For a reducing post-cracking stress, the maximum crack width is underestimated in most cases. To compensate or reduce this underestimation, the constant stress over the tensile zone must be reduced. From both cases a reduction of 10% results in a small underestimation of the maximum crack width, which is assumed to be acceptable.

Notations

Latin lower case

b	=	width of the cross section
c	=	cover of the reinforcement
d_s	=	distance to the reinforcement
f_{ctm}	=	tensile strength of the concrete
$f_{ctm,eq,300}$	=	post-cracking stress at a deflection of 1.5 mm
f_{yd}	=	yield stress of the reinforcement
h_{eff}	=	effective height
l_t	=	Transmission length
m	=	is the number of reinforcing bars
q	=	uniform distributed load
s_{rm}	=	Crack spacing, the space between two cracks
w	=	Crack width
w_{max}	=	Maximum crack width
x	=	height of the compressive zone
x_u	=	compressive height in the ultimate limit state

Latin upper case

A_c	=	Concrete area
$A_{c,eff}$	=	Effective concrete area
A_s	=	Area of the reinforcement
$A_{s,max}$	=	Area of the maximum reinforcement
$A_{s,min}$	=	Area of the minimum reinforcement
E_c	=	Young's modulus of the concrete
E_s	=	Young's modulus of the reinforcement
M	=	bending moment
M_{cr}	=	Cracking moment
M_{ed}	=	Design bending moment
M_{rd}	=	Maximum resistance moment
M_u	=	Moment in the ultimate limit state
N_c	=	Normal force in the concrete
N_{cr}	=	Cracking force of the concrete
N_s	=	Normal force in the reinforcement
W	=	Section modulus

Greek lower case

α_e	=	Ratio between Young's moduli of reinforcement steel and concrete
γ_c	=	Material factor concrete
γ_s	=	Material factor reinforcing steel
ε_c	=	Strain in the concrete
ε_{cm}	=	Mean strain in the concrete
ε_{fdc}	=	Strain of the beginning of the fully developed cracking stage
ε_s	=	Strain in the reinforcement
ε_{sm}	=	Mean strain in the reinforcement
ρ	=	Reinforcement ratio
σ_{ct}	=	concrete tensile stress
σ_f	=	Stress of the steel fibres in the tension zone

σ_{f1}	=	Stress of the steel fibres directly after cracking
σ_{f2}	=	Stress of the steel fibres at the ultimate limit state
σ_s	=	steel stress
σ_{se}	=	Steel stress in the concrete outside disturbed area
σ_{sr}	=	steel stress in the crack
τ_{bm}	=	Bond stress
Φ	=	Reinforcement bar diameter
χf_{ctm}	=	Stress in the steel fibres expressed as a part of the concrete tensile strength
ω_{max}	=	Maximum reinforcement ratio

Table of contents

1. Introduction

The tensile strength of concrete is about ten times smaller than its compressive strength. Therefore, the principal idea of designing in concrete is to make the concrete resist the compressive forces. The part of the concrete which is loaded in tension cracks. A common solution to improve the cracking behaviour of concrete is the addition of reinforcing steel. The reinforcing bars are situated at the surroundings of the concrete cracks. If the concrete is cracked, the concrete fails to transmit tensile stresses. The reinforcing steel transmits the tensile force through the crack.
Steel fibre concrete is a concrete mixture to which steel fibres are added. It contains fibres that are distributed randomly through the cross section. As the fibres are distributed through the entire cross section, this is seen as a disadvantage of steel fibre concrete. Especially in comparison to steel reinforcement bars which are situated at the location where needed. By 'strengthen' the concrete with steel fibres the concrete can resist tensile forces after cracking. This means that a part of the tensile force will be resisted by the concrete with steel fibres and the reinforcement bars carry less tensile force. This results in a reduction of the steel stress in the reinforcing bars and resulting in a smaller crack width.
A limited maximum crack width is very important for the durability of concrete, to prevent substances and chemicals to penetrate the concrete and affect it from the inside. In order to reduce the crack width, steel fibres will be added to the concrete.

1.1 Problem description

The addition of steel fibres in concrete is used in some practical cases. These projects are mainly industrial floors. For wider application of steel fibre concrete, calculation rules are needed. These are currently not available in the standards or codes.

In general, the addition of steel fibres gives a smaller crack width in reinforced concrete. To what extent the crack width decreases is still unknown. In current practise a quantity of 30 kg/m³ steel fibres is added to the concrete. The origin of this amount is empirical. Also the amount of reinforcement in steel fibre reinforced concrete is the same as that of reinforced concrete, while this could be reduced.

The starting point is to explore the steel fibre reinforced concrete and analyse the influence of steel fibres on the crack width and on the amount of required steel reinforcement is, if steel fibres are applied. The available academic knowledge of steel fibre concrete is converted into practical knowledge. This has be done by the calculation of two practical case studies.

The effective height in reinforced concrete is an important factor in the calculation of the crack width. The effective area is the area around the concrete where the crack width is controlled by the reinforcement. The behaviour of this effective area in steel fibre concrete is still not known.

1.2 Problem definition

This thesis has been focussing on the following three problem definitions.

- What is the influence of steel fibres on the crack width when combined with steel reinforcement?
- What is the amount of steel reinforcement which can be reduced if steel fibres are applied?
- What effective height can be taken into account when steel fibres are applied?

1.3 Objective

The objective is to study the cracking behaviour of steel fibre concrete and to put the available academic knowledge into practical knowledge. To focus will be on the influence of steel fibres on the crack width in combination with steel reinforcement and also on the required amount of steel reinforcement when steel fibres will used.

1.4 Work approach

The graduation thesis starts with a literature survey. After this literature study a method is formulated to calculate the maximum crack width. This method is based on an empirical derivation and by use of differential equations. The crack width models for reinforced concrete, steel fibre concrete and ultra high strength fibre concrete will be compared.
To illustrate the influence of the addition of steel fibres two case studies are carried on. The first case study is a small bridge. The bridge is loaded in bending. The second case study is a floor of a parking garage. The floor will be loaded by imposed deformations and external forces. These two case studies should give a good insight in the influence of the addition of steel fibres. The study is finished by a conclusion. This conclusion deal with the influence of steel fibres on the maximum crack width and the influence of steel fibres on the reduction of the amount of reinforcement bars.

2. Steel fibre concrete

In this chapter the behaviour of steel fibre concrete will be explained. The general behaviour is explained in 2.1 and the calculation models of steel fibre concrete is explained in 2.2.

2.1 Behaviour

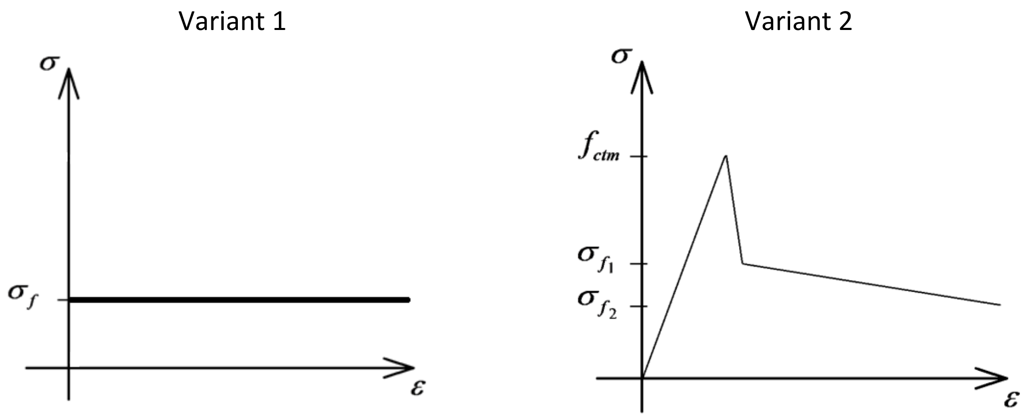
To get a better insight in the behaviour of the addition of steel fibres in concrete during cracking, the cracking behaviour of concrete should be understood. The first phase of concrete in tension is the linear elastic phase. If the tensile load of the concrete is increasing, microcracks occur near the fillers. With continuous loading these microcracks grow, until a macro-crack will occur. In plain concrete the tensile stress drops to zero in the crack, but in steel fibre concrete the tensile stress drops (or in some cases the tensile stress can increase after cracking) and the cracked cross section can transmit tensile forces by the use of steel fibres: the post-cracking strength.

The post-cracking strength has been determined by test-methods. The test-methods are presented in appendix 1.

2.2 Model

The post-cracking strength of steel fibre concrete is highly dependent on the steel fibres, in particular the amount of fibres that is used and the shape of the fibres. For the rest influences the concrete itself the behaviour of the post-cracking strength. This paragraph shows models for the calculation of steel fibre concrete.

The goals of this research are to show the effects of adding steel fibres. Therefore a performance-based stress-strain diagram is used. This means that the design is in line with a prescribed σ-ε diagram in the case of tension softening. For this performance-based stress-strain diagram two different variants are used: a constant stress over the total tensile zone (variant 1) and a more realistic post-cracking behaviour (variant 2).

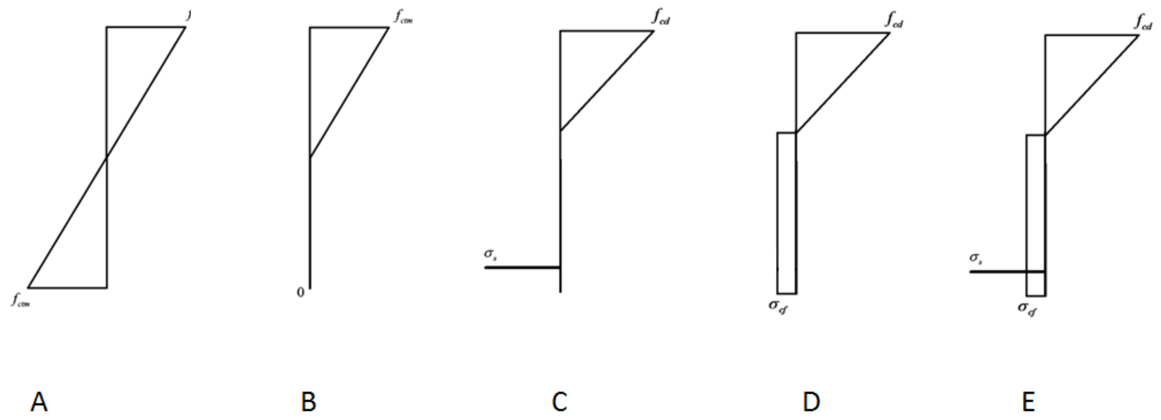


2-1 Stress-strain diagram performance-based principle

The parameters in this figure are:

f_{ctm}	=	Tensile strength of the concrete
σ_f	=	Stress of the steel fibres in the tension zone
σ_{f1}	=	Stress of the steel fibres directly after cracking
σ_{f2}	=	Stress of the steel fibres at the ultimate limit state

Figure 2-1 shows the variants of the performance-based stress-strain diagram of the tensile zone of steel fibre concrete. Variant 1 is a more simplified model of variant 2. This simplified model results in more easy calculation of steel fibre concrete. In this report is calculated with the post-cracking behaviour of variant 1. In chapter 8 the results of variant 1 will be compared with variant 2.



2-2 Stresses in the cross section, A: Uncracked concrete; B:) Plain concrete; C:) Reinforced concrete; D:) Steel fibre concrete; E:) Steel fibre reinforced concrete

Figure 2-2 shows the stresses in the cross section in several situations. In situation A the concrete is uncracked. The cross section consists in the compressive and the tensile zone the same stresses. In situation B the concrete is cracked and in the tensile zone the stresses are dropped to zero. In this situation the structure fails. Section C shows the behaviour of a concrete structure where reinforcing steel is added. The concrete cracks in the tensile zone, but in this case the reinforcing steel transmits the forces in the tensile zone. In this figure a constant stress in the fibres is assumed (variant 1) In situation D steel fibres are added to the concrete. The concrete is cracked in the tensile zone and the steel fibres consists the tensile force of the cross section. Situation E is the combination between situation C and D. Combining of both fibres and reinforcement bars results in a lower force in the reinforcement. This gives a more effective crack width control.

3. Crack width calculation model

In this chapter the calculation models for the crack width are analyzed. In 3.1 and 3.2 the models of reinforced concrete and reinforced steel fibre concrete are analyzed. Both models and the model for ultra high strength fibre concrete are compared in 3.3. The comparison with existing models of steel fibre concrete can be found in section 3.4. All these models are derived for an axial tensile bar. For the calculation of beams the effective height is needed. This can be found in 3.5.

3.1 Crack width reinforced concrete (axial bar)

The basic tool to calculate the crack width in a concrete structure is by using an axial concrete member subjected to axial tension. In this derivative a constant bond stress between the reinforcing steel and concrete is assumed with a value of $2f_{ctm}$ [6]. This assumption is based on analysis of test results.

3.1.1 Uncracked stage

In the phase before the first crack is assumed, the imposed strain increase gradually. In the uncracked stage, the strain in the reinforcing steel and in the concrete is the same. The contributions of the reinforcing steel and the concrete carrying an external force N_{tot} can be calculated with:

$$N_s = E_s A_s \varepsilon_s$$
$$N_c = E_c A_c \varepsilon_c$$
$$\varepsilon_s = \varepsilon_c = \varepsilon$$

Where:

N_s	=	Force in the reinforcement
N_c	=	Force in the concrete
E_s	=	Young's modulus of the reinforcement
E_c	=	Young's modulus of the concrete
ε_s	=	Strain in the reinforcement
ε_c	=	Strain in the concrete

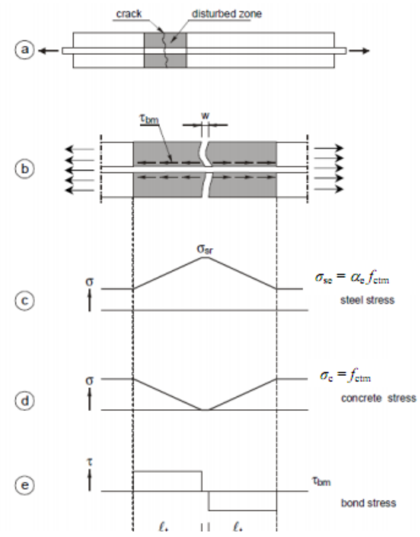
The total force is the summation of the force carried by the concrete and the reinforcing steel:

$$N_{tot} = N_s + N_c = E_s A_s \varepsilon + E_c A_c \varepsilon$$
$$N_{tot} = E_c A_c \varepsilon (1 + \rho \cdot \alpha_e)$$

Where the ratio between the Young's moduli of steel and concrete and the reinforcement ratio are:

$$\alpha_e = \frac{E_s}{E_c} \qquad \rho = \frac{A_s}{A_c}$$

By increasing the total external force, the strain in the reinforcing steel and concrete also increase. The process can be continued until the stress in the concrete reach the concrete tensile strength (expressed as $E_c \varepsilon = f_{ctm}$). If the external force is increasing more, the first crack will occur at the place with the lowest tensile strength anywhere in the bar.



3-1 Tensile axial bar model
a.) Reinforced concrete with crack b.) Stresses in transmission area c.) Steel stress d.) Concrete tensile stress e.) bond stress

In figure 3-1 the stresses in the transmission area are shown. In this transmission area the concrete will carry more of the tensile force by increasing distance of the crack, because of the bond between the reinforcing steel and the concrete.

3.1.2 Crack formation stage

At the place of the crack the stress in the concrete will reduce to zero and the stress in the reinforcing steel increase. The reinforcing steel is carrying the tensile force.

$$\sigma_{ct} = 0$$

$$\sigma_s = \sigma_{sr}$$

Where:

σ_{ct}	=	concrete tensile stress
σ_{sr}	=	steel stress in the crack
σ_s	=	steel stress

All the forces in the crack must be transmitted by the reinforcing steel. The stress in the reinforcing steel is the cracking force divided by the area of the reinforcing steel.

$$\sigma_{sr} = \frac{N_{cr}}{A_s} = \frac{E_s A_s \varepsilon + E_c A_c \varepsilon}{A_s} = \frac{E_c A_c \varepsilon}{A_s}\left(1+\rho\cdot\alpha_e\right) =>$$

$$\sigma_{sr} = \frac{E_c \varepsilon}{\rho}\left(1+\rho\cdot\alpha_e\right) = \frac{f_{ctm}}{\rho}\left(1+\rho\cdot\alpha_e\right)$$

Where:

N_{cr}	=	Cracking force of the concrete

The steel stress in the undisturbed area is directly proportional of the concrete tensile strength.

$$\varepsilon_s = \varepsilon_c => \frac{\sigma_{se}}{E_s} = \frac{f_{ctm}}{E_c} => \sigma_{se} = \frac{f_{ctm}E_s}{E_c} = \sigma_{se} = \alpha_e f_{ctm}$$

Where:

σ_{se}	=	Steel stress in the concrete outside disturbed area

Because of the bond between the reinforcing steel and the concrete, the concrete will carry more of the tensile force by increasing distance of the crack. This process is the case during the transmission length. At a certain distance (transmission length) the concrete is carrying the original part of the tensile force before cracking. Outside the transmission length, the strains of the reinforcing steel and the concrete are the same.

The force that must transmit over the transmission length:

$$N = A_c f_{ctm}$$

The force that is transmitted by the bond stresses over a distance l_t.

$$N = \tau_{bm} \cdot l_t \cdot m \cdot \pi \cdot \phi$$

Where:

A_c	=	Concrete area
τ_{bm}	=	Bond stress
l_t	=	Transmission length
Φ	=	Reinforcement bar diameter
m	=	is the number of reinforcing bars

With both expressions, the transmission length can be calculated:

$$\tau_{bm} \cdot l_t \cdot m \cdot \pi \cdot \phi = A_c f_{ctm}$$

$$\frac{\tau_{bm} \cdot l_t \cdot m \cdot \pi \cdot \phi}{A_s} = \frac{A_c f_{ctm}}{A_s} = \frac{f_{ctm}}{\rho}$$

With:

$$A_s = \frac{1}{4} \cdot m \cdot \pi \cdot \phi^2$$

This results in:

$$\frac{\tau_{bm} \cdot l_t \cdot m \cdot \pi \cdot \phi}{\frac{1}{4} \cdot m \cdot \pi \cdot \phi^2} = \frac{f_{ctm}}{\rho}$$

Rewriting this equation results in a transmission length of:

$$l_t = \frac{1}{4} \frac{f_{ctm}}{\tau_{bm}} \frac{\phi}{\rho}$$

The crack width is the difference of elongation between the reinforcing steel and the concrete over the disturbed zone. The disturbed zone has a maximum length of two times the transmission length.

$$w = 2l_t \left(\varepsilon_{sm} - \varepsilon_{cm} \right)$$

Where ε_{sm} is the mean reinforcing steel strain during the transmission length and ε_{cm} the mean concrete strain:

Mean reinforcing steel strain:

$$\varepsilon_{sm} = \frac{1}{2E_s} \left(\sigma_{sr} + \sigma_{se} \right)$$

Mean concrete strain:

$$\varepsilon_{cm} = \frac{\frac{1}{2} f_{ctm}}{E_c} = \frac{1}{2E_s} f_{ctm} \alpha_e = \frac{1}{2E_s} \sigma_{se}$$

The difference between both strains is:

$$\left(\varepsilon_{sm} - \varepsilon_{cm} \right) = \frac{1}{2E_s} \left(\sigma_{sr} + \sigma_{se} \right) - \frac{1}{2E_s} \sigma_{se} = \frac{1}{2E_s} \sigma_{sr}$$

This gives a maximum crack width of:

$$w = 2l_t \left(\varepsilon_{sm} - \varepsilon_{cm} \right) = 2 \frac{1}{4} \frac{f_{ctm}}{\tau_{bm}} \frac{\phi}{\rho} \frac{1}{2E_s} \sigma_{sr} = \frac{1}{4} \frac{f_{ctm}}{\tau_{bm}} \frac{\phi}{\rho} \frac{1}{E_s} \sigma_{sr}$$

3.1.3 Crack stabilized stage

By increasing the strain more and more, the number of cracks increases accordingly. The cracking process continues until the tensile bar consists of "disturbed regions" only. A further increase of the strain and as a result also an increase of the force N, the steel stress in the crack σ_s exceeds σ_{sr}. Because the force transmitted from the reinforcing steel to the concrete cannot increase more, the concrete strain between the cracks does not increase anymore. As a result, the increase of the crack width follows from the additional elongation of the reinforcement steel only. The maximum crack width can be found with the largest possible transmission length. Two times the transmission length is chosen for the calculation of the maximum crack width.

The elongation of the reinforcing steel is the difference in steel stress between the crack stabilized stage and the crack formation stage.

$$\Delta\varepsilon_s = \frac{\Delta\sigma_s}{E_s} = \frac{(\sigma_s - \sigma_{sr})}{E_s}$$

The increase in crack width is the elongation of the reinforcing steel over the distance of $2l_t$. The increase in crack width is:

$$\Delta w = \frac{2l_t(\sigma_s - \sigma_{sr})}{E_s} = \frac{1}{2}\frac{f_{ctm}}{\tau_{bm}}\frac{\phi}{\rho}\frac{1}{E_s}(\sigma_s - \sigma_{sr})$$

The maximum crack width is the summation of the crack width in the crack formation stage and the elongation of the reinforcing steel in the crack stabilized stage:

$$w_{\max} = w + \Delta w = \frac{1}{4}\frac{f_{ctm}}{\tau_{bm}}\frac{\phi}{\rho}\frac{1}{E_s}\sigma_{sr} + \frac{1}{2}\frac{f_{ctm}}{\tau_{bm}}\frac{\phi}{\rho}\frac{1}{E_s}(\sigma_s - \sigma_{sr}) = \frac{1}{2}\frac{f_{ctm}}{\tau_{bm}}\frac{\phi}{\rho}\frac{1}{E_s}\left(\sigma_s - \frac{1}{2}\sigma_{sr}\right)$$

This maximum crack width is calculated with constant bond stress. A better assumption is believed to be calculating with parabolic bond stresses. The result of this calculation is:

$$w_{\max} = \frac{1}{2}\frac{f_{ctm}}{\tau_{bm}}\frac{\phi}{\rho}\frac{1}{E_s}(\sigma_s - 0.6\sigma_{sr})$$

3.1.4 Long term effects

During the service life of a concrete structure, shrinkage of the concrete will occur. Furthermore, structures or parts of structure can be subjected to long term constant loads. The effects of long term constant loads and shrinkage are different for the crack formation stage and the stabilized cracking stage.

3.1.4.1 Long term effects and shrinkage in the crack formation stage

The effect of shrinkage in the crack formation stage is different from the stabilized cracking stage. When in the crack formation stage shrinkage occurs and the external imposed strain is kept constant, the external force increases. But the external force cannot exceed the cracking force and new additional cracks will develop. The additional development of cracks results that the crack width in the crack formation stage will not increase. The expression for the maximum crack width will be the same as for the expression of short term loading.

In short term loading a bond stress of $\tau_{bm} = 2 f_{ctm}$ between concrete and reinforcing steel is a good assumption. For long term constant loading, the bond stress decreases. Tests give that a bond stresses of $\tau_{bm} = 1.6 f_{ctm}$ is realistic [6][16]. Because this lower bond stress, the transmission length increases with 25%.

The maximum crack width in the crack formation stage is:

$$w_{\max} = \frac{1}{2}\frac{f_{ctm}}{\tau_{bm}}\frac{\phi}{\rho}\frac{1}{E_s}\left(\sigma_s - \frac{1}{2}\sigma_{sr}\right) = \frac{1}{2}\frac{f_{ctm}}{1.6 f_{ctm}}\frac{\phi}{\rho}\frac{1}{E_s}\left(\sigma_s - \frac{1}{2}\sigma_{sr}\right)$$

3.1.4.2 Long term effects and shrinkage in the stabilized cracking stage

In the stabilized cracking stage shrinkage influences the cracking width, because no new cracks can be developed. Shrinkage of concrete results in widening of existing cracks.
The concrete tends to shrink, but this is counteracted by the steel. If the concrete would be able to shrink freely, the concrete strain will be ε_{cs}. Because the concrete cannot shrink freely and the steel counteracted the strain, the steel stress will increase $\Delta\sigma_s = \varepsilon_{cs} E_s$.

The maximum crack width in the stabilized cracking stage as result of shrinkage is:

$$w_{\max} = \frac{1}{2}\frac{f_{ctm}}{\tau_{bm}}\frac{\phi}{\rho}\frac{1}{E_s}\left(\sigma_s - \frac{1}{2}\sigma_{sr} + \varepsilon_{cs} E_s\right)$$

In most of the cases, the concrete cracks under an applied short term load. This implies that a bond stress of $\tau_{bm} = 2 f_{ctm}$ is a good assumption for the crack spacing. The long term constant load reduced the bond stress between concrete and reinforcing steel. A reduction of the bond stress of about 40% can be assumed and the coefficient of 0.5 can be replaced for 0.3.

The maximum crack width in the stabilized cracking stage as result of long term constant loading and shrinkage is:

$$w_{\max} = \frac{1}{2}\frac{f_{ctm}}{\tau_{bm}}\frac{\phi}{\rho}\frac{1}{E_s}\left(\sigma_s - 0.3\sigma_{sr} + \varepsilon_{cs} E_s\right)$$

The more general expression for reinforced concrete is:

$$w_{\max} = \frac{1}{2}\frac{f_{ctm}}{\tau_{bm}}\frac{\phi}{\rho}\frac{1}{E_s}\left(\sigma_s - \alpha\sigma_{sr} + \beta\varepsilon_{cs} E_s\right)$$

Where:

$\alpha =$	0.3 or 0.5	dependent of short term or long term loading and the cracking stage
$\beta =$	0 or 1	dependent of shrinkage must taken into account

3.2 Crack width steel fibre reinforced concrete (axial bar)

In this calculation, it is assumed that a small volume amount of steel fibres will be added to the concrete, the called tension softening. The fibres transmit a constant stress over the total tensile zone. This is described in section 2.2.

3.2.1 Uncracked stage

The effect of steel fibres on the uncracked situation is very small. The effect of the fibres on the tensile strength is neglected. The uncracked situation of steel fibre reinforced concrete is the same as for ordinary reinforced concrete. The maximum carrying load is:

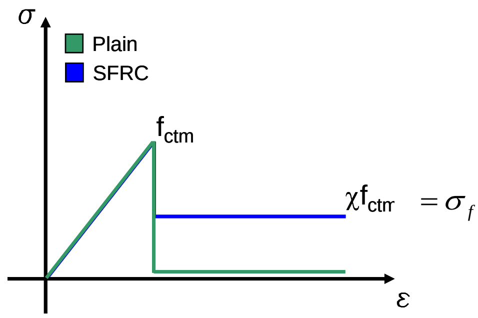
$$N_{tot} = N_s + N_c = E_s A_s \varepsilon + E_c A_c \varepsilon$$

$$N_{tot} = E_c A_c \varepsilon \left(1 + \rho \cdot \alpha_e\right)$$

3.2.2 Crack formation stage

In the crack formation stage there are some differences between reinforced concrete and steel fibre reinforced concrete. The most important difference is that the concrete is still carrying a part of the original tensile load. The steel fibres in the concrete will carry this load.

After cracking, a constant residual tensile strength of steel fibre reinforced concrete is obtained, this can be found in figure 3-2. This residual tensile strength is a constant prescribed stress. Here, no specific information is needed about the fibres.



3-2 The post-cracking behavior of concrete with and without steel fibers

The stresses in the crack can be calculated with the expression below. The concrete stress reduces not to zero, but will still have a factor of the concrete tensile strength. With a higher concrete stress at the crack, a lower steel stress is obtained.

$$\sigma_c = 0$$

$$\sigma_f = \chi f_{ctm}$$

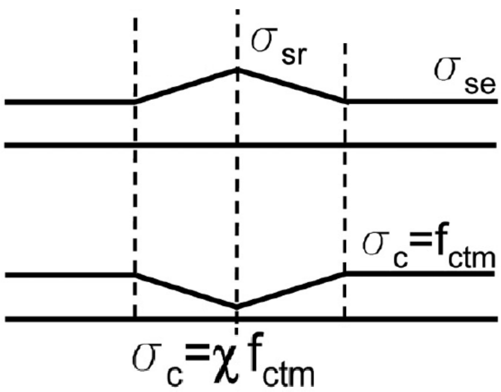
$$\sigma_s = \sigma_{sr}$$

When the concrete cracks, the tensile force must be transmitted from the concrete and reinforcement in the uncracked situation to the reinforcement and steel fibres in the cracked situation. The stress in the reinforcement is:

$$\sigma_{sr} = \frac{N_{cr}}{A_s} - \frac{\sigma_f A_c}{A_s} = \frac{N_{cr}}{A_s} - \frac{\sigma_f}{\rho}$$

The steel stress in the undisturbed area is:

$$\sigma_{se} = \alpha_e f_{ctm}$$



3-3 The course of reinforcing steel and concrete stresses at both sides of a crack during the transmission length

To divide the transmission length, the force that must be transmitted over the transmission length and the force that can be transmitted by the reinforcement must be known. The force that must be transmitted over the transmission length:

$$N = A_c(f_{ctm} - \sigma_f)$$

The force that the concrete can transmit is:

$$N = \tau_{bm} \cdot l_t \cdot m \cdot \pi \cdot \phi$$

The transmission length results from the two equations above:

$$\tau_{bm} \cdot l_t \cdot m \cdot \pi \cdot \phi = A_c(f_{ctm} - \sigma_f)$$

$$\frac{\tau_{bm} \cdot l_t \cdot m \cdot \pi \cdot \phi}{A_s} = \frac{A_c(f_{ctm} - \sigma_f)}{A_s} = \frac{(f_{ctm} - \sigma_f)}{\rho} \Rightarrow \frac{\tau_{bm} \cdot l_t \cdot m \cdot \pi \cdot \phi}{\frac{1}{4} \cdot m \cdot \pi \cdot \phi^2} = \frac{(f_{ctm} - \sigma_f)}{\rho}$$

With:

$$A_s = \frac{1}{4} \cdot m \cdot \pi \cdot \phi^2$$

This results in a transmission length of:

$$l_t = \frac{1}{4} \frac{(f_{ctm} - \sigma_f)}{\tau_{bm}} \frac{\phi}{\rho}$$

In comparison with the transmission length of ordinary reinforced concrete, one difference can be observed in the calculation of the transmission length. The post-cracking stress gives a reduction of the transmission length. A shorter transmission length is logical, because the concrete still have a part of the strength and less force has been be transmitted by the transmission length.

The crack width can be calculated with:

$$w = 2l_t(\varepsilon_{sm} - \varepsilon_{cm})$$

With a mean reinforcing steel stress is:

$$\varepsilon_{sm} = \frac{1}{2E_s}(\sigma_{sr} + \sigma_{se})$$

And the mean concrete stress:

$$\varepsilon_{cm} = \frac{\frac{1}{2} f_{ctm}}{E_c} = \frac{1}{2E_s} f_{ctm}\alpha_e = \frac{1}{2E_s}\sigma_{se}$$

The difference in elongation between the reinforcing steel and the steel fibre concrete is:

$$(\varepsilon_{sm} - \varepsilon_{cm}) = \frac{1}{2E_s}(\sigma_{sr} + \sigma_{se}) - \frac{1}{2E_s}\sigma_{se} = \frac{\sigma_{sr}}{2E_s} + \frac{1}{2E_s}(\sigma_{se} - \sigma_{se}) = \frac{1}{2E_s}\sigma_{sr}$$

The crack width in the crack formation stage is:

$$w = 2l_t(\varepsilon_{sm} - \varepsilon_{cm}) = 2\frac{1}{4}\frac{(f_{ctm} - \sigma_f)}{\tau_{bm}}\frac{\phi}{\rho}\frac{1}{2E_s}\sigma_{sr} = \frac{1}{4}\frac{(f_{ctm} - \sigma_f)}{\tau_{bm}}\frac{\phi}{\rho}\frac{1}{E_s}\sigma_{sr}$$

3.2.3 Crack stabilized stage

In comparison with ordinary reinforced concrete, there is no difference in the calculation of the crack width in the crack stabilized stage.

The difference in crack width due to the elongation of the reinforcing steel is:

$$\Delta w = \frac{2l_t\left(\sigma_s - \sigma_{sr}\right)}{E_s} = \frac{1}{2}\frac{\left(f_{ctm} - \sigma_f\right)}{\tau_{bm}}\frac{\phi}{\rho}\frac{1}{E_s}\left(\sigma_s - \sigma_{sr}\right)$$

The maximum crack width is the summation of the crack width in the crack formation stage and the elongation of the reinforcing steel in the crack stabilized stage:

$$w_{\max} = w + \Delta w = \frac{1}{4}\frac{\left(f_{ctm} - \sigma_f\right)}{\tau_{bm}}\frac{\phi}{\rho}\frac{1}{E_s}\left(\sigma_{sr} - \chi\sigma_{se}\right) + \frac{1}{2}\frac{\left(f_{ctm} - \sigma_f\right)}{\tau_{bm}}\frac{\phi}{\rho}\frac{1}{E_s}\left(\sigma_s - \sigma_{sr}\right)$$

The maximum crack width in steel fibre reinforced concrete is:

$$w_{\max} = \frac{1}{2}\frac{\left(f_{ctm} - \sigma_f\right)}{\tau_{bm}}\frac{\phi}{\rho}\frac{1}{E_s}\left(\sigma_s - \frac{1}{2}\sigma_{sr}\right)$$

With:

$$\sigma_s = \frac{N - \sigma_f A_c}{A_s}$$ Steel stress

$$\sigma_{sr} = \frac{f_{ctm}}{\rho}(1 + n\rho) - \frac{\sigma_f}{\rho}$$ Steel stress direct after cracking

This maximum crack width in steel fibre reinforced concrete can be rewritten if the post-cracking stress is rewritten as a factor of the concrete tensile strength.

$$w_{\max} = \frac{1}{2}\frac{\left(f_{ctm} - \chi f_{ctm}\right)}{\tau_{bm}}\frac{\phi}{\rho}\frac{1}{E_s}\left(\sigma_s - \frac{1}{2}\sigma_{sr}\right) = \frac{1}{2}\frac{f_{ctm}}{\tau_{bm}}\frac{\phi}{\rho}\frac{(1-\chi)}{E_s}\left(\sigma_s - \frac{1}{2}\sigma_{sr}\right)$$

With:

$$\sigma_s = \frac{N - \chi f_{ctm} A_c}{A_s}$$ Steel stress

$$\sigma_{sr} = \frac{f_{ctm}}{\rho}(1 + n\rho) - \frac{\chi f_{ctm}}{\rho}$$ Steel stress direct after cracking

This expression for the maximum crack width is also found by the differential equation (see appendix 2).

The most important differences are:

- Transmission length: the post-cracking stress results in a reduction of the transmission length. This reduction is explainable, because less force has to be transmitted by bond between the concrete and the reinforcing steel.
- As a result of the contribution of the steel fibres, the stress in the reinforcing steel reduces. Less stress in the reinforcing steel σ_s results in a lower elongation of the reinforcing steel and the crack width will also reduce.
- The stress in the reinforcement during the crack formation stage reduces as a result of the contribution of fibres. The tensile force has been transmitted partly by the steel fibres.

3.2.4 Long term effects

The long term effects for ordinary reinforced concrete are described in 3.1.4. The behaviour of the long term effects is dependent of the bond strength between concrete and reinforcing steel in all cases. Therefore it is important to understand the behaviour of the bond strength in the case of the addition of steel fibres. Bigaj and van Vliet made an overview of many researches of the bond behaviour of steel fibre concrete.

3.2.4.1 Bigaj and Van Vliet

The behaviour of the bond stress must be evaluated over a small slip distance, because the long term effects must be known for the calculation of the crack width. The crack width is in a range of 0.1-0.5 mm, a small slip is representative for the bond behaviour of the behaviour of the bond stress at the long term behaviour.

From the literature overview made by Bigaj and Van Vliet [2]: "there is no agreement with respect of fibre content on bond stiffness (the pre-peak behaviour). Conclusions vary from confirming increase of bond strength with increasing fibre volume fraction to proving any correlation false".

No difference in bond stress between plain concrete and steel fibre concrete is found for the pre-peak behaviour. Within the range of the maximum crack width the bond behaviour is in the pre-peak stage. From this can be concluded that there is no difference in the long term behaviour. The same long term behaviour can be used as for ordinary reinforced concrete.

The more general expression for steel fibre reinforced concrete is:

$$w_{\max} = \frac{1}{2}\frac{\left(f_{ctm} - \sigma_f\right)}{\tau_{bm}}\frac{\phi}{\rho}\frac{1}{E_s}\left(\sigma_s - \alpha\sigma_{sr} + \beta\varepsilon_{cs}E_s\right)$$

Where:

$\alpha =$	0.3 or 0.5	dependent of short term or long term loading and the cracking stage
$\beta =$	0 or 1	dependent of shrinkage must taken into account

3.3 The crack width equations

Below the different methods are compared. In this comparison the derivation of the ultra high strength fibre concrete is included. This derivation is done by Leutbecher [14]. The most important similarities and differences are summed below

Ordinary Reinforced Concrete	**Steel fibre Reinforced Concrete**	**Ultra high strength fibre concrete [14]**
The cracking force of ordinary reinforced concrete: $N_{tot} = E_c A_c \varepsilon (1+n\rho)$ The stress in the reinforcement direct after the formation of a crack: $\sigma_{sr} = \frac{f_{ctm} A_c}{A_s}(1+n\rho)$	The cracking force of fibre reinforced concrete: $N_{tot} = E_c A_c \varepsilon (1+n\rho)$ The stress in the reinforcement direct after the formation of a crack: $\sigma_{sr} = \frac{N_{cr} - \sigma_f A_c}{A_s}$ $\sigma_{sr} = \frac{f_{ctm} A_c}{A_s}(1+n\rho) - \frac{\sigma_f A_c}{A_s}$	The cracking force of high strength concrete: $N_{tot} = E_c A_c \varepsilon (1+n\rho)$ The stress in the reinforcement direct after the formation of a crack: $\sigma_{sr} = \frac{f_{ctm}(1+n\rho) - \sigma_f}{\rho}$ $\sigma_{sr} = \frac{f_{ctm} A_c}{A_s}(1+n\rho) - \frac{\sigma_f A_c}{A_s}$
The cracking force is the same. The obvious difference is the reduction of the steel stress by the steel fibres.		
The force that must transmit over the transmission length: $N = A_c f_{ctm}$ Transmission length: $l_t = \frac{1}{4}\frac{f_{ctm}}{\tau_{bm}}\frac{\phi}{\rho}$	The force that must transmit over the transmission length: $N = A_c (1-\chi) f_{ctm} = A_c (f_{ctm} - \sigma_f)$ Transmission length: $l_t = \frac{1}{4}\frac{(f_{ctm} - \sigma_f)}{\tau_{bm}}\frac{\phi}{\rho}$	The force that must transmit over the transmission length: $N = A_c (f_{ctm} - \sigma_f)$ Transmission length: $l_t = \frac{1}{4}\frac{(f_{ctm} - \sigma_f)}{\tau_{bm}}\frac{\phi}{\rho}$
The steel fibres consume some of the tensile force. Less force is transmitted by the reinforcement and as a consequence the transmission length of the reinforcement is reduced. Therefore the crack width will be smaller.		

<table>
<tr>
<td>The difference in strain between concrete and reinforcement in the crack formation stage:
$(\varepsilon_{sm}-\varepsilon_{cm})=\frac{1}{2E_s}(\sigma_{sr}+\sigma_{se})-\frac{1}{2E_s}\sigma_{se}$ =>
$(\varepsilon_{sm}-\varepsilon_{cm})=\frac{1}{2E_s}\sigma_{sr}$

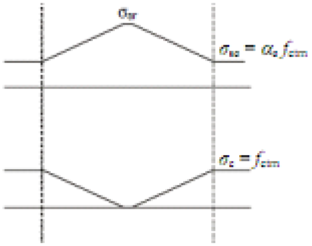
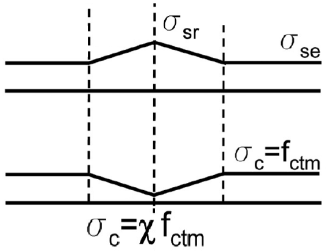



Fig. 3-4 Concrete and steel stress close to a crack in RC</td>
<td>The difference in strain between concrete and reinforcement in the crack formation stage:
$(\varepsilon_{sm}-\varepsilon_{cm})=\frac{1}{2E_s}(\sigma_{sr}+\sigma_{se})-\frac{1}{2E_s}\sigma_{se}$ =>
$(\varepsilon_{sm}-\varepsilon_{cm})=\frac{\sigma_{sr}}{2E_s}+\frac{1}{2E_s}(\sigma_{se}-\sigma_{se})=\frac{1}{2E_s}\sigma_{sr}$


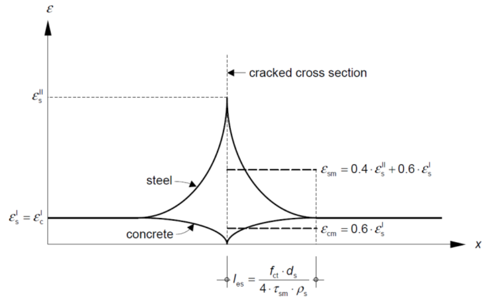
3-5 Concrete and steel stress close to a crack in SFRC</td>
<td>The difference in strain between concrete and reinforcement in the crack formation stage:
$(\varepsilon_{sm}-\varepsilon_{cm})=\frac{2}{5E_s}\sigma_{sr}+\frac{3}{5E_s}\sigma^{i}_{cf,cr}\alpha_e-\frac{3}{5E_s}\sigma^{i}_{cf,cr}\alpha_e$
$(\varepsilon_{sm}-\varepsilon_{cm})=\frac{2}{5E_s}\sigma_{sr}$


Fig. 3-6 Concrete and steel stress close to a crack in HSC</td>
</tr>
<tr>
<td colspan="3">There are no major differences. The most obvious difference is that the high fibre strength concrete is calculated with a parabolic stress progress and the other with a linear course. The ordinary reinforced concrete and the steel fibre reinforced concrete can also be calculated with a parabolic course.</td>
</tr>
<tr>
<td>The crack width in crack formation stage:
$w=2l_t(\varepsilon_{sm}-\varepsilon_{cm})=2\frac{1}{4}\frac{f_{ctm}}{\tau_{bm}}\frac{\phi}{\rho}\frac{1}{2E_s}\sigma_{sr}$
$w=\frac{1}{4}\frac{f_{ctm}}{\tau_{bm}}\frac{\phi}{\rho}\frac{1}{E_s}\sigma_{sr}$</td>
<td>The crack width in crack formation stage:
$w=2l_t(\varepsilon_{sm}-\varepsilon_{cm})=2\frac{1}{4}\frac{(f_{ctm}-\sigma_f)}{\tau_{bm}}\frac{\phi}{\rho}\frac{1}{2E_s}\sigma_{sr}$
$w=\frac{1}{4}\frac{f_{ctm}-\sigma_f}{\tau_{bm}}\frac{\phi}{\rho}\frac{1}{E_s}\sigma_{sr}$</td>
<td>The crack width in crack formation stage:
$w=2l_t(\varepsilon_{sm}-\varepsilon_{cm})=2\frac{1}{4}\frac{(f_{ctm}-\sigma_f)}{\tau}\frac{\phi}{\rho}\frac{2}{5E_s}\sigma_{sr}$
$w=\frac{1}{5}\frac{(f_{ctm}-\sigma_f)}{\tau}\frac{\phi}{\rho}\frac{1}{E_s}\sigma_{sr}$</td>
</tr>
<tr>
<td colspan="3">The crack width in the crack formation stage is the same for all of the three concretes. Only the transmission length differs. The other difference is the parabolic course of the high strength concrete, which differs from the others.</td>
</tr>
</table>

Maximum crack width in stabilized cracking stage: $$w_{\max} = \frac{1}{2}\frac{f_{ctm}}{\tau_{bm}}\frac{\phi}{\rho}\frac{1}{E_s}\left(\sigma_s - \frac{1}{2}\sigma_{sr}\right)$$	Maximum crack width in stabilized cracking stage: $$w_{\max} = \frac{1}{2}\frac{\left(f_{ctm} - \sigma_f\right)}{\tau_{bm}}\frac{\phi}{\rho}\frac{1}{E_s}\left(\sigma_s - \frac{1}{2}\sigma_{sr}\right)$$	Maximum crack width in stabilized cracking stage: $$w_{\max} = \frac{1}{2}\frac{(f_{ctm} - \sigma_f)}{\tau_{bm}}\frac{\phi}{\rho}\frac{1}{E_s}(\sigma_s - 0.6\sigma_{sr})$$
With: $$\sigma_s = \frac{N}{A_s}$$ $$\sigma_{sr} = \frac{f_{ctm}}{\rho}(1 + n\rho)$$	With: $$\sigma_s = \frac{N - \sigma_f A_c}{A_s}$$ $$\sigma_{sr} = \frac{f_{ctm}}{\rho}(1 + n\rho) - \frac{\sigma_f}{\rho}$$	With: $$\sigma_s = \frac{N}{A_s} - \frac{\sigma_f}{\rho}$$ $$\sigma_{sr} = \frac{f_{ctm}(1 + n\rho) - \sigma_f}{\rho}$$
The influence of the steel fibres can be found in the transmission length and the steel stress and the steel stress direct after cracking. This influence can be used by the addition of a factor for the stress in the steel fibres.		
Through recalculations the value of 0.5 for the steel stress direct after cracking have been modified in 0.6. [6] $$w_{\max} = \frac{1}{2}\frac{f_{ctm}}{\tau_{bm}}\frac{\phi}{\rho}\frac{1}{E_s}(\sigma_s - 0.6\sigma_{sr})$$		The maximum crack width is: $$w_{\max} = \frac{1}{2}\frac{(f_{ctm} - \sigma_f)}{\tau_{bm}}\frac{\phi}{\rho}\frac{1}{E_s}(\sigma_s - 0.6\sigma_{sr})$$

It is possible to formulate one expression for the crack width of concrete (with and without) fibres:

$$w_{\max} = \frac{1}{2} \frac{(f_{ctm} - \sigma_f)}{\tau_{bm}} \frac{\phi}{\rho} \frac{1}{E_s} (\sigma_s - \alpha \sigma_{sr})$$

With:

$$\sigma_s = \frac{N - \sigma_f A_c}{A_s}$$

$$\sigma_{sr} = \frac{f_{ctm}}{\rho}(1 + n\rho) - \frac{\sigma_f}{\rho}$$

Long term effects:
From the literature overview of tests of the bond behaviour between concrete en reinforcement, no difference in bond stress was found.
Therefore for the situation of steel fibre reinforced concrete:

$$w_{\max} = \frac{1}{2} \frac{\left(f_{ctm} - \sigma_f\right)}{\tau_{bm}} \frac{\phi}{\rho} \frac{1}{E_s} (\sigma_s - \alpha \sigma_{sr} + \beta \varepsilon_{cs} E_s)$$

Where:

$\alpha =$	0.3 or 0.5	dependent of short term or long term loading and the cracking stage
$\beta =$	0 or 1	dependent of shrinkage must taken into account

For the situation in ultra high strength fibre concrete the same results can be found. Only the long term loading in the crack formation stage is not discussed and the factor α is slightly higher. This difference is the more exact calculation of the stresses in the reinforcement and the concrete.

3.3.1 Conclusions of the crack width equations

From the comparison between the ordinary reinforced concrete tensile bar and the steel fibre reinforced concrete bars can be concluded, that despite the addition of steel fibres, one general expression can be found. The addition of steel fibres is an extra variable, which is added to the expression. This general expression is:

$$w_{\max} = \frac{1}{2}\frac{(f_{ctm} - \sigma_f)}{\tau_{bm}}\frac{\phi}{\rho}\frac{1}{E_s}(\sigma_s - \alpha\sigma_{sr})$$

With:

$$\sigma_s = \frac{N - \sigma_f A_c}{A_s}$$

$$\sigma_{sr} = \frac{f_{ctm}}{\rho}(1 + n\rho) - \frac{\sigma_f}{\rho}$$

In the first expression the steel fibres reduce the transmission length. The steel fibres are taken a part of the tensile force, less tensile force must be transmitted over the transmission length. In the second and third expressions the steel fibres consume a part of the tensile force. A reduces tensile force has to be resisted by the reinforcement. This results in a lower stress in the reinforcement.

In the case of an ordinary reinforced concrete bar, the stress consumed by the fibres is zero. The equation reduces to the original one (see section 3.1)

For the long term effects and shrinkage, no differences between reinforced concrete and reinforced steel fibre concrete were found. The maximum crack width under long term or shrinkage loading can be found:

$$w_{\max} = \frac{1}{2}\frac{\left(f_{ctm} - \sigma_f\right)}{\tau_{bm}}\frac{\phi}{\rho}\frac{1}{E_s}(\sigma_s - \alpha\sigma_{sr} + \beta\varepsilon_{cs}E_s)$$

Where:

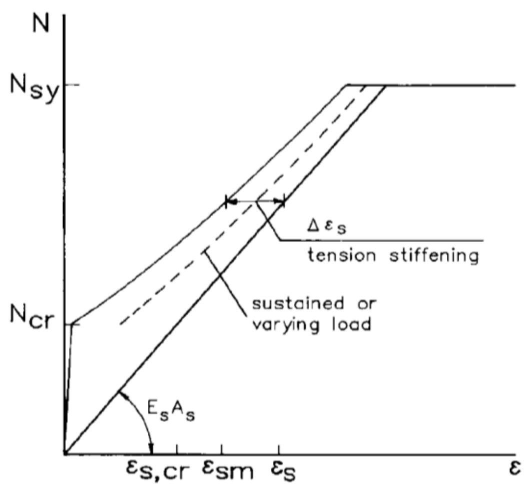
$\alpha =$	0.3 or 0.5	dependent of short term or long term loading and the cracking stage
$\beta =$	0 or 1	dependent of shrinkage must taken into account

3.4 Other models for the crack width of steel fibre concrete

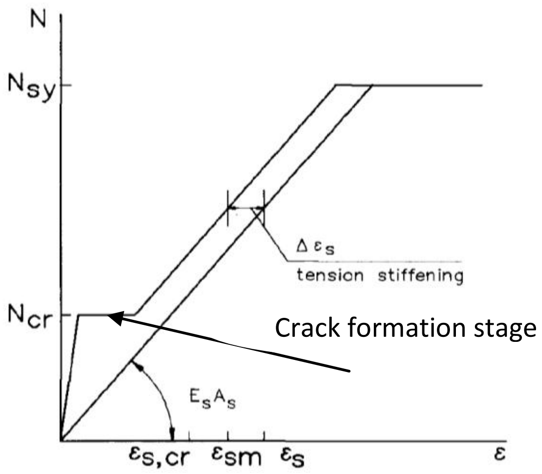
In section 3.1 and 3.2 semi-empirical relations are made for reinforced concrete and steel fibre reinforced concrete. Semi-empirical relations are relations based on experiments, where assumptions are made to simplify calculation. In this paragraph the model of the steel fibre reinforced concrete will be compared with the method of Vandewalle. The model of Vandewalle is based on the Rilem method. This method is based on the pre-version of the Eurocode. For the comparison a simple calculation is made between the semi-empirical relation as presented in 3.2 and the method Vandewalle.

3.4.1 Pre-version of the Eurocode and Eurocode

The pre-version of the Eurocode is based on the model of Leonhardt (figure 3-7), while the final version of the Eurocode is based on the model of Noakowski (figure 3-8).



3-7 Tensile force – strain cureve of a reinforced tensile member by Leonhardt



3-8 Tensile force – strain curve of a reinforced tensile member by Noakowski

Difference between pre-version of the Eurocode and the Eurocode itself is mainly the linear approach of the final version of the Eurocode. The Eurocode has the assumption of a crack formation stage and further a linear approach, see fig. 3-8. The expression of the Eurocode can be found in 3.1. The expression for the pre-version of the Eurocode is:

$$w_k = s_{rm}\varepsilon_{sm}$$

$$\varepsilon_{sm} = \frac{\sigma_s}{E_s}\left[1-\beta_1\beta_2\left(\frac{\sigma_{sr}}{\sigma_s}\right)^2\right]$$

β1:	1.0 for high bond bars	0.5 for plain bars
β2:	1.0 for single, short term loading	0.5 for many cycles

The crack spacing is:

$$s_{rm} = \left(50 + 0.25k_1k_2\frac{\phi}{\rho_r}\right)$$

k1:	0.8 for high bond bars	1.6 for plain bars
k2:	1.0 for pure tension	0.5 for bending

3.4.2 Rilem-TC162TDF / Dramix guideline

The crack width model is based on the model of Leonhardt. The total expression has the same values as in the pre-version of the Eurocode (see 3.4.1). The crack spacing is not changed compared with the crack spacing of the reinforced concrete. The calculation of the steel stresses σ_s and σ_{sr} is done taking into account a post-cracking tensile strength equal to $0.37 f_{ctm,eq,300}$. This $f_{ctm,eq,300}$ is the value of the post-cracking stress at a deflection of 1.5 mm (appendix 1). This post-cracking tensile strength is determined by the 4-points bending test. The adjustment reduces the steel stress and the crack width.

3.4.3 Model Vandewalle

From tests smaller crack spacing are observed. This effect is not taken into account in the Rilem-model. Vandewalle had made an adjustment on the formula of the Rilem method, which have an extra factor to the crack spacing.

$$s_{rm} = \left(50 + 0.25 k_1 k_2 \frac{\phi}{\rho_r}\right)\left(\frac{50}{L_f / d_f}\right)$$

In this expression the factor $\left(\frac{50}{L_f / d_f}\right)$ is the addition of the steel fibres. L_f is the length of the fibre and d_f is the thickness of the fibres. This factor must be smaller than 1. The crack spacing is independent on the amount of steel fibres. The shape of the steel fibres reduces the crack spacing.

3.4.4 Calculation of the models:

To compare the derived model in 3.2 with the models in 3.1 and 3.4 a calculation of the maximum crack width is made. The calculation is an axial tensile bar with dimensions of 100x100 mm and a concrete class of C35/45. The bar is reinforced with one reinforcement bar with a diameter of 16 mm. The bar is calculated with a post-cracking stress of 0.75 and 1.00 N/mm².
The models of Vandewalle and the Rilem-method utilize the post-cracking stress of Dramix-fibres. For the same post-cracking stresses the amount of Dramix-fibres is calculated:

$$f_{ctk,eq,300} = \left(\frac{\sigma_f}{0.37}\right)$$

Dimensions:

Width	:	100	mm	
Height	:	100	mm	
Reinforcement As	:	201	mm²	
Concrete	:	C35/45		
Dramix-fibre	:	RC-80/60-BN		
σ_f	:	1.0	N/mm²	
$F_{ctk,eq,300}$	:	2.7	N/mm²	35 kg fibres

The cracking force of the bar is:

$$N_{cr} = f_{ctm} A_c \left(1 + \rho \cdot \alpha_e\right) = 32100 N$$

Maximum crack width [mm]						
	F = 50000 N		F = 70000 N		F = 90000 N	
	No fibre	Fibre	No fibre	Fibre	No fibre	Fibre
NEN-EN-1992 Reinforced concrete	0.1444	-	0.2434	-	0.3424	-
NEN-ENV-1992 Reinforced concrete (pre-version)	0.0795	-	0.2352	-	0.3680	-
Steel fibre reinforced concrete as in 3.2	-	0.0822	-	0.1503	-	0.2184
Rilem method	-	0.0777	-	0.2253	-	0.3511
Vandewalle	-	0.0486	-	0.1408	-	0.2194

3-1 Overview of the maximum crack widths for the different calculations models

Table 3-1 shows the overview of the maximum crack width for different models for the axial tensile bar. The table shows the maximum crack width at three different tensile forces. In the table difference is made between the models without steel fibres and with the addition of steel fibres. For with and without the addition steel fibres two global models are used. For the Eurocode and the steel fibre reinforced concrete model derived in section 3.2, the Noakowski model is used. For the pre-version of the Eurocode and the Rilem and Vandewalle-method the model of Leonhardt is used. In section 3.4.1 the two different models are explained.

For tensile forces which are relative close to the cracking force a smaller crack width can be found by the global model of Leonhardt. The difference in crack width is presented in table 3-1 and the difference in strain is presented in table 3-2. For the tensile force of 50 kN a much lower crack width is found compared with the model of Noakowski. For a tensile force of 90 kN the difference between both crack widths is small.

	F = 50000 N		F = 70000 N		F = 90000 N	
	srm	εs	srm	εs	srm	εs
Steel fibre reinforced concrete as in 3.2	136,82	0,00060	136,82	0,0011	136,82	0,001596
Vandewalle	130,75	0,00037	130,75	0,00108	130,75	0,001678

3-2 Crack spacing and steel strain for 2 models

Table 3-2 shows the crack spacing and the strain in the reinforcement for the two different models which both contains the influence of the steel fibres in the transmission length and the steel stress. The crack spacing for the Vandewalle-method is slightly smaller compared with the model of steel fibre concrete. The strain in the reinforcement is at a tensile force of 50 kN much smaller in the Vandewalle-method and nearly equal at a tensile force of 90kN. The same difference is presented in table 3-1 which shows a lower crack width at a tensile force of 50 kN.

The results of both models correspond fairly well. In the case of a relative low tensile force the model for steel fibre reinforced concrete is more conservative compared with the model of Vandewalle.

A second calculation is made for steel fibres with a lower post-cracking stress:

Dramix-fibre	:	RC-80/60-BN		
σ_f	:	1.0	N/mm²	
$F_{ctk,eq,300}$	:	2.7	N/mm²	35 kg fibres

	F = 50000 N			F = 70000 N			F = 90000 N		
	srm	εs	wmax	srm	εs	wmax	srm	εs	wmax
Steel fibre reinforced concrete as in 3.2	152,36	0,0006	0,0960	152,36	0,0011	0,1721	152,36	0,0016	0,2479
Vandewalle	130,75	0,0004	0,0489	130,75	0,0011	0,1426	130,75	0,0017	0,2224

3-3 Crack spacing and steel strain for 2 models with a post-cracking stress of 0.75 N/mm²

Table 3-2 shows the crack spacing, steel strain and the maximum crack width for the Vandewalle-method and the model of steel fibre concrete. The strain in the reinforcement has the same difference between both models as in the calculation with the higher post-cracking stress. The difference in crack width compared with the higher post-cracking stress is caused by the difference in crack spacing. The crack spacing of the Vandewalle-method is independent of the volume of the steel fibres in contrast to the model of steel fibre concrete.

The model of Vandewalle results in a maximum crack width which is in general smaller compared with the model of steel fibre concrete. This major difference is the type of model which is used and this results in the difference in maximum crack width. Another difference is the crack spacing. The model of Vandewalle is based on a crack spacing which is independent on the amount of steel fibres, but only on the shape of the steel fibres. The model of steel fibre concrete is based on the amount of steel fibres and is not based on the shape of the steel fibres. It can be concluded that the model of steel fibre concrete results a more conservative maximum crack width compared with the Vandewalle-model.

3.5 Effective height

In this paragraph, attention is focused on reinforced concrete beams. The mean crack spacing was found to be depended, among the other factors, on the reinforcement ratio ρ. In the case of an axial tensile member the reinforcement ratio is defined as:

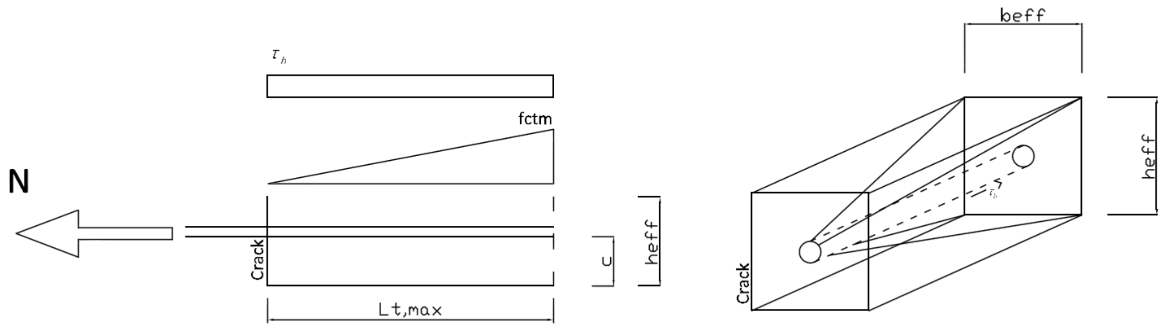
$$\rho = \frac{A_s}{A_c}$$

For beams with an increasing height, this reinforcement ratio is overestimated. From the cracking behaviour of deep beams, an 'effective concrete area' around the main reinforcement is defined:

$$\rho_{eff} = \frac{A_s}{bh_{eff}}$$

3.5.1 Effective height in reinforced concrete

The effective height in reinforced concrete has been subjected for many researches. In all of these researches, no exact solution has been found. All of the models have the same point of view, namely the effective concrete area is depending on the force which is transmitted by bond from the reinforcing steel to the concrete.



3-9 Introduction of the tensile force in a concrete beam

Figure 3-9 shows the introduction of a tensile force in a concrete beam. By the bond the tensile forces which are in the reinforcement in the crack must be transmitted to the concrete. These bond stresses have only influence on the concrete area around the reinforcement. From tests the maximum crack spacing can be found. With this crack spacing the transmission length and the tensile force in the crack can be calculated. With this tensile force the effective area can be calculated. This effective concrete area is the maximum area over which the force in the crack can be distributed due to the bond stresses. In this calculation the bond stress is assumed to be constant.

The effective height in the Eurocode is [1,16]:

$$h_{eff} = \alpha(h-d) = \alpha(h-(h-c-\phi/2)) = \alpha\left(c+\phi/2\right)$$

With:

$$\alpha = \frac{N_{\tau u}}{b(h-d)\,f_{ct}} = \frac{\pi d_s \tau_b l_t}{b\left(c+\phi/2\right)\,f_{ct}}$$

$$N = N_{\tau u} = \pi d_s \tau_b l_t$$

From the experiments a value of α=2.5 is found.

This results in an effective height of:

$$h_{eff} = 2.5(h-d) = 2.5\left(c+\phi/2\right)$$

3.5.2 Steel fibre reinforced concrete

In steel fibre reinforced concrete the force in the crack is the same as for reinforced concrete. In reinforced concrete the total tensile force is in the reinforcement. In steel fibre reinforced concrete the tensile force is divided over the steel fibres and the reinforcement.

$$N = N_\tau + N_f = \tau_{bm} \cdot l_t \cdot \pi \cdot \phi + \sigma_f A_{c,eff}$$

The only possible difference in the eurocode is the factor α:

$$\alpha = \frac{N}{b(h-d) f_{ct}}$$

The tensile force in the crack is the same as for reinforced concrete. Therefore the factor α will not change.

From this can be concluded that there is no difference in the effective height. In steel fibre concrete the same effective height can be used, as in the Eurocode for reinforced concrete, namely $h_{eff} = 2.5(h-d) = 2.5\left(c+\phi/2\right)$.

3.5.3 Effective height in plates

The effective height of the plates is different of the effective height in the beams, which is explained before. Results from experiments give smaller effective heights for plates than calculated than the effective height of beams.
The difference is that the neutral line of plates is relative low compared of the neutral line of beams. This low location of the neutral line results in a smaller effective height. From the experiments on plates the following expression can be found [6,16]:

$$h_{eff} = \frac{h-x}{3}$$

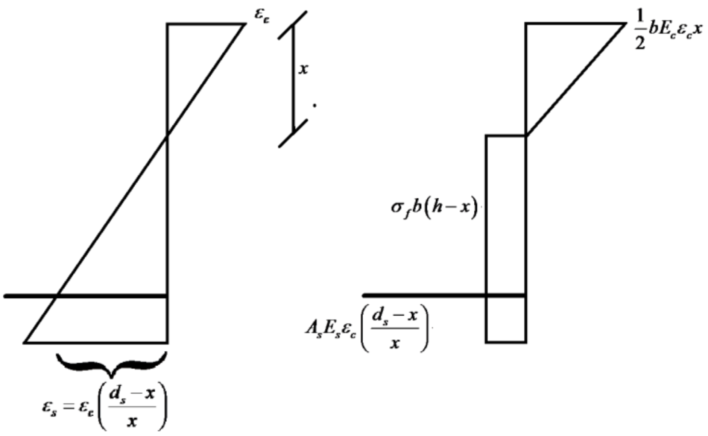
There is no clear substantiation found for this expression. No clear comparison can be made between the effective height in plates for reinforced concrete and the effective height in steel fibre reinforced concrete. Therefore in this research the effective height for plates with steel fibres is assumed to be the same as for reinforced concrete

4. Calculation of the reinforcement

In this chapter the calculation method is explained to find the amount of reinforcement and the steel stress in steel fibre concrete.

4.1 Calculation of the reinforcement

For the calculation of the reinforcement, it is necessary that the construction must be in equilibrium in each section. This equilibrium is the equilibrium in horizontal forces and in bending moments. With this equilibrium the reinforcement in the cross section can be calculated.



4-1 Strain and stress diagram of steel fibre concrete

The strain and stresses in the cross-section are shown in figure 4-1. With an assumption that the strains are linear in the cross-section, the strain in the reinforcing steel can be calculated from the concrete strain.

There are two unknown variables in the figure 4-1, the concrete strain and the height of the compressive zone. Both can be calculated with the equilibrium of horizontal forces and bending moments:

$$\Sigma H = 0 \qquad A_s E_s \varepsilon_c \left(\frac{d_s - x}{x}\right) + \sigma_f b(h-x) - \frac{1}{2} b E_c \varepsilon_c x = 0$$

$$\Sigma M = 0 \qquad A_s E_s \varepsilon_c (d_s - x)\left(\frac{d_s - x}{x}\right) + \sigma_f b(h-x)\left(\frac{h-x}{2}\right) - \frac{1}{2} b E_c \varepsilon_c x \frac{2x}{3} - M_{ed} = 0$$

With:

$$d_s = h - c - \phi_{stirrups} - \frac{\phi}{2}$$

M_{ed} = bending design moment in the cross section

By solving the expressions above the stress in the reinforcement can be found. The stress in the reinforcement is important in the calculation of the crack width (see section 3.2.3).

4.2 Boundaries of the amount of reinforcement

For a safety design of a concrete structure, the amount of reinforcing steel in a cross section has boundaries. To avoid sudden failure of the concrete, it is necessary that the cross section gives a warning. In concrete structures is cracking mostly the warning signal.

4.2.1 Calculation of the minimum reinforcement

To prevent brittle failure of the structure, it should be provided with a sufficient amount of reinforcing steel. The cracked cross section should have a bigger capacity compared with the uncracked cross section. The basis for the minimum reinforcement is that the cracked cross section at least can resist the cracking force or moment of the structure.

The minimum reinforcement in an axial bar:
The force in the axial bar at the moment of cracking:

$$N_{cr} = A_c f_{ctm}$$

The force which can resists the tensile stress:

$$N = A_{s,\min} f_{yd} + \sigma_f A_c$$

The steel stress in the reinforcement is calculated at the yielding stress. Yielding of the reinforcement results in one or more large cracks and in these cracks the fibres will be pulled out. This results in a reduction of the stress which is consumed by the steel fibres. This reduction results in an underestimation of the minimum reinforcement, while the goal of the minimum reinforcement is the guarantee of safety. Therefore the conservative approximation is made and the addition of steel fibres is not taken into account.

The minimum reinforcement in an axial bar can be calculated by the following expression:

$$A_{s,\min} f_{yd} = A_c f_{ctm}$$

$$A_{s,\min} = \frac{A_c f_{ctm}}{f_{yd}}$$

The minimum reinforcement in a bending beam:
The minimum reinforcement has to transmit the force which occurs at the cracking moment [16]:

$$M_{cr} = W f_{ctm,fl} = \frac{1}{6} b h^2 f_{ctm,fl} \qquad f_{ctm,fl} = f_{ctm}\left(\frac{1600-h}{1000}\right)$$

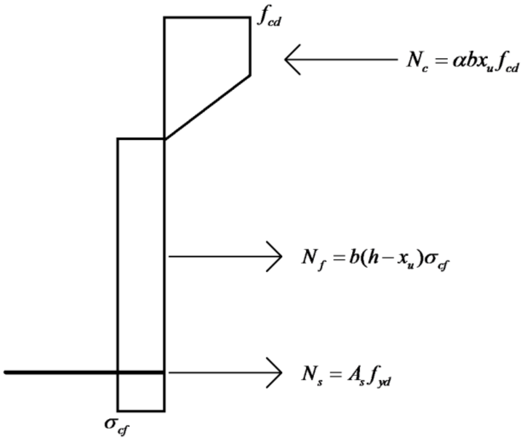
The minimum reinforcement can be calculated with the equilibrium of horizontal forces and bending moments:

$$\Sigma H = 0 \qquad A_{s,\min} f_{yd} - \frac{1}{2} b f_{ctm} x = 0$$

$$\Sigma M = 0 \qquad A_{s,\min} f_{yd}\left(\frac{d_s - x}{x}\right) + \frac{1}{2} b f_{ctm} x \frac{2x}{3} - M_{cr} = 0$$

4.2.2 Maximum reinforcement ratio

To avoid a sudden failure of the concrete in the ultimate limit state, it is necessary that the reinforcing steel yields before the maximum concrete compressive strength is reached. At the moment of yielding of the reinforcing steel, the cross section deforms and bigger cracks will occur.



4-2 Cross section of steel fibre reinforced concrete in the Ultimate Limit State

Figure 4-2 shows the forces in the cross section:

Force in the concrete : $N_c = \alpha b x_u f_{cd}$

Force in the reinforcement : $N_s = A_s f_{yd}$

Force in the steel fibres : $N_f = b(h - x_u)\sigma_f$

To calculate the maximum reinforcement, the construction must be in equilibrium. By using the described forces above, the equilibrium of horizontal forces can be calculated:

$$\sum H = 0 => N_s + N_f - N_c = 0$$

$$\sum H = 0 => A_{s,\max} f_{yd} + b(h - x_u)\sigma_f - \alpha b x_u f_{cd} = 0 \qquad A_{s,\max} = \omega_{\max} bd$$

$$\sum H = 0 => \omega_{\max} f_{yd} d + (h - x_u)\sigma_f - \alpha x_u f_{cd} = 0$$

From the equilibrium of horizontal forces, the maximum reinforcement ratio follows:

$$\omega_{\max} f_{yd} d + (h - x_u)\sigma_f - \alpha x_u f_{cd} = 0$$

$$\omega_{\max} = \frac{\alpha x_u f_{cd} - (h - x_u)\sigma_f}{f_{yd} d}$$

Where:

x_u = maximum compressive height

The maximum compressive height is dependent of redistribution of bending moments is taken into account. In this calculation no redistribution of the bending moment is used. The maximum compressive height is [16]:

Redistribution of moments gives the maximum compressive height [16]:

$$x_u = \frac{\delta - k_1}{k_2} d = \frac{1 - 0.44}{1.25\left(0.6 + \frac{0.0014}{e_{cu2}}\right)} d$$

Where:

$$\delta \geq k_1 + k_2 \frac{x_u}{d}$$

$\delta = 1$ (no redistribution) $\qquad k_1 = 0.44 \qquad k_2 = 1.25\left(0.6 + \frac{0.0014}{e_{cu2}}\right)$

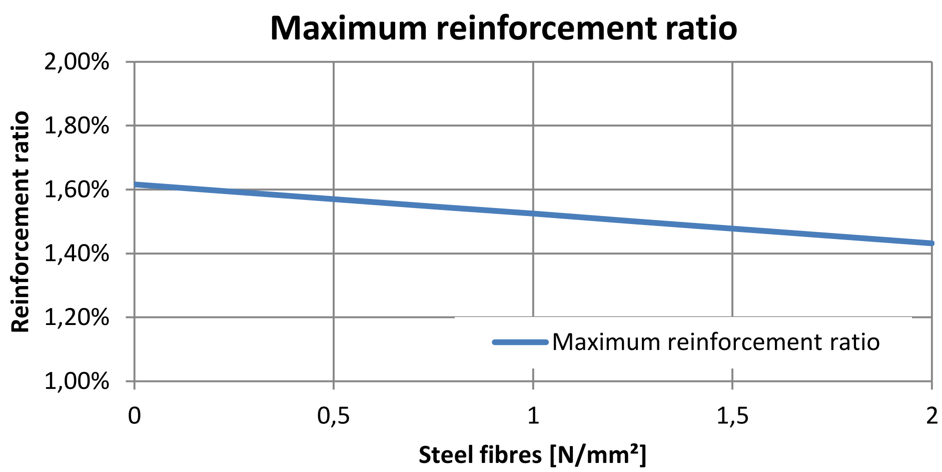
By use of the maximum reinforcement, the maximum resisting bending moment of the cross section can be calculated. The maximum resisting bending moment must be larger must be larger than the maximum design moment to avoid failure of the structure:

$$M_{rd} = A_{s,\max} f_{yd}\left[(h - x_u) - (h - d_s)\right] + \sigma_f b(h - x_u)\frac{(h - x_u)}{2} + \alpha b x_u f_{cd}(1 - 0.39)x_u$$

$$M_{rd} \geq M_{ed}$$

Example of the maximum reinforcement:

The maximum reinforcement are calculated for the dimensions and values which are given in case study 1 (chapter 6).



4-3 Maximum reinforcement ratio over the total width of the bridge (2.5m)

In figure 4-3 the maximum reinforcement ratio is calculated for the bridge of case study 1 (chapter 6). The maximum reinforcement ratio reduces if steel fibres are added. The steel fibres consume a part of the tensile forces, which results in a lower tensile force that has to been transmitted by the reinforcement. In the ultimate limit state the reinforcement yields and a lower amount of reinforcement is needed. In the table below the calculated amounts of reinforcement and the maximum reinforcement ratio are illustrated. The results of figure 4-3 are presented in the table.

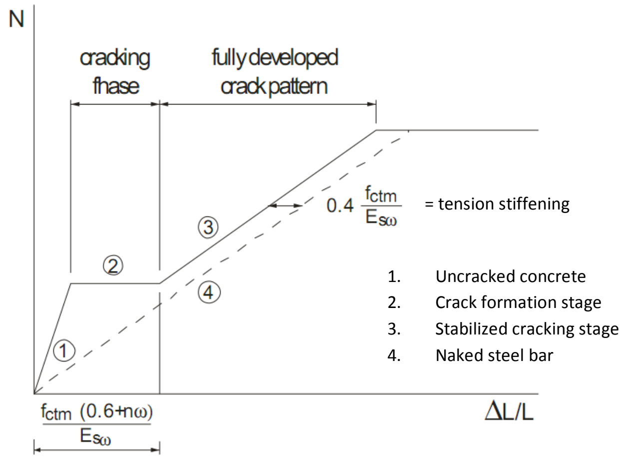
Amount of steel fibres:	Maximum reinforcement:	
0,00 N/mm²	14140 mm²	1,616%
0,25 N/mm²	13940 mm²	1,593%
0,50 N/mm²	13740 mm²	1,570%
0,75 N/mm²	13540 mm²	1,547%
1,00 N/mm²	13340 mm²	1,525%
1,25 N/mm²	13130 mm²	1,501%
1,50 N/mm²	12930 mm²	1,478%
1,75 N/mm²	12730 mm²	1,455%
2,00 N/mm²	12530 mm²	1,432%

4.3 Tension stiffening model

The tension stiffening model is important in the case of imposed deformation. In de most cases the strain due to imposed deformation is in the crack formation stage. With the tension stiffening model the length of the crack formation stage can be found and the stress in the reinforcement can be calculated.

4.3.1 Reinforced concrete

In the cracked situation the concrete around the reinforcement has still influence on the behaviour of the bar. Between the cracks in the concrete, the concrete still carrying tensile forces and gives a higher stiffness of the bar compared to the stiffness of the reinforcement only. This contribution of the concrete between the cracks to the stiffness is called tension stiffening.



4-4 Tension stiffening model axial tensile bar

Figure 4-4 shows the schematized behaviour of an axial tensile member. In the figure different stages can be found: the elastic stage, the crack formation stage and the stabilized cracking stage. The calculation of the tension stiffening is calculated for the stabilized cracking stage. An increase in the tension stiffening the crack formation stage will shorten. The crack spacing is between l_t and $2l_t$. For the average crack spacing $1.5l_t$ is assumed. The stress in the steel between two cracks is:

$$\sigma_{sx} = \sigma_s - \frac{x\tau_{bm}\pi\phi}{\frac{1}{4}\pi\phi^2}$$

- σ_s : steel stress in the crack
- $\frac{1}{4}\pi\phi^2$: Area of the reinforcement
- $x\tau_{bm}\pi\phi$: The force that can be transmitted over a length x

For the assumption of a crack spacing of $1.5l_t$, the steel stress in the mid between two cracks is:

$$\sigma_{sx} = \sigma_s - \frac{0.75 l_t \tau_{bm}\pi\phi}{\frac{1}{4}\pi\phi^2} = \sigma_s - \frac{0.75\frac{1}{4}\frac{f_{ctm}}{\tau_{bm}}\frac{\phi}{\rho}\tau_{bm}\pi\phi}{\frac{1}{4}\pi\phi^2} = \sigma_s - 0.75\frac{f_{ctm}}{\rho}$$

The mean steel stress is the average stress between crack and the steel stress between two cracks is:

$$\sigma_{sm} = \sigma_s - \frac{0.375 f_{ctm}}{\rho} \approx \sigma_s - 0.4\frac{f_{ctm}}{\rho}$$

With the use of the mean steel stress the beginning of the stabilized cracking stage can be found. The crack formation stage is defined by the cracking force (section 3.1):

$$N_{cr} = A_c f_{ctm}(1+\alpha_e \rho)$$

The stabilized cracking stage is described by:

$$N = A_s\left(\sigma_s + 0.4\frac{f_{ctm}}{\rho}\right) = A_s E_s\left(\varepsilon_s + 0.4\frac{f_{ctm}}{E_s \rho}\right)$$

The maximum strain of the crack formation stage can be formed by combining the two cracking forces:

$$A_s E_s \varepsilon_s + 0.4 f_{ctm} A_c = A_c f_{ctm}(1+\alpha_e \rho)$$

$$A_s E_s \varepsilon_s = A_c f_{ctm}(0.6+\alpha_e \rho)$$

$$\varepsilon_s = \frac{f_{ctm}(0.6+\alpha_e \rho)}{E_s \rho}$$

4.3.2 Steel fibre reinforced concrete

If steel fibres are added to the concrete matrix, the matrix will strengthen due to the steel fibres. For the tensile stiffening effect the same calculation can be done as for the ordinary reinforced concrete tensile bar. The difference with this calculation is the extra stiffening of the steel fibres.

In this calculation the same method is used as for reinforced concrete. The difference is the influence of the steel fibres. The steel fibres reduce the steel stress by a constant stress. The stress in the reinforcement between two cracks with the assumption of a crack spacing of $1.5l_t$:

$$\sigma_{sx} = \sigma_s - \frac{\sigma_f}{\rho} - \frac{0.75 f_{ctm}}{\rho} = \sigma_s - \frac{\chi f_{ctm}}{\rho} - \frac{0.75 f_{ctm}}{\rho}$$

The mean steel strain becomes:

$$\sigma_{sm} = \sigma_s - \frac{\chi f_{ctm}}{2\rho} - \frac{0.375 f_{ctm}}{\rho} \approx \sigma_s - \frac{f_{ctm}}{\rho}\left(\frac{1}{2}\chi + 0.4\right)$$

The end of the crack formation stage and the start of the stabilized cracking stage is:

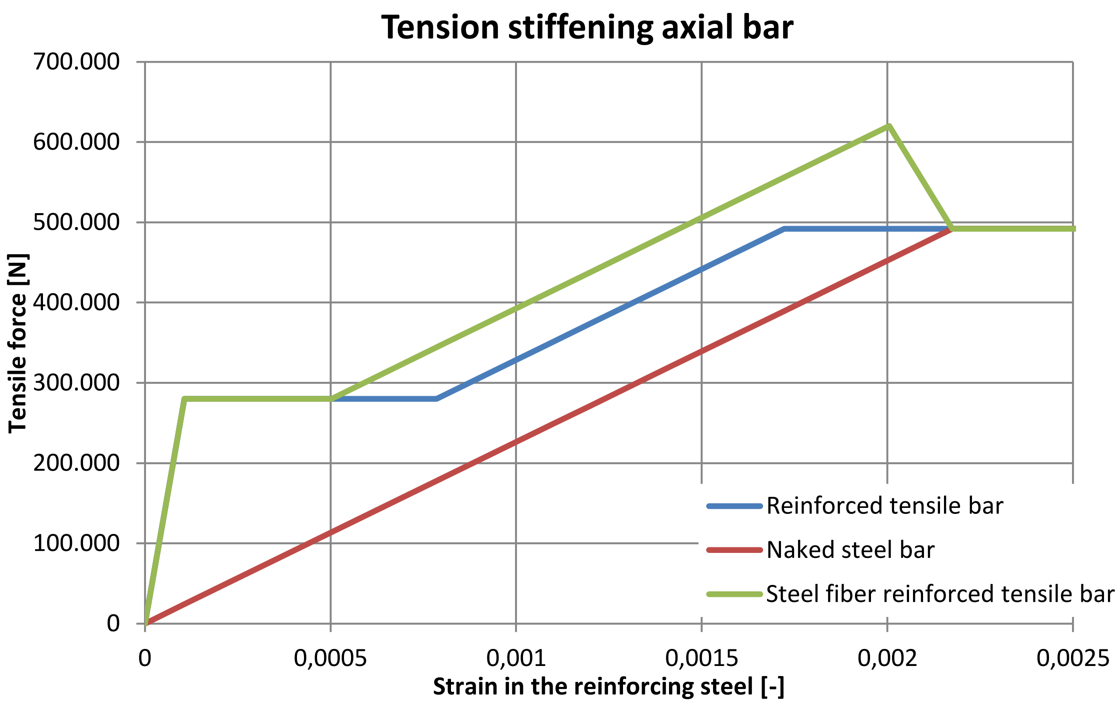
$$N = A_s\left(\sigma_s + \frac{f_{ctm}}{\rho}\left(\frac{1}{2}\chi + 0.4\right)\right) = A_s E_s\left(\varepsilon_s + \frac{f_{ctm}}{E_s \rho}\left(\frac{1}{2}\chi + 0.4\right)\right)$$

The maximum strain of the crack formation stage can be formed by combining the two cracking forces:

$$A_s E_s \varepsilon_s + f_{ctm} A_c \left(\frac{1}{2}\chi + 0.4\right) = A_c f_{ctm}(1+\alpha_e \rho) => A_s E_s \varepsilon_s = A_c f_{ctm}\left(0.6 - \frac{1}{2}\chi + \alpha_e \rho\right)$$

$$\varepsilon_s = \varepsilon_{fdc} = \frac{f_{ctm}\left(0.6 - \frac{1}{2}\chi + \alpha_e \rho\right)}{E_s \rho}$$

The maximum strain of the crack formation stage in steel fibre reinforced concrete is smaller compared with reinforced concrete. In the situation of the beginning of the stabilized cracking phase in reinforced concrete the strain and tensile force are known. The strain in the reinforcement is given, thus the addition of steel fibres causes an increase in the tensile force of the bar.



4-5 Load deformation diagram for an axially loaded tensile member with and without steel fibres

In figure 4-5 the load deformation diagram is shown for an axially loaded tensile member with and without steel fibres. It is assumed that the post-cracking tensile stress of the steel fibres is constant. The added steel fibres can transmit a part of the tensile force. With the same strain in the reinforcement a higher load can be taken. The addition of steel fibres reduces the length of the crack formation stage. The strain in the reinforcing steel becomes lower by the addition of the steel fibres in the crack formation stage. The force of the bar consists of the tensile stress in the steel fibres, the stiffening effect of the concrete and the tensile stress of the reinforcement. The tensile stress in the steel fibres and the stiffening effect of the concrete are constant values, the only variable is the increasing of the tensile stress of the reinforcement. Therefore at a lower strain a higher tensile force can be found.

When the reinforcement yields, it is assumed that the tensile stress of the steel fibres reduce to zero. The full calculation is presented in appendix 5.

4.4 Calculations

4.4.1 Calculation of the steel stress

The steel stress consists of two parts, the steel stress from the imposed deformation and the steel stress introduced by the bending moments.

The steel stress from the imposed deformation is depending on the stage.

Uncracked stage : $\sigma_s = E_s \varepsilon$

Crack formation stage : $\sigma_s = \frac{f_{ctm}}{\rho}\left(\left(1-\frac{\sigma_f}{f_{ctm}}\right)+\frac{E_s}{E_c}\rho\right)$

Stabilized cracking stage : $\sigma_s = \frac{f_{ctm}}{\rho}\left(\left(1-\frac{\sigma_f}{f_{ctm}}\right)+\frac{E_s}{E_c}\rho\right)+E_s\left(\varepsilon-\varepsilon_{fdc}\right)$

Yield stage : $\sigma_s = f_{yd}$

The steel stress causes by bending is:

$$\sigma_s = E_s \varepsilon_s = E_s \varepsilon_c \left(\frac{d_s - x}{x}\right)$$

The steel stress at cracking is:

$$\sigma_{sr} = \frac{f_{ctm}}{\rho_{eff}}\left(1+\frac{E_s}{E_c}\rho_{eff}\right)-\frac{\sigma_f}{\rho_{fibers}}$$

Here is:

$$\rho_{eff} = \frac{A_s}{h_{eff} b}$$

$$h_{eff} = \min \begin{cases} 2.5(h-d_s) & beam \\ \frac{h-x}{3} & plate \\ \frac{1}{2}h & thin\, member\, with\, 2\, layers\, reinforcement \end{cases}$$

$$\rho_{fibers} = \frac{A_s}{h_{fibers} b}$$

4.4.2 Calculation of the maximum crack width

The expression of the maximum crack width is presented in section 3.2.

$$w_{\max} = \frac{1}{2}\frac{f_{ctm}-\sigma_f}{\tau_{bm}}\frac{\phi}{\rho_{eff}}\frac{1}{E_s}\left(\sigma_s - 0.5\sigma_{sr}\right)$$

With:

$$\sigma_{sr} = \frac{f_{ctm}}{\rho_{eff}}\left(1+\frac{E_s}{E_c}\rho_{eff}\right)-\frac{\sigma_f}{\rho_{fibers}}$$

5. Some examples to illustrate the influence

In this chapter two simple calculations are done to illustrate the influence of steel fibres. The two situations are: an axial tensile bar and a beam loaded in bending.

5.1 Axial tensile bar

Given a tensile bar with a concrete area of 100x100mm and reinforced with a reinforcing bar Φ16. The bar is calculated with different amounts of steel fibres. The steel fibres are calculated by using the performance based principle and this means that there is a constant stress over the total tensile zone.

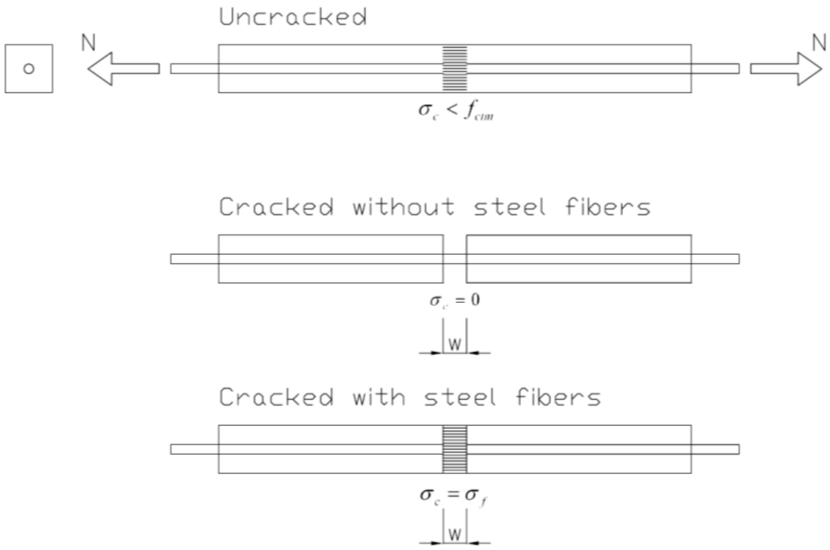


Fig. 5-1 Axial tensile bar

The cracking force of the bar is:

$$N_{cr} = E_s A_s \varepsilon + E_c A_c \varepsilon$$

$$N_{cr} = E_c A_c \varepsilon \left(1 + \rho \cdot \alpha_e\right)$$

$$N_{cr} = f_{ctm} A_c \left(1 + \rho \cdot \alpha_e\right)$$

The bar is calculated with different tensile forces. This example illustrates the influence of steel fibres on the crack width of an axial tensile bar. The influence of different amounts of steel fibres on the crack width will be illustrated.

The maximum crack width is calculated with the following expression:

$$w_{\max} = \frac{1}{2} \frac{\left(f_{ctm} - \sigma_f\right)}{\tau_{bm}} \frac{\phi}{\rho} \frac{1}{E_s} \left[\sigma_s - \frac{1}{2}\sigma_{sr}\right]$$

With:

$$\sigma_s = \frac{N - \sigma_f A_c}{A_s}$$ Steel stress

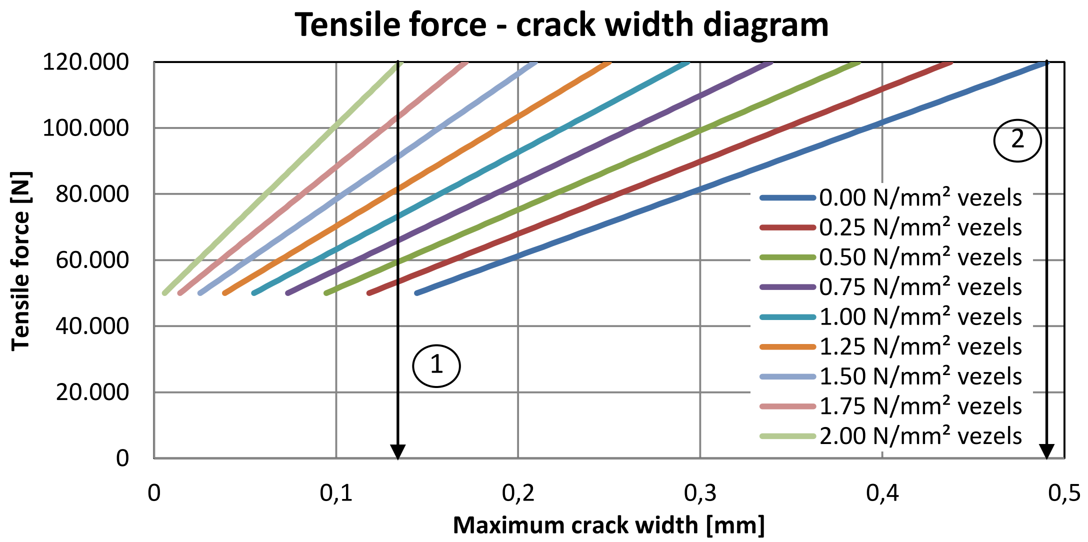
$$\sigma_{sr} = \frac{f_{ctm}}{\rho}(1 + n\rho) - \frac{\sigma_f A_c}{A_s}$$ Steel stress direct after cracking

The concrete quality of the concrete is C35/45. The reinforcing steel is FeB500 and has a bar diameter of 16 mm. The bond strength of this bar is assumed to a constant value of $2f_{ctm}$.The concrete area is 100x100mm.

For the steel fibres different values are used to get a better understanding of the influence of the steel fibres in the reduction of the crack width. The range of steel fibres is between 0.25 N/mm² and 2.00 N/mm².

Results:

The bar above is calculated with different tensile forces in the range of 50-120 kN.



5-2 Tensile force - crack width diagram axial tensile bar

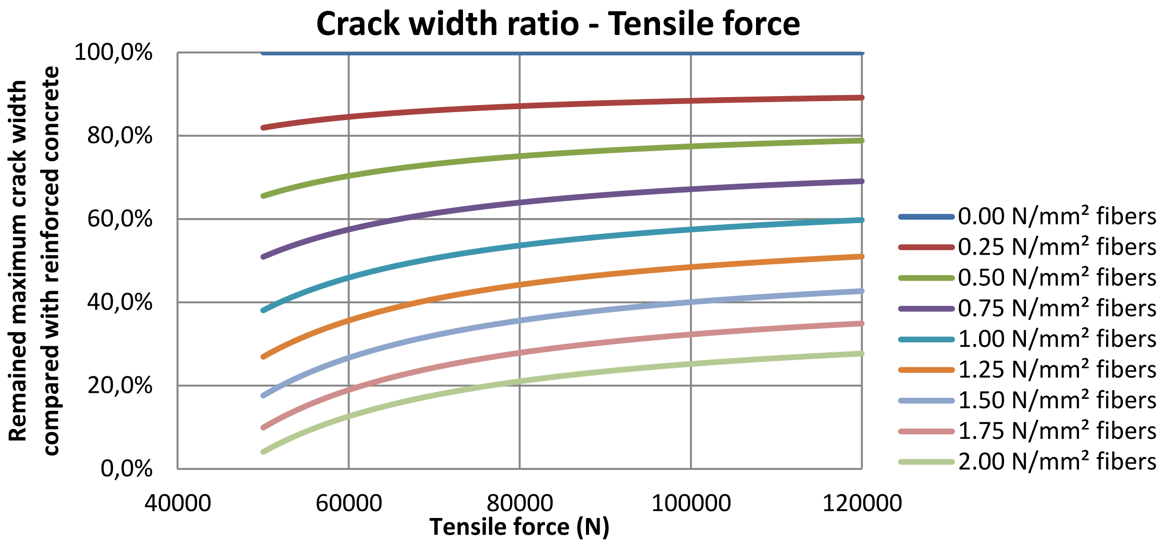
In the figure 5-2 the tensile force as function of the maximum crack width is illustrated. The maximum crack width is calculated over a tensile force between 50 and 120 kN. In the figure can be seen that the maximum crack width reduce with the same tensile force.

Example

For example: the bar is calculated without the addition of steel fibres and a maximum crack width of 0.48 mm (2) is found for a tensile force of 120 kN. By the addition of 2.00 N/mm² of steel fibres, the maximum crack width can be reduced to a value of 0.13 mm (1). The percentage of the crack width with fibres is:

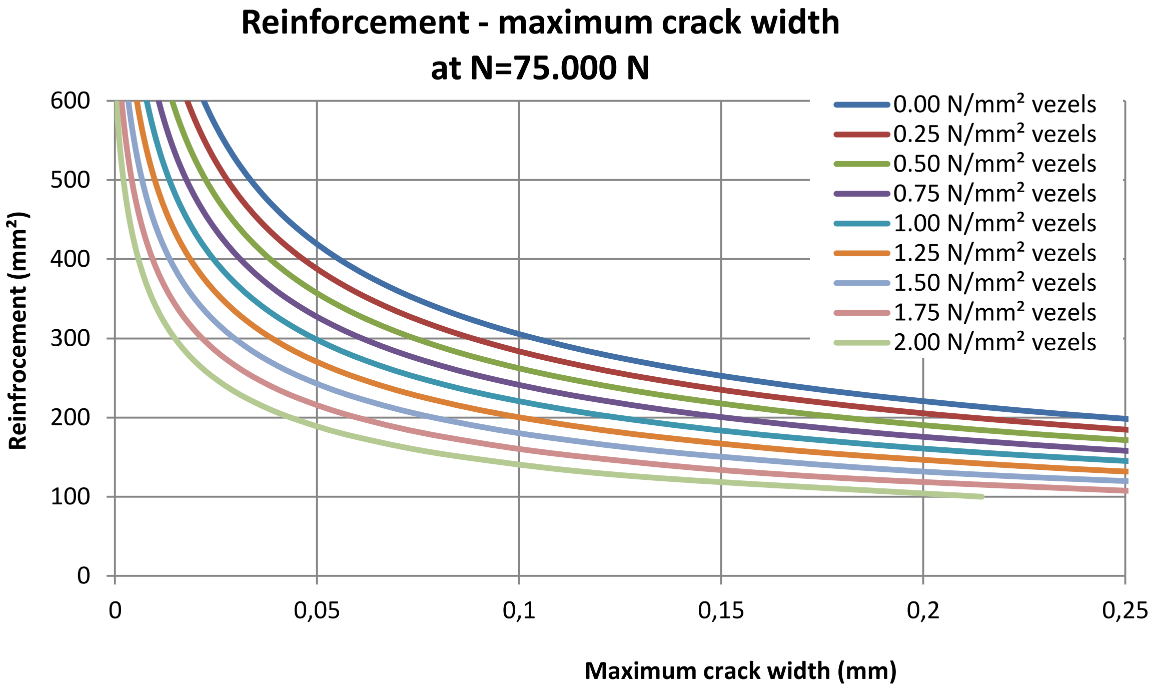
$$\frac{w_{\max,\,fibres}}{w_{\max,0\%\,fibres}} = \frac{0.13}{0.48} = 27\%$$

This is a reduction of the maximum crack width by 73%.



5-3 Maximum crack width ratio over tensile force

In figure 5-3 the maximum crack width ratio is illustrated as function of the tensile force. At low tensile forces the influence of steel fibres of the crack width is bigger compared with higher tensile forces. The reason is that the total concrete area cracks and the steel fibres transmits a constant stress. At lower tensile stresses the ratio of these forces transmitted by the steel fibres is relatively larger than in comparison with higher tensile stresses.



5-4 Reinforcement – maximum crack width at N=50.000 axial tensile bar

Figure 5-4 illustrates the required reinforcement as function of the maximum crack width at a constant tensile force of 75.000 N. In the figure the maximum crack widths are shown for the reinforced concrete and reinforced concrete with the addition of different amounts of steel fibres. A smaller amount of reinforcement is needed if steel fibres are applied. The results from figure 5-3 can also be found in figure 5-4.

Viewport of the graphs

Figure 5-4 shows just a part of the whole range of the reinforcement – maximum crack width diagram. But what happens outside this range? What happens if the reinforcement is close to zero or infinity large?

The maximum crack width is found in section 3.2:

$$w_{\max} = \frac{1}{2}\frac{(f_{ctm} - \sigma_f)}{\tau_{bm}}\frac{\phi}{\rho}\frac{1}{E_s}\left(\sigma_s - \frac{1}{2}\sigma_{sr}\right)$$

With:

$$\sigma_s = \frac{N - \sigma_f A_c}{A_s} \quad \text{and} \quad \sigma_{sr} = \frac{f_{ctm}}{\rho}(1 + n\rho) - \frac{\sigma_f}{\rho}$$

This expression is rewritten as:

$$w_{\max} = \frac{1}{2}\frac{f_{ctm} - \sigma_f}{\tau_{bm}}\frac{\phi}{A_s}\frac{h_{eff}b}{E_s}\left(\frac{F_s}{A_s} - 0.5\frac{f_{ctm}h_{eff}b}{A_s}\left(1 + \frac{E_s}{E_c}\frac{A_s}{h_{eff}b}\right) - \frac{\sigma_f h_{fibers}b}{A_s}\right)$$

If the amount of reinforcement will be reduced to nearly zero, the maximum crack width can be calculated by take the mathematical limit. To make it possible to take the limit of this function, the function must be rewritten. All the constant values should be grouped:

$$w_{\max} = \frac{1}{2}\frac{f_{ctm} - \sigma_f}{\tau}\frac{\phi}{A_s}\frac{h_{eff}b}{E_s}\left(\frac{F_s}{A_s} - 0.5\frac{f_{ctm}h_{eff}b}{A_s} - 0.5\frac{f_{ctm}h_{eff}b}{A_s}\frac{E_s}{E_c}\frac{A_s}{h_{eff}b} - \frac{\sigma_f h_{fibers}b}{A_s}\right)$$

$$w_{\max} = \frac{R}{A_s}\left(\frac{F_s}{A_s} - \frac{S}{A_s} - \frac{STA_s}{A_s} - \frac{U}{A_s}\right)$$

With the following constants:

$$R = \frac{1}{2}\frac{\left(f_{ctm} - \sigma_f\right)\phi}{\tau_{bm}}\frac{h_{eff}b}{E_s} \quad \text{and} \quad S = \frac{1}{2}f_{ctm}h_{eff}b$$

$$T = \frac{E_s}{E_c}\frac{1}{h_{eff}b} \quad \text{and} \quad U = \sigma_f h_{fibers}b$$

F_s = the tensile force in the tensile zone (constant value in this case)

It is now possible to find the limit of the function:

$$\lim_{A_s \to 0} w_{\max} = \frac{R}{A_s}\left(\frac{S}{A_s} - \frac{T}{A_s} + TU\frac{A_s}{A_s} - \frac{V}{A_s}\right)$$

$$\lim_{A_s \to 0} w_{\max} = \infty\left(\infty - \infty + TU - \infty\right) = \infty$$

And the limit for a infinity large reinforcement:

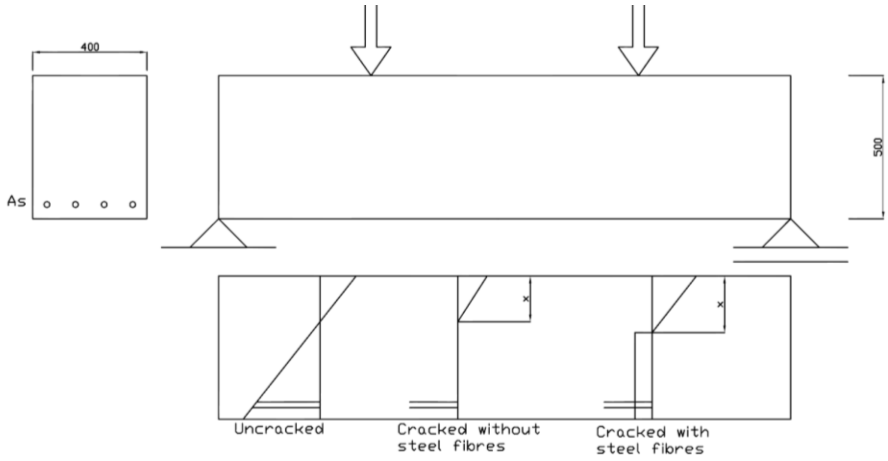
$$\lim_{A_s \to \infty} w_{\max} = \frac{R}{A_s}\left(\frac{S}{A_s} - \frac{T}{A_s} + TU\frac{A_s}{A_s} - \frac{V}{A_s}\right)$$

$$\lim_{A_s \to \infty} w_{\max} = 0\left(0 - 0 + TU - 0\right) = 0$$

The results of the limits are in agreement with figure 5-4. At a low amount of reinforcement, the crack width increases a lot and a high amount of reinforcement the crack width is very small. In practise, both situations will not occur, because a minimum and maximum reinforcement ratio is applied. The minimum and maximum reinforcement ratio is explained in section 4.2.

5.2 Beam loaded in bending

A beam is given which is loaded in bending. The beam has a height of 500 mm and a width of 400 mm. The calculation is made with different amounts of reinforcement due to a moment of 150 kNm. The beam is calculated with different volume amounts of steel fibres. The steel fibres are calculated with the performance based principle and this means that there is a constant stress over the whole tensile zone.



5-5 Bending beam

The maximum crack width is:

$$w_{\max} = \frac{1}{2}\frac{\left(f_{ctm} - \sigma_f\right)}{\tau_{bm}}\frac{\phi}{\rho}\frac{1}{E_s}\left(\sigma_s - \frac{1}{2}\sigma_{sr}\right)$$

With:

$$\sigma_s = E_s \varepsilon_s \frac{d_s - x}{x}$$ Steel stress

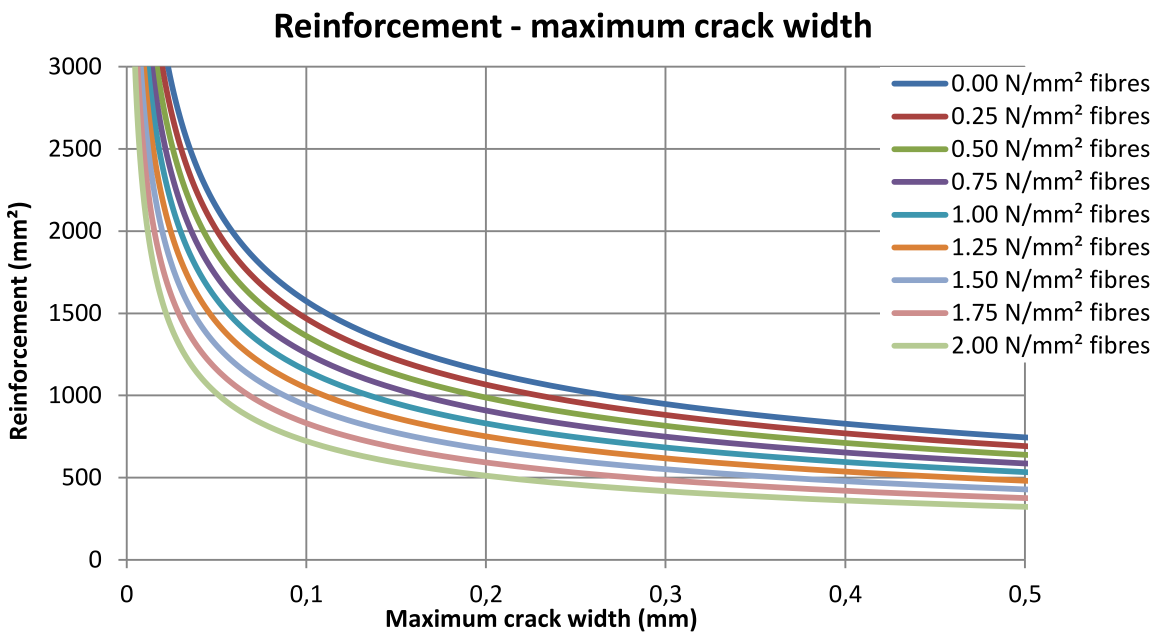
$$\sigma_{sr} = \frac{f_{ctm}}{\rho}(1 + n\rho) - \frac{\sigma_f A_c}{A_s}$$ Steel stress direct after cracking

The expression of the steel stress is the same as for reinforced concrete without the addition of steel fibres. The reduction of the steel fibres is taken into account in the horizontal and bending moment equilibrium.

Width of the beam	:	400	mm
Height of the beam	:	500	mm
Concrete class	:	C35/45	
Bending moment	:	150	kNm
Cover	:	40	mm

Results:

The beam is calculated with a constant bending moment of 150 kNm and is calculated for different amounts of reinforcement.



5-6 Reinforcement over maximum crack width diagram

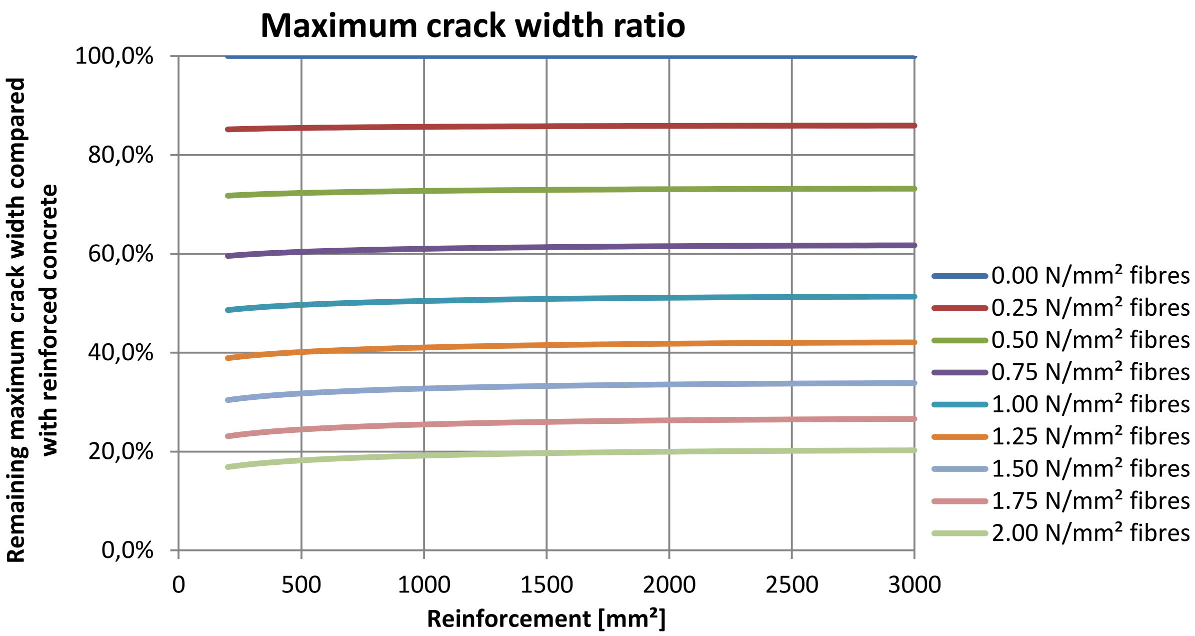
The reinforcement as function of the maximum crack width is illustrated in the figure above. Different curves are shown, each with an increasing amount of steel fibres. In the figure can be seen that with the same amount of reinforcement, the maximum crack width reduces if steel fibres are added. Also by the same maximum crack width, a lower amount of reinforcement is needed.

Example

For example: the bar is calculated without the addition of steel fibres and a reinforcement of 1600 mm² is found for a maximum crack width of 0.10 mm. By the addition of 2.00 N/mm² of steel fibres, the reinforcement can be reduced to a value of 720 mm². The percentage of the reinforcement with fibres is:

$$\frac{A_{s,\,fibres}}{A_{s,0\%\,fibres}} = \frac{720}{1600} = 45\%$$

This is a reduction of the reinforcement by 55%.



5-7 Remaining maximum crack width compared with reinforced concrete

In figure 5-7 the maximum crack width compared with the reinforced concrete beam over the reinforcement is illustrated. In the figure a percentage of the maximum crack width of the reinforced beam can be seen if steel fibres are added. This percentage is independent of the reinforcement.

5.3 Conclusions

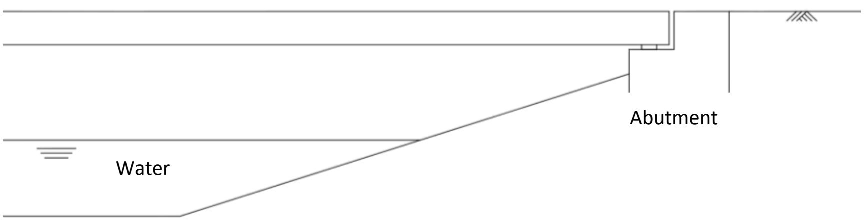
The most important conclusions are:

- The maximum crack width can be reduced if steel fibres are added. The reduction is 70 – 80% with a post-cracking stress of 2.00 N/mm². Figure 5-7 shows a near linear reduction of the maximum crack width, which is independent on the amount of reinforcement bars.
- Figures 5-4 and 5-6 show that the reinforcement can be reduced if steel fibres are added. For the beam in bending for a maximum crack width of 0.10 mm the reinforcement can be reduced with an amount of 880 mm² if steel fibres are added with a post-cracking stress of 2.00 N/mm². This results in a reduction of the reinforcement of 55%.

6. Case study 1: bridge

The users of this bridge are pedestrians and cyclists and sometimes a service vehicle. The bridge will be built in reinforced concrete and the option of the addition of steel fibres will be evaluated in this calculation. This calculation focuses on the bridge deck.

6.1 Overview of the project



6-1 Longitudinal cross section of the bridge

The bridge has the following dimensions:

length	:	13	m
width	:	2.5	m
thickness	:	350	mm
supports	:	free - free	

6.2 Structural requirements and material specifications

6.2.1 Concrete

Strength class	:	C35/45
Density reinforced concrete	:	25 kN/m³
Material factor	:	$\gamma_c = 1.50$
Concrete cover at the reinforcement	:	30 mm

There is no specific requirement for the maximum crack width. The goal of this calculation is to show the influence of steel fibres on the cracking behaviour.

6.2.2 Reinforcement

Reinforcing steel	:	FeB 500	
Yield stress	:	435 N/mm²	$\gamma_s = 1.15$

6.2.3 Steel fibres

The post-cracking behaviour of steel fibre concrete is based on the performance based principle. The post-cracking behaviour is presented in section 2.2.

6.3 Overview loads

The loads on the bridge deck consists permanent loads and variable loads. An overview of the loads is made below:

Permanent loads:

- Self weight
  The construction of the bridge will be made in reinforced concrete with a density of:
  $25\frac{kN}{m^3}$
  The distributed load due to the self weight of the construction is:
  $$q_{sw} = bh\gamma_c = 2.5 \cdot 0.35 \cdot 25 = 21.9\frac{kN}{m^1}$$

- Parapets 1.0 kN/m^1 (each side, the total is 2 x 1.0 kN/m^1 = 2.0 kN/m^1)
  The distributed load due to the parapets is:
  $$q_{parapet} = 2 \cdot 1,0 = 2.0\frac{kN}{m^1}$$

- Creep/shrinkage
  In this calculation creep is not taken into account. In this calculation the concrete will crack under the influence of short term loading (the variable loads).

Variable loads:

- Uniformly distributed traffic load
  For road bridges supporting pedestrian paths or cycle tracks, a uniformly distributed load q_{fk} should be defined (NEN-EN 1991-2: 5.3.2.1 load models; uniformly distributed load). The uniformly distributed load q_{fk} should be defined and applied only in the unfavourable parts of the surface. Load model 4 (crowd loading) consisting a uniformly distributed load of:
  $$q_{fk} = \frac{2.0+120}{L+30}$$
  $$q_{fk} = \frac{2.0+120}{13+30} = 4.8\frac{kN}{m^2}$$
  The distributed load due to crowd loading of the construction is:
  $$q_{fk} = 4.8\frac{kN}{m^2} = 2.5 \cdot 4.8 = 12.0\frac{kN}{m^1}$$

6.4 Results

In this paragraph the results of the calculation of the bridge can be found.

6.4.1 Bending moments

The first calculation is the calculation of the bending moments in the bridge deck. The calculation of the bending moments is shown below for a bridge deck, which has free supports at both ends:

For a free – free support system in the serviceability limit state:

$$M_{ed} = \frac{1}{8}(q_{sw} + q_{parapet} + q_{fk})L^2 \qquad M_{ed} = 757.3kNm$$

For a free – free support system in the ultimate limit state:

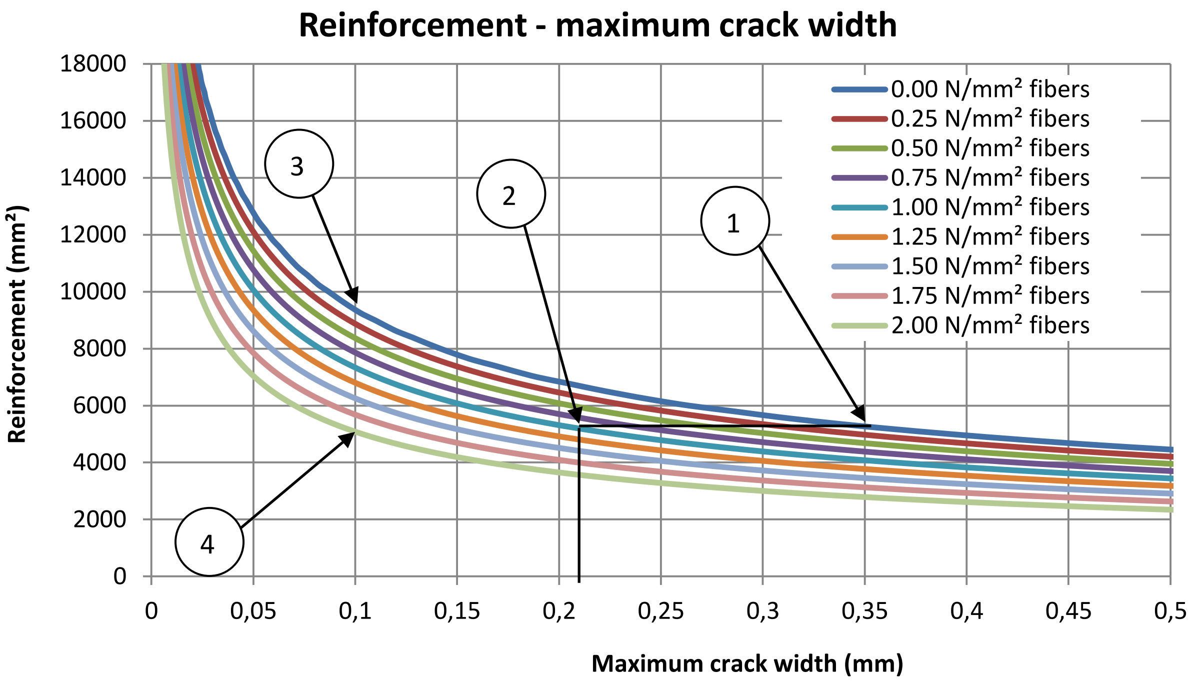
$$M_u = \frac{1}{8}(\gamma_G q_{sw} + \gamma_G q_{parapet} + \gamma_Q q_{fk})L^2 \qquad M_u = 991.0kNm$$

With:

$\gamma_G = 1.20 \qquad \gamma_Q = 1.50$

6.4.2 Calculation of the reinforcement

The maximum crack width is calculated over the full range of reinforcement, without taken the minimum and maximum reinforcement into account. The goal of this calculation is to show the influence of the addition of steel fibres to the maximum crack width. The full calculation of the maximum crack width can be seen in appendix 4.



6-2 Reinforcement over maximum crack width diagram

In figure 6-2 the reinforcement as function of the maximum crack width is shown. In the figure the curves are shown for the reinforced concrete without the addition of steel fibres and reinforced concrete with the addition of different amounts of steel fibres. The top curve is the curve of the reinforced concrete without steel fibres and the curves below are the curves with an increasing

amount of steel fibres. At a constant maximum crack width, less reinforcement is needed or at the same amount of reinforcement a lower maximum crack width can be found.
This reduction of the maximum crack width can be explained by the function of the steel fibres. In the cracked area, the steel fibres can transmit a part of the tensile forces. This part reduces the tensile force which must be transmitted by the reinforcement and this results in a reduction of the steel stress. Also by the lower tensile stress which must be transmitted over the transmission length. Both contributions can be seen in the formula below:

$$w_{\max} = 2l_t(\varepsilon_{sm} - \varepsilon_{cm}) = \frac{1}{2}\frac{\left(f_{ctm} - \sigma_f\right)}{\tau_{bm}}\frac{\phi}{\rho}\frac{1}{E_s}(\sigma_s - 0.5\sigma_{sr})$$

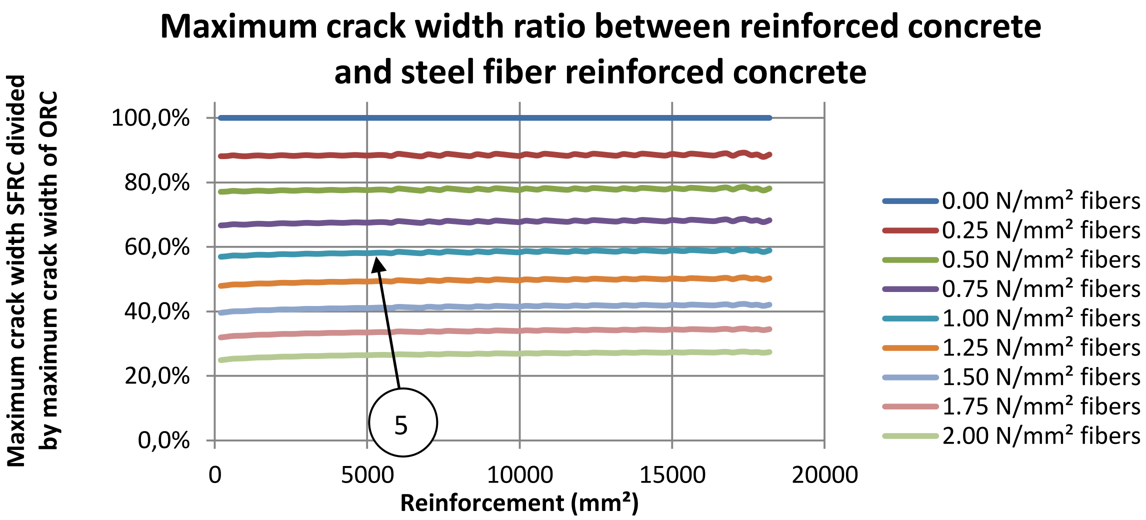
$$l_t = \frac{1}{4}\frac{f_{ctm} - \sigma_f}{\tau_{bm}}\frac{\phi}{\rho}$$

$$\sigma_s = E_s\varepsilon_c\left(\frac{d_s - x}{x}\right)$$

The contribution of the steel fibres over the transmission length is a constant value. Increasing the addition of steel fibres results in a constant decrease of the transmission length. The explanation of the decrease of the steel stress is more difficult. The formula above is dependent on the strain in the concrete compression zone and the height of the compression zone. In chapter 3.2 the tensile stress is calculated for an axial tensile bar. For this bridge, the tensile zone around the reinforcement can be seen as an axial tensile bar. This axial tensile bar has a height of which is equal to the effective height (section 3.5). The steel stress in an axial bar is:

$$\sigma_s = \frac{N - \sigma_f A_c}{A_s}$$

In this expression the influence of the steel fibres is constant. By a larger bending moment, the tensile force in the bar increases, while the steel fibres reduce this tensile force with a constant value.



6-3 Crack width ratio between reinforced concrete and steel fibre reinforced concrete

Figure 6-3 illustrates the ratio between the maximum crack width in steel fibre reinforced concrete and reinforced concrete (the maximum crack width in steel fibre reinforced concrete divided over the maximum crack width in reinforced concrete) as function of the amount of reinforcement. This

figure illustrates a near constant decrease of the maximum crack width in steel fibre concrete compared with the maximum crack width in reinforced concrete. The ratio of reinforced concrete in figure 6-3 is 100% and the concrete with the addition of fibres has a lower percentage. This near constant decrease in ratio is independent of the amount of reinforcement. The constant decrease can be explained by the constant reduction of the transmission length and the constant reduction of the steel stress.
As example can be found that the ratio for an addition of steel fibres with a tensile stress of 0.50 N/mm², the maximum crack width can be reduced with about 25%.

Example

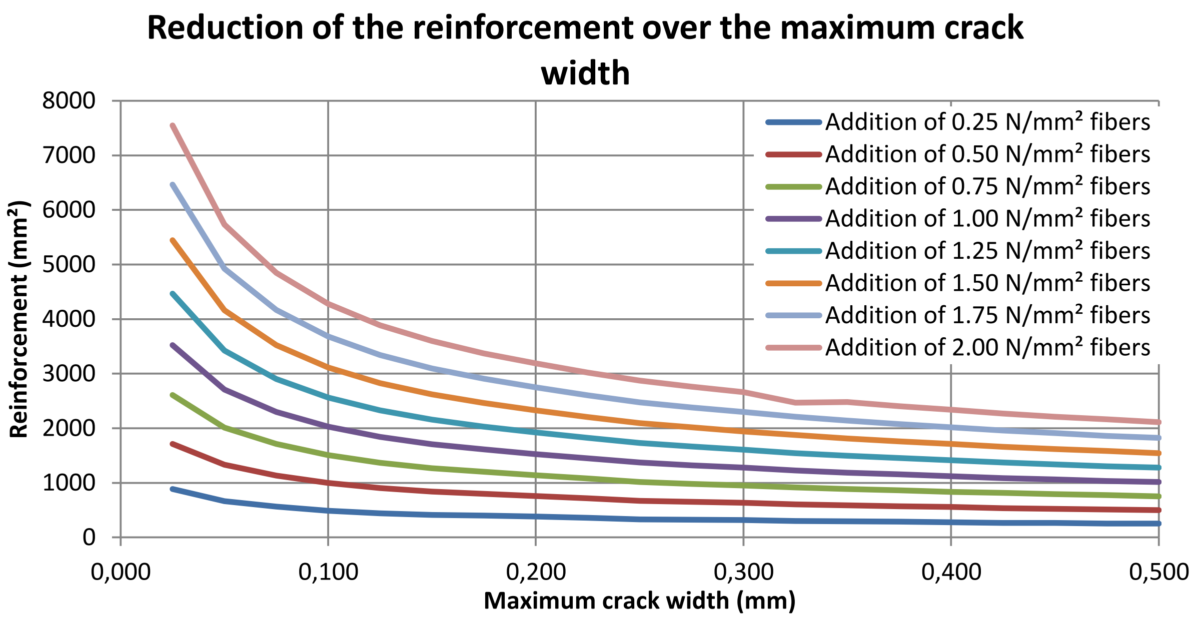
For example: the bridge is calculated without the addition of steel fibres and a maximum crack width of 0.35 mm is found (see figure 6-2 point 1). There is demanded for a smaller crack width. Steel fibres are chosen to reduce the crack width with the same amount of reinforcement. By the addition of 1.00 N/mm² of steel fibres and the same amount of reinforcement, the maximum crack width can be reduced to a value of 0.21 mm (see figure 6-2 point 2).
This maximum crack width can be rewritten as a percentage of the maximum crack width of the case without steel fibres. The percentage of the crack width with fibres is:

$$\frac{w_{\max,\,fibers}}{w_{\max,0\%\,fibers}} = \frac{0.21}{0.35} = 60\%$$

This percentage can be found in figure 6-3 at point 5.

6.4.3 Reduction of the reinforcement

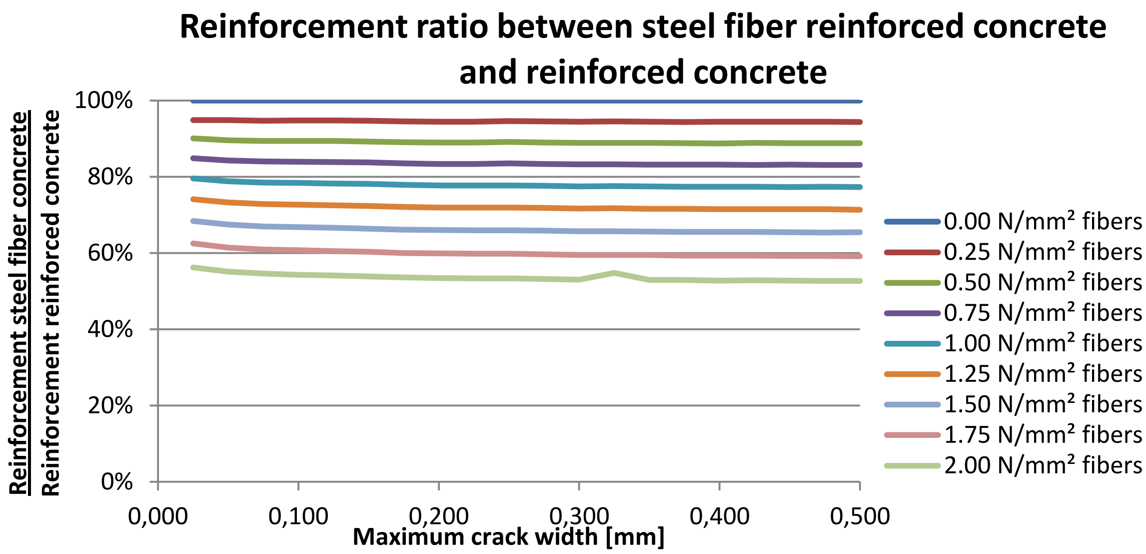
If the maximum crack width is taken equal for steel fibre reinforced concrete, less reinforcement is needed compared with ordinary reinforced concrete. The steel fibres transmit a part of the tensile force in the tensile zone and therefore the reinforcement has to transmit a lower force. Figure 6-2 shows that there is less reinforcement needed for the same maximum crack width, but it is not easy to make a comparison with ordinary reinforced concrete.



6-4 Reduction of the reinforcement over the maximum crack width

Figure 6-4 illustrates the reduction of the reinforcement as function of the maximum crack width. In this figure the reduction of the reinforcement in steel fibre concrete in comparison with ordinary concrete is easily seen. An example (see figure 6-2 point 3 and 4): if 2.00 N/mm² steel fibres is added with a maximum crack width of 0.1 mm, 4400 mm² less reinforcement is needed over the width of the bridge (2.5 meters).

The most interesting point is that there is a higher reduction of the reinforcement over a smaller maximum crack width. The reason can be found in figure 6-5.



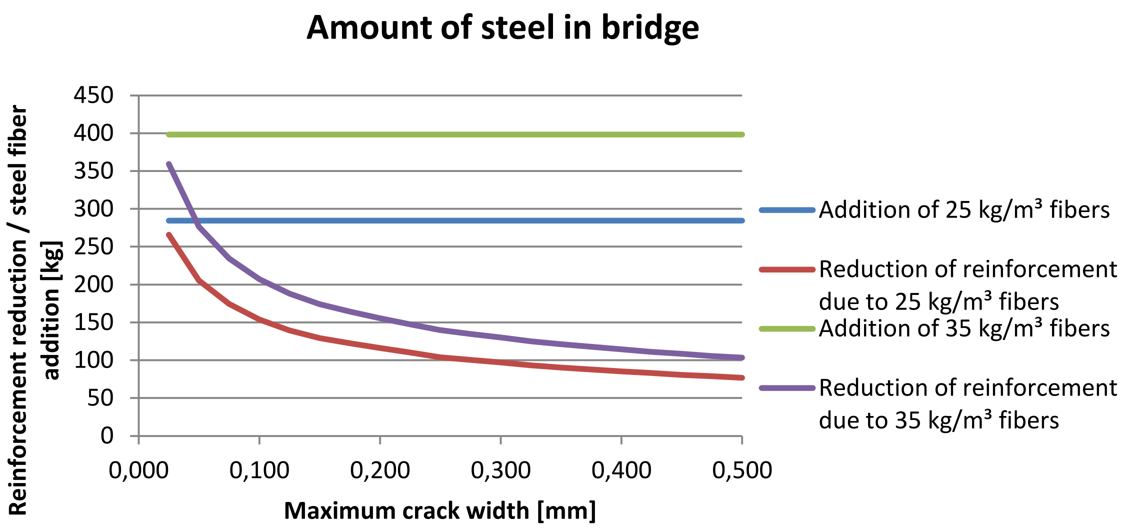
6-5 Reinforcement in Steel fibre reinforced concrete divided by reinforcement of ordinary reinforced concrete over maximum crack width

In this figure the ratio between the reinforcement of steel fibre reinforced concrete and that of reinforced concrete is a constant ratio over the maximum crack width. This constant ratio implies that there is no difference in ratio between low and high maximum crack widths. The higher reduction in figure 6-4 can be explained, namely there is more reinforcement needed at low maximum crack widths and with a constant ratio more reinforcement can be reduced.

6.4.4 Reduction of the amount of steel in the cross section

The reinforcement can be reduced if steel fibres are added. This paragraph shows the possible reduction of the total amount of steel in the structure. As reference the whole bridge is used with a thickness of 350 mm. The amounts of kilograms of steel fibres are for a given post-cracking strength:

Post-cracking strength	Steel fibres / m^3
0.75 N/mm^2	25 kg
1.00 N/mm^2	35 kg



6-6 Reduction of the amount of steel in the cross section

The prescribed amount of steel fibres gives the amounts of fibres in the cross-section: 284 and 398 kg for the whole bridge. Figure 6-6 shows the reduction of the reinforcement and the amount of added steel fibres. If the curve of the reduction is lower than the curve of the addition of the fibres, more steel is needed in the cross section if steel fibres are added. From figure 6-6 can be concluded that the addition of steel fibres causes more steel in the bridge. The increase at small crack width is bigger compared with higher crack widths. In this region more reinforcement is applied and a higher amount of reinforcement can be reduced. The amount of added steel fibres is constant.

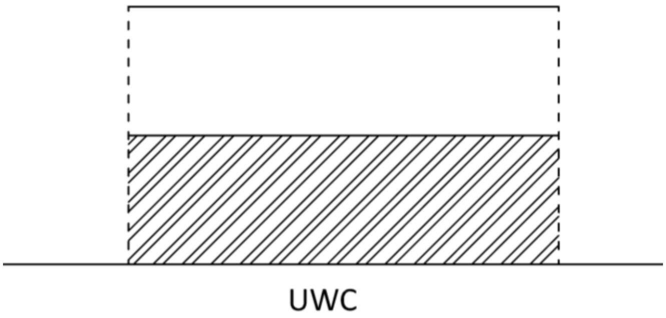
Due to the addition of steel fibres in the floor the total amount of kilograms steel in the cross section cannot be reduced. More kilograms of steel are needed in the bridge if steel fibres are added.

7. Case study 2: Floor

This case is a floor of a parking garage. This floor will be directly casted on an underwater concrete floor. This results in the case that the floor must be calculated with imposed deformation.
The floor is supported by concrete piles and on the floor columns will be placed for the structure above.

7.1 Overview of the project

This project contains the calculation of the bottom reinforcement of a floor of a parking garage. In the calculation two different calculations are made: one calculation for the imposed deformation and one calculation for the external forces. In the calculation of the imposed deformation only the bottom half of the floor is taken into account. This is because only the bottom reinforcement of the floor is calculated. The second calculation is the calculation of the external forces. In this calculation the whole cross section is taken into account, but only the bottom reinforcement is calculated.



7-1 Longitudinal cross section of the floor of the parking garage

The floor has the following dimensions:
thickness : 1200 mm
For the width and length a strip of 1000 mm is taken.
The floor is situated on an underwater concrete floor.

7.2 Structural requirements and material specifications

7.2.1 Concrete

Strength class	:	C28/35
Density reinforced concrete	:	25 kN/m³
Material factor	:	γ_c=1.50
Concrete cover at the reinforcement	:	60 mm

There is no specific requirement for the maximum crack width. The goal of this calculation is to show the influence of steel fibres on the cracking behaviour.

7.2.2 Reinforcement

Reinforcing steel	:	FeB 500	
Yield stress	:	435 N/mm²	$\gamma_s = 1.15$

7.2.3 Steel fibres

The post-cracking behaviour of steel fibre concrete is based on the performance based principle. For the post-cracking behaviour is a constant tensile stress in the fibres assumed over the cracked area.

7.3 Overview loads

- The self weight of the construction is the volume of the concrete multiplied with the density. Density of reinforced concrete: 25 kN/m³

  $$q_{sw} = bh\gamma_c = 1.0 \cdot 1.20 \cdot 25 = 30 \frac{kN}{m^2}$$

- Self weight superstructure
  The columns which support the superstructure consists point loads on the floor.
  This force differs from column to column, but the range is about 10.000-12.000 kN

- Water pressure
  From the bottom side water pressure causes a distributed load on the bottom side of the floor. This force due to the water pressure is because the groundwater level is higher than the bottom level of the floor. The groundwater level in this calculation is -3.0m NAP and the bottom level of the floor is -10.82 m NAP.
  These load case is the water pressure on the bottom side of the floor:

  $$(d - w.l.) \cdot \gamma_w$$

  $$(10.82 - 3.0) \cdot 10 = 78.2 kN / m^2$$

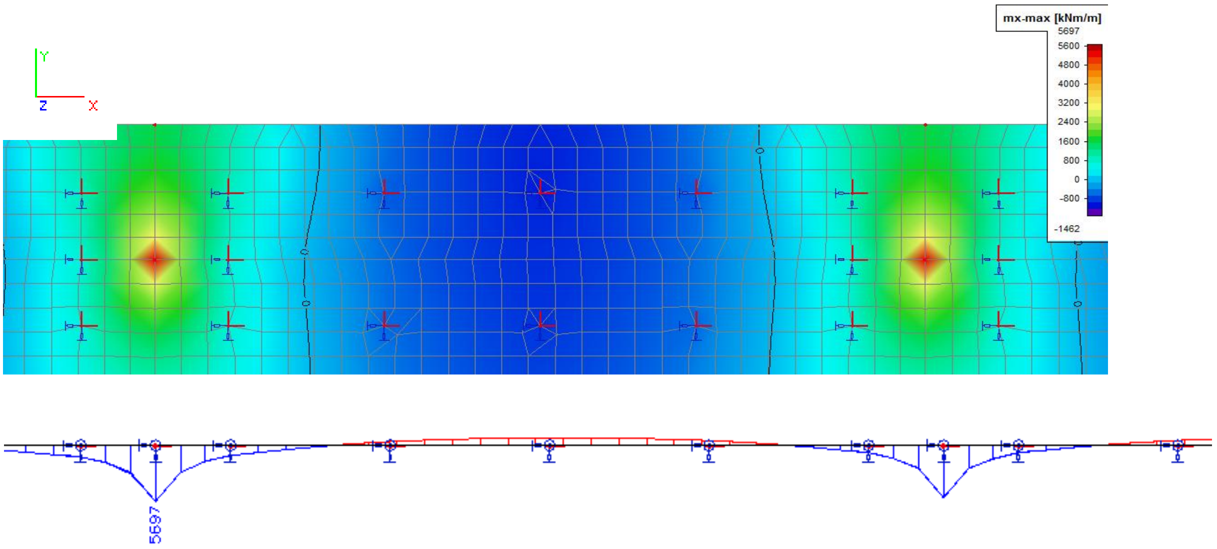
- Imposed deformation
  The strain due to the imposed deformation is explained in appendix 5.
  The imposed deformation is assumed as:

  $$\varepsilon = 0.00263$$

7.4 Bending moments in the Serviceability limit state

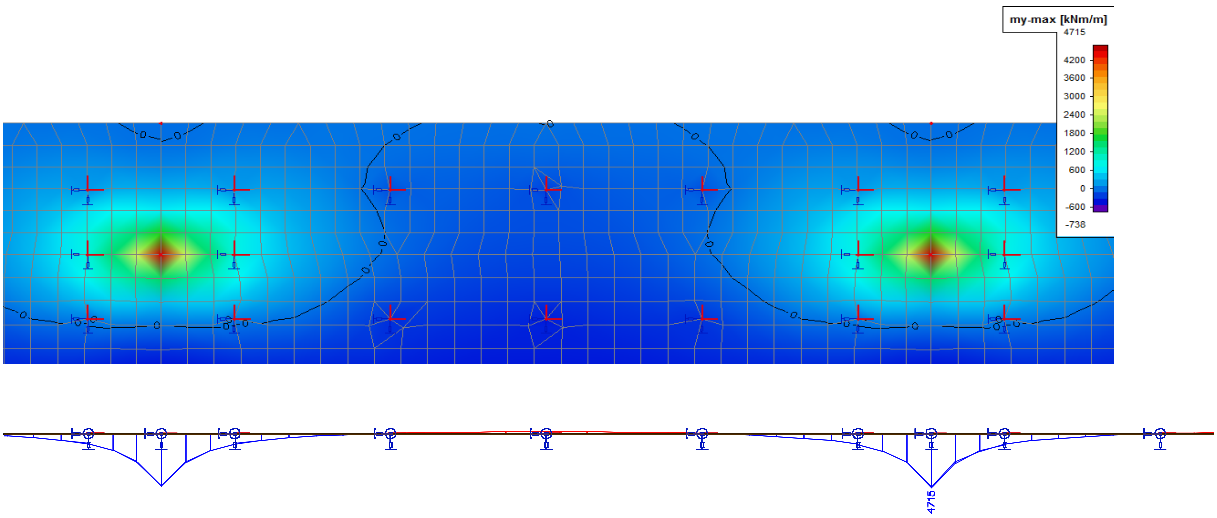
The loads described in 7.3 results in bending moments in the floor. The bending moments are:

Bending moments in the x-direction:



7-2 Bending moments in the x-direction

Bending moments in the y-direction:



7-3 Bending moments in the y-direction

The bending moments due to the external forces are shown in figure 7-2 and 7-3. The peak bending moments are caused by force in the columns. This load is schematized as a point load. In the calculation of the bending moments, due to this point load, high peaks are found. This high peaks can be distribute over a small region around it. The average moments are used in the calculation.

The maximum bending moment is:

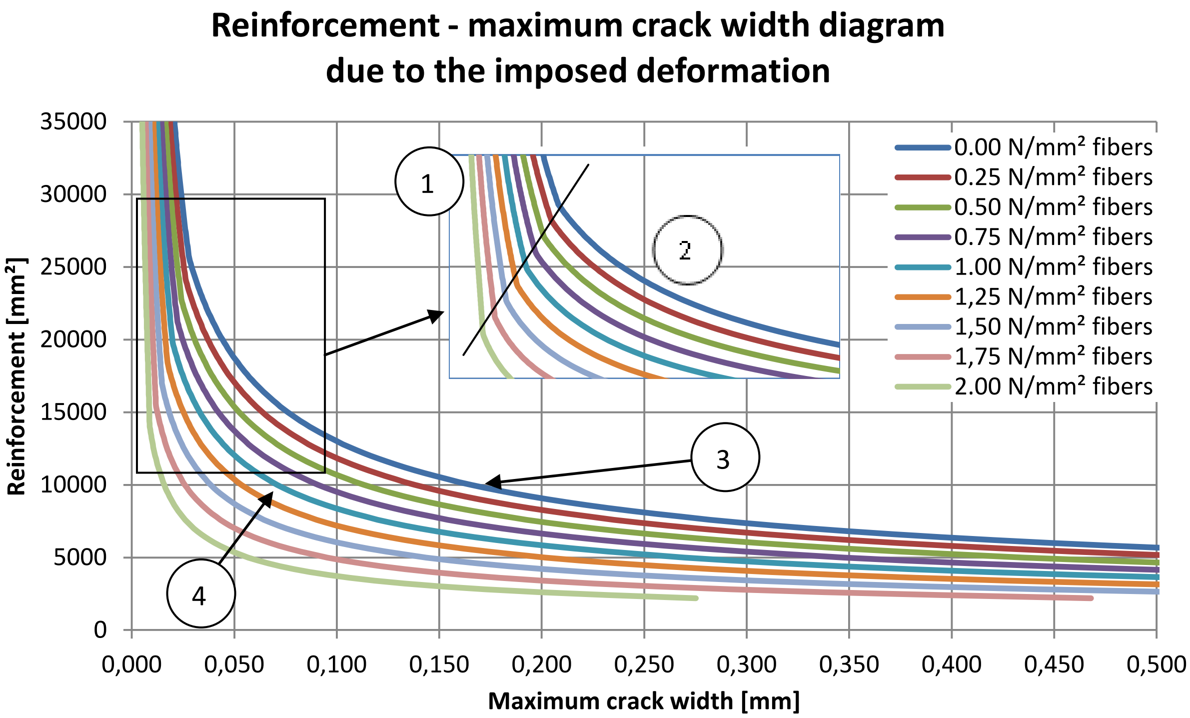
$$M_{ed} = 3540kNm$$

7.5 Results

In this paragraph the results are shown for the floor. The reinforcement and maximum crack width are calculated for the imposed deformation and the external loading apart. In this paragraph only the results of the calculation of the imposed deformation is shown. The results of the external forces can be found in appendix 5. Besides the results of the both load case apart, the results of the total loading can be seen in this paragraph.

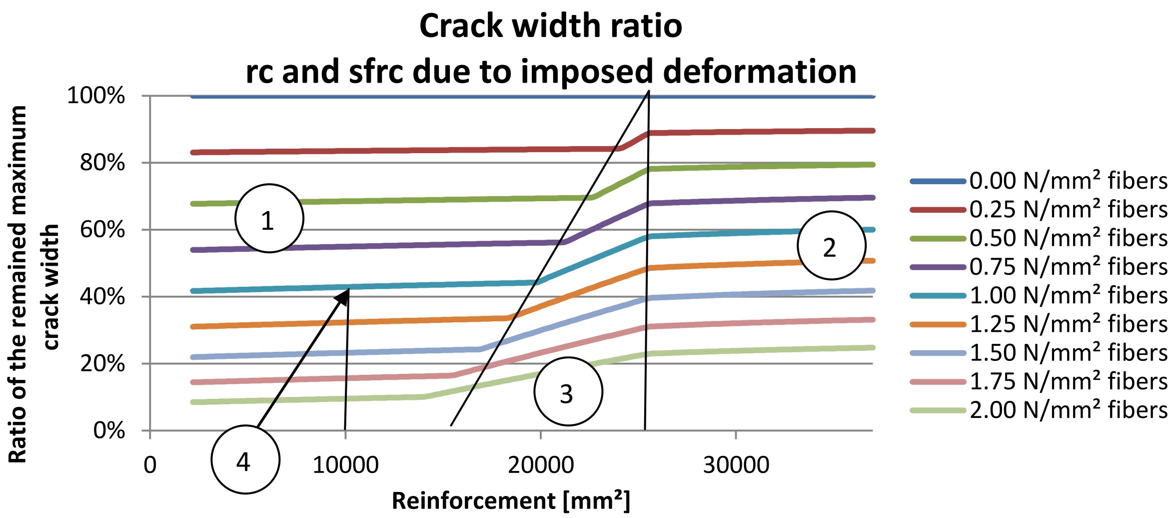
7.5.1 Calculation of the reinforcement due to imposed deformation

In this paragraph the reinforcement is calculated in the case of imposed deformation. The full calculation is presented in section 4.3 and appendix 5.



7-4 Reinforcement - maximum crack width due to imposed deformation

Figure 7-4 shows the reinforcement as function of the maximum crack width for reinforced concrete and steel fibre reinforced concrete. The effects of different amounts of steel fibres are shown. This figure has a lot of similarities with figure 6-2 which shows the reinforcement over the maximum crack width due to the external forces. The addition of steel fibres reduce the reinforcement at the same maximum crack width. The most interesting point is shown in the viewport in figure 7-4, which has a closer zoom to the graph at small crack widths. In this view the curves makes a small edge. This edge is the moment of the end of the crack formation stage and the start of the stabilized cracking stage. The stabilized cracking stage is called (1) and the crack formation stage (2). The amount of reinforcement decides in which stage the construction is and the strain, but the strain is constant in this case. At a high amount of reinforcement the beginning of the stabilized cracking stage is at a smaller strain than as for lower amounts of reinforcement. The size of the stiffening is the same as for lower amounts, but the stiffening is relative lower at higher amounts of reinforcement.



7-5 Crack width ratio due to imposed deformation

Figure 7-5 shows the crack width ratio due to the imposed deformation. In contrast to the crack width ratio due to the external forces, these ratios are not near constant. The crack width ratio increases at a higher amount of reinforcement. This means that the influence of the steel fibres reduces. At a reinforcement of about 25.000 mm² the ratio of the remaining crack width changes. This change is the difference between the stabilized cracking stage and the crack formation stage. The part with the lower ratio is the part in the crack formation stage. The crack formation stage is (1) in figure 7-5 and (2) is the stabilized cracking phase. Point (3) is the 'disturbed' area. At higher amounts of steel fibres the construction is in the stabilized cracking phase, but the reference in this figure (reinforced concrete without fibres) is in the crack formation stage. In this area a disturbed ratio is calculated.

Example

The points (3) and (4) on figure 7-4 are both points with the same amount of reinforcement, namely an amount of 10.000 mm². Point 3 is reinforced concrete without steel fibres and point 4 is the crack width with an addition of 1.00 N/mm² steel fibres. The crack width for reinforced concrete is 0.166 mm and for the one with an amount of 1.00 N/mm² fibres 0.071 mm.

$$\frac{w_{\max,\,fibers}}{w_{\max,0\%\,fibers}} = \frac{0.071}{0.166} = 42.7\%$$

This percentage can be found in figure 7-5 at point 4.

The following formulas are given in section 4.3.

The strain of the beginning of the fully development crack pattern:

$$\varepsilon_{fdc} = \frac{A_c f_{ctm}\left(1-0.5\frac{\sigma_f}{f_{ctm}}+\frac{E_c}{E_s}\frac{A_s}{A_c}\right)-0.4 f_{ctm} b h_{eff}}{A_s E_s}$$

Steel stress:

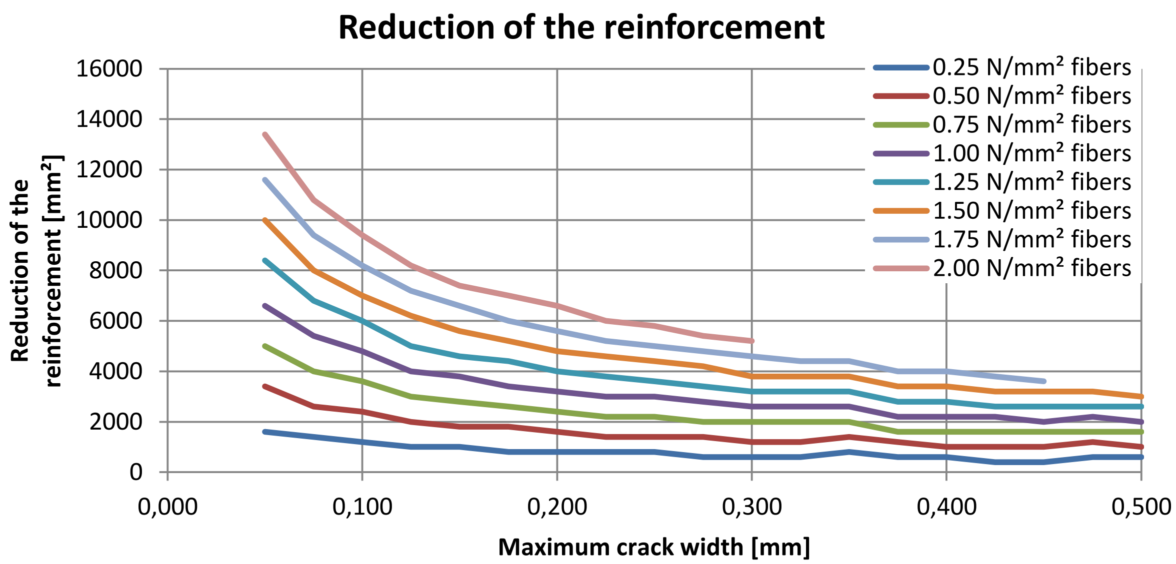
Crack formation stage : $$\sigma_s = \frac{f_{ctm}}{\rho}\left(\left(1-\frac{\sigma_f}{f_{ctm}}\right)+\frac{E_s}{E_c}\rho\right)$$

Stabilized cracking stage : $$\sigma_s = \frac{f_{ctm}}{\rho}\left(\left(1-\frac{\sigma_f}{f_{ctm}}\right)+\frac{E_s}{E_c}\rho\right)+E_s\left(\varepsilon-\varepsilon_{fdc}\right)$$

There is one factor which is the tensile stress of the steel fibres. This factor is the same for the crack formation stage as well for the stabilized cracking stage. In figure 4-5 can be seen that the strain of which place the stabilized cracking phase starts, is smaller if steel fibres are added. In the formula above for the stabilized cracking phase, this one has an extra part: $+E_s(\varepsilon - \varepsilon_{fdc})$. In this part the strain and the elasticity modulus of the steel does not change by the addition of steel fibres. Only the ε_{fdc} becomes smaller (as shown in figure 4-5) by the addition of steel fibres. This results in a relatively higher steel stress than for reinforced concrete. Therefore the ratio decreases.

7.5.2 Reduction of the maximum crack width

In the case of a prescribed maximum crack width, the addition of steel fibres results in a reduction of the reinforcement, because the tensile fibres transmit a part of the tensile forces and less reinforcement would be needed.



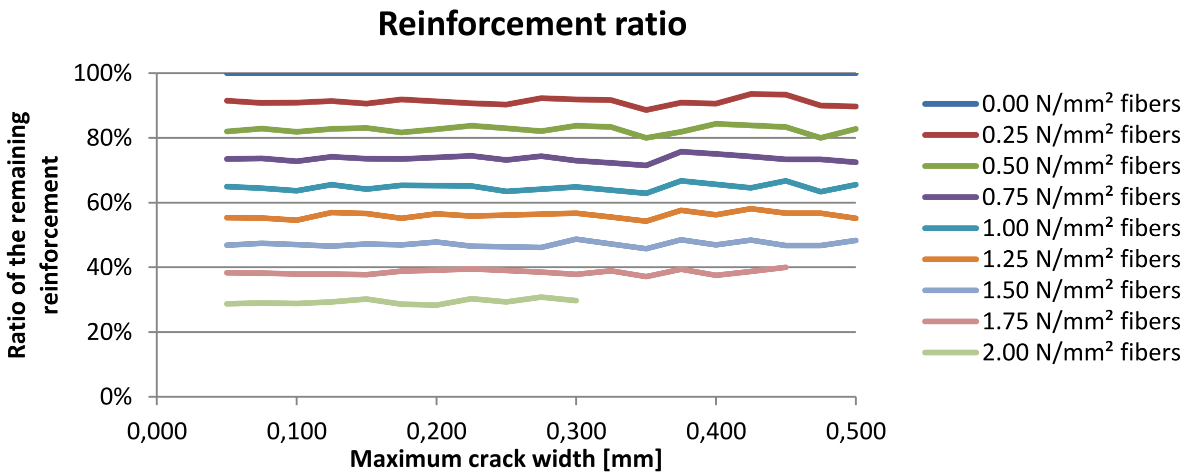
7-6 Reduction of the reinforcement in the situation of imposed deformation

Figure 7-6 shows the reduction of the reinforcement as function of the maximum crack width for the case with only imposed deformation. The reduction of the reinforcement has the same profile as the one in the situation of the external forces. For a maximum crack width of 0.20 mm with a steel tensile stress of 2.00 N/mm² the reinforcement can be reduced with 6600 mm² in this case. The reinforcement for the reinforced concrete case with this maximum crack width is 9100 mm².

The remaining reinforcement that is needed is:

$$\frac{A_{s,\,fibers}}{A_{s,0\%\,fibers}} = \frac{(9100-6600)}{9100} = 27\%$$

This percentage of the reinforcement can be seen in the figure below.



7-7 Ratio of the remaining reinforcement between RC and SFRC

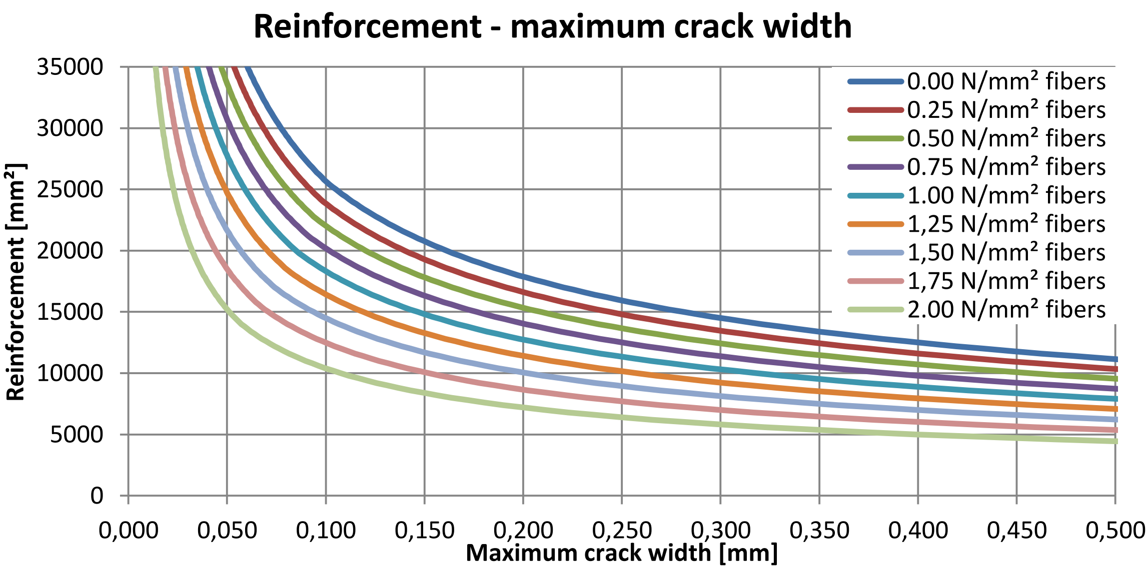
Figure 7-7 shows the remaining reinforcement ratio as function of the maximum crack width. This figure has a lot of similarities with the remaining reinforcement ratio with the external forces. The edge between the crack formation stage and the stabilized cracking stage cannot be seen in figure 7-7 other than in figure 7-5. The reason is that this point is at a crack width which is smaller than the 0.05mm. This crack width is outside the boundaries of figure 7-7.

7.5.3 Total loads

In this paragraph the crack width of the floor is calculated in the case of the total loads. The maximum crack width for the total loads (the external forces and the imposed deformation) can be found with the summation of the steel stresses of both:

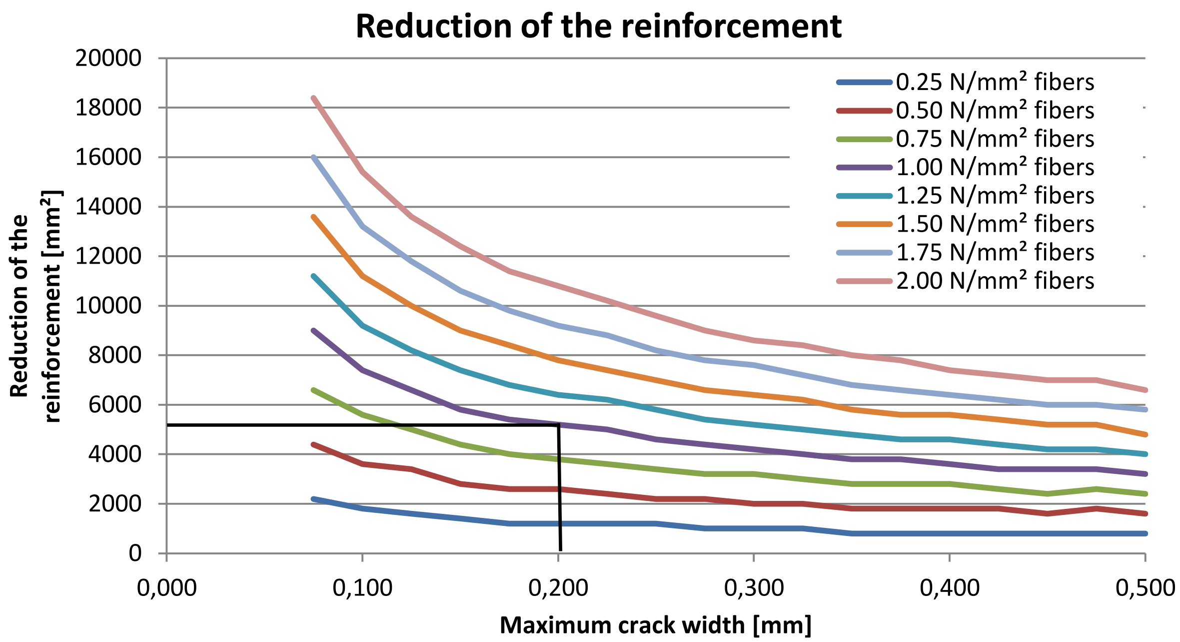
$$w_{\max} = 2l_t(\varepsilon_{sm} - \varepsilon_{cm}) = \frac{1}{2}\frac{\left(f_{ctm} - \sigma_f\right)}{\tau_{bm}}\frac{\phi}{\rho}\frac{1}{E_s}(\sigma_s - 0.5\sigma_{sr})$$

$$\sigma_s = \sigma_{s,ef} + \sigma_{s,imp}$$



7-8 Reinforcement - maximum crack width all loads

In figure 7-8 the reinforcement as function of the maximum crack width can be seen for the situation of the combination between imposed deformation and the external forces. The curves show the same curvature as for the both situations apart. This is logical, because the only difference is that this figure is calculated by summation of the both situations for the steel stress. All the other factors are equal.



7-9 Total reduction of the reinforcement

Figure 7-9 illustrates the total reduction of the reinforcement which can be reached by the addition of fibres to the floor of the parking garage. For a maximum crack width with an addition of 1.00 N/mm² fibres, a reduction of the reinforcement of 5200 mm² can be reached for the same maximum crack width. The original needed reinforcement (without steel fibres) was 18.000 mm². This is a reduction of:

$$\frac{A_{s,1.00N/mm^2}}{A_{s,tot}} = \frac{5200}{18000} = 29\%$$

29% of the reinforcement can be reduced if steel fibres are added to the construction.

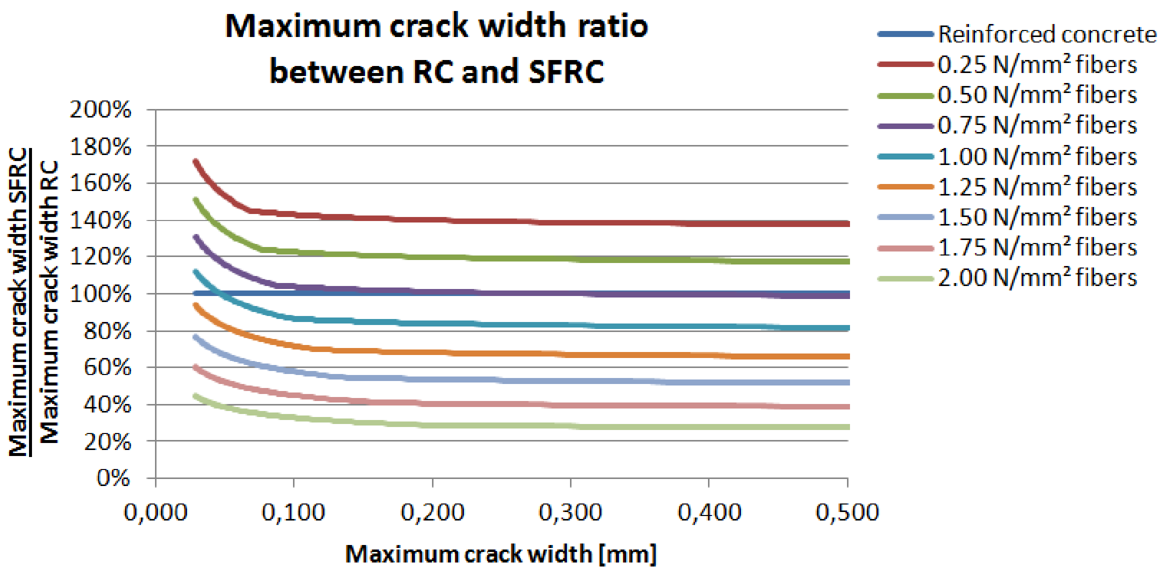
And in the case of the same amount of reinforcement for concrete with 1.00N/mm² and without steel fibres, the crack width can be reduced as:

$$\frac{w_{tot} - w_{1.00N/mm^2}}{w_{tot}} = \frac{0.2 - 0.103}{0.200} = 48\%$$

48% of the maximum crack width can be reduced if steel fibres are added.

7.5.4 Neglect ability of the imposed deformation

Is the imposed deformation neglectable if steel fibres are used? To answer this question, the comparison must be made between the maximum crack width due to the external forces in reinforced concrete and the maximum crack width due to the imposed deformation and the external forces in steel fibre reinforced concrete.



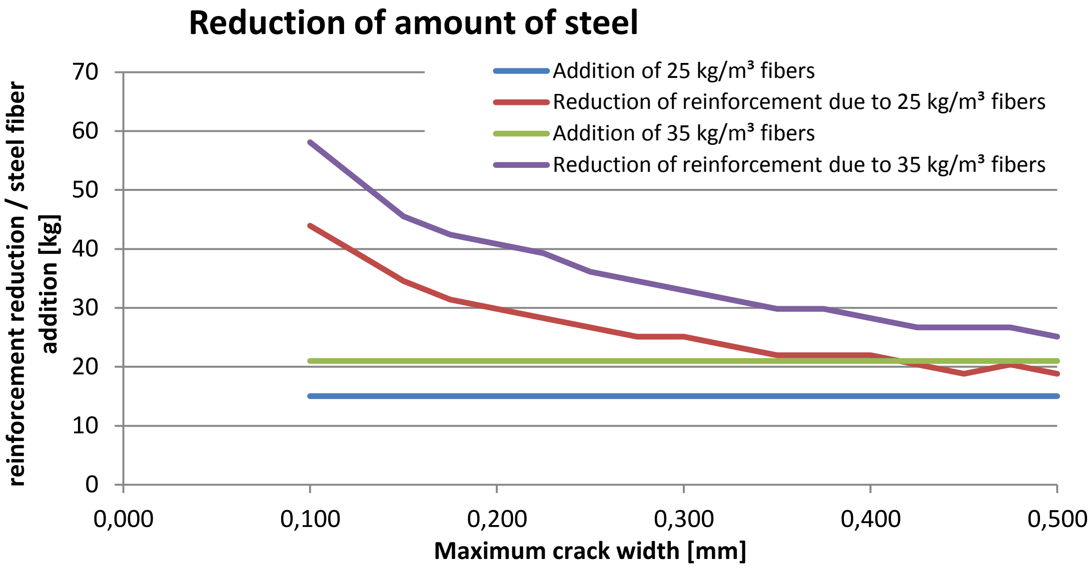
7-10 Maximum crack width SFRC compared with the maximum crack width of reinforced concrete

The figure 7-10 shows the maximum crack width of steel fibre reinforced concrete compared with reinforced concrete with only the external forces. If the concrete with steel fibres crosses the line of 100%, the maximum crack width is equal, with the same amount of reinforcement. The figure shows that for high amounts of steel fibres, the maximum crack width including the imposed deformation is smaller, than the maximum crack width in reinforced concrete with only the external forces. An addition of 0.75 N/mm² fibres gives the best comparison with the both.
The addition of 0.75 N/mm² is in this case enough to compensate the imposed deformation, if the reinforcement is calculated on the external forces. If the external forces or the imposed strain change, the ratio between the reinforcement due to the external forces and the reinforcement with steel fibres due to all the loads will change. It is not possible to use one standard amount of steel fibres to compensate the imposed deformation.

7.5.5 Reduction of the amount of steel in the cross section

The reinforcement can be reduced if steel fibres are added. This paragraph shows the possible reduction of the total amount of steel in the structure. As reference a part of 1 m² is used with a thickness of 600 mm (the bottom half of the floor is taken into account). An indication of the amounts of kilograms of steel fibres is for a given post-cracking strength:

Post-cracking strength	Steel fibres / m³
0.75 N/mm²	25 kg
1.00 N/mm²	35 kg



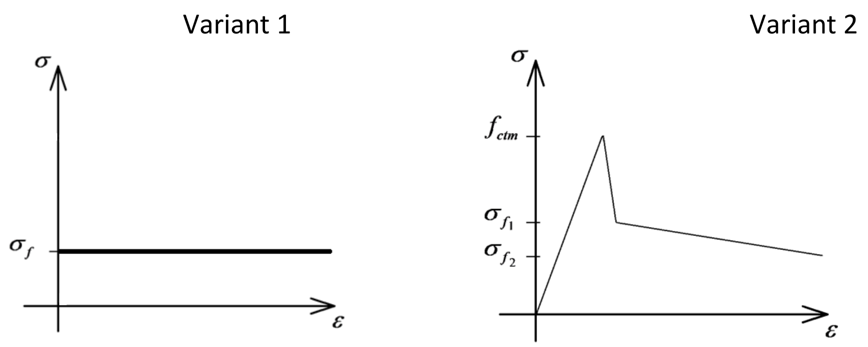
7-11 Reduction of the amount of steel in the cross section

The prescribed amount of steel fibres gives the amounts of fibres in the cross-section: 15 and 21 kg/m² (bottom half of the floor with a thickness of 600 mm is taken into account). Figure 7-11 shows the reduction of the reinforcement and the amount of added steel fibres. If the curve of the reduction is higher than the curve of the addition of the fibres, less steel is needed in the cross section. From figure 7-11 can be concluded that more reinforcement can be reduced, than that there steel fibres are added. Over the whole range of the maximum crack width (between 0.10 and 0.50 mm), the reduction is bigger than the addition of fibres. The decrease at small crack width is bigger compared with higher crack widths. In this region more reinforcement is applied and by the influence of the steel fibres more reinforcement can be reduced. The amount of added steel fibres is constant. The reduction of the reinforcement here is calculated in one direction. In this floor the peak moment is in two directions. The total reduction of the bottom side of the floor can be reduced by two times the reduction of the reinforcement which is calculated in figure 7-11.

Due to the addition of steel fibres in the floor the total amount of kilograms steel in the cross section can be reduced.

8. Comparison variant 1 and variant 2

In chapter 2 the assumption is made that the post-cracking behaviour of steel fibre concrete is constant. This results in a relative simple calculation for the maximum crack width. But how accurate is this assumption compared with the more realistic variant? In this chapter the constant post-cracking behaviour is compared with this more realistic variant. For both cases the comparison is made for the calculation of the bending moments. In the calculation of the bending moments the progress of the stress-strain diagram is in the calculation. For imposed deformation the strain is given. In variant 2 this results in a reduction of the post-cracking stress. This reduction of variant 2 can also be made by reducing the tensile stress in the fibres of variant 1. The difference between variant 1 and 2 for imposed deformations can be found in figure 7-5.

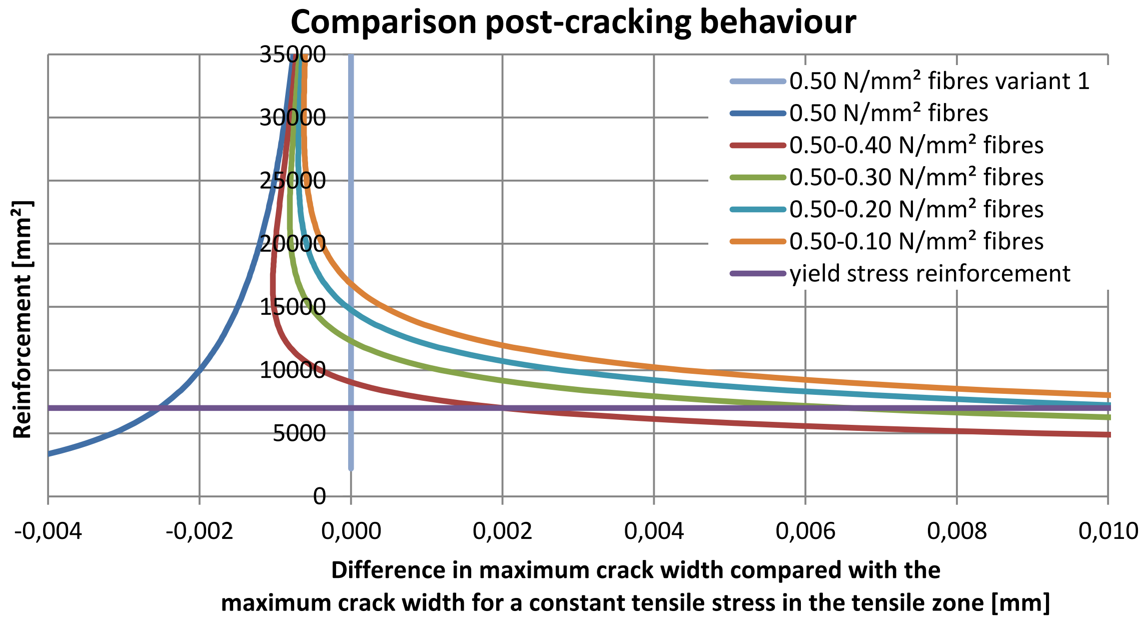


8-1 Stress strain diagram of variant the both variants

In figure 8-1 the post-cracking behaviour of both variants is shown. The full calculation sheet is presented in appendix 6 as well the results of the comparison of case study 1. For variant 2 the post-cracking stress reduces after cracking over a strain of 0.1‰ (see appendix 1) to the post-cracking stress σ_{f1}. At higher strains the post-cracking stress reduces to σ_{f2} at a strain of 20‰ [4,5].

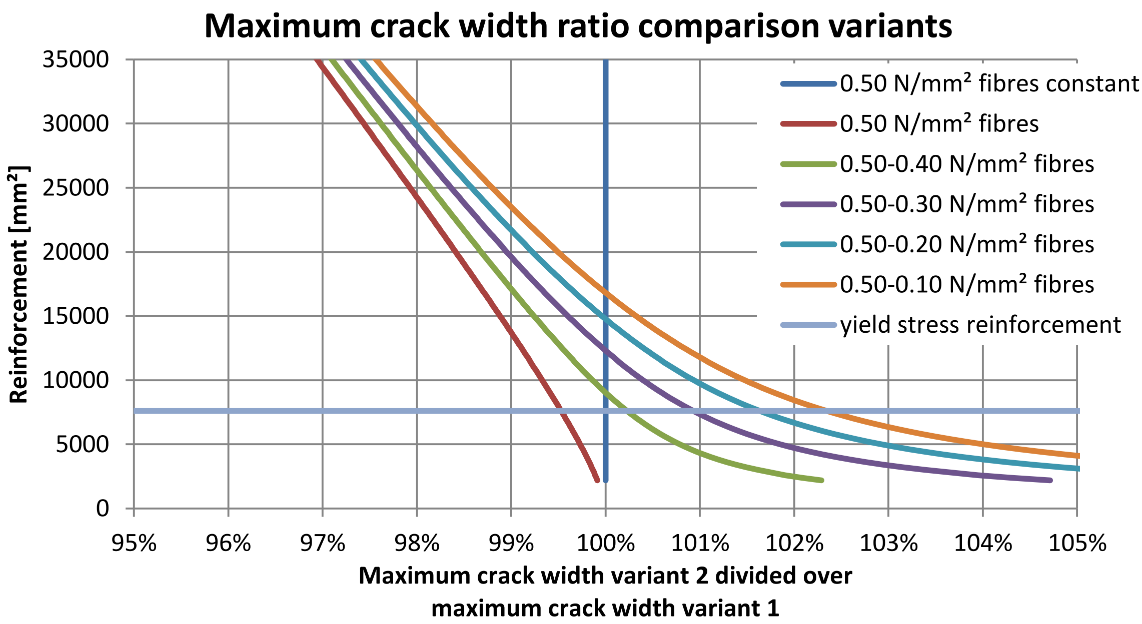
8.1 Comparison with a post-cracking stress of σ_f=0.50 N/mm²

In this paragraph the both variants are compared with a tensile stress in the steel fibres of variant 1 with a stress of 0.50 N/mm².



8-2 Comparison post-cracking behaviour with σ_f=0.50 N/mm²

In figure 8-2 the post-cracking behaviour is shown for the situation in case study 2 with a stress directly after cracking in the steel fibres of σ_f=0.50 N/mm². The maximum crack width of variant 2 is compared with the maximum crack width of variant 1. In the situation of variant 2 with a constant stress in the steel fibres a smaller maximum crack width is found in the range of 0.0007 and 0.0025 mm. The smaller crack width is caused by the higher average tensile stress over the area with the peak. In the case of a post-cracking behaviour with reducing tensile stresses at high amounts of reinforcement a reduction can be found due to the peak. At lower amounts of reinforcement an increase in the maximum crack width is found. This increase in the crack width is caused by the lower tensile stresses which are transmitted by the steel fibres. A bigger the reduction of the tensile stresses in the steel fibres results in a larger increase of the maximum crack width compared with variant 1.



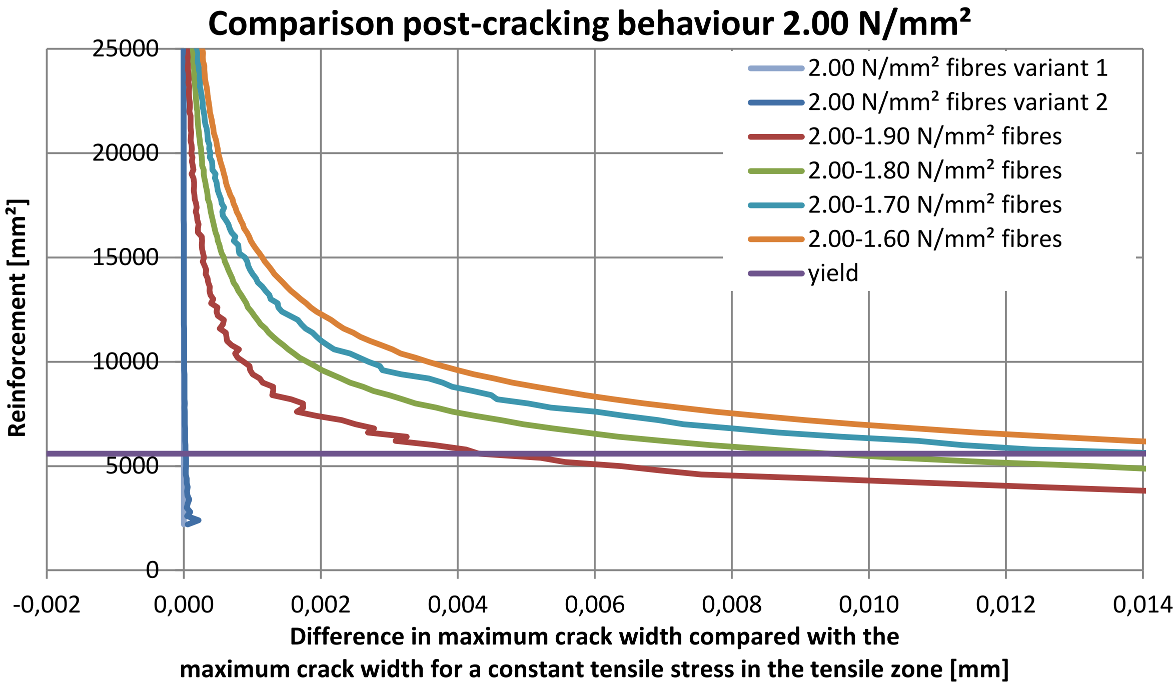
8-3 Maximum crack width ratio comparison

In figure 8-3 the difference in maximum crack width is shown expressed as a percentage of the maximum crack width of variant 1. The post-cracking behaviour of variant 2 is compared of that of variant 1. If the percentage is below the 100% the calculated maximum crack width of variant 2 is lower and this results in a overestimation of the crack width. For percentages higher than 100% the maximum crack width is underestimated. The size of this underestimation is important to know and the size is explained below with an example.

Example:
In the case of a reducing post-cracking behaviour with σ_{f1}=0.50 N/mm² and σ_{f2}=0.10 N/mm² the difference with variant 1 is 0.012 mm at an amount of reinforcement of 7400 mm². With a maximum crack width of 0.52 mm this results in a underestimation of 2.3%.

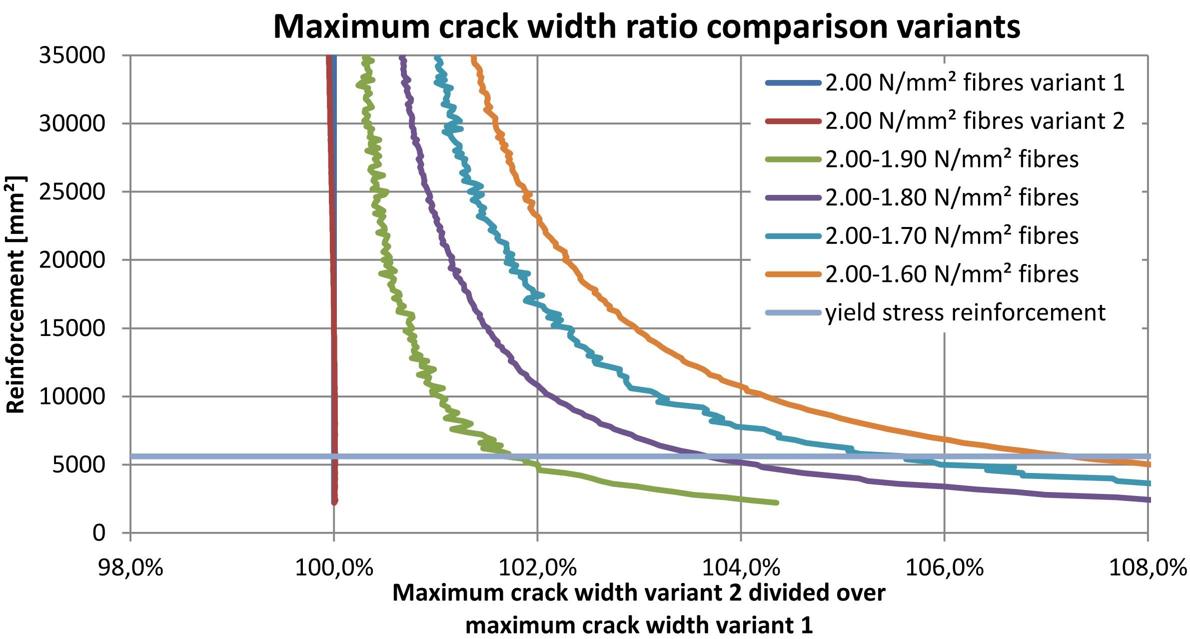
8.2 Comparison with a post-cracking stress of σ_f=2.00 N/mm²

In this paragraph the both variants are compared with a tensile stress in the steel fibres of variant 1 with a stress of 2.00 N/mm².



8-4 post-cracking behaviour with σ_f=2.00 N/mm²

In figure 8-4 the post-cracking behaviour is shown for the situation with a post-cracking stress of the steel fibres of σ_f=2.00 N/mm². The behaviour is the same as for the situation of a stress of 0.50 N/mm², but in this case the maximum crack width is higher compared with variant 1. The average tensile stress in the peak is smaller compared with the average tensile stress in the constant assumption. This results in larger crack widths.



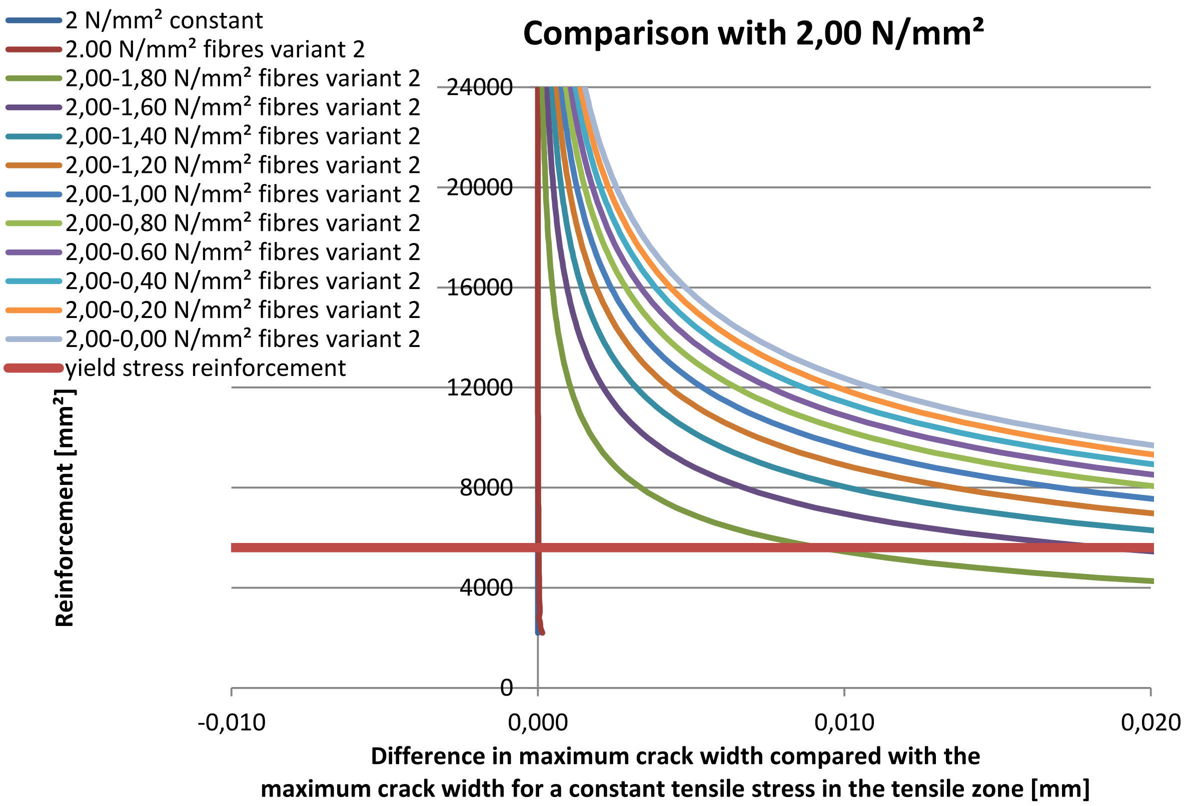
8-5 Maximum crack width ratio comparison between variants

In figure 8-5 the maximum crack width ratio is shown divided over the maximum crack width of variant 1. In the case of a reducing post-cracking stress the maximum crack width in variant 2 is bigger compared with variant 1. The size of the difference in maximum crack width is explained below with an example.

Example:
In the case of a reducing post-cracking behaviour with σ_{f1}=2.00 N/mm² to σ_{f2}=1.60 N/mm² the difference with variant 1 is 0.018 mm at an amount of reinforcement of 5600 mm². With a maximum crack width of 0.275 mm this results in a underestimation of 7.2%.

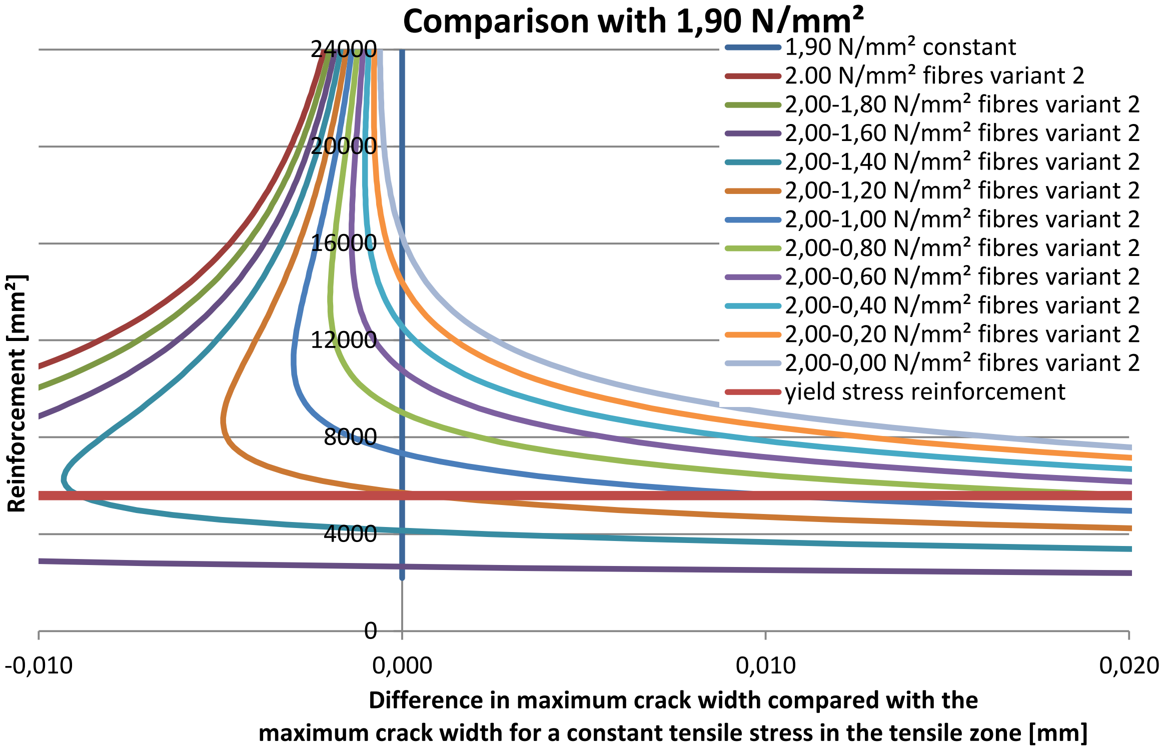
8.3 Higher reduction of the post-cracking stress variant 2

Both comparisons show that for a difference in stress between σ_{f1} and σ_{f2} of 0.40 N/mm² the difference in maximum crack width has a maximum 0.018 mm. For higher reductions of the post-cracking stress this difference in maximum crack width increases. In this section an approach is made to reduce the underestimation of the maximum crack width.



8-6 Comparison post-cracking behaviour with 2.00 N/mm²

Figure 8-6 shows the difference of the maximum crack width of variant 2 which is compared with the maximum crack width of variant 1. In this figure can be seen that for relative low amounts of reinforcement, the difference in maximum crack width is bigger. To reduce this underestimation of the maximum crack width, the tensile stress of the steel fibres in variant 1 is reduced. The results of this comparison can be found in figure 8-7.



8-7 Comparison post-cracking behaviour with 1.90 N/mm²

8.3.1 Calculation based on the maximum possible crack width

What is the point that this adjustment in the stress of the steel fibres of variant 1 should be done? For this question the following assumption is made: the maximum underestimation of the maximum crack width is maximum 0.02 mm. This assumption is done with the assumption that for practical applications mostly a maximum crack width of 0.2-0.3 mm is used. The underestimation of the maximum crack width has a maximum of 10%. The maximum difference of the crack width is calculated for the case at the amount of reinforcement that the reinforcement reaches the yield stress. The calculation of the maximum crack width is not taken into account. For maximum crack widths requested in practical applications, the difference in maximum crack width is smaller compared with the calculated maximum differences in this section.

With these assumptions, the distribution can be made of the post-cracking stresses that a reduction of the tensile stress for variant 1 is needed:

Variant 2: % of σ_{f2} / σ_{f1}	Variant 1: σ_f
100 – 80 %	0% reduction of σ_f
80 – 50 %	5% reduction of σ_{f1}
40 – 0 %	10% reduction of σ_{f1}

This table is based on the results of the case study of the floor. For the case of the bridge a lower reduction of the stress in the fibres of variant 1 is needed. The height of the construction affects mostly this difference in results. Over a larger area the lower stress is calculated, which results in a higher maximum crack width.
A reduction of the post-cracking stress in the steel fibres of variant 1 results in a reduction of the effects of steel fibres as shown in figures 6-3 and 7-5 by a few percent.

8.3.2 Calculation based on the practical maximum crack widths

The calculation in section 8.3.1 is based on the maximum possible crack widths. This calculated maximum crack widths are in the most cases above the requested maximum crack widths. The difference in maximum crack width increases a lot at amounts of reinforcement close to the yield stress of the reinforcement. In this section a overview is made for the practical maximum crack widths (0.1 – 0.2 – 0.3 mm) of both cases.

Case study 1 (bridge):

Difference in maximum crack width			
	Maximum crack width [mm]		
Post-cracking stress σf1-σf2	0,1	0,2	0,3
0,5 - 0,5	-0,0023	-0,0032	-0,0038
0,5 - 0,2	-0,0007	0,0011	0,0037
0,5 - 0,0	0,0003	0,0034	0,0082
1,0 - 1,0	-0,0015	-0,0021	-0,0024
1,0 - 0,5	0,0017	0,0063	0,0122
1,0 - 0,0			
2,0 - 2,0	0,0001	0,0001	0,0001
2,0 - 0,8	0,0128	0,0334	0,0600
2,0 - 0,0	0,0206	0,0494	0,0876

Case study 2 (floor):

Difference in maximum crack width			
	Maximum crack width [mm]		
Post-cracking stress σf1-σf2	0,1	0,2	0,3
0,5 - 0,5	-0,0014	-0,0017	-0,0020
0,5 - 0,2	-0,0004	0,0009	0,0028
0,5 - 0,0	0,0002	0,0027	0,0060
1,0 - 1,0	-0,0009	-0,0011	-0,0013
1,0 - 0,4	0,0016	0,0057	0,0112
1,0 - 0,0	0,0031	0,0096	0,0183
2,0 - 2,0	0,0000	0,0000	0,0000
2,0 - 1,4	0,0064	0,0175	0,0310
2,0 - 0,8	0,0115	0,0326	0,0515
2,0 - 0,0	0,0183	0,0470	0,0798

The differences between the both variants for ‘practical’ crack widths are small for a post-cracking stress which is 0.50 and 1.00 N/mm². For a post-cracking stress of 2.00 N/mm² the difference is higher and the difference is relative high (to 26% of the maximum crack width). From this overview of the difference in crack width at practical crack widths, no reduction is needed for a post-cracking stress of 1.00 N/mm² or lower. For the post-cracking stress of 2.00 N/mm² a reduction of 10% results in the same difference as for 1.00 N/mm².

8.4 Overview of the results

The comparison with a constant post-cracking stress the peak of the concrete tensile strength results in an overestimation of the maximum crack width for a post-cracking stress of 0.5 N/mm² and results in no difference of the maximum crack width for a post-cracking stress of 2.0 N/mm² (the difference in maximum crack width < 0.0003 mm). In the calculation with a constant post-cracking stress the peak can be neglected.

For a post-cracking stress of 0.50 N/mm² of variant 2 the difference of the maximum crack width with variant 1 is smaller compared with a post-cracking stress of 2.00 N/mm². The difference of these stresses is mostly caused by the peak in the post-cracking behaviour. This peak is independent of the post-cracking stress and has relative more influence on the maximum crack width of lower post-cracking stresses. The difference in percentage between the both post-cracking situations is mainly caused by the difference in maximum crack width. For higher post-cracking stresses lower crack widths are found and with the same crack width higher percentages.

If the post-cracking stress reduces, the height of the tensile zone and the reduction of the post-cracking behaviour are important. By using the same reduction of the post-cracking stress of variant 2, for the bridge the maximum crack width is less underestimated compared with the difference in maximum crack width of the floor.
For the bridge, the tensile stress in the fibres of variant 1 should be reduced with 5%, if the post-cracking stress of variant 2 reduces from 2.00 to 0.00 N/mm². While for the floor the reduction of the tensile stress of variant 1 should be 10%. The reduction of the tensile stress of variant 1 increases for higher heights of the construction.

Both cases show that a reduction of the tensile stress of variant 1 by a value of maximum 10% results in a small underestimation of the maximum crack width for a high reduction of the post-cracking stress of variant 2.

9. Conclusions and recommendations

9.1 Conclusions

The goal of this study was to study the cracking behaviour of steel fibre reinforced concrete in combination with steel fibres and to put the academic knowledge into practical knowledge, with a focus on the influence of the addition of the steel fibres. The goal is to use the academic knowledge from the various PhD-studies to the practical knowledge by making a derivation of the crack width expression and to use this expression in different case studies. The research questions are:

- "What is the influence of steel fibres on the crack width when combined with steel reinforcement?"
- "What is the amount of steel reinforcement which can be reduced if steel fibres are applied?"
- "What effective height can be taken into account when steel fibres are applied?"

9.1.1 Conclusions with respect to the crack width

From the case studies and the examples, it is clear that the addition of steel fibres has a significant influence on the crack width. In both case studies and the examples the addition of steel fibres reduces the maximum crack width. In the case of only imposed deformation, the reduction of the maximum crack width can be 90% in the case of a post-cracking stress of 2.00 N/mm² steel fibres. In the case of external loading, this reduction is about 75% in the case of 2.00 N/mm² steel fibres (section 6.4.2).

The reduction of the maximum crack width due to imposed deformation is dependent of the stage of the cracking phase. In the crack formation stage the reduction can be 90% in the case of a post-cracking stress of 2.00 N/mm² steel fibres. In the stabilized cracking phase the reduction can be 75%. The reduction in the crack formation stage is bigger, because the stress in the reinforcing steel is relative low and thereby steel fibres have a constant value of the post-cracking stress. The influence of the steel fibres is relative bigger for small steel stresses compared with higher steel stresses.

In practical application an amount of 25-35 kg/m³ steel fibres is used. With this amount of 25-35 kg/m³ steel fibres, the post-cracking stress is about 0.75-1.00N/mm². With this post-cracking stress the maximum crack width can be reduced by 45-55% in the crack formation stage and 30-40% in the stabilized cracking stage.

9.1.2 Conclusions with respect to the reduction of the amount of reinforcement

Not only the maximum crack width can be reduced, also the amount of reinforcement can be reduced in the case that the maximum crack width is taken equally.

In the case of imposed deformation the reinforcement can be reduced by 70%, while in the case of external loads the reduction is about 50%. The addition of steel fibres is more effective in the case of imposed deformation for the reduction of the reinforcement.

In both case studies more reinforcement can be reduced at lower maximum crack widths. At lower maximum crack widths a higher amount of reinforcement is used. The reduction of the reinforcement is a constant ratio over the maximum crack width. At low maximum crack widths, higher amounts of reinforcement are needed and more reinforcement can be reduced by the addition of steel fibres.

In practical application an amount of 25-35 kg/m³ steel fibres are used. With this amount of 25-35 kg/m³ steel fibres, the post-cracking stress is about 0.75-1.00N/mm². With this post-cracking stress the reinforcement can be reduced by 25-35% in the crack formation stage and 15-25% in the stabilized cracking stage.

9.1.3 Conclusions with respect to the effective height

As can be seen in section 3.5 the effective height of a reinforced concrete beam is the area to which the tensile forces can be transmitted from the reinforcing steel (in the crack) to the concrete by bond. From experiments the conclusion can be made, that the effective height can be written as a function of the cover of the concrete:

$$h_{eff} = \alpha(c + \phi/2) = 2.5(c + \phi/2)$$

No difference is found between plain concrete and steel fibre reinforced concrete. There is no difference in the effective height as in reinforced concrete:

$$h_{eff} = 2.5(h - d) = 2.5(c + \phi/2)$$

For plates also no differences can be found, also for plates the same expression as in reinforced concrete can be used:

$$h_{eff} = \frac{h - x}{3}$$

9.1.4 Conclusions with respect to the comparison of the variants

The comparison with a constant post-cracking stress the peak of the concrete tensile strength results in an overestimation of the maximum crack width for a post-cracking stress of 0.50 N/mm² and results in no difference for a post-cracking stress of 2.00 N/mm² (the difference in maximum crack width < 0.0003 mm). In the calculation with a constant post-cracking stress the peak can be neglected.

For a post-cracking behaviour with a reducing post-cracking stress the maximum crack width is in most cases underestimated. Therefore variant 2 cannot neglected in the case of a reducing post-cracking stress.

From both case studies the difference in maximum crack width is smaller than 0.02 mm for a reduction of 20% of the post-cracking stress direct after cracking. This is assumed to be acceptable and no reduction of the constant tensile stress of variant 1 is needed. For higher reductions of the post-cracking stress a reduction of the constant tensile stress of variant 1 is needed. For a total reduction of the post-cracking stress from 2.00 N/mm² to 0.00 N/mm², a reduction of the constant tensile stress by 10% results in a difference in maximum crack width which is smaller than the assumed 0.02 mm.

9.1.5 Conclusions with respect to Rotterdam Public Works

The question of Rotterdam Public Works was to study the behaviour of steel fibres and if possible to make some simple examples to show the influence of the addition of steel fibres. In this report is demonstrated, that it is possible to make simple calculations with steel fibre concrete. On basis of these calculations, the crack width of practical projects with steel fibre reinforced concrete can be calculated.

9.2 Future perspectives

Further optimisation of the expression of the maximum crack width may be possible through improving the results from the tests on the behaviour of the material of steel fibre concrete. The post-cracking behaviour and the bond behaviour are the biggest possible improvements.

- <u>Post-cracking behaviour</u>

In this study the post-cracking behaviour is schematized as a performance-based design. By this performance-design a constant tensile stress is taken into account for the addition of the steel fibres. In the literature survey is found that the correlation of steel fibre concrete is low. If in experiments the correlation of the results can be increased, a higher post-cracking stress can be found. The post-cracking stress is important in the calculation of the maximum crack width.

- <u>Bond behaviour</u>

Bigaj and Van Vliet made a literature overview of the influence of steel fibres on the bond behaviour. From this literature overview the following conclusion was made: "there is no agreement with respect of fibre content on bond stiffness" (the pre-peak behaviour). Conclusions vary from confirming increase of bond strength with increasing fibre volume fraction to proving any correlation false".

The bond strength is an important factor in the transmission length. If in experiments the correlation of the results can be increased, a better result of the increase in the bond strength can be found and therefore a better maximum crack width expression can be found.

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Appendices

Appendix 1: Test methods

Rilem-method (3-point bending test)

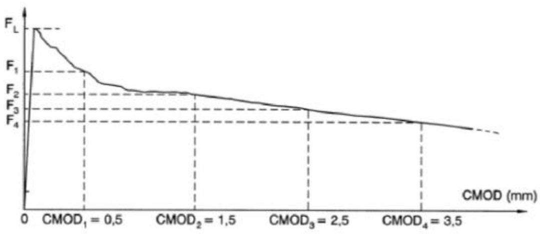
The Rilem-method is a method where the beam is loaded in bending. The standard test had a beam with a length of 550 to 600 mm and a span of 500 mm. The width and depth of the cross section is 150 mm. The beam will be loaded in the middle of the span. At this place, the cross section has a notch. This means that the crack will come at exact the middle of the span.

The test is performed under a Crack Mouth Opening Displacement (CMOD) control. At this way, the CMOD load and deflection of the prism can be measured. The beam is loaded and then the deflection of the CMOD is measured with a high frequency. The loading rate is 0.2mm/min.

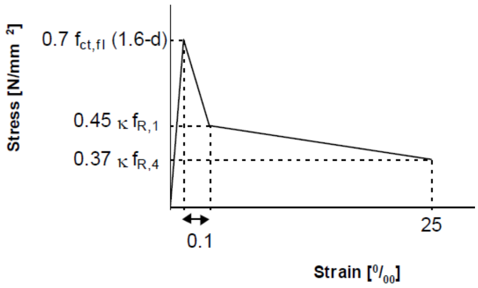
Determine the residual flexural tensile stresses

The result of the Rilem bending test is a load-CMOD diagram. From these results, it's possible to determine the residual flexural tensile stresses.
The tensile stresses are dependent of the measured load and the dimensions of the cross section of the Rilem beam. The residual flexure tensile stress is:

$$f_{R,i} = \frac{3F_{R,i}L}{2bh_{sp}^2}$$



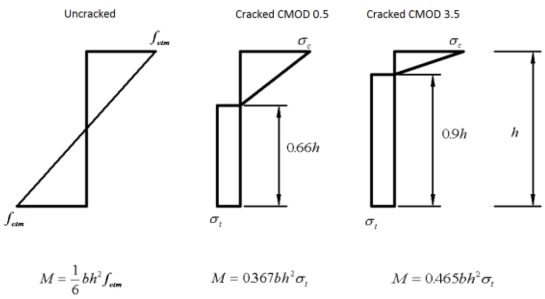
Load-CMOD diagram Rilem method



Stress strain diagram Rilem method

The flexural tensile strength is $0.7f_{ct}(1.6\text{-}d)$. Where the factor 0.7 takes the long term effects into account. The cracking stress is the flexural tensile stress and is $(1.6\text{-}d)f_{ctm}$.

The factors 0.45 and 0.37 are derived from the assumption that the height of the tensile zone at the CMOD of 0.5 and 3.5 mm are 66% and 90% respectively of the total height. See the figure right.
For the calculation of the post-cracking tensile strengths is made the assumption, which is illustrated at the left side of the figure right while the two assumptions right of this one are better assumptions and gives more realistic stress distributions in the section.

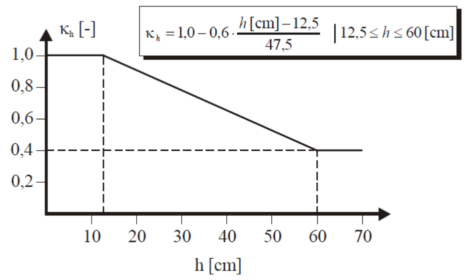


Cross sections of uncracked concrete, a CMOD of 0.5 mm and a CMOD of 3.5 mm

$$f_{fct,1} = \frac{0.166bh^2}{0.367bh^2} f_{fct} = 0.45 f_{fct}$$

$$f_{fct,4} = \frac{0.166bh^2}{0.45bh^2} f_{fct} = 0.37 f_{fct}$$

The factor κ in the stress strain diagram is the size factor. The size factor is introduced, because the method is developed without the height taking into account. In the results it was clear that the residual flexural tensile strength is overestimated. To get agreement with the tests, a reduction factor is introduced, the called size factor, which take the height of the beam into account. In a small beam the fibres are more orientated in the longitudinal direction, because the wall effects of the mould. In bigger beams, the fibres are orientated in all directions.



Size factor Rilem method

Advantages and disadvantages

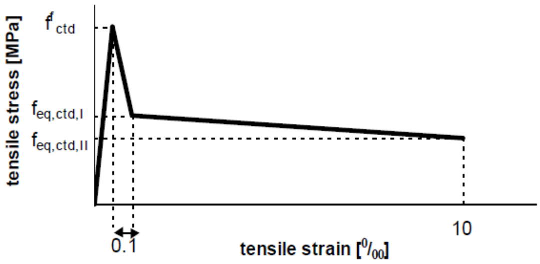
Advantage of this method is that it is a simple method. The test is performed under a CMOD-control, which is very easy to measure, because the crack is always on the same place. This is directly a disadvantage, because the crack in the middle isn't always the weakest spot of the beam. This can be resulted in too high post-cracking stresses. Also the height of the beam has a big influence in the post-cracking strength the biggest reason is described by the weakest-link model. Therefore the residual flexure stresses found in the small beam of the Rilem bending test are too high for bigger beams. Therefore a size effect factor is used in the calculation of the flexural tensile strength, because the effect of the notch and the stress gradient that forces the crack to initiate at the place of the tip of the notch.

The 4-points bending test

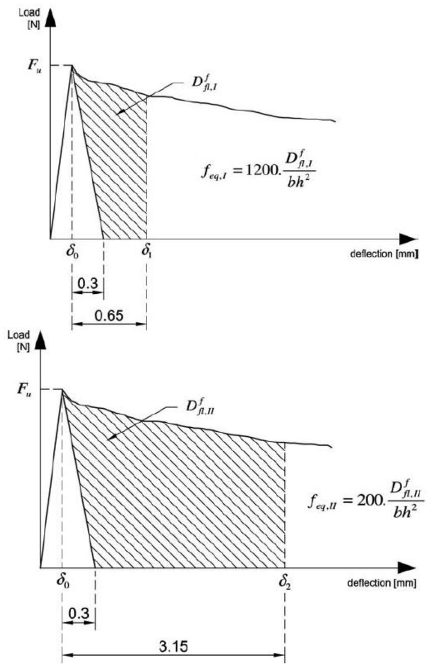
The 4-point bending test is used in several national standards. It's a standard test with a span of the beam of 450 to 600 mm, with 2 point loads at one third of the span. The beam has no notch. The loading rate and to calculation from the load-deflection curve to design parameters is dependent on the used standard.

Determine the flexural tensile stresses

The stress strain diagram follows from the measured load-deflection curve, which is the result of the 4-points bending test.



Stress strain diagram 4-points bending test



Flexural tensile stresses according the 4-points bending test

The 4-points bending test determines the flexural tensile stresses of the toughness value of the cross section. The toughness is the space underneath the load-deflection diagram.

The flexural tensile stresses calculated from the toughness values are:

$$f_{eq,1} = \frac{D_{f,1} l}{0.5bh^2}$$

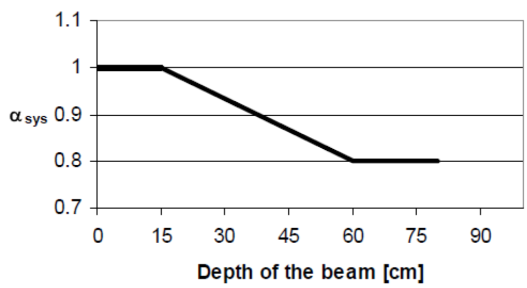
$$f_{eq,2} = \frac{D_{f,2} l}{3.0bh^2}$$

The 4-point bending test has like the Rilem method also the factors of 0.45 and 0.37. This has the same reason as the Rilem test, namely there is an assumption made, which is illustrated in part 0.

The flexural tensile stresses according the 4-points bending test are:

$$f_{eq,ctd,1} = 0.45 f_{eq,1} \alpha_c \alpha_{sys}$$

$$f_{eq,ctd,2} = 0.37 f_{eq,2} \alpha_c \alpha_{sys}$$



Size factor 4-points bending test

Here is the α_c a coefficient which takes the long term effects into account and α_{sys} is a size factor. The size factor is introduced, because the results of the calculations overestimate the tested flexural tensile stresses. The size factor of the 4-points bending test is much smaller as that one of the Rilem bending test. The reason of this is that the Rilem test the beam is notched and the crack will be at the place of the crack. In the 4-points bending test the beam cracks at the weakest spot of the beam, this gives better results and a lower size effect factor is needed. The size effect is also dependent of the orientation of the fibres. In a small beam the fibres are more orientated in the longitudinal direction, because the wall effects of the mould. In bigger beams, the fibres are orientated in all directions.

Advantage
Advantage of the test is that the beam has no notch. Also there is a zone where there is a constant maximum moment. In this zone, each section has an equal probability to failure. The crack can be occurring at the weakest place of the cross section. The absence of the notch also causes the stress gradient to be lower. These factors make that the 4-points bending test gives less difference between the axial strength and the flexural tensile strength measured with a Rilem test.
Disadvantage
Disadvantage of the test is that for specimens with small amounts of fibres, that there is often an unstable crack is formed. This means that the test can't be used. With the Rilem test, this doesn't occur.

Uni-axial test

At a uni-axial test is a cylinder of 150 mm width and 150 mm high. At mid-height there is a notch. Both sides of the cylinder are glued on a steel plate. This method is a very simple method, because the both steel plates can give the cylinder a prescribed deformation and stress can be measured. The result of this test is a stress-strain diagram. This is the diagram which easily can be used in practice. The disadvantage of the test is the uni-axial way of testing. If the test setup differs a small bit, an eccentricity arise and by this eccentricity a moment. This moment results in different values for the post-cracking strength.

Appendix 2: Differential equations

Reinforced concrete

The crack width of the concrete can also be calculated by a differential equation. In this paragraph the expression of the maximum crack width of reinforced concrete is derived.

The differential equation is based on that the first derivative of the crack width is the difference in strain between the concrete and steel:

$$\frac{dw}{dx} = \varepsilon_s - \varepsilon_c$$

The forces must be in equilibrium in the cross-section:

$$F_s + F_c = N$$

Here in is F_s the force in the steel and F_c the force in the concrete. These both forces must be in equilibrium in the situation as the concrete is uncracked and cracked. The difference of force between uncracked concrete and cracked concrete is the concrete tensile strength multiplied with the concrete surface. This force must be transmitted by bond to the reinforcing steel. The increase in force in the steel is the same bond stress, only acting in the opposite direction.

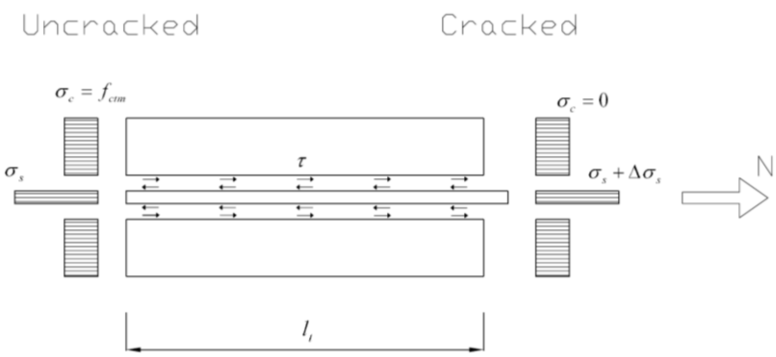
$$\Delta F_c = A_c f_{ctm} = \sigma_c A_c = -\tau \phi \pi l_t$$
$$\Delta F_s = A_s \Delta \sigma_s = \tau \phi \pi l_t$$

with l_t = transmission length
ϕ = diameter of the bar reinforcement
τ =bond strength

The crack width is the difference in strains between concrete and the reinforcing steel. The forces above can be rewritten in the strains:

$$\Delta F_c = A_c \sigma_c = A_c E_c \varepsilon_c$$
$$\varepsilon_c = \frac{F_c}{A_c E_c} = -\frac{\tau \phi \pi l_t}{A_c E_c}$$

$$\Delta F_s = A_s \Delta \sigma_s = A_s E_s \varepsilon_s$$
$$\varepsilon_s = \frac{\Delta F_s}{A_s E_s} = \frac{\tau \phi \pi l_t}{A_s E_s}$$



Overview of the stresses in the cross section

In the strains above the transmission length is still in the expression. This is an unknown variable. This transmission length can eliminate easily from the expression, namely to differentiate the both expression :

$$\frac{d\varepsilon_c}{dx} = -\frac{\tau\phi\pi}{A_c E_c} \quad \text{and} \quad \frac{d\varepsilon_s}{dx} = \frac{\tau\phi\pi}{A_s E_s}$$

The difference in strains is the first derivative of the crack width and the second derivative of the crack width is the derivative of the difference in strains:

$$\frac{dw}{dx} = \varepsilon_s - \varepsilon_c$$

$$\frac{d^2w}{dx^2} = \frac{d\varepsilon_s}{dx} - \frac{d\varepsilon_c}{dx}$$

Fill in the derivatives of the strains gives:

$$\frac{d^2w}{dx^2} = \frac{\tau\phi\pi}{E_s A_s} - -\frac{\tau\phi\pi}{E_c A_c} = \tau\phi\pi\left(\frac{1}{E_s A_s} + \frac{1}{E_c A_c}\right) = \frac{\tau\phi\pi}{E_s A_s}\left(1 + \frac{E_s A_s}{E_c A_c}\right)$$

$$\frac{d^2w}{dx^2} - \frac{\tau\phi\pi}{E_s A_s}(1 + n\rho) = 0$$

With:

$$n = \frac{E_s}{E_c} \quad \text{and} \quad \rho = \frac{A_s}{A_c}$$

And this gives the differential equation:

$$\frac{d^2w}{dx^2} - \frac{\tau\phi\pi}{E_s A_s}(1 + n\rho) = 0$$

The general solution of the differential equation can be found by integrating the equation two times:

$$\frac{d^2w}{dx^2} = \frac{\tau\phi\pi}{E_s A_s}(1 + n\rho)$$

$$\frac{dw}{dx} = x\frac{\tau\phi\pi}{E_s A_s}(1 + n\rho) + C_1$$

$$w = x^2\frac{\tau\phi\pi}{E_s A_s}(1 + n\rho) + C_1 x + C_2$$

With:

C_1 and C_2 integration constants

With the assumption of the origin of the coordinate system is in the middle between two cracks, the boundary conditions for this differential equation are:

$x = 0 \qquad w(0) = 0$

$x = l_t \qquad \sigma_c = 0 \quad => \quad \varepsilon_c = 0 \quad ; \quad \sigma_s = \frac{N}{A_s} \quad => \quad \varepsilon_s = \frac{N}{E_s A_s}$

This gives:

$$w(0) = 0^2 \frac{\tau\phi\pi}{E_s A_s}(1+n\rho) + C_1 0 + C_2 = 0$$

$$C_2 = 0$$

and:

$$\frac{dw}{dx} = l_t \frac{\tau\phi\pi}{E_s A_s}(1+n\rho) + C_1 = \varepsilon_s - \varepsilon_c = \frac{N}{E_s A_s}$$

$$\frac{\tau\phi\pi l_t}{E_s A_s}(1+n\rho) + C_1 = \frac{N}{E_s A_s}$$

$$C_1 = \frac{N}{E_s A_s} - \frac{\tau\phi\pi l_t}{E_s A_s}(1+n\rho) = \frac{N}{E_s A_s} - \frac{f_{ctm} A_c}{E_s A_s}(1+n\rho)$$

Fill in the integration constants in the general solution gives the expression of the crack width:

$$w(l_t) = l_t \frac{f_{ctm} A_c}{2 E_s A_s}(1+n\rho) + l_t \left(\frac{N}{E_s A_s} - \frac{f_{ctm} A_c}{E_s A_s}(1+n\rho) \right)$$

$$w(l_t) = l_t \frac{1}{E_s}\left[\frac{N}{A_s} - \frac{1}{2}\frac{f_{ctm}}{\rho}(1+n\rho) \right]$$

The only unknown in the calculation above is the length of the transmission length. This transmission length can be calculated with the following integral:

$$\Delta F_c = \int_0^{l_t} \tau\phi\pi dx$$

$$\Delta\sigma_c = f_{ctm} = \int_0^{l_t} \frac{\tau\phi\pi}{A_c} dx = \frac{\tau\phi\pi}{A_c} l_t - \frac{\tau\phi\pi}{A_c} 0 = \frac{\tau\phi\pi}{A_c} l_t$$

$$l_t = \frac{f_{ctm} A_c}{\tau\phi\pi} = \frac{1}{4}\frac{\phi}{\tau}\frac{f_{ctm}}{\rho}$$

Fill in the transmission length in the expression of the crack width, gives the final expression for the maximum crack width for fibre reinforced concrete.

$$w(l_t) = \frac{1}{4}\frac{\phi}{\tau}\frac{f_{ctm}}{\rho}$$

Combining the two expressions gives the maximum crack width:

$$w(l_t) = \frac{1}{4}\frac{\phi}{\tau}\frac{f_{ctm}}{\rho}\frac{1}{E_s}\left[\sigma_s - \frac{f_{ctm}}{2\rho}(1+n\rho) \right] = \frac{1}{4}\frac{\phi}{\tau}\frac{f_{ctm}}{\rho}\frac{1}{E_s}\left[\sigma_s - \frac{1}{2}\sigma_{sr} \right]$$

The maximum crack width is two times the calculated crack width above:

$$w_{\max} = 2w(l_t) = \frac{1}{2}\frac{\phi}{\tau}\frac{f_{ctm}}{\rho}\frac{1}{E_s}\left[\sigma_s - \frac{1}{2}\sigma_{sr} \right]$$

Steel fibre reinforced concrete

The crack width of the steel fibre concrete can also be calculated by a differential equation. In this paragraph the expression of the maximum crack width of steel fibre reinforced concrete is derived.

The differential equation is based on that the first derivative of the crack width is the difference in strain between the concrete and steel:

$$\frac{dw}{dx} = \varepsilon_s - \varepsilon_f - \varepsilon_c$$

The forces must be in equilibrium in the cross-section:

$$F_s + F_c = N$$

Here in is F_s the force in the steel and F_c the force in the concrete. These both forces must be in equilibrium in the situation as the concrete is uncracked and cracked. The difference of force between uncracked concrete and cracked concrete is the concrete tensile strength multiplied with the concrete surface. This force must be transmitted by bond to the reinforcing steel. The increase in force in the steel is the same bond stress, only acting in the opposite direction.

$$\Delta F_c = A_c f_{ctm} = \sigma_c A_c = -\tau \phi \pi l_t$$

$$\Delta F_s = A_s \Delta \sigma_s = \tau \phi \pi l_t$$

With: l_t = transmission length
ϕ = diameter of the bar reinforcement
τ =bond strength

The addition of the steel fibres has a direct influence on the stress in the reinforcing steel. The fibres are only active if the concrete is cracked. The steel fibres give a reduction of the stress in the reinforcing steel, if the concrete is cracked. The force which will be taken by the steel fibres is:

$$F_f = A_c \sigma_f = A_c \chi f_{ctm}$$

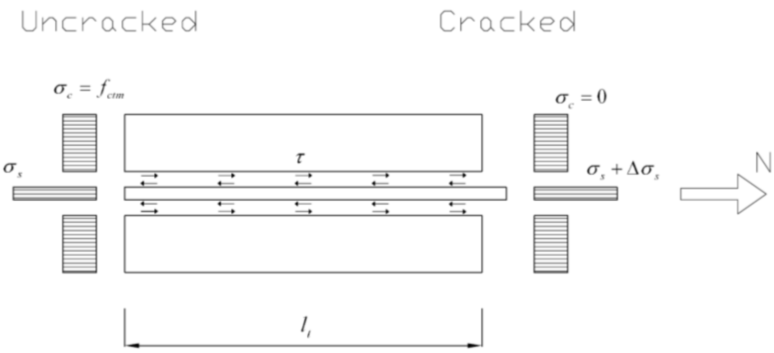
The force in the reinforcing steel is the total force in the uncracked situation minus the forces that will be taken by the steel fibres:

$$N = F_s + F_c = F_s + F_c + \Delta F_s + \Delta F_c - F_f$$

$$N = F_s + F_c = F_s + \Delta F_s - F_f$$

The expression above gives the equilibrium in forces between the uncracked and the cracked situation. If the concrete is cracked, the force what was taken by the concrete must be taken by the steel fibres and the reinforcing steel:

$$F_c = \Delta F_s - F_f$$



Stresses in cross section in cracked and uncracked situation

The crack width is the difference in strains between concrete and the reinforcing steel. The forces above can be rewritten in the strains:

$$\Delta F_c = A_c \sigma_c = A_c E_c \varepsilon_c$$

$$\varepsilon_c = \frac{F_c}{A_c E_c} = -\frac{\tau \phi \pi l_t}{A_c E_c}$$

$$\Delta F_s = A_s \Delta \sigma_s = A_s E_s \varepsilon_s$$

$$\varepsilon_s = \frac{\Delta F_s}{A_s E_s} = \frac{\tau \phi \pi l_t}{A_s E_s}$$

The strain in the steel fibres is the reduction of the steel force and is the strain dependent of the Young's modulus and reinforcing area of the reinforcement. The strains above are rewritten as a part of the force that can be transmitted by bond. Steel fibres have no influence on the bond strength and have also no influence in the force that can be transmit by bond. To get an expression which has the same approximations, the force that is taken by the steel fibres is rewritten as an expression for the force that can be transmitted by bond:

$$F_f = \chi f_{ctm} A_c = \chi \tau \phi \pi l_t$$

$$\varepsilon_f = \frac{F_f}{A_s E_s} = \frac{\chi \tau \phi \pi l_t}{A_s E_s}$$

In the strains above the transmission length is still in the expression. This is an unknown variable. This transmission length can be eliminated easily from the expression, namely to differentiate the both expression:

$$\frac{d\varepsilon_c}{dx} = -\frac{\tau \phi \pi}{A_c E_c} \quad \text{and} \quad \frac{d\varepsilon_s}{dx} = \frac{\tau \phi \pi}{A_s E_s} \quad \text{and} \quad \frac{d\varepsilon_f}{dx} = \frac{\chi \tau \phi \pi}{A_s E_s}$$

The difference in strains is the first derivative of the crack width and the second derivative of the crack width is the derivative of the difference in strains:

$$\frac{dw}{dx} = \varepsilon_s - \varepsilon_f - \varepsilon_c$$

$$\frac{d^2 w}{dx^2} = \frac{d\varepsilon_s}{dx} - \frac{d\varepsilon_f}{dx} - \frac{d\varepsilon_c}{dx}$$

Fill in the derivatives of the strains gives:

$$\frac{d^2w}{dx^2} = \frac{\tau\phi\pi}{E_sA_s} + -\frac{\chi\tau\phi\pi}{E_sA_s} - -\frac{\tau\phi\pi}{E_cA_c} = \tau\phi\pi\left(\frac{(1-\chi)}{E_sA_s} + \frac{1}{E_cA_c}\right)$$

$$\frac{d^2w}{dx^2} = \frac{\tau\phi\pi}{E_sA_s}\left(1-\chi+\frac{E_sA_s}{E_cA_c}\right)$$

$$\frac{d^2w}{dx^2} - \frac{\tau\phi\pi}{E_sA_s}\left(1-\chi+n\rho\right) = 0$$

With:

$$n = \frac{E_s}{E_c}$$

$$\rho = \frac{A_s}{A_c}$$

And this gives the differential equation:

$$\frac{d^2w}{dx^2} - \frac{\tau\phi\pi}{E_sA_s}(1-\chi+n\rho) = 0$$

The general solution of the differential equation can be found by integrating the equation two times:

$$\frac{d^2w}{dx^2} = \frac{\tau\phi\pi}{E_sA_s}(1-\chi+n\rho)$$

$$\frac{dw}{dx} = x\frac{\tau\phi\pi}{E_sA_s}(1-\chi+n\rho) + C_1$$

$$w = x^2\frac{\tau\phi\pi}{E_sA_s}(1-\chi+n\rho) + C_1x + C_2$$

With:

C_1 and C_2 integration constants

With the assumption of the origin of the coordinate system is in the middle between two cracks, the boundary conditions for this differential equation are called below. In these boundary conditions it is assumed that there will be no increase in strain of the concrete.

$$x = 0 \qquad w(0) = 0$$

$$x = l_t \qquad \sigma_c = 0 \quad => \quad \varepsilon_c = 0 \quad ; \quad \sigma_s = \frac{N - \alpha f_{ctm}A_c}{A_s} \quad => \quad \varepsilon_s = \frac{N - \alpha f_{ctm}A_c}{E_sA_s}$$

The first expression is that the crack width on x=0 must be 0. The gives:

$$w(0) = 0^2\frac{\tau\phi\pi}{E_sA_s}(1-\chi+n\rho) + C_1 0 + C_2 = 0$$

$$C_2 = 0$$

and:

$$\frac{dw}{dx}=l_t\frac{\tau\phi\pi}{E_sA_s}(1-\chi+n\rho)+C_1=\varepsilon_s-\varepsilon_c=\frac{N-\alpha f_{ctm}A_c}{E_sA_s}$$

$$\frac{\tau\phi\pi l_t}{E_sA_s}(1-\chi+n\rho)+C_1=\frac{N-\alpha f_{ctm}A_c}{E_sA_s}$$

$$C_1=\frac{N-\alpha f_{ctm}A_c}{E_sA_s}-\frac{\tau\phi\pi l_t}{E_sA_s}(1-\chi+n\rho)=\frac{N}{E_sA_s}-\frac{\alpha f_{ctm}A_c}{E_sA_s}-\frac{f_{ctm}A_c}{E_sA_s}(1-\chi+n\rho)$$

Fill in the integration constants in the general solution gives the expression of the crack width:

$$w(l_t)=l_t\frac{f_{ctm}A_c}{2E_sA_s}(1+n\rho)+l_t\left(\frac{N}{E_sA_s}-\frac{\alpha f_{ctm}A_c}{E_sA_s}-\frac{f_{ctm}A_c}{E_sA_s}(1-\chi+n\rho)\right)$$

$$w(l_t)=l_t\frac{1}{E_s}\left[\frac{N-\alpha f_{ctm}A_c}{A_s}-\frac{1}{2}\frac{f_{ctm}}{\rho}(1-\chi+n\rho)\right]$$

The only unknown in the calculation above is the length of the transmission length. This transmission length can be calculated with the following integral:

$$\Delta F_c=\int_0^{l_t}\tau\phi\pi dx$$

With:

$$\Delta F_c=\left(f_{ctm}-\chi f_{ctm}\right)A_c$$

ΔF_c = The difference in concrete force which must be transmitted by bond over the transmission length

$$A_s=\frac{1}{4}\pi\phi^2$$

$$\Delta F_c=\int_0^{l_t}\tau\phi\pi dx$$

$$\Delta\sigma_c=f_{ctm}-\alpha f_{ctm}=\int_0^{l_t}\frac{\tau\phi\pi}{A_c}dx=\frac{\tau\phi\pi}{A_c}l_t-\frac{\tau\phi\pi}{A_c}0=\frac{\tau\phi\pi}{A_c}l_t$$

From this equation the transmission length can be found by rewriting the equation and this gives:

$$f_{ctm}-\chi f_{ctm}=\frac{\tau\phi\pi}{A_c}l_t$$

$$l_t=\frac{(1-\chi)f_{ctm}A_c}{\tau\phi\pi}=\frac{(1-\chi)f_{ctm}\phi A_c}{\tau A_s}=\frac{1}{4}\frac{\phi}{\tau}\frac{(1-\chi)f_{ctm}}{\rho}$$

The transmission length is:

$$l_t=\frac{1}{4}\frac{\phi}{\tau}\frac{(1-\chi)f_{ctm}}{\rho}$$

The maximum crack width is two times the crack width, which is calculated in the differential equation. This is because the differential equation takes only the half of the crack width. Fill in all the equations gives the maximum crack width:

$$w_{\max} = 2w\left(l_t\right) = 2l_t \frac{1}{E_s}\left[\frac{N - \chi f_{ctm} A_c}{A_s} - \frac{1}{2}\frac{f_{ctm}}{\rho}(1 - \chi + n\rho)\right]$$

$$w_{\max} = 2\frac{1}{4}\frac{\phi}{\tau}\frac{(1-\chi) f_{ctm}}{\rho}\frac{1}{E_s}\left[\frac{N - \chi f_{ctm} A_c}{A_s} - \frac{1}{2}\frac{f_{ctm}}{\rho}(1 - \chi + n\rho)\right]$$

$$w_{\max} = \frac{1}{2}\frac{\phi}{\tau}\frac{(1-\chi) f_{ctm}}{\rho}\frac{1}{E_s}\left[\sigma_s - \frac{1}{2}\sigma_{sr}\right]$$

With:

$$\sigma_s = \frac{N - \chi f_{ctm} A_c}{A_s}$$ Steel stress

$$\sigma_{sr} = \frac{f_{ctm}}{\rho}(1 - \chi + n\rho) = \frac{f_{ctm}}{\rho}(1 + n\rho) - \frac{\chi f_{ctm} A_c}{A_s}$$ Steel stress direct after cracking

Appendix 3: Crack width model UHSC

The basic tool to calculate the crack width in a concrete structure, is to use an axial concrete member subjected to axial tension. In this calculation, the same assumptions are made as before.

Uncracked stage

In the phase before the first crack is assumed, that the imposed strain increase gradually. In the uncracked stage, the strains of the reinforcing steel and the concrete are the same. The contributions of the reinforcing steel and the concrete carrying an external force N_{tot} are:

$$N_s = E_s A_s \varepsilon_s$$

$$N_c = E_c A_c \varepsilon_c$$

$$\varepsilon_s = \varepsilon_c = \varepsilon$$

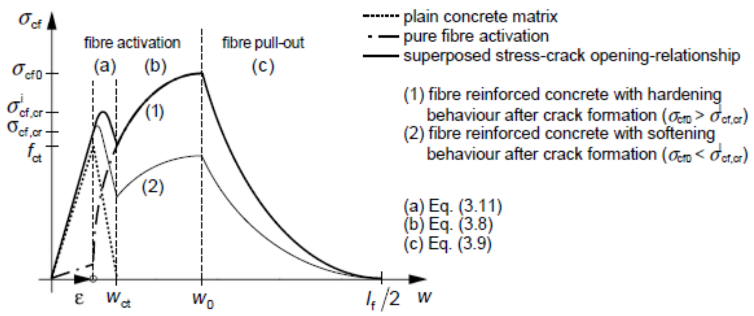
The total force is:

$$N_{tot} = N_s + N_c = E_s A_s \varepsilon + E_c A_c \varepsilon$$

$$N_{tot} = E_c A_c \varepsilon (1 + \rho \cdot \alpha_e) = A_c \sigma^i_{cf,cr} (1 + \rho \cdot \alpha_e) = \sigma_s A_s + \sigma_{cf} A_c$$

Where the ratio between the Young's moduli of steel and concrete and the reinforcement ratio are:

$$\alpha_e = \frac{E_s}{E_c} \qquad \rho = \frac{A_s}{A_c}$$



Stress – strain / crack width diagram for a ultra high strength concrete bar

$\sigma^i_{cf,cr}$ is the concrete tensile stress in case of strain hardening.

By increasing the total external force, the strain in the reinforcing steel and concrete will increase too. This process continues until the stress in the concrete reaches the concrete tensile strength ($\sigma^i_{cf,cr}$). If the external force increases, more microcracks will occur. The steel fibres participate in the load transfer. It is takes less energy to make a new micro crack than widen an existing micro crack to a macrocrack.

Crack formation stage

At the place of the crack the stress in the concrete will reduce to zero and the stress in the reinforcing steel will increase. The reinforcing steel carries the tensile force.

$$\sigma_{ct} = \sigma_{cf}$$

$$\sigma_s = \sigma_{sr}$$

$$\sigma_{sr} = \frac{N_{cr}}{A_s} = \frac{\sigma^i_{cf,cr}(1 + \rho \cdot \alpha_e) - \sigma_{cf}}{\rho_s}$$

$$\sigma_{se} = \alpha_e f_{ctm}$$

Because of the bond stresses between the reinforcing steel and the concrete, the concrete is active again to carrying the tensile force. This process is the case during the transmission length. At a certain distance (transmission length), the concrete carries its original part of the tensile force. Outside the transmission length, the strains of the reinforcing steel and the concrete are the same.

The force that must transmit over the transmission length:

$$N = A_c(\sigma^{i}_{cf,cr} - \sigma_{cf})$$

The force that is transmitted by the bond stresses over a distance l_t.

$$N = \tau_{bm} \cdot l_t \cdot m \cdot \pi \cdot \phi$$

With both expressions, the transmission length can be calculated:

$$\tau_{bm} \cdot l_t \cdot m \cdot \pi \cdot \phi = A_c(\sigma^{i}_{cf,cr} - \sigma_{cf})$$

$$\frac{\tau_{bm} \cdot l_t \cdot m \cdot \pi \cdot \phi}{A_s} = \frac{A_c(\sigma^{i}_{cf,cr} - \sigma_{cf})}{A_s} = \frac{(\sigma^{i}_{cf,cr} - \sigma_{cf})}{\rho}$$

$$A_s = \frac{1}{4} \cdot m \cdot \pi \cdot \phi^2$$

$$\frac{\tau_{bm} \cdot l_t \cdot m \cdot \pi \cdot \phi}{\frac{1}{4} \cdot m \cdot \pi \cdot \phi^2} = \frac{(\sigma^{i}_{cf,cr} - \sigma_{cf})}{\rho}$$

This gives a transmission length of:

$$l_t = \frac{1}{4} \frac{(\sigma^{i}_{cf,cr} - \sigma_{cf})}{\tau_{bm}} \frac{\phi}{\rho} \qquad \frac{dF_s}{dx} = -\frac{dF_c}{dx} = \phi \tau \pi$$

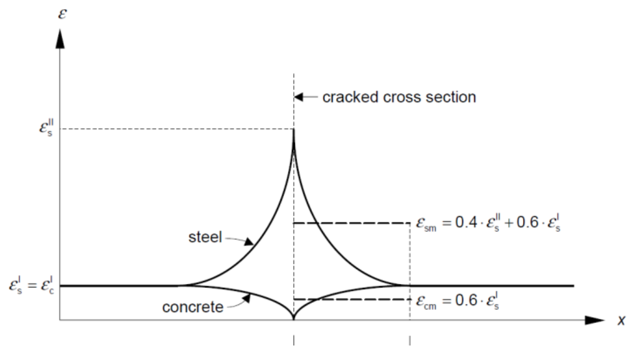
The crack width is the difference in elongation between the reinforcing steel and the concrete of the disturbed zone. The disturbed zone has a maximum length of two times the transmission length.

$$w = 2l_t\left(\varepsilon_{sm} - \varepsilon_{cm}\right)$$

Where ε_{sm} is the mean steel stress during the transmission length and ε_{cm} the mean concrete stress:

Mean steel stress:

$$\varepsilon_{sm} = 0.4\varepsilon_s + 0.6\varepsilon_c = \frac{2}{5E_s}\sigma_{sr} + \frac{3}{5E_s}\sigma^{i}_{cf,cr}\alpha$$



Steel and concrete strain diagram close to a crack

Mean concrete stress:

$$\varepsilon_{cm} = 0.6\varepsilon_c = \frac{3}{5E_s}\sigma^{i}_{cf,cr}\alpha_e$$

The difference between both is:

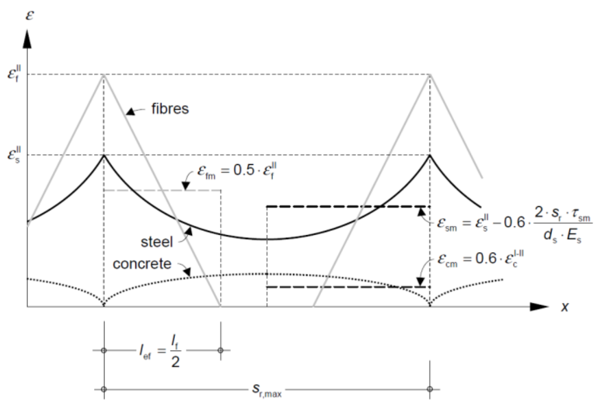
$$\left(\varepsilon_{sm} - \varepsilon_{cm}\right) = \frac{2}{5E_s}\sigma_{sr} + \frac{3}{5E_s}\sigma^{i}_{cf,cr}\alpha_e - \frac{3}{5E_s}\sigma^{i}_{cf,cr}\alpha_e = \frac{2}{5E_s}\sigma_{sr}$$

This gives a maximum crack width of:

$$w = 2l_t\left(\varepsilon_{sm} - \varepsilon_{cm}\right) = 2\frac{1}{4}\frac{f_{ctm}}{\tau_{bm}}\frac{\phi}{\rho}\frac{2}{5E_s}\sigma_{sr} = \frac{2}{5}\frac{f_{ctm}}{\tau_{bm}}\frac{\phi}{\rho}\frac{1}{E_s}\sigma_{sr}$$

Crack stabilized stage

When the strain is further increased, more and more cracks occur. The cracking process continues until the tensile bar consists of "disturbed regions" only. A further increase of the strain and as a result also an increase of the force N, the steel stress in the crack σ_s exceeds σ_{sr}. Because the force transmitted from the reinforcing steel to the concrete does not increase, the concrete strain between the cracks does not increase anymore. As a result, the increase of the crack width follows from the additional elongation of the steel only. The maximum crack width can be found with the largest possible transmission length, two times the transmission length is chosen for the calculation of the maximum crack width.



Steel and concrete strain diagram close to a crack in the crack stabilized stage

The strain in the reinforcement is:

$$\varepsilon_{sm} = \varepsilon_s - 0.6\frac{N_{trans}}{A_s E_s} = \varepsilon_s - 0.6\frac{A_c(\sigma^i_{cf,cr} - \sigma_{cf})}{A_s E_s} = \frac{\sigma_s}{E_s} - 0.6\frac{(\sigma^i_{cf,cr} - \sigma_{cf})}{\rho_s E_s}$$

Mean concrete stress:

$$\varepsilon_{cm} = 0.6\varepsilon_c = \frac{3}{5E_s}\sigma^i_{cf,cr}\alpha_e = 0.6\frac{\sigma^i_{cf,cr}\alpha_e\rho_s}{E_s\rho_s}$$

The difference between both is:

$$\left(\varepsilon_{sm} - \varepsilon_{cm}\right) = \frac{\sigma_s}{E_s} - 0.6\frac{(\sigma^i_{cf,cr} - \sigma_{cf})}{\rho_s E_s} - 0.6\frac{\sigma^i_{cf,cr}\alpha_e\rho_s}{E_s\rho_s} = \frac{1}{E_s}\left(\sigma_s - 0.6\frac{\sigma^i_{cf,cr}}{\rho_s}(1+\alpha_e\rho_s) - 0.6\frac{\sigma_{cf}}{\rho_s}\right)$$

$$\left(\varepsilon_{sm} - \varepsilon_{cm}\right) = \frac{1}{E_s}\left(\sigma_s - 0.6\sigma_{sr}\right)$$

The maximum crack width is the summation of the crack width in the crack formation stage and the elongation of the reinforcing steel in the crack stabilized stage:

$$w_{\max} = \frac{2}{4}\frac{(\sigma^i_{cf,cr} - \sigma_{cf})}{\tau_{bm}}\frac{\phi}{\rho}\frac{1}{E_s}\left(\sigma_s - 0.6\sigma_{sr}\right)$$

Appendix 4: Case study 1 (bridge)

Below the maple sheet and a hand calculation of the maximum crack width of case study 1 are shown:

Maple sheet calculation of the reinforcement

$$As := As : Es := 200000 : ds := h - c - 12 - 0.5\cdot\phi; b := 2500 : h := 350 : fcd := 23.3 : Med := 0.125\cdot\left(25\cdot\frac{b\cdot h}{10^6} + 2 + 4.79\cdot\frac{b}{1000}\right)\cdot 13^2\cdot 10^6; Ec := \frac{fcd}{1.75\cdot 10^{-3}} : fctm := 3.2 : \phi := 16 : \tau bm := 2\cdot fctm : \sigma f := \sigma f : c := 30 :$$

$$7.573312500\,10^8$$

$$H := As\cdot Es\cdot \varepsilon c\cdot\left(\frac{ds - x}{x}\right) + \sigma f\cdot b\cdot(h - x) - 0.5\cdot\varepsilon c\cdot Ec\cdot b\cdot x = 0 :$$

$$M := As\cdot Es\cdot\varepsilon c\cdot(ds - x)\cdot\left(\frac{ds - x}{x}\right) + \sigma f\cdot b\cdot(h - x)\cdot\frac{(h - x)}{2} + 0.5\cdot\varepsilon c\cdot Ec\cdot b\cdot x\cdot\frac{2\cdot x}{3} - Med = 0 :$$

$$solution := solve(\{H, M\}, \{\varepsilon c, x\}) :$$

$$assign(solution);$$

$$\sigma s := Es\cdot\varepsilon c\cdot\frac{(ds - x)}{x} :$$

$$heff := \frac{h - x}{3} : \rho eff := \frac{As}{b\cdot heff} :$$

$$\sigma sr := \frac{fctm}{\rho eff}\left(\left(1 - \frac{\sigma f}{fctm}\right) + \frac{Es}{Ec}\cdot\rho eff\right) :$$

$$wmax := \frac{1}{2}\,\frac{(fctm - \sigma f)}{\tau bm}\cdot\frac{\phi}{\rho eff}\cdot\frac{1}{Es}(\sigma s - 0.5\,\sigma sr) :$$

```
σf := 1 :
with(LinearAlgebra) :
N := rtable(1..91, 1..4) :
print(N) :

for z from 1 by 1 to 91 do
As := 200 + (z − 1)·200 :

for n from 1 by 1 to 1 do

N[z, 1] := As :
N[z, 1 + n] := evalf(wmax) :
N[z, 2 + n] := evalf(σs) :
N[z, 3 + n] := evalf(εc) :

print(N)
end do; end do;

with(ExcelTools) :
Export(N) :
```

Hand calculation case 1:

As	:	10.000	mm²
Med	:	757	kNm
Ds	:	h-c-12-0.5ϕ= 300	mm
H	:	350	mm
Ec	:	fcd/1.75e^{-3}	N/mm²
σ_f	:	0.00	N/mm²
σ_f	:	1.00	N/mm²

For σ_f=0 N/mm²:

$$\Sigma H = 0 \qquad A_s E_s \varepsilon_c \left(\frac{d_s - x}{x} \right) + \sigma_f b (h - x) - \frac{1}{2} b E_c \varepsilon_c x = 0$$

$$\Sigma M = 0 \qquad A_s E_s \varepsilon_c (d_s - x) \left(\frac{d_s - x}{x} \right) + \sigma_f b (h - x) \left(\frac{h - x}{2} \right) - \frac{1}{2} b E_c \varepsilon_c x \frac{2x}{3} - M_{ed} = 0$$

$$x = \frac{-\left(A_s E_s \varepsilon_c + \sigma_f bh\right) + \sqrt{\left(A_s E_s \varepsilon_c + \sigma_f bh\right)^2 - 4\left(\frac{1}{2} b E_c \varepsilon_c + \sigma_f b\right) - A_s E_s \varepsilon_c d_s}}{2\left(\frac{1}{2} b E_c \varepsilon_c + \sigma_f b\right)}$$

$$d_s = h - c - \phi_{stirrups} - \frac{\phi}{2}$$

This gives for x and ε_c:

$$x = 139.0668 mm$$

$$\varepsilon_c = 0.001290$$

$$\sigma_s = E_s \varepsilon_s = E_s \varepsilon_c \left(\frac{d_s - x}{x} \right) = 298 \frac{N}{mm^2}$$

$$h_{eff} = \frac{h - x}{3} = 70.31 mm$$

$$\rho_{eff} = \frac{A_s}{h_{eff} b} = 0.0569 \frac{N}{mm^2}$$

$$\sigma_{sr} = \frac{f_{ctm}}{\rho_{eff}} \left(1 + \frac{E_s}{E_c} \rho_{eff} \right) - \frac{\sigma_f}{\rho_{fibers}} = 104.31 \frac{N}{mm^2}$$

$$w_{max} = \frac{1}{2} \frac{f_{ctm} - \sigma_f}{\tau_{bm}} \frac{\phi}{\rho_{eff}} \frac{1}{E_s} (\sigma_s - 0.5 \sigma_{sr}) = 0.0866 mm$$

For σ_f=1 N/mm²:

$$\Sigma H = 0 \qquad A_s E_s \varepsilon_c \left(\frac{d_s - x}{x}\right) + \sigma_f b(h-x) - \frac{1}{2} b E_c \varepsilon_c x = 0$$

$$\Sigma M = 0 \qquad A_s E_s \varepsilon_c (d_s - x)\left(\frac{d_s - x}{x}\right) + \sigma_f b(h-x)\left(\frac{h-x}{2}\right) - \frac{1}{2} b E_c \varepsilon_c x \frac{2x}{3} - M_{ed} = 0$$

$$x = \frac{-(A_s E_s \varepsilon_c + \sigma_f bh) + \sqrt{(A_s E_s \varepsilon_c + \sigma_f bh)^2 - 4\left(\frac{1}{2} b E_c \varepsilon_c + \sigma_f b\right) - A_s E_s \varepsilon_c d_s}}{2\left(\frac{1}{2} b E_c \varepsilon_c + \sigma_f b\right)}$$

$$d_s = h - c - \phi_{stirrups} - \frac{\phi}{2}$$

This gives for x and ε_c:

$$x = 147.7501 mm$$

$$\varepsilon_c = 0.001270$$

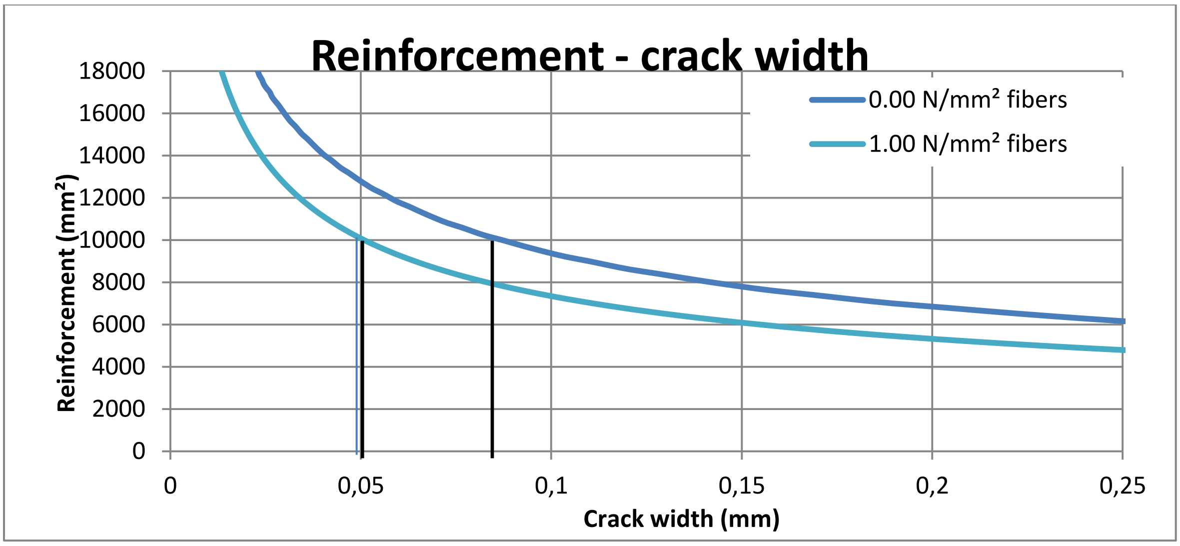
$$\sigma_s = E_s \varepsilon_s = E_s \varepsilon_c \left(\frac{d_s - x}{x}\right) = 261 \frac{N}{mm^2}$$

$$h_{eff} = \frac{h-x}{3} = 67.41 mm$$

$$\rho_{eff} = \frac{A_s}{h_{eff} b} = 0.059 \frac{N}{mm^2}$$

$$\sigma_{sr} = \frac{f_{ctm}}{\rho_{eff}}\left(1 + \frac{E_s}{E_c}\rho_{eff}\right) - \frac{\sigma_f}{\rho_{fibers}} = 85.15 \frac{N}{mm^2}$$

$$w_{\max} = \frac{1}{2} \frac{f_{ctm} - \sigma_f}{\tau_{bm}} \frac{\phi}{\rho_{eff}} \frac{1}{E_s}(\sigma_s - 0.5\sigma_{sr}) = 0.0507 mm$$

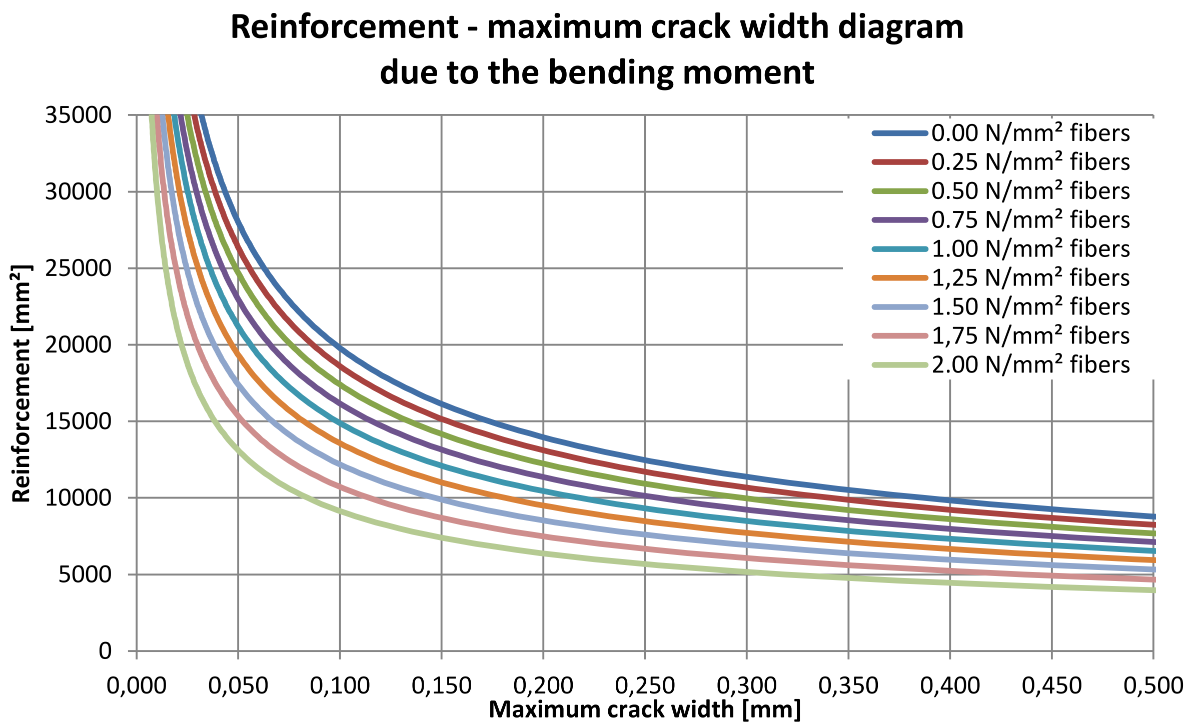


Results are found in the graph and this is correct.

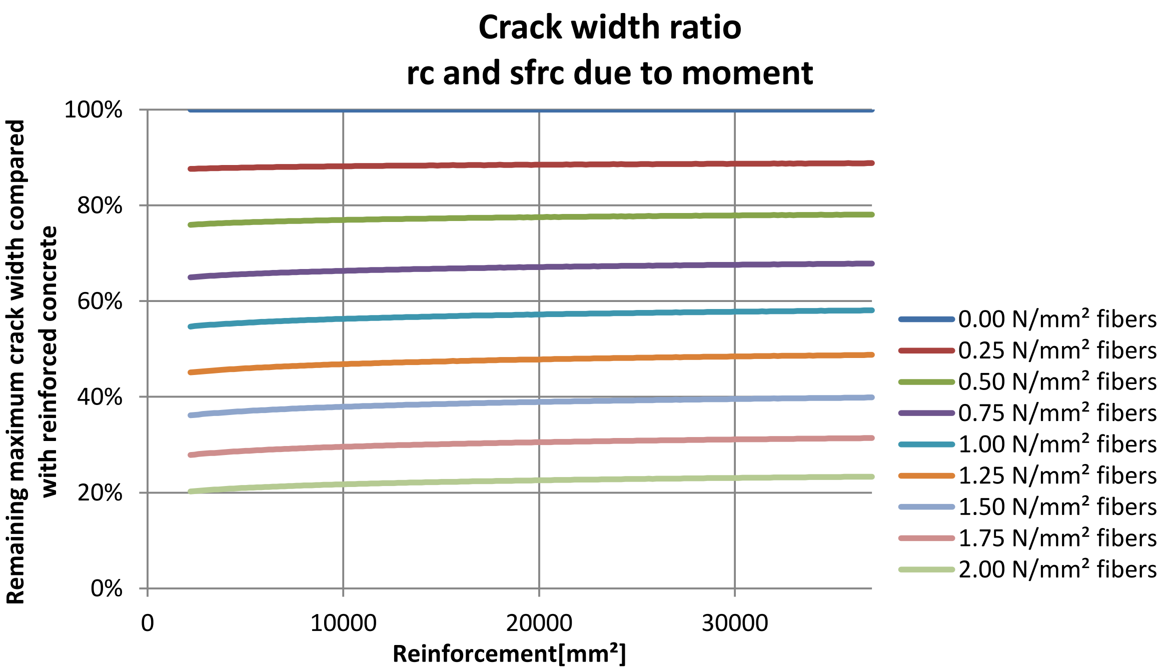
Appendix 5: Case study 2 (floor)

Calculation of the maximum crack width due to the external loads

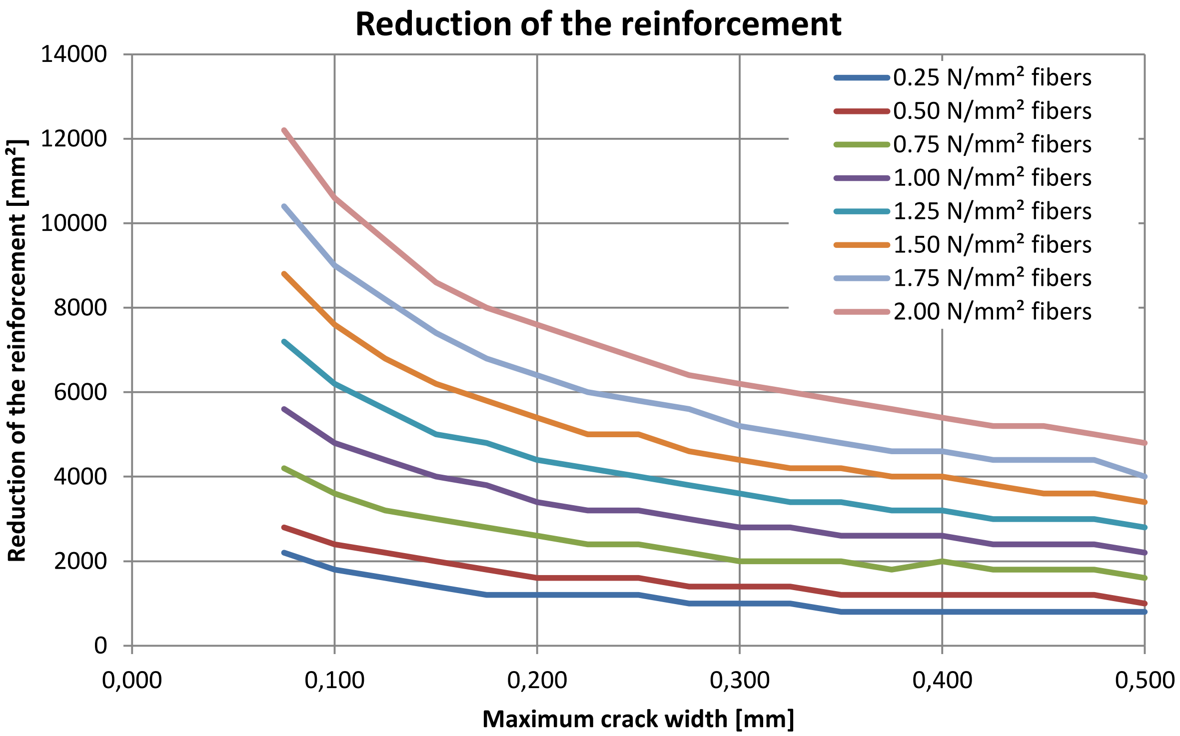
The figures for the maximum crack width and the reduction of the reinforcement for the situation of loaded by the external forces, these figures are the same as found in case 1, only with other values.



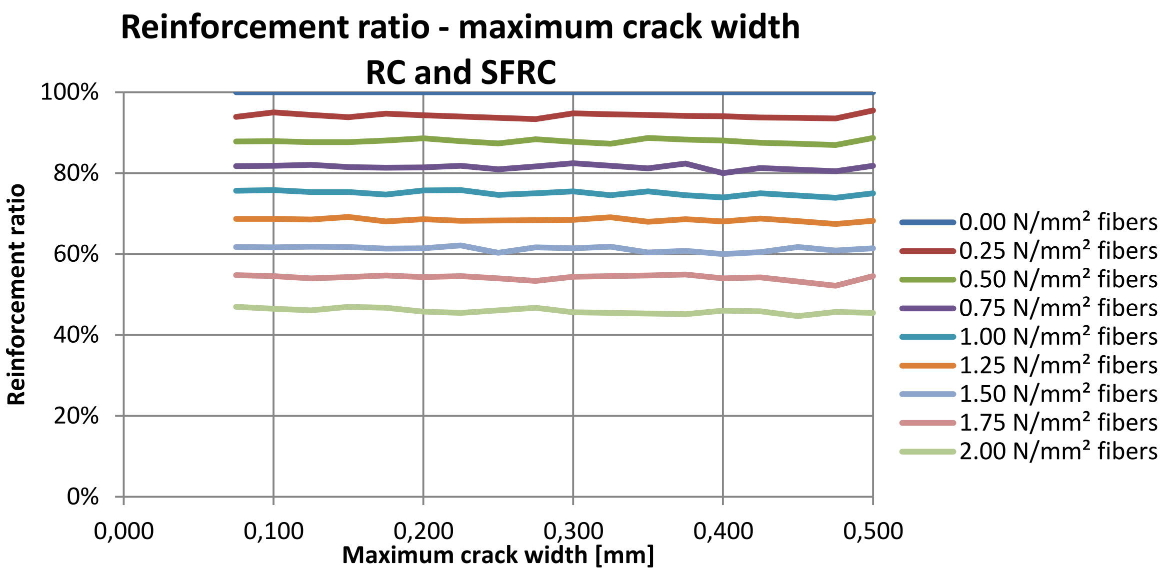
In this figure the reinforcement over the maximum crack width is plotted. In this graph the reduction of the reinforcement and the maximum crack width can be seen if steel fibres are added.



In the figure above the remaining maximum crack width is plotted if steel fibres are added. The results of the steel fibre reinforced floor are compared with the floor without steel fibres. Over the whole range of reinforcement, a near constant reduction can be seen.



The reduction of the reinforcement due to the external forces can be seen in the figure above. The same model is found as in case 1.



The reduction of the reinforcement, which is found in the figure before can be rewritten as a ratio of the reinforced concrete floor without steel fibres. In this figure can be seen that the amount of reinforcement can be reduced with about 55% in the case of 2.00 N/mm² fibres.

Overview of the imposed deformation

Imposed deformations are likely to be caused by temperature changes and shrinkage of concrete. Very often, the settlement of the subsoil layer, on which the structure is supported, is considered as being an imposed deformation. In this case, the floor is situated on an underwater concrete layer, which gives that the fresh concrete cannot shrink freely.
An imposed deformation can be written as:

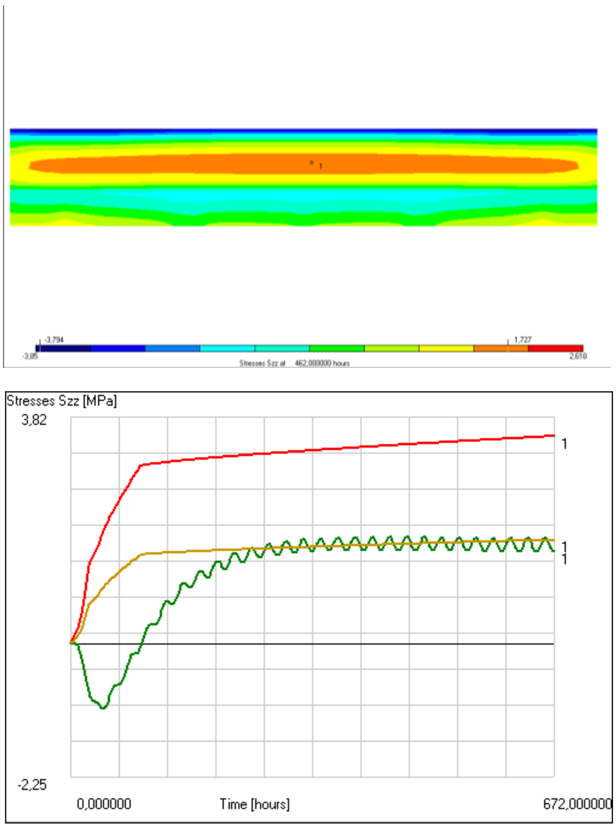
$$\varepsilon_{shr} = \alpha_c \Delta T \Rightarrow \sigma_{shr} = \alpha_c \Delta T E_s$$

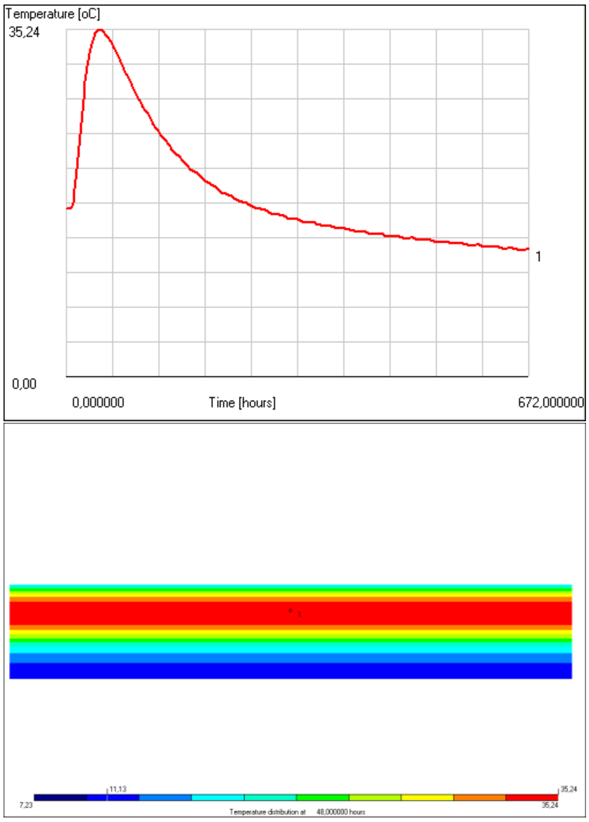
The following types of shrinkage will be considered:

- Thermal shrinkage
- Drying shrinkage
- Autogenous shrinkage

Thermal shrinkage

Hydration of cementitious materials generates heat for several days after placement. In the thick concrete elements, the internal temperature rises, but drops slowly, while the surface cools rapidly down to ambient temperature (underwater concrete layer and open air). The surface will be hardened earlier compared with the warmer internal concrete matrix. If the temperature of a volume reduced, the volume will shrink. The surface layers are hardened earlier and the internal matrix will shrink at a later time. This restraint creates tensile stresses that can crack the cross section of the concrete. These cracks are continuous cracks through the whole cross section.





Development of the stresses due to the thermal shrinkage

Development the temperature of a concrete floor cast on underwater concrete

Through cracks is a risk for floors, which must be watertight in most cases. To reduce the cracks so much as possible, the maximum tensile stress of the cross section is 50% of the tensile strength.

The tensile strength for this cementitious composition is 3.73 N/mm². This gives a maximum tensile stress due to the thermal shrinkage of 50% of 3.73 = 1.86 N/mm². This tensile stress can be rewritten as a strain:

$$\sigma = \varepsilon \cdot E_s => \varepsilon = \frac{\sigma}{E_s} = 6.2 \cdot 10^{-5}$$

Other forms of shrinkage

Shrinkage contains of more forms of shrinkage:

- Drying shrinkage
- Autogenous shrinkage

Drying shrinkage:

Drying shrinkage is the reduction of the volume of concrete, caused by the evaporation of water. This reduction of the volume is associated with the form of cracks in the case of imposed deformation. The shrinkage of the concrete is a very slow process, which can have duration of years. By the slow process, the concrete has the opportunity the resist the stresses due to the shrinkage or it has the opportunity for stress relaxation to reduce the tensile stresses. The magnitude of shrinkage mainly depends on the relative amount of cement matrix in a concrete.

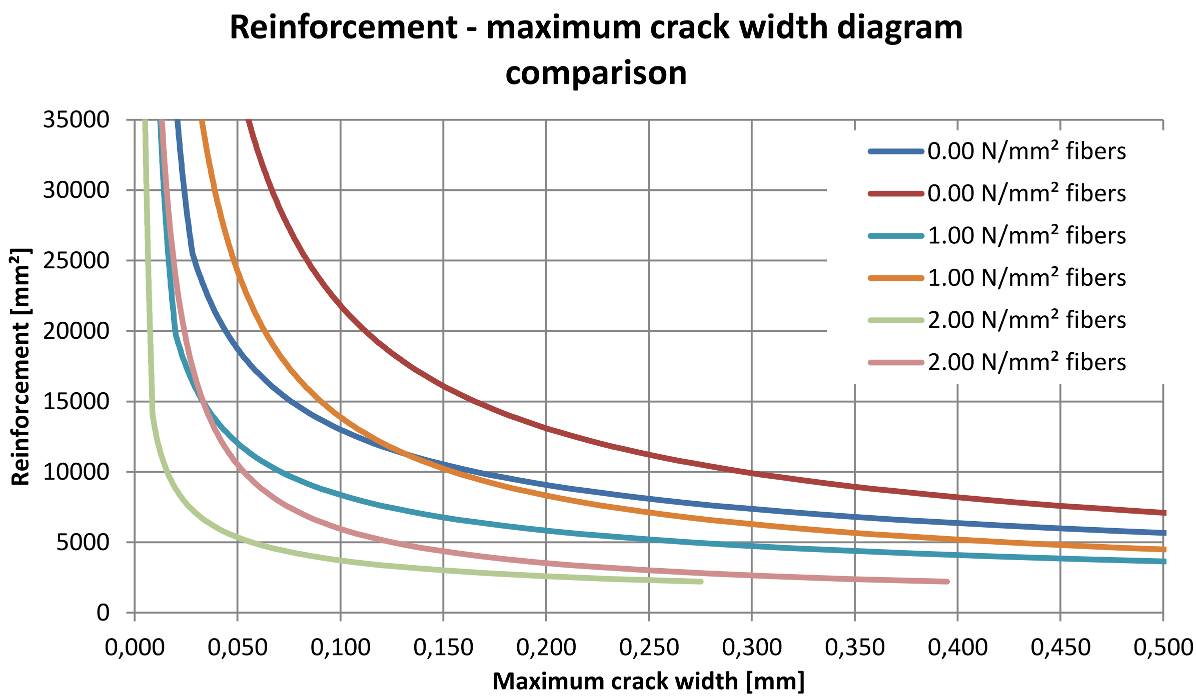
Autogenous shrinkage

Autogenous shrinkage is caused by a lack of water during the hydration. During the hydration process the cement consumed water. This process can be seen as emptying the pore system. The water in the pores is consumed by the cement and empty pores occurs. Autogenous shrinkage does not have a change in the moisture, but the shrinkage is in the concrete.

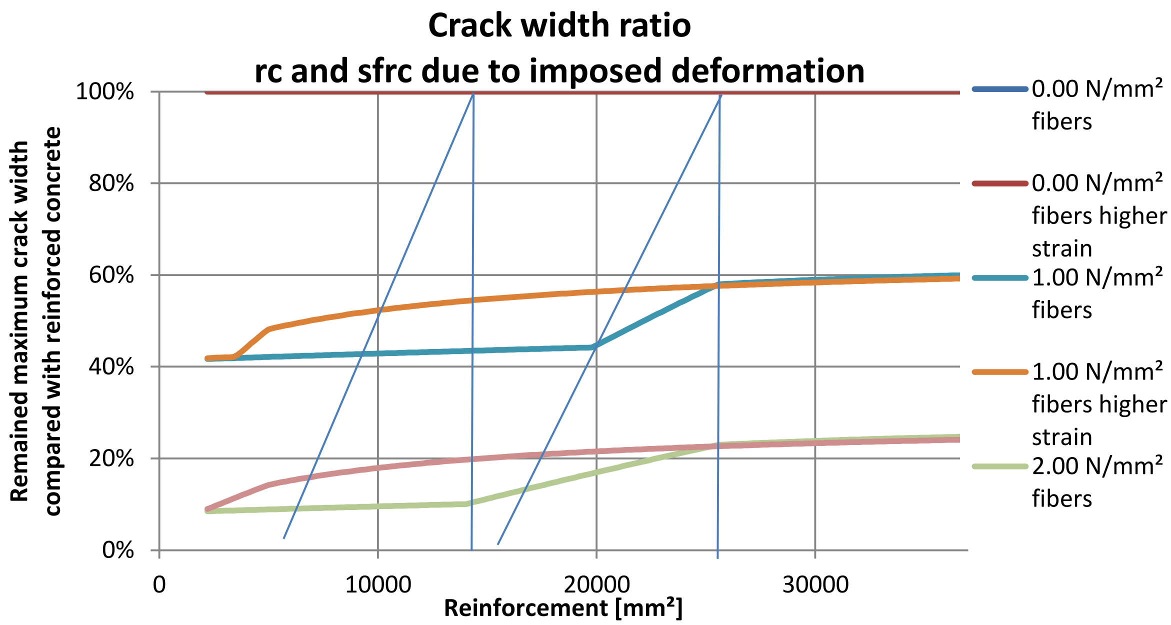
The expressions of the shrinkage can be found in the Eurocode 1992-1-1 appendix B.

Calculation with a higher imposed deformation

In the figures below calculations are done for an imposed deformation of two times the imposed deformation as calculated in 7.5.1.



In case study 2 a calculation for the imposed deformation is made for a prescribed strain. The figure above shows the influence of the strain on the reinforcement and maximum crack width. In this case a strain is assumed which is two times the original strain calculated. The both situations are plotted in the figure above. For the higher strain a higher maximum crack width can be found, if the reinforcement is taken equally. The same for the amount of reinforcement if the maximum crack width is taken equally.



The figure above shows the remaining maximum crack width in the case of a higher strain (two times the original strain), compared with the reinforced concrete floor (for both situation the maximum crack width is 100% for the reinforced concrete floor). There are quite no differences in the stabilized cracking phase. The biggest difference between the two different strains is the location of the change between the crack formation stage and the stabilized cracking stage. The situation with the higher strain is at a lower reinforcement in the stabilized cracking stage.

Hand calculation case study 2:

In this hand calculation the maximum crack width is calculated for two different amounts of reinforcement. The calculation is done with and without the addition of steel fibres.

As	:	10.000 mm²
ε_s	:	0.000263
Ds	:	h-c-12-0.5ϕ=300
H	:	1200 mm
Ec	:	31000 N/mm²
σ_f	:	0.00 N/mm²
σ_f	:	1.00 N/mm²

For σ_f=0:

$$\varepsilon_{fdc} = \frac{A_c f_{ctm}\left(1-0.5\frac{\sigma_f}{f_{ctm}}+\frac{E_c}{E_s}\frac{A_s}{A_c}\right)-0.4 f_{ctm} b h_{eff}}{A_s E_s} = 0.0006668$$

The floor is in the crack formation stage.

$$h_{eff} = 2.5(h-d_s) = 310mm \qquad \rho_{eff} = \frac{A_s}{h_{eff} b} = 0.0322\frac{N}{mm^2}$$

$$\sigma_s = \frac{f_{ctm}}{\rho}\left(\left(1-\frac{\sigma_f}{f_{ctm}}\right)+\frac{E_s}{E_c}\rho\right) = 186.06$$

$$\sigma_{sr} = \frac{f_{ctm}}{\rho_{eff}}\left(1+\frac{E_s}{E_c}\rho_{eff}\right)-\frac{\sigma_f}{\rho_{fibers}} = 104.95\frac{N}{mm^2}$$

$$w_{\max} = \frac{1}{2}\frac{f_{ctm}-\sigma_f}{\tau_{bm}}\frac{\phi}{\rho_{eff}}\frac{1}{E_s}\left(\sigma_s-0.5\sigma_{sr}\right) = 0.1657mm$$

For σ_f=1:

$$\varepsilon_{fdc} = \frac{A_c f_{ctm}\left(1-0.5\frac{\sigma_f}{f_{ctm}}+\frac{E_c}{E_s}\frac{A_s}{A_c}\right)-0.4 f_{ctm} b h_{eff}}{A_s E_s} = 0.0005185$$

The floor is in the crack formation stage.

$$h_{eff} = 2.5(h-d_s) = 310mm \qquad \rho_{eff} = \frac{A_s}{h_{eff}b} = 0.0322\frac{N}{mm^2}$$

$$\sigma_s = \frac{f_{ctm}}{\rho}\left(\left(1-\frac{\sigma_f}{f_{ctm}}\right)+\frac{E_s}{E_c}\rho\right) = 126.06$$

$$\sigma_{sr} = \frac{f_{ctm}}{\rho_{eff}}\left(1+\frac{E_s}{E_c}\rho_{eff}\right)-\frac{\sigma_f}{\rho_{fibers}} = 73.864\frac{N}{mm^2}$$

$$w_{\max} = \frac{1}{2}\frac{f_{ctm}-\sigma_f}{\tau_{bm}}\frac{\phi}{\rho_{eff}}\frac{1}{E_s}(\sigma_s-0.5\sigma_{sr}) = 0.07105mm$$

For σ_f=0 and As=30000:

$$\varepsilon_{fdc} = \frac{A_c f_{ctm}\left(1-0.5\frac{\sigma_f}{f_{ctm}}+\frac{E_c}{E_s}\frac{A_s}{A_c}\right)-0.4f_{ctm}bh_{eff}}{A_sE_s} = 0.0002243$$

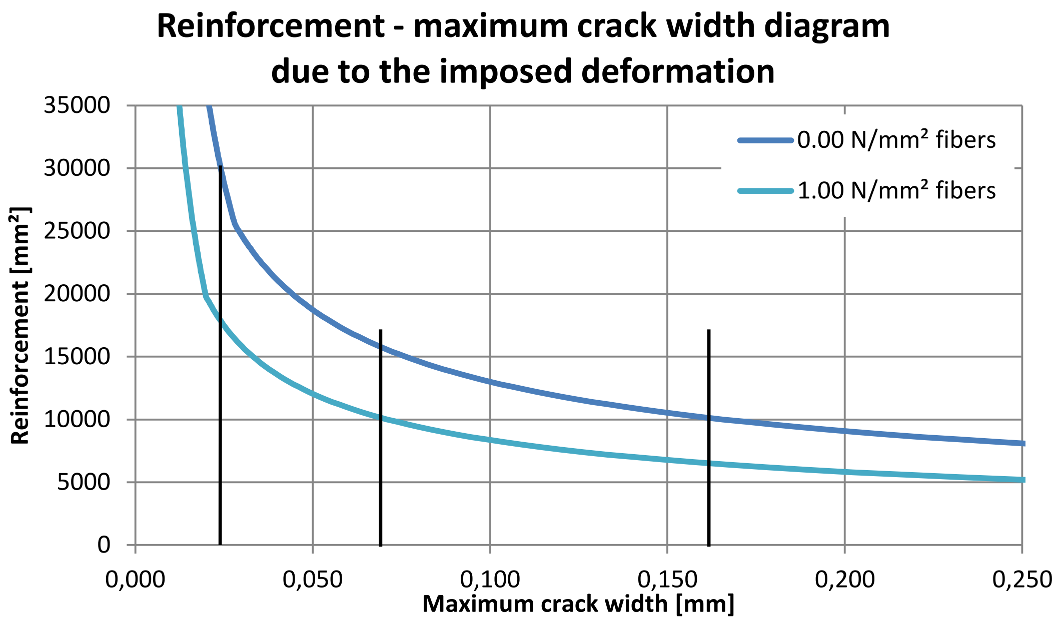
The floor is in the stabilized cracking.

$$h_{eff} = 2.5(h-d_s) = 310mm \qquad \rho_{eff} = \frac{A_s}{h_{eff}b} = 0.09677\frac{N}{mm^2}$$

$$\sigma_s = \frac{f_{ctm}}{\rho}\left(\left(1-\frac{\sigma_f}{f_{ctm}}\right)+\frac{E_s}{E_c}\rho\right)+E_s(\varepsilon-\varepsilon_{fdc}) = 81.803\frac{N}{mm^2}$$

$$\sigma_{sr} = \frac{f_{ctm}}{\rho_{eff}}\left(1+\frac{E_s}{E_c}\rho_{eff}\right)-\frac{\sigma_f}{\rho_{fibers}} = 46.997\frac{N}{mm^2}$$

$$w_{\max} = \frac{1}{2}\frac{f_{ctm}-\sigma_f}{\tau_{bm}}\frac{\phi}{\rho_{eff}}\frac{1}{E_s}(\sigma_s-0.5\sigma_{sr}) = 0.0241mm$$



The results are found in the figure above and is this correct.

Maple sheet case study 2

Below the maple sheet, which is used to calculate the maximum crack width of case study 2.

restart

$$As := As; Es := 200000 : ds := h - c - 16 - 32 - 0.5\cdot\phi; b := 1000 : h := 1200 : Med := 3540\cdot 10^6; Ec := 31000 : fctm := 2.8 : \phi := 32 : \tau bm := 2\cdot fctm : \sigma f2 := 1 : c := 60 : fyd := 435 : \varepsilon cr := \frac{fctm}{Ec}; \varepsilon fdc := \frac{Ac\cdot fctm\cdot\left(1 - 0.5\cdot\frac{\sigma f2}{fctm} + \frac{Ec}{Es}\cdot\frac{As}{Ac}\right) - 0.4\cdot fctm\cdot b\cdot heff}{As\cdot Es};$$

$$\varepsilon end := \frac{fyd}{Es}; \varepsilon := 0.000263;$$

$$H := As\cdot Es\cdot\varepsilon c\cdot\left(\frac{ds - x}{x}\right) + \sigma f\cdot b\cdot(h - x) - 0.5\cdot\varepsilon c\cdot Ec\cdot b\cdot x = 0 :$$

$$M := As\cdot Es\cdot\varepsilon c\cdot(ds - x)\cdot\left(\frac{ds - x}{x}\right) + \sigma f\cdot b\cdot(h - x)\cdot\frac{(h - x)}{2} + 0.5\cdot\varepsilon c\cdot Ec\cdot b\cdot x\cdot\frac{2\cdot x}{3} - Med = 0 :$$

$$solution := solve(\{H, M\}, \{\varepsilon c, x\}) :$$

$$assign(solution);$$

$$\sigma s1 := Es\cdot\varepsilon c\cdot\frac{(ds - x)}{x} :$$

$$heff := 2.5\cdot(h - ds) : \rho eff := \frac{As}{b\cdot heff} : Ac := b\cdot 0.5\cdot h;$$

$$\sigma sr := \frac{fctm}{\rho eff}\cdot\left(\left(1 - \frac{\sigma f}{fctm}\right) + \frac{Es}{Ec}\cdot\rho eff\right) :$$

$$lt := \frac{1}{2}\cdot\frac{(fctm - \sigma f)}{\tau bm}\cdot\frac{\phi}{\rho eff} :$$

$\sigma f := 1 :$

with(*LinearAlgebra*) :
$N := rtable(1..175, 1..5) :$
print(*N*) :

for z **from** 1 **by** 1 **to** 175 **do**
$As := 2200 + (z - 1)\cdot 200 :$

if $\varepsilon > \varepsilon end$
then $\sigma s2 := fyd$
elif $\varepsilon > \varepsilon fdc$

then $\sigma s2 := \left(\frac{fctm\cdot Ac}{As}\left(\left(1 - \frac{\sigma f2}{fctm}\right) + \frac{Es}{Ec}\cdot\frac{As}{Ac}\right)\right) + Es\cdot(\varepsilon - \varepsilon fdc)$

elif $\varepsilon > \varepsilon cr$

then $\sigma s2 := \frac{fctm\cdot Ac}{As}\left(\left(1 - \frac{\sigma f2}{fctm}\right) + \frac{Es}{Ec}\cdot\frac{As}{Ac}\right)$

else $\sigma s2 := Es\cdot\varepsilon$
end if:

```
for n from 1 by 1 to 1 do

N[z, 1] := As :
N[z, 1 + n] := evalf(σs1) :
N[z, 2 + n] := evalf(σs2) :
N[z, 3 + n] := evalf(lt) :
N[z, 4 + n] := evalf(εfdc) :

print(N)
end do; end do;

with(ExcelTools) :
Export(N) :
```

Excel sheet tension stiffening model (4.3)

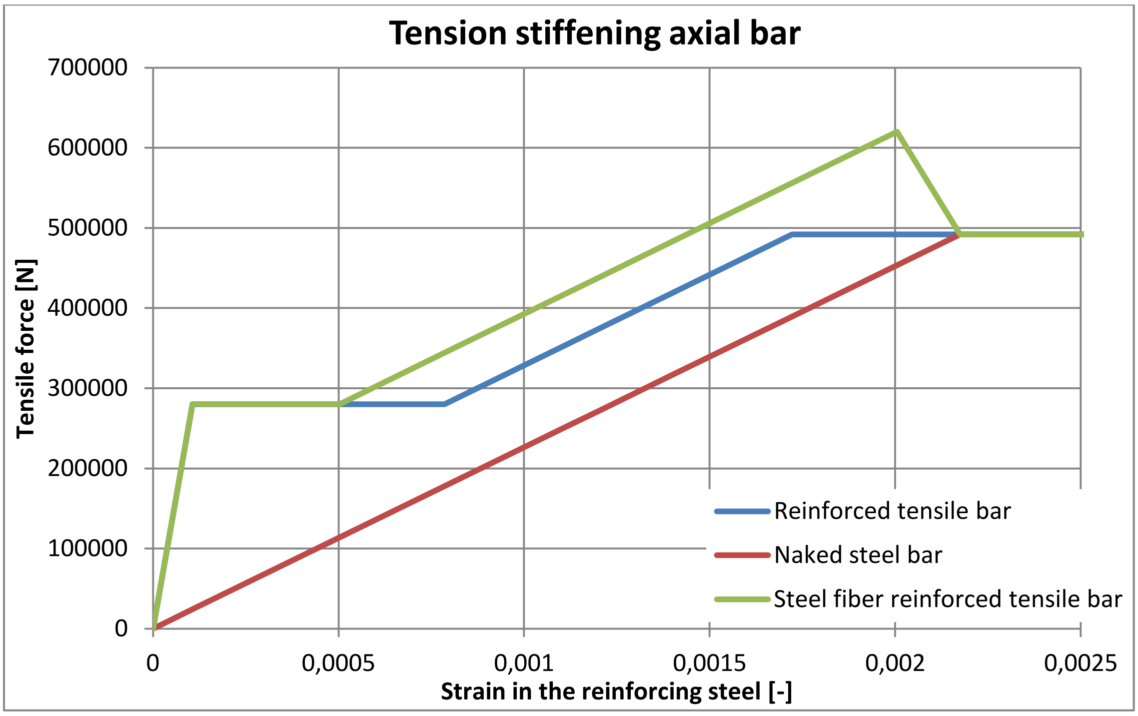
The length of the crack formation stage is calculated by

fctm	3,2	N/mm²
Ec	31000	N/mm²
Es	200000	N/mm²
Ac	80000	mm²
ϕ	12	mm²
stuks	10	stuks
As	1131	mm²
fyd	435	N/mm²
xfctm	0,5	

Ncr	279348	N
1.		
σc	3,20	N/mm²
εeind	0,00010	
σs,cr	247,0055	N/mm²
3.		
σc	1,28	N/mm²
	0 N/mm²	x N/mm²
εbegin	0,000782	0,000499
εeind	0,001722	0,002005
εyield	0,002175	

Strain and forces in the bar:

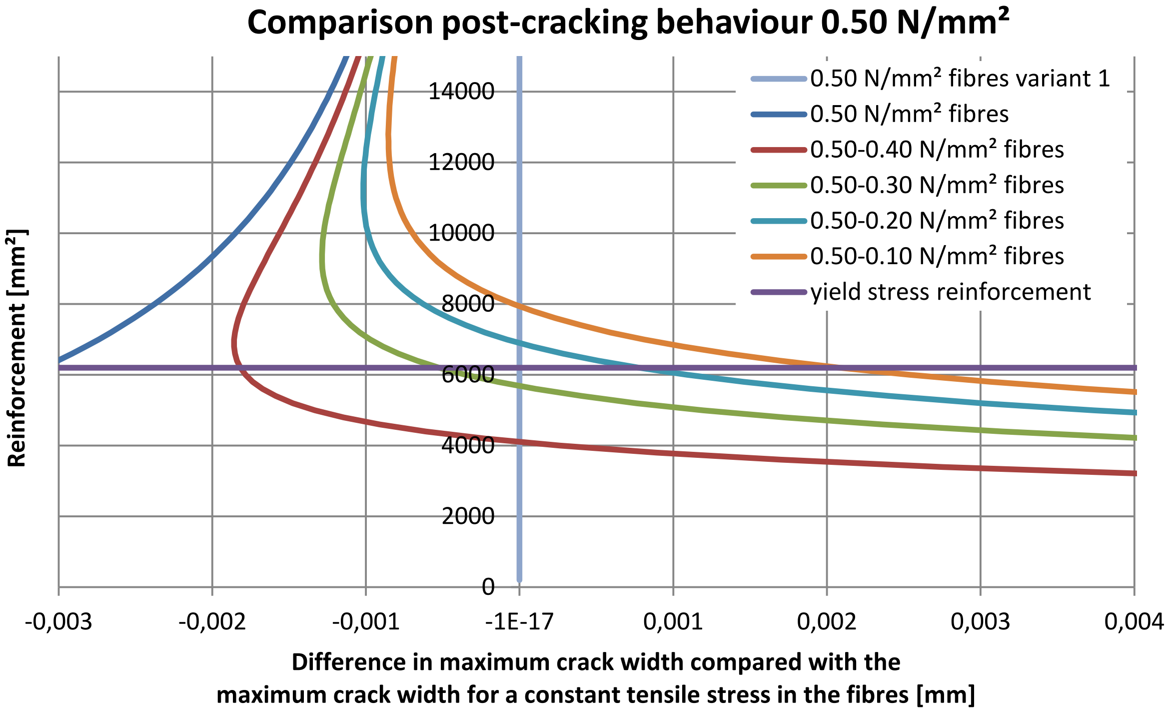
No fibres		Steel only	Fibres	
ε	N	N	ε	N
0	0	0	0	0
0,00010	279348	23348	0,00010	279348
0,000782	279348	176948	0,000499	279348
0,001722	491959	389559	0,002005	619959
0,002175	491959	491959	0,002175	491959
0,003000	491959	491959	0,003000	491959

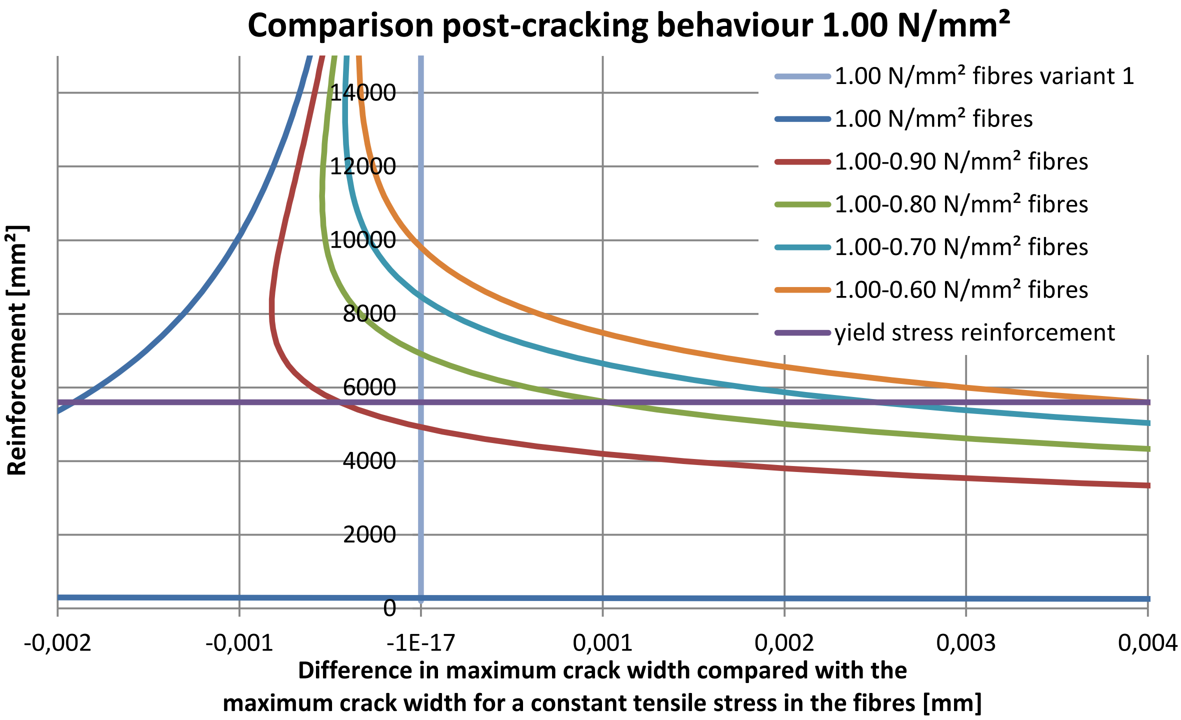


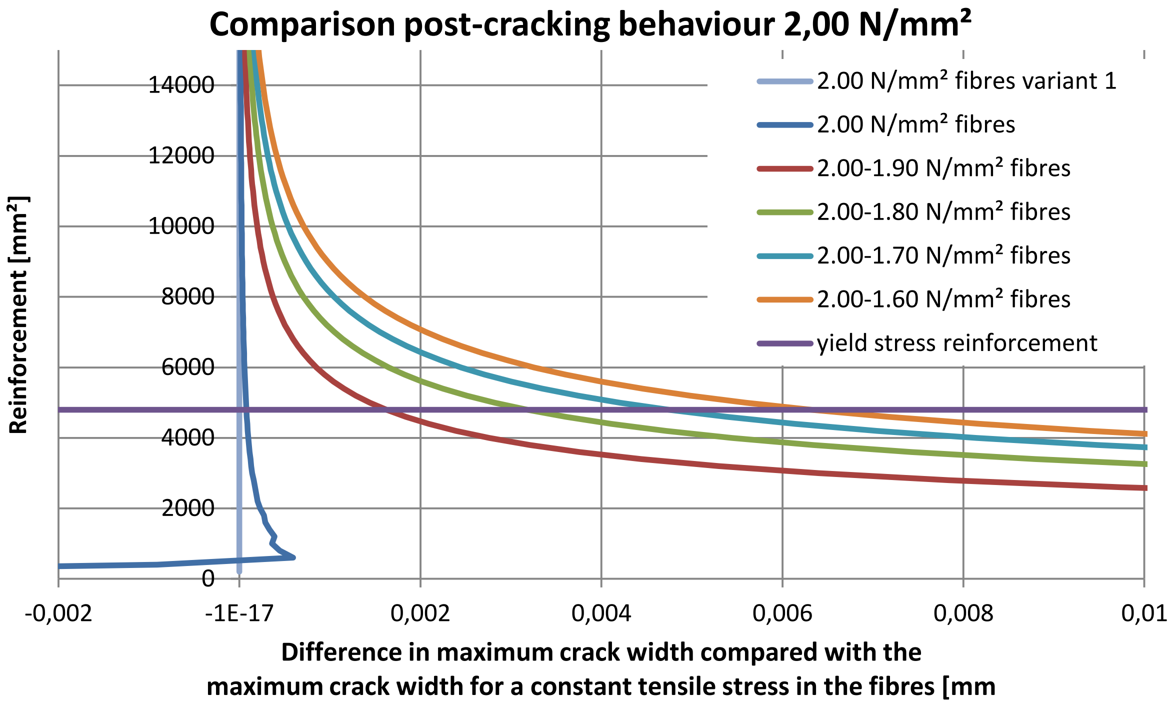
Appendix 6: Comparison of the variants

Comparison case study 1:

The figures below shows the difference in maximum crack width between the constant tensile stress over the total tension zone (variant 1) and the more realistic post-cracking behaviour (variant 2):







The figures above shows the difference in maximum crack width as function of the reinforcement for three different post-cracking stresses. In the figures with a post-cracking behaviour of 0.50 and 1.00 N/mm² the maximum crack width is overestimated for an amount of reinforcement of 10.000 mm². The maximum crack width is underestimated for the post-cracking behaviour of 2.00 N/mm². For lower amounts of reinforcement or for a higher reduction of the post-cracking stress, the difference of the maximum crack width increases.

Maple sheet variant 2 case study 1

Below the maple sheet is shown, which is calculating the maximum crack width of variant 2 in the case of case study 1.

restart

$$As := As : Es := 200000 : ds := h - c - 12 - 0.5\cdot\phi; b := 2500 : h := 350 : fcd := 23.3 : Med := 0.125\cdot\left(25\cdot\frac{b\cdot h}{10^6} + 2 + 4.79\cdot\frac{b}{1000}\right)\cdot 13^2\cdot 10^6; Ec := \frac{fcd}{1.75\cdot 10^{-3}} : fctm := 3.2 : \phi := 16 : \tau bm := 2\cdot fctm : \sigma 1 := 0.5 : \sigma 3 := 0.5 : c := 30 : \varepsilon 1 := \frac{fctm}{Ec} : \varepsilon 2 := \varepsilon 1 + 0.1\cdot 10^{-3} : \varepsilon u := 20\cdot 10^{-3} : \varepsilon 3 := \frac{(h-x)}{x}\varepsilon c : d1 := \frac{\varepsilon 1}{\varepsilon 3}(h-x) : d2 := \frac{\varepsilon 2 - \varepsilon 1}{\varepsilon 3}\cdot(h-x) : d3 := \frac{\varepsilon 3 - \varepsilon 2}{\varepsilon 3}\cdot(h-x) : \sigma 2 := \sigma 1 - \frac{\sigma 1 - \sigma 3}{\varepsilon u - \varepsilon 2}\cdot\varepsilon 3 :$$

$$H := As\cdot Es\cdot\varepsilon c\cdot\left(\frac{ds-x}{x}\right) + d1\cdot fctm\cdot b\cdot 0.5 + d2\cdot\left(\frac{\sigma 1 + fctm}{2}\right)\cdot b + d3\cdot b\cdot\left(\frac{\sigma 2 + \sigma 1}{2}\right) - 0.5\cdot\varepsilon c\cdot Ec\cdot b\cdot x = 0 :$$

$$M := As\cdot Es\cdot\varepsilon c\cdot(ds - x)\cdot\left(\frac{ds-x}{x}\right) + d1\cdot fctm\cdot b\cdot 0.5\cdot\left(\frac{2}{3}\cdot d1\right) + d2\cdot\sigma 1\cdot b\cdot\left(d1 + \frac{d2}{2}\right) + \left(\frac{fctm - \sigma 1}{2}\right)\cdot d2\cdot b\cdot\left(d1 + \frac{1}{3}d2\right) + d3\cdot b\cdot\sigma 2\cdot\left(d1 + d2 + \frac{d3}{2}\right) + \left(\frac{\sigma 1 - \sigma 2}{2}\right)\cdot d3\cdot b\cdot\left(d1 + d2 + \frac{1}{3}\cdot d3\right) + 0.5\cdot\varepsilon c\cdot Ec\cdot b\cdot x\cdot\frac{2\cdot x}{3} - Med = 0 :$$

$$solution := solve(\{H, M\}, \{\varepsilon c, x\}) :$$

$$assign(solution) :$$

$$\sigma s := Es\cdot\varepsilon c\cdot\frac{(ds - x)}{x} :$$

$$heff := \frac{h-x}{3} : \rho eff := \frac{As}{b\cdot heff} :$$

$$\sigma sr := \frac{fctm}{\rho eff}\left(\left(1 - \frac{\sigma 1}{fctm}\right) + \frac{Es}{Ec}\cdot\rho eff\right) :$$

$$wmax := \frac{1}{2}\frac{(fctm - \sigma 2)}{\tau bm}\cdot\frac{\phi}{\rho eff}\cdot\frac{1}{Es}(\sigma s - 0.5\,\sigma sr) :$$