Delft, University of Technology
Engineering office of Public works Rotterdam

Interaction between plate and column buckling

Master Thesis

Name: Alex van Ham
Student number: 1306138
Email: vanham.alex@gmail.com

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Graduation Committee

Chairman
prof. ir. F.S.K. Bijlaard
Department of Structural Engineering, Steel Structures
Faculty of Civil Engineering and Geosciences
Delft University of Technology

Committee members
ir. R. Abspoel
Department of Structural Engineering, Steel Structures
Faculty of Civil Engineering and Geosciences
Delft University of Technology

dr. ir. P.C.J. Hoogenboom
Department of Structural Engineering, Structural Mechanics
Faculty of Civil Engineering and Geosciences
Delft University of Technology

External committee member
ir. J.H. Reusink
Senior consultant Bridges and Steel Structures
Engineering office Public Works Rotterdam

Secretary
ir. L.J.M. Houben
Department of Structural Engineering, Road and Railway Engineering
Faculty of Civil Engineering and Geosciences
Delft University of Technology

Preface

This master thesis has been written as the final part of my study Civil Engineering at Delft University of Technology. My master is at the department of Structural Engineering and the specialization is Steel and Timber Structures. Also an honours track has been done to include courses on both Concrete Structures and Structural Mechanics.

My master thesis project is done at the Engineering Office of Public Works Rotterdam. They offered me the opportunity to study the subject of plate buckling in steel structures. Their main question at the Engineering Office was to provide insight in the Eurocode on plate buckling (NEN-EN1993-1-5) and how it should be applied. This resulted in a tool which can be used to check class 4 steel cross-sections. This question is combined with the subject of the interaction of plate and column buckling which is a theoretical study.

I would like to thank the entire MSc graduation committee for their input and feedback.

Alex van Ham
Rotterdam, January 2012

Abstract

Dimensioning and verification of steel structures is often governed by the demands for stability and the plastic capacity of the material is not fully utilized. There are a several forms of instability such as column buckling, lateral-torsional buckling and plate buckling.

Interaction of plate and column buckling has not been extensively studied before and that is the main subject of this master thesis. In present verification codes in The Netherlands it is allowed to separately calculate the effects of both and then combine them in a certain way. Question is whether this is indeed correct. Analytical calculations have been done on an I-column to derive the theoretical buckling load combining the effects of plate and column buckling. In general the interaction is very small but for uneconomic cross-sections with a large area of the web compared to the area of the flange this interaction can be large. The numerical answer is not very relevant because the post-buckling behaviour of plate and column buckling is totally different but the knowledge gained is more important. The most important conclusion from the analytical calculation is that interaction is only present when the number of half sine waves in the web is equal to the number of half sine waves in the entire column.

Also calculations have been performed using a finite element method. Many cross-sections have been considered and the general trend is that the bearing capacity according to the verification regulations are closely related to the results from the finite element analysis. For a certain type of cross-section the results are significantly different compared to the verification regulations. This is for uneconomic cross-sections with a large area of the web compared to the area of the flange. The difference can be up to 20% of the bearing capacity. This type of cross-sections is not used very often because these would be very uneconomic but the verification regulations overestimate the bearing capacity. In general a designer is free to design any sort of structure. After that the structure should be verified for structural safety using the regulations. These regulations should either provide the correct verification regulations or inform that a certain type of structure is outside the scope of the regulations. Neither is done in the current verification regulations. A formula is developed to calculate the reduced bearing capacity for this type of cross-sections.

A probabilistic design has been done on an economic and an uneconomic type of cross-section. The required safety was reached for the economic cross-section ($\beta = 3{,}85$) but for the uneconomic cross-section ($\beta = 3{,}03$) the safety was not reached. No general conclusions may be drawn because only two situations have been investigated but the difference is striking. It confirms the unsafe nature of the verification regulations for the uneconomic cross-sections.

A design recommendation has been made on how to design a class 4 cross-section. It is shown that it is not efficient to add longitudinal stiffeners to the web when verification regulations of the effective cross-section method are used. Only the effective area of the web is increased which is in general small and for bending moments also close to the centre of gravity.

A design tool has been developed which checks class 4 cross-sections according to the verification regulations in NEN-EN1993-1-5. The tool can check I cross-sections as well as a box or a π cross-section. Longitudinal stiffeners to the web, through decks and concrete toppings can be added.

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Chapter 1: Introduction

General introduction

In modern days structural engineers are often challenged by architects to design more slender and more challenging steel structures. Therefore, not always the most economic or common construction is chosen which creates challenges for structural engineers. To be able to design uncommon constructions it is necessary to have a proper understanding of the structural behaviour of those structures.

One important aspect of this is the concept of plate buckling. The new Eurocode on plate buckling (NEN-EN1993-1-5, 2006) recommends the use of the effective width method. There is an issue which needs to be investigated. This is an I-column with small flanges compared to the size of the web in which it is assumed that the flanges fully support the web against plate buckling. NEN-EN1993-1-5 prescribes no requirements for the flanges to be able to fully support the web. It is assumed that the stability of the flanges is covered in the checks on column buckling and lateral-torsional buckling. The question is whether the flanges can indeed be seen as stiff supports for the web no matter what dimensions are chosen.

Van der Burg (2011) has done research on this subject and calculated an I-column for a certain configuration according to the NEN-EN1993-1-5 and compared these results to the collapse load according to Ansys using a geometrical and physical non-linear model. From that it was concluded that there is no additional requirement necessary for flanges of an I-column. However, no analytical calculations have been done which provide insight in the interaction of column buckling and plate buckling. Also no calculations have been done using a bending moment instead of axial compression which provides information about the interaction of lateral-torsional buckling and plate buckling.

Plate buckling is a complicated subject in NEN-EN1993-1-5. The Engineering Office of Public Works of Rotterdam is in need of a tool to calculate plate buckling according to the NEN-EN1993-1-5. The verification rules are not clearly specified in the regulations and the procedure is quite cumbersome. The NEN-EN1993-1-5 does not clearly prescribe why some rules are needed and what the background is according to structural mechanics. This information is required about the background of the design rules to accommodate the design checks for situations which are not exactly prescribed in the NEN-EN1993-1-5.

Objectives

Therefore the purpose of this master thesis is as follows:

1. Gain insight in the background of plate buckling according to the NEN-EN1993-1-5 using literature.
2. Produce a versatile tool in Mathcad to check plate buckling for I- and box-profiles according to the NEN-EN1993-1-5. A check of this tool should also be performed.
3. Produce guidelines about whether flanges of an I-column are stiff or not using analytical solutions and also FEM-analyses. These should be compared to the results from NEN-EN1993-1-5.

Working in this way will be a natural way to get familiar with the NEN-EN1993-1-5 on plate buckling because first literature is studied and then the knowledge gained is applied in creating a tool. After that phase there should be a proper understanding of the NEN-EN1993-1-5 and then an investigation can be set up on the required stiffness of flanges.

Thesis outline

Chapter 1 gives a general introduction of the subject of this thesis. The problem definition is stated and an outline is given of this thesis.

Chapter 2 is a general introduction on the subject of buckling as a short introduction for the reader that is not familiar with the subject of buckling. The conventions used in this thesis are described such as the coordinate system. Also the derivations for column buckling, lateral-torsional buckling and plate buckling are given.

Chapter 3 describes how plate buckling is dealt with in the NEN-EN1993-1-5. The difference between the effective cross-section method and the reduced stress method is explained. A general design recommendation is made for class 4 cross-sections. A calculation tool has been developed which is necessary to check cross-sections according to the NEN-EN1993-1-5. This tool is explained here.

Chapter 4 describes the buckling behaviour of a welded I-column under uniform compression. This is done for a theoretical structure that has no imperfections and no residual stresses. Therefore the theoretical buckling load is the result of this derivation. This theoretical buckling load is for the interaction of column buckling and plate buckling.

Chapter 5 provides the verification method for a welded I-column according to the regulations in the Netherlands as they are now and as they were before. The difference is explained.

Chapter 6 is about the finite element analysis of the welded I-column. Different forms of imperfections are investigated and the interaction between column buckling and plate buckling is studied.

Chapter 7 sums up the conclusions for the welded I-column. General conclusions are drawn based on the results of the finite element analysis. The interaction of plate buckling and column buckling is examined here. It is shown that the ratio of the area of the web to the ratio of the area of the flange is important for the load bearing capacity.

Chapter 8 is about the probabilistic analysis that is done for two different cross-sections. An economic cross-section is compared to an uneconomic cross-section. The safety of both is investigated and especially the difference is important.

Chapter 9 describes the buckling behaviour of a welded I-beam under uniform bending moment. This is done for a theoretical structure that has no imperfections and no residual stresses. Therefore the theoretical buckling load is the result of this derivation. This buckling load is the theoretical buckling load for the interaction of lateral-torsional buckling and plate buckling.

Chapter 10 provides the verification method for a welded I-beam according to the regulations in the Netherlands as they are now and as they were before.

Chapter 11 is about the finite element analysis of the welded I-beam. Different forms of imperfections are investigated and the interaction between lateral-torsional buckling and plate buckling is studied.

Chapter 12 sums up the conclusions for the welded I-beam. General conclusions are drawn based on the results of the finite element analysis. The interaction of plate buckling and lateral-torsional buckling is examined here. It is shown that the ratio of the area of the web to the ratio of the area of the flange is important for the load bearing capacity.

Chapter 13 provides a general conclusion about the interaction forms of buckling that exist. Also recommendations are done for the future whether regulations should be changed concerning the interaction of buckling. Also recommendations are done for future research on this subject.

Annex A to Annex J contain the analytical calculations, the finite element analyses and the calculation tool.

Chapter 2: Buckling

Plate buckling is a form of buckling in which a plated structure buckles if the width to thickness ratio is large. Buckling means a loss of stability so first the concept of stability and buckling is explained before plate buckling is introduced. This is only a general introduction. Literature provides more information (Vrouwenvelder, 2003).

Buckling

Buckling is a phenomenon which occurs in structures which are stiff in the loaded direction and slender in another direction. Initially equilibrium is stable but when the load is increased there is a sudden increase in deflection in loading direction due to a displacement in the slender direction. This is at the location of the bifurcation point. For structural behaviour this is a very important point because it is for slender constructions often governing and the plastic or elastic cross-sectional capacity is not reached. When buckling does not occur at a certain load level, a structure is considered stable.

There are some important differences between the behaviour of shells, plates and columns. Perfect Euler columns have a slight post buckling strength. For common dimensions this is generally around 3% for an out of plane deflection of 20% of the length of the column. (Vrouwenvelder, 2003) So for Euler columns the buckling load is considered to be the load bearing capacity. However, if buckling occurs, the load can still be carried and large deflections occur which is a warning for the users. Imperfections and residual stresses change the behaviour from the theoretically perfect column. Lateral deflections occur at a lower loading than the buckling load and the lateral deformation increases until the buckling load is reached or the section fails due to large stresses.

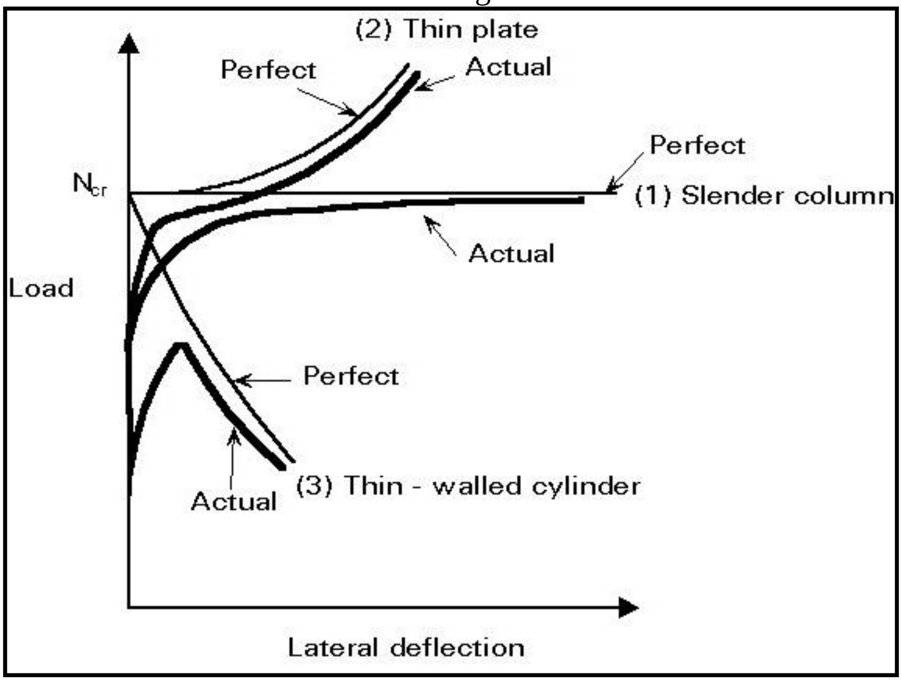


Figure 1: Buckling of shells, columns and plates for geometrical (non)linear and physically linear behaviour (University of Ljubljani, 2011)

Shells have a very efficient way of carrying loading but buckling is a sudden occurrence and is immediately the end of the load bearing capacity. This is a dangerous occurrence because the load bearing capacity is immediately lost when buckling occurs. Also, because of imperfections and residual stresses, the theoretical buckling load cannot be reached. Shells are not considered in this thesis so no further information is supplied.

Plates have an additional post-buckling strength. A plate can buckle and the load can be increased further until final collapse. This difference may be quite large and can be exploited in structural calculations. Small imperfections do not have a large influence on the behaviour like it is the case with shells. Von Karman has developed a formula to take account of the post-buckling strength and is derived in the following way.

The maximum stress on a plate is given by the following formula which will be derived later.

$$\sigma_{cr} = k * \frac{\pi^2 * E}{12 * (1 - v^2)} * \left(\frac{t}{b}\right)^2 \quad (2.1)$$

If the critical stress is replaced by the yield stress in equation (2.1) the effective width can be calculated.

$$f_y = k * \frac{\pi^2 * E}{12 * (1 - v^2)} * \left(\frac{t}{b_{eff}}\right)^2 \quad (2.2)$$

These two equations result in the following relation.

$$f_y * \left(\frac{1}{b}\right)^2 = \sigma_{cr} * \left(\frac{1}{b_{eff}}\right)^2 \quad (2.3)$$

Therefore the effective width is given by the following relation.

$$b_{eff} = b * \sqrt{\frac{\sigma_{cr}}{f_y}} = \rho * b \quad (2.4)$$

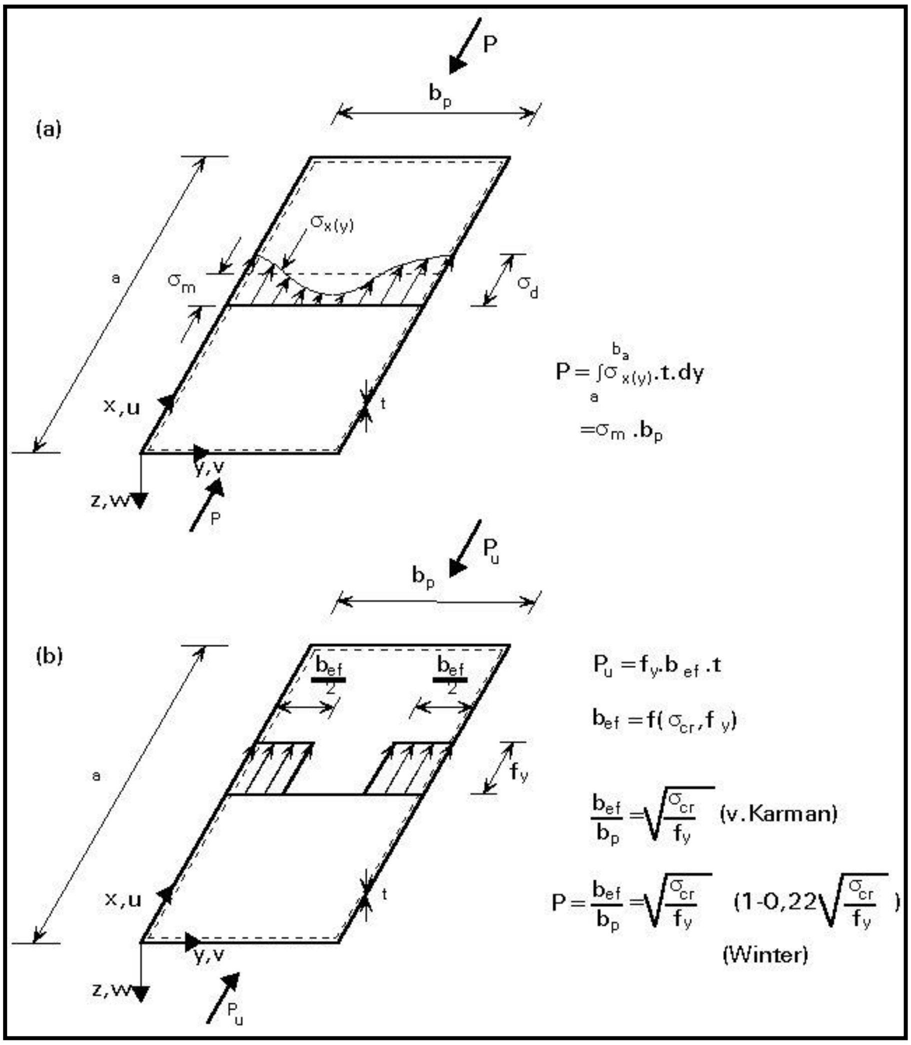


Figure 2: Effective width of a compressed plate (University of Ljubljani, 2011)

The reduction factor ρ can be written in the following way.

$$\rho = \sqrt{\frac{\sigma_{cr}}{f_y}} = \frac{1}{\lambda_{rel}} \tag{2.5}$$

Therefore the effective width can be calculated in the following way:

$$b_{eff} = \frac{1}{\lambda_{rel}} * b \tag{2.6}$$

This is based on a theoretic case. Tests by Winter resulted in a slightly different formula which is:

$$b_{eff} = \frac{\lambda_{rel} - 0{,}22}{\lambda_{rel}^2} * b \tag{2.7}$$

However, for structures that are loaded by a variable loading, this post-buckling strength cannot be utilized. If a variable loading is causing the plate to buckle and the load is removed, the plate will have large plastic deformations and will not return to its original shape. If the variable load is applied and removed numerous times, the post-buckling strength cannot be utilized anymore. This is caused by significant secondary bending stresses near edges of the web and at discontinuities like transverse and longitudinal stiffeners. They cause cumulative fatigue damage to the welded structure and therefore the post-buckling strength cannot be utilized. This is called web-breathing.

In this thesis there is often a mention of local, global and lateral-torsional buckling. A clear definition is presented to guide the reader.
Global buckling: Global buckling or column buckling (Figure 3) is buckling of an entire member out of plane under uniform compression. This is a lateral deflection.

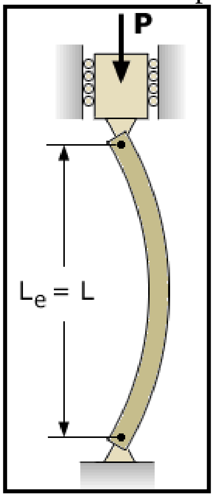


Figure 3: Global buckling or column buckling (FEA Optimization, 2011)

Local buckling: Local buckling or plate buckling (Figure 4) is buckling of a single plate in one or multiple sines where the edges do not have a displacement out of plane.

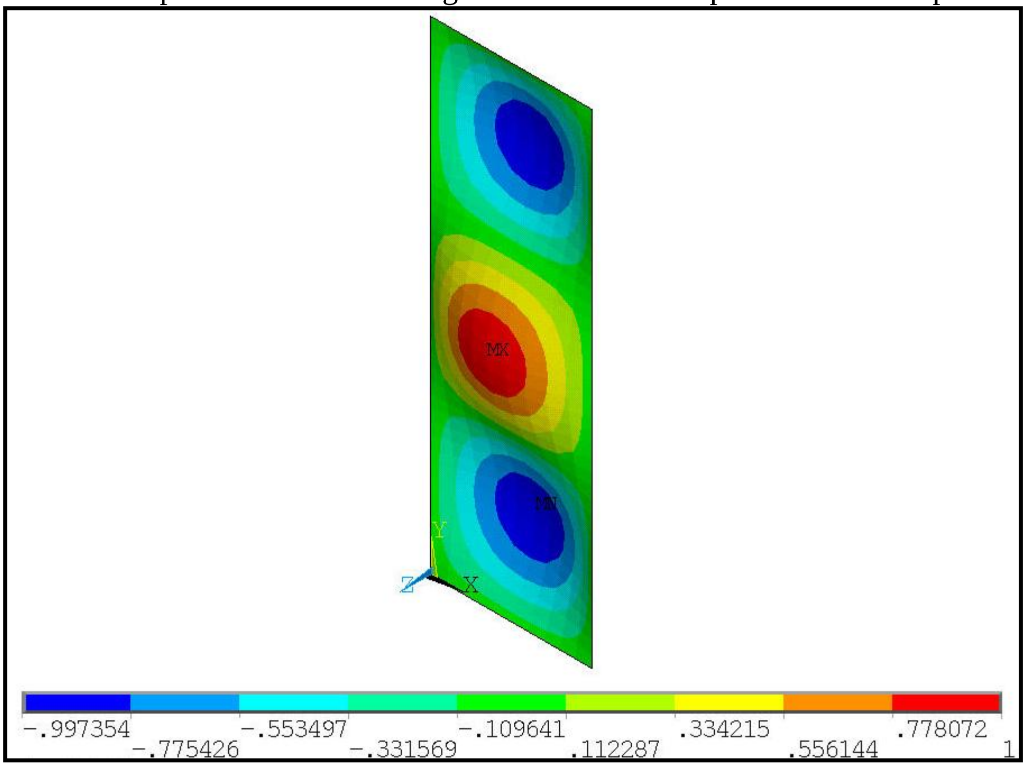


Figure 4: Plate buckling or local buckling for a ratio of $a/b = 3$ (displacement out of plane plotted)

Lateral-torsional buckling: Lateral-torsional buckling (Figure 5) is buckling of an entire member due to a bending moment. This is a lateral deflection combined with a torsional rotation.

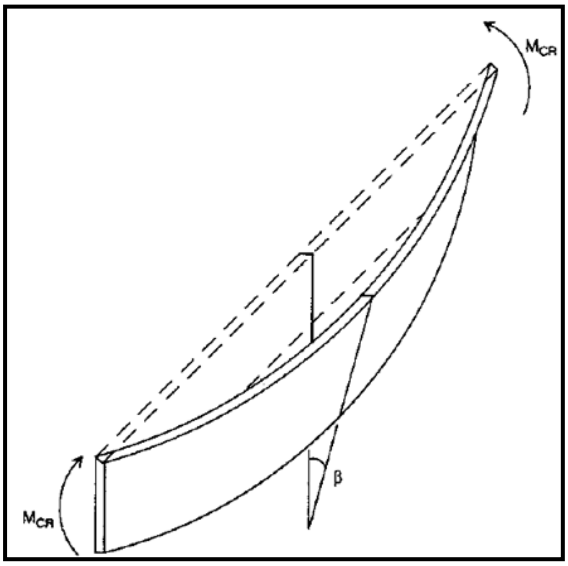


Figure 5: Lateral-torsional buckling (Civil park, 2011)

Rayleigh method

Rayleigh defined a method to derive the theoretical buckling load. He defined buckling as the following: (Welleman, 2007)

Stability is guaranteed when every change to a kinematical possible configuration results in a lower amount of virtual work done by the loading than the virtual energy that is needed to deform the structure.

This method is used to derive all buckling loads in this thesis. It is important to note that this method is an upper bound method because not all possible kinematic configurations can be examined with a hand calculation. If not all configurations are calculated there may always be another configuration which leads to a lower buckling load.

Conventions in thesis

Bernoulli assumptions

In this thesis all cross-sections are calculated using beam theory as it is prescribed by Euler. For these cross-sections there is the assumption of Bernoulli that cross-sections remain flat. This is only applicable for cross-sections with approximately l/h>4. Therefore results that do not satisfy this condition should be neglected even if they are given in the results.

Economic design

There can be many definitions of an economic design depending upon many factors including the effects of material cost and labour cost. For this thesis an economic design is a design with a slender web and most material in the flanges which ensures that the material is where it is most efficient. In general this is a design with a low material usage but this is not necessarily the design that should be applied because many other demands such as aesthetics can change the design.

Coordinate system

The coordinate system is a bit different from the normal conventions for steel structures. In this case the formulas from plate theory are important because they describe the behaviour of the web. Plate theory always prescribes the use of the x-axis and y-axis in respectively longitudinal and transverse direction of a plate. The z-axis is the direction out of plane. The axis system is defined so that the web has this coordinate system as given in Figure 6. The moments of inertia are therefore defined as:

$$I_{zz} = I_y = weak\ direction$$
$$I_{yy} = I_z = strong\ direction$$

This is in contradiction to the normal conventions for steel structures and mechanics. However, this is much more convenient for the derivations of the buckling loads.

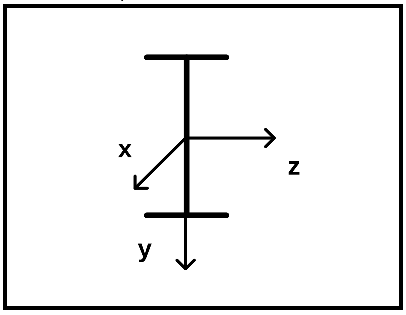


Figure 6: Coordinate system 1

In several cases it is more convenient to define the axis system at the connection of the top flange and the web as given in Figure 7. This is mainly necessary because the behaviour of the web can then be described using a sine-function for the web in transverse direction. The definitions for the moments of inertia as above are still valid. The shift of the axis should not be included when calculating those moments of inertia.

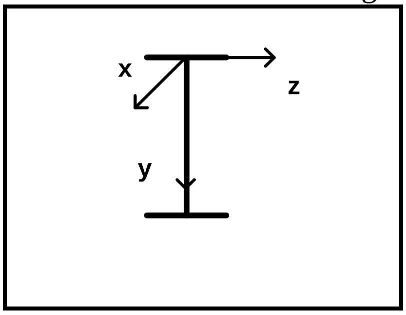


Figure 7: Coordinate system 2

Furthermore are compressive stresses in the theoretical buckling analyses defined as positive. This is done as mostly only compressive stresses are present and those stresses are the destabilizing component.

Derivation column buckling

The derivation of column buckling is done using the Rayleigh method described before. The configuration is very simple and given in Figure 8.

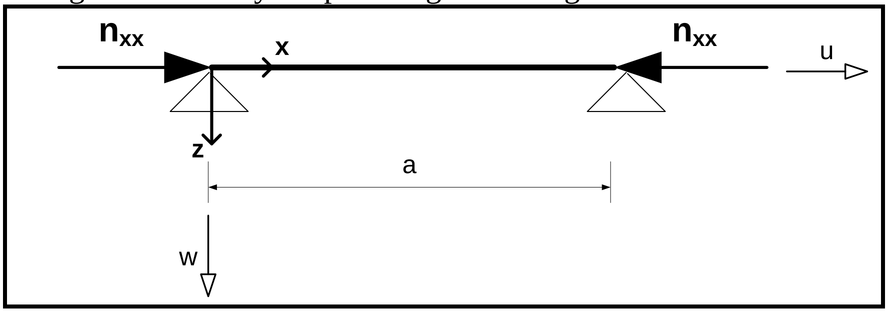


Figure 8: Description of situation for column buckling

The energy that is needed to deform the structure into a certain configuration is the following:

$$U = \int_0^a \frac{1}{2} * \left(EA * \left(\frac{\partial u}{\partial x}\right)^2 + EI * \left(\frac{\partial^2 w}{\partial x^2}\right)^2 \right) * dx \qquad (2.8)$$

However, just before buckling there is already energy in the structure due to axial strain. This is equal to the energy due to axial strain after buckling because there is no increase in loading from just before buckling to just after buckling. The energy due to axial strain before buckling is:

$$U = \int_0^a \frac{1}{2} * EA * \left(\frac{\partial u}{\partial x}\right)^2 * dx \qquad (2.9)$$

Therefore the energy that is additionally added in the structure due to a change in configuration is:

$$U = \int_0^a \frac{1}{2} * EI * \left(\frac{\partial^2 w}{\partial x^2}\right)^2 * dx \qquad (2.10)$$

Later a relationship (equation (14.15)) is derived to explain the shortening at the point of loading due to the lateral deflection. The result is the following:

$$\frac{\partial u}{\partial x} = -\frac{1}{2} * \left(\frac{\partial w}{\partial x}\right)^2 \qquad (2.11)$$

Therefore the term describing the virtual work done by the point load is the following:

$$T = \int_0^a F * \frac{\partial u}{\partial x} * dx = \int_0^a -\frac{1}{2} * F * \left(\frac{\partial w}{\partial x}\right)^2 dx \qquad (2.12)$$

Now the energy and the virtual work equations are known a possible kinematic configuration is chosen. For global buckling it is known that buckling will be in the form of a sine so this is applied here. Any other configuration will lead to a higher buckling force. n is an integer which gives the number of half sine waves in the deflection field.

$$w(x) = \sum_{n=1}^{\infty} A_n * \sin\left(\frac{n\pi x}{a}\right) \qquad (2.13)$$

Applying the derivative of equation (2.13) to equation (2.10) gives the energy in the structure:

$$U = \int_0^a \frac{1}{2} * EI * \left(-\sum_{n=1}^{\infty} A_n * \left(\frac{n\pi}{a}\right)^2 * \sin\left(\frac{n\pi x}{a}\right) \right)^2 * dx$$
$$= \sum_{n=1}^{\infty} \frac{1}{2} * EI * A_n^2 * \left(\frac{n\pi}{a}\right)^4 * \frac{a}{2} \qquad (2.14)$$

Applying the derivative of equation (2.13) to equation (2.12) gives the virtual work done on the structure:

$$T = \int_0^a -\frac{1}{2} * F * \left(\sum_{n=1}^{\infty} A_n * \frac{n\pi}{a} * \cos\left(\frac{n\pi x}{a}\right) \right)^2 dx$$
$$= -\sum_{n=1}^{\infty} \frac{1}{2} * F * A_n^2 * \left(\frac{n\pi}{a}\right)^2 * \frac{a}{2} \qquad (2.15)$$

Now the energy in the structure is equated to the virtual work done $U + T = 0$ and that results in:

$$\frac{1}{2} * EI * A_n^2 * \left(\frac{n\pi}{a}\right)^4 * \frac{a}{2} - \frac{1}{2} * F * A_n^2 * \left(\frac{n\pi}{a}\right)^2 * \frac{a}{2} = 0 \tag{2.16}$$

The final result is:

$$F = \frac{n^2 * \pi^2 * EI}{a^2} \tag{2.17}$$

The lowest buckling load is obtained when $n = 1$ is applied resulting in the well known formula for buckling, which is also known as the Euler buckling load:

$$F = \frac{\pi^2 * EI}{a^2} \tag{2.18}$$

Derivation lateral torsional buckling

Lateral-torsional buckling is the loss of stability due to a bending moment in a combined lateral deflection ($w_0 = \sum_{n=1}^{\infty} A_n * \sin(n\pi x/a)$) and torsional rotation ($\theta = \sum_{t=1}^{\infty} F_t * \sin(t\pi x/a)$). The derivation is a modified version of (Vrouwenvelder, 2003). The general displacement field in a double symmetric cross-section, assuming small rotations, can be described as:

$$w = w_0 - \theta * y \tag{2.19}$$

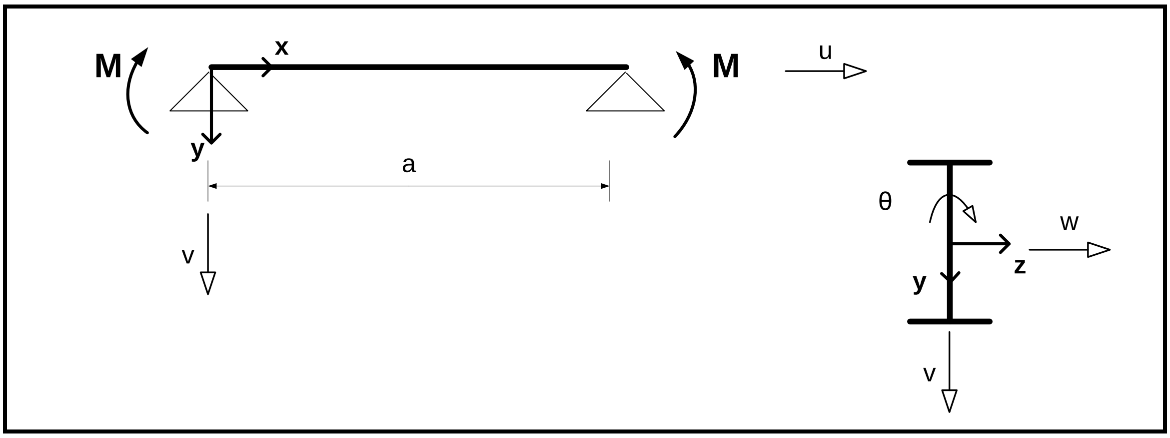


Figure 9: Description of situation for lateral-torsional buckling

The energy in the structure is defined as before but with two additional terms describing the free rotation (Saint Venant torsion) and the restrained warping.

$$U = \int_0^l \frac{1}{2} * \left(EI_{zz} * \left(\frac{\partial^2 w_0}{\partial x^2}\right)^2 + S_t * \left(\frac{\partial \theta}{\partial x}\right)^2 + E * C_w * \left(\frac{\partial^2 \theta}{\partial x^2}\right)^2\right) * dx \tag{2.20}$$

The resistance against the free rotation is: (Hoogenboom, 2010)

$$S_t = G * \sum \frac{1}{3} * b * t^3 \tag{2.21}$$

The resistance against warping for I cross-sections is: (Hoogenboom, 2010)

$$E * C_w = E * \frac{1}{4} * h^2 * I_{zz} \tag{2.22}$$

The result of this integral is the following:

$$U = \sum_{n=1}^{\infty} \frac{1}{2} * EI_{zz} * A_n^2 * \left(\frac{n\pi}{a}\right)^4 * \frac{a}{2} + \sum_{t=1}^{\infty} \frac{1}{2} * S_t * F_t^2 * \left(\frac{t\pi}{a}\right)^2 * \frac{a}{2} + \sum_{t=1}^{\infty} \frac{1}{2} * E * C_w * F_t^2 * \left(\frac{t\pi}{a}\right)^4 * \frac{a}{2} \tag{2.23}$$

The virtual work done is the displacement at the end of the beam multiplied by the stress in that certain point.

$$T = \iiint -\frac{1}{2} * \sigma * \left(\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2\right) * dx * dy * dz \tag{2.24}$$

The stress in the fibre can be described with the following relation:

$$\sigma = -\frac{M}{I_{yy}} * y \tag{2.25}$$

Inserting the equations for σ and w results in:

$$T = \frac{1}{2} * \int_0^a \frac{\mathrm{M}}{I_{yy}} \iint \mathrm{y} * \left(\left(\frac{\partial(w_0 - \theta * y)}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2\right) * dy * dz * dx \tag{2.26}$$

Expanding of this results into:

$$T = \frac{1}{2} * \int_0^a \frac{\mathrm{M}}{I_{yy}} \iint \left(y * \left(\frac{\partial w_0}{\partial x}\right)^2 + y^3 * \left(\frac{\partial \theta}{\partial x}\right)^2 - 2 * y^2 * \frac{\partial w_0}{\partial x} * \frac{\partial \theta}{\partial x} + y * \left(\frac{\partial v}{\partial x}\right)^2\right) * dy * dz * dx \tag{2.27}$$

Terms with y or y^3 result in 0 for symmetric cross-sections when integrated over the height of the section. Therefore only one term is non zero. Integration of y^2 over dy and dz is the same as I_{yy}.

$$T = -\int_0^a \mathrm{M} * \frac{\partial w_0}{\partial x} * \frac{\partial \theta}{\partial x} * dx \tag{2.28}$$

Evaluating this will result in the following which is only valid for $n = t$. If this is not the case, the result is zero.

$$T = -\sum_{n=1}^{\infty}\sum_{t=1}^{\infty} \mathrm{M} * \mathrm{A_n} * \mathrm{F_t} * \frac{\mathrm{n * t * \pi^2}}{\mathrm{a^2}} * \frac{a}{2} \tag{2.29}$$

Now the equation $(U + T = 0)$ can be obtained which gives the lateral-torsional buckling moment.

$$U + T = \sum_{n=1}^{\infty} \frac{1}{2} * EI_{zz} * A_n^2 * \left(\frac{n\pi}{a}\right)^4 * \frac{a}{2} + \sum_{t=1}^{\infty} \frac{1}{2} * S_t * F_t^2 * \left(\frac{t\pi}{a}\right)^2 * \frac{a}{2} + \sum_{t=1}^{\infty} \frac{1}{2} * E * C_w * F_t^2 * \left(\frac{t\pi}{a}\right)^4 * \frac{a}{2} - \sum_{n=1}^{\infty}\sum_{t=1}^{\infty} \mathrm{M} * \mathrm{A_n} * \mathrm{F_t} * \frac{\mathrm{n * t * \pi^2}}{\mathrm{a^2}} * \frac{a}{2} = 0 \tag{2.30}$$

However, this equation cannot be solved because it is dependent on both $\mathrm{A_n}$ and $\mathrm{F_t}$. Therefore the values of $\mathrm{A_n}$ and $\mathrm{F_t}$ need to be such that the bending moment is minimum. The summation is not included anymore because the terms are only valid for $n = t$. Therefore the derivatives with respect to $\mathrm{A_n}$ and $\mathrm{F_t}$ are taken and equated to zero.

$$\frac{\partial(U+T)}{\partial \mathrm{A_n}} = EI_{zz} * \mathrm{A_n} * \left(\frac{n\pi}{a}\right)^4 * \frac{a}{2} - \mathrm{M} * \mathrm{F_t} * \frac{\mathrm{n} * \mathrm{t} * \pi^2}{\mathrm{a}^2} * \frac{a}{2} = 0 \tag{2.31}$$

The derivative to F_t is:

$$\frac{\partial(U+T)}{\partial \mathrm{F_t}} = S_t * \mathrm{F_t} * \left(\frac{t\pi}{a}\right)^2 * \frac{a}{2} + E * C_w * \mathrm{F_t} * \left(\frac{t\pi}{a}\right)^4 * \frac{a}{2} - \mathrm{M} * \mathrm{A_n} * \frac{\mathrm{n} * \mathrm{t} * \pi^2}{\mathrm{a}^2} * \frac{a}{2} = 0 \tag{2.32}$$

These equations need to be written in matrix notation and can then be solved by equating the determinant to zero. This will give the non-trivial solutions to the equation. This will result in a matrix that looks like this where x are non-zero terms:

$$\begin{bmatrix} x & x & 0 & 0 \\ x & x & 0 & 0 \\ 0 & 0 & x & x \\ 0 & 0 & x & x \end{bmatrix} * \begin{bmatrix} \mathrm{A_n} \\ \mathrm{F_n} \\ \mathrm{A_{n+1}} \\ \mathrm{F_{n+1}} \end{bmatrix} = 0 \tag{2.33}$$

The determinant will be zero when only one value of $n = t$ is used. Any other result using multiple values of $n = t$ will result in an average buckling stress. Therefore the matrix can be written as:

$$\begin{bmatrix} EI_{zz} * \left(\frac{n\pi}{a}\right)^4 * \frac{a}{2} & -\mathrm{M} * \frac{\mathrm{n} * \mathrm{t} * \pi^2}{\mathrm{a}^2} * \frac{a}{2} \\ -\mathrm{M} * \frac{\mathrm{n} * \mathrm{t} * \pi^2}{\mathrm{a}^2} * \frac{a}{2} & S_t * \left(\frac{t\pi}{a}\right)^2 \frac{a}{2} + E * C_w * \left(\frac{t\pi}{a}\right)^4 \frac{a}{2} \end{bmatrix} * \begin{bmatrix} \mathrm{A_n} \\ \mathrm{F_t} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{2.34}$$

The determinant of equation (2.34) equated to zero is the following:

$$EI_{zz} * \left(\frac{n\pi}{a}\right)^4 * \frac{a}{2} * \left(S_t * \left(\frac{t\pi}{a}\right)^2 * \frac{a}{2} + E * C_w * \left(\frac{t\pi}{a}\right)^4 * \frac{a}{2}\right) - \left(\mathrm{M} * \frac{\mathrm{n} * \mathrm{t} * \pi^2}{\mathrm{a}^2} * \frac{a}{2}\right)^2 = 0 \tag{2.35}$$

This is only valid for $n = t$ as stated before so all t are changed to n and the equation is solved for M.

$$M = \frac{\pi}{a} * \sqrt{EI_{zz} * n^2 * \left(S_t + E * C_w * \left(\frac{n\pi}{a}\right)^2\right)} \tag{2.36}$$

Now it is easily seen that the lowest lateral-torsional buckling moment will be obtained for $n = 1$. This gives the well known formula:

$$M = \frac{\pi}{a} * \sqrt{EI_{zz} * \left(S_t + E * C_w * \left(\frac{\pi}{a}\right)^2\right)} \tag{2.37}$$

When the warping of the cross-section is not prevented ($C_w = 0$) this will result in the formula that is generally used:

$$M = \frac{\pi}{a} * \sqrt{EI_{zz} * S_t} \tag{2.38}$$

Derivation plate buckling

Before the interaction of plate buckling and column buckling is derived, first the derivation of the plate buckling load is presented. This is a modification of the derivation as it is presented in (Abspoel & Bijlaard, 2005). An energy method is applied to calculate the buckling behaviour. The entire derivation is presented in Annex A and here only the general principle and the results are explained.

The aim is to determine the buckling load of a rectangular plate under a uniform loading from one side as presented in Figure 10.

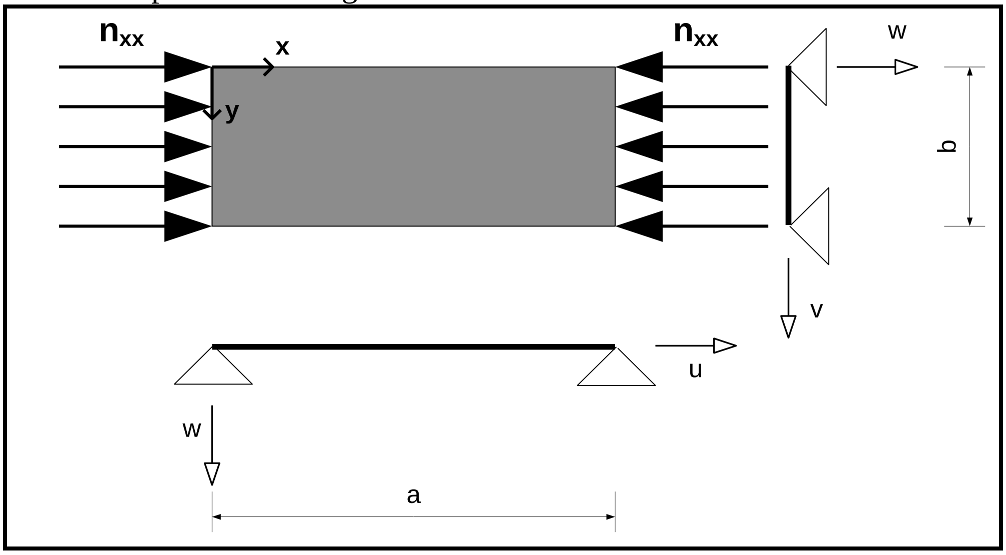


Figure 10: Description of situation for plate buckling

In a flow chart (Figure 11) the steps are given that will result in the buckling load.

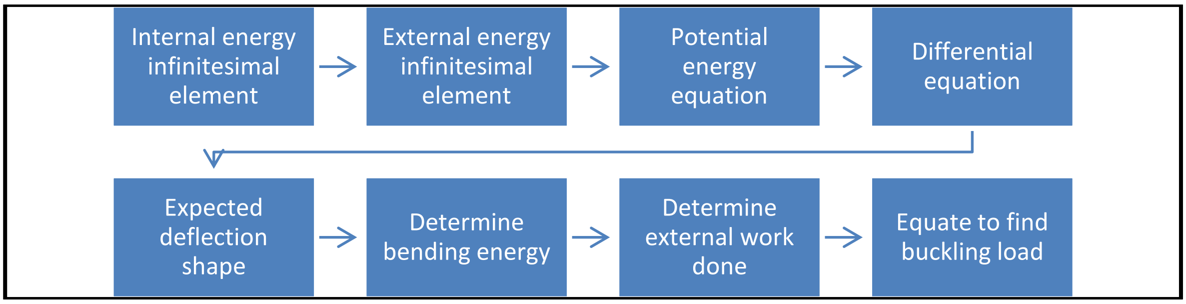


Figure 11: Derivation of the plate buckling load

The entire derivation is presented in Annex A and that will result in a buckling load of:

$$n_{xx} = \frac{D * \pi^2}{b^2} * \left(\frac{m}{\alpha} + \frac{\alpha}{m}\right)^2 = k * \frac{D * \pi^2}{b^2} \tag{2.39}$$

The aspect ratio α of a plate is defined as:

$$\alpha = \frac{a}{b} \tag{2.40}$$

The buckling factor k is defined as:

$$k = \left(\frac{m}{\alpha} + \frac{\alpha}{m}\right)^2 \tag{2.41}$$

The value of m can be all real and positive integers so all buckling modes of the steel plate are given by this equation. The value of m is the amount of half-sine waves in a steel plate when it buckles. In Figure 12 the buckling mode with $m = 3$ and $\alpha = 3$ is presented.

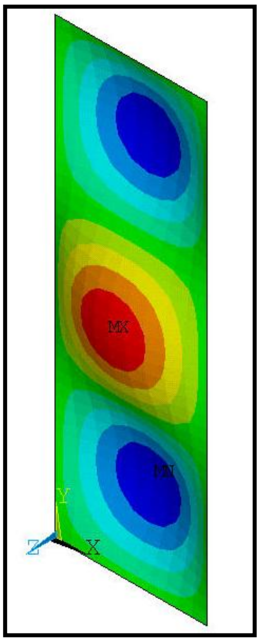


Figure 12: Plate buckling shape for $m = 3$ and $\alpha = 3$.

The buckling factor is dependent on the aspect ratio of the plate and the number of half sine waves and this can be plotted in a graph as done in Figure 13.

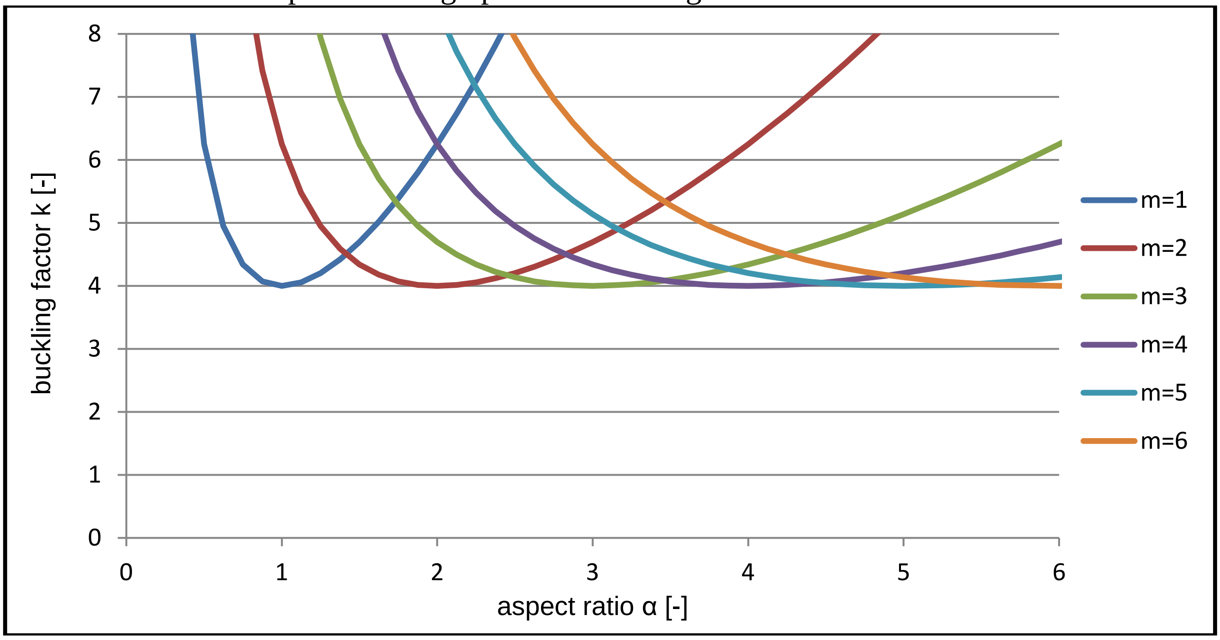


Figure 13: Buckling factor as a function of the aspect ratio for uniform compression

From Figure 13 it is clear that the buckling factor is 4 except when the aspect ratio is smaller than 1 because then the buckling load is higher.

Chapter 3: Plate buckling in Eurocode

The NEN-EN1993, a part of the Eurocode, is the standard for the verification of steel structures in Civil Engineering in The Netherlands since 31st of March 2010. Before the NEN-EN1993 was introduced, the NEN6770, a part of the TGB, was the standard in The Netherlands. However, because of uniformity across Europe, a new standard was developed. The NEN-EN1993 is an accumulation of design rules from different standards from the participating countries combined with research results from the last years. In some cases, the mechanical background is completely left out and only some formulas are supplied which have been fitted to test results. A good engineer does not calculate always according to the rules as they are but knows the mechanical background so he can make a proper judgement whether a certain formula is applicable or not in a specific situation.

In the Eurocode 3 (NEN-EN1993) the structural verification of steel structures is described. In particular in NEN-EN1993-1-5 the structural verification of plated structures are dealt with for structural applications. This standard is a complicated standard because the mechanical background is sometimes lost. Literature is available to guide an engineer in calculating plate buckling. Relevant examples are Beg et al. (2010), Johansson et. al. (2007) and Van der Burg (2011).

NEN-EN1993-1-5 has two ways for dealing with plate buckling. The recommended way is to use the effective cross-section method and the other is the reduced stress method. Both start with the same calculation which is to calculate the reduction factors for the individual elements based on the width to thickness ratio and the stress distribution for each individual element.

Effective cross-section method

The effective cross-section method reduces each section with its own reduction factor. The result is a new, smaller cross-section which is then utilized as a class 3 cross-section. This means that an elastic cross-sectional verification has to be done. All verifications are done using the new effective cross-section. In this case the weakest link is not governing and it is assumed to be a parallel system. This method allows the plate to buckle and therefore utilizes the post buckling strength that is available. Therefore this is the recommended method because the largest possible strength is used.

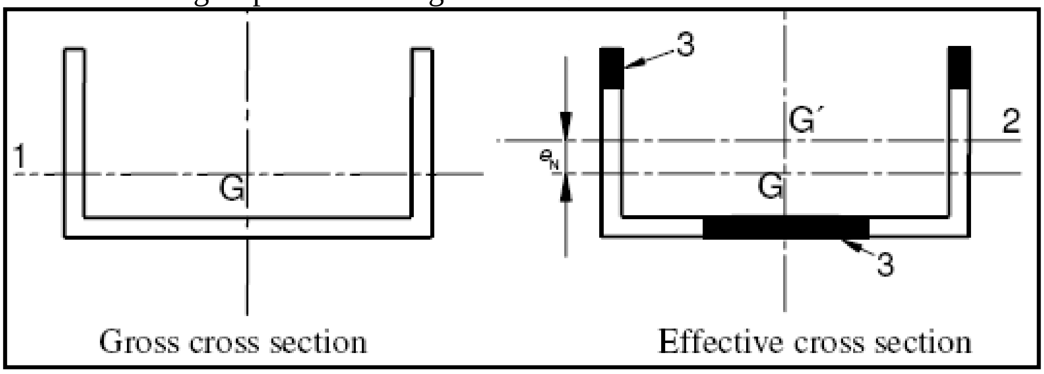


Figure 14: The effective cross-section method for axial compression (NEN-EN1993-1-5, 2006)(p. 14)

Reduced stress method

The reduced stress method also determines the reduction factors for all individual elements. Then the cross-section is used as the gross section but the yield stress is reduced by the largest reduction factor. All verifications are done using a lower allowable stress with the gross cross-section. This is an elastic cross-sectional verification. The element with the largest reduction factor is governing and all other elements are not fully utilized. Therefore the weakest link is governing as it is seen as a serial system. This method does not allow the governing plate to buckle and therefore does not utilize the post buckling strength that is available. This method is therefore not the recommended method. However, in the case of fatigue it is recommended to use this method because buckling of the plate causes significant secondary bending stresses in the welds. Repeated loading and unloading will result in fatigue damage because of those secondary bending stresses. This effect is called web breathing.

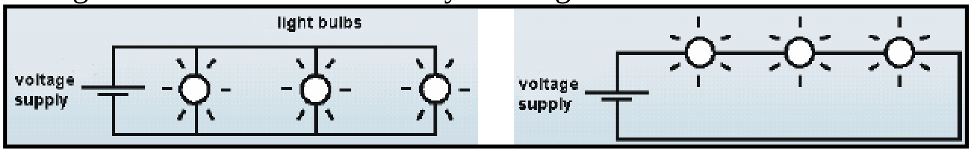


Figure 15: Analogy to effective cross-section method (left, parallel system) and reduced stress method (right, serial system) (Stichting CUR, 1997)

Design of a cross-section against plate buckling

Designing a cross-section against plate buckling used to be done according to the NEN6771. This method recommends and explains the reduced stress method properly and the effective cross-section method was not frequently applied. Now the NEN-EN1993-1-5 is applicable for the verification of steel structures the effective cross-section method is recommended. The main design issues are discussed here because especially the design of the web is significantly different.

Using the reduced stress method in general the web is decisive for the strength of the entire cross-section. If the web is very slender, the flanges may not be exploited up to the yield strength. Applying a longitudinal stiffener will result in a higher reduced stress that may be applied and therefore the flanges can have a much higher capacity.

Using the effective cross-section method the web is not decisive for the strength of the entire cross-section. If the web is very slender, no reduction of the flanges is needed and they are fully exploited. Applying a longitudinal stiffener will only result in a larger effective part of the web but the web is in general small compared to the flanges so the additional capacity is only small.

This reasoning can be clarified with a theoretical example. In Figure 16 a theoretical cross-section is analyzed for axial compression. The case without a stiffener and with a stiffener is examined using both methods. The areas and reduction factors are given.

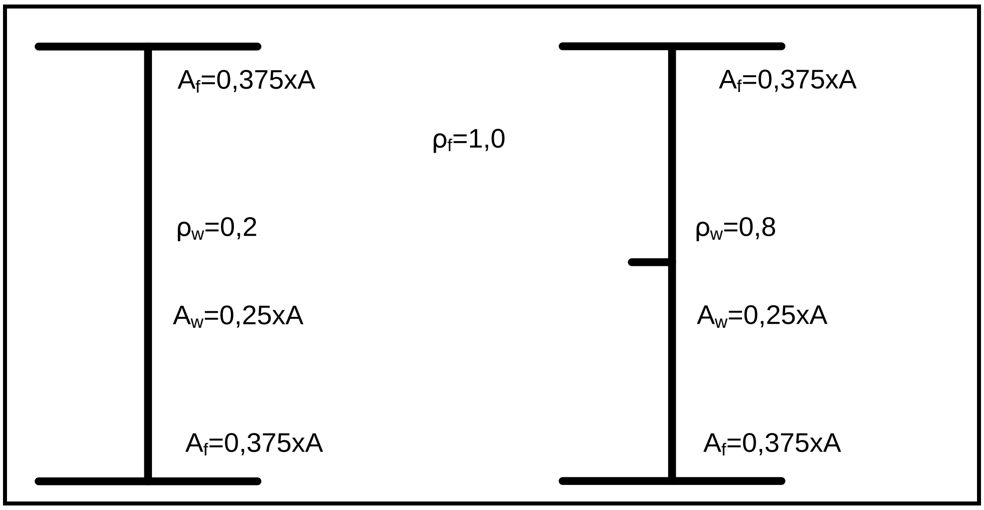


Figure 16: Cross-section for theoretical case between reduced stress and effective cross-section method

For the reduced stress method the axial capacity can be determined using:

$$N_{reduced\ stress} = \rho_{min} * A * f_y \tag{3.1}$$

The capacity using the effective cross-section method can be determined using:

$$N_{effective\ cross-section} = (\rho_{web} * A_{web} + 2 * \rho_{flange} * A_{flange}) * f_y \tag{3.2}$$

Applying those formulas to the cross-section of Figure 16 gives the results as presented in Table 1.

	Without a stiffener	With one stiffener
Reduced stress method (NEN6771)	$0{,}2 * A * f_y$	$0{,}8 * A * f_y$
Effective cross-section method (NEN-EN1993-1-5)	$0{,}8 * A * f_y$	$0{,}95 * A * f_y$

Table 1: Results using the reduced stress and the effective cross-section method

It is immediately clear that the stiffener gives an increased capacity using the reduced stress method of 300%. However, for the effective cross-section method the increase is only 18,8%. This shows the difference between both methods and it is clear that in the NEN6771 using the reduced stress method a stiffener is extremely effective to increase the axial capacity. Applying the NEN-EN1993-1-5 the stiffener is probably not the most economic solution because the increase is only small for a quite labour intensive handling. It would probably be more economic to just increase the area of the flanges.

Some general remarks are given for consideration when designing a cross-section using the effective cross-section method.

- For the effective cross-section method stiffeners to the web are not very useful for the static strength against axial compression and bending moments. In general the web is slender and a stiffener only increases the area of the web that is effective. For axial compression this is usually a small increase of the area and for bending moments the material is close to the centre of gravity resulting in a small increase of the effective section modulus.
- It would be much more useful to apply larger flanges because the costs of adding a stiffener are in general high in developed countries.

- Fatigue loading can influence the design of the web. If variable loading is present, the effect of web breathing should be examined and for slender webs a reduced stress method should be applied. (NEN-EN1993-2, 2007)
- Shear force influences the design of the web. The shear force is carried by the web and the web should be designed for that including the effect of shear buckling.
- Flange-induced buckling influences the design of the web. Depending on the area of the flanges, the web needs a minimum thickness to withstand the inward forces of the flange in bending.

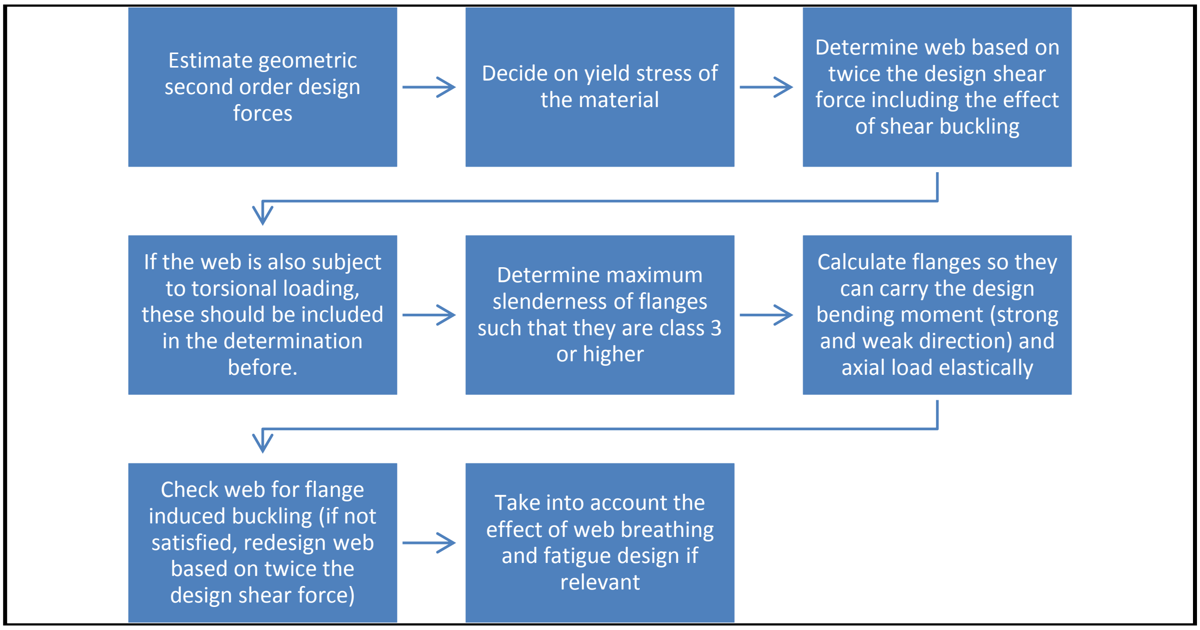


Figure 17: Design of a class 4 cross-section

In Figure 17 a very simple design procedure is given for a class 4 cross-section. This can be applied in many situations to get very quickly to approximately the right dimensions of the cross-section. The interaction formulas in the NEN-EN1993-1-5 are based on second order geometric forces. It would also be possible to estimate the buckling factor and increase the first order forces by that factor. According to the NEN-EN1993-1-5 there is no interaction check necessary when the shear force is 50% of the shear force capacity including the effects of shear buckling. Therefore the web should be designed using twice the design shear force.

Then the slenderness of the flanges should be calculated such that they fit just into class 3. A higher class would not be beneficial because a plastic design is not allowed using the effective cross-section method. A lower class would result in a reduction of the effective area of the flange and is only recommended when column buckling or lateral-torsional buckling may be a decisive problem. The area of the flanges can be determined using the bending moments in both directions and the axial force. This should be based on an elastic calculation of the stresses using only the flanges. The small beneficial effect of the effective web is ignored and could be exploited for the verification calculation.

$$\sigma_{el} = \frac{N}{2 * A_f} + \frac{M_{strong}}{h_w * A_f} + \frac{M_{weak}}{2 * \frac{1}{6} * t_f * h_f^2} \tag{3.3}$$

Now the flanges and web are designed the web should be checked for flange induced buckling. If the web is too slender, the web should be changed. If the height of the web is changed, also the area of the flanges changes. When this is all done, this is sufficient for a preliminary static design. If there is a large variable loading, also a fatigue preliminary design should be applied.

Classes in NEN-EN1993-1-1

NEN-EN1993-1-1 has defined classes to internal and external compression elements that indicate whether the full plastic capacity, an elastic capacity or a reduced elastic capacity needs to be used. The limit for internal elements under pure compression, e.g. a web, is a ratio of 42ε to ensure class 3 behaviour according to table 5.2 in NEN-EN1993-1-1. However, if a ratio of $\frac{b}{t} = 42\varepsilon$ is inserted into the plate buckling formula this results into a effective cross-section of 95% as shown in the calculation below.

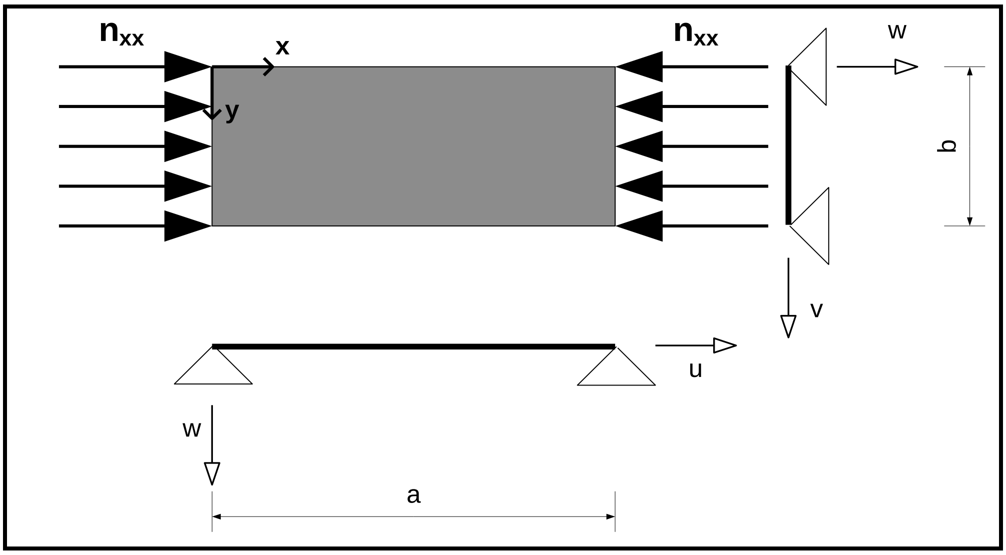


Figure 18: Plate under uniform axial compression

The factor ε is dependent on the yield stress:

$$\varepsilon = \sqrt{\frac{235}{f_y}} \tag{3.4}$$

The slenderness for plate buckling is defined as follows.

$$\lambda = \sqrt{\frac{f_y}{\sigma_{cr}}} \tag{3.5}$$

In this formula the critical plate buckling stress can be inserted.

$$\lambda = \sqrt{\frac{f_y}{k * \frac{\pi^2 * E}{12 * (1 - v^2)} * \left(\frac{t}{b}\right)^2}} \tag{3.6}$$

The relevant numbers can be applied to the equation resulting into a slenderness.

$$\lambda = \sqrt{\frac{f_y}{4 * \frac{\pi^2 * 210000}{12 * (1 - 0{,}3^2)} * \left(\frac{1}{42\varepsilon}\right)^2}} = 0{,}739 \tag{3.7}$$

Now the formula for the reduction of the cross-section is as follows for internal compression elements. This is the same as the Winter formula written in a slightly different way.

$$\rho = \frac{\lambda - 0{,}055 * (3 + \psi)}{\lambda^2} \tag{3.8}$$

Applying the numerical figures to this equation results in:

$$\rho = \frac{0{,}739 - 0{,}055 * (3 + 1)}{0{,}739^2} = 0{,}95 \tag{3.9}$$

This is a very remarkable result because an infinitesimal increase of the length of a web just above the ratio 42ε reduces the capacity by 5%. This seems a very strange conclusion because an increase of material generally does not lead to a reduction of the capacity if the effective cross-section method is applied. This is plotted for a web of 15 mm thick and $f_y = 235\ N/mm^2$ in Figure 19.

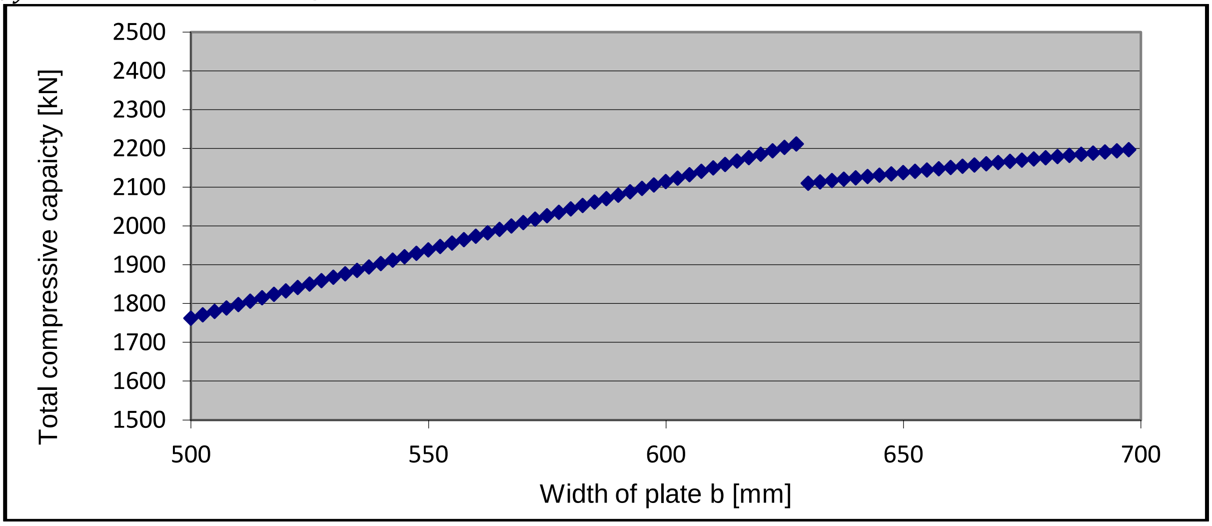


Figure 19: Jump at slenderness of 42ε

NEN-EN1993-1-5 does prescribe the correct solution because there the limiting slenderness is different for which the reduction factor ρ is 1.

$$\lambda_{limit} = 0{,}5 + \sqrt{0{,}085 - 0{,}055 * \psi} \tag{3.10}$$

Applying $\psi = 1$ for uniform compression gives:

$$\lambda_{limit} = 0{,}5 + \sqrt{0{,}085 - 0{,}055 * 1} = 0{,}673 \tag{3.11}$$

The formula for the reduction factor ρ is:

$$\rho = \frac{\lambda - 0{,}055 * (3 + \psi)}{\lambda^2} \tag{3.12}$$

The resulting reduction factor therefore is:

$$\rho = \frac{0{,}673 - 0{,}055 * (3 + 1)}{0{,}673^2} = 1{,}0 \tag{3.13}$$

However, NEN-EN1993-1-5 is only applicable for class 4 cross-sections so if the ratio of a web is lower than 42ε plate buckling does not have to be calculated. An increase of material always results in an equal or higher capacity if the effective cross-section method is applied. This is not represented in the NEN-EN1993-1-1.

Therefore the calculation of NEN-EN1993-1-5 seems to be correct whereas the assumption of class 3 behaviour for 42ε for internal compression elements is not consistent with the design of plated structures.

Development of calculation tool

The Engineering Office of Public works of Rotterdam is in need of a tool to calculate class 4 cross-sections according to the NEN-EN1993-1-5. The procedure in the NEN-EN1993-1-5 is very cumbersome and is difficult to do quickly and correctly if this is not done on a regular basis. Therefore a tool is needed to check an I cross-section. They are looking for an extremely versatile tool which can be used for nearly all class 4 cross-sections. This means that there should be several possibilities available:

- 0, 1 or 2 flat longitudinal stiffeners in the web
- A fictitious plate at the top or at the bottom of the girder (e.g. a concrete deck)
- A trough plate at the top or at the bottom of the girder (e.g. a steel deck)

Another requirement is that there should be several types of loading and also combinations possible:

- Axial compression or tension
- Bending moment in the stiff direction
- Shear force
- Transverse load introduction

This means that there are many configurations possible. The tool is presented in Annex B and the explanation of the tool is also in Annex B in Dutch. Dutch is the language used at the engineering office so is also most suitable for the explanation of the procedure. A short outline is presented here in English to guide the reader.

The input is an I cross-section which can have four different flanges. (topleft, topright, lowerleft and lowerright) This does not mean that the tool is only applicable for an I cross-section because a box girder may be cut in two and then it is similar to an I cross-section. Also a π-girder can be used when it is split in two.

There is one main advantage of the NEN-EN1993-1-5 that is applied in the tool. The NEN-EN1993-1-5 allows the resistance of the cross-section to be determined without any knowledge of the applied loading for I and box cross-sections. This was not true in the old NEN6771 codes where the loading configuration influences the resistance of the cross-section. The NEN-EN1993-1-5 allows the calculation of the axial resistance based on only the geometry and axial compression. Then the resistance against bending moments can be calculated using the geometry and the stresses are based on pure bending. Afterwards there are interaction formulas to combine the effects of axial forces and bending moments.

The resistance against shear forces is calculated based on only the web. NEN-EN1993-1-5 allows the increase of the resistance using the flanges but this reduces the capacity for axial compression and bending moments. Therefore this is not utilized.

The entire process of the calculation sheet in Mathcad is explained in Figure 20.

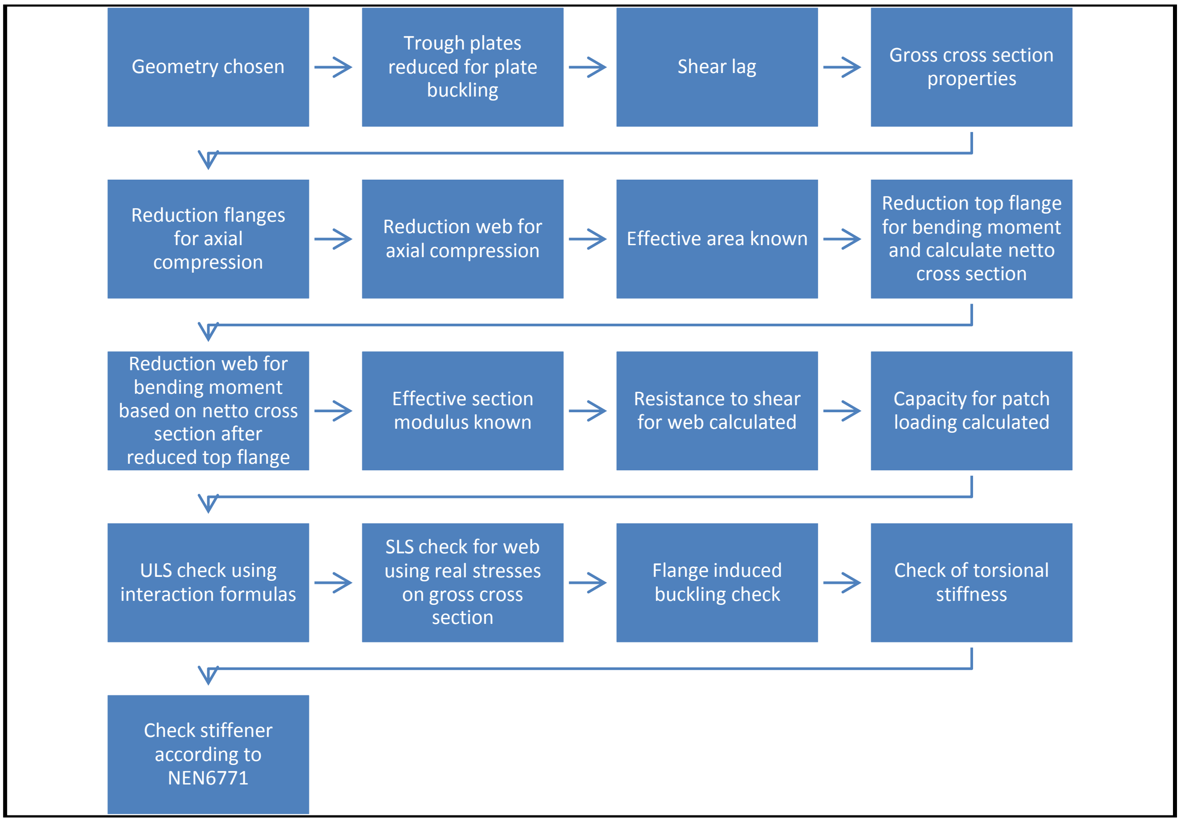


Figure 20: Process of checking I cross-section in Mathcad sheet

Chapter 4: Interaction between plate and column buckling

Welded I-columns are often designed to be very slender instead of the hot-rolled sections which are quite stocky and are mostly in class 3 or higher. Class 3 means that plate buckling will not occur before yielding occurs and therefore the entire cross-section is effective. Therefore only global buckling or yielding of the cross-section will be governing. This is not the case for welded I-columns which often have a class 4 web. Welded columns are applied when rolled columns will not fit the requirements of the structure. The use of material is much more efficient because all parts of the cross-section can be optimized.

I-columns with class 4 webs need to be reduced to an effective cross-section according to NEN-EN1993-1-5. This states that for welded I-columns under uniform compression the web is always considered to be simply supported along all edges and a k-factor of 4 is applied. Then the NEN-EN1993-1-1 is applied to calculate the global buckling behaviour of the column. The relative slenderness is calculated using the gross cross-section and then a reduction is applied using the ratio of the effective to the gross cross-section. This relationship follows from the following derivation.

The plastic capacity of the section is:

$$N_{pl} = A_{eff} * f_y \tag{4.1}$$

The critical buckling load is based on the gross cross-section equal to:

$$N_{cr} = \frac{\pi^2 * E * I}{a^2} \tag{4.2}$$

Therefore the resulting relative slenderness is the following:

$$\lambda = \sqrt{\frac{N_{pl}}{N_{cr}}} = \sqrt{\frac{A_{eff}}{A} * \frac{f_y}{\sigma_{buck}}} \tag{4.3}$$

However, when the flange is very small this is a remarkable assumption because a very small flange can never support a web for out of plane deformation. Research (Burg, 2011) has shown that for the static ultimate limit state this is covered by the check of global column buckling. This means that if the flanges are not stiff enough, the entire column will buckle and local plate buckling of the web is not governing. If the flanges are stiff enough, the column will not buckle and local plate buckling will be governing.

However, Van der Burg has not done analytical calculations which describe the behaviour of the I-column. Analytical calculations will result in a proper understanding of the behaviour of a welded I-column.

Analytical calculations are done to show the interaction between column and plate buckling. Three calculations are done that describe the interaction.

1. Stability of a flange. A flange is considered to be connected to the web with a hinge and stability is calculated.
2. Welded I-column under uniform compression. The buckling load of a welded I-column is calculated where plate buckling as well as global buckling is possible. The welds are again modelled as hinges for simplicity.
3. Welded I-column under uniform compression. The buckling load of a welded I-column is calculated where plate buckling as well as global buckling is possible. The welds are modelled as fixed connections.

Of course the analytical calculations are done for a perfectly straight structure and later using finite element methods the calculations can be done including the effects of imperfections and plasticity. Then the results can be compared to the calculations according to the NEN-EN1993-1-5 and conclusions can be drawn whether the NEN-EN1993-1-5 provides the correct method for the verification of class 4 cross-sections.

Stability of a flange

For stiffeners there are strict guidelines to determine when a stiffener is considered stiff and when a stiffener is considered flexible. A stiff stiffener is a stiffener which provides sufficient support to ensure that torsional buckling of the stiffener does not occur before yielding when it is loaded axially. This requirement is in NEN-EN1993-1-5 article 9.2.1 (8). It is further explained and derived in the background document. (ECCS, 2007, pp. 112-113) The requirements are quite strict and therefore it is strange that these requirements are not imposed upon the flanges of an I-column. The entire derivation of the transverse and longitudinal stiffeners is followed as it was done by (Bijlaard, 1982) and is applied upon the flanges of an I-column that is axially compressed.

This flange (Figure 21) can be seen as a longitudinal stiffener at the edge which is supported in the middle. This simplifies the calculation because all eccentricities are zero. It is here assumed that the web gives sufficient support to prevent any displacement in y-direction but does not give any support against a rotation around the origin of the axis system.

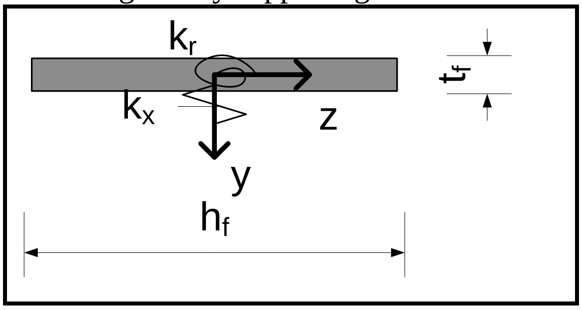


Figure 21: Flange dimensions

The torsional buckling stress of any long structure is defined as:

$$\sigma_{cr,t} = G * \frac{I_t}{I_p} \tag{4.4}$$

Applying the relevant definitions of I_t and I_p for a rectangular strip results in:

$$\sigma_{cr,t} = G * \frac{\frac{1}{3} * h_f * t_f^3}{\frac{1}{12} * h_f^3 * t_f + \frac{1}{12} * h_f * t_f^3} \tag{4.5}$$

When the requirement is that yielding has to occur before torsional buckling occurs the limiting dimensions can be derived.

$$\sqrt{\frac{f_y}{\sigma_{cr,t}}} = \lambda_{rel} \leq 0.2 \tag{4.6}$$

Applying equation (4.5) to equation (4.6) results in:

$$\sqrt{\frac{f_y}{G * \frac{\frac{1}{3} * h_f * t_f^3}{\frac{1}{12} * h_f^3 * t_f + \frac{1}{12} * h_f * t_f^3}}} \leq 0.2 \tag{4.7}$$

Now equation (4.7) can be multiplied by the factor ε and simplified which results in:

$$\sqrt{\frac{235 * h_f^2}{G * 4 * t_f^2}} \leq 0.2\varepsilon \tag{4.8}$$

Rewriting this equation and applying the definition of G ($G = \frac{E}{2*(1+v)}$) will result in the demand for the slenderness of the flange:

$$\frac{h_f}{t_f} \leq 7{,}42 * \varepsilon \tag{4.9}$$

However, in this analysis a piano hinge was assumed which is a very conservative approach. Therefore the result is indeed a very strict guideline for the width to thickness ratio of the flange. However, this is no realistic approach and the results for the buckling analysis need to be more accurate.

Stability of a welded I-column under uniform compression

A loss of stability of a welded I-column may occur in multiple ways. Local buckling may be the governing mechanism which creates multiple half sine waves in the web. If local buckling occurs the flanges are stiff enough to support the web. Local buckling is not a failure of the entire structure. Global buckling of the entire column may be governing which would create a single half sine in the weakest direction of the column. If global column buckling occurs the flanges are not stiff enough to support the web. Global buckling is a failure mechanism. However, also an intermediate form may occur which is an interaction between both forms. The flanges are quite stiff but do not fully prevent local buckling.

This interaction is especially important for fatigue loaded structures. If a structure is loaded in fatigue the post plate buckling strength may not be utilized so the strength is governed by the lowest of the plate buckling strength and the column buckling strength. When either one is much higher than the other, the lowest will be the governing mechanism. However, if they are quite close to each other, there may be some interaction which is possibly unfavourable.

The calculation of the critical buckling stress of the welded I-column is equivalent to the calculation made earlier for the plate simply supported along all four edges. Except this time the local buckling behaviour is combined with the global buckling behaviour.

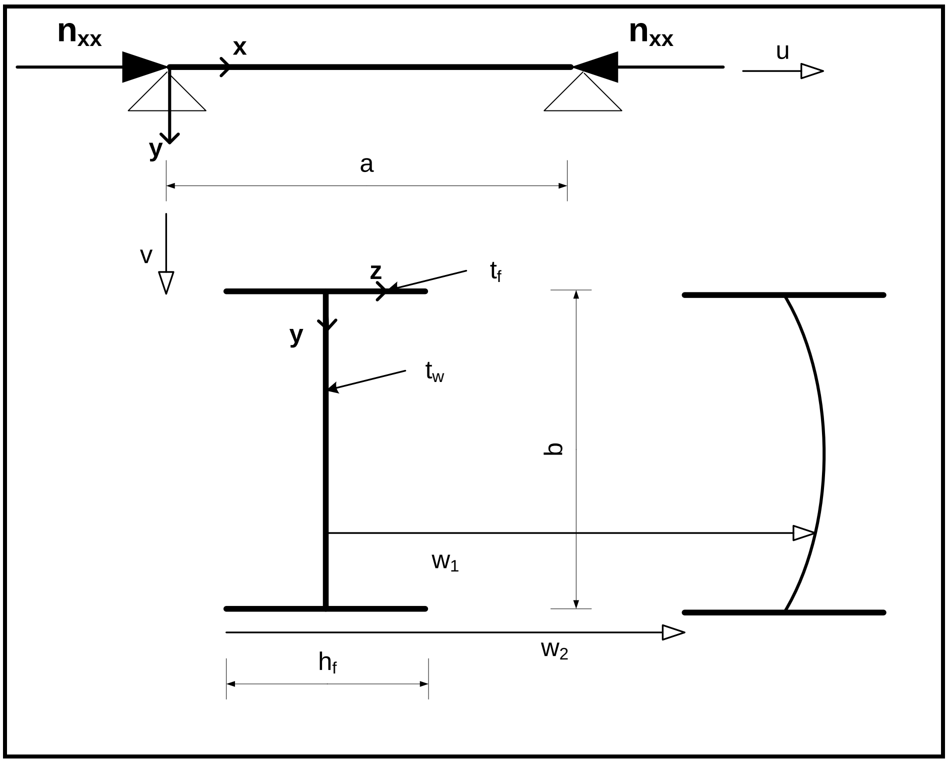


Figure 22: Description of situation for uniform compression of welded I-column using hinged welds

The process to derive the buckling load is given in Figure 23.

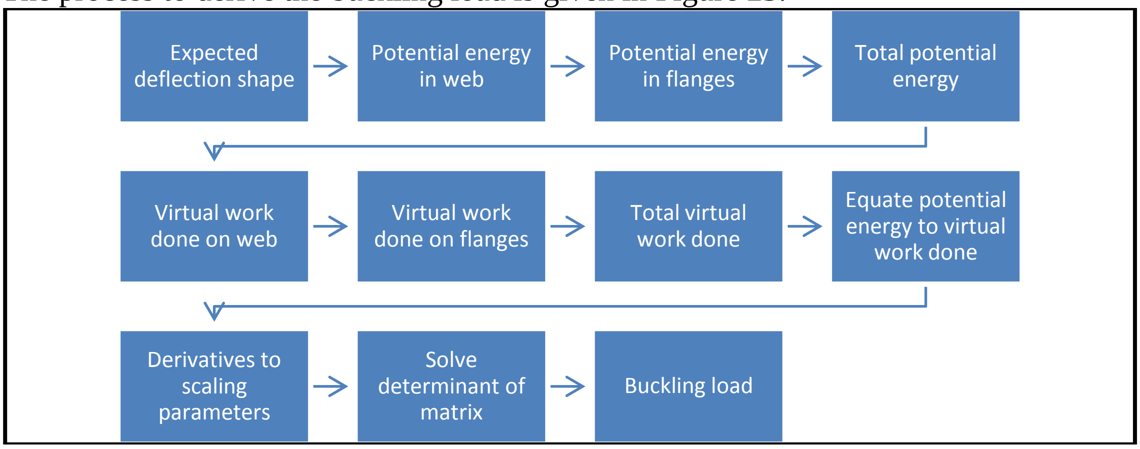


Figure 23: Flow chart to derive the buckling load of an I-column

Deflection shape as a sum of global and plate buckling behaviour

The deformation shape of the column may be described in the following way. For the global behaviour a series of sines is used and for the local behaviour a series of sines multiplied by another series of sines in transverse direction is used. These deformations are the ones of the individual buckling shapes of the global and the local behaviour. The deformation of the web (w_1) is a summation of the local and the global behaviour. The deformation of the flange (w_2) is described by the global behaviour. At $y = 0$ and $y = b$ the web and the flange have the same deformation for all x so compatibility between the web and the flange is ensured.

Therefore the deformation shape is given in Figure 24.

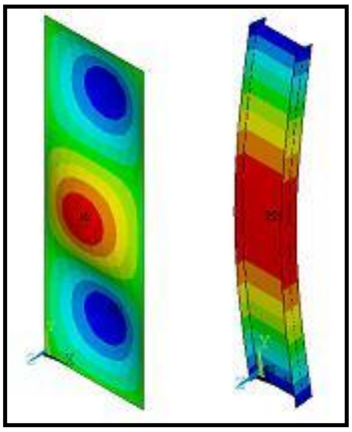


Figure 24: The local behaviour ($m = 3, s = 1$) (left) and the global behaviour ($n = 1$) (right) (displacement out of plane plotted)

The deformation shape of the web is:

$$w_1 = \sum_{n=1}^{\infty} A_n * \sin\left(\frac{n\pi x}{a}\right) + \sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms} * \sin\left(\frac{m\pi x}{a}\right) * \sin\left(\frac{s\pi y}{b}\right) \quad (4.10)$$

The deformation shape of the flanges is:

$$w_2 = \sum_{n=1}^{\infty} A_n * \sin\left(\frac{n\pi x}{a}\right) \quad (4.11)$$

Flanges normally have a moment-resistant connection to the flanges. However, the flanges have a very small resistance to rotation for smaller flanges which are important for this thesis.

Timoshenko (1963) described the buckling behaviour of columns and plates separately using an energy method. The strain energy in the construction needs to be smaller than the virtual work done by the external forces to ensure that the construction is stable. This is again similar to the applied derivation of the plate buckling load earlier. However, this thesis combines the effects of local and global buckling.

Determine equilibrium condition

The derivation of the potential energy and the virtual work done is presented in Annex C. The limit state where the construction is just stable is the point where the total potential energy in the column is equal to the virtual work done by the external forces. If the load is increased anymore at this point the structure will buckle.

$$U_{total} + T_{total} = 0 \quad (4.12)$$

From this equilibrium condition the buckling stress can be computed. This derivation is also in Annex C. However, this buckling stress is still dependent on A_n and B_{ms}.

First of all, higher values of s will lead to a higher amount of energy in the structure and a lower amount of virtual work done so only $s = 1$ is applied here. This means there is only a single half sine wave in transverse direction in the web.

Second, multiple values of A_n and B_{m1} will lead to an average value of σ_{xx} as explained before for the case of lateral-torsional buckling. Therefore only one of those is used to find the minimum.

The derivatives of the equilibrium equation are taken with respect to A_n and B_{m1}. This results in two equations that need to be equated to zero to find the minimum buckling stress. These equations are given in matrix form and the determinant equated to zero will result in the buckling stress. This matrix is still dependent on m $(= n)$ and the result needs to be calculated for several values of m and the lowest is the buckling load.

$$\begin{bmatrix} 2*n^4*\pi^4*\frac{1}{a^3}*\frac{EI_{zz}}{4}-\sigma_{xx}*2*n^2*\pi^2*\frac{1}{a}*\left(\frac{b*t_w}{4}+\frac{t_f*h_f}{2}\right) & n^2*\frac{\pi^4}{a^2}*\frac{a*b}{\pi*1}*D*\left(\frac{m^2}{a^2}+\frac{1^2*v}{b^2}\right)-\sigma_{xx}*n*m*\left(\frac{\pi}{a}\right)^2*\frac{\mathrm{a*b}}{\pi*1}*t_w \\ n^2*\frac{\pi^4}{a^2}*\frac{a*b}{\pi*1}*D*\left(\frac{m^2}{a^2}+\frac{1^2*v}{b^2}\right)-\sigma_{xx}*n*m*\left(\frac{\pi}{a}\right)^2*\frac{\mathrm{a*b}}{\pi*1}*t_w & 2*\pi^4*\frac{ab}{8}*D*\left(\frac{m^2}{a^2}+\frac{1^2}{b^2}\right)^2-\sigma_{xx}*2*\left(\frac{m\pi}{a}\right)^2*\frac{\mathrm{ab}}{8}*t_w \end{bmatrix}*\begin{bmatrix} A_n \\ B_{m1} \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Verification of the result

To verify whether calculations have been done correctly, only the local buckling behaviour is checked which means that $A_n = 0$. This means that the term in the second row and second column is the only one that remains and is equated to zero.

$$\sigma_{xx} = \frac{\pi^2 * D * \left(\frac{m^2}{a^2} + \frac{1^2}{b^2}\right)^2}{t_w * \left(\frac{m}{a}\right)^2} \tag{4.13}$$

This is slightly changed into the formula that is recognisable.

$$n_{xx} = \frac{\pi^2 * D * \left(\frac{m}{\alpha} + \frac{\alpha}{m}\right)^2}{b^2} \tag{4.14}$$

The result is indeed the traditional plate buckling formula that was derived earlier and that is equation (14.52). Another way to verify is to assume only global buckling behaviour which means that $B_{ms} = 0$. This means that the term in the first row and first column is the only one that remains and is equated to zero.

$$\sigma_{xx} = \frac{n^2 * \pi^2 * \left(\frac{D * b}{4} + \frac{EI_f}{2}\right)}{a^2 * \left(\frac{b * t_w}{4} + \frac{t_f * h_f}{2}\right)} \tag{4.15}$$

This is slightly changed into the formula that is recognisable.

$$\sigma_{xx} = \pi^2 * n^2 * \frac{EI_{cross\ section}}{a^2 * A_{cross\ section}} \tag{4.16}$$

This is the same as the known Euler buckling load (equation (2.18)) if n is chosen equal to 1:

$$\sigma_{xx} = \frac{\pi^2}{a^2} * \frac{EI}{A} \tag{4.17}$$

This is also correct because this is the classical Euler buckling formula for columns which is known from literature.

This analysis is also performed using fixed connections between the flanges and the web for the welds as presented in Figure 25. This is presented in Annex D and only the results are included in the next paragraph.

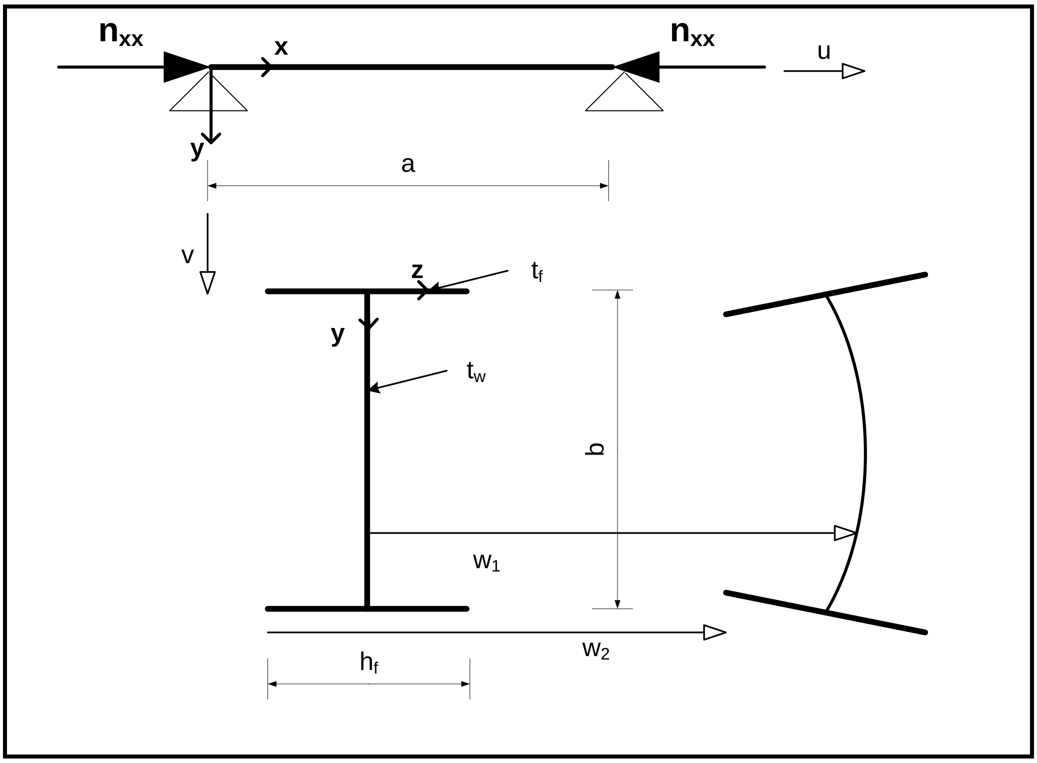


Figure 25: Description of situation for uniform compression of welded I-column using fixed welds

Results

Analytical results for IPE200

First of all the analytical results for a standard rolled cross-section have been calculated. This is a class 1 profile so plate buckling is not governing and the entire cross-section is effective. The added mass of the welds are neglected so it is actually a welded cross-section with flanges 100x8,5 mm and web 191,5x5,6 mm. The calculations for local plate buckling and global column buckling have been added to the results to make a comparison as done in Figure 26. In Annex D the analytical calculation has been done including the effects of a fixed connection between the flanges and the web. The results from that analysis are also plotted in Figure 26. In Chapter 6 a finite element model is developed and the theoretical buckling stress according to that model is also included in Figure 26. Chapter 6 explains the model and also a verification of the results.

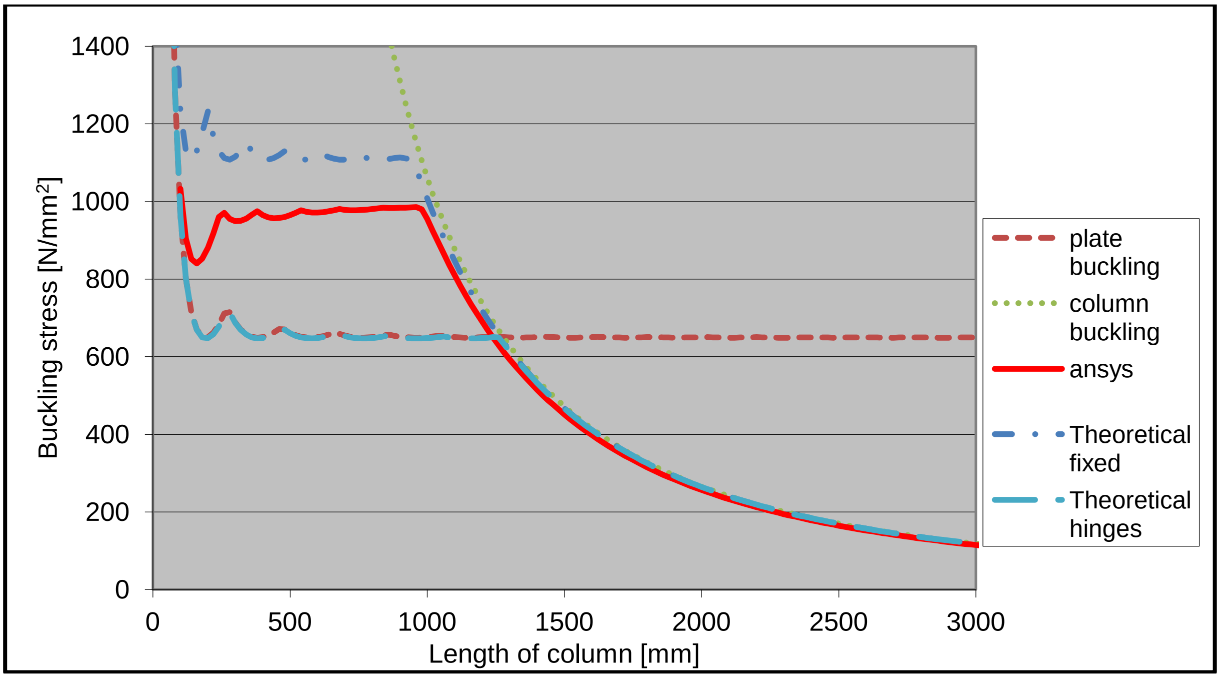


Figure 26: Theoretical buckling curve for IPE200

The theoretical buckling stress using hinges and fixed welds compared to the results from the finite element model are plotted in Figure 27. It is clear that the model using fixed welds gives the best results but does overestimate the buckling load especially for shorter lengths.

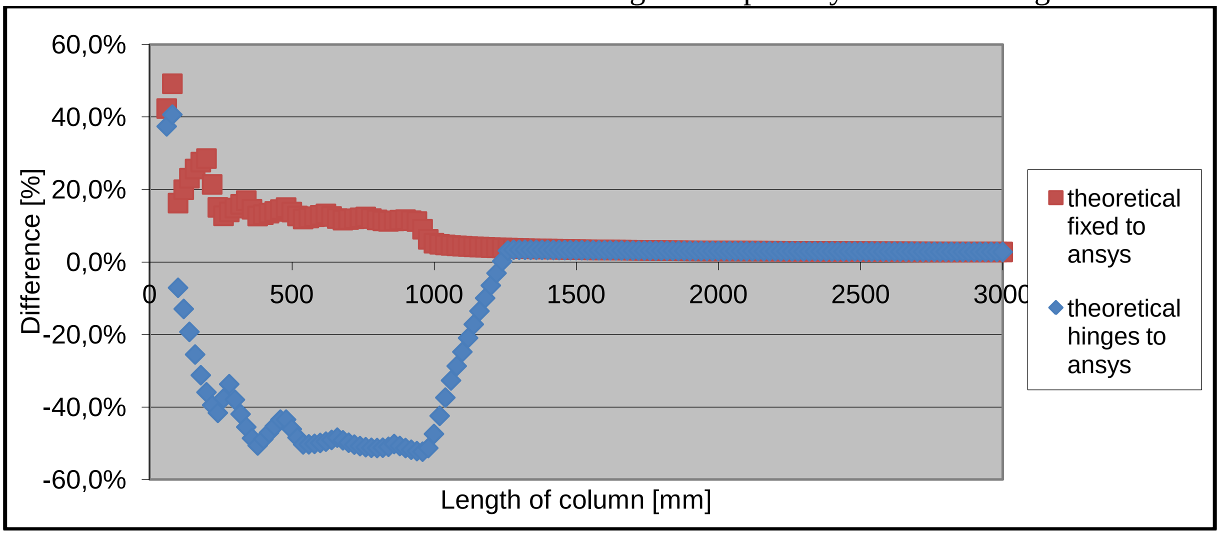


Figure 27: Difference caused by interaction of local and global buckling for IPE200

From Figure 26 it is clear that the plate buckling stress is well above the yield stress of mild steel. The analytical solution using fixed connection between the flanges and the web gives a more accurate result but is an overestimate as shown in Figure 27. This overestimation is caused by the fact that for the fixed welds the buckling shape often has buckles that do not have the same magnitude as the others. This is not included in the model because not all kinematical possible shapes can be examined.

For longer lengths the theoretical buckling load is always slightly overestimated because not exactly the most unfavourable kinematical possible shape is used as is shown is Annex D.

Analytical results for flanges 150x15 mm and web 1500x15 mm

The previous cross-section is not sensitive to plate buckling so now a cross-section is used with flanges 150x15 mm and a web of 1500x15 mm. This is a cross-section with a very slender web and flanges that have a small cross-sectional area compared to the web.

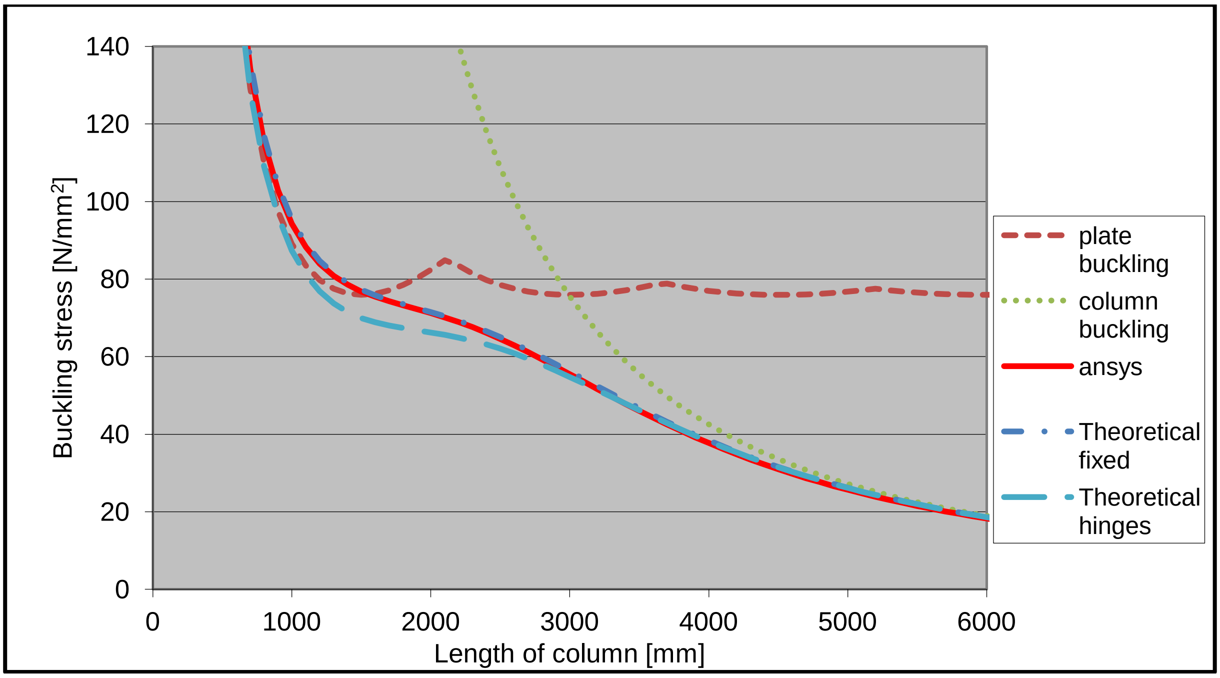


Figure 28: Theoretical buckling curve for flanges 150x15 mm and web 1500x15 mm

In Figure 28 it is clear that the theoretical curve (with fixed and hinged welds) based on the interaction of plate buckling and column buckling is well below the behaviour that is predicted by taking the minimum of plate buckling and column buckling. In Figure 29 it is shown that the analytical calculations using fixed welds produces an accurate result and only slightly overestimates the buckling load according to the finite element model.

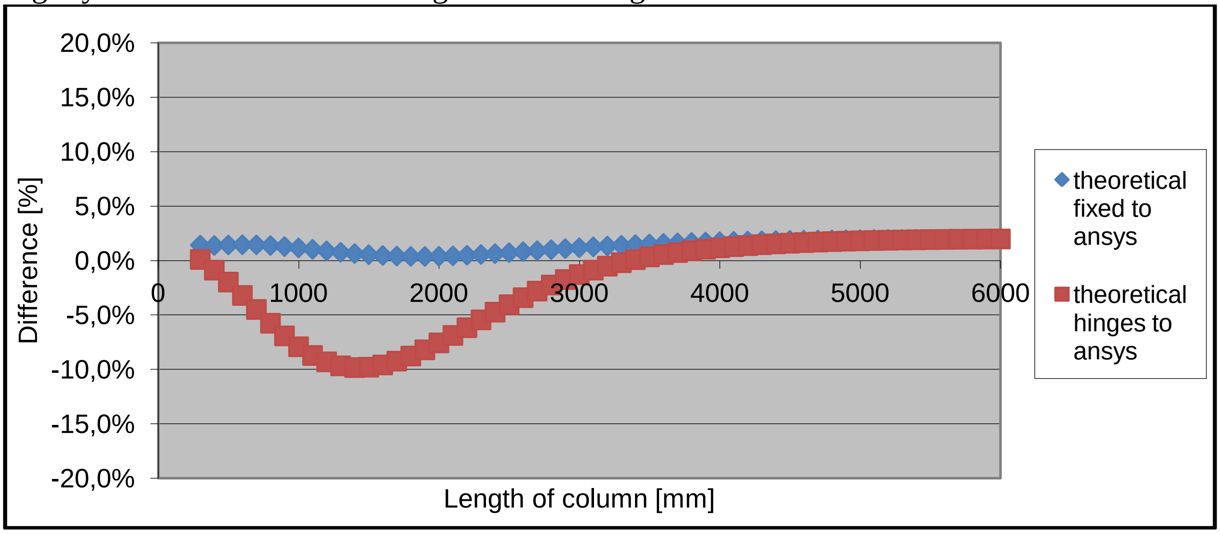


Figure 29: Difference caused by interaction of local and global buckling for flanges 150x15 mm and web 1500x15 mm

The results for a column length of 3000 mm are given in Table 2. To illustrate the interaction buckling behaviour also a plot is made of the displacement out of plane in Figure 30.

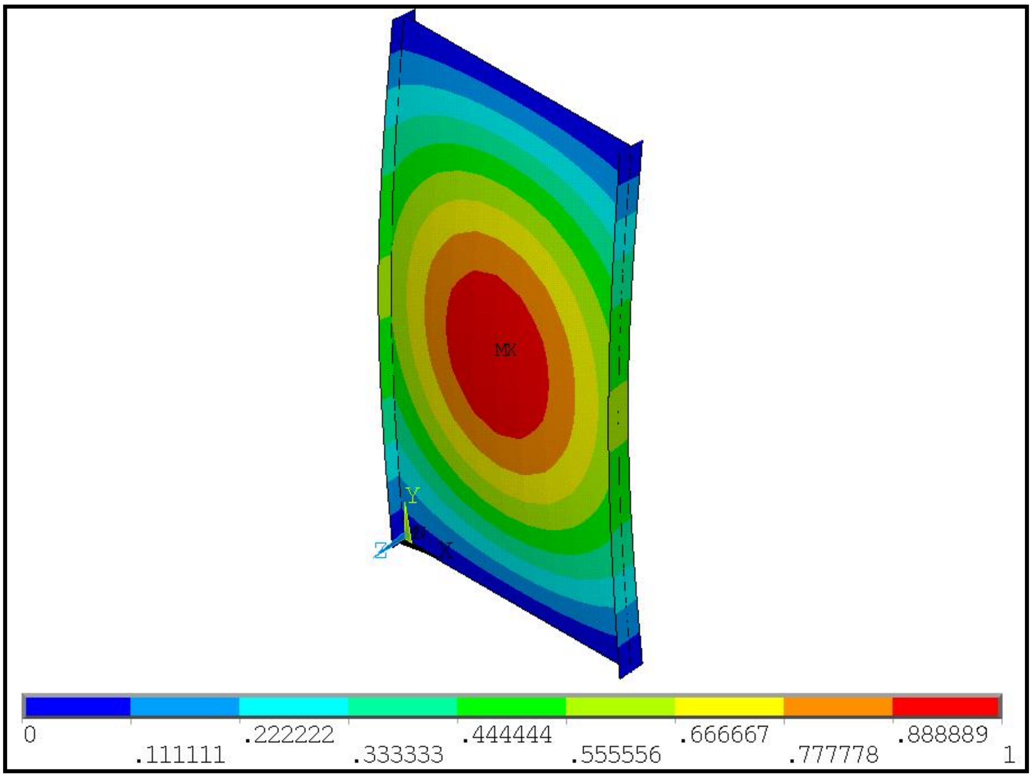


Figure 30: Buckling of column with flanges 150x15 and web 1500x15 and length 3000 (displacement out of plane plotted)

The difference between the local and global behaviour should be according to equation (16.49):

$$C_3 = 1{,}13 \tag{4.18}$$

The calculation in Ansys results in:

$$C_{3,ansys} = \frac{0{,}56}{1 - 0{,}56} = 1{,}27 \tag{4.19}$$

There is a difference because this calculation is very sensitive to the length of the column. For a column length of 3100 mm the analytical result is 1,28.

Calculation	Buckling stress
Theoretical hinges	54,78 N/mm^2
Theoretical fixed	56,14 N/mm^2
Column buckling	75,60 N/mm^2
Plate buckling	75,92 N/mm^2
Ansys model	55,48 N/mm^2

Table 2: Buckling stress for column with flanges 150x15 and web 1500x15 and length 3000

Analytical results for flanges 500x25 mm and web 1500x15 mm

The previous cross-section is not a very common cross-section because proper engineering judgement would result in a cross-section with relatively large flanges and a slender web. For this example flanges of 500x25 mm and a web of 1500x15 mm are applied.

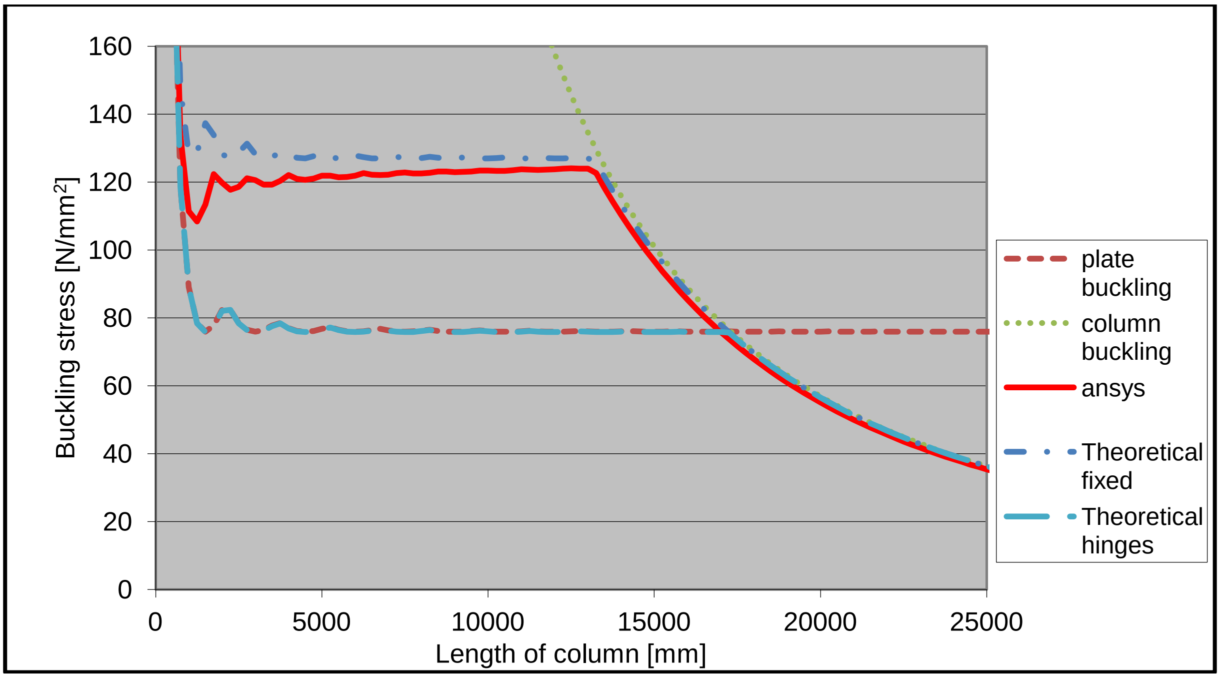


Figure 31: Theoretical buckling curve for flanges 500x25 mm and web 1500x15 mm

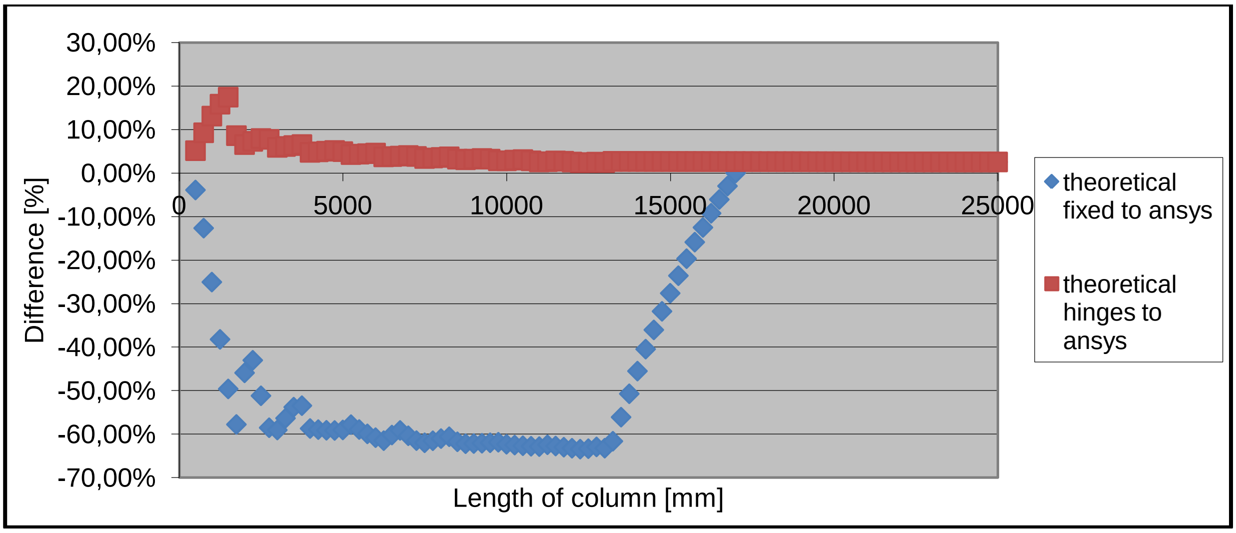


Figure 32: Difference caused by interaction of local and global buckling for flanges 500x25 mm and web 1500x15 mm

In Figure 31 the theoretical buckling curve is plotted and the difference between the theoretical curve and the result in Ansys is given in Figure 32. It is clear that the model using fixed connections between the flanges and the web is much more accurate but does again overestimate the buckling load as in the previous cases. For a column length of 17500 mm the results are given in Table 3.

Calculation	Buckling stress
Theoretical hinges	73,58 N/mm^2
Theoretical fixed	73,62 N/mm^2
Column buckling	74,27 N/mm^2
Plate buckling	75,98 N/mm^2
Ansys model	71,69 N/mm^2

Table 3: Buckling stress for column with flanges 500x25 and web 1500x15 and length 17500

Conclusion

General conclusion

The buckling stress calculated using the analytical equations with hinged welds is always lower than the plate buckling stress and the column buckling stress. This is not true for the analytical calculations with fixed welds.

In the case of stiff flanges it is clear that the model using hinged welds is almost equal to the minimum of the plate and column buckling stress. However, as seen in the model using fixed welds and the model in Ansys, it is clear that the buckling stress is much higher when plate buckling is governing because the flanges prevent a large rotation at the welds.

The most important conclusion is that there is only interaction between the local and the global behaviour if they have the same number of half-waves in the length of the column. ($m = n$ in this thesis) This conclusion can be used later when the imperfections need to be modelled to predict the real behaviour of an I-column because the NEN-EN1993-1-5 does not prescribe that these type of imperfections need to be modelled when calculating the restraint of a class 4 I-column. It is also important to note that the interaction is larger when the cross-sectional area of the web is large compared to the cross-sectional area of the flanges.

Rotational restraint

The buckling curve calculated has an important assumption that the flanges do not give any rotational restraint to the web. This is realistic for I-columns with a relatively large width of the web and flanges with a low rotational stiffness. It has been shown that the buckling curve calculated is a good approximation of the buckling curve according to the model in Ansys in those cases. However, when stiff flanges are applied the buckling curve is underestimated and the model in Annex D using fixed welds is a much more accurate result but does overestimate the theoretical buckling load. The disadvantage is that for the model using fixed welds a larger matrix has to be solved.

Theoretical model

The behaviour calculated now gives a proper inside in the way buckling happens in an I-column. It is clear that the buckling load is much better approximated using the method described here but always requires a determinant of a matrix to be solved or an approximation formula should be developed. It is questionable if this is necessary especially as it is a combination of two buckling phenomena which have a completely different post-buckling behaviour. Therefore the understanding of the interaction of plate and column buckling is more important than the actual result of the calculation.

Chapter 5: I-column according to the NEN-EN1993-1-1 and NEN6771

NEN-EN1993 gives requirements on the design of steel structures. The requirements are concerned with the static strength and the stability of steel structures. The static strength of steel structures is not part of this thesis. Therefore the section on the stability of steel structures is used here and applied upon the I-column in compression. In NEN-EN1993-1-1 the section 6.3 describes the calculation for stability of steel members. For the calculation of class 4 cross-sections a reference is made to NEN-EN1993-1-5 to calculate the effective cross-section.

Stability of a welded I-column under uniform compression

According to NEN-EN1993-1-1

A welded I-column under uniform compression has to be checked using equation 6.46 from NEN-EN1993-1-1:

$$\frac{N_{ed}}{N_{b,Rd}} \leq 1 \tag{5.1}$$

In this case only the resistance is interesting and this is described by equation 6.48 from NEN-EN1993-1-1 for class 4 cross-sections:

$$N_{b,Rd} = \frac{\chi * A_{eff} * f_y}{\gamma_{M1}} \tag{5.2}$$

It is assumed that only the web has to be reduced for plate buckling. Therefore the minimum requirement of the flanges is that:

$$\frac{h_f - t_w}{2 * t_f} \leq 14 * \varepsilon \tag{5.3}$$

For the calculation of A_{eff} NEN-EN1993-1-5 has to be considered. This calculation has been shown before and is repeated here. The critical buckling stress for the web is:

$$\sigma_{cr} = 4 * \frac{\pi^2 * E}{12 * (1 - \nu^2)} * \left(\frac{t}{b}\right)^2 \tag{5.4}$$

Therefore the relative slenderness for plate buckling of the web is:

$$\lambda_p = \sqrt{\frac{f_y}{\sigma_{cr}}} \tag{5.5}$$

The reduction factor for plate buckling of the web is:

$$\rho = \frac{\lambda_p - 0{,}055 * (3 + 1)}{\lambda_p^2} \leq 1 \tag{5.6}$$

Therefore the effective area of the I-column is the following:

$$A_{eff} = 2 * h_f * t_f + \rho * b * t_w \tag{5.7}$$

According to equation 6.49 from NEN-EN1993-1-1 the relative slenderness of the I-column is:

$$\lambda = \sqrt{\frac{A_{eff} * f_y}{N_{cr}}} \tag{5.8}$$

The critical buckling load is calculated using the gross cross-section and is given by:

$$N_{cr} = \frac{\pi^2 * E * I}{a^2} \tag{5.9}$$

Using the relative slenderness the reduction factor can be calculated using the following equation:

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \lambda^2}} \leq 1 \tag{5.10}$$

In which Φ is:

$$\Phi = 0,5 * (1 + \alpha * (\lambda - 0,2) + \lambda^2) \tag{5.11}$$

The factor α depends on the type of cross-section that is applied. The correct buckling curve can be obtained from Figure 33.

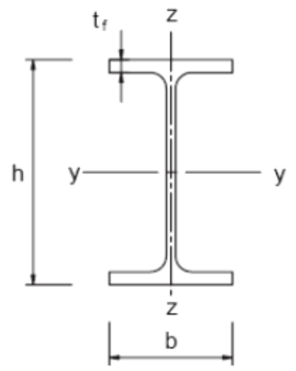
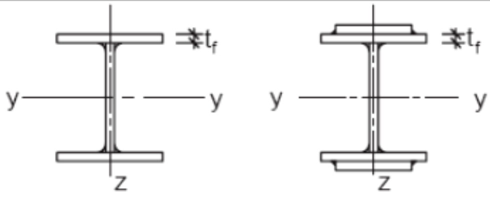
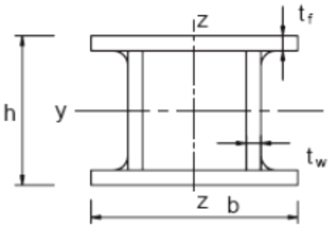
Doorsnede		Begrenzingen		Knik om de as	Knikkromme	
					S 235 S 275 S 355 S 420	S 460
Gewalste profielen		h/b > 1,2	$t_f \leq 40$ mm	y – y	a	a_0
				z – z	b	a_0
			40 mm < $t_f \leq 100$ mm	y – y	b	a
				z – z	c	a
		h/b ≤ 1,2	$t_f \leq 100$ mm	y – y	b	a
				z – z	c	a
			$t_f > 100$ mm	y – y	d	c
				z – z	d	c
Gelaste profielen		$t_f \leq 40$ mm		y – y	b	b
				z – z	c	c
		$t_f > 40$ mm		y – y	c	c
				z – z	d	d
Buisprofielen		warmvervaardigd		elke as	a	a_0
		koudgevormd en gelast		elke as	c	c
Gelaste kokerprofielen		algemeen (behalve in het hieronder gegeven geval)		elke as	b	b
		dikke lassen: $a > 0{,}5t_f$ $b/t_f < 30$ $h/t_w < 30$		elke as	c	c

Figure 33: Choice of buckling curve according to NEN-EN1993-1-1 table 6.2 (in Dutch)

When the buckling curve is known the correct value for α can be obtained from Figure 34.

Knikkromme	a_0	a	b	c	d
Imperfectiefactor α	0,13	0,21	0,34	0,49	0,76

Figure 34: Factor α dependent on the buckling curve chosen according to NEN-EN1993-1-1 table 6.1 (in Dutch)

Now the resistance can be calculated. This resulting resistance can be compared to the calculations using FEM methods later on.

According to NEN6771

The old Dutch codes for the design of steel structures are the NEN6770 and the NEN6771. The approach according to those codes is different and is also explained to show the major difference in the calculation of a welded I-column. The NEN6771 states that the reduction factors for plate buckling and column buckling need to be determined for the gross cross-section separately. Then the reduction factors need to be multiplied with the yield stress and this is the maximum allowable stress in compression. This is the prescribed method.

The critical buckling stress for the plate is:

$$\sigma_{cr} = 4 * \frac{\pi^2 * E}{12 * (1 - \nu^2)} * \left(\frac{t}{b}\right)^2 \tag{5.12}$$

The critical stress for column buckling is based on the gross cross-section and is given by:

$$\sigma_{buck} = \frac{\pi^2 * E * I}{a^2 * \left(b * t_w + 2 * h_f * t_f\right)} \tag{5.13}$$

The relative slenderness for the plate is:

$$\lambda_{plaat,rel} = \sqrt{\frac{f_y}{\sigma_{cr}}} \tag{5.14}$$

The reduction factor for the stress in the plate is dependent on the slenderness of the plate.

$$\sigma_{plooi,rel} = 1 \; if \; 0 \leq \lambda_{plaat,rel} \leq 0{,}7$$
$$\sigma_{plooi,rel} = 1{,}474 - 0{,}677 * \lambda_{plaat,rel} \; if \; 0{,}7 < \lambda_{plaat,rel} \leq 1{,}291$$
$$\sigma_{plooi,rel} = \frac{1}{\lambda^2_{plaat,rel}} + 0{,}132 * \lambda_{plaat,rel} - 0{,}170 \; if \; 1{,}291 < \lambda_{plaat,rel} \leq 2{,}5 \tag{5.15}$$
$$\sigma_{plooi,rel} = \frac{1}{\lambda^2_{plaat,rel}} \; if \; 2{,}5 < \lambda_{plaat,rel}$$

Now the reduction factor is known it is possible to determine the relative column slenderness.

$$\lambda_{rel} = \sqrt{\frac{\sigma_{plooi,rel} * f_y}{\sigma_{buck}}} \tag{5.16}$$

The reduction factor for column buckling is calculated using:

$$\omega_{buc} = \frac{1 + \alpha_k * (\lambda_{rel} - \lambda_0) + \lambda^2_{rel}}{2 * \lambda^2_{rel}} - \frac{\sqrt{(1 + \alpha_k * (\lambda_{rel} - \lambda_0) + \lambda^2_{rel})^2 - 4 * \lambda^2_{rel}}}{2 * \lambda^2_{rel}} \tag{5.17}$$

The ultimate bearing capacity is therefore:

$$N_{c,u,d} = \omega_{buc} * \sigma_{plooi,rel} * f_y * \left(b * t_w + 2 * h_f * t_f\right) \tag{5.18}$$

However, NEN6771 also describes that the effective cross-section method may be used. This is not the recommended method but is allowed and is exactly the same as the procedure described in the NEN-EN1993-1-1. This means that the method according to the NEN6771 has the same safety as the NEN-EN1993-1-1 when the effective cross-section method is used. When the reduced stress method is used the results of the NEN6771 are more conservative then the result using the NEN-EN1993-1-1.

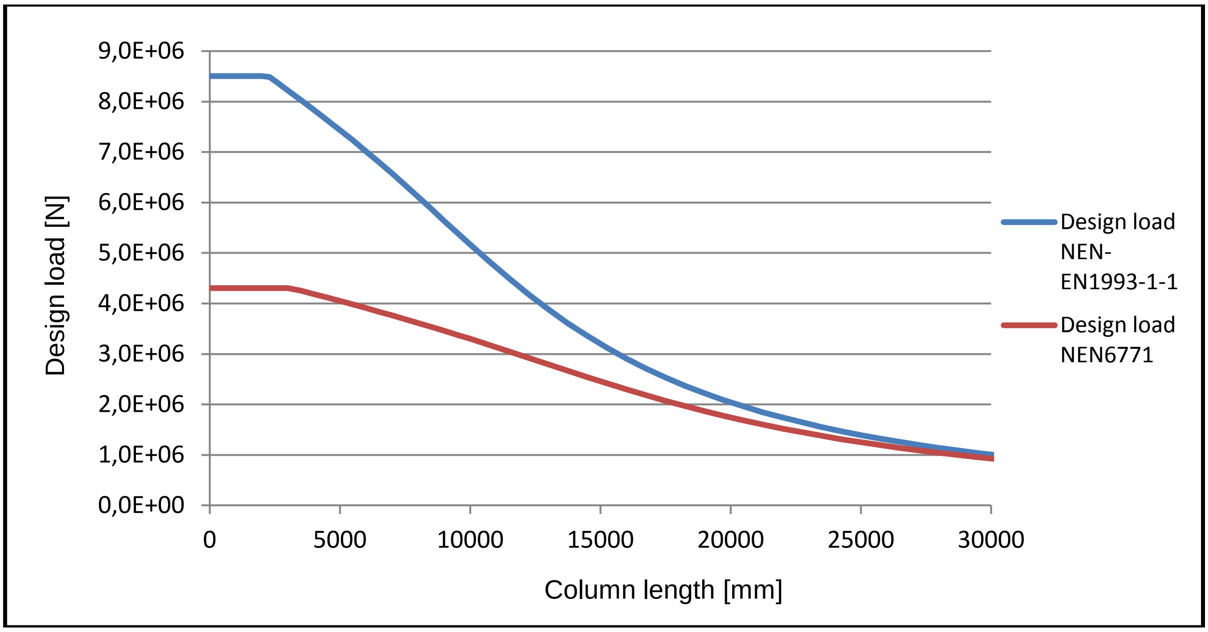


Figure 35: Design load for column with flanges 500x25 and web 1500x15 according to NEN and Eurocode

The difference between the NEN-EN1993-1-1 and the NEN6771 is illustrated in Figure 35 for a certain cross-section. The major difference for the NEN6771 is immediately clear because for smaller lengths the capacity is almost twice as high for the NEN-EN1993-1-1. This is indeed the expected behaviour because the reduced stress method prescribes that the flanges need to be reduced with the same factor as the web. The effective cross-section method prescribes that the flanges can be exploited up to the yield strength for short columns. For much longer lengths the column buckling factor is governing and the difference is very small because the procedures for column buckling are identical in the codes. For cross-sections with an almost fully effective web, the reduction is very small, therefore, the difference will be small. If the slenderness of the web increases, the reduction factor increases and therefore the difference between the NEN-EN1993-1-1 and the NEN6771 increases. More extensive research has been done on the major differences between the NEN-EN1993-1-5 and the NEN6771 considering plate buckling. (Burg, 2011)

Chapter 6: I-column in FEM

Now the bearing capacity of the I-column is known according to the NEN-EN1993-1-5 it is necessary to determine the bearing capacity according to finite element calculations. Finite element methods are capable of calculating constructions including the effects of geometrical non-linear behaviour (large displacements and rotations) and physical non-linear behaviour (plasticity). This is not in the analytical calculation of Chapter 4.

A real structure is never perfectly straight and has to be welded and therefore has geometrical imperfections and residual stresses. Geometrical imperfections can easily be included in a finite element model but residual stresses are much more difficult to include. Therefore often the combination of geometrical imperfections and residual stresses is included using a single geometric imperfection.

NEN-EN1993-1-5 gives a recommendation for the imperfections that need to be used. This is a global buckling behaviour of the column combined with a local buckling behaviour of the web. However, as shown in Chapter 4 this is not the lowest buckling load and therefore the I-column will also be calculated according to the imperfections of Chapter 4. The main goal is to find out if the buckling curves according to the NEN-EN1993-1-1 represent the same behaviour as the finite element calculations with imperfections according to the NEN-EN1993-1-1. Then the question is whether the use of the imperfections according to the NEN-EN1993-1-5 results in the same bearing load as the imperfections according to Chapter 4.

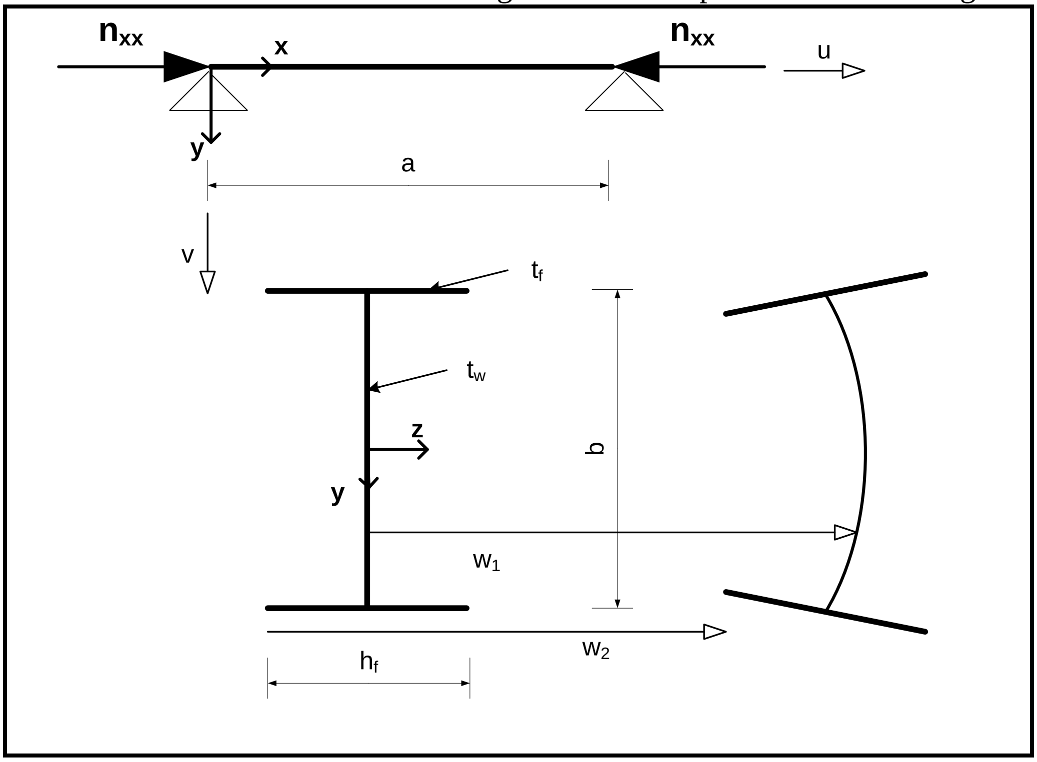


Figure 36: Welded I-column in FEM

The model is a simply supported column as given in Figure 36. This is the same as Figure 22 where the situation is described except that in FEM it is possible to rotate the flanges in a 3D-model.

Displacement control vs Force control

An important difference is between the use of displacement control and force control. First of all this is an essential part as a form of snap through behaviour may not be detected if force control is applied. However, there is a difference in modelling the real behaviour. Displacement controlled means that the displacement is prescribed at one end of the column for the entire cross-section. Therefore if a part of the cross-section, most likely the web, has a smaller stiffness the other parts of the cross-section, most likely the flanges, will take more loading. This means that a distribution of loading has to take place according to the largest stiffness out of plane of the elements. This distribution is only possible if a thick end plate is used. If there is no thick end plate, forces may not be distributed according to stiffness. In this case a more realistic approach would be a force controlled behaviour where each element of the cross-section is loaded and forces are only transferred through the welds between the flanges and the web. This is a completely different behaviour and should be investigated separately.

It is also clear that if a relative high buckling factor is applicable for the column the connection needs to be able to carry a relative high percentage of the plastic capacity. Therefore the connection also needs to introduce load onto the flanges otherwise yielding of the web is governing for the column. This is in general through a stiff connection.

In general it can be stated that the forces are introduced through a thick end plate and distributed according to the load bearing capacity of the individual elements. Therefore in this thesis a displacement control is applied. However, one should be wary when using the effective cross-section method that this is an essential assumption in the method.

Model description

For the calculation of the welded I-column in FEM the chosen program is Ansys. This program has been used in previous research (Burg, 2011) and is used as a starting point for this research. The model is described in previous research and only the changes that are applied are discussed here. The general concept of the model is explained in Figure 37 and the corresponding command grouping is given.

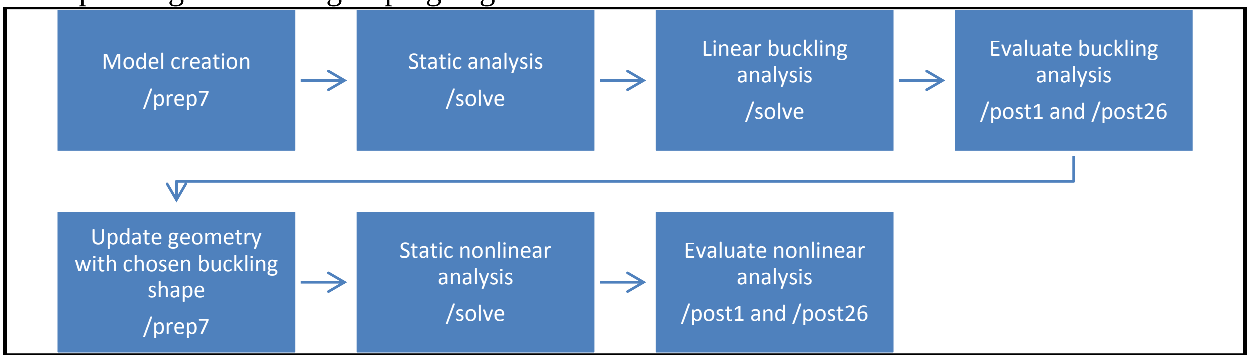


Figure 37: Flow chart of the process in FEM including the Ansys group commands

First of all the model needed is a hinged column and not a fixed column at both ends. In a 3D-model this is a complex model because a line support at the flanges will create a fixed end. To overcome this problem a very stiff beam is applied at the flanges and is supported at the web with a support. Therefore the flanges can rotate but act as a support through the beam. This is shown in Figure 38. This model does prevent local forms of instability at the point of load introduction. That is not a subject in this thesis and is therefore correct.

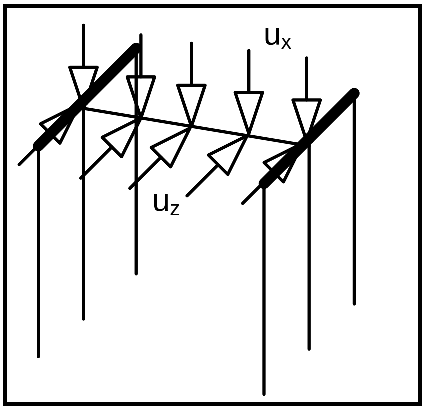


Figure 38: Hinged column support in 3D model. Displacement described at web and thick line is the very stiff beam.

Second is the use of do commands to calculate many cases and save the data to a file which can be used in Excel. It is possible to insert the data for a certain cross-section and the column is calculated for a number of lengths so the buckling curve is known by issuing two commands.

The verification of the model is presented in Annex F.

Single geometrical imperfections

The third important change is in the way the imperfections are modelled. The imperfections need to be modelled according to the NEN-EN1993-1-5 and according to the real buckling behaviour described in Chapter 4.

The single geometric imperfection is a method to replace the real geometrical imperfections and the residual stresses. For a welded I-column the correct buckling curve depends on the thickness of the flanges. If the flanges are thinner than 40 mm the correct buckling curve is c and for flanges thicker than 40 mm the buckling curve is d. Thicker flanges have higher residual stresses because of the larger temperature differences. Thin flanges have a more uniform temperature in the cross-section. Therefore the residual stresses are lower. However, for thicker flanges the single geometric imperfection replacing the residual stresses is not higher. Therefore in the FEM-results the effect of higher residual stresses in thicker flanges is not included. Because it is not included, the application of buckling curve d is not valid for the comparison of the FEM-results to the NEN-EN1993-1-1. Therefore all calculations are done using buckling curve c for axial compression of welded I-columns.

Eurocode imperfections

First the web is modelled with supports along its edges as described for a perfect plate. Then the lowest eigenvalue is calculated (Figure 39) and applied upon the plate with magnitude:

$$w_{web,max} = \frac{1}{200} * b * 0{,}7 \qquad (6.1)$$

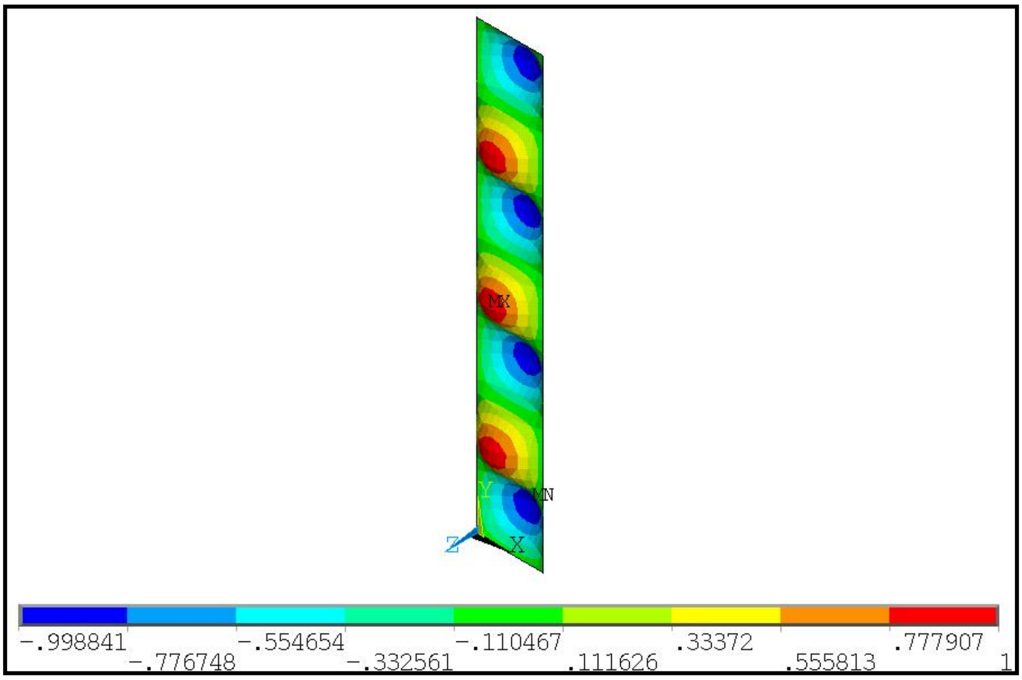


Figure 39: The local imperfection of the web according to the Eurocode

The factor 0,7 is used because only the governing mechanism should be applied at full scale and all other instability configurations may be scaled to 70% of the given limit. The major displacement is the global buckling behaviour and plate buckling is the secondary buckling mode.
Then the flanges are added to the construction and the entire model is forced to displace only in z-direction. The buckling load is the global buckling behaviour (Figure 40) and is applied upon the entire column with magnitude equal to:

$$w_{flange,max} = \frac{1}{200} * a \tag{6.2}$$

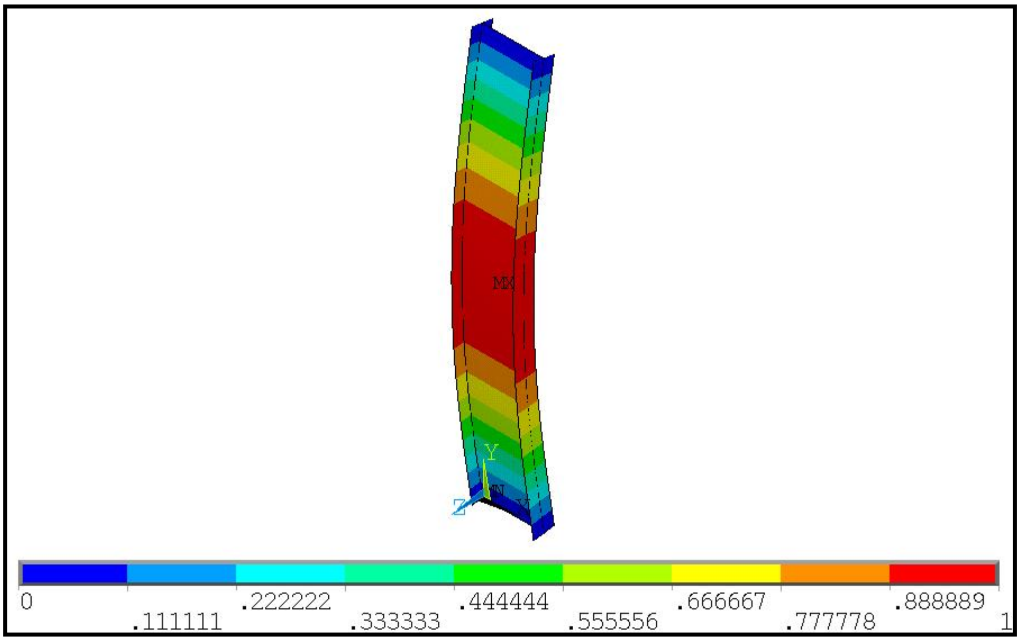


Figure 40: The global imperfection of the entire section according to the Eurocode

The combination of the two displacements are applied to the construction and then the non-linear calculation can be done.

Imperfections according to lowest buckling load

The imperfections according to the buckling load are more difficult. The buckling load with $m = n = 1$ should be applied because otherwise global buckling is not initiated. The behaviour with $m = 1$ is the displacement of the I-column out of plane in a half-sine and an additional displacement of the web in a half-sine wave is added to the displacement. The behaviour with $m = 1$ is not always the lowest buckling load as shown in Chapter 4 and should therefore be found. This is done by calculating many buckling loads and calculating the total displacement in z-direction of all nodes for each buckling load. The buckling load with the largest total displacement in z-direction is the buckling load with $m = 1$. (Figure 41) This buckling load is applied.

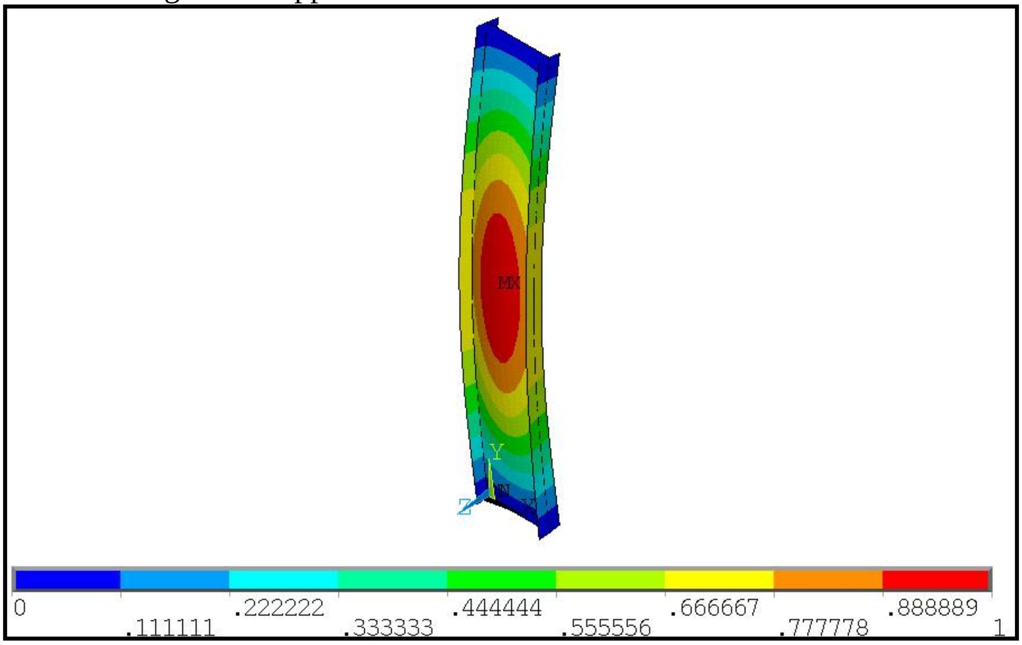


Figure 41: Imperfection of the entire section according to Chapter 4

Scaling this buckling load has to be done depending on the displacement of the web and the displacement of the flange. The displacement has to be scaled according to the maximum allowable displacement of the flange and the maximum allowable displacement of the web as they are given in the NEN-EN1993-1-5.

If the local displacement is large compared to the global displacement the scaling factor will depend on the local behaviour. This is given by:

$$e = \frac{b}{200} * \frac{1}{w_{web} - w_{flange}} \quad if \quad \frac{b}{200} \frac{1}{w_{web} - w_{flange}} \leq \frac{0,7 * a}{200} \frac{1}{w_{flange}} \qquad (6.3)$$

If the global displacement is large compared to the local behaviour the scaling factor will depend on the global behaviour.

$$e = \frac{a}{200} * \frac{1}{w_{flange}} \quad if \quad \frac{0,7 * b}{200} \frac{1}{w_{web} - w_{flange}} \geq \frac{a}{200} \frac{1}{w_{flange}} \qquad (6.4)$$

If the displacement is not specified by the above equations the scaling factor will depend on both and therefore the following two equations are provided to calculate the scaling factor if there is no governing displacement.

$$e = \frac{0{,}7 * b}{200} * \frac{1}{w_{web} - w_{flange}} \; if \; \frac{0{,}7 * b}{200} \frac{1}{w_{web} - w_{flange}} \geq \frac{0{,}7 * a}{200} \frac{1}{w_{flange}} \quad (6.5)$$

$$e = \frac{0{,}7 * a}{200} * \frac{1}{w_{flange}} \; if \; \frac{0{,}7 * b}{200} \frac{1}{w_{web} - w_{flange}} \leq \frac{0{,}7 * a}{200} \frac{1}{w_{flange}} \quad (6.6)$$

The imperfection is therefore known and a non-linear calculation can be done.

Imperfections according to Eurocode using m=1

Imperfections according to the buckling load give a problem that either the local or the global imperfection is scaled down below the level of 70% of the maximum which is prescribed for a secondary imperfection. Therefore another imperfection model is added which works the same as the previously described model which generates imperfections according to the NEN-EN1993-1-5. However, it has been shown in Chapter 4 that there is only interaction in the theoretical buckling load when the number of half sine waves in the flanges is equal to the number of half sine waves in the web. This is chosen at $m = 1$ here.

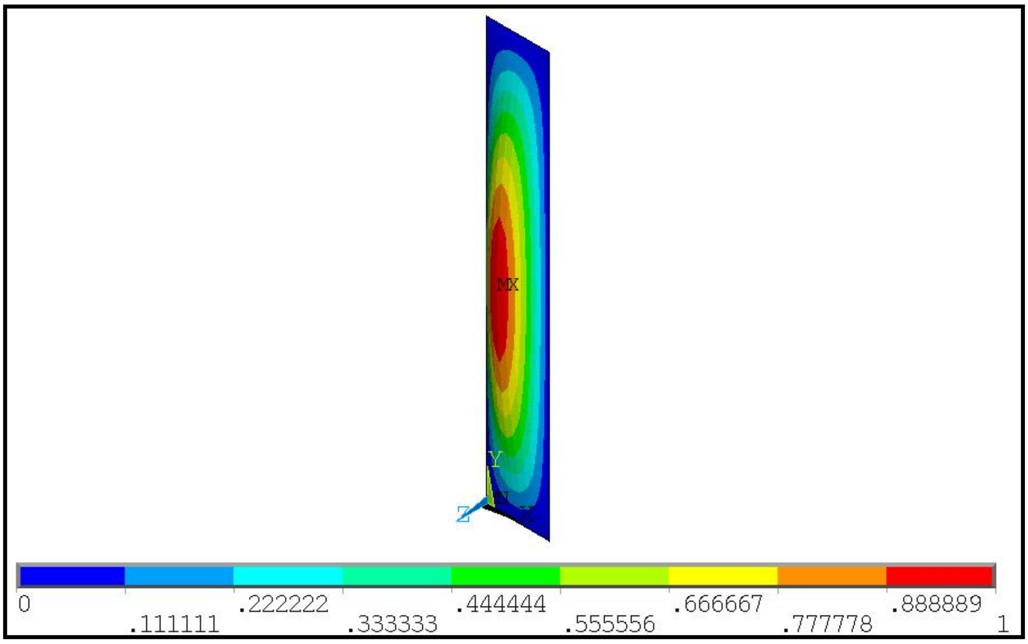


Figure 42: The local imperfection of the web according to the Eurocode using $m = 1$

For this model the buckling modes of the web are calculated and for each mode the total deflection out of plane is calculated. The mode with the largest total imperfection out of plane is applied at the web. (Figure 42) Then the flanges are added to the model and the buckling modes are again calculated. For the buckling modes the total deflection out of plane is calculated and the mode with the largest out of plane deflection is added to the entire column. (Figure 40)

Results

Many cases using different dimensions for the flange and web have been calculated and the results are presented in Annex F. Only for two configurations the results are presented and discussed.

Class 4 profile with large flanges (flanges 500x25 mm and web 1500x15 mm)

This profile has been calculated before for the theoretical buckling load and it proved that the interaction between column buckling and plate buckling was very small and reduced the buckling load by only 1%. Now the geometrical imperfections and residual stresses can be taken into account using a single geometric imperfection. For a certain configuration several plots can be made such as Figure 43 which shows the axial compressive load as a function of the displacement that is applied at the top of the column.

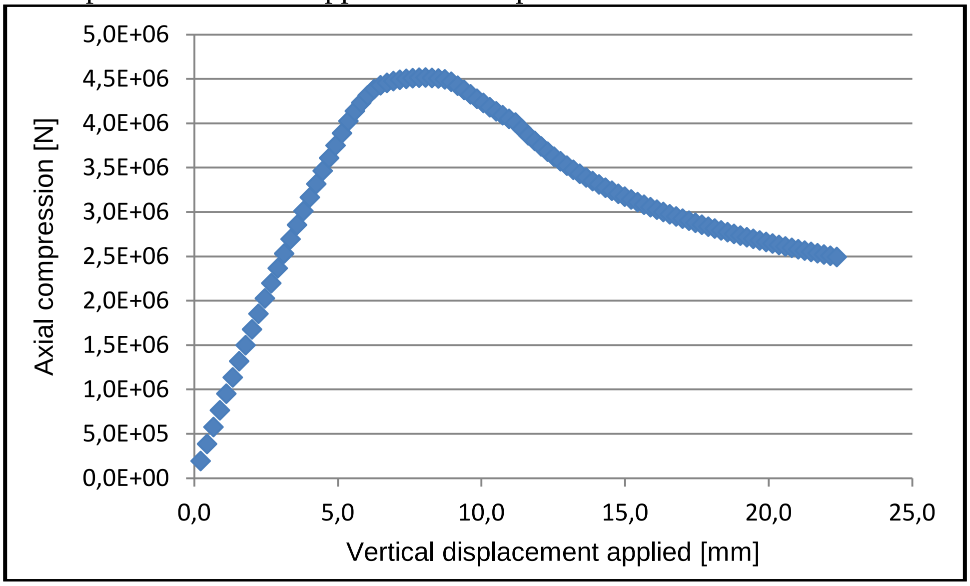


Figure 43: Load-in plane displacement graph for flanges 500x25 and web 1500x15 and length 10000

Also plots such as Figure 44 can be made which shows the axial compressive load as a function of the out of plane displacement of the web. This plot does not start in the origin because of the single geometric imperfection that is applied before loading.

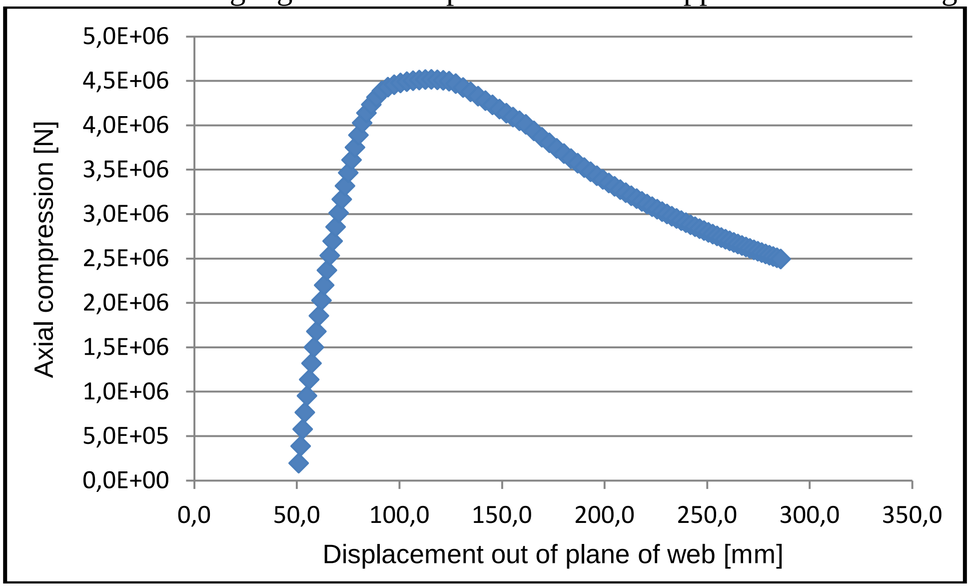


Figure 44: Load-out of plane displacement graph for flanges 500x25 and web 1500x15 and length 10000

These are just to illustrate the results for one situation and can be made for all situations. These results give the maximum axial compressive load and the real buckling curve can be determined. This is done by running the model a number of times in Ansys with a different column length. This will result in plots as given in Figure 45.

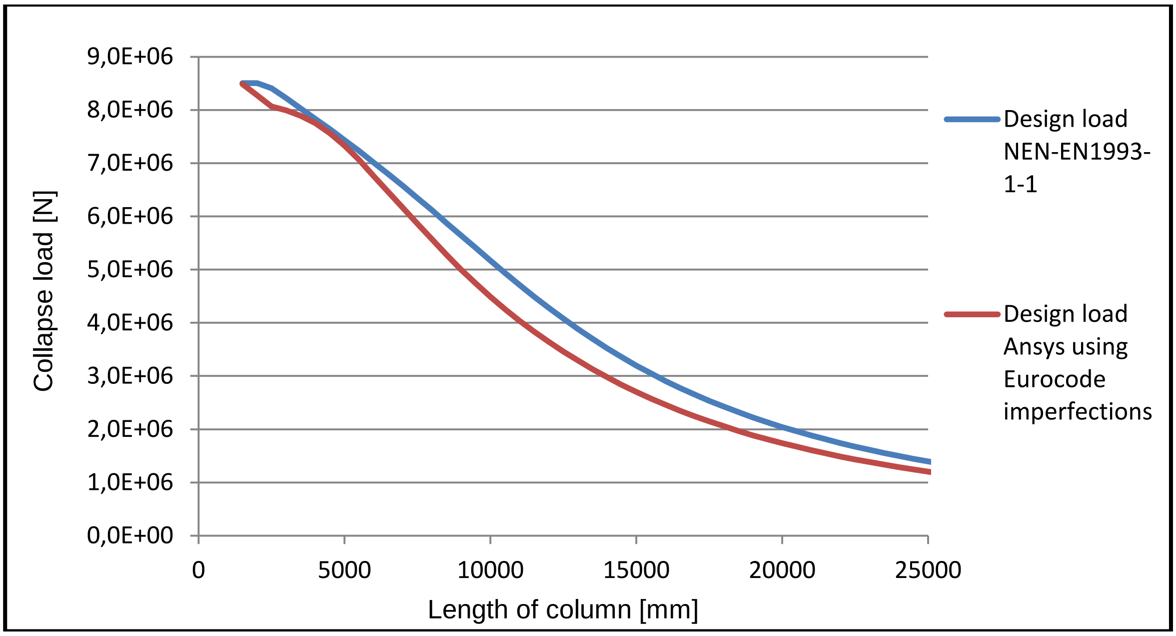


Figure 45: Buckling curve including imperfections with flanges 500x25 and web 1500x15

This profile is calculated using the three types of imperfections described before and this results in the collapse load. This collapse load is compared to the results from the NEN-EN1993-1-1 in Figure 46. These kind of figures give more information of the behaviour of the column than the ones before and are therefore used. If the result in such a figure is above 100% the collapse load in Ansys is higher than the design load according to the NEN1993-1-1. If the value is below 100% the collapse load in Ansys is lower than the design load according to the NEN1993-1-1.

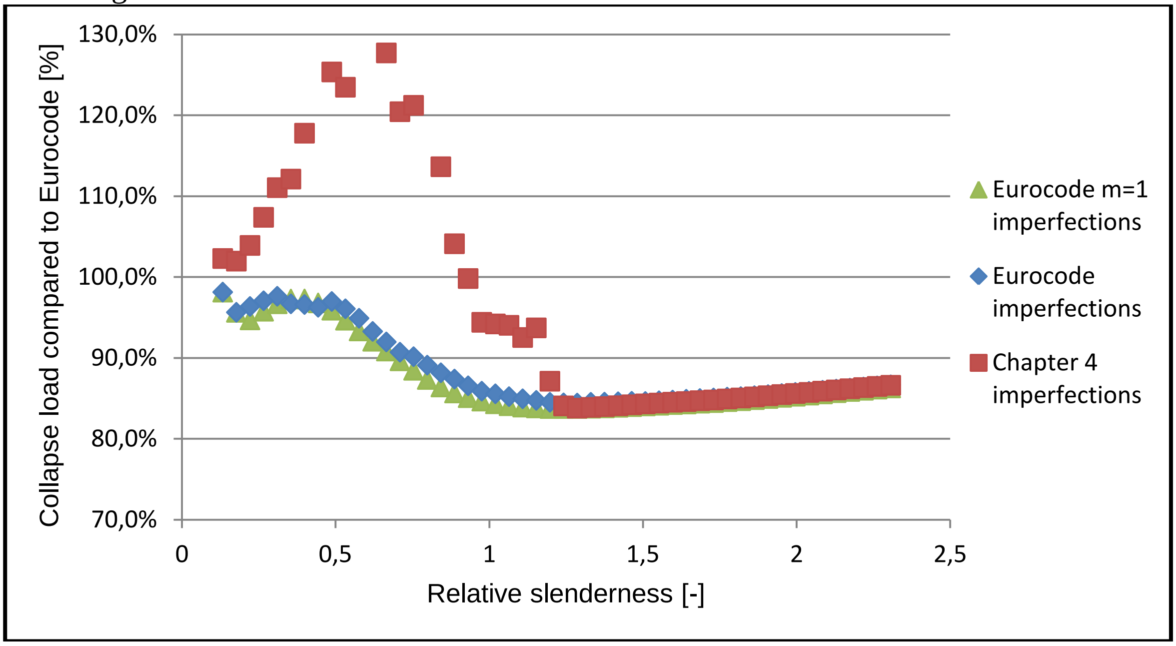


Figure 46: Collapse loads for column with flanges 500x25 and web 1500x15

The results according to Chapter 4 give a remarkable result because for shorter lengths the local imperfection is governing. Therefore the global imperfection is too small compared to the maximum allowable global imperfection and the destabilizing eccentricity is too small resulting in higher collapse loads.

The difference between the results from the NEN-EN1993-1-1 verification and the results with a single half sine wave (NEN-EN1993-1-5 with $m = 1$) are small and the difference is at most 2%. In general the collapse loads using $m = 1$ are smaller but the difference is small. The difference is plotted in Figure 47.

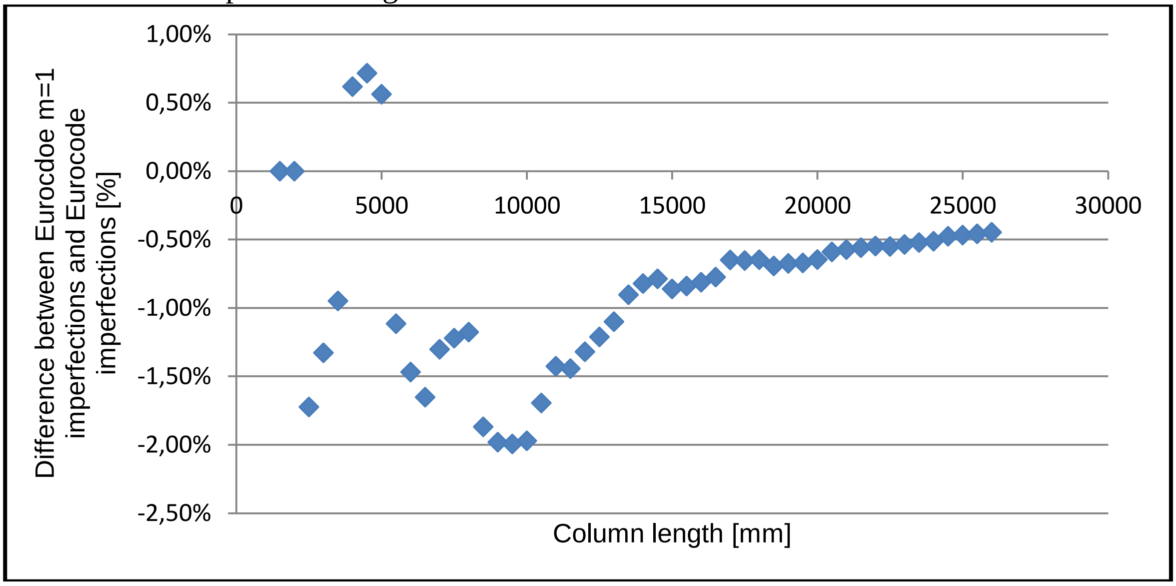


Figure 47: Percentual difference between Eurocode $m = 1$ imperfections and Eurocode imperfections

The general trend shows a correct collapse load according to the verification regulations for a relative slenderness up to 0,6 but a higher relative slenderness results in an overestimate of the collapse load of up to 17%.

Class 4 profile with small flanges (flanges 150x15 mm and web 1500x15 mm)

The profile with small flanges that has been calculated in Chapter 4 is also calculated in Ansys using finite elements. The interaction between plate and column buckling resulted in a 24% lower theoretical buckling load. The collapse load as it is calculated in Ansys compared to the results according to the NEN-EN1993-1-1 verification is presented in Figure 48.

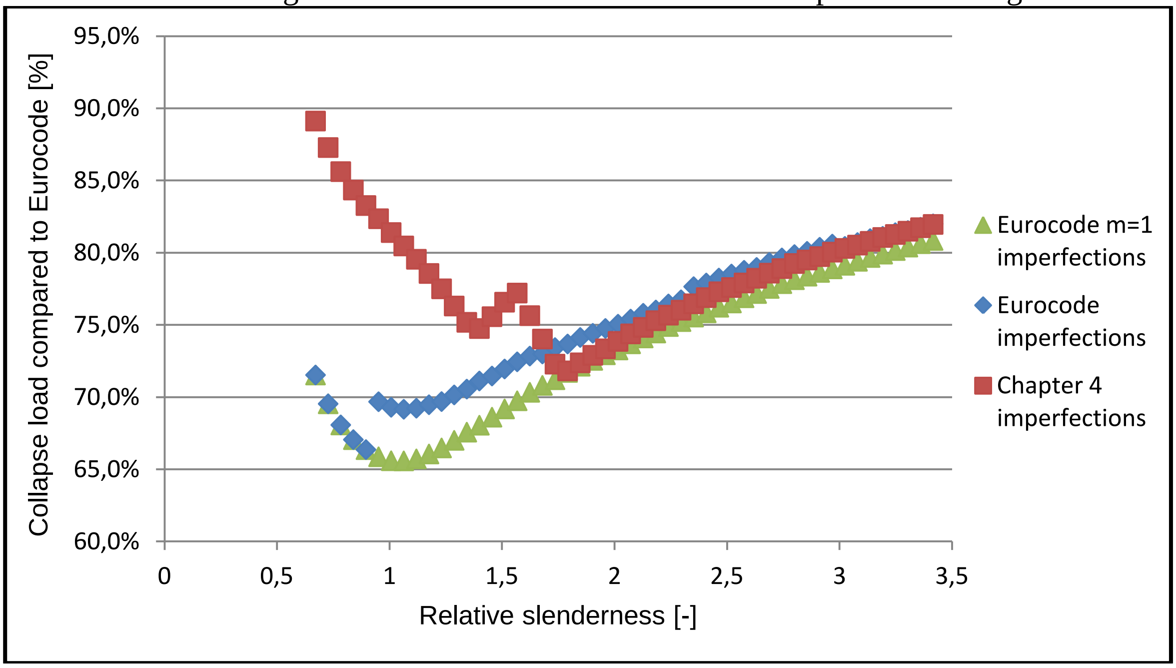


Figure 48: Collapse loads for column with flanges 150x15 and web 1500x15

The results using the imperfections from Chapter 4 give an overestimate for a low relative slenderness as explained before. For a higher relative slenderness the result is about equal to the results using imperfections according to the NEN-EN1993-1-1 verification. So the result using the imperfections from Chapter 4 are from here on omitted.

The lowest relative slenderness calculated is a result from a length of 1500 mm so a square column is applied. This is not in accordance with the Bernoulli theory that flat cross-sections remain flat and therefore the results for very short columns should be ignored up to a length of approximately 4500 mm. The collapse load using the imperfections in the NEN-EN1993-1-5 give a 33% difference compared to the NEN-EN1993-1-1 verification.

The imperfections using $m = 1$ gives a significant difference compared to the imperfections according to the NEN-EN1993-1-5. For a very small length this is equal because the lowest plate buckling mode is also a single half sine wave. However, when that length is exceeded the difference is 5,5% in the collapse load which does indicate that there is a significant difference when a single half sine wave is chosen as the imperfection. This difference is presented in Figure 49.

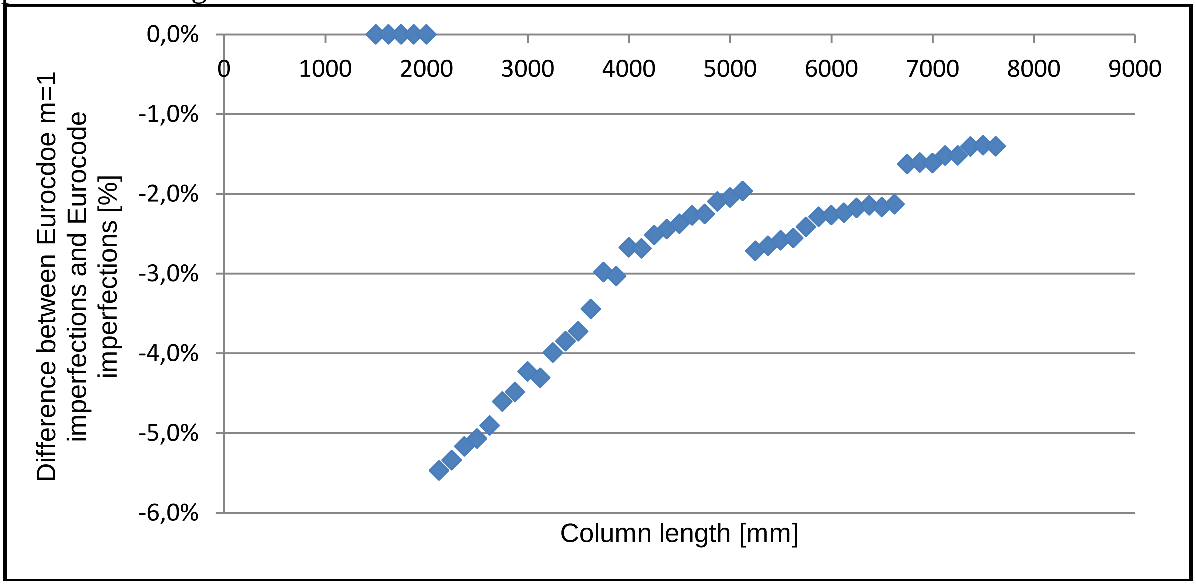


Figure 49: Percentual difference between Eurocode $m = 1$ imperfections and Eurocode imperfections

Sensitivity analysis

The model is applied in a number of cases but the sensitivity with respect to a number of variables needs to be investigated. The buckling curves are applicable for many situations and before conclusions are drawn on the basis of the previous results it is necessary to investigate the sensitivity of the variables. The important variables that will be investigated are the yield stress and the size of the imperfections.

Yield stress

All previous results have been obtained using a yield stress of 235 N/mm^2 which is the most common steel grade applied. However, in recent years higher strength steel is applied more frequently and the sensitivity to the yield stress should be investigated. For the column with flanges 500x25 mm and web 1500x15 mm, using the imperfections in the NEN-EN1993-1-5, the difference between the Ansys collapse load and the NEN-EN1993-1-1 verification is plotted in Figure 50 for three different steel grades. (235 N/mm^2, 355 N/mm^2 and 460 N/mm^2)

It is clear from Figure 50 that the yield stress does influence the behaviour and the difference between the model and the NEN-EN1993-1-1 verification is smaller when the yield stress is higher. This is consistent with the NEN-EN1993-1-1 because for many types of profiles S460 has a higher buckling curve so the reduction is smaller. This is given in table 6.2 of NEN-EN1993-1-1. However, this is not true for welded I-columns so here the same buckling curve is applied for all calculations.

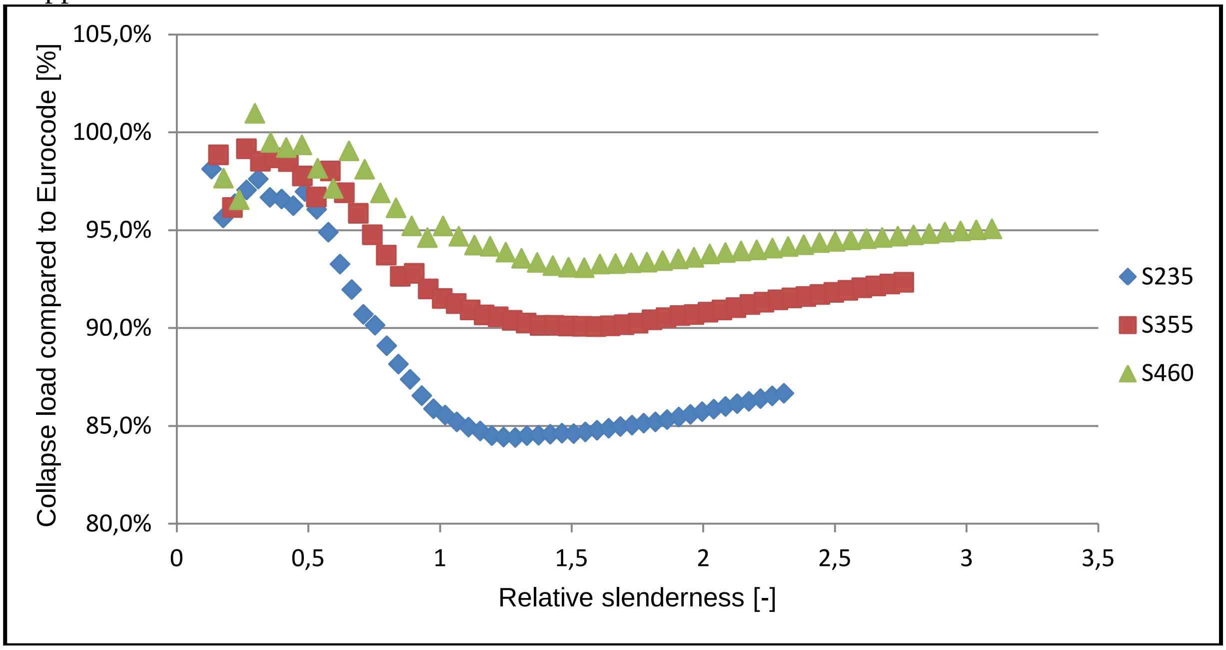


Figure 50: Sensitivity of non-linear analysis to yield stress

Imperfections

The imperfections are added to the structure according to the limits given in the NEN-EN1993-1-5 for calculations using finite elements. These single geometric imperfections represent the real geometrical imperfections and the residual stresses. The question is whether a slight increase or decrease of the imperfections results in a considerable change of load bearing capacity. For the column with flanges 500x25 mm and web 1500x15 mm the difference between the Ansys collapse load and the NEN-EN1993-1-1 verification is plotted in Figure 51 for three different imperfections. (1/150, 1/200 and 1/250) Both the local imperfection of the web and the global imperfection of the entire cross-section is changed. All verifications are done using buckling curve c as is prescribed for welded I cross-sections.

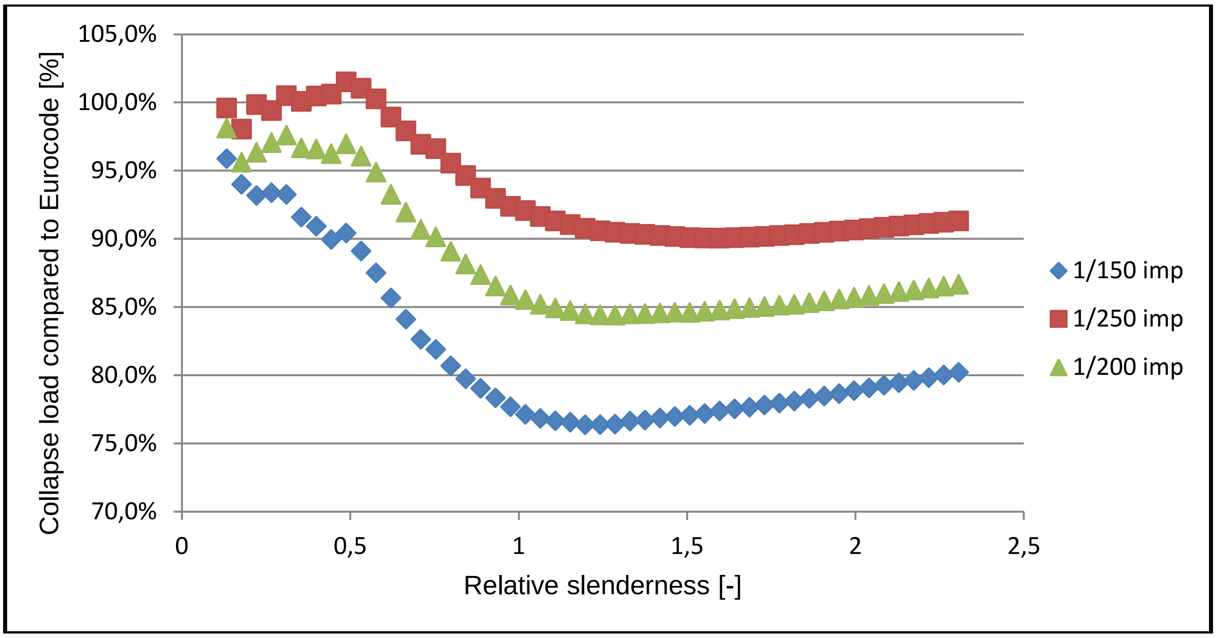


Figure 51: Collapse load compared to Eurocode verifications assuming different imperfection parameters.

It is clear that a lower imperfection will result in a higher collapse load because the driving force is smaller. However, it does not matter what the imperfection size is, the curves all show the same behaviour that the relative load bearing capacity at a relative slenderness of 1-1,5 is lower than at a different relative slenderness. A decrease of imperfection by 20% results in a higher load bearing capacity by at most 7,5%. An increase of imperfection by 33,3% results in a lower load bearing capacity by at most 9,8%.

Chapter 7: Comparison of analytical, FEM and Eurocode results for I-column

General results

For all cross-sections it is remarkable that the collapse load according to Ansys is structurally lower than the load bearing capacity according to the NEN-EN1993-1-1. This is generally in the order of 10-15% at a relative slenderness around 1-1,5. At a very low relative slenderness the bearing capacity is about equal because this is equal to the plastic capacity and for a higher relative slenderness the difference is smaller than at a relative slenderness of 1-1,5. This seems to indicate that the NEN-EN1993-1-5 does not provide the same level of safety for certain configurations. However, there are several reasons that indicate that this is not true.

- The buckling curves of the NEN-EN1993-1-1 have been derived using experimental results. Those curves have been fitted to the test results (Politecnico di Milano, 2011) and for the general cases it may be assumed that the buckling curves fit the test results. These experimental results have geometric imperfections and residual stresses. The NEN-EN1993-1-5 replaces these geometric imperfections and residual stresses with a single geometric imperfection for finite element methods because introducing residual stresses is very difficult and cumbersome in finite elements. This single geometric imperfection has to be on the safe side because the FEA is an equivalent method for the use of the buckling curves. Therefore the single geometric imperfection is larger than it actually should be. Therefore the collapse loads using FEA is lower than the use of the buckling curves. Literature (Beg, Kuhlmann, Davaine, & Braun, 2010, p. 169) also indicates that this method is not as accurate as the calculation using both geometric imperfections and residual stresses. However, the results are still very suitable for comparative studies.

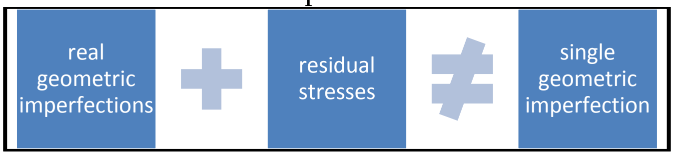


Figure 52: Method to combine geometric imperfection and residual stresses in FEM.

- Experimental results implicitly include strain hardening. In the FEA a bilinear stress-strain diagram is used which does include plasticity but not strain hardening. This does make a difference in the collapse load. However, this is only relevant for very short columns. For longer columns the strain at maximum loading is approximately $2 * \varepsilon_{yield}$ or even smaller. This means that there is no strain hardening because the effect of strain hardening is only noticed from a strain of $7 * \varepsilon_{yield}$ $(= 7 * 235/210000 = 0{,}00783)$. (NEN6771) For a certain configuration the Von Mises strains at maximum loading are plotted in Figure 53.

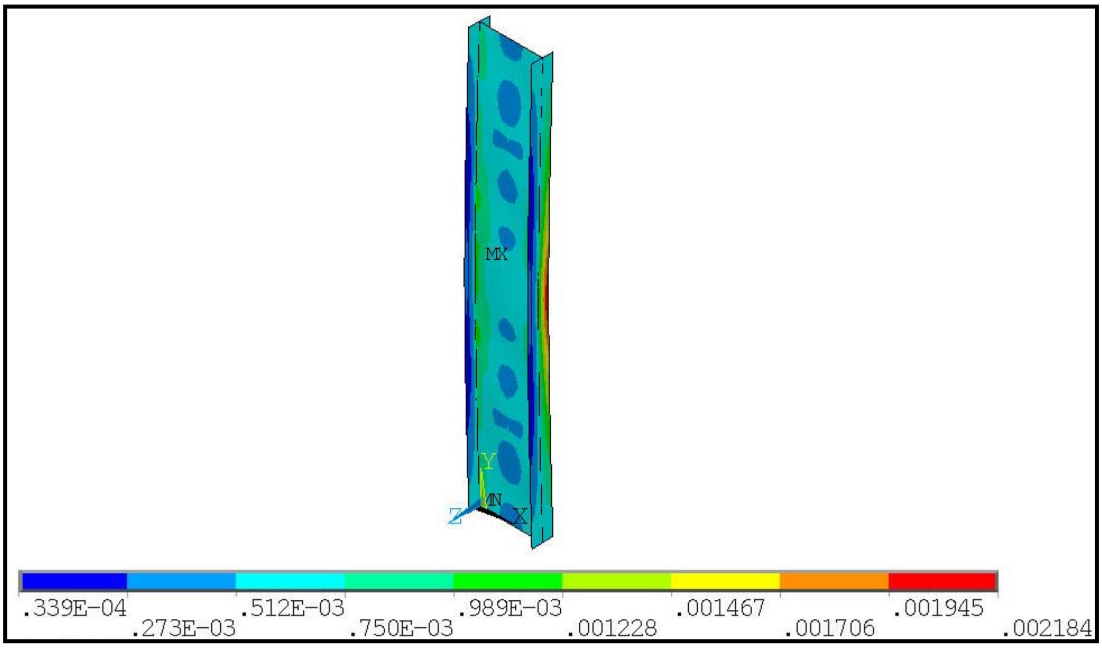


Figure 53: Von Mises strains at maximum loading of I-column with flanges 500x50 and web 1500x10 and length 10000

For these reasons a slight difference of 10-15% between the collapse load in FEA and NEN-EN1993-1-1 verification is accepted as reasonable. All buckling curves show the same behaviour with the largest difference at a relative slenderness between 1,0 and 1,5.

Influence ratio web to flange

However, for some cross-sections the largest difference is around 30-35% and this cannot be explained as this is only true for some cross-sections and it is not a general trend. It is noticed that the difference is larger if the web has a larger area compared to the flanges. This does make sense because the web has nearly no resistance against global buckling. However, if the web is larger, the flanges need to be able to carry a larger driving force of the web. For a cross-section with flanges 500x50 mm and a web with a width of 1500 mm and a variable thickness the difference between the collapse load in Ansys and the NEN-EN1993-1-1 verification are plotted in Figure 54. It is clear that the difference between the collapse in Ansys and the NEN-EN1993-1-1 verification increases with an increasing thickness of the web. This difference is not only present for class 4 cross-sections but does also arise for class 3 and higher classifications.

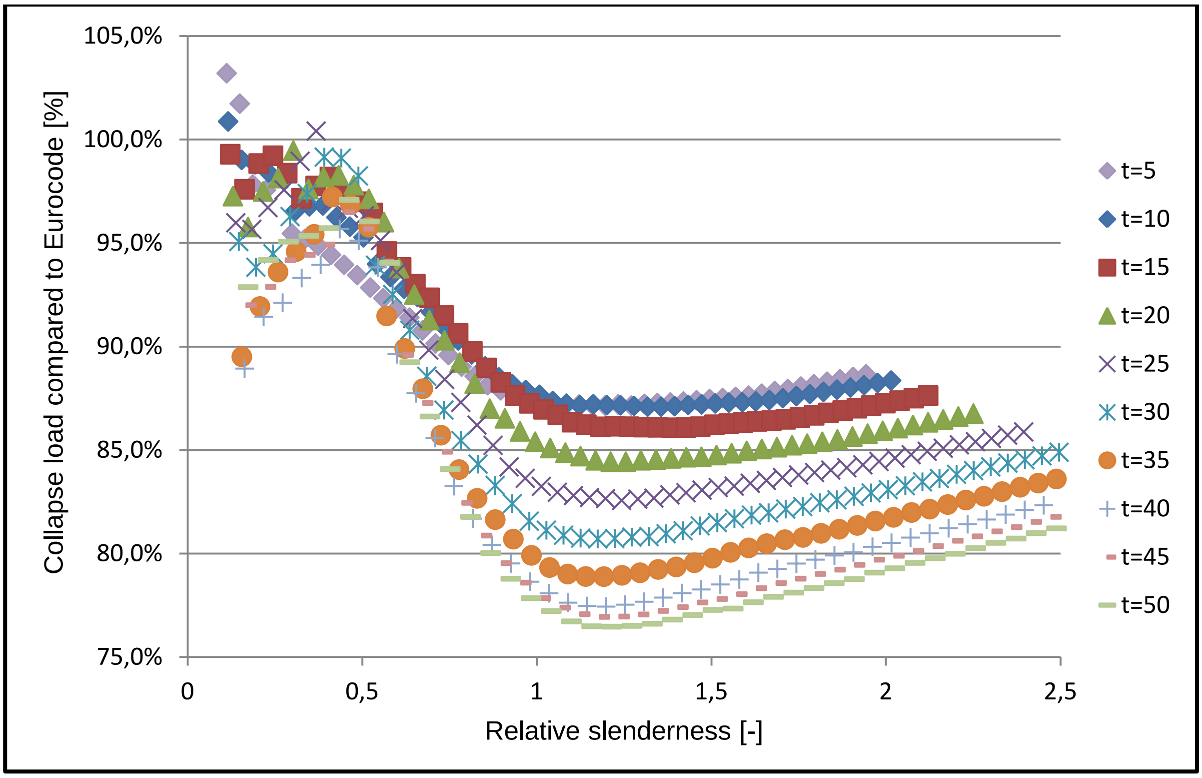


Figure 54: Collapse load compared to the Eurocode for flanges 500x50 mm and web with a width of 1500 mm

The general trend is that the largest difference is around a relative slenderness of 1 to 1,5. For each cross-section the largest difference is plotted as a function of the web thickness in Figure 55.

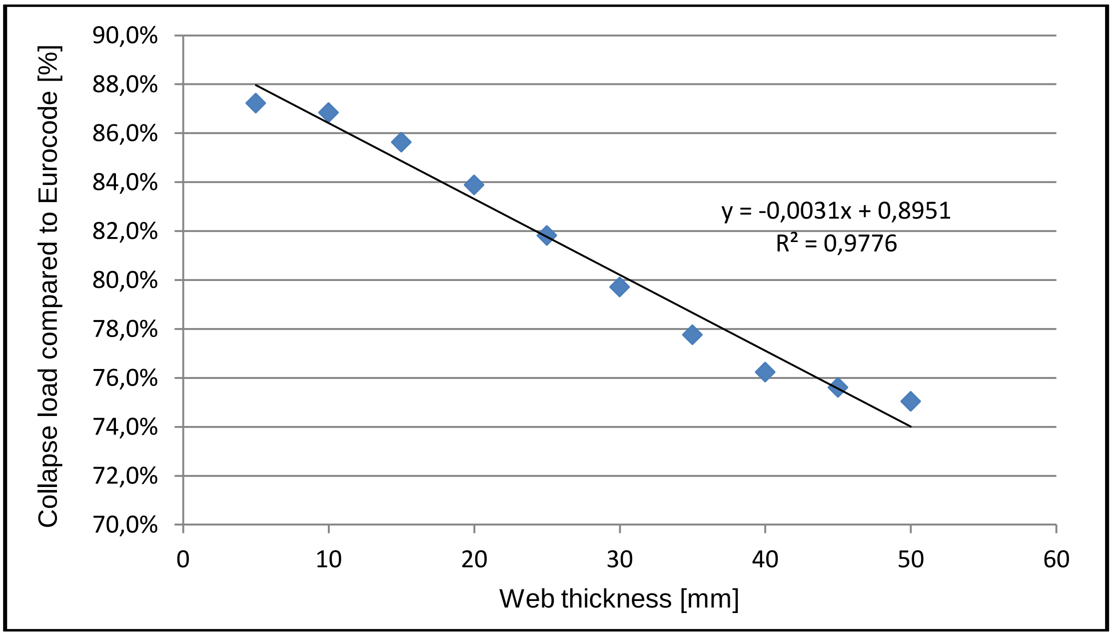


Figure 55: Largest difference collapse load compared to Eurocode for flanges 500x50 mm and web with a width of 1500 mm

More cases have been calculated using different configurations and therefore the difference in collapse load is plotted as a function of the ratio of the area of the web to the area of the flange in Figure 56. The dimensionless scaling parameter is named:

$$\omega = \frac{A_{web}}{A_{flange}} \tag{7.1}$$

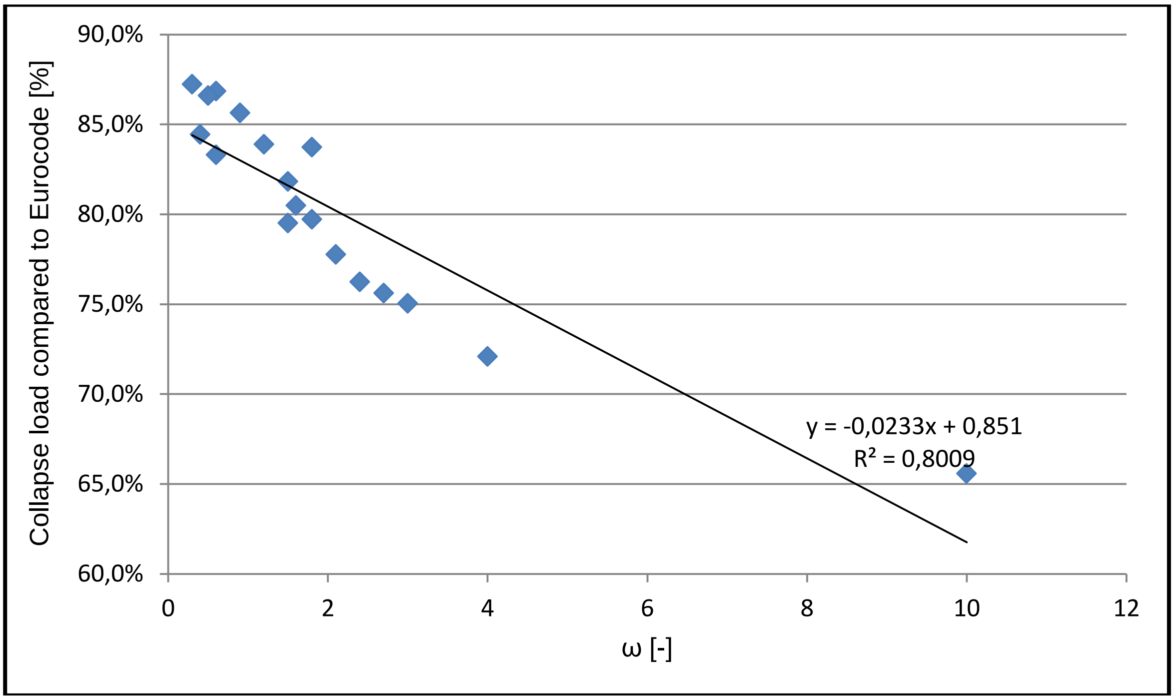


Figure 56: Collapse load compared to Eurocode for several cases

The collapse load therefore is plotted as a function of parameter ω on a logarithmic scale in Figure 57.

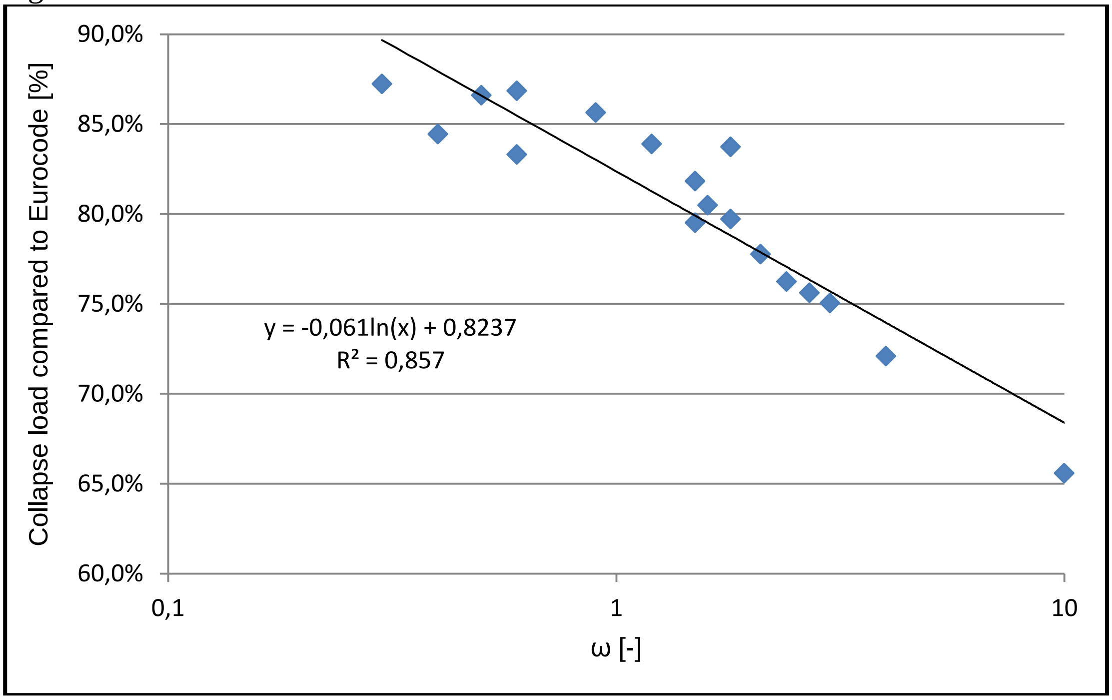


Figure 57: Collapse load compared to Eurocode as a function of ω

This gives reason to narrow the margins for the applicability of the buckling load according to the NEN-EN1993-1-1. If the parameter is $\omega \leq 1$ the difference between the Ansys and the

NEN-EN1993-1-1 verification are small and it seems to be a cut-off limit. This is plotted in Figure 58.

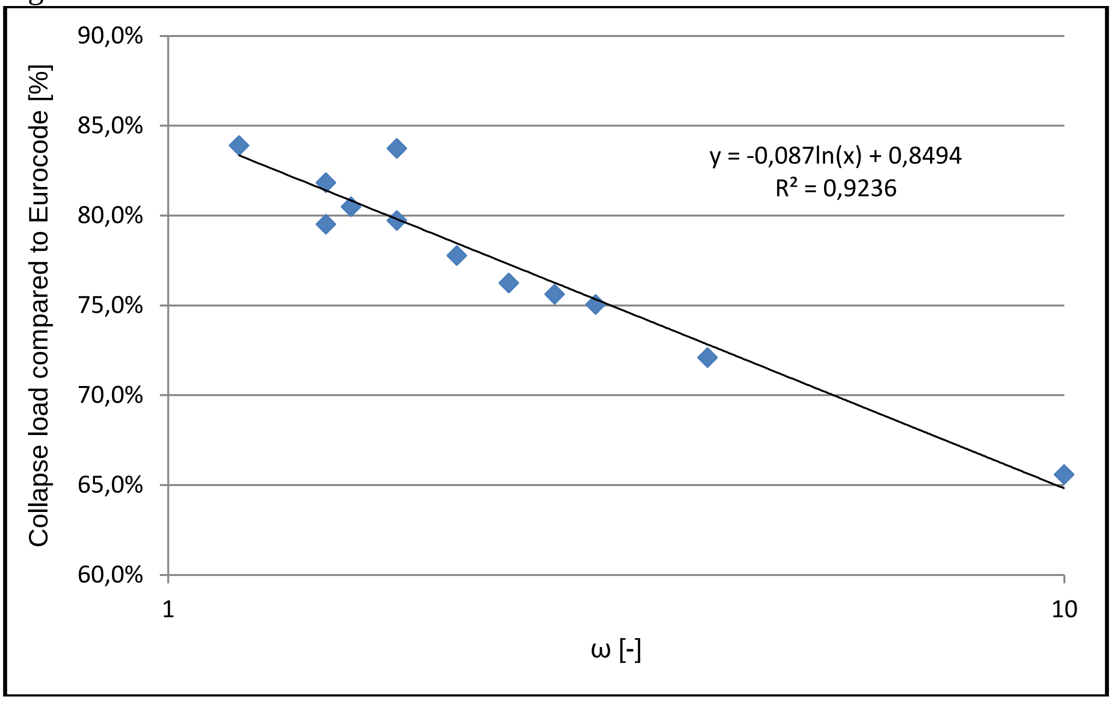


Figure 58: Collapse load compared to Eurocode as a function of ω for restricted values

The results certainly indicate that the Eurocode does not provide the same safety level for cross-sections with a large web compared to the results with a small web. This is not a very advanced finite element model because the geometrical imperfections and residual stresses are added as a single geometric imperfection. Also strain hardening is not included in the model. The results certainly give reason for further experimental research to validate the conclusions above.

Influence type of imperfections

Another remarkable effect is the difference between the three types of imperfections. The imperfections according to Chapter 4 give a too small global imperfection for a low relative slenderness because the local imperfection is governing and the global imperfection is scaled down too much. This explains why the results for a low relative slenderness using the Chapter 4 imperfections overestimate the collapse load.

The differences between the imperfections according to the NEN-EN1993-1-5 (using multiple half sine waves in the web) and according to the NEN-EN1993-1-5 using $m = 1$ (one single half sine wave in the web) are in general quite small ($\pm 1\%$). Especially for cross-sections with a small ratio of the area of the web to the area of the flange this is small. However, for cross-sections with a large web compared to the flange this difference is quite significant. This is similar to the theoretical analysis in Chapter 4. The theoretical buckling curve is significantly different when the area of the web is large compared to the area of the flange because of the interaction when there is a single half wave in the web. Therefore it would be reasonable if the NEN-EN1993-1-5 does prescribe this for finite elements analysis when the area of the web is large compared to the area of the flange as it may cause a significant difference ($\geq 5\%$) for some uncommon cross-sections.

Chapter 8: Probabilistic design on an economic and an uneconomic I-column

Probabilistic design in Ansys

The previous analyses have been done using deterministic values as input for the Ansys model. The deterministic values used are the average values for the dimensions and the characteristic value for the yield strength. However, the variables in this analysis are in general probabilistic design variables. The probabilistic design analysis is described in Figure 59. The batch file that describes the analysis for a single set of variables is the same as the batch file for the deterministic analysis except for the parameters that are defined in the probabilistic design file.

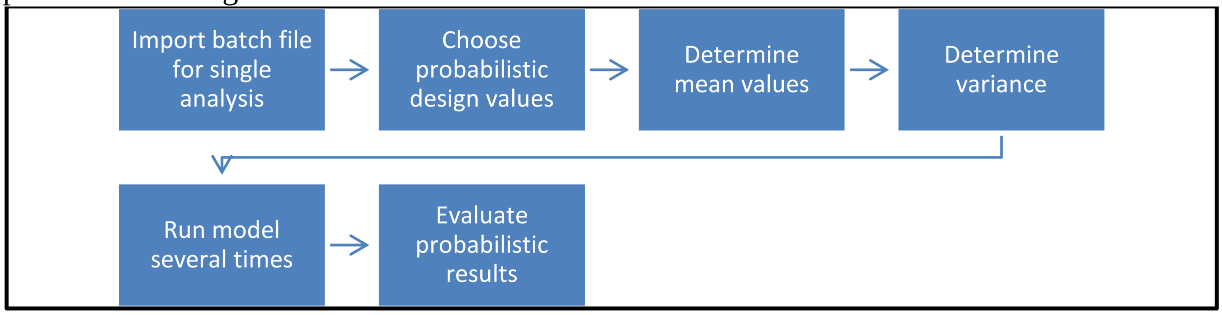


Figure 59: Flow chart of the process in FEM for probabilistic analysis including the Ansys group commands

Design variables

The most difficult part is to estimate the mean values and the variance of the design variables. There are mechanical design variables, geometrical design variables, imperfection design variables and a load design variable. All parameters are assumed to be uncorrelated to each other.

The mechanical design variables can be gained from literature (Stichting CUR, 1997) where the mean and the variance are given. For the yield strength a lognormal distribution is applied and for the Young's modulus a normal distribution is used.

For the geometrical design variables there are regulations that describe the allowable fabrication tolerances. (NEN-EN1090-2, 2008) The tolerances are given for two types namely the fundamental and the functional requirements. The first is concerned with the strength and stability of the construction and the second is concerned with fitting and aesthetics. In general both are important for a steel construction so the minimum of both requirements is applied. The lowest and highest allowable tolerances are used as the 95% confidence interval. The following formula calculates the upper boundary.

$$\mu + 2 * \sigma = x_{upper} = \mu + \Delta_{tol} \tag{8.1}$$

The following formula calculates the lower boundary.

$$\mu - 2 * \sigma = x_{lower} = \mu - \Delta_{tol} \tag{8.2}$$

The regulations (NEN-EN1090-2, 2008) provide the geometrical tolerances in article 11. The width of the web is given in D.1.1 and in D.2.1. The most severe requirement results in a tolerance of $\frac{b}{300}$ if it is between the boundaries of 3 mm and 6 mm:

$$\Delta_{width,web} = 3\ mm \leq \frac{b}{300} \leq 6\ mm \tag{8.3}$$

For the width of the flange the requirement is:

$$\Delta_{width,flange} = 3\ mm \leq \frac{h_w}{100} \tag{8.4}$$

The length of the column is given by:

$$\Delta_{length,column} = \frac{a}{5000} + 2\ mm \tag{8.5}$$

Tolerances of the material are specified in article 5.3.2 which refers to a different regulation (EN10029, 1991) where class A has to be applied. The tolerances for the thickness are given in Table 4.

Nominal thickness	Lower margin	Upper margin
$3 \leq t < 5$	$-0{,}4$	$+0{,}8$
$5 \leq t < 8$	$-0{,}4$	$+1{,}1$
$8 \leq t < 15$	$-0{,}5$	$+1{,}2$
$15 \leq t < 25$	$-0{,}6$	$+1{,}3$
$25 \leq t < 40$	$-0{,}8$	$+1{,}4$
$40 \leq t < 80$	$-1{,}0$	$+1{,}8$
$80 \leq t < 150$	$-1{,}0$	$+2{,}2$
$150 \leq t < 250$	$-1{,}2$	$+2{,}4$

Table 4: Margins on thickness (all dimensions in mm) of a steel plate according to EN10029 Table 1.

The imperfection design variables are the representation of residual stresses and geometrical imperfections. These are non-measurable parameters because it is a method to replace the real structural behaviour. Therefore only an estimate can be made of the mean and the variation. The variation is estimated at 0,15 because residual stresses and geometrical imperfections are far from constant. The mean is set such that the characteristic value at three times the standard deviation (probability at three times standard deviation is 0,13%) is $\frac{1}{200}$ as described in the NEN-EN1993-1-5. This can be calculated using:

$$e_{upper} - 3 * V * \mu = \mu \tag{8.6}$$

The relevant numbers applied gives:

$$\frac{1}{200} - 3 * 0{,}15 * \mu = \mu \tag{8.7}$$

Therefore the result is:

$$\mu_{global} = \frac{1}{290} \tag{8.8}$$

Since the local imperfection is highly dependent on the welding process the variation of the local imperfection is estimated at 0,20. This will result in a mean of the local imperfection:

$$\mu_{local} = \frac{1}{320} \tag{8.9}$$

Also the load is a design variable. In general the characteristic load is defined as the load that will occur once in the lifetime of the structure with an exceedance probability of 5%. This is without any safety factors. In this case nothing is known about the loading so it is assumed that the design load (including safety factors) is equal to the capacity according to the NEN-EN1993-1-1. The load factor is assumed to be around 1,4 as a combination of the dead load and the live load. The mean of the load is given by: (Stichting CUR, 1997, pp. 5-52, 5-53)

$$\mu_s = \frac{N_{ed}}{1 + \alpha * \beta * V} \tag{8.10}$$

The safety factors for loading are described by: (Stichting CUR, 1997, pp. 5-52, 5-53)

$$\gamma_L = \frac{1 + \alpha * \beta * V}{1 + k * V} \tag{8.11}$$

The β-factor required is 3,8 for a reference period of 50 years in consequence class 2 (NEN-EN1990, 2002). The α-factor for dominant loading is equal to 1,0 (NEN-EN1990, 2002). Therefore the variance of the load is estimated by:

$$V = \frac{\gamma_L - 1}{\alpha * \beta - k * \gamma_L} = \frac{1{,}40 - 1}{1 * 3{,}8 - 1{,}64 * 1{,}4} = 0{,}27 \tag{8.12}$$

Now the loading and resistance design variables are all known it is possible to apply the probabilistic design in Ansys. It is possible to include the load and simply calculate the probability in the following way using a Monte Carlo analysis:

$$P_f = \frac{n_{failures}}{n_{total}} \tag{8.13}$$

However, the probability of failure should be very small and the number of simulations for a reliability of 95% and a relative error of 0,1 should be at least: (Stichting CUR, 1997)

$$n \geq 400 * \left(\frac{1}{P_f} - 1\right) \tag{8.14}$$

For a probability of 10^{-4} this will result in at least 4 million simulations. This is not very efficient because a single calculation in Ansys takes about 1 minute. Therefore only the resistance is calculated in Ansys and from that the distribution of the resistance is estimated. Then the combination of loading and resistance can be calculated using an analytical calculation. The reliability function $Z\ (= R - S)$ indicates whether a structure is safe or not. The probability of failure is:

$$P_f = P(Z < 0) = P(R < S) \tag{8.15}$$

When it is assumed that both are normally distributed the reliability function is also normally distributed. The mean is given by the following relations assuming the load is independent from the resistance:

$$\mu_z = \mu_R - \mu_S \tag{8.16}$$

The standard deviation is given by:

$$\sigma_z = \sqrt{\sigma_R^2 + \sigma_S^2} \tag{8.17}$$

Then the β-factor is calculated using the following relation and this factor should be at least 3,8 to satisfy the safety demands.

$$\beta = \frac{\mu_z}{\sigma_z} \tag{8.18}$$

From this β-value a probability can be calculated and this can be compared to the probability required. The required probability is related to a β-value of 3,8.

$$P_{f,max} = \Phi(-\beta) = \Phi(-3{,}8) = 7{,}2 * 10^{-5} \tag{8.19}$$

Economic I-column

An economic I-column with flanges 500x50 mm and web 1500x10 mm and length 15000 mm is applied for the probabilistic design analysis. The calculation according to Chapter 5 using the verification regulations of NEN-EN1993-1-5 results in a design strength of:

$$N_{b,rd} = 5863\ kN \tag{8.20}$$

Therefore the mean load is equal to:

$$\mu_S = \frac{N_{ed}}{1 + \alpha * \beta * V} = \frac{5863}{1 + 1 * 3{,}8 * 0{,}27} = 2893\ kN \tag{8.21}$$

The standard deviation for the load is given by:

$$\sigma_S = \mu_S * V_s = 2893 * 0{,}27 = 781\ kN \tag{8.22}$$

The probabilistic design variables are already specified above and for this configuration the relevant numbers are calculated in Table 5.

Parameter	Description	Statistical spread	Mean μ	Standard deviation σ	Variation
a	Length of column	Normal	$15000\ mm$	$2{,}5\ mm$	0,000167
b	Width of web	Normal	$1500\ mm$	$2{,}5\ mm$	0,00167
t_w	Thickness of web	Normal	$10{,}35\ mm$	$0{,}425\ mm$	0,0411
t_f	Thickness of flange	Normal	$50{,}4\ mm$	$0{,}7\ mm$	0,0139
h_w	Width of flange	Normal	$500\ mm$	$2{,}5\ mm$	0,005
E	Young's modulus	Normal	$210000\ \frac{N}{mm^2}$	$8400\ \frac{N}{mm^2}$	0,04
f_y	Yield strength	Lognormal	$280\ \frac{N}{mm^2}$	$22{,}4\ \frac{N}{mm^2}$	0,08
e/a	Column imperfection	Normal	$\frac{1}{290}$	$\frac{1}{290} * 0{,}15$	0,15
e_{loc}/b	Web imperfection	Normal	$0{,}7 * \frac{1}{320}$	$0{,}7 * \frac{1}{320} * 0{,}20$	0,20

Table 5: Design variables for economic I-column

Ansys can perform this probabilistic design analysis and produces a plot of the results on a Gauss plot as shown in Figure 60. This means that a perfectly straight line would fit a normal distribution. The resistance is assumed to be normally distributed given the approximate straight line on a Gaussian cumulative distribution plot. The tail of the distribution is not straight so this is an assumption which is not entirely correct but this may also be due to the amount of calculations done.

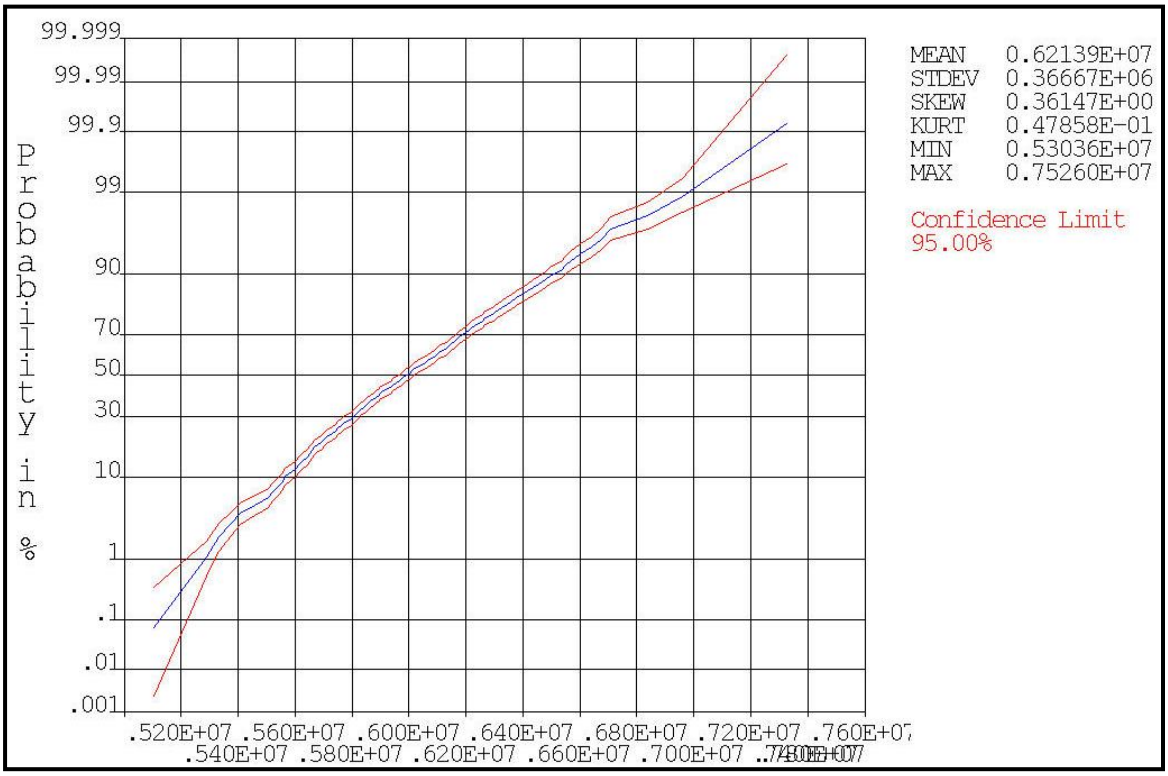


Figure 60: Cumulative distribution of collapse load according to Ansys for economic cross-section

The dependence of the collapse load on the variables is also calculated by Ansys and is given in Figure 61. The most important variables are the global imperfection, the mechanical properties and the width and thickness of the flange. The variance of the others apparently does not influence the collapse load significantly.

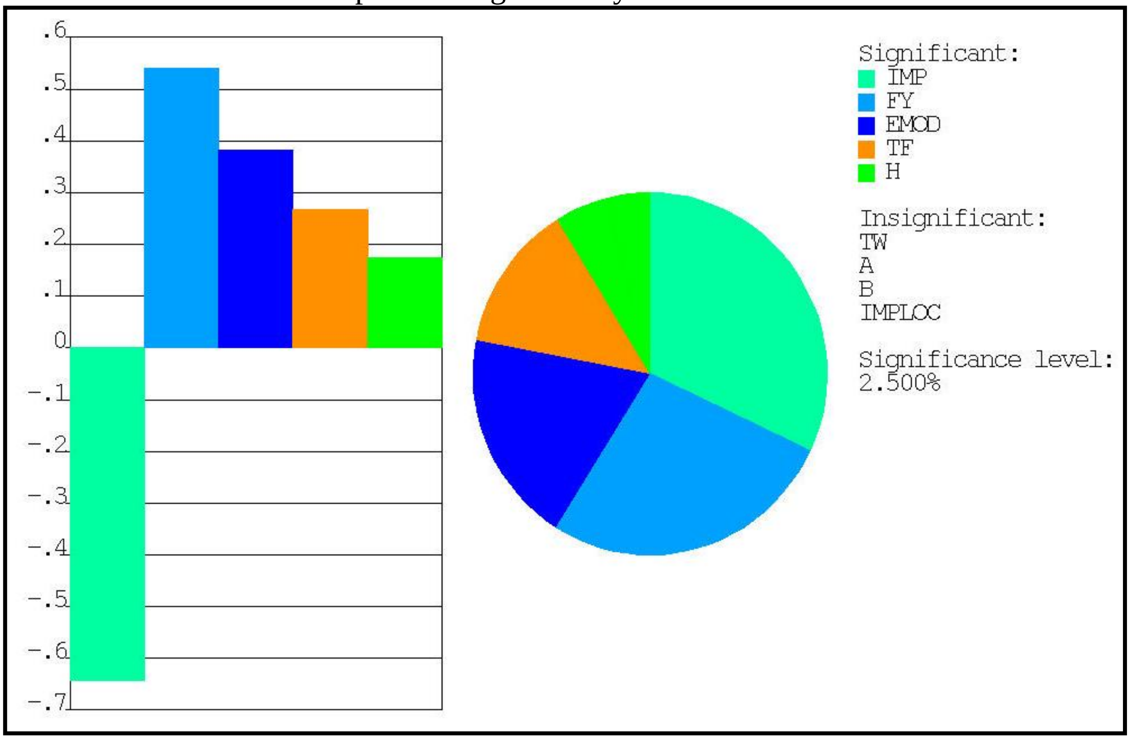


Figure 61: Significance of the design variables to the collapse load.

The mean for the resistance is:

$$\mu_R = 6214\ kN \tag{8.23}$$

The variance for the resistance is:

$$\sigma_R = 367\ kN \tag{8.24}$$

Now the mean for the reliability function can be calculated.

$$\mu_z = \mu_R - \mu_S = 6214 - 2893 = 3321\ kN \tag{8.25}$$

The standard deviation is given by:

$$\sigma_z = \sqrt{\sigma_R^2 + \sigma_S^2} = \sqrt{367^2 + 781^2} = 863\ kN \tag{8.26}$$

Then the β-factor is calculated using the following relation and this factor should be at least 3,8 to satisfy the safety demand required.

$$\beta = \frac{\mu_z}{\sigma_z} = \frac{3321}{863} = 3{,}85 \tag{8.27}$$

Therefore the probability for failure can be calculated:

$$P_f = P(Z < 0) = \Phi(-\beta) = \Phi(-3{,}85) = 5{,}91 * 10^{-5} \tag{8.28}$$

Uneconomic I-column

An uneconomic I-column with flanges 500x50 mm and web 1500x50 mm and length 15000 mm is applied for the probabilistic design analysis. This is the same as the economic I-column except for the thickness of the web. In this case, not an extremely uneconomic I-column is used as in the case of the analytical calculation. The calculation according to Chapter 5 using the verification regulations of NEN-EN1993-1-5 results in a design load of:

$$N_{b,rd} = 7310\ kN \tag{8.29}$$

Therefore the characteristic load is equal to:

$$\mu_S = \frac{N_{ed}}{1 + \alpha * \beta * V} = \frac{7310}{1 + 1 * 3{,}8 * 0{,}27} = 3608\ kN \tag{8.30}$$

The standard deviation for the load is given by:

$$\sigma_S = \mu_S * V_s = 3608 * 0{,}27 = 974\ kN \tag{8.31}$$

The probabilistic design variables are already specified above and for this configuration the relevant numbers are calculated in Table 6.

Parameter	Description	Statistical spread	Mean μ	Standard deviation σ	Variation
a	Length of column	Normal	$15000\ mm$	$2{,}5\ mm$	0,000167
b	Width of web	Normal	$1500\ mm$	$2{,}5\ mm$	0,00167
t_w	Thickness of web	Normal	$50{,}4\ mm$	$0{,}7\ mm$	0,0139
t_f	Thickness of flange	Normal	$50{,}4\ mm$	$0{,}7\ mm$	0,0139
h_w	Width of flange	Normal	$500\ mm$	$2{,}5\ mm$	0,005
E	Young's modulus	Normal	$210000\ \frac{N}{mm^2}$	$8400\ \frac{N}{mm^2}$	0,04
f_y	Yield strength	Lognormal	$280\ \frac{N}{mm^2}$	$22{,}4\ \frac{N}{mm^2}$	0,08
e/a	Column imperfection	Normal	$\frac{1}{290}$	$\frac{1}{290} * 0{,}15$	0,15
e_{loc}/b	Web imperfection	Normal	$0{,}7 * \frac{1}{320}$	$0{,}7 * \frac{1}{320} * 0{,}20$	0,20

Table 6: Design variables for uneconomic I-column

Ansys can perform this probabilistic design analysis and produces a plot of the results on a Gauss plot as shown in Figure 62. This means that a perfectly straight line would fit a normal distribution. The resistance is assumed to be normally distributed given the approximate straight line on a Gaussian cumulative distribution plot. The tail of the distribution is not straight so this is an assumption which is not entirely correct but this may also be due to the amount of calculations done.

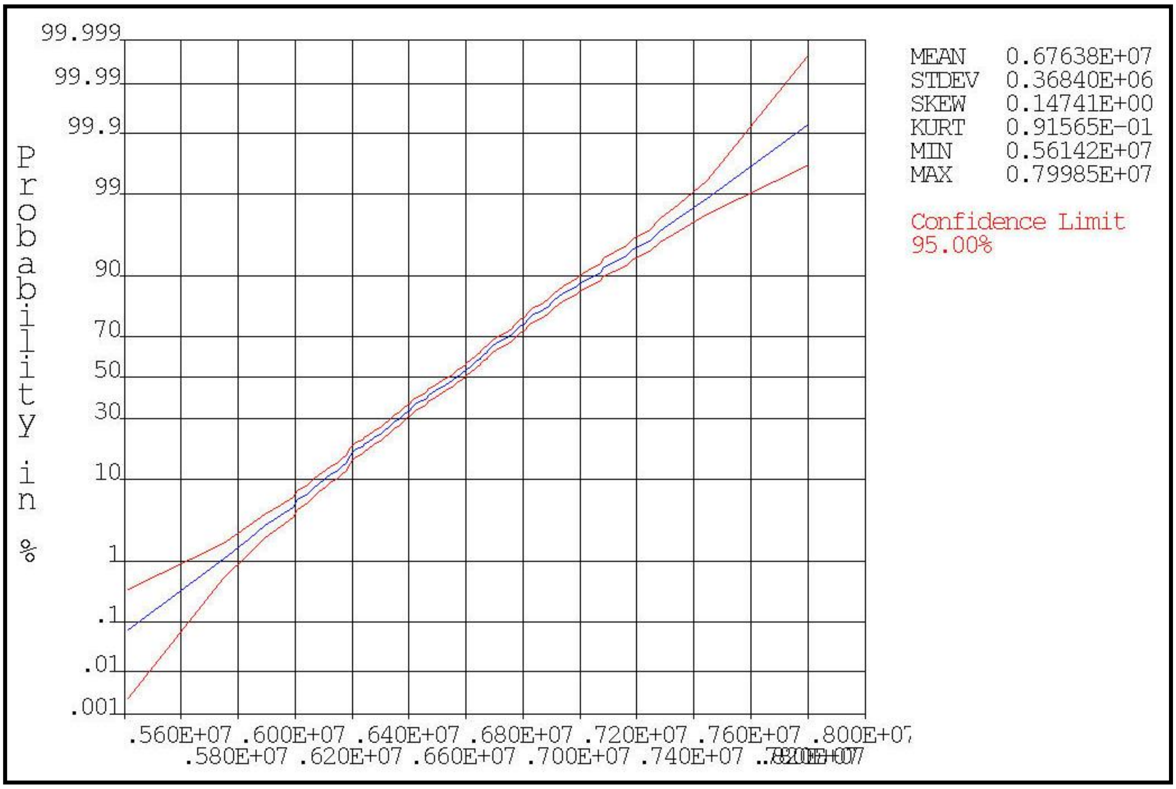


Figure 62: Cumulative distribution of collapse load according to Ansys for uneconomic cross-section

The dependence of the collapse load on the variables is also calculated by Ansys and is given in Figure 63. The most important variables are the global imperfection, the mechanical properties and the width and the thickness of the flange. The variance of the others apparently does not influence the collapse load significantly.

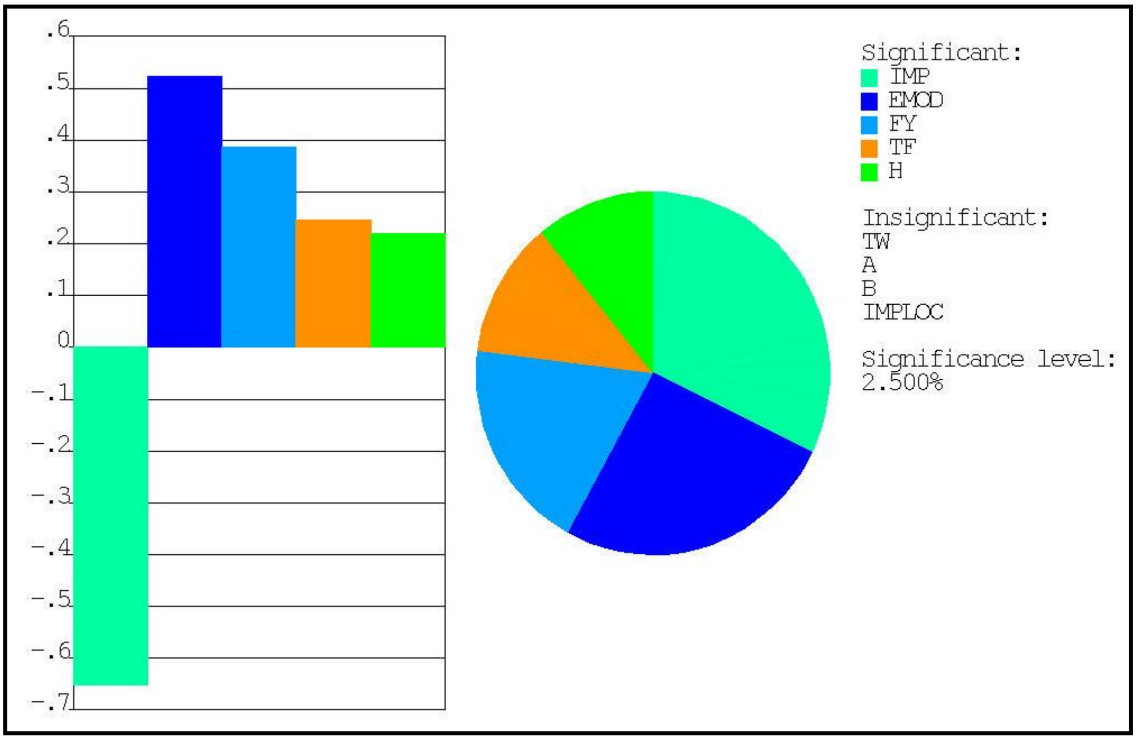


Figure 63: Significance of the design variables to the collapse load.

The mean for the resistance is:

$$\mu_R = 6764\ kN \tag{8.32}$$

The variance for the resistance is:

$$\sigma_R = 368\ kN \tag{8.33}$$

Now the mean for the reliability function can be calculated.

$$\mu_z = \mu_R - \mu_S = 6764 - 3608 = 3156\ kN \tag{8.34}$$

The standard deviation is given by:

$$\sigma_z = \sqrt{\sigma_R^2 + \sigma_S^2} = \sqrt{368^2 + 974^2} = 1041\ kN \tag{8.35}$$

Then the β-factor is calculated using the following relation and this factor should be at least 3,8 to satisfy the safety demand required.

$$\beta = \frac{\mu_z}{\sigma_z} = \frac{3156}{1041} = 3{,}03 \tag{8.36}$$

Therefore the probability for failure can be calculated:

$$P_f = P(Z < 0) = \Phi(-\beta) = \Phi(-3{,}03) = 1{,}22 * 10^{-3} \tag{8.37}$$

Conclusion

The NEN-EN1990 prescribes the required safety that a certain type of structure should have. This is expressed in the β-factor which is 3,8 for residential buildings, offices and public buildings. From that β-factor the probabilistic values of the load can be determined. For the imperfections and residual stresses not a lot is known about the mean values and the variance. In this analysis these are replaced by a single geometric imperfection so for that probabilistic design value an estimate has to be made about the mean and the variance. Therefore the comparison between the economic and the uneconomic cross-section is more important than the exact results.

It is immediately noticed that the economic cross-section has a β-factor of $3{,}85$ and the uneconomic cross-section has a β-factor of $3{,}03$. This difference is quite remarkable.

Chapter 9: Interaction between plate and lateral-torsional buckling

Stability of a welded I-beam under uniform bending moment

A welded I-beam can also lose stability under a uniform bending moment. Local buckling may occur in the form of multiple sines in the web. In that case the flanges are stiff enough to support the web and only a loss of stability of the web occurs. Also lateral-torsional buckling may occur which is a lateral deflection and a torsional rotation of the beam. The flanges are not stiff enough to support the web. This means that the girder as a whole will buckle. Also an interaction form may occur which may be unfavourable for the girder. Especially for fatigue loaded structures and for serviceability limit state it is important to have a proper estimate of the buckling load of the girder as explained before for the case of uniform compression.

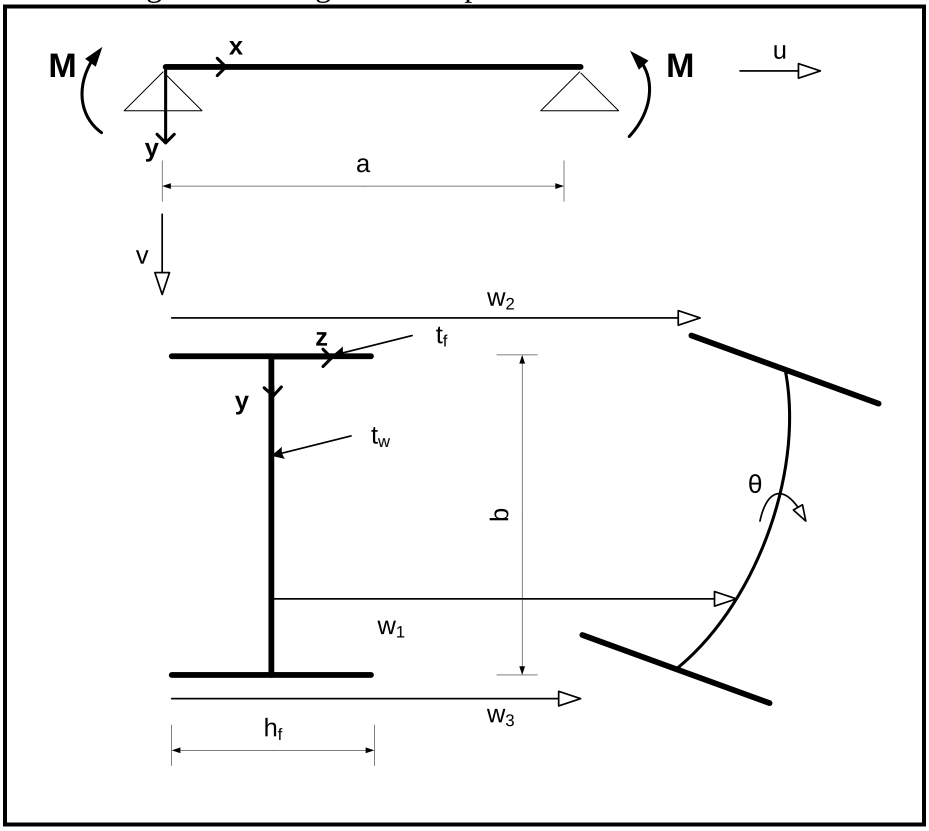


Figure 64: Description of situation for uniform bending moment of welded I-beam

Initial deflection as a sum of lateral-torsional and plate buckling behaviour

The deformation shape of the beam may be described in the following way. For the global behaviour a series of sines is used and for the local behaviour a series of sines multiplied by a series of sines in transverse direction is used. In contradiction to the case of uniform compression a series of sines as rotation is added. These deformations are the ones of the individual buckling shapes of the lateral-torsional and the local behaviour. The deformation of the web (w_1) is a summation of the local and the global behaviour. The deformation of the top flange (w_2) is described by the global behaviour. The behaviour of the bottom flange (w_3) is also described by the global behaviour. At $y = 0$ and $y = b$ the web and the flange have the same deformation for all x so compatibility between the web and the flange is ensured.

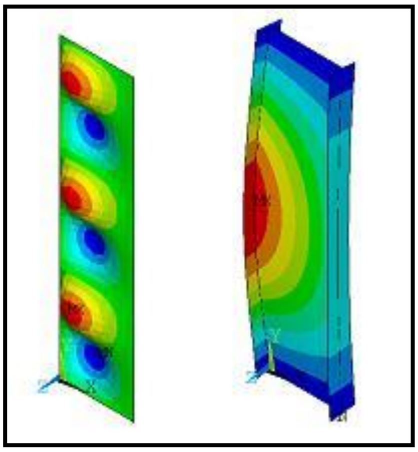


Figure 65: The local behaviour ($m = 6$) (left) and the global behaviour ($n = 1, t = 1$) (right) (displacement out of plane plotted)

The behaviour of the web is described by:

$$w_1 = \sum_{n=1}^{\infty} A_n * \sin\left(\frac{n\pi x}{a}\right) + \sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms} * \sin\left(\frac{m\pi x}{a}\right) * \sin\left(\frac{s\pi y}{b}\right) + \left(1 - 2 * \frac{y}{b}\right) * \frac{b}{2} * \sum_{t=1}^{\infty} F_t * \sin\left(\frac{t\pi x}{a}\right) \quad (9.1)$$

The behaviour of the top flange is described by:

$$w_2 = \sum_{n=1}^{\infty} A_n * \sin\left(\frac{n\pi x}{a}\right) + \frac{b}{2} * \sum_{t=1}^{\infty} F_t * \sin\left(\frac{t\pi x}{a}\right) \quad (9.2)$$

The behaviour of the bottom flange is described by:

$$w_3 = \sum_{n=1}^{\infty} A_n * \sin\left(\frac{n\pi x}{a}\right) - \frac{b}{2} * \sum_{t=1}^{\infty} F_t * \sin\left(\frac{t\pi x}{a}\right) \quad (9.3)$$

Determine equilibrium condition

The entire derivation is presented in Annex G. The limit state where the construction is just stable is the point where the total potential energy in the column is equal to the virtual work done by the external forces. This equation is $U_{total} + T_{total} = 0$ but is not shown here because of the length of the equation. The equation is still depending on A_n, B_{ms} and F_t. Now it is clear that only one value of n, m and t should be used. However, there are multiple values of s that may be in the solution. The accuracy of the solution decreases when the ratio a/b increases. (Timoshenko & Gere, 1963, pp. 354-355) This calculation is also made for relatively long girders so therefore a check should be performed to show that the calculation is sufficiently accurate by increasing the maximum value of s by 1 and the difference should be sufficiently small.

The minimum of M is obtained when the derivatives are taken with respect to A_n, B_{ms} and F_t and equated to zero. This is presented in Annex G. This is written in matrix form below. The buckling moment is obtained by solving the determinant for several values of m $(= n = t)$. The lowest result is the critical buckling moment.

$$\begin{bmatrix} \frac{1}{2}n^4\pi^4\frac{1}{a^3}I_{zz} & -\frac{M}{2a}nt\pi^2 & n^2D\frac{\pi^3 b}{a*1}\left(\frac{m^2}{a^2}+\frac{1^2*v}{b^2}\right) & -\frac{Mb^2t_w\pi}{2I_{yy}a}nm\frac{1}{2} & n^2D\frac{\pi^3 b}{a*3}\left(\frac{m^2}{a^2}+\frac{3^2*v}{b^2}\right) \\ -\frac{M}{2a}nt\pi^2 & S_t\left(\frac{t\pi}{a}\right)^2\frac{a}{2}+EC_w\left(\frac{t\pi}{a}\right)^4\frac{a}{2} & -\frac{1}{4}*\frac{Mb^3t_w}{I_{yy}}mt\frac{\pi}{a}\left(\frac{1}{1}-\frac{8}{\pi^2*1^3}\right) & 0 & -\frac{1}{4}*\frac{Mb^3t_w}{I_{yy}}mt\frac{\pi}{a}\left(\frac{1}{3}-\frac{8}{\pi^2*3^3}\right) \\ n^2D\frac{\pi^3 b}{a*1}\left(\frac{m^2}{a^2}+\frac{1^2*v}{b^2}\right) & -\frac{1}{4}*\frac{Mb^3t_w}{I_{yy}}mt\frac{\pi}{a}\left(\frac{1}{1}-\frac{8}{\pi^2*1^3}\right) & \pi^4\frac{ab}{4}D\left(\frac{m^2}{a^2}+\frac{1^2}{b^2}\right)^2 & -2\frac{Mt_wb^2}{aI_{yy}}m^2\frac{1*2}{(1^2-2^2)^2} & 0 \\ -\frac{Mb^2t_w\pi}{2I_{yy}a}nm\frac{1}{2} & 0 & -2\frac{Mt_wb^2}{aI_{yy}}m^2\frac{1*2}{(1^2-2^2)^2} & \pi^4\frac{ab}{4}D\left(\frac{m^2}{a^2}+\frac{2^2}{b^2}\right)^2 & -2\frac{Mt_wb^2}{aI_{yy}}m^2\frac{3*2}{(3^2-2^2)^2} \\ n^2D\frac{\pi^3 b}{a*3}\left(\frac{m^2}{a^2}+\frac{3^2*v}{b^2}\right) & -\frac{1}{4}*\frac{Mb^3t_w}{I_{yy}}mt\frac{\pi}{a}\left(\frac{1}{3}-\frac{8}{\pi^2*3^3}\right) & 0 & -2\frac{Mt_wb^2}{aI_{yy}}m^2\frac{3*2}{(3^2-2^2)^2} & \pi^4\frac{ab}{4}D\left(\frac{m^2}{a^2}+\frac{3^2}{b^2}\right)^2 \end{bmatrix} * \begin{bmatrix} A_n \\ F_t \\ B_{m1} \\ B_{m2} \\ B_{m3} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Verification of the result

To verify whether calculations have been done correctly, only the lateral-torsional buckling behaviour is checked which means that $B_{ms} = 0$.

$$\begin{bmatrix} \frac{1}{2}n^4\pi^4\frac{1}{a^3}I_{zz} & -\frac{M}{2a}nt\pi^2 \\ -\frac{M}{2a}nt\pi^2 & S_t\left(\frac{t\pi}{a}\right)^2\frac{a}{2}+EC_w\left(\frac{t\pi}{a}\right)^4\frac{a}{2} \end{bmatrix} * \begin{bmatrix} A_n \\ F_t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{9.4}$$

This is the same as the matrix for lateral-torsional buckling (equation (2.34)). Therefore this is correct. The verification for plate buckling can be done by calculating the theoretical buckling factor of 23,9 for a bending moment. This has been done and this is correct.

Results

Analytical results for flanges 150x15 mm and web 1500x15 mm

The calculations have been done for a profile with very small flanges compared to the web. This cross-section is extremely sensitive to the interaction of plate buckling and lateral-torsional buckling.

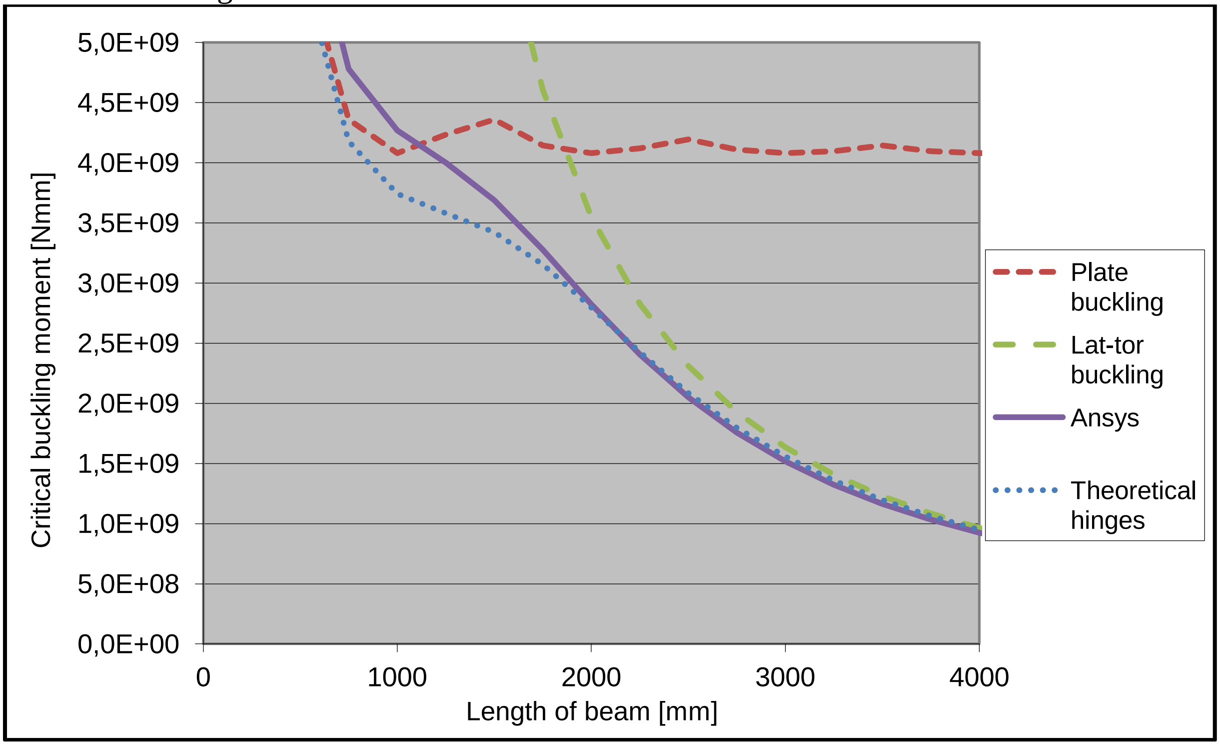


Figure 66: Theoretical buckling curve for flanges 150x15 mm and web 1500x15 mm

In Figure 66 it is clear that the theoretical curve based on the interaction of plate buckling and column buckling is well below the behaviour that is predicted by taking the minimum of plate buckling and column buckling. In Figure 67 it is shown that the analytical calculations underestimate the theoretical buckling load for small lengths. For longer lengths the theoretical buckling load is always slightly overestimated but this may be solved by mesh refinement.

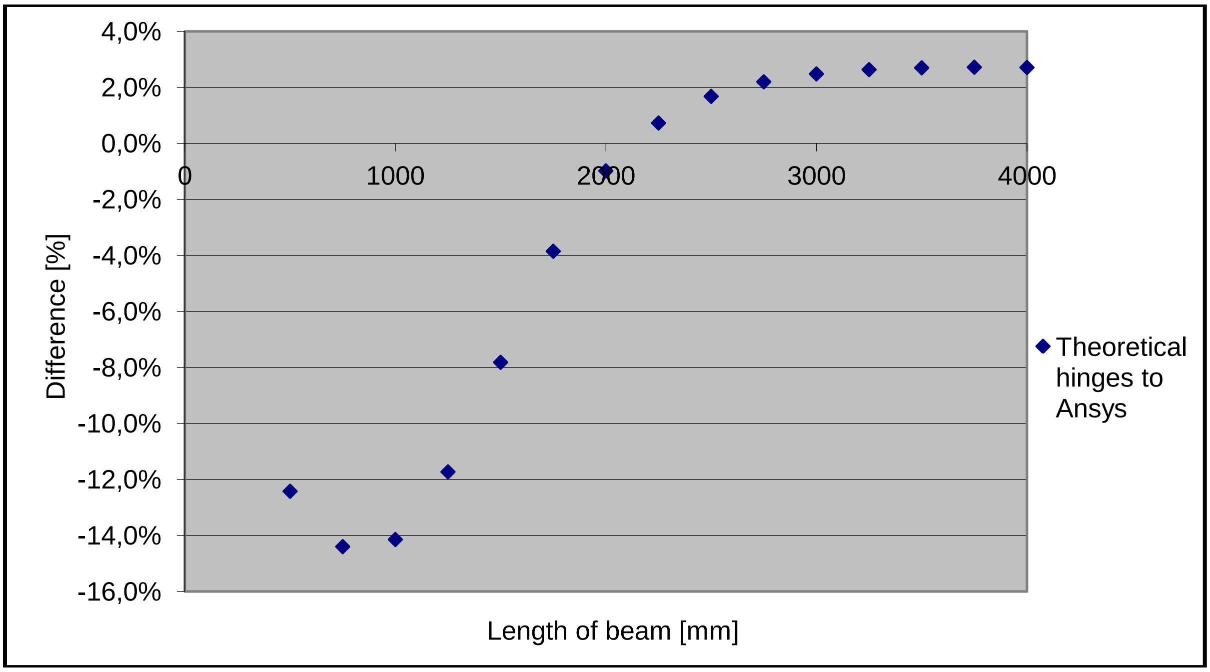


Figure 67: Difference caused by interaction of local and lateral-torsional buckling for flanges 150x15 mm and web 1500x15 mm

The results for a beam length of 2000 mm are given in Table 7.

Calculation	Buckling moment
Theoretical hinges	$2{,}80 * 10^9\ Nmm$
Column buckling	$3{,}55 * 10^9\ Nmm$
Plate buckling	$4{,}08 * 10^9\ Nmm$
Ansys model	$2{,}82 * 10^9\ Nmm$

Table 7: Buckling stress for beam with flanges 150x15 and web 1500x15 and length 2000

To illustrate the plate buckling behaviour also a plot is made of the displacement out of plane in Figure 68. This is indeed the behaviour of a lateral displacement and a rotation which is the global behaviour. Next to that behaviour also a local displacement of the web is observed. This combination is indeed applied in Chapter 9.

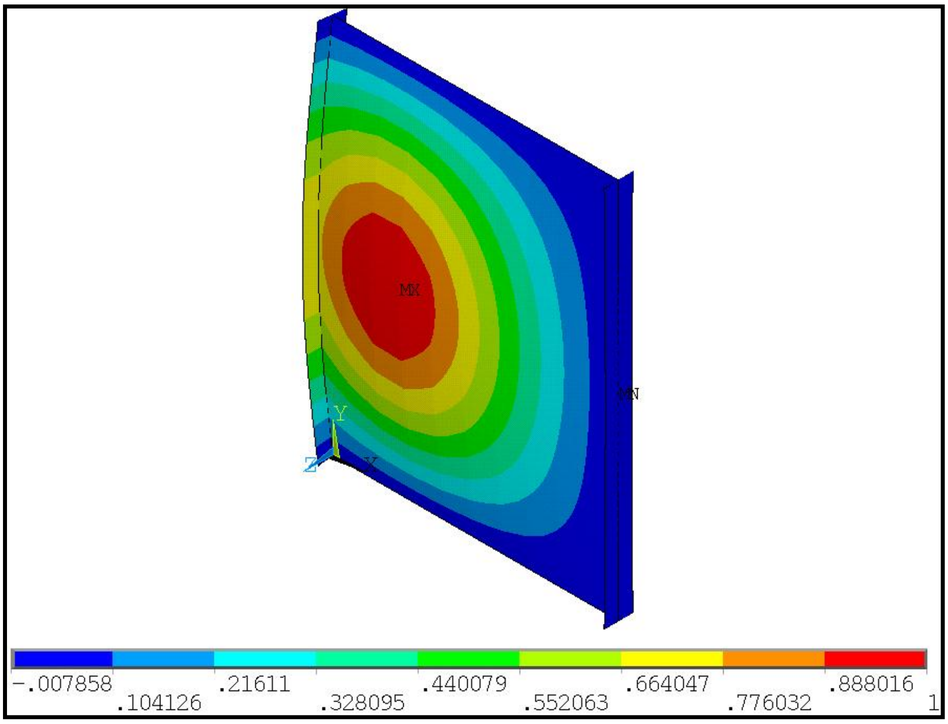


Figure 68: Buckling of beam with flanges 150x15 and web 1500x15 and length 3000 (displacement out of plane plotted)

Analytical results for flanges 500x25 mm and web 1500x15 mm

The previous cross-section is not a very common cross-section because proper engineering judgement would result in a cross-section with relatively large flanges and a slender web. For this example flanges of 500x25 mm and a web of 1500x15 mm are applied.

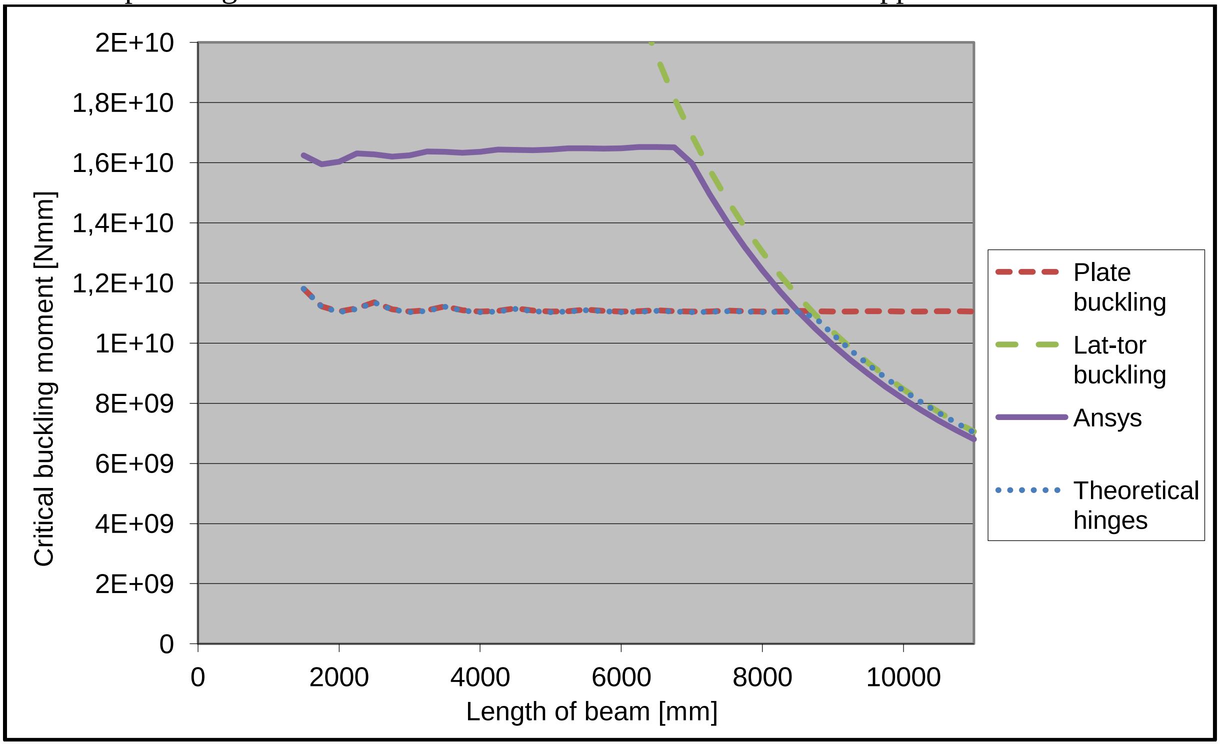


Figure 69: Theoretical buckling curve for flanges 500x25 mm and web 1500x15 mm

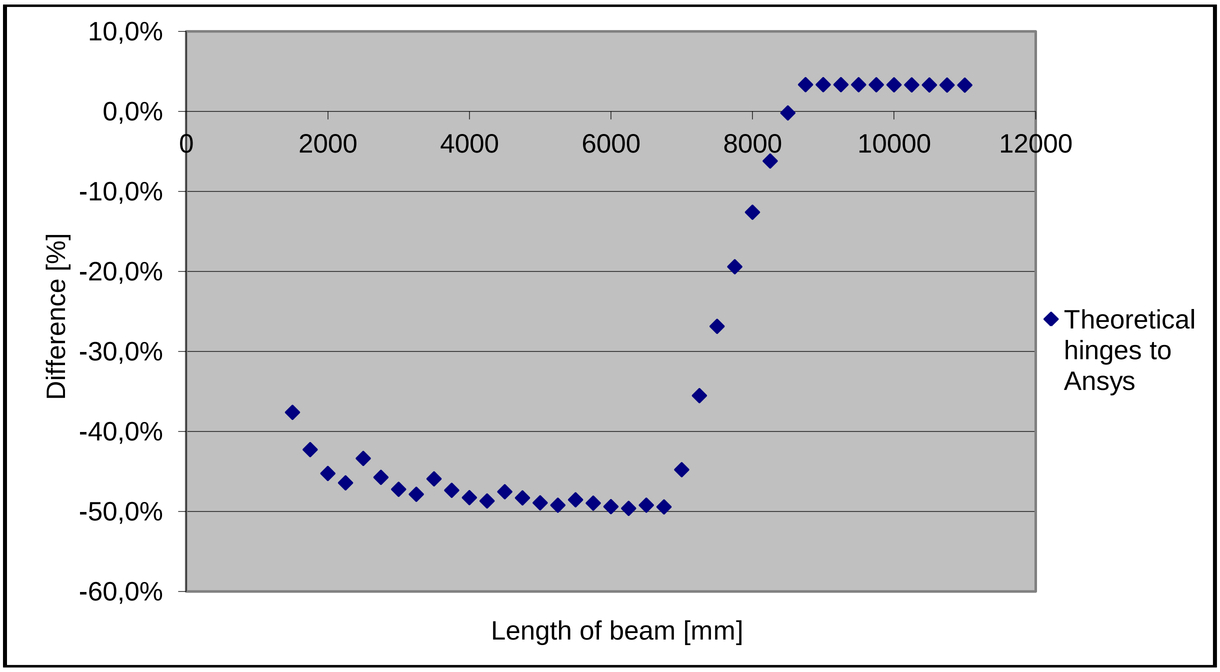


Figure 70: Difference caused by interaction of local and lateral-torsional buckling for flanges 500x25 mm and web 1500x15 mm

In Figure 69 the theoretical buckling curve is plotted and the difference between the theoretical curve and the result in Ansys is given in Figure 70. It is clear that for shorter lengths the theoretical buckling moment is underestimated due to the restraining effect of the flanges which is not included in the analytical calculations.

For a column length of 9000 mm the results are given in Table 8.

Calculation	Buckling stress
Theoretical hinges	$1{,}03 * 10^{10}\ Nmm$
Column buckling	$1{,}04 * 10^{10}\ Nmm$
Plate buckling	$1{,}11 * 10^{10}\ Nmm$
Ansys model	$0{,}99 * 10^{10}\ Nmm$

Table 8: Buckling stress for column with flanges 500x25 and web 1500x15 and length 9000

Conclusion

General conclusion

The buckling stress calculated using the analytical equations with hinged welds is always lower than the plate buckling stress and the column buckling stress.

In the case of stiff flanges it is clear that the model using hinged welds is almost equal to the minimum of the plate and column buckling stress. However, as seen in the model in Ansys, it is clear that the buckling stress is much higher when plate buckling is governing because the flanges prevent a large rotation at the welds.

The most important conclusion is that there is only interaction between the local and the global behaviour if they have the same number of half-waves in the length of the column. ($m = n = t$ in this thesis) This conclusion can be used later when the imperfections need to be modelled to predict the real behaviour of an I-beam because the NEN-EN1993-1-5 does not prescribe that these type of imperfections need to be modelled when calculating the restraint of a class 4 I-beam.

Rotational restraint

The buckling curve calculated has an important assumption that the flanges do not give any rotational restraint to the web. This is realistic for I cross-sections with a relatively large width of the web and flanges with a low rotational stiffness. It has been shown that the buckling curve calculated is a good approximation of the buckling curve according to the model in Ansys in those cases. However, when stiff flanges are applied the buckling curve is underestimated.

Theoretical model

The behaviour calculated now gives a proper inside in the way buckling happens in an I-beam. It is clear that the buckling load is much better approximated using the method described here but always requires a determinant of a matrix to be solved or an approximation formula should be developed. It is questionable if this is necessary especially as it is a combination of two buckling phenomena which have a completely different post-buckling behaviour.

Chapter 10: I-beam according to the Eurocode and NEN

NEN-EN1993-1-1 gives requirements on the design of steel structures. The requirements are concerned with the static strength and the stability of steel structures. The static strength of steel structures is not part of this thesis. Therefore the section on the stability of steel structures is used here and applied upon the I-beam in bending. In NEN-EN1993-1-1 the section 6.3 describes the calculation for stability of steel members. For the calculation of class 4 cross-sections a reference is made to NEN-EN1993-1-5 to calculate the effective cross-section.

Stability of a welded I-beam under uniform bending moment

According to NEN-EN1993-1-1

A welded I-beam under uniform bending moment has to be checked using equation 6.46 from NEN-EN1993-1-1:

$$\frac{M_{ed}}{M_{b,Rd}} \leq 1 \tag{10.1}$$

In this case only the resistance is interesting and this is described by equation 6.48 from NEN-EN1993-1-1 for class 4 cross-sections:

$$M_{b,Rd} = \frac{\chi_{LT} * W_{eff} * f_y}{\gamma_{M1}} \tag{10.2}$$

It is assumed that only the web has to be reduced for plate buckling. Therefore the minimum requirement of the flanges is that:

$$\frac{h_f - t_w}{2 * t_f} \leq 14 * \varepsilon \tag{10.3}$$

For the calculation of W_{eff} NEN-EN1993-1-5 has to be considered. This calculation has been shown before and is repeated here. The critical buckling stress for the web is: (assuming the flanges are equal in dimensions)

$$\sigma_{cr} = 23{,}9 * \frac{\pi^2 * E}{12 * (1 - v^2)} * \left(\frac{t}{b}\right)^2 \tag{10.4}$$

Therefore the relative slenderness for plate buckling of the web is:

$$\lambda_p = \sqrt{\frac{f_y}{\sigma_{cr}}} \tag{10.5}$$

The reduction factor for plate buckling of the web is:

$$\rho = \frac{\lambda_p - 0{,}055 * (3 - 1)}{\lambda_p^2} \leq 1 \tag{10.6}$$

Using the effective cross-section (Figure 71) the effective moment of inertia can be calculated. This formula is not given here. Then the effective section modulus can be calculated which is needed for the resistance of the beam.

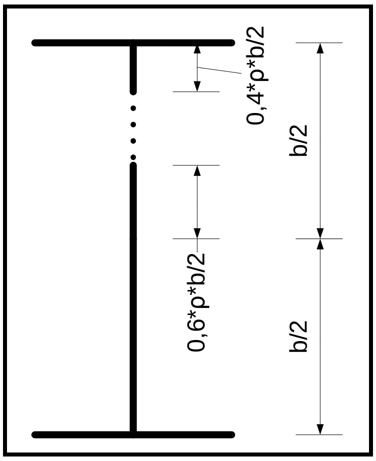


Figure 71: Effective cross-section for I-beam under uniform bending moment

According to equation 6.49 from NEN-EN1993-1-1 the relative slenderness of the I-beam is:

$$\lambda_{LT} = \sqrt{\frac{W_{eff} * f_y}{M_{cr}}} \tag{10.7}$$

The critical buckling load is given by:

$$M_{cr} = \frac{\pi}{a} * \sqrt{EI_{zz} * \left(S_t + E * C_w * \left(\frac{\pi}{a}\right)^2\right)} \tag{10.8}$$

In the critical buckling load the warping term C_w is often neglected. This is not applied here because it is also not neglected in the real behaviour and in the FEM model later on. The method for rolled cross-sections and equivalent welded cross-sections is used as it is presented in NEN-EN1993-1-1 article 6.3.2.3. Using the relative slenderness the reduction factor can be calculated using the following equation:

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{{\Phi_{LT}}^2 - 0{,}75 * {\lambda_{LT}}^2}} \leq 1 \tag{10.9}$$

In which Φ_{LT} is:

$$\Phi_{LT} = 0{,}5 * (1 + \alpha * (\lambda_{LT} - 0{,}4) + 0{,}75 * {\lambda_{LT}}^2) \tag{10.10}$$

The buckling curve depends on the dimensions of the welded I-section as given in Figure 72.

Doorsnede	Begrenzing	Kipkromme
Gewalste I-profielen	h/b ≤ 2	b
	h/b > 2	c
Gelaste I-profielen	h/b ≤ 2	c
	h/b > 2	d

Figure 72: Choice of buckling curve according to NEN-EN1993-1-1 table 6.5 (in Dutch)

The factor α is dependent on the buckling curve as given in Figure 73.

Knikkromme	a_0	a	b	c	d
Imperfectiefactor α	0,13	0,21	0,34	0,49	0,76

Figure 73: Factor α dependent on the buckling curve chosen according to NEN-EN1993-1-1 table 6.1 (in Dutch)

Now the resistance can be calculated. This resulting resistance can be compared to the calculations using FEM methods later on.

According to NEN6771

The old Dutch codes for the design of steel structures are the NEN6770 and the NEN6771. The approach according to those codes is different and is also explained to show the major difference in the calculation of a welded I-beam. The NEN6771 states that the reduction factors for plate buckling and lateral-torsional buckling need to be determined for the gross cross-section separately. Then the reduction factors need to be multiplied with the yield stress and this is the maximum allowable stress in compression. This is the prescribed method.

The critical buckling stress for the plate is:

$$\sigma_{cr} = 23{,}9 * \frac{\pi^2 * E}{12 * (1 - v^2)} * \left(\frac{t}{b}\right)^2 \tag{10.11}$$

The critical moment for lateral-torsional buckling is based on the gross cross-section and is given by:

$$M_{lattor} = \frac{\pi}{a} * \sqrt{EI_{zz} * \left(S_t + E * C_w * \left(\frac{\pi}{a}\right)^2\right)} \tag{10.12}$$

The relative slenderness for the plate is:

$$\lambda_{plaat,rel} = \sqrt{\frac{f_y}{\sigma_{cr}}} \tag{10.13}$$

The reduction factor for the stress in the plate is dependent on the slenderness of the plate.

$$\begin{gathered}
\sigma_{plooi,rel} = 1 \; if \; 0 \leq \lambda_{plaat,rel} \leq 0{,}7 \\
\sigma_{plooi,rel} = 1{,}474 - 0{,}677 * \lambda_{plaat,rel} \; if \; 0{,}7 < \lambda_{plaat,rel} \leq 1{,}291 \\
\sigma_{plooi,rel} = \frac{1}{\lambda_{plaat,rel}^2} + 0{,}132 * \lambda_{plaat,rel} - 0{,}170 \; if \; 1{,}291 < \lambda_{plaat,rel} \leq 2{,}5 \\
\sigma_{plooi,rel} = \frac{1}{\lambda_{plaat,rel}^2} \; if \; 2{,}5 < \lambda_{plaat,rel}
\end{gathered} \tag{10.14}$$

Now the reduction factor is known it is possible to determine the relative slenderness.

$$\lambda_{rel} = \sqrt{\frac{\sigma_{plooi,rel} * f_y * W_{el}}{M_{lattor}}} \tag{10.15}$$

The reduction factor for lateral-torsional buckling is calculated using:

$$\omega_{kip} = \frac{1 + \alpha_k * (\lambda_{rel} - \lambda_0) + \lambda_{rel}^2}{2 * \lambda_{rel}^2} - \frac{\sqrt{(1 + \alpha_k * (\lambda_{rel} - \lambda_0) + \lambda_{rel}^2)^2 - 4 * \lambda_{rel}^2}}{2 * \lambda_{rel}^2} \tag{10.16}$$

The ultimate bearing capacity is therefore:

$$M_{u,d} = \omega_{kip} * \sigma_{plooi,rel} * f_y * W_{el} \quad (10.17)$$

Chapter 11: I-beam in FEM

The I-column has been calculated but also bending moments occur in real structures which could lead to the instability of lateral-torsional buckling. The theoretical lateral-torsional buckling load has been derived in Chapter 9 but in real structures there are also geometrical imperfections and residual stresses which are represented by a single geometric imperfection as explained for the I-column in Chapter 6. The single geometric imperfection is a global deflection and rotation combined with a local displacement of the web.

The model is given in Figure 74. This is the same as Figure 64 except in FEM it is possible to assume a fixed connection at the location of the welds and this is applied.

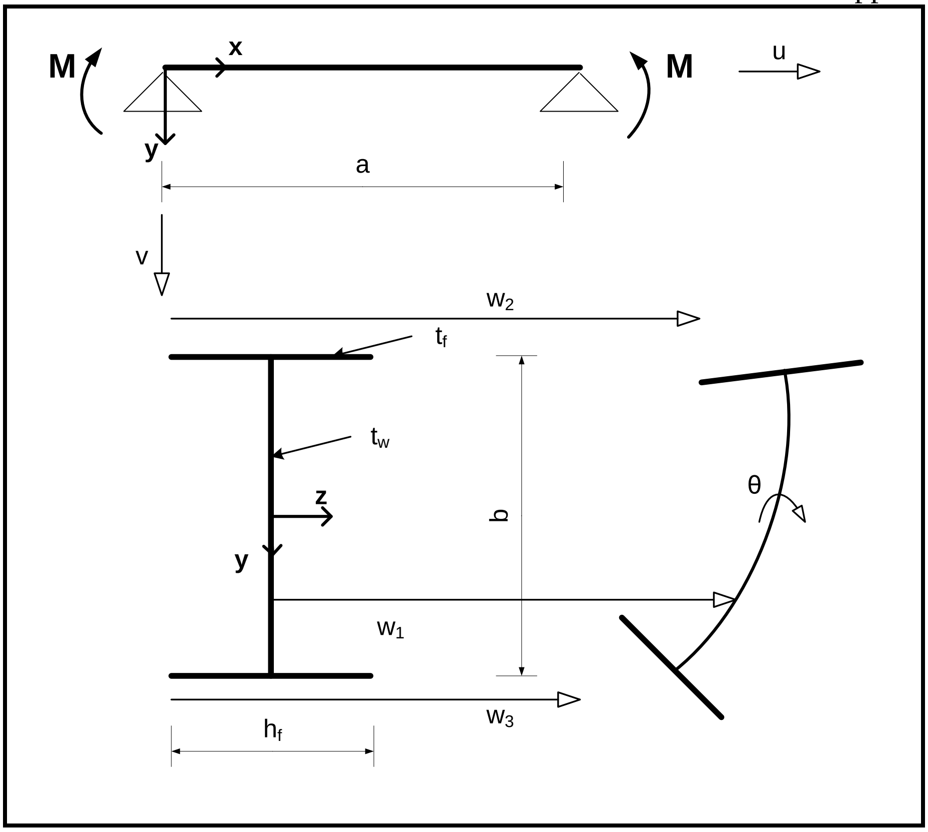


Figure 74: Welded I-beam in FEM

Displacement control vs Force control

It has been described for the I-column that displacement control is applied at the end of the beam to ensure that the stiff parts take the most loading. This is not possible for an I-beam. If the displacement is specified at both ends as a linear displacement over the height of the beam a tension part will develop in the web. The compressed flange will displace out of plane and the tension flange will only increase in length due to axial strain. Therefore the compressed flange will shorten more than the tension flange will lengthen. This means that the entire I-beam will shorten at the centre line. However, it is not possible to include this in the model. Therefore a load is applied at the ends of the I-beam. However, the end conditions are modelled in such a way that the stiffest part of the cross-section carry the largest load. This is described next.

Model description

The model for axial compression as described before is the start of the model for bending moments. The geometry remains unchanged but the boundary conditions at the end differ. As explained before it is necessary to apply a load at the end of the beam. This is done at both ends to ensure equilibrium. The end support is given in Figure 75. To ensure that the stiffer parts take the most loading three very stiff beams are applied at the end and provide the distribution as if there is thick end plate.

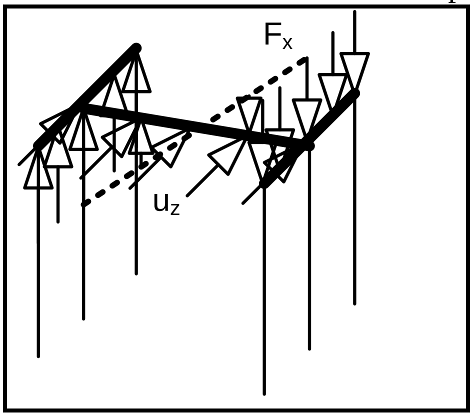


Figure 75: Hinged beam support in 3D model. Displacement described at web and thick line is the very stiff beam.

The load is a linear line load. For a given bending moment, the load per node is calculated as followed:

$$F_{node} = \frac{M}{2 * b * \left(nc + \frac{(na+1)*(na+2)}{12 * na}\right)} \frac{y_{node}}{y_{max}} \tag{11.1}$$

The value of the force is determined by the number of nodes in y-direction (na) and the number of nodes on a single side of the flange (nc). This factor is calculated by assuming a linear load distribution considering the individual leveller arms.

Single geometrical imperfections

It has been shown for the I-column that the imperfections according to Chapter 4 do not give a lower result than the imperfections according to the NEN-EN1993-1-5 using a single half wave in the web and the imperfections according to the NEN-EN1993-1-5 using multiple half waves in the web. Therefore the imperfections according to Chapter 9 are not included for the I-beam.

The single geometric imperfection is a method to replace the real geometrical imperfections and the residual stresses. For a welded I-beam the correct buckling curve depends on the height to width ratio. If the ratio is larger than 2 the correct buckling curve is d and for a ratio smaller than 2 the buckling curve is c. This influences the imperfections because for curve c $\frac{1}{200}$ is the correct ratio and for curve d $\frac{1}{150}$ is the correct ratio. This is not included and all cross-sections are calculated using curve c and a ratio of $\frac{1}{200}$. This allows a better comparison of the results.

Eurocode imperfections

First the web is modelled with supports along its edges as described for a perfect plate. Then the lowest eigenvalue is calculated and applied upon the plate with magnitude:

$$w_{web,max} = \frac{1}{200} * b * 0{,}7 \tag{11.2}$$

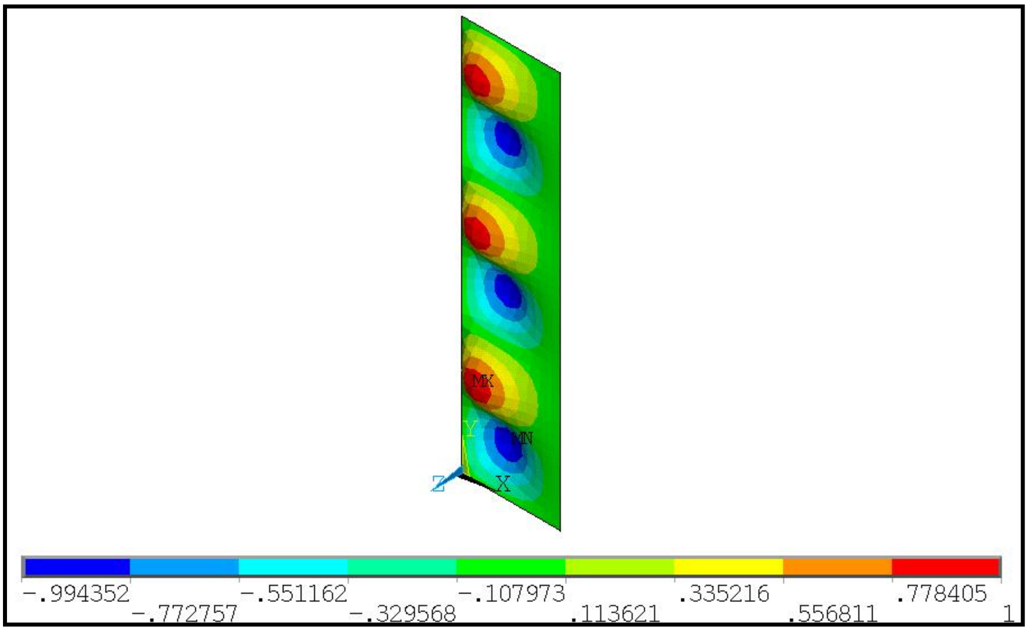


Figure 76: The local imperfection of the web according to the Eurocode

The factor 0,7 is used because only the governing mechanism should be applied at full scale and all other instability configurations may be scaled to 70% of the given limit. The major displacement is the lateral-torsional buckling behaviour and plate buckling is the secondary buckling mode.

Then the flanges are added to the construction and the entire model is loaded as described before. The buckling load is the lateral-torsional buckling behaviour and is applied upon the entire beam with magnitude equal to:

$$w_{flange,max} = \frac{1}{200} * a \tag{11.3}$$

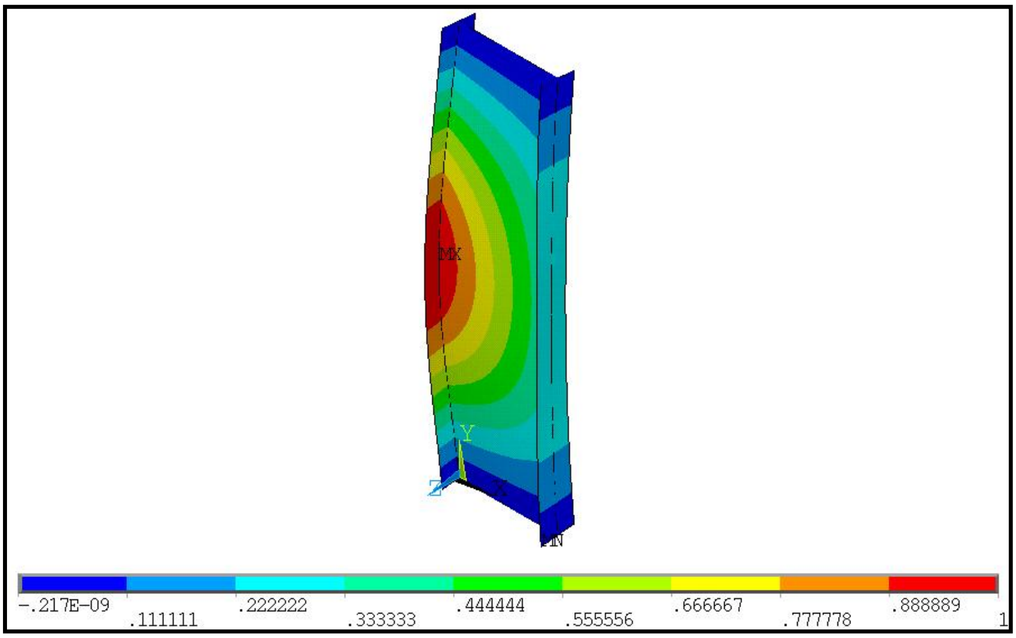


Figure 77: The global imperfection of the entire section according to the Eurocode

One should be aware that the maximum displacement is at the compressed flange. Therefore the tension flange is only given a displacement lower than the maximum displacement. For one case it has been investigated whether an imperfection in the form of a uniformly compressed I-column is unfavourable but the difference was small. The imperfection in the form of lateral-torsional buckling was slightly more unfavourable. NEN-EN1993-1-5 is not clear about whether the maximum imperfection is at the compressed flange or as an average for the cross-section. That would mean that the centre of the web is given the maximum imperfection and then the cross-section is rotated which would result in a larger imperfection at the compressed flange. It is applied such that the compressed flange is scaled to the maximum imperfection.

The combination of the two displacements are applied to the construction and then the non-linear calculation can be done.

Imperfections according to Eurocode using m=1

Another imperfection model is added which works the same as the previously described model which generates imperfections according to the NEN-EN1993-1-5. However, it has been shown in Chapter 9 that there is only interaction in the theoretical buckling load when the number of half sine waves in the flanges is equal to the number of half sine waves in the web. This is chosen at $m = 1$ here.

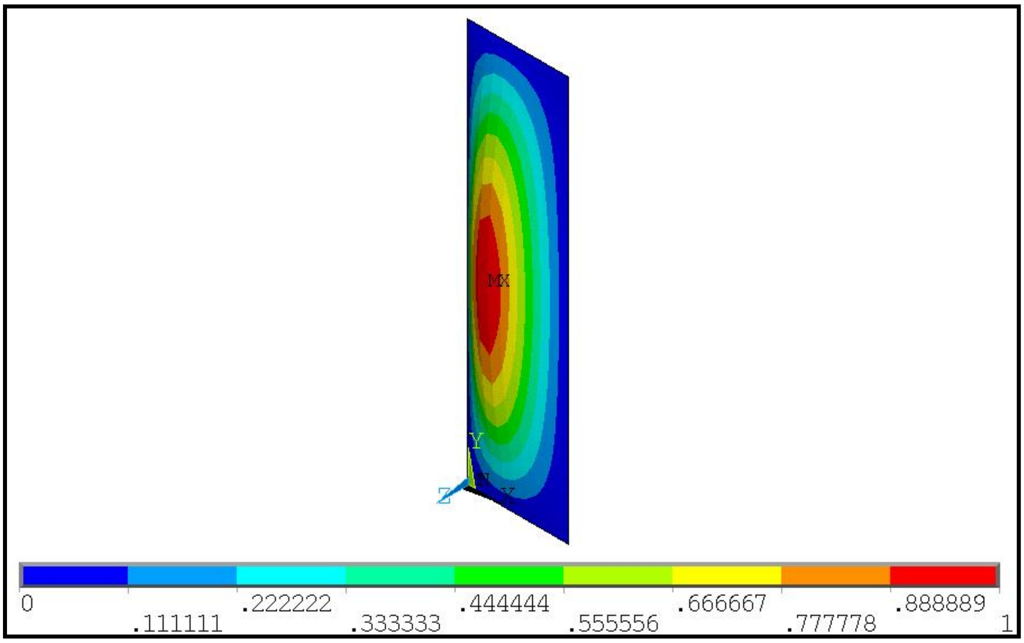


Figure 78: The local imperfection of the web according to the Eurocode using $m = 1$

For this model the eigenvalues of the web are calculated and for each mode the total deflection out of plane is calculated. The mode with the largest total imperfection out of plane is applied at the web. (Figure 78) Then the flanges are added to the model and the buckling modes are again calculated. For the buckling modes the total deflection out of plane is calculated and the mode with the largest out of plane deflection is added to the entire column. This is given in Figure 77.

Results

Many cases using different dimensions for the flange and web have been calculated and the results are presented in Annex I. Only for two configurations the results are presented and discussed.

Economic class 4 profile (flanges 500x25 mm and web 1500x10 mm)

For an I-column it has been shown that multiple calculations for a number of lengths of the column using Ansys results in a buckling curve which can be compared to the design curve according to the NEN-EN1993-1-1. This is also done for the I-beam and the results are compared to the calculation of the I-beam in Chapter 10. If the result in such a figure is above 100% the collapse load in Ansys is higher than the design load according to the NEN-EN1993-1-1. If the value is below 100% the collapse load in Ansys is lower than the design load according to the NEN-EN1993-1-1. The results for certain dimensions are given in Figure 79.

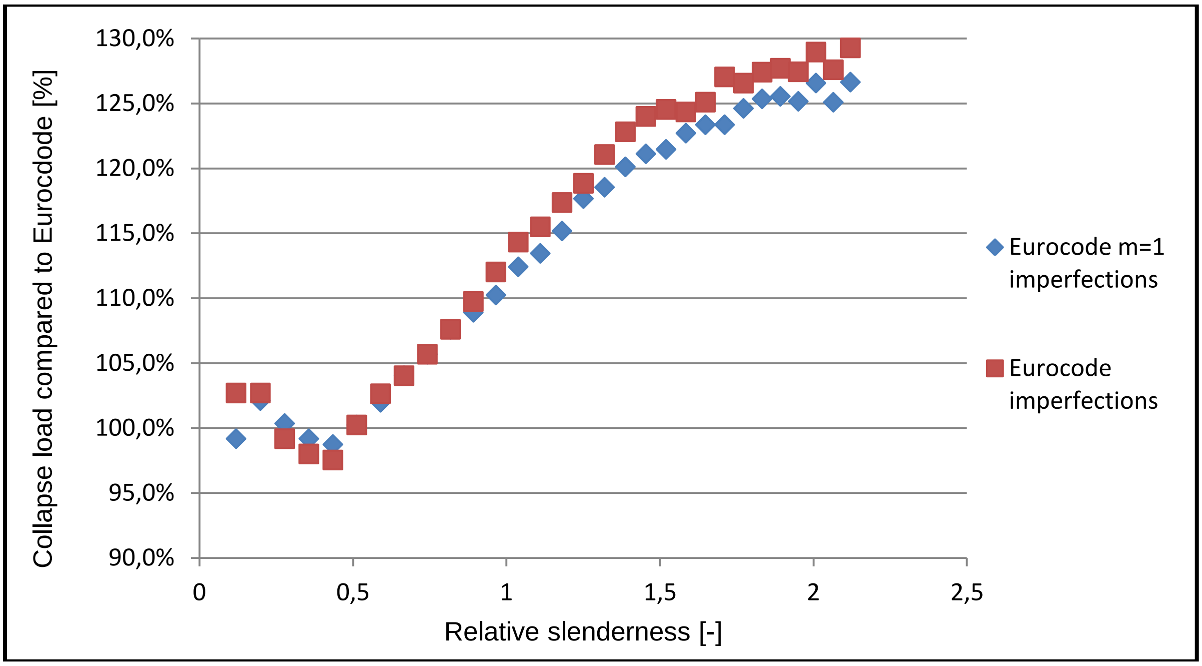


Figure 79: Collapse loads for beam with flanges 500x25 and web 1500x10

The results show that the collapse load is in general underestimated by the NEN-EN1993-1-1. For very short I-beams the collapse load is approximately equal which is the cross-sectional capacity depending on the class of the cross-section. For longer I-beams the collapse load is underestimated by the NEN-EN1993-1-1. This is the general trend observed for almost all cross-sections calculated.

For very short I-beams the results using the different type of imperfections are almost equal. When the slenderness increases the difference increases up to approximately 2%. The imperfections using a single half sine wave in the web is therefore unfavourable compared to the multiple half sine waves in the web.

Uneconomic class 1 profile (flanges 500x25 mm and web 1500x50 mm)

Now a more uneconomic cross-section is calculated because the web is much thicker. In general for I-beams a slender web is the most economic cross-section for bending moments because the material is then allocated in the flanges where it has a large level arm to the centre of the cross-section. The results for an uneconomic cross-section are presented in Figure 80.

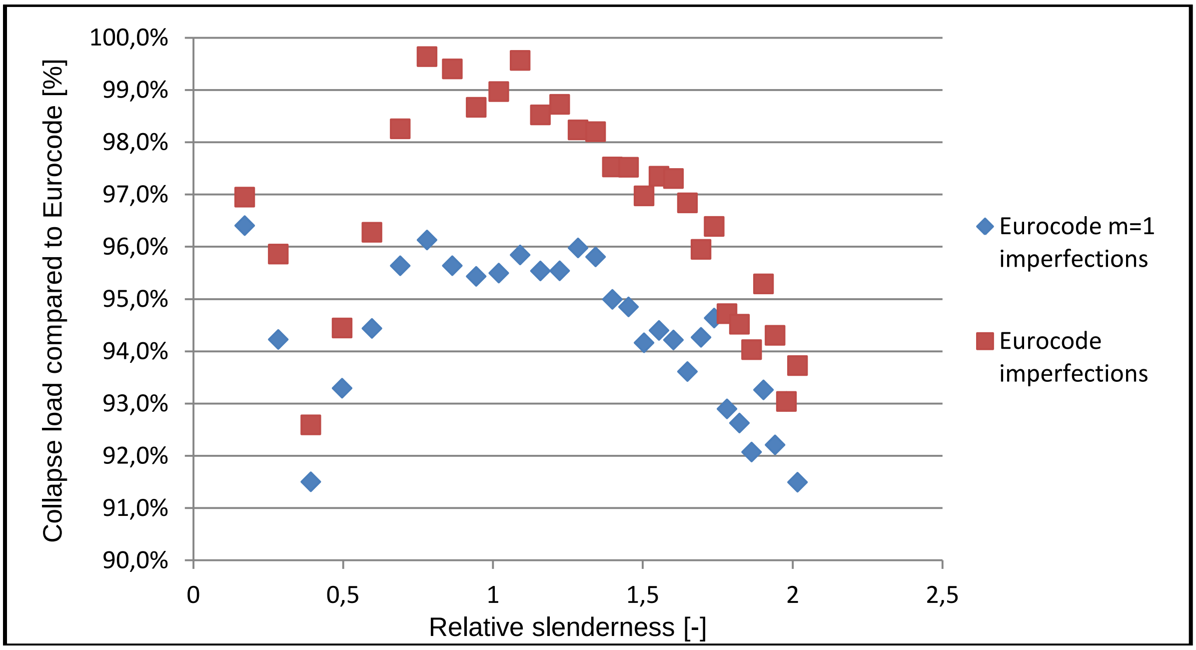


Figure 80: Collapse loads for beam with flanges 500x25 and web 1500x50

For very short I-beams there are some numerical errors and these results should be treated with caution. This is also the region where the Bernoulli assumption is not applicable. The general trend is that the NEN-EN1993-1-1 verification is in accordance with the results from Ansys. For a higher slenderness the difference increases slightly.

A more important conclusion is that the imperfections using a single half sine wave in the web is unfavourable compared to multiple half sine waves in the web. The difference is approximately 3% for a high slenderness.

Sensitivity analysis

The model is applied in a number of cases but the sensitivity with respect to a number of variables needs to be investigated. The buckling curves are applicable for many situations and before conclusions are drawn on the basis of the previous results it is necessary to investigate the sensitivity of the variables. The important variables that will be investigated are the yield stress and the size of the imperfections.

Yield stress

All previous results have been obtained using a yield stress of 235 N/mm^2 which is the most common steel grade applied. However, in recent years higher strength steel is applied more frequently and the sensitivity to the yield stress should be investigated. For the beam with flanges 500x25 mm and web 1500x15 mm, using the imperfections in the NEN-EN1993-1-5, the difference between the Ansys collapse load and the NEN-EN1993-1-5 verification is plotted in Figure 81 for three different steel grades. (235 N/mm^2, 355 N/mm^2 and 460 N/mm^2)

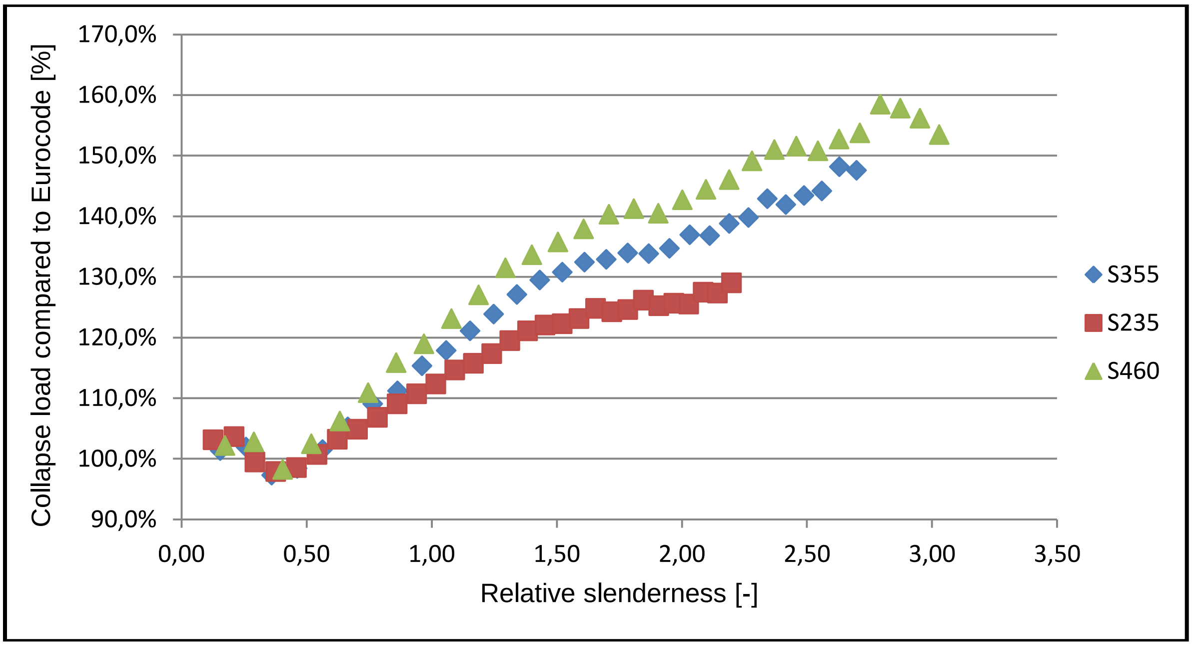


Figure 81: Sensitivity of non-linear analysis to yield stress

It is clear from Figure 81 that the yield stress does influence the behaviour and the safety level is higher when the yield stress is higher. This is consistent with the NEN-EN1993-1-1 because for many types of profiles S460 has a higher buckling curve so the reduction is smaller. This is given in table 6.2 of NEN-EN1993-1-1. However, this is not true for welded I-beams so here the same buckling curve is applied for all calculations.

Imperfections

The single geometric imperfections are added to the structure according to the limits given in the NEN-EN1993-1-5 for calculations using finite elements. These single geometric imperfections represent the geometrical imperfections and the residual stresses. The question is whether a slight increase or decrease of the single geometric imperfection results in a considerable change of load bearing capacity. For the beam with flanges 500x25 mm and web 1500x15 mm the difference between the Ansys collapse load and the NEN-EN1993-1-1 verification is plotted in Figure 82 for three different imperfections. (1/150, 1/200 and 1/250) Both the local imperfection of the web and the global imperfection of the entire cross-section is changed.

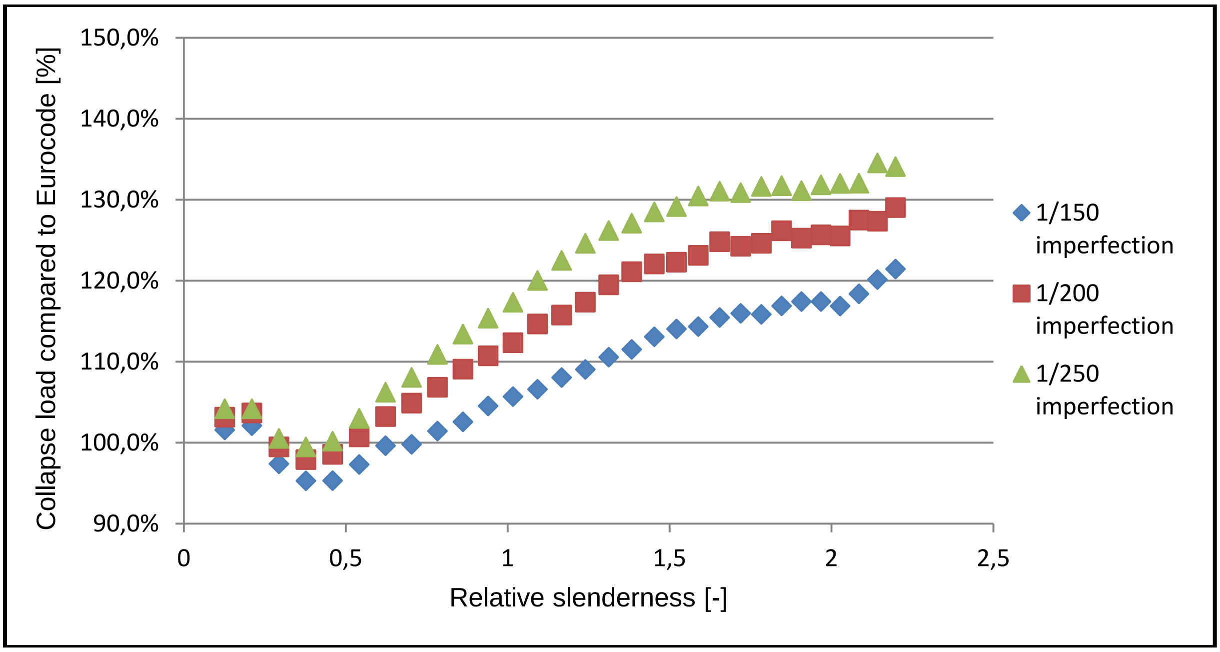


Figure 82: Collapse load compared to Eurocode verification assuming different imperfection parameters.

Chapter 12: Comparison of analytical, FEM and Eurocode results for I-beam

General results

For all cross-sections it is remarkable that in general the collapse load in Ansys is higher than the collapse load according to the NEN-EN1993-1-1 verification. This is not the same behaviour as for I-columns because there the collapse load in Ansys was always lower. However, this can still be explained using the same criteria as for the I-column.

- The most important one is that the single geometric imperfection is not equal to the real structure with real geometric imperfections and residual stresses. The buckling curves have been fitted to the test results for I-beams under uniform bending moment. However, as shown in literature (Mateescu & Ungureanu), the results for I-beams are slightly underestimated for higher slenderness's.
- The second is the effect of strain hardening which is present in real structures but not in Ansys because a bilinear model has been used for the stress-strain diagram.
- The NEN-EN1993-1-5 does not prescribe whether the maximum global displacement which should be applied in the finite element model is an average maximum which would mean that the centre of the web is given the prescribed displacement. If it is an absolute maximum the compressed flange is given the maximum displacement. This is applied here but the other method would result in lower collapse loads in Ansys.

Therefore it may be assumed that the results may be slightly different but the results are certainly comparable to each other.

Influence ratio web to flange

In the case of uniform compression on an I-column it was concluded that the ratio web to flange is important for the bearing capacity. The NEN-EN1993-1-5 proved to be less conservative when the web is relatively large compared to the flange. This relationship is also examined for the I-beam. For a certain cross-section the web thickness is varied and the collapse loads in Ansys are compared to the design loads according to the NEN-EN1993-1-5. The results are given in Figure 83. One should be aware that for a thickness up to 18,1 mm the cross-section is class 3 or lower so an elastic cross-sectional capacity should be used in the verification. For a class 2 or higher a plastic cross-sectional capacity is allowed. Therefore the larger difference between the thickness of 15 mm and the thickness of 20 mm is explained.

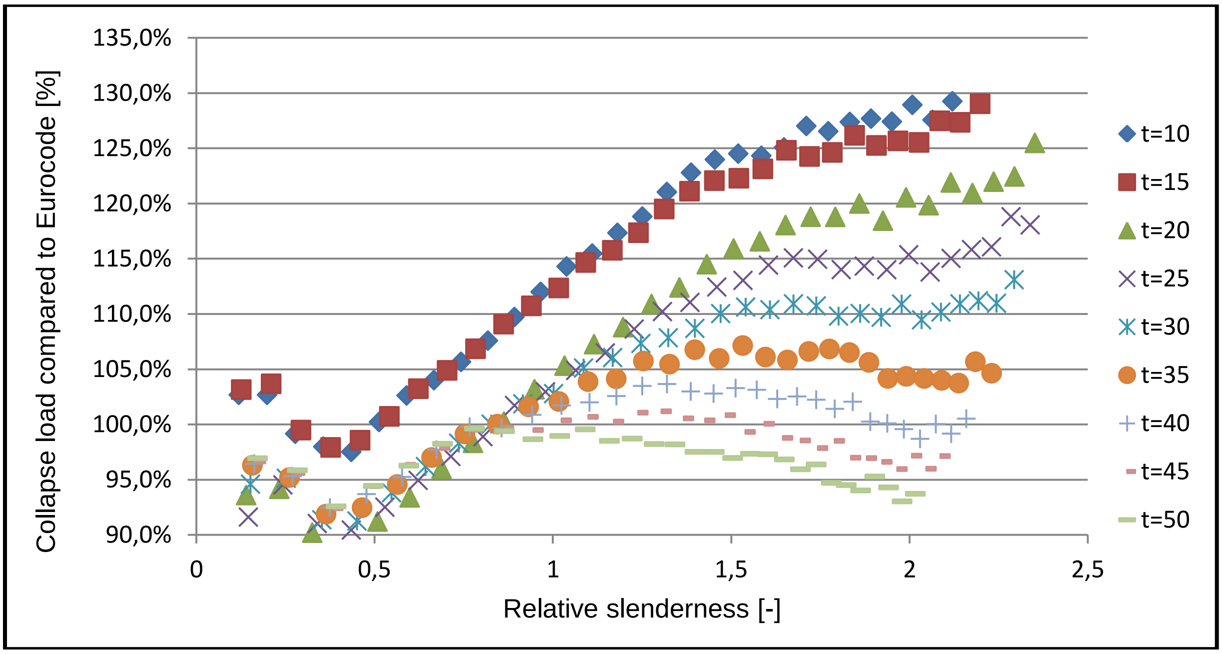


Figure 83: Collapse load compared to the Eurocode for flanges 500x25 mm and web with a width of 1500 mm

It is clear that the margins between the collapse loads in Ansys and the NEN-EN1993-1-5 is larger when the thickness of the web is smaller. This is the same conclusion as drawn for the case of uniform compression on an I-column.

However, for the case of a uniform bending moment on an I-beam no clear relationship has been found as is done for the case of the I-column where the ratio of areas was a clear parameter.

Influence type of imperfections

Another remarkable effect is the difference between the three types of imperfections. The imperfections according to Chapter 9 give a too small global imperfection for a low relative slenderness because the local imperfection is governing and the global imperfection is scaled down too much. This result was already found for the I-column and is not again investigated for the I-beam.

The differences between the imperfections according to the Eurocode (using multiple half sine waves in the web) and according to the Eurocode using $m = 1$ (one single half sine wave in the web) is calculated for only two cross-sections. It is clear that for an uneconomic cross-section the difference is larger than for an economic cross-section. The web has a larger area and therefore the difference is larger. In general the single half sine wave in the web is unfavourable compared to the multiple half sine waves just as it was for the I-column.

Chapter 13: Conclusion, recommendations and future research

Conclusions

First conclusions are drawn for the I-column and the interaction of column buckling and plate buckling. Later the I-beam is discussed shortly because the results are in general similar to the I-column. Structural engineers that need to design a class 4 cross-section are also advised to read the recommendations.

Verification regulations

The old Dutch codes for the verification of plate buckling (NEN6771) have strict regulations and recommend the reduced stress method. This is a weakest link calculation and the post-buckling strength is not exploited. The NEN-EN1993-1-5 provides a much more liberate verification for plate buckling. The recommended method is the use of the effective width method where the post-buckling strength is exploited and it is calculated as a parallel system where the weakest link is not governing. Using a finite element method the results for I-columns are verified and in general the results from the finite element analysis and the results from the verification of the NEN-EN1993-1-1 are closely related. The differences for commonly used cross-sections is in the order of 10-15%. The differences can be explained because the finite element analysis has some simplifications concerning the residual stresses and the geometric imperfections whereas the buckling curves are fitted to real test results.

Also verifications for plates including stiffeners is much more liberate in the NEN-EN1993-1-5. The NEN6771 prescribes the use of a stiffener such that only individual plate panels may buckle. The NEN-EN1993-1-5 verifies the stiffened plate panel using interaction formulas for the individual sub-panels as well as for the entire stiffened panel.

Theoretical buckling load

Analytical calculations have been performed to calculate the theoretical buckling load. This is done using an energy method and virtual work. The assumed displaced shape is a combination of the plate buckling and the column buckling shape. It is important to note that the post-buckling behaviour is entirely different and therefore the understanding of the interaction is more important than the actual value of the theoretical buckling load.

Interaction plate buckling and column buckling

The most important conclusion from the analytical calculation is that there is only interaction when the number of half sine waves in the web is equal to the number of half sine waves in the entire column. The interaction is in general small for the theoretical buckling load. ($<$ 1%) However, if the area of the web is large compared to the area of the flange, the theoretical buckling load is significantly lower. ($2\% - 30\%$)

Influence ratio web to flange

The finite element analysis includes the effects of residual stresses and geometric imperfections. These are included as a representative single geometric imperfection. The finite element model is not an extremely advanced model and therefore the results should not be seen as the exact answer for the collapse load but the results are especially suitable for a comparative study.

The finite element analysis has been done on many cross-sections with different lengths. From the results it is clear that the interaction is also present for structures with imperfections and residual stresses. For a certain cross-section with a variable thickness of the web, the difference between the finite element analysis and the NEN-EN1993-1-1 verification increases when the thickness of the web increases. For an I-column under uniform compression a relationship has been developed which describes the decrease of load bearing capacity for a cross-section with a large web compared to the flanges. When the area of the web is larger than the area of the flange a reduction factor is calculated in Chapter 7. The reduction can be up to 20% for extremely uneconomic cross-sections. The results are consistent with the results from the analytical calculation where the interaction of plate buckling and column buckling is also only significant for uneconomic types of cross-sections.

Imperfection shape in finite element model

The single geometric imperfection that is applied in the finite element analysis is a representation of the geometric imperfection and the residuals stresses. The NEN-EN1993-1-5 prescribes the use of the global buckling shape and the local buckling shape combined in the model. However, it has been shown that in general the local buckling shape can be made more unfavourable if the web is displaced in a single half sine wave instead of multiple half sine waves. This is not included in the NEN-EN1993-1-5. The difference is in general small ($\pm 1\%$) but can be significant for very uneconomic cross-sections. ($> 5\%$)

Probabilistic design

Using a probabilistic design method two types of cross-sections have been calculated. For an economic cross-section it has been shown that the safety margin required ($\beta = 3{,}80$) in the verification regulations is reached. ($\beta = 3{,}85$) For an uneconomic cross-section the safety margin required is not reached. ($\beta = 3{,}03$) As stated before, the results should not be seen as exact answers but the difference is striking. Only two cross-sections have been examined so no general conclusions can be drawn from this analysis but apparently the safety is lower for uneconomic types of cross-sections.

I-beam

Not only I-columns have been studied. Also I-beams under uniform bending moment have been studied. The results for those I-beams is very similar to the I-columns.

- The finite element analysis results are slightly higher compared to the NEN-EN1993-1-1 verification.
- For the theoretical buckling load there is only interaction when the number of half sine waves in the web is equal to the entire beam.
- The theoretical buckling load is only significantly lower when the web is large compared to the flanges.
- The finite element results for an increasing thickness results in a reducing load bearing capacity compared to the NEN-EN1993-1-1 verification.

Recommendations

Validate conclusions

The most important recommendation is that experimental test results are needed to validate the conclusions drawn before. The conclusions before are gained from results of a fairly simple finite element model and analytical calculations. The finite element model and the analytical calculations are consistent with each other and therefore the conclusions do seem reasonable. However, to indicate whether the verification of these uneconomic type of cross-sections has indeed a lower safety margin cannot be concluded based on only finite element calculations. These test results could be used to set boundaries for the validity of the buckling curves. Therefore experiments are necessary and this would be suitable for another master thesis.

Also these results could be studied in a more advanced finite element model. The model used does not include the residual stresses and geometrical imperfections as they really are but uses a single geometric imperfection to replace the effects of those. Also a bilinear stress-strain model has been used which could be refined by the use of the real stress-strain curve. It is perhaps possible to validate such a model against existing test results and apply this also to the more uneconomic cross-sections.

Design recommendation for structural engineer

For the structural engineer it is recommended to use the verification method provided in the NEN-EN1993-1-5 using the effective cross-section method. The finite element analysis shows that the results are closely related and the NEN-EN1993-1-5 is based upon experimental results. However, it is recommended to be wary of the assumptions done. Therefore some general remarks are made which could be taken into consideration. For more information one is recommended to read Chapter 3 of this thesis and other resources.

- The effective cross-section method assumes a parallel system which means an accumulation of the individual bearing capacities. The reduced stress method assumes a serial system which means that the weakest link is governing.
- For very uneconomic cross-sections the conclusions drawn before should be taken into account. In general one is advised to optimize the cross-section such that the effects of the uneconomic cross-section are removed. However, if this is not possible, it is recommended to reduce the design resistance.
- For the effective cross-section method stiffeners to the web are not very useful for the static strength against axial compression and bending moments. In general the web is slender and a stiffener only increases the area of the web that is effective. For axial compression this is usually a small increase of the area and for bending moments the material is close to the centre of gravity resulting in a small increase of the effective section modulus.
- It would be much more useful to apply larger flanges because the costs of adding a stiffener are in general high in developed countries. The size of the web is only decided by the fatigue loading, the shear force, the shear plate buckling stability and the possibility of flange-induced buckling.

Future research

There are several subjects which would be very interesting relating to the subject of plate buckling. Some possibilities are mentioned here.

- A very interesting and challenging subject is the interaction of plate buckling and fatigue damage. This phenomena is called web breathing and a requirement is made in the EN1993-2 which is the verification of steel bridges. The general problem is that if a web often buckles and returns to its original shape secondary bending stresses occur that cause fatigue damage.
    - The verification regulation states that if the web is below a certain slenderness no additional requirements are necessary. If the slenderness of the web is above the limit, a verification has to be done using the reduced stress method. This is a sharp boundary which could be refined. This is also only mentioned in the verification of steel bridges. For other types of variable loading this is

not referred to. So the question also is, how many repetitive cycles is variable loading.
  - Also it states that the limiting slenderness is also valid for individual sub-panels of stiffened panels. There are no requirements made for the stiffeners of the web so an extremely small stiffener would be sufficient to prevent web breathing according to the verification codes. This is not in accordance with proper engineering judgement and regulations could be developed to derive a minimum size of the stiffener.
- The imperfections could be examined much more accurately.
  - First of all one could study whether the prescribed values in the NEN-EN1993-1-5 for the single geometric imperfection are accurate. This value represents the residual stresses and real geometric imperfections and could possibly be refined.
  - Secondly, the sensitivity to the real geometric imperfections could be studied. There are some situations where the geometric imperfection is larger than the prescribed value that results from the fabrication tolerances. These should be measured in practise. This may for example be the case in closed box girders that are subject to overpressure on the inside. Question is whether this significantly lowers the bearing capacity of the box girder.

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Annex A: Derivation theoretical plate buckling load

In Chapter 2 the theoretical plate buckling load is given. The derivation of this buckling load is presented here. This is a modification of the derivation as it is presented in (Abspoel & Bijlaard, 2005). An energy method is applied to calculate the buckling behaviour.

The aim is to determine the buckling load of a rectangular plate under a uniform loading from one side as presented in Figure 84.

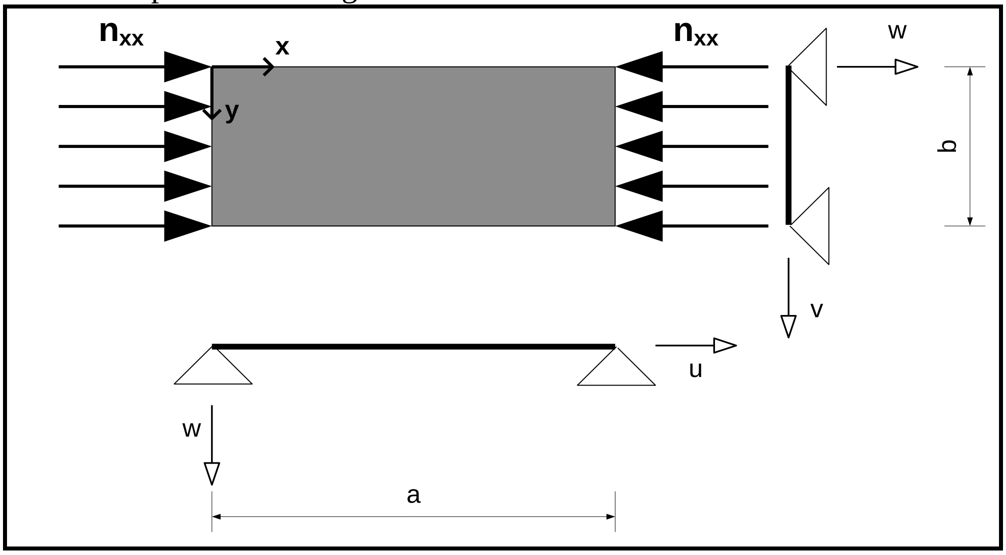


Figure 84: Description of situation for plate buckling

In a flow chart (Figure 85) the steps are given that will result in the buckling load.

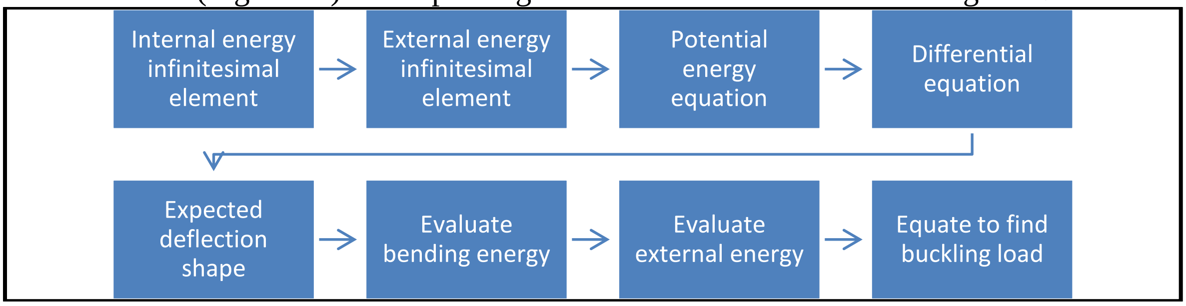


Figure 85: Derivation of the plate buckling load

Internal energy

First of all the internal energy due to bending strains of a plate needs to be determined. This is caused by bending only. Shear strains and axial strains are neglected in this analysis because they are small compared to the bending strains. For this an infinitesimal element of the plate of size dx and dy is considered.

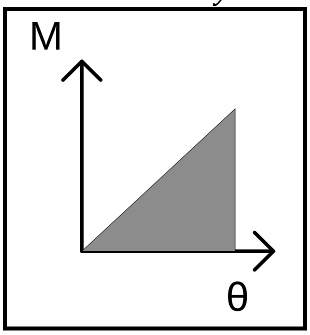


Figure 86: Bending energy of a plate (shaded area)

The bending energy due to a bending moment is given by the shaded area in Figure 86:

$$U = \frac{1}{2} * M * \theta \tag{14.1}$$

Bending in x-direction

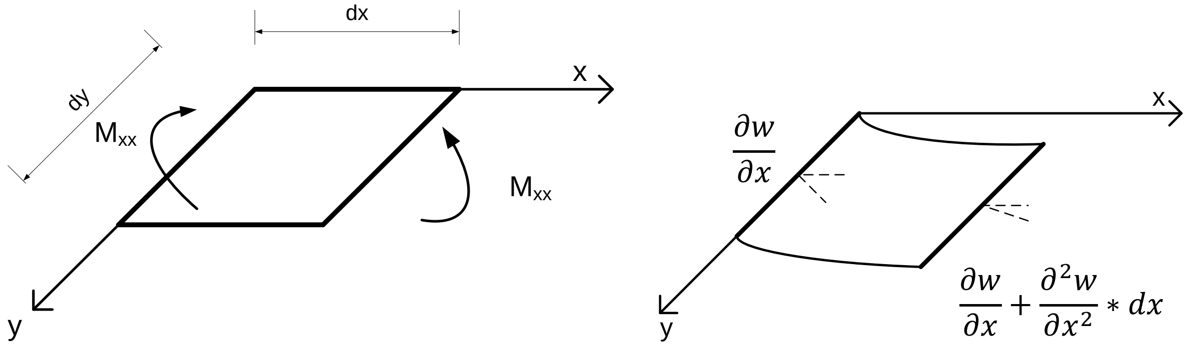


Figure 87: Bending of a plate in x-direction

The bending energy due to bending of the infinitesimal element is:

$$dU_{xx} = \frac{1}{2} * M_{xx} * dy * \left(\frac{\partial w}{\partial x} + \frac{\partial^2 w}{\partial x^2} * dx - \frac{\partial w}{\partial x}\right)$$
$$= \frac{1}{2} * M_{xx} * \frac{\partial^2 w}{\partial x^2} * dy * dx \tag{14.2}$$

Bending in y-direction

The same reasoning holds for the y-direction.

$$dU_{yy} = \frac{1}{2} * M_{yy} * \frac{\partial^2 w}{\partial y^2} * dx * dy \tag{14.3}$$

Bending in xy-direction

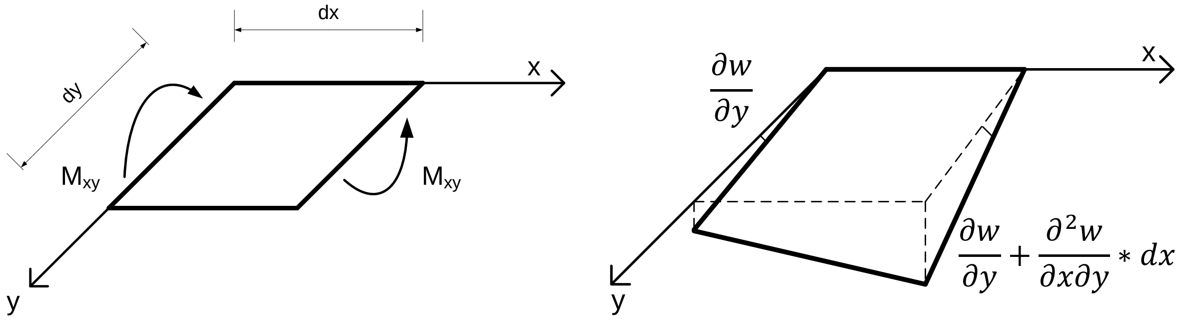


Figure 88: Twisting of a plate

The bending energy due to bending of the infinitesimal element is:

$$dU_{xy} = \frac{1}{2} * M_{xy} * dy * \left(\frac{\partial w}{\partial y} + \frac{\partial^2 w}{\partial x \partial y} * dx - \frac{\partial w}{\partial y}\right)$$
$$+ \frac{1}{2} * M_{xy} * dx * \left(\frac{\partial w}{\partial x} + \frac{\partial^2 w}{\partial x \partial y} * dy - \frac{\partial w}{\partial x}\right) \tag{14.4}$$

The evaluation of equation (14.4) results in:

$$dU_{xy} = 2 * \frac{1}{2} * M_{xy} * \frac{\partial^2 w}{\partial x \partial y} * dy * dx \tag{14.5}$$

Total bending energy

The bending moments are related to the curvature in standard plate theory.

$$M_{xx} = D * (\kappa_{xx} + v * \kappa_{yy}) \tag{14.6}$$

$$M_{yy} = D * (\kappa_{yy} + v * \kappa_{xx}) \tag{14.7}$$

$$M_{xy} = D * (1 - v) * \kappa_{xy} \tag{14.8}$$

The formulas (14.6), (14.7) and (14.8) can be introduced in the derived formulas for the bending energy in equations (14.2), (14.3) and (14.5).

$$\begin{aligned} dU_{xx} &= \frac{1}{2} * M_{xx} * \frac{\partial^2 w}{\partial x^2} * dy * dx \\ &= \frac{1}{2} * D * \left(\frac{\partial^2 w}{\partial x^2} + v * \frac{\partial^2 w}{\partial y^2}\right) * \frac{\partial^2 w}{\partial x^2} * dx * dy \end{aligned} \tag{14.9}$$

$$\begin{aligned} dU_{yy} &= \frac{1}{2} * M_{yy} * \frac{\partial^2 w}{\partial y^2} * dy * dx \\ &= \frac{1}{2} * D * \left(\frac{\partial^2 w}{\partial y^2} + v * \frac{\partial^2 w}{\partial x^2}\right) * \frac{\partial^2 w}{\partial y^2} * dx * dy \end{aligned} \tag{14.10}$$

$$\begin{aligned} dU_{xy} &= 2 * \frac{1}{2} * M_{xy} * \frac{\partial^2 w}{\partial x \partial y} * dy * dx \\ &= 2 * \frac{1}{2} * (1 - v) * D * \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 * dx * dy \end{aligned} \tag{14.11}$$

The total internal energy is the summation of the above equations (14.9), (14.10) and (14.11) integrated over the total area of the plate.

$$U = \int_0^y \int_0^x \left(dU_{xx} + dU_{yy} + dU_{xy}\right) * dx * dy \tag{14.12}$$

The result is:

$$\begin{aligned} U = \frac{1}{2} * D * \int_0^b \int_0^a \Bigg(&\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)^2 \\ &- 2 * (1 - v) * \left(\frac{\partial^2 w}{\partial x^2} * \frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right)\Bigg) * dx * dy \end{aligned} \tag{14.13}$$

This is the total internal energy due to bending in the plate.

External energy

Now the external energy needs to be determined. This is the load multiplied by the displacement. Only loading in x-direction is present. An infinitesimal element is considered which will displace and rotate and Pythagoras theorem is applied.

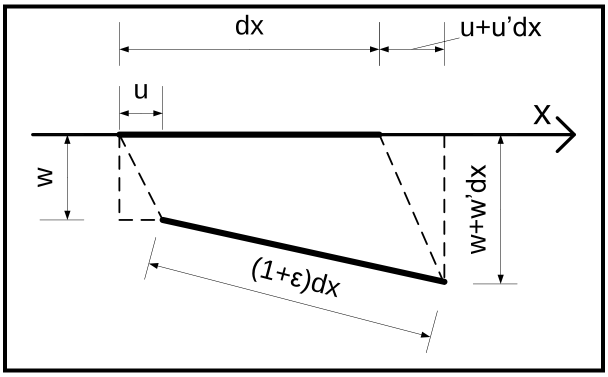
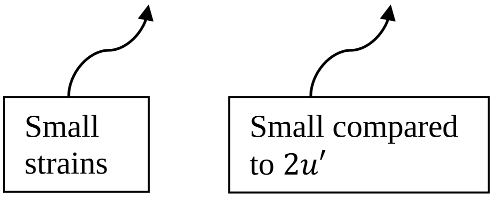


Figure 89: Rotation and elongation of an infinitesimal element

$$(1+\varepsilon)^2 * dx^2 = (w' * dx)^2 + (dx + u' * dx)^2$$
$$(1+\varepsilon)^2 = (w')^2 + (1+u')^2$$
$$1 + 2\varepsilon + \varepsilon^2 = w'^2 + 1 + 2u' + u'^2$$
$$2\varepsilon + \varepsilon^2 = w'^2 + 2u' + u'^2$$



Therefore the relation between strain and deformation of the element is:

$$\varepsilon = u' + \frac{1}{2} * w'^2 \tag{14.14}$$

The total strain of the element is considered to be negligible because the deformation will be mostly in bending.

$$\varepsilon = 0 \ \rightarrow \ u' = -\frac{1}{2} * w'^2 \tag{14.15}$$

The energy is the stress at the end multiplied by the displacement at the end. The displacement at the end is the derivative of the axial displacement integrated over the total length.

$$T = \int_0^b \int_0^a \sigma_{xx} * t * u' * dx * dy \tag{14.16}$$

Applying equation (14.15) to equation (14.16) gives:

$$T = -\frac{1}{2} * \int_0^b \int_0^a n_{xx} * \left(\frac{\partial w}{\partial x}\right)^2 * dx * dy \tag{14.17}$$

Potential energy

The sum of the internal energy and the external energy is the equation for the potential energy.

$$P = \frac{1}{2} * D * \int_0^b \int_0^a \left(\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)^2 - 2 * (1 - v) * \left(\frac{\partial^2 w}{\partial x^2} * \frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \right) \right) * dx * dy - \frac{1}{2} * \int_0^b \int_0^a n_{xx} * \left(\frac{\partial w}{\partial x} \right)^2 * dx * dy \tag{14.18}$$

Differential equation

By rewriting the potential energy equation the differential equation will be derived.

$$P = \int_0^b \int_0^a \left(\frac{1}{2} * D * \left(\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)^2 - 2 * (1 - v) * \left(\frac{\partial^2 w}{\partial x^2} * \frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \right) \right) - \frac{1}{2} * n_{xx} * \left(\frac{\partial w}{\partial x} \right)^2 \right) * dx * dy \tag{14.19}$$

The equation is slightly changed into the following:

$$P = \int_0^b \int_0^a \left(\frac{1}{2} * D * \left(\left(\frac{\partial^2 w}{\partial x^2} \right)^2 + \left(\frac{\partial^2 w}{\partial y^2} \right)^2 + 2 * v * \frac{\partial^2 w}{\partial x^2} * \frac{\partial^2 w}{\partial y^2} + 2 * \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 - 2 * v * \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \right) - \frac{1}{2} * n_{xx} * \left(\frac{\partial w}{\partial x} \right)^2 \right) * dx * dy \tag{14.20}$$

The first variation of P (δP) must be equal to zero to satisfy that the potential energy is stationary.

$$\delta P = \int_0^b \int_0^a \left(\frac{1}{2} * D * \left(2 * \frac{\partial^2 w}{\partial x^2} * \delta \frac{\partial^2 w}{\partial x^2} + 2 * \frac{\partial^2 w}{\partial y^2} * \delta \frac{\partial^2 w}{\partial y^2} + 2 * v * \frac{\partial^2 w}{\partial x^2} * \delta \frac{\partial^2 w}{\partial y^2} + 2 * v * \frac{\partial^2 w}{\partial y^2} * \delta \frac{\partial^2 w}{\partial x^2} + 2 * 2 * \frac{\partial^2 w}{\partial x \partial y} * \delta \frac{\partial^2 w}{\partial x \partial y} - 2 * v * 2 * \frac{\partial^2 w}{\partial x \partial y} * \delta \frac{\partial^2 w}{\partial x \partial y} \right) - \frac{1}{2} * 2 * n_{xx} * \frac{\partial w}{\partial x} * \delta \frac{\partial w}{\partial x} \right) * dx * dy = 0 \tag{14.21}$$

Integration by parts will remove the derivatives behind the δ-symbol.

$$\delta P = \int_0^a \int_0^b \left(\frac{1}{2} * D * \left(2 * \frac{\partial^4 w}{\partial x^4} * \delta w + 2 * \frac{\partial^4 w}{\partial y^4} * \delta w + 2 * \nu * \frac{\partial^4 w}{\partial x^2 \partial y^2} * \delta w + 2 * \nu * \frac{\partial^4 w}{\partial x^2 \partial y^2} * \delta w + 2 * 2 * \frac{\partial^4 w}{\partial x^2 \partial y^2} * \delta w - 2 * \nu * 2 * \frac{\partial^4 w}{\partial x^2 \partial y^2} * \delta w\right) + \frac{1}{2} * 2 * n_{xx} * \frac{\partial^2 w}{\partial x^2} * \delta w\right) * dy * dx = 0 \quad (14.22)$$

Evaluating this function will lead to this equation:

$$\delta P = \int_0^b \int_0^a \left(\left(D * \frac{\partial^4 w}{\partial x^4} * \delta w + 2 * D * \frac{\partial^2 w}{\partial x^2 \partial y^2} * \delta w + D * \frac{\partial^4 w}{\partial y^4} * \delta w\right) + n_{xx} * \frac{\partial^2 w}{\partial x^2} * \delta w\right) * dx * dy = 0 \quad (14.23)$$

There are two solutions to equation (14.23). The variation of w (δw) can be zero which is not interesting. Therefore the differential equation is the result.

$$D * \frac{\partial^4 w}{\partial x^4} + D * \frac{\partial^4 w}{\partial y^4} + 2 * D * \frac{\partial^2 w}{\partial x^2 \partial y^2} + n_{xx} * \frac{\partial^2 w}{\partial x^2} = 0 \quad (14.24)$$

This is not the ordinary expression for the differential equation because n_{xx} is defined as a compressive edge load. The ordinary expression uses n_{xx} as a tensile edge load which would lead to a minus in front of the loading term.

Buckling load

To derive the buckling load the equation for the potential energy is considered.

$$P = \frac{1}{2} * D * \int_0^b \int_0^a \left(\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)^2 - 2 * (1 - \nu) * \left(\frac{\partial^2 w}{\partial x^2} * \frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right)\right) * dx * dy - \frac{1}{2} * \int_0^b \int_0^a n_{xx} * \left(\frac{\partial w}{\partial x}\right)^2 * dx * dy \quad (14.25)$$

The bending energy needs to be equal to the virtual work applied. This means that $P = 0$ is the condition. The bending of the plate will be in the form of a double series of sine functions. The factor B_{ms} is a constant which is the amplitude of the sinusoidal.

$$w(x, y) = \sum_{m=1}^{\infty} \sum_{s=1}^{\infty} B_{ms} * \sin\left(\frac{m\pi x}{a}\right) * \sin\left(\frac{s\pi y}{b}\right) \quad (14.26)$$

Bending energy

First the equation for the bending energy is considered.

$$U = \frac{1}{2} * D * \int_0^b \int_0^a \left(\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)^2 - 2 * (1 - \nu) * \left(\frac{\partial^2 w}{\partial x^2} * \frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right)\right) * dx * dy \quad (14.27)$$

This equation is written in a different form which simplifies the calculation later on.

$$U = \frac{1}{2} * D \int_0^b \int_0^a \left(\left(\frac{\partial^2 w}{\partial x^2} \right)^2 + 2 * \frac{\partial^2 w}{\partial x^2} * \frac{\partial^2 w}{\partial y^2} + \left(\frac{\partial^2 w}{\partial y^2} \right)^2 - 2 * (1 - v) * \left(\frac{\partial^2 w}{\partial x^2} * \frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \right) \right) * dx * dy \tag{14.28}$$

To be able to compute the calculation of U some derivatives are needed.

$$\frac{\partial w}{\partial x} = \sum_{m=1}^{\infty} \sum_{s=1}^{\infty} B_{ms} * \frac{m\pi}{a} * \cos\left(\frac{m\pi x}{a}\right) * \sin\left(\frac{s\pi y}{b}\right) \tag{14.29}$$

$$\frac{\partial w}{\partial y} = \sum_{m=1}^{\infty} \sum_{s=1}^{\infty} B_{ms} * \frac{s\pi}{b} * \sin\left(\frac{m\pi x}{a}\right) * \cos\left(\frac{s\pi y}{b}\right) \tag{14.30}$$

$$\frac{\partial^2 w}{\partial x^2} = \sum_{m=1}^{\infty} \sum_{s=1}^{\infty} -B_{ms} * \left(\frac{m\pi}{a}\right)^2 * \sin\left(\frac{m\pi x}{a}\right) * \sin\left(\frac{s\pi y}{b}\right) \tag{14.31}$$

$$\frac{\partial^2 w}{\partial y^2} = \sum_{m=1}^{\infty} \sum_{s=1}^{\infty} -B_{ms} * \left(\frac{s\pi}{b}\right)^2 * \sin\left(\frac{m\pi x}{a}\right) * \sin\left(\frac{s\pi y}{b}\right) \tag{14.32}$$

$$\frac{\partial^2 w}{\partial x \partial y} = \sum_{m=1}^{\infty} \sum_{s=1}^{\infty} B_{ms} * \frac{m * s * \pi^2}{a * b} * \cos\left(\frac{m\pi x}{a}\right) * \cos\left(\frac{s\pi y}{b}\right) \tag{14.33}$$

These derivatives can be inserted in the equation for the bending energy.

$$U = \frac{1}{2} * D * \int_0^b \int_0^a \Bigg(\left(\sum_{m=1}^{\infty} \sum_{s=1}^{\infty} -B_{ms} * \left(\frac{m\pi}{a}\right)^2 * \sin\left(\frac{m\pi x}{a}\right) * \sin\left(\frac{s\pi y}{b}\right) \right)^2 + 2 * \left(\sum_{m=1}^{\infty} \sum_{s=1}^{\infty} -B_{ms} * \left(\frac{m\pi}{a}\right)^2 * \sin\left(\frac{m\pi x}{a}\right) * \sin\left(\frac{s\pi y}{b}\right) \right) * \left(\sum_{m=1}^{\infty} \sum_{s=1}^{\infty} -B_{ms} * \left(\frac{s\pi}{b}\right)^2 * \sin\left(\frac{m\pi x}{a}\right) * \sin\left(\frac{s\pi y}{b}\right) \right) + \left(\sum_{m=1}^{\infty} \sum_{s=1}^{\infty} -B_{ms} * \left(\frac{s\pi}{b}\right)^2 * \sin\left(\frac{m\pi x}{a}\right) * \sin\left(\frac{s\pi y}{b}\right) \right)^2 - 2 * (1-v) * \Bigg(\left(\sum_{m=1}^{\infty} \sum_{s=1}^{\infty} -B_{ms} * \left(\frac{m\pi}{a}\right)^2 * \sin\left(\frac{m\pi x}{a}\right) * \sin\left(\frac{s\pi y}{b}\right) \right) * \left(\sum_{m=1}^{\infty} \sum_{s=1}^{\infty} -B_{ms} * \left(\frac{s\pi}{b}\right)^2 * \sin\left(\frac{m\pi x}{a}\right) * \sin\left(\frac{s\pi y}{b}\right) \right) - \left(\sum_{m=1}^{\infty} \sum_{s=1}^{\infty} B_{ms} * \frac{m * s * \pi^2}{a * b} * \cos\left(\frac{m\pi x}{a}\right) * \cos\left(\frac{s\pi y}{b}\right) \right)^2 \Bigg) \Bigg) * dx * dy \tag{14.34}$$

The following mathematical equations help to solve the integral above by hand.

$$\int_0^a \sin^2\left(\frac{m * \pi * x}{a}\right) dx = \frac{1}{2} * \int_0^a \left(1 - \cos\left(\frac{2 * m * \pi * x}{a}\right)\right) dx = \frac{a}{2} \tag{14.35}$$

The following is valid for $m \neq n$.

$$\int_0^a \sin\left(\frac{m * \pi * x}{a}\right) * \sin\left(\frac{n * \pi * x}{a}\right) dx = \frac{1}{2} * \int_0^a \left(\cos\left(\frac{\pi * x}{a}(m-n)\right) - \cos\left(\frac{\pi * x}{a}(n-m)\right)\right) dx = 0 \tag{14.36}$$

Using these relations the equation for the bending energy can be rewritten.

$$U = \frac{1}{2} * D * \left(\sum_{m=1}^{\infty} \sum_{s=1}^{\infty} B_{ms}^2 * \left(\frac{m\pi}{a}\right)^4 * \frac{a}{2} * \frac{b}{2} \right.$$
$$+ 2 * \sum_{m=1}^{\infty} \sum_{s=1}^{\infty} B_{ms}^2 * \frac{m^2 * s^2 * \pi^4}{a^2 * b^2} * \frac{a}{2} * \frac{b}{2}$$
$$+ \sum_{m=1}^{\infty} \sum_{s=1}^{\infty} B_{ms}^2 * \left(\frac{s\pi}{b}\right)^4 * \frac{a}{2} * \frac{b}{2} \tag{14.37}$$
$$- 2 * (1 - v) * \left(\sum_{m=1}^{\infty} \sum_{s=1}^{\infty} B_{ms}^2 * \frac{m^2 * s^2 * \pi^4}{a^2 * b^2} * \frac{a}{2} * \frac{b}{2} \right.$$
$$\left.\left. - \sum_{m=1}^{\infty} \sum_{s=1}^{\infty} B_{ms}^2 * \frac{m^2 * s^2 * \pi^4}{a^2 * b^2} * \frac{a}{2} * \frac{b}{2} \right) \right)$$

Then U simplifies to the following:

$$U = \frac{D}{2} * \left(\sum_{m=1}^{\infty} \sum_{s=1}^{\infty} B_{ms}^2 * \left(\frac{m\pi}{a}\right)^4 * \frac{a}{2} * \frac{b}{2} \right.$$
$$+ 2 * \sum_{m=1}^{\infty} \sum_{s=1}^{\infty} B_{ms}^2 * \frac{m^2 * s^2 * \pi^4}{a^2 * b^2} * \frac{a}{2} * \frac{b}{2} \tag{14.38}$$
$$\left. + \sum_{m=1}^{\infty} \sum_{s=1}^{\infty} B_{ms}^2 * \left(\frac{s\pi}{b}\right)^4 * \frac{a}{2} * \frac{b}{2} \right)$$

The equation can be simplified into:

$$U = \frac{D}{2} * \pi^4 * \frac{a * b}{4} * \left(\sum_{m=1}^{\infty} \sum_{s=1}^{\infty} B_{ms}^2 * \left(\left(\frac{m}{a}\right)^4 + \frac{m^2 * s^2}{a^2 * b^2} + \left(\frac{s}{b}\right)^4 \right) \right) \tag{14.39}$$

Therefore U becomes:

$$U = \frac{D}{8} * \pi^4 * a * b * \left(\sum_{m=1}^{\infty} \sum_{s=1}^{\infty} B_{ms}^2 * \left(\left(\frac{m}{a}\right)^2 + \left(\frac{s}{b}\right)^2 \right)^2 \right) \tag{14.40}$$

External energy

Now the external energy equation is considered.

$$T = -\frac{1}{2} * \int_0^b \int_0^a n_{xx} * \left(\frac{\partial w}{\partial x}\right)^2 * dx * dy \tag{14.41}$$

Applying the derivative to equation (14.41) will result in:

$$T = -\frac{1}{2} * n_{xx}$$
$$* \int_0^b \int_0^a \left(\sum_{m=1}^{\infty} \sum_{s=1}^{\infty} B_{ms} * \frac{m\pi}{a} * \cos\left(\frac{m\pi x}{a}\right) * \sin\left(\frac{s\pi y}{b}\right) \right)^2 * dx * dy \tag{14.42}$$

Evaluating this integral will lead to:

$$T = -\frac{1}{2} * n_{xx} * \sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms}^2 * \frac{m^2 * \pi^2}{a^2} * \frac{a}{2} * \frac{b}{2} \tag{14.43}$$

Rewriting this equation will result into:

$$T = -\frac{\pi^2 * b}{8 * a} * n_{xx} * \sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms}^2 * m^2 \tag{14.44}$$

Combining equations

Now the stability equation ($U + T = 0$) can be solved:

$$\begin{aligned} &\frac{D * ab * \pi^4}{8} * \sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms}^2 * \left(\left(\frac{m}{a}\right)^2 + \left(\frac{s}{b}\right)^2\right)^2 \\ &- \frac{\pi^2 * b}{8 * a} * n_{xx} * \sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms}^2 * m^2 = 0 \end{aligned} \tag{14.45}$$

This results in the critical load:

$$n_{xx} = D * a^2 * \pi^2 * \frac{\sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms}^2 * \left(\left(\frac{m}{a}\right)^2 + \left(\frac{s}{b}\right)^2\right)^2}{\sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms}^2 * m^2} \tag{14.46}$$

The critical load is a minimum. The minimum of the function n_{xx} will be reached when all values of B_{ms} are taken equal to zero except for one. Therefore the critical load will be:

$$n_{xx} = \frac{D * a^2 * \pi^2}{m^2} * \left(\frac{m^2}{a^2} + \frac{s^2}{b^2}\right)^2 \tag{14.47}$$

Rewriting this equation results into the following:

$$n_{xx} = \frac{D * \pi^2}{a^2} * \left(m + \frac{a^2 * s^2}{b^2 * m}\right)^2 \tag{14.48}$$

As mentioned before, the critical load is a minimum. This will obviously be reached when $s = 1$. This means there is a single half sine wave in transverse direction.

$$n_{xx} = \frac{D * \pi^2}{a^2} * \left(m + \frac{a^2}{b^2 * m}\right)^2 \tag{14.49}$$

This equation has two parts. The first part is the column buckling load of a rectangular strip with a length of a and unit width. The load is also per unit width. The factor behind that term is the increase of the buckling load due to the supports along the length of the column.

Now the equation can be rewritten again.

$$n_{xx} = \frac{D * \pi^2}{b^2} * \left(m * \frac{b}{a} + \frac{a}{b} * \frac{1}{m}\right)^2 \tag{14.50}$$

The aspect ratio α of a plate is defined as:

$$\alpha = \frac{a}{b} \tag{14.51}$$

This results in:

$$n_{xx} = \frac{D * \pi^2}{b^2} * \left(\frac{m}{\alpha} + \frac{\alpha}{m}\right)^2 = \frac{D * \pi^2}{b^2} * k \tag{14.52}$$

In which k is the buckling factor:

$$k = \left(\frac{m}{\alpha} + \frac{\alpha}{m}\right)^2 \tag{14.53}$$

The value of m can be all real and positive integers so all buckling modes of the steel plate are given by this equation. The value of m determines the amount of half sine waves in a steel plate when it buckles. In Figure 90 the buckling mode with $m = 3$ and $\alpha = 3$ is presented.

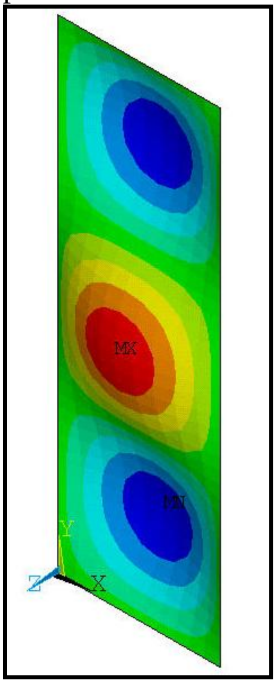


Figure 90: Plate buckling shape for $m = 3$ and $\alpha = 3$

The buckling factor is dependent on the aspect ratio of the plate and the number of half sine waves and this can be plotted in a graph as done in Figure 91.

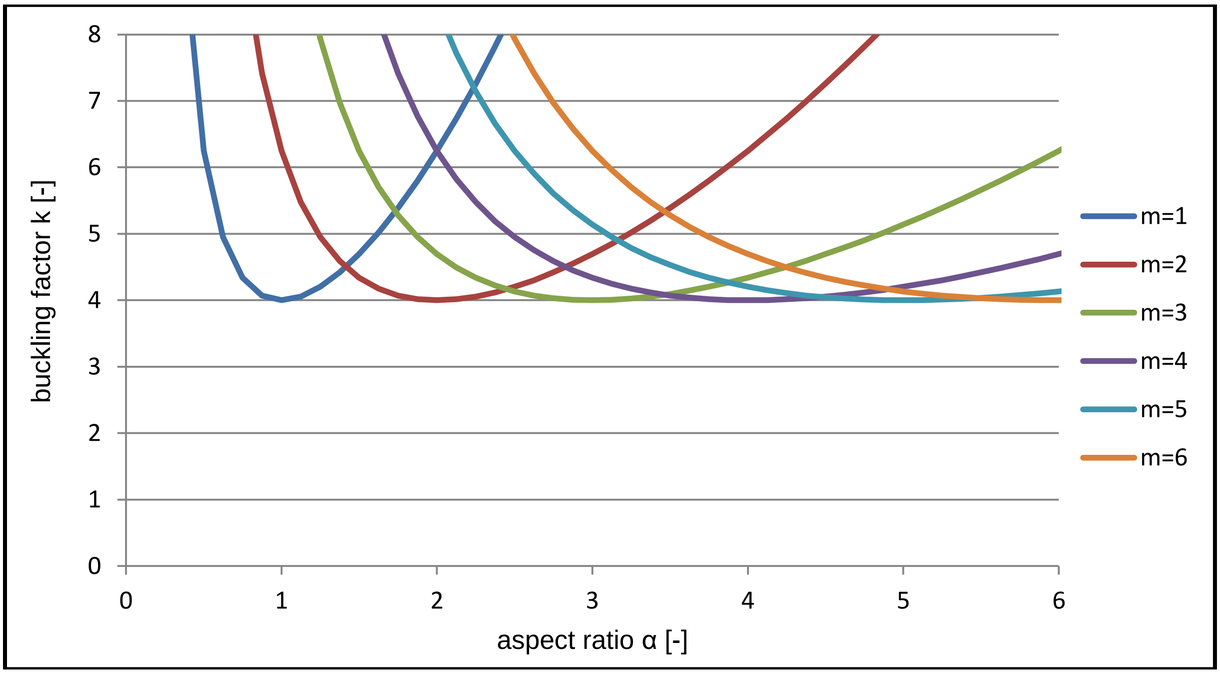


Figure 91: Buckling factor as a function of the aspect ratio for uniform compression

From Figure 91 it is clear that the buckling factor is approximately 4 except when the aspect ratio is smaller than 1 because then the buckling load is higher.

Annex B: Tool for checking class 4 I cross-sections

Guide to tool (in Dutch)

Plooicontrole volgens NEN-EN1993-1-5

Dit document beschrijft de werking van de toetsingtool voor gelaste, klasse 4 I-profielen volgens de NEN-EN1993-1-5. Voor een uitgebreide toelichting op de gebruikte norm en de daarin toegepaste principes wordt verwezen naar literatuur. (Beg, Kuhlmann, Davaine, & Braun, 2010) (Burg, 2011) (ECCS, 2007)

Deze is gebaseerd op de NEN-EN1993-1-5 en werkt op basis van de effectieve doorsnede methode. Dit betekent dat voor elk deel een effectief oppervlak wordt bepaald en op basis daarvan wordt de doorsnede getoetst als een klasse 3 profiel. Volgens art 4.3.3 en 4.3.4 mag er op basis van alleen axiale druk een effectief oppervlak worden uitgerekend en op basis van alleen een buigend moment een effectief weerstandsmoment worden uitgerekend. De verplaatsing van het zwaartepunt ten gevolge van axiale druk wordt meegenomen. Van dit principe is hier gebruik gemaakt dus de bepaling van de weerstand van de doorsnede hangt niet af van de spanningsverdeling in de ligger.

In deze berekening wordt uitgegaan van een dubbelsymmetrisch profiel waarbij de momenten om de horizontale as werken. In het geval van profielen met andere hoofdrichtingen dan de verticale en horizontale dient er te worden nagegaan in hoeverre de berekening nog correct is.

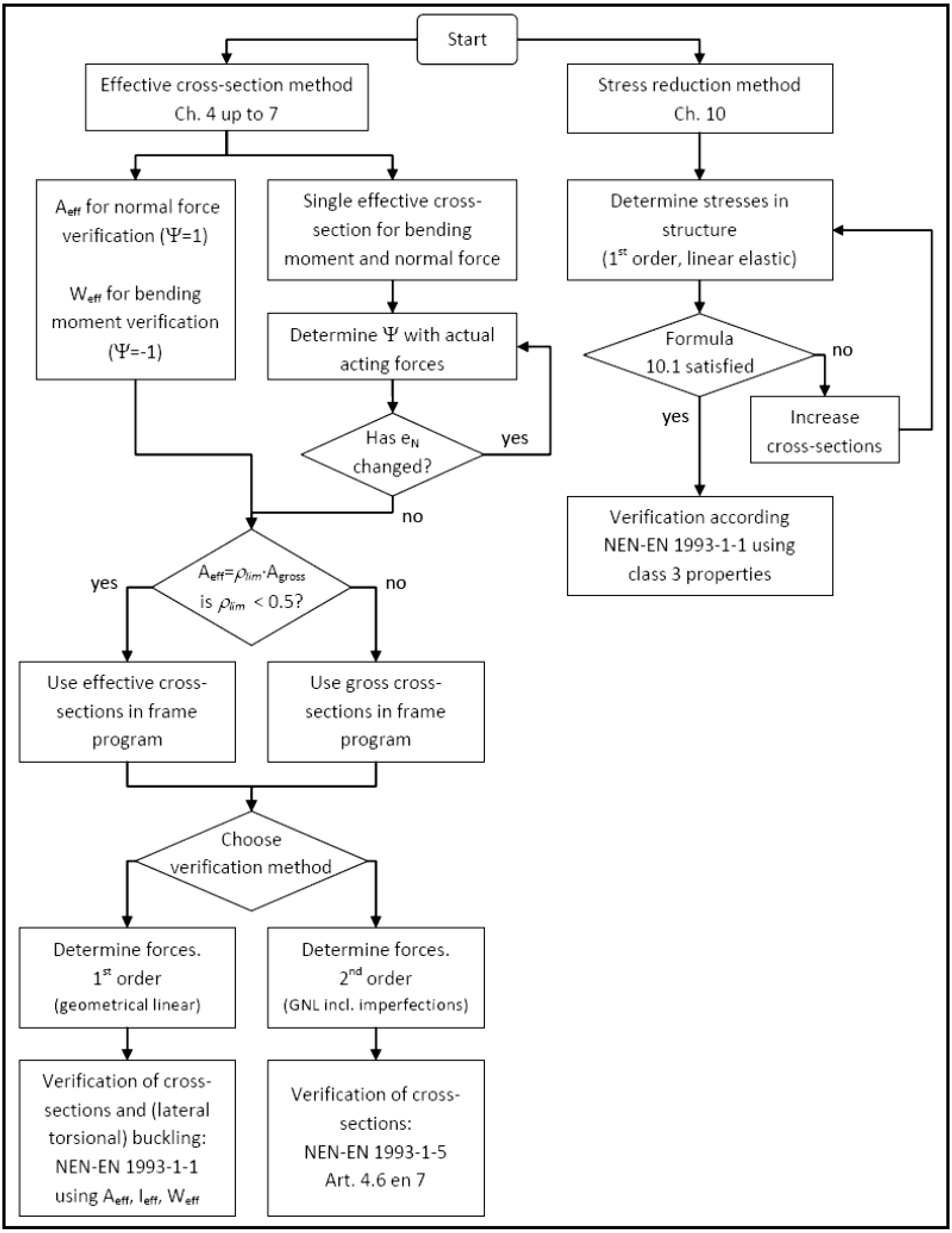


Figure 92: Proces voor het toepassen van NEN-EN1993-1-5 (Burg, 2011)

Algemene factoren
Hier worden de algemene factoren ingevoerd zoals de vloeisterkte en de elasticiteitsmodulus.

Invoer geometrie
Hier wordt de geometrie van het I-profiel ingevoerd.

Verstijvingen van het lijf
Als er geen verstijvingen aanwezig zijn, dienen de vinkjes bij die onderdelen te worden uitgezet.

Verstijvingen zijn in deze sheet rechthoekige strips. Verstijvingen van het lijf zijn alleen effectief voor plooistabiliteit van het lijf als ze in de drukzone zitten. Dit betekent dat ze niet worden meegenomen als verstijvende werking als ze in de trekzone zitten. Daarna worden ze wel meegenomen bij het bepalen van de effectieve doorsnedegrootheden. Verwaarlozen betekent in dit geval dat ze automatisch worden weggelaten voor de weerstand tegen buigende momenten als ze in de trekzone zitten.

Verstijvingen dienen altijd in dezelfde volgorde te worden ingevoerd. Dat betekent dat verstijving 1 altijd dichter bij de bovenflens zit dan verstijving 2. Als er slechts één verstijving aanwezig is, dient verstijving 1 te worden gebruikt. Uiteraard dienen de verstijvingen altijd tussen de boven- en onderflens te worden geplaatst.

Fictieve boven- en of onderplaat
Er is de mogelijkheid om een fictieve plaat in te voeren zoals bijvoorbeeld een betonnen dek. Als deze niet aanwezig is, dan dienen de vinkjes bij die onderdelen te worden uitgezet. Hiervoor dienen de oppervlakte, het eigen traagheidsmoment en de afstand van het hart van de plaat tot de bovenkant van de ligger te worden ingevuld. Dit kan ook een negatieve afstand zijn voor de bovenplaat maar de bovenplaat moet wel in de drukzone van het profiel zitten. De onderplaat dient altijd te worden ingevuld zodanig dat deze in de trekzone zit onder. Als het nodig is om twee verschillende boven- of onderplaten in te vullen dan kunnen deze handmatig worden opgeteld en dan als één fictieve boven- of onderplaat worden ingevuld.

De reductiefactoren voor axiale druk en voor momenten zijn om respectievelijk de effectieve oppervlakte en het effectieve eigen traagheidsmoment uit te rekenen.

Troggen
Troggen kunnen ook worden ingevoerd. Er is de mogelijkheid om twee bovenplaten (troggen 1 en 2) en twee onderplaten (troggen 3 en 4) in te voeren. Alleen als er trogplaten aanwezig zijn, dienen deze te worden aangevinkt. De troggen worden automatisch gereduceerd om de effectieve doorsnede te bepalen alsof ze onderworpen zijn aan axiale druk. Dit is een conservatieve benadering voor buigende momenten omdat er een klein spanningsverloop is over de hoogte van de trog. Dit wordt hier verwaarloosd maar zou handmatig kunnen worden toegevoegd als de troggen niet klein zijn ten opzichte van de totale hoogte van de ligger. Alle platen worden gereduceerd voor axiale druk en de bovenplaten voor buigende momenten.
De trog wordt alleen gereduceerd op de individuele onderdelen en het globale knik- en/of plooigedrag van de trogplaat als geheel en het effect van shear lag wordt niet berekend omdat dit afhangt van de specifieke geometrie. Dit kan de effectieve doorsnede van de trogplaat reduceren en dient dus bekeken te worden. De waarde van n geeft aan hoeveel troggen er aanwezig zijn zoals getekend.

Shear lag effect

Hier wordt shear lag berekend voor de boven- en onderflens van het profiel. Dit wordt gedaan door de breedte te reduceren. Deze dient dan handmatig opnieuw te worden ingevoerd direct onder dit blok. Shear lag wordt niet berekend voor de verstijvingen, fictieve platen en de troggen. Hier dient zelf een keuze in te worden gemaakt door deze verstandig te kiezen of door deze handmatig correct bij de geometrie in te voeren.

Bruto doorsnede eigenschappen

Hier wordt het oppervlak, zwaartepunt en traagheidsmoment van de bruto doorsnede berekend.

Reduceren doorsnede flenzen t.g.v. axiale druk

Hierbij worden de flenzen gereduceerd voor axiale druk dus elk plaatdeel wordt onder uniforme druk berekend.

Lijf (Druk)

Daarna wordt voor het lijf de reductie toegepast voor 0, 1 en 2 verstijvingen ten gevolge van axiale druk en daarbij worden ook de nieuwe oppervlaktes, zwaartepunten en traagheidsmomenten uitgerekend. Later wordt pas gekozen of er inderdaad verstijvingen aanwezig zijn. Dan wordt het juiste oppervlak, zwaartepunt en traagheidsmoment gekozen.

Reduceren doorsnede flenzen t.g.v. buigend moment

De doorsnede reduceren voor buigende momenten werkt op een andere manier. Volgens art. 4.4.3 mag dit worden gedaan door eerst de gedrukte flens te reduceren voor buigende momenten en dan vanuit de nieuwe spanningsverdeling het lijf te reduceren. Dit principe wordt hier ook toegepast.

Eerst worden de bovenflenzen, de fictieve bovenplaat en de troggen 1 en 2 gereduceerd waarbij dezelfde reductie is toegepast als voor axiale druk omdat het spanningsverloop over de hoogte minimaal is. Daarna worden er nieuwe doorsnede eigenschappen bepaald met de gereduceerde bovenflens. Dit is de basis voor de nieuwe spanningsverdeling.

Lijf (Moment)

Daarmee wordt de spanningsverdeling in het lijf uitgerekend en op basis daarvan wordt het lijf gereduceerd. De spanningsverdeling is dus niet de werkelijk aanwezige spanningsverdeling maar de spanningsverdeling ten gevolge van alleen een buigend moment. Na het reduceren van het lijf worden steeds de nieuwe oppervlaktes, zwaartepunten en traagheidsmomenten bepaald en die worden dan weer later gekozen zoals dat ook voor axiale druk werkt.

Reduceren lijf met verstijvingen

Het reduceren van het lijf met verstijvingen is een ingewikkeld principe in de NEN-EN1993-1-5. De gehele procedure wordt beschreven in annex A van NEN-EN1993-1-5. Dit werkt door eerst de individuele delen te reduceren op basis van de spanningsverdeling. Dus van alle subpanelen wordt een effectieve breedte bepaald. Daarna wordt naar het gehele plaatveld gekeken waarbij de verstijvingen in het plaatveld aanwezig zijn. Deze verstijvingen met meewerkende delen worden berekend als liggers met een verende ondersteuning van het lijf. Daar zijn interactieformules om de plaatknik van de gehele plaat te bepalen. Dit levert een reductiefactor op voor de plaat. Deze wordt toegepast op de dikte van de onderdelen die worden beïnvloed door de verstijvingen. Dit zijn dus de subpanelen en de verstijvingen zelf behalve de randen van de subpanelen die aan de flenzen zitten. Hierbij wordt er vanuit gegaan dat torsieknik van de verstijvingen volledig wordt voorkomen. Dit zal later worden getoetst.

Bij het bepalen van de weerstand van het lijf met twee verstijvingen als “lumped stiffener” wordt er vanuit gegaan dat deze zit op het zwaartepunt van de twee verstijvingen samen inclusief de meewerkende delen. Als dus een van de veldafstanden klein is ten opzichte van de andere veldafstanden kan deze aanname niet correct zijn.

Effectieve schuifkracht bepalen
De effectieve schuifkracht wordt bepaald door een reductiefactor uit te rekenen voor het lijf. Deze hangt af van de verstijvingen en wordt drie maal bepaald voor 0,1 en 2 verstijvingen. Later wordt dan weer de juiste gekozen.

Lastintroductie
Voor de lastintroductie is een aparte berekening toegevoegd en deze wordt meegenomen in de toetsing van het profiel. Als er geen sprake is van lastintroductie kan de belasting op 0 worden gezet.

ULS Toets
De ULS toets zoekt de juiste eigenschappen afhankelijk van het aantal aanwezige verstijvingen. Als er slechts 1 verstijving aanwezig is, dient altijd verstijving 1 te worden gebruikt. Als verstijving 2 wordt aangevinkt, is verstijving 1 ook altijd actief.
Hier wordt ook bepaald of een verstijving in de trekzone ligt van het profiel en als dat zo is dan wordt die verwaarloosd voor de bepaling van de weerstand tegen plooien van het lijf.

Daarna worden de interactie formules voor axiale druk, buigende momenten en schuifkracht toegepast zoals die in artikel 7.1 en 7.2 staan. Hierbij wordt in principe uitgegaan van krachten en momenten uit een tweede-orde berekening. Als een eerste-orde berekening wordt toegepast dient er te worden gerekend volgens NEN-EN1993-1-1 waarbij knik en kip berekeningen dienen te worden gedaan met de effectieve doorsnedegrootheden.

SLS toets
Bij de SLS toets (gereduceerde spanningen methode) wordt voor de individuele subpanelen van het lijf een reductiefactor uitgerekend op basis van de echte spanningen die aanwezig zijn. Dit wordt berekend conform artikel 10 uit NEN-EN1993-1-5. Daarna wordt aan de hand van die reductiefactor de vloeisterkte verlaagd en dit wordt toegepast op het bruto profiel. Deze toets zal altijd gelijk zijn of hoger uitkomen als/dan de gereduceerde doorsnede methode bij gelijke belasting. Hierbij wordt dus niet gekeken of de verstijvingen voldoen aan de eisen die nodig zijn om alleen individuele subpanelen te toetsen. Ook wordt niet bekeken of de flenzen wellicht een grotere reductiefactor hebben. Dit is dus meer ter indicatie of de individuele panelen goed zijn gekozen of dat de onderlinge afstanden beter kunnen worden aangepast.

Eigenlijk moet op basis van de geometrie van de plaat met verstijvingen een kritische knikspanning worden bepaald. Dit kan bijvoorbeeld met het programma EBPlate wat gratis beschikbaar is op internet. Aan de hand van die kritische spanning kan direct de plaat worden getoetst volgens de gereduceerde spanningen methode. Dit is niet verwerkt in de tool omdat de tool gemaakt is voor de uiterste grenstoestand. De gereduceerde spanningen toets is slechts ter indicatie van de individuele plaatdelen in het lijf.

Ook wordt hier de spanning getoetst alsof het een klasse 3 profiel is in de uiterste vezel van het profiel. Deze spanning is vooral om een indicatie te hebben of het überhaupt mogelijk is om de doorsnede te laten voldoen met de huidige geometrie uitgaande van een volledig effectieve doorsnede. Hierbij wordt de schuifspanning in het lijf opgeteld omdat de axiale spanning in het lijf nauwelijks kleiner is dan de axiale spanning in de uiterste vezel. Dit is dus een conservatieve benadering.

Flange induced buckling
Het buigen van de ligger als geheel geeft een verticale kracht vanuit de flens op het lijf. Deze spanning kan leiden tot het plooien van het lijf door die kracht. Daarom wordt er een minimale eis gesteld aan het profiel. Daarbij wordt uitgegaan van vier aannames:

- Symmetrische doorsnede. (Als de flenzen niet even groot zijn, wordt degene met het grootste oppervlak gekozen. Hiermee wordt de berekening niet geheel correct aangezien deze ook van invloed zijn op de kromming.)
- Maximaal 50% van de vloeisterkte als residuele spanning in de flenzen.
- Beide flenzen vloeien volledig op het moment dat knik optreedt.
- Weerstand tegen buiging wordt elastisch berekend.

Wanneer er verstijvingen zijn gelast aan het lijf is deze berekening niet correct maar wel conservatief.

Torsiestijfheid van langsverstijvingen
Langsverstijvingen zijn volledig meegenomen in het bepalen van de effectieve doorsnede en hoeven niet apart te worden uitgerekend voor de gegeven belasting. Bij deze berekening is er wel van uitgegaan dat torsieknik niet kan optreden. Voor een strip als verstijving is er een eenvoudige formule die de kritische torsieknikspanning bepaald. Als deze formule niet voldoet, kan er voor worden gekozen om niet de vloeispanning als grens aan te houden maar de actueel optredende spanning. (Beg, Kuhlmann, Davaine, & Braun, 2010, p. 139) Deze wordt ook bepaald. De actuele spanning is hier met gebruik van de effectieve doorsnede omdat dit ook de methode is die wordt gebruikt voor de berekening volgens de NEN-EN1993-1-5.

Opmerkingen
- Verstijvingen van het lijf dienen daadwerkelijk verstijvingen te zijn en niet te zware platen die een significante bijdrage leveren aan het constructief draagvermogen. Dit komt omdat ze als dunwandige onderdelen worden uitgerekend.
- Dwarsverstijvingen worden niet getoetst.

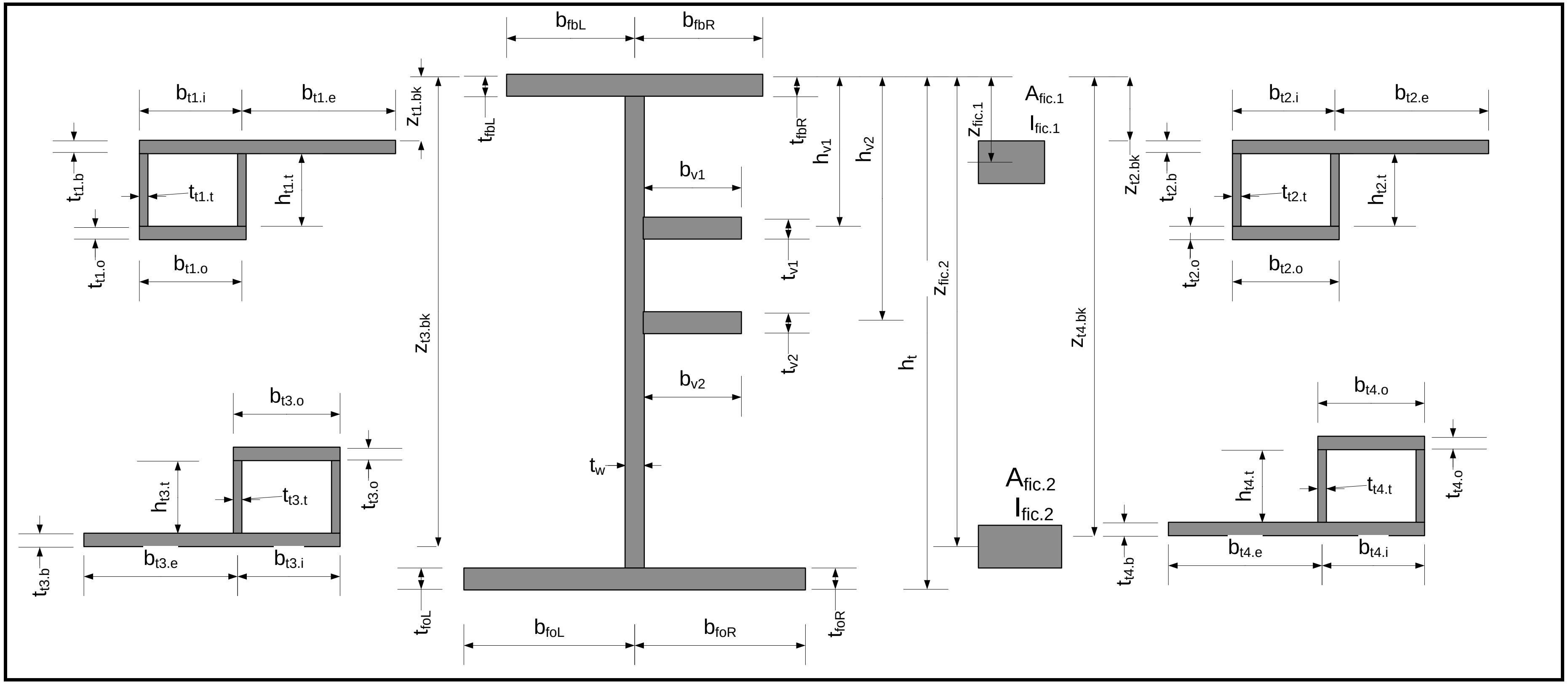


Figure 93: Dimensions of the tool for checking class 4 cross-sections

Tool

The calculation tool is in total approximately 75 pages so this tool is not included here but is included at the end in Annex J.

Validation of tool

The tool is able to verify class 4 I cross-sections according to the verification rules given in NEN-EN1993-1-5. The tool contains approximately 75 pages and is difficult to check entirely. There is always a risk of interpretation errors, certain configurations that do not fit the assumptions, typing errors or errors in the NEN-EN1993-1-5. No guarantees can be given that these errors are removed but some checks are done to validate the tool. Ansys is used to calculate a certain configuration with and without a stiffener included. The results according to Ansys can be compared to the calculations according to the NEN-EN1993-1-5.

Validation of compressed plate with two stiffeners

In literature (Beg, Kuhlmann, Davaine, & Braun, 2010, pp. 68-71) calculations have been performed on a plate with two stiffeners in uniform compression. The dimensions of the plate are 1800x12 mm and the stiffeners are at equivalent distances of 600 mm and the dimension of the stiffeners is 100x10 mm.

In literature the critical plate buckling stress is computed of the global plate. This is done according to the NEN-EN1993-1-5. The tool also computes these stresses and the results should be equal.

The critical plate buckling stress of the plate for one stiffener is computed at:

$$\sigma_{cr,p}{}^{I} = 322\ N/mm^2$$

The critical plate buckling stress of the lumped stiffener is computed at:

$$\sigma_{cr,p}{}^{lumped} = 290\ N/mm^2$$

These stresses are also computed using the tool and the result for one stiffener is:

$$\sigma_{cr,p,tool}{}^{I} = 323\ N/mm^2$$

The result for the lumped stiffener is:

$$\sigma_{cr,p,tool}{}^{lumped} = 292\ N/mm^2$$

The results are almost equal because in the tool the assumption is made that the stiffener is a thin-walled component. This assumption is also clearly stated in the guide so the user is aware of the assumption.

I-column without stiffener

This is a tool based on cross-sectional level and only takes into account plate buckling. Therefore also a hand calculation needs to be included to calculate the column buckling behaviour according to the NEN-EN1993-1-5. The chosen cross-section has a web of 1500x10 mm, flanges of 500x50 mm and a length of 10000 mm. The stiffener is a rectangular strip of 80x6 mm. The strip is located at 750 mm of the compressed flange. The yield stress is $235\ \frac{N}{mm^2}$.

The calculation excluding a stiffener is as given in Chapter 5 so only the numerical results are given here.

The critical buckling stress for the web is:

$$\sigma_{cr} = 4 * \frac{\pi^2 * E}{12 * (1 - v^2)} * \left(\frac{t}{b}\right)^2 = 33{,}74 \, \frac{N}{mm^2} \tag{15.1}$$

Therefore the relative slenderness for plate buckling in the web is:

$$\lambda_p = \sqrt{\frac{f_y}{\sigma_{cr}}} = 2{,}64 \tag{15.2}$$

The reduction factor for plate buckling of the web is:

$$\rho = \frac{\lambda_p - 0{,}055 * (3 + 1)}{\lambda_p^2} = 0{,}347 \tag{15.3}$$

Therefore the effective area of the I-column is the following:

$$A_{eff} = 2 * h_f * t_f + \rho * b * t_w = 55210 \, mm^2 \tag{15.4}$$

This effective area is also calculated using the tool and this results in 55193 mm^2. The small difference is caused by the effect that the calculation above uses the assumption of thin-walled cross-sections. This assumption is not included in the tool.
According to equation 6.49 from NEN-EN1993-1-1 the relative slenderness of the I-column is:

$$\lambda = \sqrt{\frac{A_{eff} * f_y}{N_{cr}}} = 0{,}775 \tag{15.5}$$

The critical buckling load is calculated using the gross cross-section and is given by:

$$N_{cr} = \frac{\pi^2 * E * I}{a^2} = 21592 \, kN \tag{15.6}$$

Using the relative slenderness the reduction factor can be calculated using the following equation:

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \lambda^2}} = 0{,}678 \tag{15.7}$$

In which Φ is:

$$\Phi = 0{,}5 * (1 + \alpha * (\lambda - 0{,}2) + \lambda^2) = 0{,}941 \tag{15.8}$$

The design load therefore is:

$$N_{b,Rd} = \frac{\chi * A_{eff} * f_y}{\gamma_{M1}} = 8793 \, kN \tag{15.9}$$

The calculation has also been performed using Ansys and this results in:

$$N_{d,ansys} = 7957 \, kN \tag{15.10}$$

The difference is therefore 9,6%. This is an acceptable difference as explained in Chapter 7.

I-column with one stiffener

The calculation including one stiffener is as given in NEN-EN1993-1-5 so only the numerical results are given here. The calculation of the effective area is done using the tool and results in:

$$A_{eff} = 57709 \, mm^2 \tag{15.11}$$

According to equation 6.49 from NEN-EN1993-1-1 the relative slenderness of the I-column is:

$$\lambda = \sqrt{\frac{A_{eff} * f_y}{N_{cr}}} = 0{,}793 \tag{15.12}$$

The critical buckling load is calculated using the gross cross-section and is given by:

$$N_{cr} = \frac{\pi^2 * E * I}{a^2} = 21592\ kN \tag{15.13}$$

Using the relative slenderness the reduction factor can be calculated using the following equation:

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \lambda^2}} = 0{,}666 \tag{15.14}$$

In which Φ is:

$$\Phi = 0{,}5 * (1 + \alpha * (\lambda - 0{,}2) + \lambda^2) = 0{,}960 \tag{15.15}$$

The design load therefore is:

$$N_{b,Rd} = \frac{\chi * A_{eff} * f_y}{\gamma_{M1}} = 9032\ kN \tag{15.16}$$

The calculation has also been performed using Ansys. The imperfections are the same as for the case without any stiffeners. This results in:

$$N_{d,ansys} = 8044\ kN \tag{15.17}$$

The difference is therefore 10,9%. This difference is slightly larger so the increase of capacity from 0 to 1 stiffener is smaller in Ansys than in the NEN-EN1993-1-5.

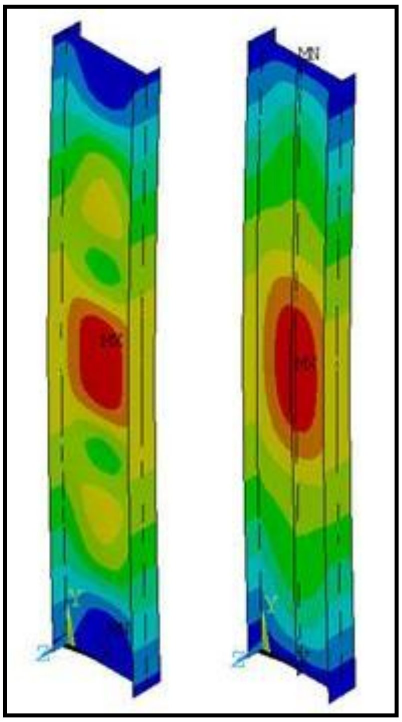


Figure 94: Out of plane displacement at maximum loading. (left without stiffener, right with stiffener)

The out of plane displacement at the maximum load is displayed in Figure 94. It is clear that without any stiffeners the web will show buckles as predicted by the analytical calculation. Even a small stiffener of 80x6 mm will prevent these buckles and forces the web to participate in the load carrying behaviour. However, the increase of load bearing capacity is only from 8793 kN to 9032 kN. This is an increase of 2,7% which is very small. The additional amount of costs is probably not acceptable given the small increase of capacity. One should therefore be alert when adding stiffeners and also consider the option of an increased thickness of the web or of the flanges.

Only when plate buckling is not allowed, the use of stiffeners is probably useful. This is only useful when the post-buckling capacity may not be explored. Relevant examples of these kind of situations are fatigue loading, aesthetic reasons or large temperature differences.

I-beam without stiffener

For the beam another cross-section is used with a thinner web. The chosen cross-section has a web of 1500x8 mm, flanges of 500x50 mm and a length of 10000 mm. The stiffener is a rectangular strip of 80x6 mm. The strip is located at 450 mm of the compressed flange. The yield stress is $235\ \frac{N}{mm^2}$.

The calculation excluding a stiffener is as given in Chapter 10 so only the numerical results are given here.

The critical buckling stress for the web is:

$$\sigma_{cr} = 23{,}9 * \frac{\pi^2 * E}{12 * (1 - v^2)} * \left(\frac{t}{b}\right)^2 = 129{,}0\ \frac{N}{mm^2} \qquad (15.18)$$

Therefore the relative slenderness for plate buckling in the web is:

$$\lambda_p = \sqrt{\frac{f_y}{\sigma_{cr}}} = 1{,}35 \qquad (15.19)$$

The reduction factor for plate buckling of the web is:

$$\rho = \frac{\lambda_p - 0{,}055 * (3 - 1)}{\lambda_p^2} = 0{,}68 \qquad (15.20)$$

Using the effective cross-section (Figure 95) the effective moment of inertia can be calculated. This formula is not given here. Then the effective section modulus can be calculated which is needed for the resistance of the beam.

$$I_{yy,eff} = 3{,}00 * 10^{10}\ mm^4 \qquad (15.21)$$

This effective moment of inertia is also calculated using the tool and this results in $2{,}98 * 10^{10}\ mm^4$. The small difference is caused by the effect that the calculation above uses the assumption of thin-walled cross-sections. This assumption is not included in the tool.

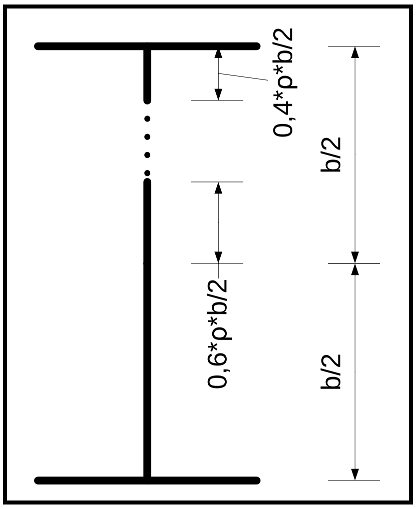


Figure 95: Effective cross-section for I-beam under uniform bending moment

According to equation 6.49 from NEN-EN1993-1-1 the relative slenderness of the I-beam is:

$$\lambda_{LT} = \sqrt{\frac{W_{eff} * f_y}{M_{cr}}} = 0{,}71 \quad (15.22)$$

The critical buckling load is given by:

$$M_{cr} = \frac{\pi}{a} * \sqrt{EI_{zz} * \left(S_t + E * C_w * \left(\frac{\pi}{a}\right)^2\right)} = 1{,}83 * 10^{10}\ Nmm \quad (15.23)$$

In the critical buckling load the warping term C_w is often neglected. This is not applied here because it is also not neglected in the real behaviour and in the FEM model later on. The method for rolled cross-sections and equivalent welded cross-sections is used as it is presented in NEN-EN1993-1-1 article 6.3.2.3. In this validation of the tool the correct buckling curve (d) is used in contradiction with the rest of this thesis where for all calculations buckling curve c is used for comparative reasons.

Using the relative slenderness the reduction factor can be calculated using the following equation:

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{{\Phi_{LT}}^2 - 0{,}75 * {\lambda_{LT}}^2}} = 0{,}752 \quad (15.24)$$

In which Φ_{LT} is:

$$\Phi_{LT} = 0{,}5 * \left(1 + \alpha * (\lambda_{LT} - 0{,}4) + 0{,}75 * {\lambda_{LT}}^2\right) = 0{,}807 \quad (15.25)$$

The design load therefore is:

$$M_{b,Rd} = \frac{\chi_{LT} * W_{eff} * f_y}{\gamma_{M1}} = 6948\ kNm \quad (15.26)$$

The calculation has also been performed using Ansys. The imperfections are the same as for the case without any stiffeners. This results in:

$$M_{d,ansys} = 7700\ kNm \quad (15.27)$$

The difference is therefore 10,8%. This is an acceptable difference as explained in Chapter 12.

I-beam with one stiffener

The calculation including one stiffener is as given in NEN-EN1993-1-5 so only the numerical results are given here. The calculation of the effective moment of inertia is done using the tool and results in:

$$I_{yy,eff} = 3{,}02 * 10^{10}\ mm^4 \tag{15.28}$$

According to equation 6.49 from NEN-EN1993-1-1 the relative slenderness of the I-beam is:

$$\lambda_{LT} = \sqrt{\frac{W_{eff} * f_y}{M_{cr}}} = 0{,}71 \tag{15.29}$$

The critical buckling load is given by:

$$M_{cr} = \frac{\pi}{a} * \sqrt{EI_{zz} * \left(S_t + E * C_w * \left(\frac{\pi}{a}\right)^2\right)} = 1{,}83 * 10^{10}\ Nmm \tag{15.30}$$

In the critical buckling load the warping term C_w is often neglected. This is not applied here because it is also not neglected in the real behaviour and in the FEM model later on. The method for rolled cross-sections and equivalent welded cross-sections is used as it is presented in NEN-EN1993-1-1 article 6.3.2.3. In this validation of the tool the correct buckling curve (d) is used in contradiction with the rest of this thesis where for all calculations buckling curve c is used for comparative reasons.

Using the relative slenderness the reduction factor can be calculated using the following equation:

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}{}^2 - 0{,}75 * \lambda_{LT}{}^2}} = 0{,}752 \tag{15.31}$$

In which Φ_{LT} is:

$$\Phi_{LT} = 0{,}5 * \left(1 + \alpha * (\lambda_{LT} - 0{,}4) + 0{,}75 * \lambda_{LT}{}^2\right) = 0{,}807 \tag{15.32}$$

The design load therefore is:

$$M_{b,Rd} = \frac{\chi_{LT} * W_{eff} * f_y}{\gamma_{M1}} = 6998\ kNm \tag{15.33}$$

The calculation has also been performed using Ansys. The imperfections are the same as for the case without any stiffeners. This results in:

$$N_{d,ansys} = 7800\ kN \tag{15.34}$$

The difference is therefore 11,5%. This difference is slightly larger so the increase of capacity from 0 to 1 stiffener is larger in Ansys than in the NEN-EN1993-1-5.

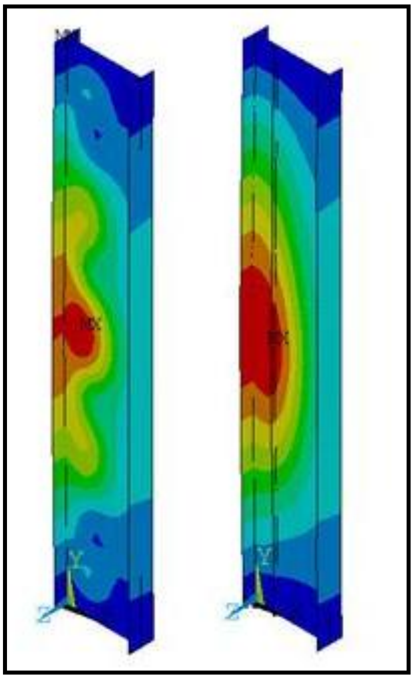


Figure 96: Out of plane displacement at maximum loading. (left without stiffener, right with stiffener)

The out of plane displacement at the maximum load is displayed in Figure 96. It is clear that without any stiffeners the web will show buckles as predicted by the analytical calculation. Even a small stiffener of 80x6 mm will prevent these buckles and forces the web to participate in the load carrying behaviour. However, the increase of load bearing capacity is only from $6948\ kNm$ to $6998\ kNm$. This is an increase of 0,7% which is very small. The additional amount of costs is probably not acceptable given the small increase of capacity. One should therefore be alert when adding stiffeners and also consider the option of an increased thickness of the web or of the flanges.

Only when plate buckling is not allowed, the use of stiffeners is probably useful. This is only useful when the post-buckling capacity may not be explored. Relevant examples of these kind of situations are fatigue loading, aesthetic reasons or large temperature differences.

Annex C: Analytical calculation of I-column under uniform compression without rotational restraint

Stability of a welded I-column under uniform compression

A loss of stability of a welded I-column may occur in multiple ways. Local buckling may be the governing mechanism which creates multiple half sine waves in the web. If local buckling occurs the flanges are stiff enough to support the web. Local buckling is not a failure of the entire structure. Global buckling of the entire column may be governing which would create a single half sine wave in the weakest direction of the column. If global column buckling occurs the flanges are not stiff enough to support the web. Global buckling is a failure mechanism. However, also an intermediate form may occur which is an interaction between both forms. The flanges are quite stiff but do not fully prevent local buckling.

This interaction is especially important for fatigue loaded structures. If a structure is loaded in fatigue the post plate buckling strength may not be utilized so the strength is governed by the lowest of the plate buckling strength and the column buckling strength. When either one is much higher than the other, the lowest will be the governing mechanism. However, if they are quite close to each other, there may be some interaction which is possibly unfavourable.

The calculation of the critical buckling stress of the welded I-column is equivalent to the calculation made earlier for the plate simply supported along all four edges. Except this time the local buckling behaviour is combined with the global buckling behaviour.

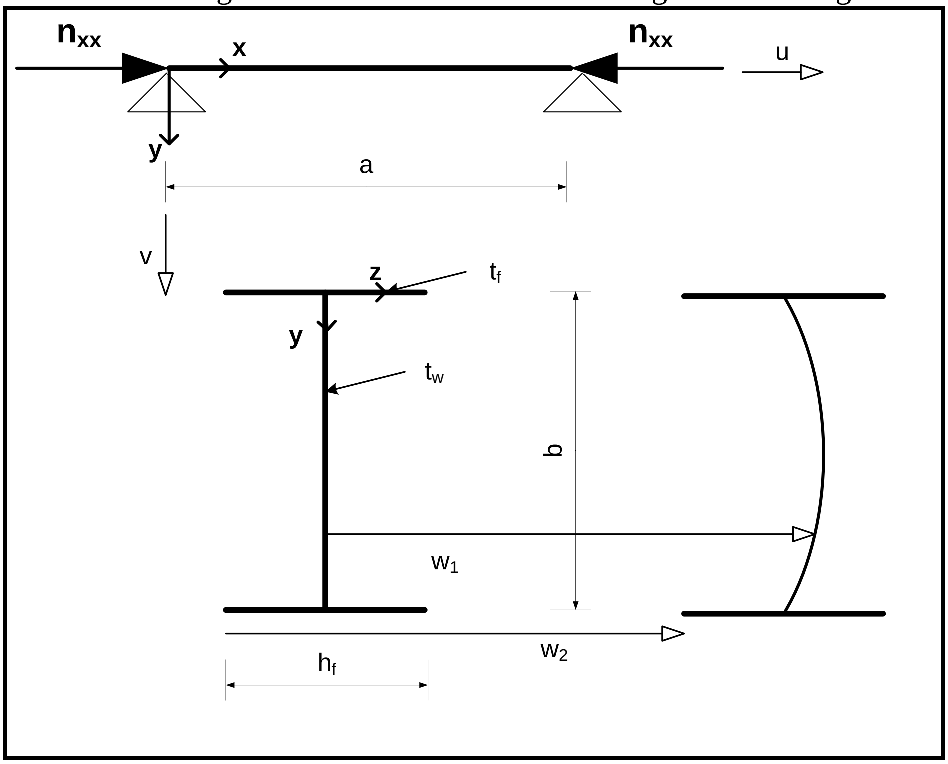


Figure 97: Description of situation for uniform compression of welded I-column

The process to derive the buckling load is given in Figure 23.

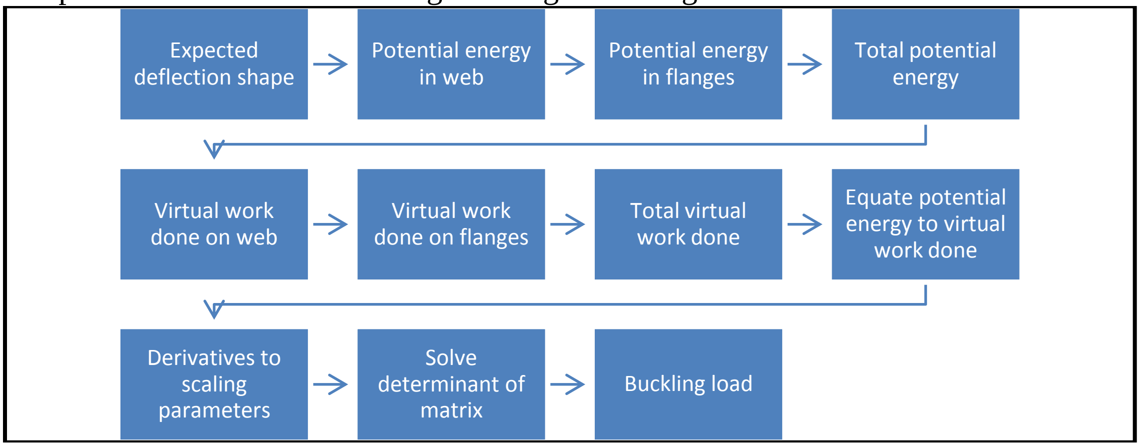


Figure 98: Flow chart to derive the buckling load of an I-column

Deflection shape as a sum of global and plate buckling behaviour

The deformation shape of the column may be described in the following way. For the global behaviour a series of half sines is used and for the local behaviour a series of half sines multiplied by another series of half sines in transverse direction is used. These deformations are the ones of the individual buckling shapes of the global and the local behaviour. The deformation of the web (w_1) is a summation of the local and the global behaviour. The deformation of the flange (w_2) is described by the global behaviour. At $y = 0$ and $y = b$ the web and the flange have the same deformation for all x so compatibility between the web and the flange is ensured.

Therefore the deformation shape is given in Figure 99.

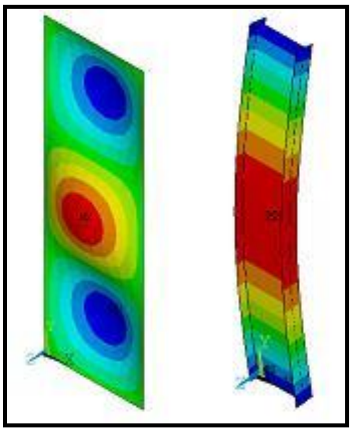


Figure 99: The local behaviour ($m = 3, s = 1$) (left) and the global behaviour ($n = 1$) (right) (displacement out of plane plotted)

The deformation shape of the web is:

$$w_1 = \sum_{n=1}^{\infty} A_n * \sin\left(\frac{n\pi x}{a}\right) + \sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms} * \sin\left(\frac{m\pi x}{a}\right) * \sin\left(\frac{s\pi y}{b}\right) \tag{16.1}$$

The deformation shape of the flanges is:

$$w_2 = \sum_{n=1}^{\infty} A_n * \sin\left(\frac{n\pi x}{a}\right) \tag{16.2}$$

Flanges normally have a moment-resistant connection to the flanges. However, the flanges have a very small resistance to rotation for smaller flanges which are important for this thesis.

Timoshenko (1963) described the buckling behaviour of columns and plates separately using an energy method. The strain energy in the construction needs to be smaller than the virtual work done by the external forces to ensure that the construction is stable. This is again similar to the applied derivation of the plate buckling load earlier. However, this thesis combines the effects of local and global buckling.

Determine potential energy in the web

The potential energy in a plate can be described using the following equation.

$$U_1 = \frac{1}{2} * D \int_0^b \int_0^a \left(\left(\frac{\partial^2 w_1}{\partial x^2} + \frac{\partial^2 w_1}{\partial y^2} \right)^2 - 2 * (1 - \upsilon) * \left(\frac{\partial^2 w_1}{\partial x^2} * \frac{\partial^2 w_1}{\partial y^2} - \left(\frac{\partial^2 w_1}{\partial x \partial y} \right)^2 \right) \right) * dx * dy \tag{16.3}$$

Equation (16.3) can be written in a more convenient form to simplify calculations later on.

$$U_1 = \frac{1}{2} * D \int_0^b \int_0^a \left(\left(\frac{\partial^2 w_1}{\partial x^2} \right)^2 + \left(\frac{\partial^2 w_1}{\partial y^2} \right)^2 + 2 * (1 - \upsilon) * \left(\frac{\partial^2 w_1}{\partial x \partial y} \right)^2 + 2 * \upsilon * \frac{\partial^2 w_1}{\partial x^2} * \frac{\partial^2 w_1}{\partial y^2} \right) * dx * dy \tag{16.4}$$

First some derivatives are computed to be able to calculate the potential energy.

$$\frac{\partial^2 w_1}{\partial x^2} = -\sum_{n=1}^{\infty} A_n * \left(\frac{n\pi}{a} \right)^2 * \sin\left(\frac{n\pi x}{a} \right) - \sum_{m=1}^{\infty} \sum_{s=1}^{\infty} B_{ms} * \left(\frac{m\pi}{a} \right)^2 * \sin\left(\frac{m\pi x}{a} \right) * \sin\left(\frac{s\pi y}{b} \right) \tag{16.5}$$

$$\frac{\partial^2 w_1}{\partial y^2} = -\sum_{m=1}^{\infty} \sum_{s=1}^{\infty} B_{ms} * \left(\frac{s\pi}{b} \right)^2 * \sin\left(\frac{m\pi x}{a} \right) * \sin\left(\frac{s\pi y}{b} \right) \tag{16.6}$$

$$\frac{\partial^2 w_1}{\partial x \partial y} = \sum_{m=1}^{\infty} \sum_{s=1}^{\infty} B_{ms} * \frac{\pi^2 * m * s}{a * b} * \cos\left(\frac{m\pi x}{a} \right) * \cos\left(\frac{s\pi y}{b} \right) \tag{16.7}$$

Several derivates need to be squared. These are computed here.

$$\begin{aligned}\left(\frac{\partial^2 w_1}{\partial x^2}\right)^2 &= \left(-\sum_{n=1}^{\infty} A_n * \left(\frac{n\pi}{a}\right)^2 * \sin\left(\frac{n\pi x}{a}\right)\right)^2 \\ &+ \left(-\sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms} * \left(\frac{m\pi}{a}\right)^2 * \sin\left(\frac{m\pi x}{a}\right) * \sin\left(\frac{s\pi y}{b}\right)\right)^2 \\ &+ 2 * \left(-\sum_{n=1}^{\infty} A_n * \left(\frac{n\pi}{a}\right)^2 * \sin\left(\frac{n\pi x}{a}\right)\right) \\ &* \left(-\sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms} * \left(\frac{m\pi}{a}\right)^2 * \sin\left(\frac{m\pi x}{a}\right) * \sin\left(\frac{s\pi y}{b}\right)\right)\end{aligned} \quad (16.8)$$

$$\left(\frac{\partial^2 w_1}{\partial y^2}\right)^2 = \left(-\sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms} * \left(\frac{s\pi}{b}\right)^2 * \sin\left(\frac{m\pi x}{a}\right) * \sin\left(\frac{s\pi y}{b}\right)\right)^2 \quad (16.9)$$

$$\left(\frac{\partial^2 w_1}{\partial x \partial y}\right)^2 = \left(\sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms} * \frac{\pi^2 * m * s}{a * b} * \cos\left(\frac{m\pi x}{a}\right) * \cos\left(\frac{s\pi y}{b}\right)\right)^2 \quad (16.10)$$

$$\begin{aligned}\frac{\partial^2 w_1}{\partial x^2} * \frac{\partial^2 w_1}{\partial y^2} &= \left(-\sum_{n=1}^{\infty} A_n * \left(\frac{n\pi}{a}\right)^2 * \sin\left(\frac{n\pi x}{a}\right)\right) \\ &* \left(-\sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms} * \left(\frac{s\pi}{b}\right)^2 * \sin\left(\frac{m\pi x}{a}\right) * \sin\left(\frac{s\pi y}{b}\right)\right) \\ &+ \left(-\sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms} * \left(\frac{m\pi}{a}\right)^2 * \sin\left(\frac{m\pi x}{a}\right) * \sin\left(\frac{s\pi y}{b}\right)\right) \\ &* \left(-\sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms} * \left(\frac{s\pi}{b}\right)^2 * \sin\left(\frac{m\pi x}{a}\right) * \sin\left(\frac{s\pi y}{b}\right)\right)\end{aligned} \quad (16.11)$$

General integration is needed to be able to calculate the integrals needed.

$$\int_0^a \sin^2\left(\frac{m * \pi * x}{a}\right) dx = \frac{1}{2} * \int_0^a \left(1 - \cos\left(\frac{2 * m * \pi * x}{a}\right)\right) dx = \frac{a}{2} \quad (16.12)$$

$$\int_0^a \cos^2\left(\frac{m * \pi * x}{a}\right) dx = \frac{1}{2} * \int_0^a \left(1 + \cos\left(\frac{2 * m * \pi * x}{a}\right)\right) dx = \frac{a}{2} \quad (16.13)$$

The following relationship is only applicable for $s = odd$.

$$\int_0^b \sin\left(\frac{s * \pi * y}{b}\right) dy = \frac{2 * b}{\pi * s} \quad (16.14)$$

For $m \neq n$ the following relations are applicable:

$$\int_0^a \sin\left(\frac{m*\pi*x}{a}\right)*\sin\left(\frac{n*\pi*x}{a}\right)dx$$
$$= \frac{1}{2}*\int_0^a \left(\cos\left(\frac{\pi*x}{a}(m-n)\right)-\cos\left(\frac{\pi*x}{a}(n+m)\right)\right)dx = 0 \tag{16.15}$$

$$\int_0^a \cos\left(\frac{m*\pi*x}{a}\right)*\cos\left(\frac{n*\pi*x}{a}\right)dx$$
$$= \frac{1}{2}*\int_0^a \left(\cos\left(\frac{\pi*x}{a}(m-n)\right)+\cos\left(\frac{\pi*x}{a}(n+m)\right)\right)dx = 0 \tag{16.16}$$

Using the general integrals, the terms of the potential energy can be computed. The triple summations over m and n are only applicable if $m = n$. If $m \neq n$ that part is equal to 0.

The third term of the first integral is only valid for $s = odd$.

$$\int_0^b \int_0^a \left(\frac{\partial^2 w_1}{\partial x^2}\right)^2 * dx * dy$$
$$= \sum_{n=1}^{\infty} A_n^2 * \left(\frac{n\pi}{a}\right)^4 * \frac{ab}{2} + \sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms}^2 * \left(\frac{m\pi}{a}\right)^4 * \frac{\mathrm{ab}}{4}$$
$$+ 2 * \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sum_{s=1}^{\infty} A_n * B_{ms} * n^2 * m^2 * \left(\frac{\pi}{a}\right)^4 * \frac{a}{2} * \frac{2*b}{\pi * s} \tag{16.17}$$

$$\int_0^b \int_0^a \left(\frac{\partial^2 w_1}{\partial y^2}\right)^2 * dx * dy = \sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms}^2 * \left(\frac{s\pi}{b}\right)^4 * \frac{ab}{4} \tag{16.18}$$

$$\int_0^b \int_0^a \left(\frac{\partial^2 w_1}{\partial x \partial y}\right)^2 * dx * dy = \sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms}^2 * \frac{\pi^4 * m^2 * s^2}{a^2 * b^2} * \frac{ab}{4} \tag{16.19}$$

The second part of this integral is only valid for $s = odd$ and $m = n$.

$$\int_0^b \int_0^a \frac{\partial^2 w_1}{\partial x^2} * \frac{\partial^2 w_1}{\partial y^2} * dx * dy$$
$$= \sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms}^2 * \frac{\pi^4 * m^2 * s^2}{a^2 * b^2} * \frac{ab}{4}$$
$$+ \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sum_{s=1}^{\infty} A_n * B_{ms} * n^2 * s^2 * \frac{\pi^4}{a^2 * b^2} * \frac{a}{2} * \frac{2*b}{\pi * s} \tag{16.20}$$

Therefore the potential energy in the web is the following result. The triple summations are only applicable if $m = n$. If $m \neq n$ that part is equal to 0. The terms with the triple summations are only valid if $s = odd$.

$$U_1 = \frac{1}{2} * D * \left(\sum_{n=1}^{\infty} A_n^2 * \left(\frac{n\pi}{a}\right)^4 * \frac{ab}{2} + \sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms}^2 * \left(\frac{m\pi}{a}\right)^4 * \frac{\text{ab}}{4} \right.$$
$$+ 2 * \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sum_{s=1}^{\infty} A_n * B_{ms} * n^2 * m^2 * \left(\frac{\pi}{a}\right)^4 * \frac{a}{2} * \frac{2 * b}{\pi * s}$$
$$+ \sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms}^2 * \left(\frac{s\pi}{b}\right)^4 * \frac{ab}{4}$$
$$+ 2 * (1 - v) * \sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms}^2 * \frac{\pi^4 * m^2 * s^2}{a^2 * b^2} * \frac{ab}{4} \tag{16.21}$$
$$+ 2 * v * \sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms}^2 * \frac{\pi^4 * m^2 * s^2}{a^2 * b^2} * \frac{ab}{4}$$
$$\left. + 2 * v * \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sum_{s=1}^{\infty} A_n * B_{ms} * n^2 * s^2 * \frac{\pi^4}{a^2 * b^2} * \frac{a}{2} * \frac{2 * b}{\pi * s} \right)$$

Simplification of this results into the bending energy in the web. The triple summations over m and n are only applicable if $m = n$. If $m \neq n$ that part is equal to 0. The third term of this integral is only applicable for $s = odd$.

$$U_1 = \frac{1}{2} * D * \left(\sum_{n=1}^{\infty} A_n^2 * \left(\frac{n\pi}{a}\right)^4 * \frac{ab}{2} \right.$$
$$+ \sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms}^2 * \pi^4 * \frac{ab}{4} * \left(\frac{m^2}{a^2} + \frac{s^2}{b^2}\right)^2 \tag{16.22}$$
$$\left. + 2 * \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sum_{s=1}^{\infty} A_n * B_{ms} * n^2 * \frac{\pi^4}{a^2} * \frac{a * b}{\pi * s} * \left(\frac{m^2}{a^2} + \frac{s^2 * v}{b^2}\right) \right)$$

Determine potential energy in the flange

The potential energy in both flanges can be described using the following equation.

$$U_2 = 2 * \frac{EI_f}{2} * \int_0^a \left(\frac{\partial^2 w_2}{\partial x^2}\right)^2 * dx \tag{16.23}$$

First the derivative is computed to be able to calculate the potential energy.

$$\frac{\partial^2 w_2}{\partial x^2} = -\sum_{n=1}^{\infty} A_n * \left(\frac{n\pi}{a}\right)^2 * \sin\left(\frac{n\pi x}{a}\right) \tag{16.24}$$

The derivate needs to be squared. This is computed here.

$$\left(\frac{\partial^2 w_1}{\partial x^2}\right)^2 = \left(-\sum_{n=1}^{\infty} A_n * \left(\frac{n\pi}{a}\right)^2 * \sin\left(\frac{n\pi x}{a}\right)\right)^2 \tag{16.25}$$

Using the general integrals, the terms of the potential energy can be computed.

$$\int_0^a \left(\frac{\partial^2 w_1}{\partial x^2}\right)^2 * dx = \sum_{n=1}^{\infty} A_n^2 * \left(\frac{n\pi}{a}\right)^4 * \frac{a}{2} \tag{16.26}$$

Therefore the potential energy in both flanges combined is the following:

$$U_2 = EI_f * \left(\sum_{n=1}^{\infty} A_n^2 * \left(\frac{n\pi}{a}\right)^4 * \frac{a}{2} \right) \tag{16.27}$$

Total potential energy in the welded I-column

The results for the potential energy in the web (equation (16.22)) and in the flanges (equation (16.27)) can be combined and results in the total potential energy in the welded I-column. The triple summations are only applicable if $m = n$. If $m \neq n$ that part is equal to 0. The third term is only applicable for $s = odd$.

$$\begin{aligned} U_{total} = U_1 + U_2 = \frac{1}{2} * D * \Bigg(&\sum_{n=1}^{\infty} A_n^2 * \left(\frac{n\pi}{a}\right)^4 * \frac{ab}{2} \\ &+ \sum_{m=1}^{\infty} \sum_{s=1}^{\infty} B_{ms}^2 * \pi^4 * \frac{ab}{4} * \left(\frac{m^2}{a^2} + \frac{s^2}{b^2}\right)^2 \\ &+ 2 * \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \sum_{s=1}^{\infty} A_n * B_{ms} * n^2 * \frac{\pi^4}{a^2} * \frac{a * b}{\pi * s} * \left(\frac{m^2}{a^2} + \frac{s^2 * v}{b^2}\right) \Bigg) \\ &+ EI_f * \left(\sum_{n=1}^{\infty} A_n^2 * \left(\frac{n\pi}{a}\right)^4 * \frac{a}{2} \right) \end{aligned} \tag{16.28}$$

This is simplified into the following where the triple summations are only applicable if $m = n$. If $m \neq n$ that part is equal to 0. The third term is only applicable for $s = odd$.

$$\begin{aligned} U_{total} = &\sum_{n=1}^{\infty} A_n^2 * n^4 * \pi^4 * \frac{1}{a^3} * \left(\frac{D * b}{4} + \frac{EI_f}{2}\right) \\ &+ \sum_{m=1}^{\infty} \sum_{s=1}^{\infty} B_{ms}^2 * \pi^4 * \frac{ab}{8} * D * \left(\frac{m^2}{a^2} + \frac{s^2}{b^2}\right)^2 \\ &+ \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \sum_{s=1}^{\infty} A_n * B_{ms} * n^2 * \frac{\pi^4}{a^2} * \frac{a * b}{\pi * s} * D * \left(\frac{m^2}{a^2} + \frac{s^2 * v}{b^2}\right) \end{aligned} \tag{16.29}$$

Virtual work done by the external forces on the web

The virtual work done by the external forces on the web can be described using the following equation.

$$T_1 = -\frac{1}{2} * \sigma_{xx} * t_w * \int_0^b \int_0^a \left(\frac{\partial w_1}{\partial x}\right)^2 * dx * dy \tag{16.30}$$

First the derivative is computed to be able to calculate virtual work.

$$\begin{aligned} \frac{\partial w_1}{\partial x} = &\sum_{n=1}^{\infty} A_n * \frac{n\pi}{a} * \cos\left(\frac{n\pi x}{a}\right) \\ &+ \sum_{m=1}^{\infty} \sum_{s=1}^{\infty} B_{ms} * \frac{m\pi}{a} * \cos\left(\frac{m\pi x}{a}\right) * \sin\left(\frac{s\pi y}{b}\right) \end{aligned} \tag{16.31}$$

The derivative needs to be squared. This is computed here.

$$\left(\frac{\partial w_1}{\partial x}\right)^2 = \left(\sum_{n=1}^{\infty} A_n * \frac{n\pi}{a} * \cos\left(\frac{n\pi x}{a}\right)\right)^2 + \left(\sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms} * \frac{m\pi}{a} * \cos\left(\frac{m\pi x}{a}\right) * \sin\left(\frac{s\pi y}{b}\right)\right)^2 + 2 * \left(\sum_{n=1}^{\infty} A_n * \frac{n\pi}{a} * \cos\left(\frac{n\pi x}{a}\right)\right) * \left(\sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms} * \frac{m\pi}{a} * \cos\left(\frac{m\pi x}{a}\right) * \sin\left(\frac{s\pi y}{b}\right)\right) \tag{16.32}$$

Using the general integrals, the terms of the virtual work can be computed. The triple summations are only applicable if $m = n$. If $m \neq n$ that part is equal to 0. The third term is only applicable for $s = odd$.

$$\int_0^b \int_0^a \left(\frac{\partial w_1}{\partial x}\right)^2 * dx * dy = \sum_{n=1}^{\infty} A_n^2 * \left(\frac{n\pi}{a}\right)^2 * \frac{ab}{2} + \sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms}^2 * \left(\frac{m\pi}{a}\right)^2 * \frac{\mathrm{ab}}{4} + 2 * \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sum_{s=1}^{\infty} A_n * B_{ms} * n * m * \left(\frac{\pi}{a}\right)^2 * \frac{\mathrm{a}}{2} * \frac{2 * \mathrm{b}}{\pi * \mathrm{s}} \tag{16.33}$$

Therefore the virtual work done on the web is the following result. The triple summations are only applicable if $m = n$. If $m \neq n$ that part is equal to 0. The third term is only applicable for $s = odd$.

$$T_1 = -\frac{1}{2} * \sigma_{xx} * t_w * \left(\sum_{n=1}^{\infty} A_n^2 * \left(\frac{n\pi}{a}\right)^2 * \frac{ab}{2} + \sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms}^2 * \left(\frac{m\pi}{a}\right)^2 * \frac{\mathrm{ab}}{4} + 2 * \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sum_{s=1}^{\infty} A_n * B_{ms} * n * m * \left(\frac{\pi}{a}\right)^2 * \frac{\mathrm{a} * \mathrm{b}}{\pi * \mathrm{s}}\right) \tag{16.34}$$

Virtual work done by the external forces on the flange

The virtual work done by the external forces on both flanges can be described using the following equation.

$$T_2 = -2 * \frac{1}{2} * \sigma_{xx} * t_f * h_f * \int_0^a \left(\frac{\partial w_2}{\partial x}\right)^2 * dx \tag{16.35}$$

First the derivative is computed to be able to calculate virtual work.

$$\frac{\partial w_2}{\partial x} = \sum_{n=1}^{\infty} A_n * \frac{n\pi}{a} * \cos\left(\frac{n\pi x}{a}\right) \tag{16.36}$$

The derivative needs to be squared. This is computed here.

$$\left(\frac{\partial w_2}{\partial x}\right)^2 = \left(\sum_{n=1}^{\infty} A_n * \frac{n\pi}{a} * \cos\left(\frac{n\pi x}{a}\right)\right)^2 \tag{16.37}$$

Using the general integrals, the terms of the virtual work can be computed.

$$\int_0^a \left(\frac{\partial w_2}{\partial x}\right)^2 * dx = \sum_{n=1}^{\infty} A_n^2 * \left(\frac{n\pi}{a}\right)^2 * \frac{a}{2} \tag{16.38}$$

Therefore the virtual work done on both flanges combined is the following result.

$$T_2 = -\sigma_{xx} * t_f * h_f * \sum_{n=1}^{\infty} A_n^2 * \left(\frac{n\pi}{a}\right)^2 * \frac{a}{2} \tag{16.39}$$

Virtual work done by the external forces on the welded I-column

The results for the virtual work done on the web (equation (16.34)) and on both flanges (equation (16.39)) can be combined and results in the total virtual work done on the welded I-column. The triple summations are only applicable if $m = n$. If $m \neq n$ that part is equal to 0. The third term is only applicable for $s = odd$.

$$\begin{aligned} T_{total} = T_1 + T_2 = -\frac{1}{2} * \sigma_{xx} * t_w * \Bigg(\sum_{n=1}^{\infty} A_n^2 * \left(\frac{n\pi}{a}\right)^2 * \frac{ab}{2} \\ + \sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms}^2 * \left(\frac{m\pi}{a}\right)^2 * \frac{\mathrm{ab}}{4} \\ + 2 * \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sum_{s=1}^{\infty} A_n * B_{ms} * n * m * \left(\frac{\pi}{a}\right)^2 * \frac{\mathrm{a} * \mathrm{b}}{\pi * \mathrm{s}}\Bigg) \\ - \sigma_{xx} * t_f * h_f * \sum_{n=1}^{\infty} A_n^2 * \left(\frac{n\pi}{a}\right)^2 * \frac{a}{2} \end{aligned} \tag{16.40}$$

This is simplified into the following where the triple summations are only applicable if $m = n$. If $m \neq n$ that part is equal to 0. The third term is only applicable for $s = odd$.

$$\begin{aligned} T_{total} = -\sigma_{xx} * \sum_{n=1}^{\infty} A_n^2 * n^2 * \pi^2 * \frac{1}{a} * \left(\frac{b * t_w}{4} + \frac{t_f * h_f}{2}\right) \\ - \sigma_{xx} * \sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms}^2 * \left(\frac{m\pi}{a}\right)^2 * \frac{\mathrm{ab}}{8} * t_w \\ - \sigma_{xx} * \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sum_{s=1}^{\infty} A_n * B_{ms} * n * m * \left(\frac{\pi}{a}\right)^2 * \frac{\mathrm{a} * \mathrm{b}}{\pi * \mathrm{s}} * t_w \end{aligned} \tag{16.41}$$

Determine equilibrium condition

The limit state where the construction is just stable is the point where the total potential energy in the column is equal to the virtual work done by the external forces. If the load is increased anymore at this point the structure will buckle. The triple summations are only applicable if $m = n$. If $m \neq n$ that part is equal to 0.

$$U_{total} + T_{total} = 0 \tag{16.42}$$

From this equilibrium condition the buckling stress can be computed.

$$\sigma_{xx} = \frac{\sum_{n=1}^{\infty} A_n^2 * n^4 * \pi^4 * \frac{1}{a^3} * \left(\frac{D*b}{4} + \frac{EI_f}{2}\right) + \sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms}^2 * \pi^4 * \frac{ab}{8} * D * \left(\frac{m^2}{a^2} + \frac{s^2}{b^2}\right)^2 + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sum_{s=1}^{\infty} A_n * B_{ms} * n^2 * \frac{\pi^4}{a^2} * \frac{a*b}{\pi * s} * D * \left(\frac{m^2}{a^2} + \frac{s^2 * v}{b^2}\right)}{\sum_{n=1}^{\infty} A_n^2 * n^2 * \pi^2 * \frac{1}{a} * \left(\frac{b * t_w}{4} + \frac{t_f * h_f}{2}\right) + \sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms}^2 * \left(\frac{m\pi}{a}\right)^2 * \frac{\mathrm{ab}}{8} * t_w + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sum_{s=1}^{\infty} A_n * B_{ms} * n * m * \left(\frac{\pi}{a}\right)^2 * \mathrm{a} * \frac{\mathrm{b}}{\pi * \mathrm{s}} * t_w}$$

It would be very difficult to derive an equation for the minimum buckling load. This is obtained when the derivative of σ_{xx} is taken with respect to A_n or B_{ms}. The equation $U + T = 0$ is still depending on A_n and B_{ms}. The minimum of σ_{xx} is obtained when the derivatives to A_n and B_{ms} are equated to zero. The derivative with respect to A_n is where $m = n$:

$$\begin{aligned}\frac{\partial(U+T)}{\partial A_n} &= 2 * A_n * n^4 * \pi^4 * \frac{1}{a^3} * \left(\frac{D * b}{4} + \frac{EI_f}{2}\right) \\ &+ \sum_{s=1}^{\infty} B_{ms} * n^2 * \frac{\pi^4}{a^2} * \frac{a * b}{\pi * s} * D * \left(\frac{m^2}{a^2} + \frac{s^2 * v}{b^2}\right) \\ &- \sigma_{xx} * 2 * A_n * n^2 * \pi^2 * \frac{1}{a} * \left(\frac{b * t_w}{4} + \frac{t_f * h_f}{2}\right) \\ &- \sigma_{xx} * \sum_{s=1}^{\infty} B_{ms} * n * m * \left(\frac{\pi}{a}\right)^2 * \frac{\mathrm{a} * \mathrm{b}}{\pi * \mathrm{s}} * t_w\end{aligned} \tag{16.43}$$

The derivative with respect to B_{ms} is where $m = n$:

$$\begin{aligned}\frac{\partial(U+T)}{\partial B_{ms}} &= \sum_{s=1}^{\infty} 2 * B_{ms} * \pi^4 * \frac{ab}{8} * D * \left(\frac{m^2}{a^2} + \frac{s^2}{b^2}\right)^2 \\ &+ \sum_{s=1}^{\infty} A_n * n^2 * \frac{\pi^4}{a^2} * \frac{a * b}{\pi * s} * D * \left(\frac{m^2}{a^2} + \frac{s^2 * v}{b^2}\right) \\ &- \sigma_{xx} * \sum_{s=1}^{\infty} 2 * B_{ms} * \left(\frac{m\pi}{a}\right)^2 * \frac{\mathrm{ab}}{8} * t_w \\ &- \sigma_{xx} * \sum_{s=1}^{\infty} A_n * n * m * \left(\frac{\pi}{a}\right)^2 * \frac{\mathrm{a} * \mathrm{b}}{\pi * \mathrm{s}} * t_w\end{aligned} \tag{16.44}$$

Higher values of s will lead to a higher amount of energy in the structure and a lower amount of virtual work done so only $s = 1$ is applied here.

Multiple values of A_n and B_{m1} will lead to an average value of σ_{xx} as explained before for the case of lateral-torsional buckling. Therefore only one of those is used to find the minimum.

$$\begin{bmatrix} 2*n^4*\pi^4*\frac{1}{a^3}*\frac{EI_{zz}}{4}-\sigma_{xx}*2*n^2*\pi^2*\frac{1}{a}*\left(\frac{b*t_w}{4}+\frac{t_f*h_f}{2}\right) & n^2*\frac{\pi^4}{a^2}*\frac{a*b}{\pi*1}*D*\left(\frac{m^2}{a^2}+\frac{1^2*v}{b^2}\right)-\sigma_{xx}*n*m*\left(\frac{\pi}{a}\right)^2*\frac{a*b}{\pi*1}*t_w \\ n^2*\frac{\pi^4}{a^2}*\frac{a*b}{\pi*1}*D*\left(\frac{m^2}{a^2}+\frac{1^2*v}{b^2}\right)-\sigma_{xx}*n*m*\left(\frac{\pi}{a}\right)^2*\frac{a*b}{\pi*1}*t_w & 2*\pi^4*\frac{ab}{8}*D*\left(\frac{m^2}{a^2}+\frac{1^2}{b^2}\right)^2-\sigma_{xx}*2*\left(\frac{m\pi}{a}\right)^2*\frac{ab}{8}*t_w \end{bmatrix}*\begin{bmatrix} A_n \\ B_{m1} \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

This can be written symbolically in the following way.

$$\begin{bmatrix} Z_1-\sigma_{xx}*Z_2 & Z_3-\sigma_{xx}*Z_4 \\ Z_5-\sigma_{xx}*Z_6 & Z_7-\sigma_{xx}*Z_8 \end{bmatrix}*\begin{bmatrix} A_n \\ B_{m1} \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{16.45}$$

When the determinant is equated to zero and solved for σ_{xx} this will result in the buckling stress.

$$\sigma_{xx}=\frac{-\frac{1}{2}(-Z_1Z_8-Z_2Z_7+Z_3Z_6+Z_4Z_5)}{Z_2Z_8-Z_4Z_6} \pm \frac{\frac{1}{2}\sqrt{(-Z_1Z_8-Z_2Z_7+Z_3Z_6+Z_4Z_5)^2-4*(Z_2Z_8-Z_4Z_6)*(Z_1Z_7-Z_3Z_5)}}{Z_2Z_8-Z_4Z_6} \tag{16.46}$$

Now it is applied that $Z_3=Z_5$ and $Z_4=Z_6$.

$$\sigma_{xx}=\frac{-\frac{1}{2}(-Z_1Z_8-Z_2Z_7+2*Z_3Z_4)}{Z_2Z_8-Z_4^2} \pm \frac{\frac{1}{2}\sqrt{(-Z_1Z_8-Z_2Z_7+2*Z_3Z_4)^2-4*(Z_2Z_8-Z_4^2)*(Z_1Z_7-Z_3^2)}}{Z_2Z_8-Z_4^2} \tag{16.47}$$

This equation is only dependent on m $(=n)$. Therefore for the several buckling modes $m=1,2,3..$ the buckling stress σ_{xx} can be calculated. The lowest value will result in the buckling stress.

It is also interesting to calculate the ratio of global to local buckling. The result of σ_{xx} is known so either equation of the matrix can be solved and this will result in the ratio of global to local buckling.

$$C_3 = \frac{A_n}{B_{m1}} \tag{16.48}$$

Using the first equation of the matrix will result in:

$$C_3 = \frac{n^2 * \frac{\pi^4}{a^2} * \frac{a*b}{\pi} * D * \left(\frac{m^2}{a^2} + \frac{1^2 * v}{b^2}\right) - \sigma_{xx} * n * m * \left(\frac{\pi}{a}\right)^2 * \frac{a*b}{\pi} * t_w}{2 * n^4 * \pi^4 * \frac{1}{a^3} * \frac{EI_{zz}}{4} - \sigma_{xx} * 2 * n^2 * \pi^2 * \frac{1}{a} * \left(\frac{b * t_w}{4} + \frac{t_f * h_f}{2}\right)} \tag{16.49}$$

Verification of the result

To verify whether calculations have been done correctly, only the local buckling behaviour is checked which means that $A_n = 0$. This means that the term in the second row and second column is the only one that remains and is equated to zero.

$$\sigma_{xx} = \frac{\pi^2 * D * \left(\frac{m^2}{a^2} + \frac{1^2}{b^2}\right)^2}{t_w * \left(\frac{m}{a}\right)^2} \tag{16.50}$$

This is slightly changed into the formula that is recognisable.

$$n_{xx} = \frac{\pi^2 * D * \left(\frac{m}{\alpha} + \frac{\alpha}{m}\right)^2}{b^2} \tag{16.51}$$

The result is indeed the traditional plate buckling formula that was derived earlier and that is equation (14.52). Another way to verify is to assume only global buckling behaviour which means that $B_{ms} = 0$.

$$\sigma_{xx} = \frac{n^2 * \pi^2 * \left(\frac{D * b}{4} + \frac{EI_f}{2}\right)}{a^2 * \left(\frac{b * t_w}{4} + \frac{t_f * h_f}{2}\right)} \tag{16.52}$$

This is slightly changed into the formula that is recognisable.

$$\sigma_{xx} = \pi^2 * n^2 * \frac{EI_{cross\ section}}{a^2 * A_{cross\ section}} \tag{16.53}$$

This is the same as the known Euler buckling load (equation (2.18)) if n is chosen equal to 1:

$$\sigma_{xx} = \frac{\pi^2}{a^2} * \frac{EI}{A} \tag{16.54}$$

This is also correct because this is the classical Euler buckling formula for columns which is known from literature.

Annex D: Analytical calculation of I-column under uniform compression including rotational restraint

The I-column under uniform compression has been calculated in Chapter 4 and Annex C under the assumption that the welds are hinges. This assumption is safe but it is also possible to include the rotational restraint of the welds. The configuration is described in Figure 100. The only difference with Chapter 4 is that the rotation of the flange is included and only a single half sine wave and a single cosine wave is included in the web in transverse direction. The extended explanations are given in Annex C and here only the formulas are given as the procedures is exactly alike.

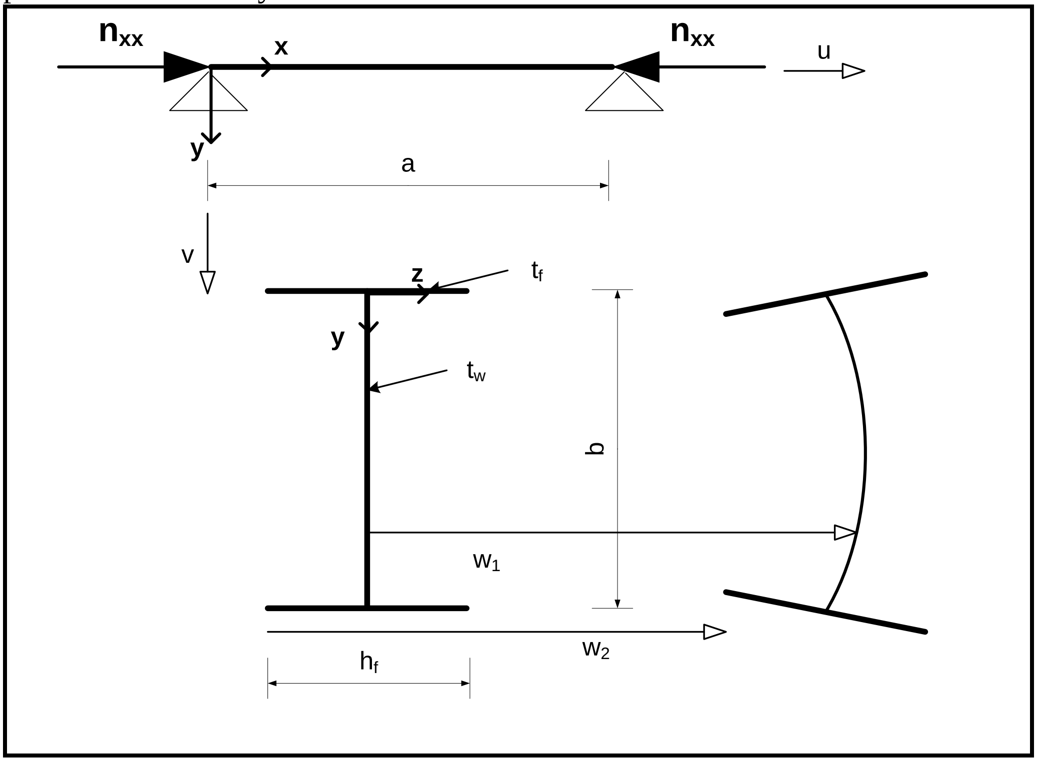


Figure 100: Situation for the welded I-column including rotational restraint

The deformation shape of the web is:

$$w_1 = \sum_{n=1}^{\infty} A_n * \sin\left(\frac{n\pi x}{a}\right) + \sum_{m=1}^{\infty} B_{m1} * \sin\left(\frac{m\pi x}{a}\right) * \sin\left(\frac{\pi y}{b}\right) + \sum_{k=1}^{\infty} C_k * \sin\left(\frac{k\pi x}{a}\right) * \left(1 - \cos\left(\frac{2\pi y}{b}\right)\right) \quad (17.1)$$

The deformation shape of the flange is:

$$w_2 = \sum_{n=1}^{\infty} A_n * \sin\left(\frac{n\pi x}{a}\right) \quad (17.2)$$

The rotation of the flanges is calculated by differentiating w_1 to y and applying $y = 0$ or $y = b$ which gives the same result except for the sign.

$$\theta = \frac{\partial w_1}{\partial y}|_{y=0} = \sum_{m=1}^{\infty} B_{m1} * \frac{\pi}{b} * \sin\left(\frac{m\pi x}{a}\right) \quad (17.3)$$

Determine potential energy in the web

First the derivatives are needed:

$$\frac{\partial^2 w_1}{\partial x^2} = -\sum_{n=1}^{\infty} A_n * \left(\frac{n\pi}{a}\right)^2 * \sin\left(\frac{n\pi x}{a}\right) - \sum_{m=1}^{\infty} B_{m1} * \left(\frac{m\pi}{a}\right)^2 * \sin\left(\frac{m\pi x}{a}\right) * \sin\left(\frac{\pi y}{b}\right) - \sum_{k=1}^{\infty} C_k * \left(\frac{k\pi}{a}\right)^2 * \sin\left(\frac{k\pi x}{a}\right) * \left(1 - \cos\left(\frac{2\pi y}{b}\right)\right) \quad (17.4)$$

$$\frac{\partial^2 w_1}{\partial y^2} = -\sum_{m=1}^{\infty} B_{m1} * \left(\frac{\pi}{b}\right)^2 * \sin\left(\frac{m\pi x}{a}\right) * \sin\left(\frac{\pi y}{b}\right) + \sum_{k=1}^{\infty} C_k * \left(\frac{2\pi}{b}\right)^2 * \sin\left(\frac{k\pi x}{a}\right) * \cos\left(\frac{2\pi y}{b}\right) \quad (17.5)$$

$$\frac{\partial^2 w_1}{\partial x \partial y} = \sum_{m=1}^{\infty} B_{m1} * \frac{\pi^2 * m}{a * b} * \cos\left(\frac{m\pi x}{a}\right) * \cos\left(\frac{\pi y}{b}\right) + \sum_{k=1}^{\infty} C_k * \frac{2 * k * \pi^2}{a * b} * \cos\left(\frac{k\pi x}{a}\right) * \sin\left(\frac{2\pi y}{b}\right) \quad (17.6)$$

Several derivates need to be squared. These are not shown. The integrals that need to be calculated are given here:

$$\int_0^b \int_0^a \left(\frac{\partial^2 w_1}{\partial x^2}\right)^2 * dx * dy = \sum_{n=1}^{\infty} A_n^2 * \left(\frac{n\pi}{a}\right)^4 * \frac{ab}{2} + \sum_{m=1}^{\infty} B_{m1}^2 * \left(\frac{m\pi}{a}\right)^4 * \frac{\mathrm{ab}}{4} + \sum_{k=1}^{\infty} C_k^2 * \left(\frac{k\pi}{a}\right)^4 * \frac{\mathrm{a}}{2} * \frac{3 * \mathrm{b}}{2} + 2 * \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_n * B_{m1} * n^2 * m^2 * \left(\frac{\pi}{a}\right)^4 * \frac{a}{2} * \frac{2 * b}{\pi} + 2 * \sum_{n=1}^{\infty}\sum_{k=1}^{\infty} A_n * C_k * n^2 * k^2 * \left(\frac{\pi}{a}\right)^4 * \frac{a}{2} * b + 2 * \sum_{m=1}^{\infty}\sum_{k=1}^{\infty} B_{m1} * C_k * m^2 * k^2 * \left(\frac{\pi}{a}\right)^4 * \frac{a}{2} * \frac{8 * b}{3 * \pi} \quad (17.7)$$

$$\int_0^b \int_0^a \left(\frac{\partial^2 w_1}{\partial y^2}\right)^2 * dx * dy = \sum_{m=1}^{\infty} B_{m1}^2 * \left(\frac{\pi}{b}\right)^4 * \frac{ab}{4} + \sum_{k=1}^{\infty} C_k^2 * \left(\frac{2\pi}{b}\right)^4 * \frac{\mathrm{a}}{2} * \frac{\mathrm{b}}{2} + 2 * \sum_{m=1}^{\infty}\sum_{k=1}^{\infty} B_{m1} * C_k * 4 * \left(\frac{\pi}{b}\right)^4 * \frac{a}{2} * \frac{2 * b}{3 * \pi} \quad (17.8)$$

$$\int_0^b \int_0^a \left(\frac{\partial^2 w_1}{\partial x \partial y}\right)^2 * dx * dy = \sum_{m=1}^{\infty} B_{m1}^2 * \frac{\pi^4 * m^2}{a^2 * b^2} * \frac{ab}{4} + \sum_{k=1}^{\infty} C_k^2 * \frac{4 * k^2 * \pi^4}{a^2 * b^2} * \frac{\mathrm{a}}{2} * \frac{\mathrm{b}}{2} + 2 * \sum_{m=1}^{\infty} \sum_{k=1}^{\infty} B_{m1} * C_k * \frac{2 * m * k * \pi^4}{a^2 * b^2} * \frac{a}{2} * \frac{4 * b}{3 * \pi} \tag{17.9}$$

$$\int_0^b \int_0^a \frac{\partial^2 w_1}{\partial x^2} * \frac{\partial^2 w_1}{\partial y^2} * dx * dy = \sum_{m=1}^{\infty} B_{m1}^2 * \frac{\pi^4 * m^2}{a^2 * b^2} * \frac{ab}{4} + \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} A_n * B_{m1} * n^2 * \frac{\pi^4}{a^2 * b^2} * \frac{a}{2} * \frac{2 * b}{\pi} + \sum_{m=1}^{\infty} \sum_{k=1}^{\infty} B_{m1} * C_k * \frac{4 * m^2 * \pi^4}{a^2 * b^2} * \frac{a}{2} * \frac{2 * b}{3 * \pi} + \sum_{m=1}^{\infty} \sum_{k=1}^{\infty} B_{m1} * C_k * \frac{k^2 * \pi^4}{a^2 * b^2} * \frac{a}{2} * \frac{8 * b}{3 * \pi} + \sum_{k=1}^{\infty} C_k^2 * \frac{4 * k^2 * \pi^4}{a^2 * b^2} * \frac{\mathrm{a}}{2} * \frac{\mathrm{b}}{2} \tag{17.10}$$

Therefore the total potential energy in the web is:

$$U_1 = \frac{1}{2} * D * \left(\sum_{n=1}^{\infty} A_n^2 * \left(\frac{n\pi}{a}\right)^4 * \frac{ab}{2} + \sum_{m=1}^{\infty} B_{m1}^2 * \left(\frac{m\pi}{a}\right)^4 * \frac{\mathrm{ab}}{4} \right.$$
$$+ \sum_{k=1}^{\infty} C_k^2 * \left(\frac{k\pi}{a}\right)^4 * \frac{\mathrm{a}}{2} * \frac{3 * \mathrm{b}}{2}$$
$$+ 2 * \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} A_n * B_{m1} * n^2 * m^2 * \left(\frac{\pi}{a}\right)^4 * \frac{a}{2} * \frac{2 * b}{\pi}$$
$$+ 2 * \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} A_n * C_k * n^2 * k^2 * \left(\frac{\pi}{a}\right)^4 * \frac{a}{2} * b$$
$$+ 2 * \sum_{m=1}^{\infty} \sum_{k=1}^{\infty} B_{m1} * C_k * m^2 * k^2 * \left(\frac{\pi}{a}\right)^4 * \frac{a}{2} * \frac{8 * b}{3 * \pi}$$
$$+ \sum_{m=1}^{\infty} B_{m1}^2 * \left(\frac{\pi}{b}\right)^4 * \frac{ab}{4} + \sum_{k=1}^{\infty} C_k^2 * \left(\frac{2\pi}{b}\right)^4 * \frac{\mathrm{a}}{2} * \frac{\mathrm{b}}{2}$$
$$+ 2 * \sum_{m=1}^{\infty} \sum_{k=1}^{\infty} B_{m1} * C_k * 4 * \left(\frac{\pi}{b}\right)^4 * \frac{a}{2} * \frac{2 * b}{3 * \pi}$$
$$+ 2 * (1 - \nu) * \left(\sum_{m=1}^{\infty} B_{m1}^2 * \frac{\pi^4 * m^2}{a^2 * b^2} * \frac{ab}{4} \right. \tag{17.11}$$
$$+ \sum_{k=1}^{\infty} C_k^2 * \frac{4 * k^2 * \pi^4}{a^2 * b^2} * \frac{\mathrm{a}}{2} * \frac{\mathrm{b}}{2}$$
$$\left. + 2 * \sum_{m=1}^{\infty} \sum_{k=1}^{\infty} B_{m1} * C_k * \frac{2 * m * k * \pi^4}{a^2 * b^2} * \frac{a}{2} * \frac{4 * b}{3 * \pi} \right)$$
$$+ 2 * \nu * \left(\sum_{m=1}^{\infty} B_{m1}^2 * \frac{\pi^4 * m^2}{a^2 * b^2} * \frac{ab}{4} \right.$$
$$+ \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} A_n * B_{m1} * n^2 * \frac{\pi^4}{a^2 * b^2} * \frac{a}{2} * \frac{2 * b}{\pi}$$
$$+ \sum_{m=1}^{\infty} \sum_{k=1}^{\infty} B_{m1} * C_k * \frac{4 * m^2 * \pi^4}{a^2 * b^2} * \frac{a}{2} * \frac{2 * b}{3 * \pi}$$
$$+ \sum_{m=1}^{\infty} \sum_{k=1}^{\infty} B_{m1} * C_k * \frac{k^2 * \pi^4}{a^2 * b^2} * \frac{a}{2} * \frac{8 * b}{3 * \pi}$$
$$\left. \left. + \sum_{k=1}^{\infty} C_k^2 * \frac{4 * k^2 * \pi^4}{a^2 * b^2} * \frac{\mathrm{a}}{2} * \frac{\mathrm{b}}{2} \right) \right)$$

The potential energy in the web is:

$$U_1=\frac{1}{2}*D*\left(\sum_{n=1}^{\infty}A_n^2*\left(\frac{n\pi}{a}\right)^4*\frac{ab}{2}+\sum_{m=1}^{\infty}B_{m1}^2*\pi^4*\frac{\mathrm{ab}}{4}*\left(\frac{\mathrm{m}^2}{\mathrm{a}^2}+\frac{1}{\mathrm{b}^2}\right)^2\right.$$
$$+\sum_{k=1}^{\infty}C_k^2*\pi^4*\frac{\mathrm{a}}{2}*\frac{\mathrm{b}}{2}*\left(3*\left(\frac{k}{a}\right)^4+8*\left(\frac{k}{ab}\right)^2+\left(\frac{2}{b}\right)^4\right)$$
$$+2*\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}A_n*B_{m1}*n^2*\frac{\pi^4}{a^2}*\frac{a*b}{\pi}*(\frac{m^2}{a^2}+\frac{v}{b^2}) \tag{17.12}$$
$$+2*\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}A_n*C_k*n^2*k^2*\left(\frac{\pi}{a}\right)^4*\frac{a}{2}*b$$
$$\left.+2*\sum_{m=1}^{\infty}\sum_{k=1}^{\infty}B_{m1}*C_k*\pi^4*a*\frac{8*b}{6*\pi}*\left(\frac{mk}{a^2}+\frac{1}{b^2}\right)^2\right)$$

Determine potential energy in the flange

The potential energy in the flange is the same as in Annex C with the addition of the rotational resistance. This is calculated using:

$$U_{\theta,flanges}=2*\frac{1}{2}*\int_0^a\left(S_{t,fl}*\left(\frac{\partial\theta}{\partial x}\right)^2+EC_{w,fl}*\left(\frac{\partial^2\theta}{\partial x^2}\right)^2\right)dx \tag{17.13}$$

The necessary derivatives are:

$$\frac{\partial\theta}{\partial x}=\sum_{m=1}^{\infty}B_{m1}*\frac{m*\pi^2}{a*b}*\cos\left(\frac{m\pi x}{a}\right) \tag{17.14}$$

$$\frac{\partial^2\theta}{\partial x^2}=-\sum_{m=1}^{\infty}B_{m1}*\frac{m^2*\pi^3}{a^2*b}*\sin\left(\frac{m\pi x}{a}\right) \tag{17.15}$$

The integral results in:

$$\int_0^a\left(S_{t,fl}*\left(\frac{\partial\theta}{\partial x}\right)^2+EC_{w,fl}*\left(\frac{\partial^2\theta}{\partial x^2}\right)^2\right)dx$$
$$=S_{t,fl}*\sum_{m=1}^{\infty}B_{m1}^2*\left(\frac{m*\pi^2}{a*b}\right)^2*\frac{\mathrm{a}}{2} \tag{17.16}$$
$$+EC_{w,fl}*\sum_{m=1}^{\infty}B_{m1}^2*\left(\frac{m^2*\pi^3}{a^2*b}\right)^2*\frac{\mathrm{a}}{2}$$

So the total potential energy in the flange is:

$$U_2 = EI_f * \sum_{n=1}^{\infty} A_n^2 * \left(\frac{n\pi}{a}\right)^4 * \frac{a}{2} + S_{t,fl} * \sum_{m=1}^{\infty} B_{m1}^2 * \left(\frac{m*\pi^2}{a*b}\right)^2 * \frac{\mathrm{a}}{2} + EC_{w,fl} * \sum_{m=1}^{\infty} B_{m1}^2 * \left(\frac{m^2*\pi^3}{a^2*b}\right)^2 * \frac{\mathrm{a}}{2} \quad (17.17)$$

Virtual work done by the external forces on the web

The derivative needed is:

$$\frac{\partial w_1}{\partial x} = \sum_{n=1}^{\infty} A_n * \left(\frac{n\pi}{a}\right) * \cos\left(\frac{n\pi x}{a}\right) + \sum_{m=1}^{\infty} B_{m1} * \left(\frac{m\pi}{a}\right) * \cos\left(\frac{m\pi x}{a}\right) * \sin\left(\frac{\pi y}{b}\right) + \sum_{k=1}^{\infty} C_k * \left(\frac{k\pi}{a}\right) * \cos\left(\frac{k\pi x}{a}\right) * \left(1 - \cos\left(\frac{2\pi y}{b}\right)\right) \quad (17.18)$$

Integration will result in:

$$\int_0^b \int_0^a \left(\frac{\partial w_1}{\partial x}\right)^2 * dx * dy = \sum_{n=1}^{\infty} A_n^2 * \left(\frac{n\pi}{a}\right)^2 * \frac{ab}{2} + \sum_{m=1}^{\infty} B_{m1}^2 * \left(\frac{m\pi}{a}\right)^2 * \frac{\mathrm{ab}}{4} + \sum_{k=1}^{\infty} C_k^2 * \left(\frac{k\pi}{a}\right)^2 * \frac{\mathrm{a}}{2} * \frac{3*\mathrm{b}}{2} + 2 * \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_n * B_{m1} * n * m * \left(\frac{\pi}{a}\right)^2 * \frac{a}{2} * \frac{2*b}{\pi} + 2 * \sum_{n=1}^{\infty}\sum_{k=1}^{\infty} A_n * C_k * n * k * \left(\frac{\pi}{a}\right)^2 * \frac{a}{2} * b + 2 * \sum_{m=1}^{\infty}\sum_{k=1}^{\infty} B_{m1} * C_k * m * k * \left(\frac{\pi}{a}\right)^2 * \frac{a}{2} * \frac{8*b}{3*\pi} \quad (17.19)$$

Therefore the total virtual work done on the web is:

$$T_1 = -\frac{1}{2} * \sigma_{xx} * t_w * \left(\sum_{n=1}^{\infty} A_n^2 * \left(\frac{n\pi}{a}\right)^2 * \frac{ab}{2} + \sum_{m=1}^{\infty} B_{m1}^2 * \left(\frac{m\pi}{a}\right)^2 * \frac{\mathrm{ab}}{4} + \sum_{k=1}^{\infty} C_k^2 * \left(\frac{k\pi}{a}\right)^2 * \frac{\mathrm{a}}{2} * \frac{3 * \mathrm{b}}{2} + 2 * \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} A_n * B_{m1} * n * m * \left(\frac{\pi}{a}\right)^2 * \frac{a}{2} * \frac{2 * b}{\pi} + 2 * \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} A_n * C_k * n * k * \left(\frac{\pi}{a}\right)^2 * \frac{a}{2} * b + 2 * \sum_{m=1}^{\infty} \sum_{k=1}^{\infty} B_{m1} * C_k * m * k * \left(\frac{\pi}{a}\right)^2 * \frac{a}{2} * \frac{8 * b}{3 * \pi} \right) \quad (17.20)$$

Virtual work done by the external forces on the flange

This is the same as the virtual work done on the flanges in Annex C. This is:

$$T_2 = -\sigma_{xx} * t_f * h_f * \sum_{n=1}^{\infty} A_n^2 * \left(\frac{n\pi}{a}\right)^2 * \frac{a}{2} \quad (17.21)$$

Determine equilibrium condition

The equation $U + T = 0$ gives the buckling load. However, it still depends on three variables. Therefore the derivatives with respect to the variables A_n, B_{m1} and C_k need to be determined and this can be represented in matrix form. Interaction only occurs when a single value of n $(= m = k)$ is chosen.

The matrix from which the determinant should be solved to find the buckling load is: $(m = n = t)$

$$\begin{bmatrix} 2*n^4*\pi^4*\frac{1}{a^3}*\frac{EI_{zz}}{4}-\sigma_{xx}*2*n^2*\pi^2*\frac{1}{a}*\left(\frac{b*t_w}{4}+\frac{t_f*h_f}{2}\right) & n^2*\frac{\pi^4}{a^2}*\frac{ab}{\pi}*D*\left(\frac{m^2}{a^2}+\frac{1^2*v}{b^2}\right)-\sigma_{xx}*n*m*\left(\frac{\pi}{a}\right)^2*\mathrm{a}*\frac{\mathrm{b}}{\pi}*t_w & D*n^2*k^2*\frac{\pi^4}{a^4}*\frac{ab}{2}-\sigma_{xx}*t_w*n*k*\frac{\pi^2}{a^2}*\frac{a}{2}*b \\ n^2*\frac{\pi^4}{a^2}*\frac{ab}{\pi}*D*\left(\frac{m^2}{a^2}+\frac{1^2*v}{b^2}\right)-\sigma_{xx}*n*m*\left(\frac{\pi}{a}\right)^2*\mathrm{a}*\frac{\mathrm{b}}{\pi}*t_w & 2*\pi^4\frac{ab}{8}D\left(\frac{m^2}{a^2}+\frac{1^2}{b^2}\right)^2-\sigma_{xx}*2*\left(\frac{m\pi}{a}\right)^2\frac{\mathrm{ab}}{8}t_w+2*S_{t,fl}*\frac{m^2\pi^4}{a^2b^2}\frac{a}{2}+2*EC_{w,fl}*\frac{m^4\pi^6}{a^4b^2}\frac{a}{2} & D*\pi^4*\frac{ab}{\pi}*\frac{8}{6}*\left(\frac{mk}{a^2}+\frac{1}{b^2}\right)^2-\sigma_{xx}*t_w*\frac{mk}{a^2}*\pi^2a\frac{8b}{6\pi} \\ D*n^2*k^2*\frac{\pi^4}{a^4}*\frac{ab}{2}-\sigma_{xx}*t_w*n*k*\frac{\pi^2}{a^2}*\frac{a}{2}*b & D*\pi^4*\frac{ab}{\pi}*\frac{8}{6}*\left(\frac{mk}{a^2}+\frac{1}{b^2}\right)^2-\sigma_{xx}*t_w*\frac{mk}{a^2}*\pi^2a\frac{8b}{6\pi} & D*\pi^4\frac{ab}{4}*\left(3\left(\frac{k}{a}\right)^4+8\left(\frac{k}{ab}\right)^2+\left(\frac{2}{b}\right)^4\right)-\sigma_{xx}t_w\left(\frac{k\pi}{a}\right)^2\frac{a}{2}\frac{3b}{2} \end{bmatrix}$$

$$*\begin{bmatrix} A_n \\ B_{m1} \\ C_k \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Verification of result

The matrix given of which the determinant should be solved is an expansion of the matrix that was derived in Chapter 4 and Annex C. It is therefore only necessary to verify the result of the web when it has clamped edges. ($A_n = 0$ and $B_{m1} = 0$)

$$D * \pi^4 \frac{ab}{4} * \left(3\left(\frac{k}{a}\right)^4 + 8\left(\frac{k}{ab}\right)^2 + \left(\frac{2}{b}\right)^4\right) - \sigma_{xx} * t_w * \left(\frac{k\pi}{a}\right)^2 \frac{a}{2}\frac{3b}{2} = 0 \quad (17.22)$$

When this equation is solved to determine the buckling stress it results in:

$$\sigma_{xx} = \frac{D * \pi^2 * \left(\frac{1}{\alpha^2} + \frac{8}{3} + \frac{16}{3} * \alpha^2\right)}{b^2 * t_w} = k * \frac{D * \pi^2}{b^2 * t_w} \quad (17.23)$$

The value of k can be solved by taking the derivative of k with respect to α and equate this to zero. This will result in:

$$\frac{\partial k}{\partial \alpha} = \frac{-2}{\alpha^3} + \frac{32}{3} * \alpha = 0 \rightarrow \alpha = \sqrt[4]{\frac{2 * 3}{32}} = 0{,}658 \quad (17.24)$$

Therefore the minimum buckling factor will be:

$$k = \left(\frac{1}{0{,}658^2} + \frac{8}{3} + \frac{16}{3} * 0{,}658^2\right) = 7{,}29 \quad (17.25)$$

Literature (Timoshenko & Gere, 1963) provides the correct result for buckling with clamped edges which is $k = 7$. Therefore the chosen cosine as the imperfection form is not exactly the correct one but only results in an overestimation of the buckling load with 4,1%. As seen in the results of Chapter 4 this is a proper estimate of the buckling load.

Annex E: Finite element model for I-column

The model consists always of a main file where parameters can be changed. To run the model, Ansys has to be started and the main file is loaded. The main file automatically refers to the calculation file to repeat the calculation numerous times for a different column length. For every calculation the following data is stored into a file:

- Time
- Displacement applied at the top of the column
- Axial compressive force in the column
- Displacement out of plane of the web

This file can be loaded into Excel for further processing.

Model using imperfections according to Eurocode

Main file

```
finish
/clear
/batch,list
/filnam,plaatmodel
/title,plastisch gnl plaatmodel
/units,mpa
/nerr,1,99999999
/uis,msgpop,3

aint=1000
adelta=250
delta=50
tw=40
tf=25
b=250
hw=250

na=10
nb=40
nc=3
imp=200
imploc=200/0.7
nmax=100
buckm=100
fy=235

*do,i,1,delta,1

/input,c://acer/nonlinkolomv3impeurocode
disp,txt,,

*enddo

finish
/clear
```

Calculation file

```
parsav,scalar,,
/clear
/batch,list
/filnam,plaatmodel
/title,plastisch gnl plaatmodel
/units,mpa
/nerr,1,99999999
/uis,msgpop,3

parres,new,,

a=aint+(i-1)*adelta
e=a/imp
eloc=b/imploc
dispmax=fy/210000*a*2

/view,,1,1,1
/prep7
et,1,shell181
r,1,tw
r,2,tf
et,2,beam4
r,3,1e2,1e9,1e9,100,100,0,0,100

mp,ex,1,2.1e5
mp,prxy,1,0.3
tb,biso,1
tbdata,1,fy,21
tbpl,biso,1

k,,0,0,0
k,,0,0,hw/2
k,,0,0,-hw/2
k,,b,0,0
```

```
k,,b,0,hw/2
k,,b,0,-hw/2
k,,0,a,0
k,,0,a,hw/2
k,,0,a,-hw/2
k,,b,a,0
k,,b,a,hw/2
k,,b,a,-hw/2
kplot

mshape,0

real,1
a,1,4,10,7

real,1
lsel,s,tan1,x,1
lsel,a,tan1,x,-1
lesize,all,,,na
lsel,s,tan1,y,1
lsel,a,tan1,y,-1
lesize,all,,,nb
asel,s,area,,1,1,1
amesh,all

nsel,s,loc,x,b/2
nsel,r,loc,y,0
nsel,r,loc,z,0
d,all,ux,0

nsel,s,loc,x,b/2
nsel,r,loc,y,a
nsel,r,loc,z,0
d,all,ux,0

nsel,s,loc,y,0
nsel,r,loc,z,0
d,all,uz,0
d,all,uy,0

nsel,s,loc,y,a
nsel,r,loc,z,0
d,all,uz,0
d,all,uy,-0.1

nsel,s,loc,x,0
nsel,a,loc,x,b
nsel,r,loc,z,0
d,all,uz,0

nsel,s,loc,x,b/2
d,all,ux,0

nmidmid=node(0.5*b,0.5*a,0)
nmiddenboven=node(0.5*b,a,0)
save
finish

/solve
antype,static
pstress,on
allsel
sbctran
solve
finish

/solve
antype,buckle
bucopt,lanb,1
mxpand,1
solve
finish

/post1
set,last
pldisp,1
plnsol,u,z,0,,
/wait,1
finish

/prep7
/inquire,myjobname,jobname
allsel
upgeom,eloc,1,1,'%myjobname(1)%',rst
save

real,2
a,1,2,8,7
a,1,3,9,7
a,4,5,11,10
a,4,6,12,10

real,3
l,1,2
l,1,3
l,4,5
l,4,6
l,7,8
```

```
l,7,9
l,10,11
l,10,12

real,2
lsel,s,tan1,y,1
lsel,a,tan1,y,-1
lesize,all,,,nb
lsel,s,tan1,z,1
lsel,a,tan1,z,-1
lesize,all,,,nc

real,2
asel,s,area,,2,5,1
amesh,all

real,3
lmesh,all

nsel,s,loc,y,0
nsel,a,loc,y,a
nsel,invert
ddele,all,uz
ddele,all,ux

nsel,all
d,all,roty,0

save
finish

/solve
antype,static
pstress,on
allsel
sbctran
solve
finish

/solve
antype,buckle
bucopt,lanb,buckm
mxpand,buckm
solve
finish

/post26
numvar,200
nsel,all
*get,nmin,node,0,nxth
*get,ntot,node,,count
nsol,3,nmin,u,z
*do,ndo,2,ntot
*get,nmin,node,nmin,nxth
nsol,196,nmin,u,z
add,3,3,196,,uz_tot
*enddo

*dim,uz_tot,array,buckm
vget,uz_tot(1),3
buckuz=uz_tot(1)
bucktoe=1
*do,j,2,buckm,1
*if,uz_tot(j),gt,buckuz,then
buckuz=uz_tot(j)
bucktoe=j
*else
*endif
*enddo
finish

/solve
antype,static
pstress,on
allsel
sbctran
solve
finish

/solve
antype,buckle
bucopt,lanb,bucktoe
mxpand,bucktoe
solve
finish

/post1
set,last
pldisp,1
plnsol,u,z,0,,
/wait,1
finish

/prep7
/inquire,myjobname,jobname
allsel
upgeom,e,1,bucktoe,'%myjobname(1)%',rs
t
```

```
nsel,all
ddele,all,rotz
ddele,all,roty

nsel,s,loc,y,a
ddele,all,uy,0
nsel,s,loc,y,a
nsel,r,loc,z,0
d,all,uy,-dispmax

save
finish

/solve
antype,static
nlgeom,on
outres,all,all
allsel
sbctran
nsubst,nmax,nmax,nmax
kbc,0
solve
save
finish

/post26
numvar,200
allsel
nsol,200,nmiddenboven,u,y,uyneg
filldata,199,,,,-1,0
prod,2,200,199,,uy
nsel,s,loc,y,0
*get,nmin,node,0,nxth
*get,ntot,node,,count
rforce,3,nmin,f,y
*do,ndo,2,ntot
*get,nmin,node,nmin,nxth
rforce,196,nmin,f,y
add,3,3,196,,fy_tot
*enddo

nsol,198,nmidmid,u,z,uz_echt
filldata,197,,,,e,0
add,4,198,197,,uz

save

*del,_p26_export
*dim,_p26_export,table,nmax,2
vget,_p26_export(1,0),1
vget,_p26_export(1,1),3
vget,_p26_export(1,2),4
/output,'blabla','','.',append
*vwrite,'time','uz'
%14c %14c %14c
*vwrite,_p26_export(1,0),_p26_export(1,1
),_p26_export(1,2)
%14.5g %14.5g %14.5g
/output,term

finish
```

Model using imperfections according to Eurocode using m=1

Main file

```
finish
/clear
/batch,list
/filnam,plaatmodel
/title,plastisch gnl plaatmodel
/units,mpa
/nerr,1,99999999
/uis,msgpop,3

aint=1000
adelta=250
delta=50
tw=40
tf=25
b=250
hw=250

na=10
nb=40
nc=3
imp=200
imploc=200/0.7
nmax=100
buckm=250
buckmgl=100
fy=235

*do,i,1,delta,1
```

```
/input,c://acer/nonlinkolomv3impeurocode
dispmis1,txt,,

*enddo

finish
/clear
```

Calculation file

```
parsav,scalar,,
/clear
/batch,list
/filnam,plaatmodel
/title,plastisch gnl plaatmodel
/units,mpa
/nerr,1,99999999
/uis,msgpop,3

parres,new,,

a=aint+(i-1)*adelta
e=a/imp
eloc=b/imploc
dispmax=fy/210000*a*2

/view,,1,1,1
/prep7
et,1,shell181
r,1,tw
r,2,tf
et,2,beam4
r,3,1e2,1e9,1e9,100,100,0,0,100

mp,ex,1,2.1e5
mp,prxy,1,0.3
tb,biso,1
tbdata,1,fy,21
tbpl,biso,1

k,,0,0,0
k,,0,0,hw/2
k,,0,0,-hw/2
k,,b,0,0
k,,b,0,hw/2
k,,b,0,-hw/2
k,,0,a,0
k,,0,a,hw/2
k,,0,a,-hw/2
k,,b,a,0
k,,b,a,hw/2
k,,b,a,-hw/2
kplot

mshape,0

real,1
a,1,4,10,7

real,1
lsel,s,tan1,x,1
lsel,a,tan1,x,-1
lesize,all,,,na
lsel,s,tan1,y,1
lsel,a,tan1,y,-1
lesize,all,,,nb
asel,s,area,,1,1,1
amesh,all

nsel,s,loc,x,b/2
nsel,r,loc,y,0
nsel,r,loc,z,0
d,all,ux,0

nsel,s,loc,x,b/2
nsel,r,loc,y,a
nsel,r,loc,z,0
d,all,ux,0

nsel,s,loc,y,0
nsel,r,loc,z,0
d,all,uz,0
d,all,uy,0

nsel,s,loc,y,a
nsel,r,loc,z,0
d,all,uz,0
d,all,uy,-0.1

nsel,s,loc,x,0
nsel,a,loc,x,b
nsel,r,loc,z,0
d,all,uz,0

nsel,s,loc,x,b/2
d,all,ux,0

nmidmid=node(0.5*b,0.5*a,0)
```

```
nmiddenboven=node(0.5*b,a,0)
save
finish

/solve
antype,static
pstress,on
allsel
sbctran
solve
finish

/solve
antype,buckle
bucopt,lanb,buckm
mxpand,buckm
solve
finish

/post26
numvar,200
nsel,all
*get,nmin,node,0,nxth
*get,ntot,node,,count
nsol,3,nmin,u,z
*do,ndo,2,ntot
*get,nmin,node,nmin,nxth
nsol,196,nmin,u,z
add,3,3,196,,uz_tot
*enddo

*dim,uz_tot,array,buckm
vget,uz_tot(1),3
buckuz=uz_tot(1)
bucktoe=1
*do,j,2,buckm,1
*if,uz_tot(j),gt,buckuz,then
buckuz=uz_tot(j)
bucktoe=j
*else
*endif
*enddo
finish

/solve
antype,static
pstress,on
allsel
sbctran
solve
finish

/solve
antype,buckle
bucopt,lanb,bucktoe
mxpand,bucktoe
solve
finish

/post1
set,last
pldisp,1
plnsol,u,z,0,,
finish

/prep7
/inquire,myjobname,jobname
allsel
upgeom,eloc,1,bucktoe,'%myjobname(1)%
',rst
save

real,2
a,1,2,8,7
a,1,3,9,7
a,4,5,11,10
a,4,6,12,10

real,3
l,1,2
l,1,3
l,4,5
l,4,6
l,7,8
l,7,9
l,10,11
l,10,12

real,2
lsel,s,tan1,y,1
lsel,a,tan1,y,-1
lesize,all,,,nb
lsel,s,tan1,z,1
lsel,a,tan1,z,-1
lesize,all,,,nc

real,2
asel,s,area,,2,5,1
```

```
amesh,all

real,3
lmesh,all

nsel,s,loc,y,0
nsel,a,loc,y,a
nsel,invert
ddele,all,uz
ddele,all,ux

nsel,all
d,all,roty,0

save
finish

/solve
antype,static
pstress,on
allsel
sbctran
solve
finish

/solve
antype,buckle
bucopt,lanb,buckmgl
mxpand,buckmgl
solve
finish

/post26
numvar,200
nsel,all
*get,nmin,node,0,nxth
*get,ntot,node,,count
nsol,3,nmin,u,z
*do,ndo,2,ntot
*get,nmin,node,nmin,nxth
nsol,196,nmin,u,z
add,3,3,196,,uz_tott
*enddo

*dim,uz_tott,array,buckmgl
vget,uz_tott(1),3
buckuz=uz_tott(1)
bucktoegl=1
*do,j,2,buckmgl,1
*if,uz_tott(j),gt,buckuz,then
buckuz=uz_tott(j)
bucktoegl=j
*else
*endif
*enddo
finish

/solve
antype,static
pstress,on
allsel
sbctran
solve
finish

/solve
antype,buckle
bucopt,lanb,bucktoegl
mxpand,bucktoegl
solve
finish

/post1
set,last
pldisp,1
plnsol,u,z,0,,
/wait,1
finish

/prep7
/inquire,myjobname,jobname
allsel
upgeom,e,1,bucktoegl,'%myjobname(1)%',
rst

nsel,all
ddele,all,rotz
ddele,all,roty

nsel,s,loc,y,a
ddele,all,uy,0
nsel,s,loc,y,a
nsel,r,loc,z,0
d,all,uy,-dispmax

save
finish
```

```
/solve
antype,static
nlgeom,on
outres,all,all
allsel
sbctran
nsubst,nmax,nmax,nmax
kbc,0
solve
save
finish

/post26
numvar,200
allsel
nsol,200,nmiddenboven,u,y,uyneg
filldata,199,,,,-1,0
prod,2,200,199,,uy

nsel,s,loc,y,0
*get,nmin,node,0,nxth
*get,ntot,node,,count
rforce,3,nmin,f,y
*do,ndo,2,ntot
*get,nmin,node,nmin,nxth
rforce,196,nmin,f,y
add,3,3,196,,fy_tot
*enddo

nsol,198,nmidmid,u,z,uz_echt
filldata,197,,,,e,0
add,4,198,197,,uz

save

*del,_p26_export
*dim,_p26_export,table,nmax,2
vget,_p26_export(1,0),1
vget,_p26_export(1,1),3
vget,_p26_export(1,2),4
/output,'blabla','','.',append
*vwrite,'time','uz'
%14c %14c %14c
*vwrite,_p26_export(1,0),_p26_export(1,1
),_p26_export(1,2)
%14.5g %14.5g %14.5g
/output,term

finish
```

Model using imperfections according to lowest buckling load

Main file

```
finish
/clear
/batch,list
/filnam,plaatmodel
/title,plastisch gnl plaatmodel
/units,mpa
/nerr,1,99999999
/uis,msgpop,3

aint=12000
adelta=500
delta=29
tw=40
tf=30
b=1500
hw=500

na=10
nb=40
nc=3
eloc=200
eglob=200
nmax=100
fy=235
buckm=40

*do,i,1,delta,1

/input,c://acer/nonlinkolomv3,txt,,

*enddo

finish
```

Calculation file

```
parsav,scalar,,
/clear
/batch,list
/filnam,plaatmodel
/title,plastisch gnl plaatmodel
/units,mpa
/nerr,1,99999999
```

```
/uis,msgpop,3

parres,new,,

a=aint+(i-1)*adelta
dispmax=fy/210000*a*2

/view,,1,1,1
/prep7
et,1,shell181
r,1,tw
r,2,tf
et,2,beam4
r,3,1e2,1e9,1e9,100,100,0,0,100

mp,ex,1,2.1e5
mp,prxy,1,0.3
tb,biso,1
tbdata,1,fy,21
tbpl,biso,1

k,,0,0,0
k,,0,0,hw/2
k,,0,0,-hw/2
k,,b,0,0
k,,b,0,hw/2
k,,b,0,-hw/2
k,,0,a,0
k,,0,a,hw/2
k,,0,a,-hw/2
k,,b,a,0
k,,b,a,hw/2
k,,b,a,-hw/2
kplot

mshape,0

real,1
a,1,4,10,7

real,2
a,1,2,8,7
a,1,3,9,7
a,4,5,11,10
a,4,6,12,10

real,3
l,1,2
l,1,3
l,4,5
l,4,6
l,7,8
l,7,9
l,10,11
l,10,12

real,1
lsel,s,tan1,x,1
lsel,a,tan1,x,-1
lesize,all,,,na
lsel,s,tan1,y,1
lsel,a,tan1,y,-1
lesize,all,,,nb
lsel,s,tan1,z,1
lsel,a,tan1,z,-1
lesize,all,,,nc
asel,s,area,,1,1,1
amesh,all

real,2
asel,s,area,,2,5,1
amesh,all

real,3
lmesh,all

nsel,s,loc,x,b/2
nsel,r,loc,y,0
nsel,r,loc,z,0
d,all,ux,0

nsel,s,loc,x,b/2
nsel,r,loc,y,a
nsel,r,loc,z,0
d,all,ux,0

nsel,s,loc,y,0
nsel,r,loc,z,0
d,all,uz,0
d,all,uy,0

nsel,s,loc,y,a
nsel,r,loc,z,0
d,all,uz,0
d,all,uy,-0.1

save
finish
```

```
/solve
antype,static
pstress,on
allsel
sbctran
solve
finish

/solve
antype,buckle
bucopt,lanb,buckm
mxpand,buckm
solve
finish

/post26
numvar,200
nsel,all
*get,nmin,node,0,nxth
*get,ntot,node,,count
nsol,3,nmin,u,z
*do,ndo,2,ntot
*get,nmin,node,nmin,nxth
nsol,196,nmin,u,z
add,3,3,196,,uz_tot
*enddo

*dim,uz_tot,array,buckm
vget,uz_tot(1),3
buckuz=uz_tot(1)
bucktoe=1
*do,j,2,buckm,1
*if,uz_tot(j),gt,buckuz,then
buckuz=uz_tot(j)
bucktoe=j
*else
*endif
*enddo
finish

/solve
antype,static
pstress,on
allsel
sbctran
solve
finish

/solve
antype,buckle
bucopt,lanb,bucktoe
mxpand,bucktoe
solve
finish

/post1
set,last
pldisp,1
plnsol,u,z,0,,
nmiddenboven=node(0.5*b,a,0)
nweb=node(0.5*b,0.5*a,0)
nflange=node(0,0.5*a,0)
*get,uweb,node,nweb,u,z
*get,uflange,node,nflange,u,z
finish

/prep7
*if,1/eglob*0.7*a/uflange,gt,1/eloc*b/(uwe
b-uflange),then
e=1/eloc*b/(uweb-uflange)
*elseif,1/eloc*0.7*b/(uweb-
uflange),gt,1/eglob*a/uflange,then
e=1/eglob*a/uflange
*elseif,1/eloc*0.7*b/(uweb-
uflange),gt,1/eglob*0.7*a/uflange,then
e=1/eloc*0.7*b/(uweb-uflange)
*else
e=1/eglob*0.7*a/uflange
*endif
/inquire,myjobname,jobname
allsel
upgeom,e,1,bucktoe,'%myjobname(1)%',rs
t

nsel,s,loc,y,a
ddele,all,uy
ddele,all,uz

nsel,s,loc,y,a
nsel,r,loc,z,0
d,all,uz,0
d,all,uy,-dispmax

save
finish

/solve
```

```
antype,static
nlgeom,on
outres,all,all
allsel
sbctran
nsubst,nmax,nmax,nmax
kbc,0
solve
save
finish

/post26
numvar,200
allsel
nsol,200,nmiddenboven,u,y,uyneg
filldata,199,,,,-1,0
prod,2,200,199,,uy

nsel,s,loc,y,0
*get,nmin,node,0,nxth
*get,ntot,node,,count
rforce,3,nmin,f,y
*do,ndo,2,ntot
*get,nmin,node,nmin,nxth
rforce,196,nmin,f,y
add,3,3,196,,fy_tot
*enddo

nsol,198,nweb,u,z,uz_echt
filldata,197,,,,e,0
add,4,198,197,,uz

save

*del,_p26_export
*dim,_p26_export,table,nmax,2
vget,_p26_export(1,0),1
vget,_p26_export(1,1),3
vget,_p26_export(1,2),4
/output,'blabla','','.',append
*vwrite,'time','uz'
%14c %14c %14c
*vwrite,_p26_export(1,0),_p26_export(1,1
),_p26_export(1,2)
%14.5g %14.5g %14.5g
/output,term

finish
```

Model for probabilistic design

The model for the probabilistic design analysis works in the same way as for the deterministic calculation. The single calculation file describes the model and the calculations that need to be done. The main file sets the probabilistic design variables and performs the analysis a set number of times.

Main file

```
/pds

pdanl,pds3bar,pdan

pdvar,tw,gaus,50.4,0.7
pdvar,tf,gaus,50.4,0.7
pdvar,a,gaus,15000,2.5
pdvar,b,gaus,1500,2.5
pdvar,hw,gaus,500,2.5
pdvar,imp,gaus,1/290,1/290*0.15
pdvar,imploc,gaus,1/320*0.7,1/320*0.20*
0.7
pdvar,fy,log1,280,22.4
pdvar,emod,gaus,210000,8400

pdvar,designload,resp
pdmeth,mcs,dir
pddmcs,100,none,all,,,,123457

pdexe,mcs3bar

pdsens,mcs3bar,designload,both,rank,0.2
pdcdf,mcs3bar,designload,gaus,,,
pdsave,probeersel
```

Single calculation file

```
*create,pds3bar,pdan

/batch,list
/filnam,plaatmodel
/title,plastisch gnl plaatmodel
/units,mpa
/nerr,1,99999999
/uis,msgpop,3
seltol
```

```
*set,tw,10.35
*set,tf,50.4
*set,a,10000
*set,b,1500
*set,hw,500
*set,imp,1/500
*set,imploc,1/500*0.7
*set,fy,280
*set,emod,210000

/view,,1,1,1
/prep7

na=10
nb=40
nc=3
nmax=50
buckm=100
dispmax=fy/210000*a
e=a*imp
eloc=b*imploc

et,1,shell181
r,1,tw
r,2,tf
et,2,beam4
r,3,1e2,1e9,1e9,100,100,0,0,100

mp,ex,1,emod
mp,prxy,1,0.3
tb,biso,1
tbdata,1,fy,21
tbpl,biso,1

k,,0,0,0
k,,0,0,hw/2
k,,0,0,-hw/2
k,,b,0,0
k,,b,0,hw/2
k,,b,0,-hw/2
k,,0,a,0
k,,0,a,hw/2
k,,0,a,-hw/2
k,,b,a,0
k,,b,a,hw/2
k,,b,a,-hw/2
kplot
mshape,0

real,1
a,1,4,10,7

real,1
lsel,s,tan1,x,1
lsel,a,tan1,x,-1
lesize,all,,,na
lsel,s,tan1,y,1
lsel,a,tan1,y,-1
lesize,all,,,nb
asel,s,area,,1,1,1
amesh,all

nmiddenboven=node(0.5*b,a,0)
finish

/solve
antype,static
pstress,on

nsel,s,loc,x,b/2
nsel,r,loc,y,0
nsel,r,loc,z,0
d,all,ux,0

nsel,s,loc,x,b/2
nsel,r,loc,y,a
nsel,r,loc,z,0
d,all,ux,0

nsel,s,loc,y,0
nsel,r,loc,z,0
d,all,uz,0
d,all,uy,0

nsel,s,loc,y,a
nsel,r,loc,z,0
d,all,uz,0
d,all,uy,-0.1

nsel,s,loc,x,0
nsel,a,loc,x,b
nsel,r,loc,z,0
d,all,uz,0

nsel,s,loc,x,b/2
d,all,ux,0
```

```
allsel
sbctran
solve
finish

/post1
plnsol,u,y,0,,
plnsol,epel,y,0,,
finish

/solve
antype,buckle
bucopt,lanb,1
mxpand,1
solve
finish

/post1
set,last
pldisp,1
plnsol,u,z,0,,
finish

/prep7
/inquire,myjobname,jobname
allsel
upgeom,eloc,1,1,'%myjobname(1)%',rst
nsel,s,loc,z,eloc
*get,nmidmid,node,,num,min

real,2
a,1,2,8,7
a,1,3,9,7
a,4,5,11,10
a,4,6,12,10

real,3
l,1,2
l,1,3
l,4,5
l,4,6
l,7,8
l,7,9
l,10,11
l,10,12

real,2
lsel,s,tan1,y,1
lsel,a,tan1,y,-1
lesize,all,,,nb
lsel,s,tan1,z,1
lsel,a,tan1,z,-1
lesize,all,,,nc

real,2
asel,s,area,,2,5,1
amesh,all

real,3
lmesh,all

nsel,s,loc,y,0
nsel,a,loc,y,a
nsel,invert
ddele,all,uz
ddele,all,ux

nsel,all
d,all,roty,0

save
finish

/solve
antype,static
pstress,on

fcum,add
allsel
sbctran

solve
finish

/solve
antype,buckle
bucopt,lanb,buckm
mxpand,buckm
solve
finish

/post26
numvar,200
nsel,all
*get,nmin,node,0,nxth
*get,ntot,node,,count
nsol,3,nmin,u,z
```

```
*do,ndo,2,ntot
*get,nmin,node,nmin,nxth
nsol,196,nmin,u,z
add,3,3,196,,uz_tot
*enddo

*dim,uz_tot,array,buckm
vget,uz_tot(1),3
buckuz=uz_tot(1)
bucktoe=1
*do,j,2,buckm,1
*if,uz_tot(j),gt,buckuz,then
buckuz=uz_tot(j)
bucktoe=j
*else
*endif
*enddo

nsel,all
finish

/solve
antype,buckle
bucopt,lanb,bucktoe
mxpand,bucktoe
solve
finish

/post1
set,last
pldisp,1
plnsol,u,z,0,,
finish

/prep7
/inquire,myjobname,jobname
allsel
upgeom,e,1,bucktoe,'%myjobname(1)%',rs
t
nsel,s,loc,z,e
*get,nmidmid,node,,num,min

nsel,all
ddele,all,rotz
ddele,all,roty

nsel,s,loc,y,a
ddele,all,uy,0
nsel,s,loc,y,a
nsel,r,loc,z,0
d,all,uy,-dispmax

save
finish

/solve
antype,static
nlgeom,on
outres,all,all

allsel
sbctran

fcum

nsubst,nmax,nmax,nmax
kbc,0
solve
finish

/post1
allsel
set,last
/dscale,all,10
shell,mid
plnsol,s,y,2
finish

/post26
numvar,200
allsel
nsol,200,nmiddenboven,u,y,uyneg
filldata,199,,,,-1,0
prod,2,200,199,,uy

nsel,s,loc,y,0
*get,nmin,node,0,nxth
*get,ntot,node,,count
rforce,3,nmin,f,y
*do,ndo,2,ntot
*get,nmin,node,nmin,nxth
rforce,196,nmin,f,y
add,3,3,196,,fy_tot
*enddo

nsol,198,nmidmid,u,z,uz_echt
filldata,197,,,,e,0
add,4,198,197,,uz
```

```
filldata,191,,,,1,1
realvar,191,191
prvar,3,
*dim,fytot,array,nmax,1
vget,fytot(1,1,1),3

designload=fytot(1)
*do,j,2,nmax,1
*if,fytot(j),gt,designload,then
designload=fytot(j)
*else
*endif
*enddo

finish
*end
```

Annex F: Verification and results finite element model for I-column

Verification of the model

Finite element models should never be used as a black box and the results will depend on the input and on the assumptions done when creating the model. Certain checks will be needed to check the model and verify that indeed the modelled behaviour does approach the real behaviour. All checks will be done on the profile with flanges 500x25 mm and a web of 1500x15 mm and a length of 10000 mm.

Linear elastic calculation

First a linear elastic calculation is performed. The load is a displacement at one end of the column. This displacement is set at 0,1 mm. The displacement at the other end is set at 0 mm. The stress in the column should be:

$$\sigma_{el} = E * \frac{\Delta l}{a} = 210000 * \frac{0{,}1}{10000} = 2{,}1 \frac{N}{mm^2} \tag{19.1}$$

The stress can be checked by requesting a plot of the stresses as done in Figure 101. This should be a uniform stress distribution. It is acceptable that there is a slight deviation over the cross-section at the ends of the column because the beam at the end creating the hinge support is not infinitely stiff. This does indeed happen and the difference is only small as shown by the scale.

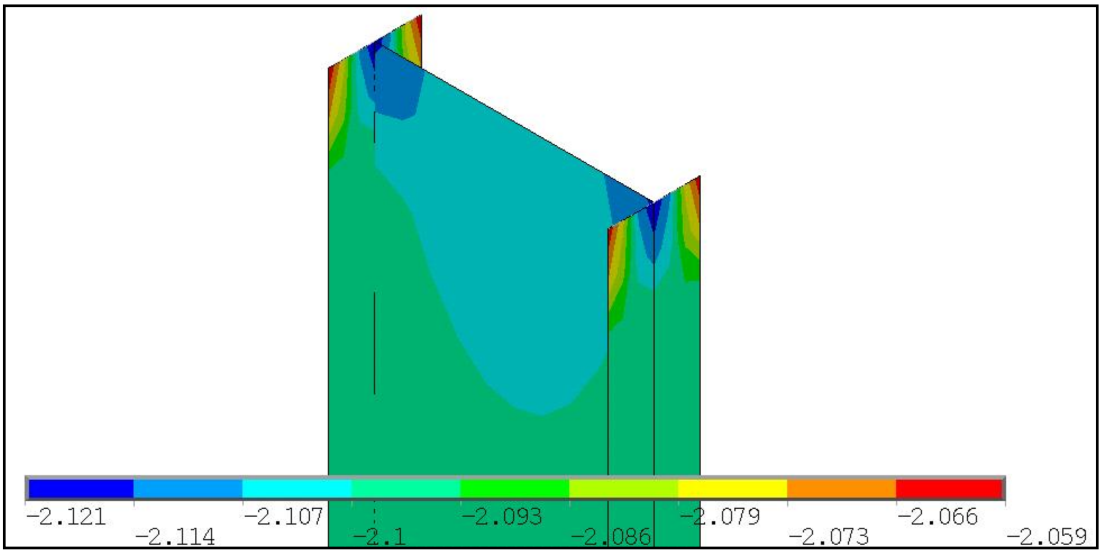


Figure 101: Stresses at one end of the I-column

The reaction forces are summed up at one end and should be equal to the load applied which can be calculated using the strain in the column.

$$N_{el} = E * A * \frac{\Delta l}{a} \tag{19.2}$$

Using the correct figures gives as a result:

$$N_{el} = 210000 * (500 * 25 * 2 + 1500 * 15) * \frac{0{,}1}{10000} = 99{,}75\ kN \tag{19.3}$$

The load according to Ansys is:

$$N_{el,ansys} = 99{,}70\ kN \tag{19.4}$$

This is equal to the load so the linear elastic calculation is correct. The small difference (0,05%) can be explained because of the hinge support that is created that is not infinitely stiff.

Plastic calculation

A plastic calculation is a non-linear calculation in physical sense. The result should be a fully yielded cross-section resulting in a force equal to:

$$N_{pl} = A * f_y = (500 * 25 * 2 + 1500 * 15) * 235 = 11162{,}5\ kN \quad (19.5)$$

The result in Ansys is:

$$N_{pl,ansys} = 11171{,}4\ kN \quad (19.6)$$

This is equal to the load so the plastic calculation is correct. The small difference (0,07%) can again be explained because of the hinge support and the very small increase in stiffness after yielding.

Theoretical buckling load

The theoretical buckling load is calculated for a different configuration because the difference between the column buckling load and the buckling load according to Chapter 4 is very small for the given configuration. Therefore the configuration with flanges 150x15 mm and a web of 1500x15 mm and a length of 4000 mm. The theoretical buckling load according to Chapter 4 is:

$$\sigma_{cr} = 38{,}15 \frac{N}{mm^2} \quad (19.7)$$

The displacement in Ansys is set at 0,1 mm so the stress is equal to the stress in equation (19.1). An eigenvalue linear buckling analysis is done and a load factor of 7,181 is the result. Therefore the critical stress according to Ansys is:

$$\sigma_{cr,ansys} = E * \frac{\Delta l}{a} * k_{ansys} = 210000 * \frac{0{,}1}{4000} * 7{,}181 = 37{,}70 \frac{N}{mm^2} \quad (19.8)$$

This is approximately equal to the stress as calculated in equation (19.7) so the result is correct. The slight difference (1,19%) in buckling load is probably due to a slightly different deflection shape which is not included in the analytical model.

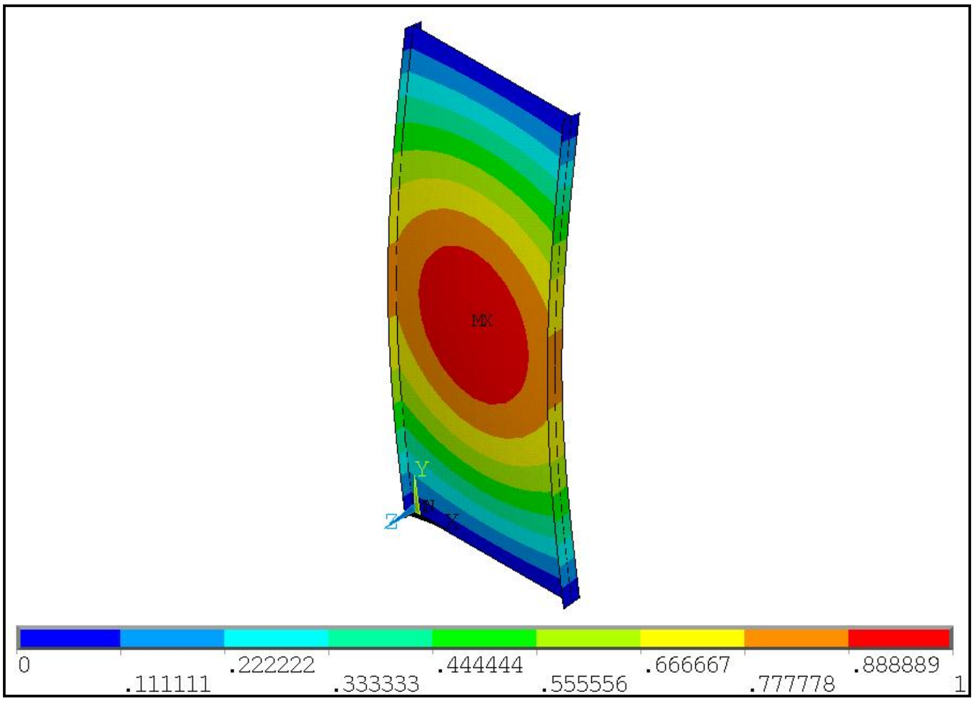


Figure 102: Displacement of I-column for first buckling mode

In Figure 102 the buckling mode is shown for the given geometry. This is indeed the behaviour that was expected based on Chapter 4. The column has a global deflection and the web has an additional local deflection with a single half-sine wave. The ratio of the deflection from the flange to the web is:

$$C_{3,ansys} = \frac{w_{flange}}{w_{web} - w_{flange}} = \frac{0{,}801}{1 - 0{,}801} = 4{,}03 \tag{19.9}$$

The calculated value according to Chapter 4 is:

$$C_3 = 3{,}62 \tag{19.10}$$

This difference (11,3%) is explained before because the sensitivity to a different column or beam length is very large. The calculated value for 4100 mm is 4,02 which is about right.

Mesh refinement

The mesh is defined by applying 10 elements in the width of the web. The flanges are divided in width by 6 elements. There are 40 elements in length direction. A refinement of the mesh should lead to a result that is similar to the calculation with a coarse mesh. Therefore the given profile with imperfections according to the buckling load of Chapter 4 is calculated in FEM with a non-linear calculation. The mesh may be refined but it may also be coarser and then the same results should be obtained. The mesh is coarsened by a factor 2 and the results are presented in Figure 103. It does indeed look the same and the maximum load only changes from $5477\ kN$ to $5470\ kN$. This is a minor difference (0,13%) so the fine mesh is sufficient for the analysis and probably the coarse mesh would also be sufficient.

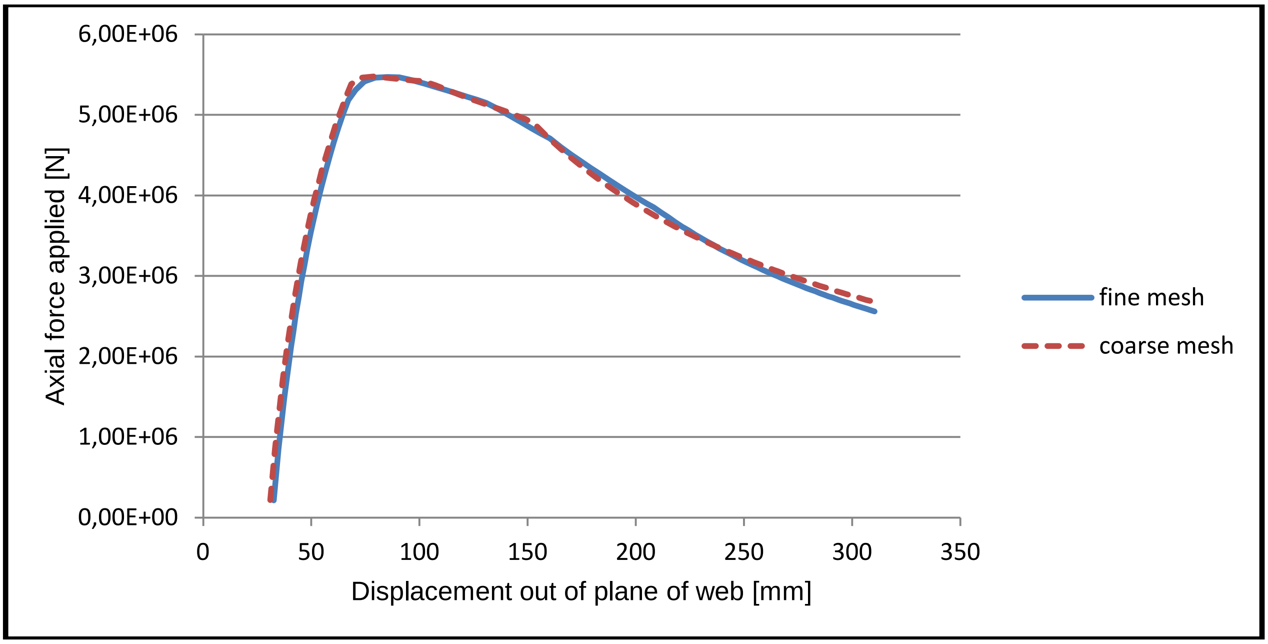


Figure 103: Results using a fine and a coarse mesh

Load step reduced

The load step (which is actually a displacement step) is important because it allows the step by step increase until failure. The load steps should be small enough to allow plastic strains to develop and second order displacements to occur. Therefore for the given profile the load step is applied and also a load step of 25% of the original load step is applied. The results are presented as a load-displacement graph in Figure 104. The lines overlap in the graph so the difference is negligible. This means that a load step of 0,01 (100 steps) is sufficient for the analysis.

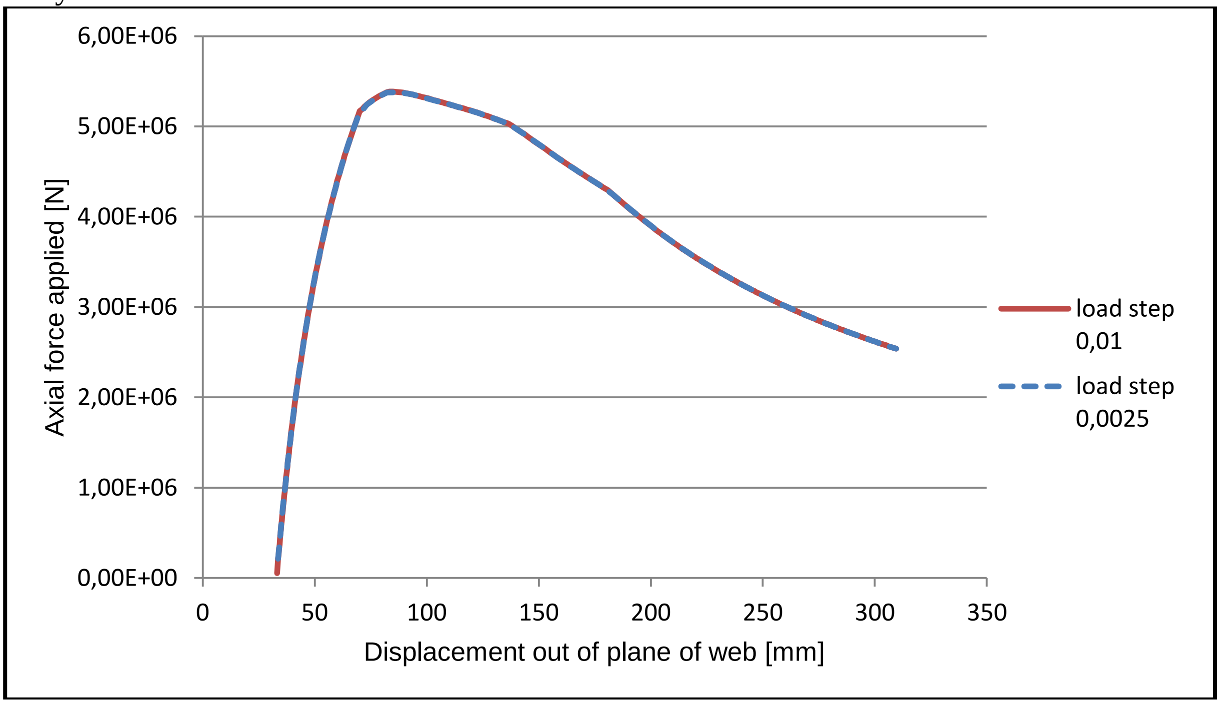


Figure 104: Results using a large and a small load step (lines overlap)

Simple checks in FEM

There are several fairly simple and fast checks to be able to investigate whether the model is producing accurate results. The first one is to check whether the maximum stress is indeed below or equal to the yield stress. Therefore a plot is requested of the Von Mises stress at a large displacement of:

$$\Delta l = \frac{f_y}{E} * 2 * a = \frac{235}{210000} * 2 * 10000 = 22{,}38\, mm \tag{19.11}$$

The Von Mises stress is plotted in Figure 105.

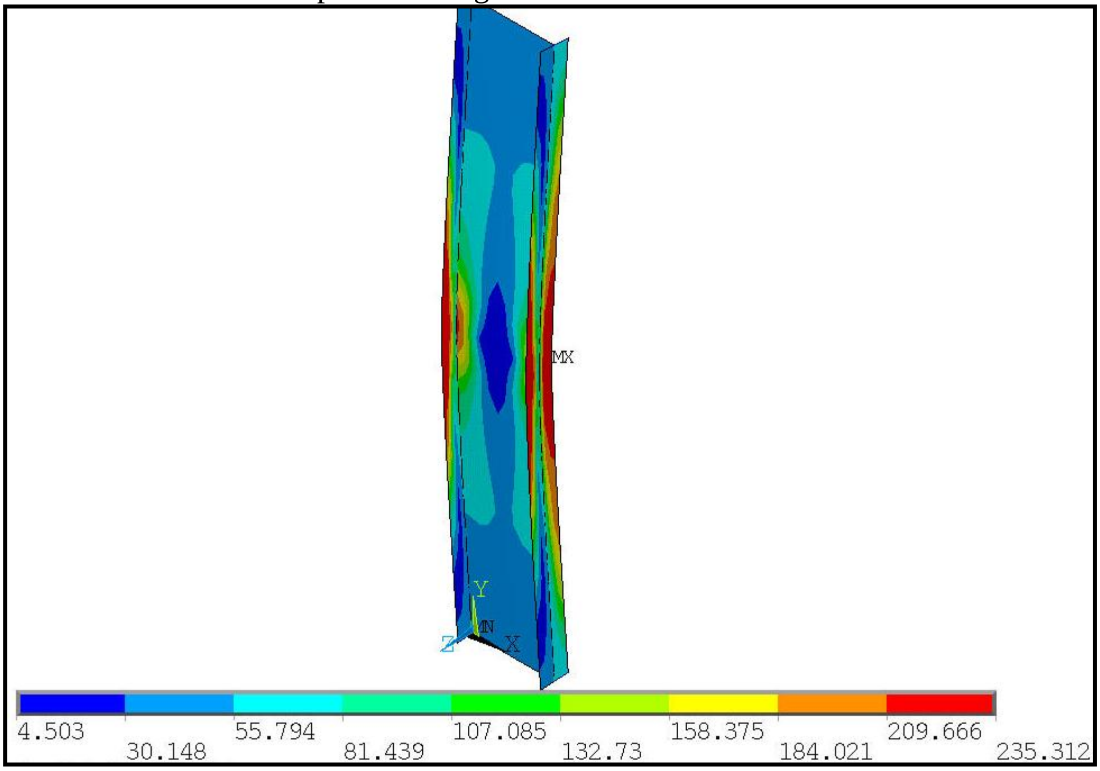


Figure 105: Von Mises stress at large displacement

It is clear that the Von Mises stress is equal to the yield stress in the flanges where the bending moment is largest. If the stress is slightly over the yield stress, this can be explained by the very small increase of the Young's modulus after yielding and by the extrapolation of the stresses from the integration points in the elements to the edges of the elements.

Another very easy check is to check whether the total reactions are zero at the large displacement used above. This is given in Table 9. The results are indeed very small so this is correct.

Reaction force	Value [N]
F_x	$-0{,}14 * 10^{-11}$
F_y	$-0{,}22 * 10^{-8}$
F_z	$0{,}18 * 10^{-11}$

Table 9: Total reactions for all nodes

Also a calculation using a different iteration method for the non-linear calculation is a check whether the calculations performed are correct. The calculation has been performed using a full Newton-Raphson procedure and a modified Newton-Raphson procedure and this gave the same result.

Results

All results are here provided. All dimensions are given in mm and N. The first column is the length of the column. The second column is the result in Ansys using the imperfections in the NEN-EN1993-1-5 and a single half sine wave in the web. The third column is the result in Ansys using the imperfections according to the NEN-EN1993-1-5.

Flanges 250x25, web 250x10

a	m=1	m=?
1000	3462840	3465020
1250	3366190	3367270
1500	3301410	3304190
1750	3168540	3172550
2000	3101020	3103650
2250	3028460	3030760
2500	2949440	2952700
2750	2863160	2866840
3000	2773080	2776570
3250	2680780	2684790
3500	2583490	2587380
3750	2483150	2487510
4000	2382380	2386870
4250	2284260	2288600
4500	2190170	2194140
4750	2095090	2098990
5000	2000340	2004050
5250	1909260	1912180
5500	1823890	1826930
5750	1742940	1744940
6000	1666290	1668670
6250	1592710	1594980
6500	1521830	1524080
6750	1456430	1458430
7000	1393170	1395020
7250	1333060	1334910
7500	1276110	1277690
7750	1222160	1223580
8000	1169990	1171380
8250	1121590	1122780
8500	1074860	1076090
8750	1031070	1032090
9000	989647	990604
9250	950207	951100
9500	912784	913621
9750	877252	878035
10000	843604	844337
10250	811766	812449
10500	781578	782219
10750	752839	753578
11000	725904	726466
11250	700394	700883
11500	676088	676594
11750	653063	653530
12000	631172	631607
12250	610405	610820
12500	590598	590990
12750	571783	572154
13000	553718	554069
13250	536546	536875

Flanges 250x25,web 250x40

a	m=1	m=?
1000	5113720	5126210
1250	5010610	5022490
1500	4801870	4845360
1750	4702680	4702680
2000	4577190	4569580
2250	4415980	4417480
2500	4245680	4246780
2750	4059720	4060170
3000	3860330	3860940
3250	3660490	3659690
3500	3461300	3462700
3750	3261550	3262250
4000	3072010	3071150
4250	2884680	2886900
4500	2706880	2707070
4750	2543070	2543310
5000	2391030	2390490
5250	2247650	2248720
5500	2120790	2120800
5750	2003530	2004030
6000	1896010	1896580
6250	1797160	1796870
6500	1703540	1704260
6750	1617410	1618330
7000	1536870	1537800
7250	1462010	1462850
7500	1391780	1392600
7750	1326250	1327050
8000	1264970	1265490
8250	1207240	1207570
8500	1154490	1153480
8750	1102460	1102770
9000	1054130	1054420

9250	1009370	1009360
9500	966817	966803
9750	927222	927332
10000	889766	889666
10250	854775	854686
10500	821548	821480
10750	790503	790438
11000	761093	761032
11250	733365	733334
11500	707167	707044
11750	682179	682093
12000	658440	658337
12250	636084	635991
12500	614854	614760
12750	594664	594582
13000	575502	575427
13250	557139	557069

Flanges 150x15, web 1500x15

a	m=1	m=?
1500	1956500	1956500
1625	1813080	1813080
1750	1686830	1686830
1875	1574710	1574710
2000	1473350	1473350
2125	1379740	1459560
2250	1294290	1367290
2375	1217730	1284080
2500	1147570	1208840
2625	1084510	1140450
2750	1026180	1075710
2875	972880	1018560
3000	923391	964146
3125	875927	915355
3250	832524	867124
3375	792160	823856
3500	754430	783605
3625	719422	745076
3750	685710	706778
3875	653463	673888
4000	624427	641561
4125	596639	613096
4250	570496	585222
4375	545905	559568
4500	522541	535253
4625	501266	512912
4750	481199	492288
4875	461958	471848
5000	444517	453802
5125	427491	436056
5250	411301	422776
5375	395946	406729
5500	381940	392063
5625	368258	377911
5750	355635	364434
5875	343394	351434
6000	332051	339751
6125	321206	328556
6250	310691	317604
6375	300602	307190
6500	291060	297503
6625	281946	288080
6750	273276	277796
6875	265127	269458
7000	257300	261521
7125	249750	253604
7250	242591	246330
7375	235720	239083
7500	229148	232379
7625	222821	225993

Flanges 500x25, web 1500x15

a	m=1	m=?
1500	8344780	8344780
2000	8132440	8132440
2500	7963420	8103060
3000	7870250	7976100
3500	7760130	7834410
4000	7618320	7571500
4500	7427000	7374170
5000	7194170	7153920
5500	6928930	7007010
6000	6638780	6737680
6500	6342410	6448920
7000	6049980	6129770
7500	5763030	5834210
8000	5477730	5542910
8500	5197970	5296930
9000	4924670	5024170
9500	4667820	4762760
10000	4426770	4515750
10500	4200420	4272800
11000	3988430	4046110
11500	3787960	3843410
12000	3599320	3647450
12500	3421430	3463370
13000	3254440	3290630
13500	3096620	3124840
14000	2949310	2973720
14500	2810070	2832350

15000	2679110	2702360
15500	2555680	2577310
16000	2440260	2460220
16500	2331680	2349860
17000	2229340	2243890
17500	2132800	2146820
18000	2041960	2055250
18500	1955760	1969400
19000	1875320	1888040
19500	1798790	1810930
20000	1727620	1738850
20500	1660170	1670020
21000	1596820	1606020
21500	1537080	1545720
22000	1480610	1488750
22500	1426960	1434850
23000	1376210	1383620
23500	1328210	1335170
24000	1282710	1289310
24500	1239480	1245400
25000	1198330	1203950
25500	1159210	1164540
26000	1121940	1126970

Flanges 500x30, web 1500x40

a	m=1	m=?
1500	8344780	8344780
2000	8132440	8132440
2500	7963420	8103060
3000	7870250	7976100
3500	7760130	7834410
4000	7618320	7571500
4500	7427000	7374170
5000	7194170	7153920
5500	6928930	7007010
6000	6638780	6737680
6500	6342410	6448920
7000	6049980	6129770
7500	5763030	5834210
8000	5477730	5542910
8500	5197970	5296930
9000	4924670	5024170
9500	4667820	4762760
10000	4426770	4515750
10500	4200420	4272800
11000	3988430	4046110
11500	3787960	3843410
12000	3599320	3647450
12500	3421430	3463370
13000	3254440	3290630
13500	3096620	3124840
14000	2949310	2973720
14500	2810070	2832350
15000	2679110	2702360
15500	2555680	2577310
16000	2440260	2460220
16500	2331680	2349860
17000	2229340	2243890
17500	2132800	2146820
18000	2041960	2055250
18500	1955760	1969400
19000	1875320	1888040
19500	1798790	1810930
20000	1727620	1738850
20500	1660170	1670020
21000	1596820	1606020
21500	1537080	1545720
22000	1480610	1488750
22500	1426960	1434850
23000	1376210	1383620
23500	1328210	1335170
24000	1282710	1289310
24500	1239480	1245400
25000	1198330	1203950
25500	1159210	1164540
26000	1121940	1126970

Flanges 500x40, web 500x10

a	m=1	m=?
1500	10453700	10434700
2000	10255000	10218500
2500	10101600	10064100
3000	9913100	9897980
3500	9547570	9537590
4000	9323250	9312170
4500	9115020	9098530
5000	8889700	8880620
5500	8645020	8643380
6000	8386400	8386530
6500	8124540	8123760
7000	7854230	7852540
7500	7574800	7575620
8000	7295820	7300420
8500	7019380	7025270
9000	6746180	6749710
9500	6469020	6475690
10000	6195160	6198280
10500	5927380	5932030
11000	5674870	5679550
11500	5435880	5440110

12000	5206970	5211080
12500	4987200	4991200
13000	4778050	4781960
13500	4576790	4580570
14000	4384230	4387860
14500	4201050	4204520
15000	4026250	4029260
15500	3861150	3863640
16000	3699440	3702240
16500	3550840	3553160
17000	3404510	3407080
17500	3268390	3270410
18000	3139410	3141320
18500	3016220	3017910
19000	2898020	2899730
19500	2786640	2788240
20000	2681750	2683260
20500	2581270	2582690
21000	2486030	2487350
21500	2395690	2396940
22000	2310680	2311850
22500	2229920	2231030
23000	2153200	2154260
23500	2080270	2081240
24000	2011150	2012070
24500	1945360	1946230
25000	1882550	1883380
25500	1822740	1823520
26000	1765600	1766330

Flanges 500x40, web 1000x10

a	m=1	m=?
1500	10513100	10755100
2000	10347100	10567700
2500	10330400	10403500
3000	10278700	10252100
3500	10171700	9861280
4000	9619170	9666980
4500	9566990	9389100
5000	9386820	9198500
5500	9022730	8981720
6000	8733400	8709970
6500	8477830	8460390
7000	8278440	8170270
7500	7988130	7898310
8000	7683790	7583740
8500	7368910	7299880
9000	7052250	6988950
9500	6737510	6705980
10000	6430370	6408130
10500	6138040	6128400
11000	5861880	5862010
11500	5601440	5604140
12000	5353810	5359280
12500	5117030	5118610
13000	4891270	4896940
13500	4676870	4684490
14000	4473330	4476890
14500	4278900	4283780
15000	4094940	4100230
15500	3919400	3926190
16000	3754160	3760360
16500	3596730	3601890
17000	3445980	3452590
17500	3305740	3311940
18000	3172280	3177490
18500	3045340	3050260
19000	2924990	2930020
19500	2810800	2815410
20000	2702720	2707400
20500	2600240	2604770
21000	2503280	2507380
21500	2411420	2415340
22000	2325140	2328860
22500	2243130	2246690
23000	2165310	2168700
23500	2091310	2094560
24000	2021270	2024370
24500	1954670	1957620
25000	1891140	1893730
25500	1830500	1833020
26000	1772740	1775080

Flanges 500x40, web 1000x30

a	m=1	m=?
1500	15821900	15797200
2000	15625800	15769100
2500	15435600	15342300
3000	14817700	15011900
3500	14460400	14481100
4000	14047500	14112100
4500	13586500	13608100
5000	13085900	13134200
5500	12534300	12594100
6000	11932600	12019000
6500	11291900	11393100
7000	10631000	10740300
7500	9973510	10090300
8000	9360720	9459470
8500	8796840	8888920

9000	8278100	8353250
9500	7792470	7862880
10000	7334300	7401190
10500	6918580	6979660
11000	6539460	6595240
11500	6191000	6239970
12000	5867250	5911440
12500	5566120	5607220
13000	5284850	5323010
13500	5023100	5058120
14000	4776610	4808610
14500	4545980	4575090
15000	4331040	4357440
15500	4128270	4152780
16000	3939320	3960970
16500	3761560	3781010
17000	3594240	3612240
17500	3437150	3453830
18000	3289400	3304620
18500	3149290	3163750
19000	3018350	3031020
19500	2895090	2907910
20000	2778740	2790260
20500	2669580	2680750
21000	2567060	2576850
21500	2470070	2479660
22000	2378840	2387730
22500	2292360	2300780
23000	2210530	2218600
23500	2132820	2140440
24000	2059470	2066590
24500	1989850	1996540
25000	1923700	1929950
25500	1860750	1866620
26000	1800820	1806370

Flanges 500x50, web 1500x5

a	m=1	m=?
1500	12456200	12456200
2000	12279000	12279000
2500	11650200	11808000
3000	11367600	11628600
3500	11128600	4230240
4000	10910500	10941300
4500	10683500	10691900
5000	10441800	10433100
5500	10157500	10159300
6000	9880420	9879960
6500	9606380	9596230
7000	9295650	9299710
7500	9007150	9009150
8000	1266170	8717220
8500	8419980	8429380
9000	3598960	8122820
9500	7806850	7815670
10000	7501090	7515160
10500	7199790	7215770
11000	6939280	6927900
11500	3222200	6645040
12000	3559610	6380730
12500	2291980	6122820
13000	1834420	5876260
13500	2012480	5632570
14000	2582670	5402830
14500	3596770	5183970
15000	2123310	4977020
15500	1900430	4775310
16000	1870020	4581250
16500	1838170	4399540
17000	4224900	4220530
17500	849648	4056230
18000	3902560	3897350
18500	3745960	3745770
19000	1667700	3602160
19500	801563	3465920
20000	1859650	3336090
20500	1693210	3212410
21000	1989970	3095060
21500	1726970	2983580
22000	2878930	2878580
22500	2779540	2779840
23000	1993190	2684690
23500	1930240	2594540
24000	2508200	2508720
24500	2426430	2427100
25000	2348510	2349200
25500	2274030	2274670
26000	2203240	2203950

Flanges 500x50, web 1500x15

a	m=1	m=?
1500	14277300	14277300
2000	14034000	14034000
2500	13930900	14183500
3000	13680000	13941700
3500	13408100	13530600
4000	13243700	13072100
4500	13010300	12856000
5000	12700100	12609900
5500	12325100	12196600

6000	11905600	11838600
6500	11461700	11454300
7000	11009300	10912900
7500	10554300	10499400
8000	10104100	10080600
8500	9649640	9676660
9000	9199730	9253270
9500	8759150	8833810
10000	8337230	8417680
10500	7941400	8016050
11000	7568200	7632720
11500	7214750	7263930
12000	6879050	6927750
12500	6561620	6608620
13000	6259830	6302320
13500	5974230	6002340
14000	5703910	5730820
14500	5449080	5472390
15000	5205510	5234110
15500	4978010	5004010
16000	4760650	4785520
16500	4554450	4578770
17000	4362610	4382010
17500	4180390	4197380
18000	4007870	4023720
18500	3844420	3861720
19000	3689980	3706610
19500	3543840	3559610
20000	3405840	3420630
20500	3275230	3288230
21000	3151860	3163810
21500	3035320	3046540
22000	2925680	2936260
22500	2821550	2831740
23000	2722570	2732400
23500	2628990	2638190
24000	2540250	2548860
24500	2455940	2463760
25000	2375340	2382740
25500	2298700	2305710
26000	2225620	2232320

Flanges 500x50, web 1500x10

a	m=1	m=?
1500	13087400	13087400
2000	12846700	12846700
2500	12681500	12819200
3000	12460500	12554900
3500	12051200	12264800
4000	11865000	11827500
4500	11657600	11602200
5000	11410000	11353300
5500	11131500	11029200
6000	10860300	10716900
6500	10336500	10393600
7000	10023900	9985030
7500	9700560	9645960
8000	9371510	9310940
8500	9022550	8990130
9000	8667050	8634980
9500	8304550	8282380
10000	7956740	7940110
10500	7619020	7597490
11000	7302275	7268250
11500	6957900	6949040
12000	6663150	6652130
12500	6376670	6365460
13000	6093480	6090800
13500	5826780	5820470
14000	5573480	5570660
14500	5332570	5333280
15000	5105690	5113720
15500	4884110	4897540
16000	4680950	4693220
16500	4482480	4496850
17000	4298440	4309760
17500	4123500	4133370
18000	3956450	3967130
18500	3798270	3810330
19000	3648500	3660300
19500	3506720	3518480
20000	3372350	3383590
20500	3244730	3254740
21000	3123970	3133340
21500	3010210	3018940
22000	2902690	2910960
22500	2800450	2808690
23000	2703280	2711240
23500	2611500	2618910
24000	2524140	2531060
24500	2441060	2447240
25000	2361810	2367670
25500	2286130	2291680
26000	2214310	2219480

Flanges 500x50, web 1500x20

a	m=1	m=?
1500	15759000	15759000
2000	15517400	15517400
2500	15419800	15663100

3000	15210000	15417100
3500	15094200	15270800
4000	14869200	14633700
4500	14531400	14361400
5000	14085700	14007700
5500	13564600	13566500
6000	13003300	13093500
6500	12415800	12556300
7000	11833400	11879300
7500	11263400	11327400
8000	10711800	10784100
8500	10176700	10275300
9000	9654920	9760060
9500	9151670	9259780
10000	8674080	8752280
10500	8233380	8330840
11000	7820930	7904950
11500	7434440	7505220
12000	7072200	7136620
12500	6729330	6789340
13000	6406690	6461100
13500	6103220	6145600
14000	5817390	5854770
14500	5547870	5583260
15000	5293980	5330490
15500	5055610	5088710
16000	4829850	4861390
16500	4616730	4646690
17000	4417160	4441950
17500	4228930	4252020
18000	4051270	4072930
18500	3883410	3905080
19000	3725160	3745450
19500	3575720	3594730
20000	3434790	3452640
20500	3301620	3317500
21000	3175540	3190320
21500	3057010	3070930
22000	2945550	2958650
22500	2839610	2852260
23000	2739240	2751290
23500	2644110	2655380
24000	2554180	2564740
24500	2468660	2478300
25000	2387110	2396240
25500	2309600	2318230
26000	2235750	2244010

Flanges 500x50, web 1500x25

a	m=1	m=?
1500	17619800	17619800
2000	17561900	17561900
2500	17189800	17481000
3000	17130400	17216000
3500	16909900	17033100
4000	16527000	16844900
4500	16023800	15847800
5000	15416300	15348000
5500	14741400	14839500
6000	14031000	14197500
6500	13309300	13505400
7000	12594300	12720200
7500	11907000	12043200
8000	11259500	11388400
8500	10645800	10782700
9000	10057600	10189400
9500	9493460	9624110
10000	8968210	9069290
10500	8486900	8584240
11000	8040230	8133240
11500	7624460	7710170
12000	7237010	7316360
12500	6874250	6946520
13000	6533000	6598880
13500	6213990	6266790
14000	5914450	5963940
14500	5633830	5676850
15000	5369780	5413900
15500	5122770	5162520
16000	4889340	4926910
16500	4670180	4705300
17000	4464020	4494110
17500	4270560	4298740
18000	4088730	4115050
18500	3916980	3942380
19000	3755530	3778760
19500	3602470	3624540
20000	3459170	3478970
20500	3323290	3341850
21000	3196200	3213210
21500	3076040	3092410
22000	2962960	2978360
22500	2855560	2870430
23000	2753960	2768060
23500	2657520	2670790
24000	2566570	2578970
24500	2480030	2491320
25000	2397740	2408480
25500	2319470	2329650
26000	2244940	2254540

Flanges 500x50, web 1500x30

a	m=1	m=?
1500	19741400	19741400
2000	19483600	19483600
2500	19189100	19171800
3000	18972700	19039500
3500	18584700	18752000
4000	18056900	18565000
4500	17416700	18019400
5000	16669900	17315600
5500	15863300	16016900
6000	15013200	15230500
6500	14148500	14395600
7000	13305200	13493300
7500	12505000	12695500
8000	11759600	11936100
8500	11068100	11238900
9000	10417100	10574100
9500	9798380	9949440
10000	9228130	9351610
10500	8708960	8824300
11000	8232870	8340710
11500	7791300	7890490
12000	7381370	7472320
12500	6999540	7082270
13000	6644850	6718450
13500	6310510	6372590
14000	5999380	6056350
14500	5708890	5759440
15000	5435670	5487250
15500	5181410	5227030
16000	4941690	4984470
16500	4717200	4756840
17000	4506430	4539780
17500	4307380	4339680
18000	4121750	4149090
18500	3945310	3973260
19000	3779900	3805920
19500	3625350	3649640
20000	3479480	3502680
20500	3343310	3363750
21000	3214790	3234120
21500	3093380	3111850
22000	2979070	2995350
22500	2870400	2886690
23000	2767320	2783090
23500	2670260	2685130
24000	2578260	2592270
24500	2490900	2503670
25000	2408030	2420110
25500	2329110	2340470
26000	2253880	2264640

Flanges 500x50, web 1500x35

a	m=1	m=?
1500	20888800	20888800
2000	21374400	21374400
2500	21299300	21185700
3000	20683000	20826900
3500	20170300	20415200
4000	19527100	20182400
4500	18757200	19470100
5000	17873600	18607000
5500	16927300	17139300
6000	15919200	16200900
6500	14900100	15204800
7000	13930200	14175000
7500	13037600	13263200
8000	12212500	12416000
8500	11446000	11645900
9000	10728100	10913600
9500	10062200	10230800
10000	9460740	9603190
10500	8908180	9038460
11000	8403620	8524820
11500	7939570	8050930
12000	7509890	7611500
12500	7111460	7204130
13000	6741850	6825130
13500	6397130	6467170
14000	6076370	6139160
14500	5776210	5833620
15000	5495520	5552590
15500	5234340	5285400
16000	4989440	5035120
16500	4756980	4800470
17000	4542860	4576390
17500	4337980	4371520
18000	4147200	4179710
18500	3970170	3999410
19000	3801980	3831470
19500	3647050	3672380
20000	3499520	3525180
20500	3362200	3383760
21000	3232780	3252950
21500	3109210	3129480
22000	2993650	3011750
22500	2884470	2902020
23000	2780660	2797860
23500	2682800	2699000
24000	2590140	2605280

24500	2502050	2515770
25000	2418520	2431210
25500	2338910	2351000
26000	2263120	2274570

Flanges 500x50, web 1500x40

a	m=1	m=?
1500	22989400	22989400
2000	23420500	23420500
2500	22837200	22933800
3000	22344000	22560400
3500	21731000	22028000
4000	20974000	21717800
4500	20057000	20856900
5000	19034000	19826400
5500	17908100	18202700
6000	16715600	17077000
6500	15532000	15914400
7000	14480900	14750200
7500	13490400	13749300
8000	12601000	12828100
8500	11754100	11996700
9000	10994500	11200800
9500	10295400	10487400
10000	9666890	9823600
10500	9087300	9232970
11000	8559440	8691820
11500	8074540	8197020
12000	7626710	7738630
12500	7214200	7316010
13000	6831480	6922920
13500	6477380	6553340
14000	6146260	6214600
14500	5838940	5899040
15000	5551560	5611510
15500	5284630	5333200
16000	5030050	5079320
16500	4791020	4838050
17000	4572380	4613830
17500	4366140	4401570
18000	4173130	4209390
18500	3994400	4025610
19000	3824690	3856480
19500	3668300	3695690
20000	3519580	3546530
20500	3380620	3404100
21000	3249880	3271290
21500	3125270	3146790
22000	3008860	3028830
22500	2898820	2917670
23000	2794820	2812500
23500	2695980	2713030
24000	2602330	2618510
24500	2513620	2528350
25000	2429190	2442930
25500	2349110	2361990
26000	2272800	2285170

Flanges 500x50, web 1500x45

a	m=1	m=?
1500	25402100	25402100
2000	25321600	25321600
2500	24562400	24929500
3000	23973500	24247000
3500	23255100	23608500
4000	22366300	23194000
4500	21322300	22181000
5000	20094500	20982400
5500	18769200	19154900
6000	17383200	17837200
6500	16101900	16538700
7000	14961200	15255700
7500	13896500	14175400
8000	12922300	13175800
8500	12022400	12279000
9000	11229500	11453900
9500	10507100	10710600
10000	9847770	10028500
10500	9248190	9410050
11000	8702270	8847650
11500	8198970	8331620
12000	7736920	7857580
12500	7311560	7417890
13000	6916920	7011990
13500	6553550	6630570
14000	6214180	6281500
14500	5897170	5961400
15000	5598380	5661160
15500	5326940	5379240
16000	5065940	5118180
16500	4824820	4873710
17000	4603540	4647460
17500	4393890	4437280
18000	4200090	4237780
18500	4018850	4053460
19000	3849160	3880980
19500	3690020	3720350
20000	3541330	3567660
20500	3399370	3425410
21000	3267570	3290370

21500	3143330	3164980
22000	3025160	3046520
22500	2914300	2934900
23000	2809510	2828110
23500	2710390	2727080
24000	2615930	2632020
24500	2526520	2541290
25000	2441430	2455730
25500	2360760	2374270
26000	2283680	2296950

Flanges 500x50, web 1500x50

a	m=1	m=?
1500	27279800	27279800
2000	27221600	27221600
2500	26302900	26656000
3000	25578300	25902600
3500	24748900	25151900
4000	23730200	24623300
4500	22514800	23449700
5000	21045800	22037700
5500	19480500	20011100
6000	17995600	18522900
6500	16647300	17089400
7000	15383500	15747400
7500	14243800	14560500
8000	13209800	13510200
8500	12290500	12558100
9000	11460900	11703700
9500	10700200	10914500
10000	10022200	10209300
10500	9398980	9576730
11000	8837740	8993050
11500	8315050	8456760
12000	7841880	7968740
12500	7403320	7518170
13000	6999900	7102200
13500	6625270	6700990
14000	6275190	6349500
14500	5947470	6019910
15000	5648870	5709800
15500	5366220	5423370
16000	5101740	5158230
16500	4861130	4909000
17000	4635380	4683930
17500	4423350	4468140
18000	4229900	4269660
18500	4044690	4082420
19000	3875360	3906610
19500	3713570	3744730

20000	3562410	3592110
20500	3421540	3446780
21000	3287160	3312600
21500	3161750	3184760
22000	3043630	3065060
22500	2932110	2951880
23000	2825730	2845150
23500	2725070	2743840
24000	2629970	2647990
24500	2539930	2556030
25000	2454590	2469350
25500	2373330	2387570
26000	2296200	2309650

Flanges 1000x100, web 1500x40

a	m=1	m=?
1500	59169200	59169200
2000	59454600	59454600
2500	58423400	57592600
3000	58286300	58761000
3500	58252700	58385500
4000	57778400	57813200
4500	57307600	57459800
5000	56833500	57021200
5500	56301700	56320700
6000	55773100	55827100
6500	55229000	55310000
7000	54663000	54551600
7500	54061000	54003200
8000	53445900	53420200
8500	52803900	52843100
9000	52111900	52172000
9500	51381200	51481500
10000	50627200	50726600
10500	49841500	49945200
11000	49032800	49148400
11500	48221300	48310800
12000	47394200	47482400
12500	46536600	46638700
13000	45649200	45768600
13500	44737700	44827100
14000	43802600	43909500
14500	42850600	42972400
15000	41887700	42019300
15500	40938100	41076900
16000	39993000	40138700
16500	39074900	39218700
17000	38188400	38319000
17500	37303600	37431400
18000	36434600	36576400

18500	35570700	35692500
19000	34715500	34835800
19500	33871600	33988500
20000	33045000	33158900
20500	32232300	32336200
21000	31445900	31546600
21500	30690600	30788500
22000	29960500	30055500

22500	29255300	29347200
23000	28571900	28659700
23500	27906900	27992700
24000	27256700	27339600
24500	26630200	26711600
25000	26018500	26097600
25500	25415400	25495800
26000	24833300	24908400

Annex G: Analytical calculation of I-beam under uniform bending moment without rotational restraint

Stability of a welded I-beam under uniform bending moment

A welded I-beam can also lose stability under a uniform bending moment. Local buckling may occur in the form of multiple half sine waves in the web. In that case the flanges are stiff enough to support the web and only a loss of stability of the web occurs. Also lateral-torsional buckling may occur which is a lateral deflection and a torsional rotation of the beam. The flanges are not stiff enough to support the web. This means that the girder as a whole will buckle. Also an interaction form may occur which may be unfavourable for the girder. Especially for fatigue loaded structures and for serviceability limit state it is important to have a proper estimate of the buckling load of the girder as explained before for the case of uniform compression.

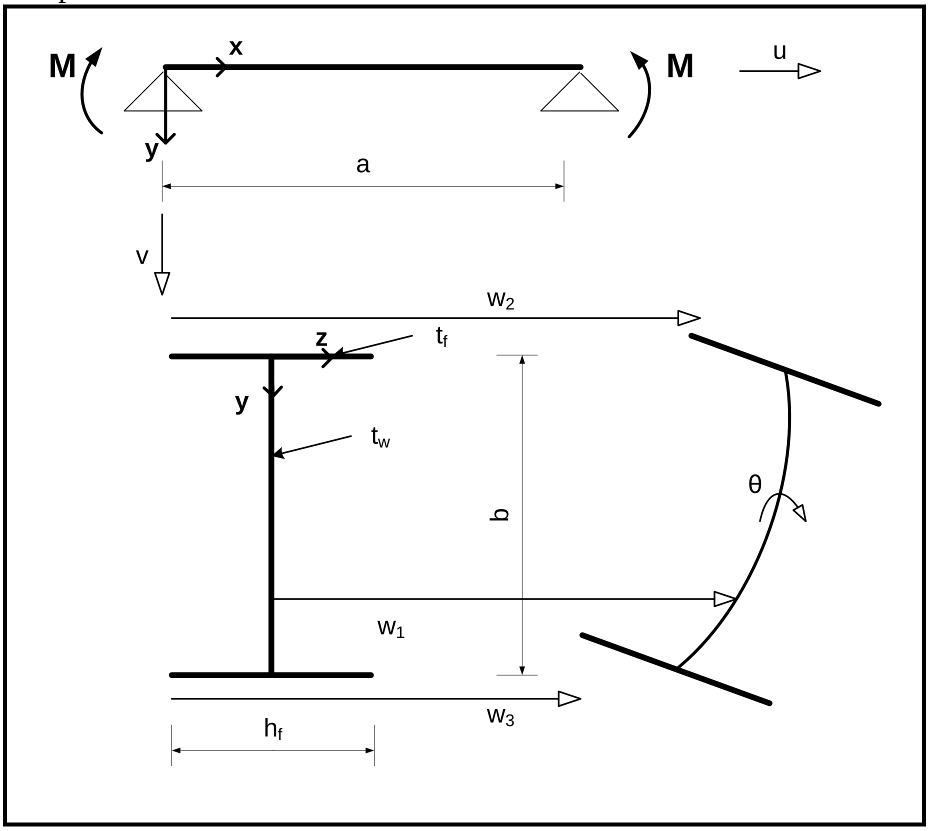


Figure 106: Description of situation for uniform bending moment of welded I-beam

Initial deflection as a sum of lateral-torsional and plate buckling behaviour

The deformation shape of the beam may be described in the following way. For the global behaviour a series of sines is used and for the local behaviour a series of sines multiplied by a series of sines in transverse direction is used. In contradiction to the case of uniform compression also a series of sines as rotation is used. These deformations are the ones of the individual buckling shapes of the lateral-torsional and the local behaviour. The deformation of the web (w_1) is a summation of the local and the global behaviour. The deformation of the top flange (w_2) is described by the global behaviour. The behaviour of the bottom flange (w_3) is also described by the global behaviour. At $y = 0$ and $y = b$ the web and the flange have the same deformation for all x so compatibility between the web and the flange is ensured.

The behaviour of the web is described by:

$$w_1 = \sum_{n=1}^{\infty} A_n * \sin\left(\frac{n\pi x}{a}\right) + \sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms} * \sin\left(\frac{m\pi x}{a}\right) * \sin\left(\frac{s\pi y}{b}\right) + \left(1 - 2 * \frac{y}{b}\right) * \frac{b}{2} * \sum_{t=1}^{\infty} F_t * \sin\left(\frac{t\pi x}{a}\right) \quad (20.1)$$

The behaviour of the top flange is described by:

$$w_2 = \sum_{n=1}^{\infty} A_n * \sin\left(\frac{n\pi x}{a}\right) + \frac{b}{2} * \sum_{t=1}^{\infty} F_t * \sin\left(\frac{t\pi x}{a}\right) \quad (20.2)$$

The behaviour of the bottom flange is described by:

$$w_3 = \sum_{n=1}^{\infty} A_n * \sin\left(\frac{n\pi x}{a}\right) - \frac{b}{2} * \sum_{t=1}^{\infty} F_t * \sin\left(\frac{t\pi x}{a}\right) \quad (20.3)$$

Total potential energy in the welded I-beam

The potential energy in the welded I-beam is a combined calculation. The energy in the I-beam consists of the energy due to the global and local deflections combined with the energy due to rotation of the cross-section. All these terms have been calculated before and are repeated here.

The energy due to the local and global deflection is the same as the total energy in the structure for the case of uniform compression in the welded I-column with hinged welds. This is equation (16.29). The triple summations over m and n are only applicable if $m = n$. If $m \neq n$ that part is equal to 0. The third term of this integral is only applicable for $s = odd$.

$$U_{def} = \sum_{n=1}^{\infty} A_n^2 * n^4 * \pi^4 * \frac{1}{a^3} * \left(\frac{D * b}{4} + \frac{EI_f}{2}\right) + \sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms}^2 * \pi^4 * \frac{ab}{8} * D * \left(\frac{m^2}{a^2} + \frac{s^2}{b^2}\right)^2 + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sum_{s=1}^{\infty} A_n * B_{ms} * n^2 * \frac{\pi^4}{a^2} * \frac{a * b}{\pi * s} * D * \left(\frac{m^2}{a^2} + \frac{s^2 * v}{b^2}\right) \quad (20.4)$$

The energy due to rotation has already been calculated for the derivation of lateral-torsional buckling. This is a part of equation (2.23).

$$U_{rot} = \sum_{t=1}^{\infty} \frac{1}{2} * S_t * F_t^2 * \left(\frac{t\pi}{a}\right)^2 * \frac{a}{2} + \sum_{t=1}^{\infty} \frac{1}{2} * E * C_w * F_t^2 * \left(\frac{t\pi}{a}\right)^4 * \frac{a}{2} \quad (20.5)$$

The total energy due to deflection and rotation is the sum of equation (20.4) and equation (20.5).

$$U_{total} = \sum_{n=1}^{\infty} A_n^2 * n^4 * \pi^4 * \frac{1}{a^3} * \left(\frac{D*b}{4} + \frac{EI_f}{2}\right) + \sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms}^2 * \pi^4 * \frac{ab}{8} * D * \left(\frac{m^2}{a^2} + \frac{s^2}{b^2}\right)^2 + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sum_{s=1}^{\infty} A_n * B_{ms} * n^2 * \frac{\pi^4}{a^2} * \frac{a*b}{\pi*s} * D * \left(\frac{m^2}{a^2} + \frac{s^2*v}{b^2}\right) + \sum_{t=1}^{\infty} \frac{1}{2} * S_t * F_t^2 * \left(\frac{t\pi}{a}\right)^2 * \frac{a}{2} + \sum_{t=1}^{\infty} \frac{1}{2} * E * C_w * F_t^2 * \left(\frac{t\pi}{a}\right)^4 * \frac{a}{2} \tag{20.6}$$

Virtual work done by the external forces on the web

The virtual work done on the web is a more complicated calculation than for the case of uniform compression because the applied stress is not the same everywhere.

The general expression for the work done is the following:

$$T_1 = -\frac{1}{2} * \iiint \sigma * \left(\frac{\partial w_1}{\partial x}\right)^2 dx * dy * dz \tag{20.7}$$

The stress in a fibre can be described by the following equation.

$$\sigma = \frac{M}{I_{yy}} * \frac{b}{2} * \left(1 - 2 * \frac{y}{b}\right) \tag{20.8}$$

Integration over dz is just a multiplication of the equation by the thickness of the web (t_w).

$$T_1 = -\frac{1}{2} * \frac{M}{I_{yy}} * \frac{b}{2} * t_w \int_0^a \int_0^b \left(1 - 2 * \frac{y}{b}\right) * \left(\frac{\partial w_1}{\partial x}\right)^2 dx * dy \tag{20.9}$$

First the derivative of w_1 with respect to x needs to be determined.

$$\frac{\partial w_1}{\partial x} = \sum_{n=1}^{\infty} A_n * \left(\frac{n\pi}{a}\right) * \cos\left(\frac{n\pi x}{a}\right) + \sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms} * \left(\frac{m\pi}{a}\right) * \cos\left(\frac{m\pi x}{a}\right) * \sin\left(\frac{s\pi y}{b}\right) + \left(1 - 2 * \frac{y}{b}\right) * \frac{b}{2} * \sum_{t=1}^{\infty} F_t * \left(\frac{t\pi}{a}\right) * \cos\left(\frac{t\pi x}{a}\right) \tag{20.10}$$

Now the derivative needs to be squared.

$$\begin{aligned}\left(\frac{\partial w_1}{\partial x}\right)^2 &= \left(\sum_{n=1}^{\infty} A_n * \left(\frac{n\pi}{a}\right) * \cos\left(\frac{n\pi x}{a}\right)\right)^2 \\ &+ \left(\sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms} * \left(\frac{m\pi}{a}\right) * \cos\left(\frac{m\pi x}{a}\right) * \sin\left(\frac{s\pi y}{b}\right)\right)^2 \\ &+ \left(\left(1 - 2 * \frac{y}{b}\right) * \frac{b}{2} * \sum_{t=1}^{\infty} F_t * \left(\frac{t\pi}{a}\right) * \cos\left(\frac{t\pi x}{a}\right)\right)^2 \\ &+ 2 * \left(\sum_{n=1}^{\infty} A_n * \left(\frac{n\pi}{a}\right) * \cos\left(\frac{n\pi x}{a}\right)\right) \\ &* \left(\sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms} * \left(\frac{m\pi}{a}\right) * \cos\left(\frac{m\pi x}{a}\right) * \sin\left(\frac{s\pi y}{b}\right)\right) \\ &+ 2 * \left(\sum_{n=1}^{\infty} A_n * \left(\frac{n\pi}{a}\right) * \cos\left(\frac{n\pi x}{a}\right)\right) \\ &* \left(\left(1 - 2 * \frac{y}{b}\right) * \frac{b}{2} * \sum_{t=1}^{\infty} F_t * \left(\frac{t\pi}{a}\right) * \cos\left(\frac{t\pi x}{a}\right)\right) \\ &+ 2 * \left(\sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms} * \left(\frac{m\pi}{a}\right) * \cos\left(\frac{m\pi x}{a}\right) * \sin\left(\frac{s\pi y}{b}\right)\right) \\ &* \left(\left(1 - 2 * \frac{y}{b}\right) * \frac{b}{2} * \sum_{t=1}^{\infty} F_t * \left(\frac{t\pi}{a}\right) * \cos\left(\frac{t\pi x}{a}\right)\right)\end{aligned} \tag{20.11}$$

Equation (20.9) can now be evaluated using equation (20.11). Equation (20.9) contains six terms which are evaluated separately for clarification.

The first term is equal to zero.

$$\int_0^a \int_0^b \left(1 - 2 * \frac{y}{b}\right) * \left(\sum_{n=1}^{\infty} A_n * \left(\frac{n\pi}{a}\right) * \cos\left(\frac{n\pi x}{a}\right)\right)^2 dx * dy = 0 \tag{20.12}$$

The second term is only valid if $s \neq i$ and if $s + i = odd$.

$$\begin{aligned}&\int_0^a \int_0^b \left(1 - 2 * \frac{y}{b}\right) \\ &* \left(\sum_{m=1}^{\infty}\sum_{s=1}^{\infty} B_{ms} * \left(\frac{m\pi}{a}\right) * \cos\left(\frac{m\pi x}{a}\right) * \sin\left(\frac{s\pi y}{b}\right)\right)^2 * dx * dy \\ &= \sum_{m=1}^{\infty}\sum_{s=1}^{\infty}\sum_{i=1}^{\infty} \left(\frac{m\pi}{a}\right)^2 * \frac{a}{2} * \frac{B_{ms} * B_{mi} * s * i}{(s^2 - i^2)^2} * \frac{8 * b}{\pi^2} * 2\end{aligned} \tag{20.13}$$

The third term is equal to zero.

$$\int_0^a \int_0^b \left(1 - 2 * \frac{y}{b}\right) * \left(\left(1 - 2 * \frac{y}{b}\right) * \frac{b}{2} * \sum_{t=1}^{\infty} F_t * \left(\frac{t\pi}{a}\right) * \cos\left(\frac{t\pi x}{a}\right)\right)^2 dx * dy = 0 \tag{20.14}$$

The fourth term is only applicable if $n = m$ and if $s = even$.

$$\int_0^a \int_0^b \left(1 - 2 * \frac{y}{b}\right) * 2 * \left(\sum_{n=1}^{\infty} A_n * \left(\frac{n\pi}{a}\right) * \cos\left(\frac{n\pi x}{a}\right)\right) * \left(\sum_{m=1}^{\infty} \sum_{s=1}^{\infty} B_{ms} * \left(\frac{m\pi}{a}\right) * \cos\left(\frac{m\pi x}{a}\right) * \sin\left(\frac{s\pi y}{b}\right)\right) dx * dy = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \sum_{s=1}^{\infty} 2 * A_n * B_{ms} * n * m * \left(\frac{\pi}{a}\right)^2 * \frac{2 * b}{\pi * s} * \frac{a}{2} \tag{20.15}$$

The fifth term is only valid if $n = t$.

$$\int_0^a \int_0^b \left(1 - 2 * \frac{y}{b}\right) * 2 * \left(\sum_{n=1}^{\infty} A_n * \left(\frac{n\pi}{a}\right) * \cos\left(\frac{n\pi x}{a}\right)\right) * \left(\left(1 - 2 * \frac{y}{b}\right) * \frac{b}{2} * \sum_{t=1}^{\infty} F_t * \left(\frac{t\pi}{a}\right) * \cos\left(\frac{t\pi x}{a}\right)\right) dx * dy = \sum_{n=1}^{\infty} \sum_{t=1}^{\infty} 2 * \frac{b}{3} * \frac{b}{2} * A_n * F_t * \frac{a}{2} * n * t * \left(\frac{\pi}{a}\right)^2 \tag{20.16}$$

The sixth term is only valid if $m = t$ and if $s = odd$.

$$\int_0^a \int_0^b \left(1 - 2 * \frac{y}{b}\right) * 2 * \left(\sum_{m=1}^{\infty} \sum_{s=1}^{\infty} B_{ms} * \left(\frac{m\pi}{a}\right) * \cos\left(\frac{m\pi x}{a}\right) * \sin\left(\frac{s\pi y}{b}\right)\right) * \left(\left(1 - 2 * \frac{y}{b}\right) * \frac{b}{2} * \sum_{t=1}^{\infty} * \left(\frac{t\pi}{a}\right) * \cos\left(\frac{t\pi x}{a}\right)\right) dx * dy = \sum_{t=1}^{\infty} \sum_{m=1}^{\infty} \sum_{s=1}^{\infty} 2 * \frac{b}{2} * F_t * B_{ms} * m * t * \left(\frac{\pi}{a}\right)^2 * \left(\frac{2 * b}{\pi * s} - \frac{16 * b}{\pi^3 * s^3}\right) * \frac{a}{2} \tag{20.17}$$

Therefore the virtual work done on the web can be calculated. All the terms in the total equation are still related to the conditions that are stated before for when they are valid.

$$T_1 = -\frac{1}{2}*\frac{M}{I_{yy}}*\frac{b}{2}*t_w *\left(\sum_{m=1}^{\infty}\sum_{s=1}^{\infty}\sum_{i=1}^{\infty}\left(\frac{m\pi}{a}\right)^2*\frac{a}{2}*\frac{B_{ms}*B_{mi}*s*i}{(s^2-i^2)^2}*\frac{8*b}{\pi^2}*2 + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sum_{s=1}^{\infty}2*A_n*B_{ms}*n*m*\left(\frac{\pi}{a}\right)^2*\frac{2*b}{\pi*s}*\frac{a}{2} + \sum_{n=1}^{\infty}\sum_{t=1}^{\infty}2*\frac{b}{3}*\frac{b}{2}*A_n*F_t*\frac{a}{2}*n*t*\left(\frac{\pi}{a}\right)^2 + \sum_{t=1}^{\infty}\sum_{m=1}^{\infty}\sum_{s=1}^{\infty}2*\frac{b}{2}*F_t*B_{ms}*m*t*\left(\frac{\pi}{a}\right)^2*\left(\frac{2*b}{\pi*s}-\frac{16*b}{\pi^3*s^3}\right)*\frac{a}{2}\right) \tag{20.18}$$

Virtual work done by the external forces on the flange

The general expression for the work done on the top flange is the following:

$$T_2 = -\frac{1}{2}*\iiint \sigma*\left(\frac{\partial w_2}{\partial x}\right)^2 dx*dy*dz \tag{20.19}$$

The stress in the top flange is the following:

$$\sigma = \frac{M}{I_{yy}}*\frac{b}{2} \tag{20.20}$$

Integration over dy and dz is just a multiplication of the equation by the area of the flange.

$$T_2 = -\frac{1}{2}*\frac{M}{I_{yy}}*\frac{b}{2}*t_f*h_f\int_0^a\left(\frac{\partial w_2}{\partial x}\right)^2 dx \tag{20.21}$$

First the derivative of w_2 with respect to x needs to be determined.

$$\frac{\partial w_2}{\partial x} = \sum_{n=1}^{\infty}A_n*\left(\frac{n\pi}{a}\right)*\cos\left(\frac{n\pi x}{a}\right)+\frac{b}{2}*\sum_{t=1}^{\infty}F_t*\left(\frac{t\pi}{a}\right)*\cos\left(\frac{t\pi x}{a}\right) \tag{20.22}$$

Now the derivative needs to be squared.

$$\left(\frac{\partial w_2}{\partial x}\right)^2 = \left(\sum_{n=1}^{\infty}A_n*\left(\frac{n\pi}{a}\right)*\cos\left(\frac{n\pi x}{a}\right)\right)^2 + \left(\frac{b}{2}*\sum_{t=1}^{\infty}F_t*\left(\frac{t\pi}{a}\right)*\cos\left(\frac{t\pi x}{a}\right)\right)^2 + 2*\left(\sum_{n=1}^{\infty}A_n*\left(\frac{n\pi}{a}\right)*\cos\left(\frac{n\pi x}{a}\right)\right)*\left(\frac{b}{2}*\sum_{t=1}^{\infty}F_t*\left(\frac{t\pi}{a}\right)*\cos\left(\frac{t\pi x}{a}\right)\right) \tag{20.23}$$

This allows the calculation of T_2 where the third term is only applicable if $n = t$.

$$T_2 = -\frac{1}{2} * \frac{M}{I_{yy}} * \frac{b}{2} * t_f * h_f * \left(\sum_{n=1}^{\infty} A_n^2 * \left(\frac{n\pi}{a}\right)^2 * \frac{a}{2} + \left(\frac{b}{2}\right)^2 * \sum_{t=1}^{\infty} F_t^2 * \left(\frac{t\pi}{a}\right)^2 * \frac{a}{2} + 2 * \frac{b}{2} \sum_{n=1}^{\infty} \sum_{t=1}^{\infty} A_n * F_t * \left(\frac{n\pi}{a}\right) * \left(\frac{t\pi}{a}\right) * \frac{a}{2} \right) \tag{20.24}$$

The general expression for the work done on the bottom flange is the following:

$$T_3 = -\frac{1}{2} * \iiint \sigma * \left(\frac{\partial w_3}{\partial x}\right)^2 dx * dy * dz \tag{20.25}$$

The stress in the bottom flange is the following:

$$\sigma = -\frac{M}{I_{yy}} * \frac{b}{2} \tag{20.26}$$

Integration over dy and dz is just a multiplication of the equation by the area of the flange.

$$T_3 = \frac{1}{2} * \frac{M}{I_{yy}} * \frac{b}{2} * t_f * h_f \int_0^a \left(\frac{\partial w_3}{\partial x}\right)^2 dx \tag{20.27}$$

First the derivative of w_3 with respect to x needs to be determined.

$$\frac{\partial w_3}{\partial x} = \sum_{n=1}^{\infty} A_n * \left(\frac{n\pi}{a}\right) * \cos\left(\frac{n\pi x}{a}\right) - \frac{b}{2} * \sum_{t=1}^{\infty} F_t * \left(\frac{t\pi}{a}\right) * \cos\left(\frac{t\pi x}{a}\right) \tag{20.28}$$

Now the derivative needs to be squared.

$$\left(\frac{\partial w_3}{\partial x}\right)^2 = \left(\sum_{n=1}^{\infty} A_n * \left(\frac{n\pi}{a}\right) * \cos\left(\frac{n\pi x}{a}\right) \right)^2 + \left(-\frac{b}{2} * \sum_{t=1}^{\infty} F_t * \left(\frac{t\pi}{a}\right) * \cos\left(\frac{t\pi x}{a}\right) \right)^2 + 2 * \left(\sum_{n=1}^{\infty} A_n * \left(\frac{n\pi}{a}\right) * \cos\left(\frac{n\pi x}{a}\right) \right) * \left(-\frac{b}{2} * \sum_{t=1}^{\infty} F_t * \left(\frac{t\pi}{a}\right) * \cos\left(\frac{t\pi x}{a}\right) \right) \tag{20.29}$$

This allows the calculation of T_3 where the third term is only applicable if $n = t$.

$$T_3 = \frac{1}{2} * \frac{M}{I_{yy}} * \frac{b}{2} * t_f * h_f * \left(\sum_{n=1}^{\infty} A_n^2 * \left(\frac{n\pi}{a}\right)^2 * \frac{a}{2} + \left(\frac{b}{2}\right)^2 * \sum_{t=1}^{\infty} F_t^2 * \left(\frac{t\pi}{a}\right)^2 * \frac{a}{2} - 2 * \frac{b}{2} \sum_{n=1}^{\infty} \sum_{t=1}^{\infty} A_n * F_t * \left(\frac{n\pi}{a}\right) \left(\frac{t\pi}{a}\right) * \frac{a}{2} \right) \tag{20.30}$$

Now the total virtual work done on the flanges can be calculated. That is the sum of T_2 (equation (20.24)) and T_3 (equation (20.30)).

$$T_{flanges} = T_2 + T_3$$
$$= -2 * \frac{1}{2} * \frac{M}{I_{yy}} * \frac{b}{2} * t_f * h_f * \left(2 * \frac{b}{2} \sum_{n=1}^{\infty} \sum_{t=1}^{\infty} A_n * F_t * \left(\frac{n\pi}{a}\right)\left(\frac{t\pi}{a}\right) * \frac{a}{2} \right) \quad (20.31)$$

Virtual work done by the external forces on the welded I-beam

The results for the virtual work done on the web (equation (20.18)) and on both flanges (equation (20.31)) can be combined and results in the total virtual work done on the welded I-beam. All the previously mentioned conditions for all the specific terms are still valid but not repeated here.

$$\begin{aligned} T_{total} = & -\frac{1}{2} * \frac{M}{I_{yy}} * \frac{b}{2} * t_w \\ & * \left(\sum_{m=1}^{\infty} \sum_{s=1}^{\infty} \sum_{i=1}^{\infty} \left(\frac{m\pi}{a}\right)^2 * \frac{a}{2} * \frac{B_{ms} * B_{mi} * s * i}{(s^2 - i^2)^2} * \frac{8 * b}{\pi^2} * 2 \right. \\ & + \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \sum_{s=1}^{\infty} 2 * A_n * B_{ms} * n * m * \left(\frac{\pi}{a}\right)^2 * \frac{2 * b}{\pi * s} * \frac{a}{2} \\ & + \sum_{n=1}^{\infty} \sum_{t=1}^{\infty} 2 * \frac{b}{3} * \frac{b}{2} * A_n * F_t * \frac{a}{2} * n * t * \left(\frac{\pi}{a}\right)^2 \\ & \left. + \sum_{t=1}^{\infty} \sum_{m=1}^{\infty} \sum_{s=1}^{\infty} 2 * \frac{b}{2} * F_t * B_{ms} * m * t * \left(\frac{\pi}{a}\right)^2 * \left(\frac{2 * b}{\pi * s} - \frac{16 * b}{\pi^3 * s^3}\right) * \frac{a}{2} \right) \\ & - 2 * \frac{1}{2} * \frac{M}{I_{yy}} * \frac{b}{2} * t_f * h_f \left(2 * \frac{b}{2} \sum_{n=1}^{\infty} \sum_{t=1}^{\infty} A_n * F_t * nt \left(\frac{\pi}{a}\right)^2 * \frac{a}{2} \right) \end{aligned} \quad (20.32)$$

Determine equilibrium condition

The limit state where the construction is just stable is the point where the total potential energy in the beam is equal to the virtual work done by the external forces. This equation is $U_{total} + T_{total} = 0$ but is not shown here because of the length of the equation. The equation is still depending on A_n, B_{ms} and F_t. Now it is clear that only one value of n, m and t should be used. However, there are multiple values of s that may be in the solution. The accuracy of the solution decreases when the ratio a/b increases. (Timoshenko & Gere, 1963, pp. 354-355) This calculation is also made for relatively long girders so therefore a check should be performed to show that the calculation is sufficiently accurate by increasing the maximum value of s by 1 and the difference should be sufficiently small.

The minimum of M is obtained when the derivatives are taken with respect to A_n, B_{ms} and F_t and equated to zero.

$$\begin{aligned}\frac{\partial(U+T)}{\partial A_n} &= 0 = 2*A_n*n^4*\pi^4*\frac{1}{a^3}*\left(\frac{D*b}{4}+\frac{EI_f}{2}\right)\\
&+\sum_{s=1}^{\infty} B_{ms}*n^2*\frac{\pi^4}{a^2}*\frac{a*b}{\pi*s}*D*\left(\frac{m^2}{a^2}+\frac{s^2*v}{b^2}\right)\\
&-\frac{1}{2}*\frac{M}{I_{yy}}*\frac{b}{2}*t_w*\left(\sum_{s=1}^{\infty} 2*B_{ms}*n*m*\left(\frac{\pi}{a}\right)^2*\frac{2*b}{\pi*s}*\frac{a}{2}\right.\\
&\left.+2*\frac{b}{3}*\frac{b}{2}*F_t*\frac{a}{2}*n*t*\left(\frac{\pi}{a}\right)^2\right)\\
&-2*\frac{1}{2}*\frac{M}{I_{yy}}*\frac{b}{2}*t_f*h_f*\left(2*\frac{b}{2}*F_t*n*t*\left(\frac{\pi}{a}\right)^2*\frac{a}{2}\right)\end{aligned} \tag{20.33}$$

$$\begin{aligned}\frac{\partial(U+T)}{\partial B_{ms}} &= 0 = 2*B_{ms}*\pi^4*\frac{ab}{8}*D*\left(\frac{m^2}{a^2}+\frac{s^2}{b^2}\right)^2\\
&+A_n*n^2*\frac{\pi^4}{a^2}*\frac{a*b}{\pi*s}*D*\left(\frac{m^2}{a^2}+\frac{s^2*v}{b^2}\right)\\
&-\frac{1}{2}*\frac{M}{I_{yy}}*\frac{b}{2}*t_w*\left(\sum_{i=1}^{\infty}\left(\frac{m\pi}{a}\right)^2*\frac{a}{2}*\frac{B_{mi}*s*i}{(s^2-i^2)^2}*\frac{8*b}{\pi^2}*2\right.\\
&+2*A_n*n*m*\left(\frac{\pi}{a}\right)^2*\frac{2*b}{\pi*s}*\frac{a}{2}\\
&\left.+2*\frac{b}{2}*F_t*m*t*\left(\frac{\pi}{a}\right)^2*\left(\frac{2*b}{\pi*s}-\frac{16*b}{\pi^3*s^3}\right)*\frac{a}{2}\right)\end{aligned} \tag{20.34}$$

$$\begin{aligned}\frac{\partial(U+T)}{\partial F_t} &= 0 = S_t*F_t*\left(\frac{t\pi}{a}\right)^2*\frac{a}{2}+E*C_w*F_t*\left(\frac{t\pi}{a}\right)^4*\frac{a}{2}\\
&-\frac{1}{2}*\frac{M}{I_{yy}}*\frac{b}{2}*t_w*\left(2*\frac{b}{3}*\frac{b}{2}*A_n*\frac{a}{2}*n*t*\left(\frac{\pi}{a}\right)^2\right.\\
&\left.+\sum_{s=1}^{\infty} 2*\frac{b}{2}*B_{ms}*m*t*\left(\frac{\pi}{a}\right)^2*\left(\frac{2*b}{\pi*s}-\frac{16*b}{\pi^3*s^3}\right)*\frac{a}{2}\right)\\
&-2*\frac{1}{2}*\frac{M}{I_{yy}}*\frac{b}{2}*t_f*h_f\left(2*\frac{b}{2}*A_n*n*t*\left(\frac{\pi}{a}\right)^2*\frac{a}{2}\right)\end{aligned} \tag{20.35}$$

The derivatives may be written in matrix form. All equations are only valid for a single $n = m = t$.

$$\begin{bmatrix} \frac{1}{2}n^4\pi^4\frac{1}{a^3}I_{zz} & -\frac{M}{2a}nt\pi^2 & n^2D\frac{\pi^3 b}{a*1}\left(\frac{m^2}{a^2}+\frac{1^2*v}{b^2}\right) & -\frac{Mb^2t_w\pi}{2I_{yy}a}nm\frac{1}{2} & n^2D\frac{\pi^3 b}{a*3}\left(\frac{m^2}{a^2}+\frac{3^2*v}{b^2}\right) \\ -\frac{M}{2a}nt\pi^2 & S_t\left(\frac{t\pi}{a}\right)^2\frac{a}{2}+EC_w\left(\frac{t\pi}{a}\right)^4\frac{a}{2} & -\frac{1}{4}*\frac{Mb^3t_w}{I_{yy}}mt\frac{\pi}{a}\left(\frac{1}{1}-\frac{8}{\pi^2*1^3}\right) & 0 & -\frac{1}{4}*\frac{Mb^3t_w}{I_{yy}}mt\frac{\pi}{a}\left(\frac{1}{3}-\frac{8}{\pi^2*3^3}\right) \\ n^2D\frac{\pi^3 b}{a*1}\left(\frac{m^2}{a^2}+\frac{1^2*v}{b^2}\right) & -\frac{1}{4}*\frac{Mb^3t_w}{I_{yy}}mt\frac{\pi}{a}\left(\frac{1}{1}-\frac{8}{\pi^2*1^3}\right) & \pi^4\frac{ab}{4}D\left(\frac{m^2}{a^2}+\frac{1^2}{b^2}\right)^2 & -2\frac{Mt_wb^2}{aI_{yy}}m^2\frac{1*2}{(1^2-2^2)^2} & 0 \\ -\frac{Mb^2t_w\pi}{2I_{yy}a}nm\frac{1}{2} & 0 & -2\frac{Mt_wb^2}{aI_{yy}}m^2\frac{1*2}{(1^2-2^2)^2} & \pi^4\frac{ab}{4}D\left(\frac{m^2}{a^2}+\frac{2^2}{b^2}\right)^2 & -2\frac{Mt_wb^2}{aI_{yy}}m^2\frac{3*2}{(3^2-2^2)^2} \\ n^2D\frac{\pi^3 b}{a*3}\left(\frac{m^2}{a^2}+\frac{3^2*v}{b^2}\right) & -\frac{1}{4}*\frac{Mb^3t_w}{I_{yy}}mt\frac{\pi}{a}\left(\frac{1}{3}-\frac{8}{\pi^2*3^3}\right) & 0 & -2\frac{Mt_wb^2}{aI_{yy}}m^2\frac{3*2}{(3^2-2^2)^2} & \pi^4\frac{ab}{4}D\left(\frac{m^2}{a^2}+\frac{3^2}{b^2}\right)^2 \end{bmatrix} * \begin{bmatrix} A_n \\ F_t \\ B_{m1} \\ B_{m2} \\ B_{m3} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Verification of the result

To verify whether calculations have been done correctly, only the lateral-torsional buckling behaviour is checked which means that ($B_{ms} = 0$).

$$\begin{bmatrix} \frac{1}{2} n^4 \pi^4 \frac{1}{a^3} I_{zz} & -\frac{M}{2a} nt\pi^2 \\ -\frac{M}{2a} nt\pi^2 & S_t \left(\frac{t\pi}{a}\right)^2 \frac{a}{2} + EC_w \left(\frac{t\pi}{a}\right)^4 \frac{a}{2} \end{bmatrix} * \begin{bmatrix} A_n \\ F_t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad (20.36)$$

This is the same as the matrix for lateral-torsional buckling (equation (2.34)). Therefore this is correct.

The verification for plate buckling ($A_n = 0$ and $F_t = 0$) can be done by calculating the theoretical buckling factor of 23,9 for a bending moment. This has been done and this is correct.

Annex H: Finite element model for I-beam

The model consists always of a main file where parameters can be changed. To run the model, Ansys has to be started and the main file is loaded. The main file automatically refers to the calculation file to repeat the calculation numerous times for a different beam length. For every calculation the following data is stored into a file:

- Time (From which the percentage of the maximum moment applied can be calculated)
- Displacement out of plane of the web

This file can be loaded into Excel for further processing.

Model using imperfections according to Eurocode

Main file

```
finish
/clear
/batch,list
/filnam,plaatmodel
/title,plastisch gnl plaatmodel
/units,mpa
/nerr,1,99999999
/uis,msgpop,3
seltol

aint=1500
adelta=1000
delta=30
tw=10
tf=25
b=1000
hw=500

na=10
nb=40
nc=3
imp=200
imploc=200/0.7
nmax=200
buckm=100
fy=235
m=5.0e9
iyy=1/12*tw*b*b*b
iyy=iyy+2*1/12*hw*tf*tf*tf
iyy=iyy+2*tf*hw*(b/2)*(b/2)
loadmax=m/(2*nc*b+2*b*(na+1)*(na+2)/
12/na)

*do,i,1,delta,1

/input,c://acer/nonlinliggerv3impeurocoded
isp,txt,,

*enddo

finish
/clear
```

Calculation file

```
parsav,scalar,,
/clear
/batch,list
/filnam,plaatmodel
/title,plastisch gnl plaatmodel
/units,mpa
/nerr,1,99999999
/uis,msgpop,3

parres,new,,

seltol

a=aint+(i-1)*adelta
e=a/imp
eloc=b/imploc

/view,,1,1,1
/prep7
et,1,shell181
r,1,tw
r,2,tf
et,2,beam4
r,3,1e10,1e10,1e10,100,100,0,0,100

mp,ex,1,2.1e5
mp,prxy,1,0.3
tb,biso,1
tbdata,1,fy,21
tbpl,biso,1
/wait,1
```

```
k,,0,0,0
k,,0,0,hw/2
k,,0,0,-hw/2
k,,b,0,0
k,,b,0,hw/2
k,,b,0,-hw/2
k,,0,a,0
k,,0,a,hw/2
k,,0,a,-hw/2
k,,b,a,0
k,,b,a,hw/2
k,,b,a,-hw/2
kplot

mshape,0

real,1
a,1,4,10,7

real,1
lsel,s,tan1,x,1
lsel,a,tan1,x,-1
lesize,all,,,na
lsel,s,tan1,y,1
lsel,a,tan1,y,-1
lesize,all,,,nb
asel,s,area,,1,1,1
amesh,all

real,1

nsel,s,loc,x,b/2
nsel,r,loc,y,0
nsel,r,loc,z,0
d,all,ux,0

nsel,s,loc,x,b/2
nsel,r,loc,y,a
nsel,r,loc,z,0
d,all,ux,0

*do,k,1,na+1,1
nsel,s,loc,y,a
nsel,r,loc,x,(k-1)*b/na
d,all,uz,0
d,all,uy,0.1*(2*(k-1)/na-1)
*enddo

*do,k,1,na+1,1
nsel,s,loc,y,0
nsel,r,loc,x,(k-1)*b/na
d,all,uz,0
d,all,uy,-0.1*(2*(k-1)/na-1)
*enddo

nsel,s,loc,x,0
nsel,a,loc,x,b
nsel,r,loc,z,0
d,all,uz,0

allsel
sbctran

nmiddenboven=node(0.5*b,a,0)
save
finish

/solve
antype,static
pstress,on
allsel
sbctran
solve
finish

/post1
plnsol,u,y,0,,
/wait,2
plnsol,epel,y,0,,
finish

/solve
antype,buckle
bucopt,lanb,1
mxpand,1
solve
finish

/post1
set,last
pldisp,1
plnsol,u,z,0,,
/wait,1
finish

/prep7
/inquire,myjobname,jobname
allsel
```

```
upgeom,eloc,1,1,'%myjobname(1)%',rst
nsel,s,loc,z,eloc
*get,nmidmid,node,,num,min
save

real,2
a,1,2,8,7
a,1,3,9,7
a,4,5,11,10
a,4,6,12,10

real,3
l,1,2
l,1,3
l,4,5
l,4,6
l,7,8
l,7,9
l,10,11
l,10,12
l,1,4
l,7,10

real,2
lsel,s,tan1,y,1
lsel,a,tan1,y,-1
lesize,all,,,nb
lsel,s,tan1,z,1
lsel,a,tan1,z,-1
lesize,all,,,nc

real,2
asel,s,area,,2,5,1
amesh,all

real,3
lsel,s,tan1,z,1
lsel,a,tan1,z,-1
lsel,a,tan1,x,1
lsel,a,tan1,x,-1
lmesh,all

nsel,s,loc,y,0
nsel,a,loc,y,a
nsel,invert
ddele,all,uz

nsel,all
fdele,all
ddele,all,uy

nsel,s,loc,x,b/2
nsel,r,loc,y,a/2
d,all,uy,0

*do,k,1,na+1,1
nsel,s,loc,y,a
nsel,r,loc,x,(k-1)*b/na
f,all,fy,10000*(2*(k-1)/na-1)
*enddo

*do,k,1,na+1,1
nsel,s,loc,y,0
nsel,r,loc,x,(k-1)*b/na
f,all,fy,-10000*(2*(k-1)/na-1)
*enddo

*do,k,1,nb+1,1
nsel,s,loc,y,0+(k-1)*a/nb
cp,next,roty,all
*enddo

save
finish

/solve
antype,static
pstress,on
allsel
sbctran
solve
finish

/solve
antype,buckle
bucopt,lanb,buckm
mxpand,buckm
solve
finish

/post1
set,last
pldisp,1
plnsol,u,z,0,,
/wait,1
finish

/post26
```

```
numvar,200
nsel,s,loc,x,0
nsel,r,loc,y,a/2
*get,nmin,node,0,nxth
*get,ntot,node,,count
nsol,3,nmin,u,z
*do,ndo,2,ntot
*get,nmin,node,nmin,nxth
nsol,196,nmin,u,z
add,3,3,196,,uz_tot
*enddo

*dim,uz_tot,array,buckm
vget,uz_tot(1),3
buckuz=uz_tot(1)
bucktoe=1
*do,j,2,buckm,1
*if,uz_tot(j),gt,buckuz,then
buckuz=uz_tot(j)
bucktoe=j
*else
*endif
*enddo

nsel,all
finish

/solve
antype,buckle
bucopt,lanb,bucktoe
mxpand,bucktoe
solve
finish

/post1
set,last
pldisp,1
plnsol,u,z,0,,
/wait,1
finish

/prep7
/inquire,myjobname,jobname
allsel
upgeom,e,1,bucktoe,'%myjobname(1)%',rs
t
nsel,s,loc,z,e
*get,nmidmid,node,,num,min
nsel,all
cpdele,all

nsel,all
fdele,all

seltol,10
*do,k,1,na+1,1
nsel,s,loc,y,a
nsel,r,loc,x,(k-1)*b/na
f,all,fy,loadmax*(2*(k-1)/na-1)
*enddo

*do,k,1,na+1,1
nsel,s,loc,y,0
nsel,r,loc,x,(k-1)*b/na
f,all,fy,-loadmax*(2*(k-1)/na-1)
*enddo

save
finish

/solve
antype,static
nlgeom,on
outres,all,all
allsel
sbctran

nsubst,nmax,nmax,nmax
kbc,0
solve
save
finish

/post26
numvar,200
allsel
nsol,200,nmiddenboven,u,y,uyneg
filldata,199,,,,-1,0
prod,2,200,199,,uy

nsel,s,loc,y,0
*get,nmin,node,0,nxth
*get,ntot,node,,count
rforce,3,nmin,f,y
*do,ndo,2,ntot
*get,nmin,node,nmin,nxth
rforce,196,nmin,f,y
```

```
add,3,3,196,,fy_tot
*enddo

nsol,198,nmidmid,u,z,uz_echt
filldata,197,,,,e,0
add,4,198,197,,uz

save

*del,_p26_export
*dim,_p26_export,table,nmax,2
vget,_p26_export(1,0),1
vget,_p26_export(1,1),3
vget,_p26_export(1,2),4
/output,'blabla','','.',append
*vwrite,'time','uz'
%14c %14c %14c
*vwrite,_p26_export(1,0),_p26_export(1,1
),_p26_export(1,2)
%14.5g %14.5g %14.5g
/output,term

finish
```

Model using imperfections according to Eurocode using m=1

Main file

```
finish
/clear
/batch,list
/filnam,plaatmodel
/title,plastisch gnl plaatmodel
/units,mpa
/nerr,1,99999999
/uis,msgpop,3
seltol

aint=1500
adelta=500
delta=50
tw=10
tf=50
b=1500
hw=500

na=10
nb=40
nc=3
imp=200
imploc=200/0.7
nmax=200
buckm=250
buckmgl=100
fy=235
m=1.0e10
iyy=1/12*tw*b*b*b
iyy=iyy+2*1/12*hw*tf*tf*tf
iyy=iyy+2*tf*hw*(b/2)*(b/2)
loadmax=m/(2*nc*b+2*b*(na+1)*(na+2)/
12/na)

*do,i,1,delta,1

/input,c://acer/nonlinliggerv3impeurocoded
ispmis1,txt,,

*enddo

finish
/clear
```

Calculation file

```
parsav,scalar,,
/clear
/batch,list
/filnam,plaatmodel
/title,plastisch gnl plaatmodel
/units,mpa
/nerr,1,99999999
/uis,msgpop,3
seltol

parres,new,,

a=aint+(i-1)*adelta
e=a/imp
eloc=b/imploc

/view,,1,1,1
/prep7
et,1,shell181
r,1,tw
r,2,tf
et,2,beam4
r,3,1e2,1e9,1e9,100,100,0,0,100
```

```
mp,ex,1,2.1e5
mp,prxy,1,0.3
tb,biso,1
tbdata,1,fy,21
tbpl,biso,1
/wait,1

k,,0,0,0
k,,0,0,hw/2
k,,0,0,-hw/2
k,,b,0,0
k,,b,0,hw/2
k,,b,0,-hw/2
k,,0,a,0
k,,0,a,hw/2
k,,0,a,-hw/2
k,,b,a,0
k,,b,a,hw/2
k,,b,a,-hw/2
kplot
/wait,1

mshape,0

real,1
a,1,4,10,7

real,1
lsel,s,tan1,x,1
lsel,a,tan1,x,-1
lesize,all,,,na
lsel,s,tan1,y,1
lsel,a,tan1,y,-1
lesize,all,,,nb
asel,s,area,,1,1,1
amesh,all

real,1

nsel,s,loc,x,b/2
nsel,r,loc,y,0
nsel,r,loc,z,0
d,all,ux,0

nsel,s,loc,x,b/2
nsel,r,loc,y,a
nsel,r,loc,z,0
d,all,ux,0

*do,k,1,na+1,1
nsel,s,loc,y,a
nsel,r,loc,x,(k-1)*b/na
d,all,uz,0
d,all,uy,0.1*(2*(k-1)/na-1)
*enddo

*do,k,1,na+1,1
nsel,s,loc,y,0
nsel,r,loc,x,(k-1)*b/na
d,all,uz,0
d,all,uy,-0.1*(2*(k-1)/na-1)
*enddo

nsel,s,loc,x,0
nsel,a,loc,x,b
nsel,r,loc,z,0
d,all,uz,0

allsel
sbctran

nmiddenboven=node(0.5*b,a,0)
save
finish

/solve
antype,static
pstress,on
allsel
sbctran
solve
finish

/post1
plnsol,u,y,0,,
/wait,2
plnsol,epel,y,0,,
finish

/solve
antype,buckle
bucopt,lanb,buckm
mxpand,buckm
solve
finish

/post1
```

```
set,last
pldisp,1
plnsol,u,z,0,,
/wait,1
finish

/post26
numvar,200
nsel,all
*get,nmin,node,0,nxth
*get,ntot,node,,count
nsol,3,nmin,u,z
*do,ndo,2,ntot
*get,nmin,node,nmin,nxth
nsol,196,nmin,u,z
add,3,3,196,,uz_tot
*enddo

*dim,uz_tot,array,buckm
vget,uz_tot(1),3
buckuz=uz_tot(1)
bucktoe=1
*do,j,2,buckm,1
*if,uz_tot(j),gt,buckuz,then
buckuz=uz_tot(j)
bucktoe=j
*else
*endif
*enddo
finish

/solve
antype,buckle
bucopt,lanb,bucktoe
mxpand,bucktoe
solve
finish

/post1
set,last
pldisp,1
plnsol,u,z,0,,
/wait,1
finish

/prep7
/inquire,myjobname,jobname
allsel
upgeom,eloc,1,bucktoe,'%myjobname(1)%
',rst
nsel,s,loc,z,eloc
*get,nmidmid,node,,num,min
save

real,2
a,1,2,8,7
a,1,3,9,7
a,4,5,11,10
a,4,6,12,10

real,3
l,1,2
l,1,3
l,4,5
l,4,6
l,7,8
l,7,9
l,10,11
l,10,12
l,1,4
l,7,10

real,2
lsel,s,tan1,y,1
lsel,a,tan1,y,-1
lesize,all,,,nb
lsel,s,tan1,z,1
lsel,a,tan1,z,-1
lesize,all,,,nc

real,2
asel,s,area,,2,5,1
amesh,all

real,3
lsel,s,tan1,z,1
lsel,a,tan1,z,-1
lsel,a,tan1,x,1
lsel,a,tan1,x,-1
lmesh,all

nsel,s,loc,y,0
nsel,a,loc,y,a
nsel,invert
ddele,all,uz

nsel,all
```

```
fdele,all
ddele,all,uy

nsel,s,loc,x,b/2
nsel,r,loc,y,a/2
d,all,uy,0

*do,k,1,na+1,1
nsel,s,loc,y,a
nsel,r,loc,x,(k-1)*b/na
f,all,fy,100000*(2*(k-1)/na-1)
*enddo

*do,k,1,na+1,1
nsel,s,loc,y,0
nsel,r,loc,x,(k-1)*b/na
f,all,fy,-100000*(2*(k-1)/na-1)
*enddo

*do,k,1,nb+1,1
nsel,s,loc,y,0+(k-1)*a/nb
cp,next,roty,all
*enddo

save
finish

/solve
antype,static
pstress,on
allsel
sbctran

solve
finish

/solve
antype,buckle
bucopt,lanb,buckmgl
mxpand,buckmgl
solve
finish

/post26
numvar,200
nsel,s,loc,x,0
nsel,r,loc,y,a/2
*get,nmin,node,0,nxth
*get,ntot,node,,count
nsol,3,nmin,u,z
*do,ndo,2,ntot
*get,nmin,node,nmin,nxth
nsol,196,nmin,u,z
add,3,3,196,,uz_tott
*enddo

*dim,uz_tott,array,buckmgl
vget,uz_tott(1),3
buckuz=uz_tott(1)
bucktoegl=1
*do,j,2,buckmgl,1
*if,uz_tott(j),gt,buckuz,then
buckuz=uz_tott(j)
bucktoegl=j
*else
*endif
*enddo

nsel,all
finish

/solve
antype,buckle
bucopt,lanb,bucktoegl
mxpand,bucktoegl
solve
finish

/post1
set,last
pldisp,1
plnsol,u,z,0,,
/wait,1
finish

/prep7
/inquire,myjobname,jobname
allsel
upgeom,e,1,bucktoegl,'%myjobname(1)%',
rst
nsel,s,loc,z,e
*get,nmidmid,node,,num,min

nsel,all
cpdele,all

nsel,all
fdele,all
```

```
seltol,10
*do,k,1,na+1,1
nsel,s,loc,y,a
nsel,r,loc,x,(k-1)*b/na
f,all,fy,loadmax*(2*(k-1)/na-1)
*enddo

*do,k,1,na+1,1
nsel,s,loc,y,0
nsel,r,loc,x,(k-1)*b/na
f,all,fy,-loadmax*(2*(k-1)/na-1)
*enddo

save
finish

/solve
antype,static
nlgeom,on
outres,all,all
allsel
sbctran

nsubst,nmax,nmax,nmax
kbc,0
solve
save
finish

/post26
numvar,200
allsel
nsol,200,nmiddenboven,u,y,uyneg
filldata,199,,,,-1,0
prod,2,200,199,,uy

nsel,s,loc,y,0
*get,nmin,node,0,nxth
*get,ntot,node,,count
rforce,3,nmin,f,y
*do,ndo,2,ntot
*get,nmin,node,nmin,nxth
rforce,196,nmin,f,y
add,3,3,196,,fy_tot
*enddo

nsol,198,nmidmid,u,z,uz_echt
filldata,197,,,,e,0
add,4,198,197,,uz

save

*del,_p26_export
*dim,_p26_export,table,nmax,2
vget,_p26_export(1,0),1
vget,_p26_export(1,1),3
vget,_p26_export(1,2),4
/output,'blabla','','.',append
*vwrite,'time','uz'
%14c %14c %14c
*vwrite,_p26_export(1,0),_p26_export(1,1
),_p26_export(1,2)
%14.5g %14.5g %14.5g
/output,term

finish
```

Annex I: Verification and results finite element model for I-beam

Verification of the model

Finite element models should never be used as a black box and the results will depend on the input and on the assumptions done when creating the model. Certain checks will be needed to check the model and verify that indeed the modelled behaviour does approach the real behaviour. All checks will be done on the cross-section with flanges 500x25 mm and a web of 1500x15 mm and a length of 10000 mm.

Linear elastic calculation

First a linear elastic calculation is performed. Therefore a bending moment is applied of 20 kNm. This will result in a stress of:

$$\sigma_{el} = \frac{M}{I} * y_{max} = \frac{20 * 10^6}{1{,}828 * 10^{10}} * 750 = 0{,}82 \frac{N}{mm^2} \tag{22.1}$$

The stress can be checked by requesting a plot of the stresses as done in Figure 107. This should be a linear changing stress distribution over the height of the cross-section. A slight deviation is acceptable because the beams at the end are not infinitely stiff. This can be seen in the plot because the stress is slightly higher at the edges of the flange and slightly lower in the flange at the location of the welds.

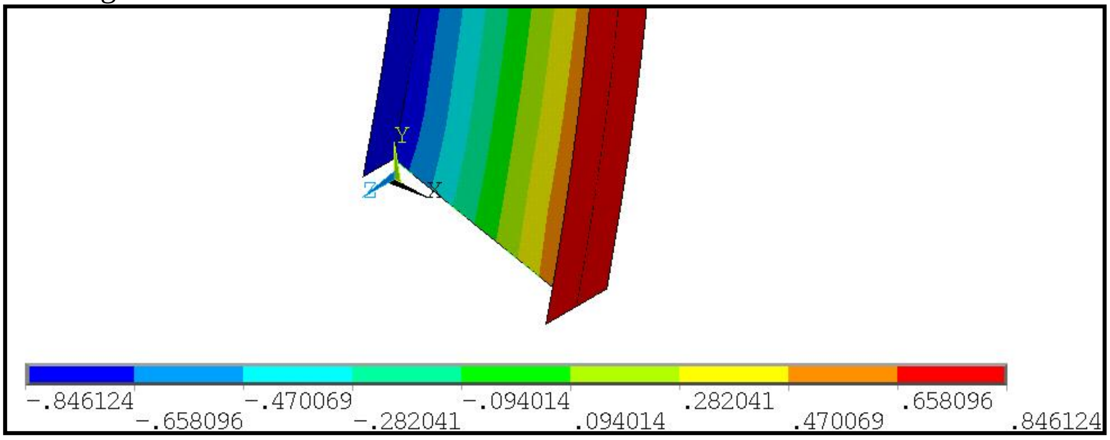


Figure 107: Stresses at one end of the I-beam

At the middle of the I-beam the stresses are requested and the result is equal for all nodes in the flange:

$$\sigma_{el,ansys} = \pm 0{,}82 \frac{N}{mm^2} \tag{22.2}$$

Also the reactions at one end summed up and multiplied by its individual lever arm results in a bending moment that Ansys can calculate automatically:

$$M_{el,ansys} = 0{,}20 * 10^8 Nmm \tag{22.3}$$

This is equal to the bending moment that was applied by calculating the forces. This means that the calculation of the forces and the interpretation of Ansys is correct.

Plastic calculation

A plastic calculation is a non-linear calculation in physical sense. The result should be the plastic bending moment of the cross-section. The centre of gravity of a half of the cross-section is:

$$z_{half} = \frac{t_w * \frac{b}{2} * \frac{b}{4} + t_f * h_f * \frac{b}{2}}{t_w * \frac{b}{2} + t_f * h_f} \quad (22.4)$$

Applying the correct figures results in:

$$z_{half} = \frac{15 * \frac{1500}{2} * \frac{1500}{4} + 25 * 500 * \frac{1500}{2}}{15 * \frac{1500}{2} + 25 * 500} = 572{,}4\ mm \quad (22.5)$$

Therefore the plastic bending moment capacity is:

$$M_{pl} = \left(t_w * \frac{b}{2} + t_f * h_f\right) * z_{half} * f_y \quad (22.6)$$

Applying the correct figures results in:

$$M_{pl} = \left(15 * \frac{1500}{2} + 25 * 500\right) * 2 * 572{,}4 * 235 = 6{,}39 * 10^9\ Nmm \quad (22.7)$$

The bending moment as input in Ansys is $12 * 10^9\ Nmm$ and the load factor at collapse is 0,53 so the plastic load is:

$$M_{pl} = 0{,}53 * 12 * 10^9 = 6{,}36 * 10^9\ Nmm \quad (22.8)$$

This is approximately equal to the plastic bending moment capacity so the plastic analysis is correct. A smaller load step would approximate the plastic capacity even more.

Mesh refinement

The mesh for the I-beam is the same as it is for the I-column. However, the mesh should again be checked by using a coarser mesh or a finer mesh which should have no significant influence on the result. The coarser mesh and the fine mesh are applied and the results are given in Figure 108. Both meshes give approximately the same result and the collapse load differs 1,09%. This is sufficiently small and would probably be even smaller if the load step was reduced. The major difference with the I-column is that the post-critical behaviour is not displayed in this graph as it is a load-controlled analysis.

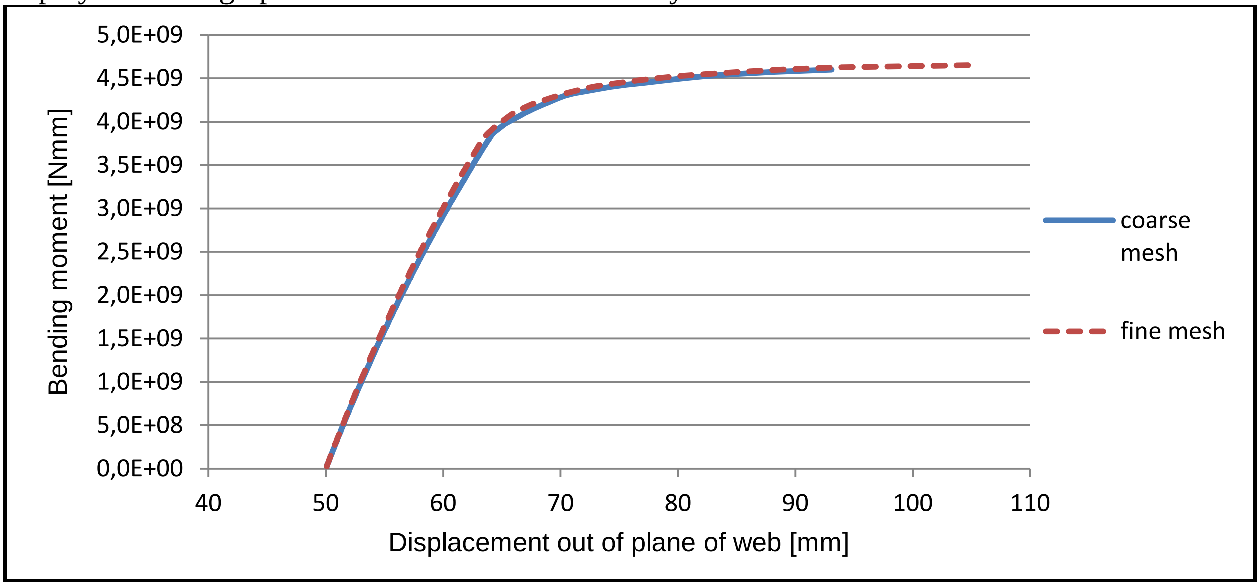


Figure 108: Results using a fine and a coarse mesh

Load step reduced

The load step is also important as explained for the I-column for the development of plastic strains and second order displacements. However, the load step is also important because the analysis is load-controlled. Therefore the accuracy is determined by the load step. The load step for this analysis is set at 0,005 (200 steps) and the reduced load step is set at 0,001 (1000 steps). The results are given in Figure 109.

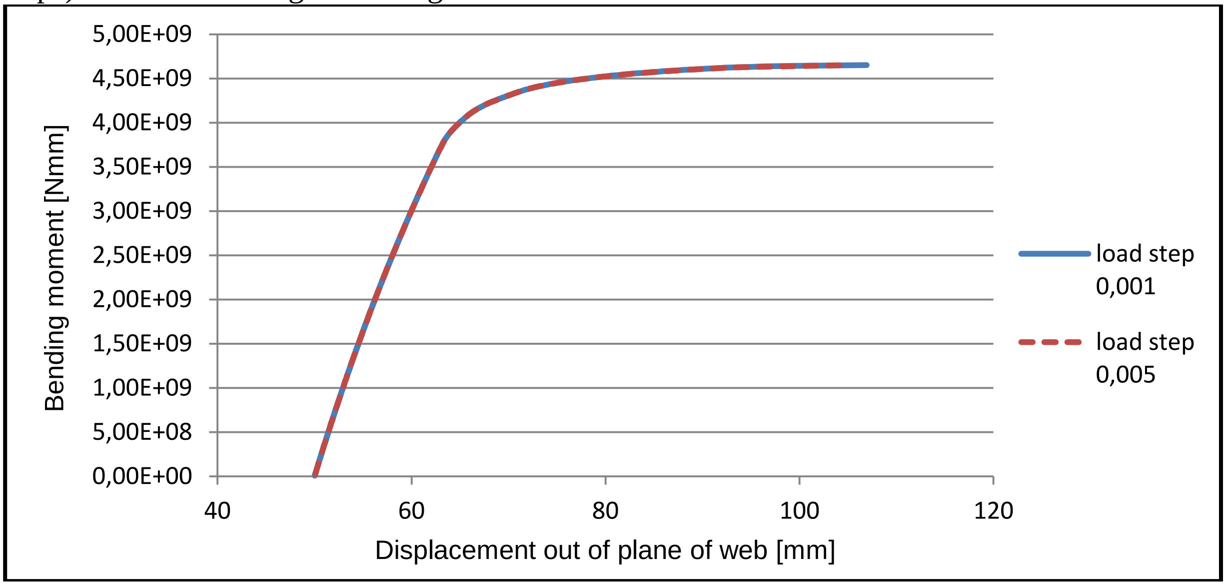


Figure 109: Results using a large and a small load step

Simple checks in FEM

There are several fairly simple and fast checks to be able to investigate whether the model is producing accurate results. The first one is to check whether the maximum stress is indeed below or equal to the yield stress. The Von Mises stress is plotted in Figure 105 at the maximum bending moment.

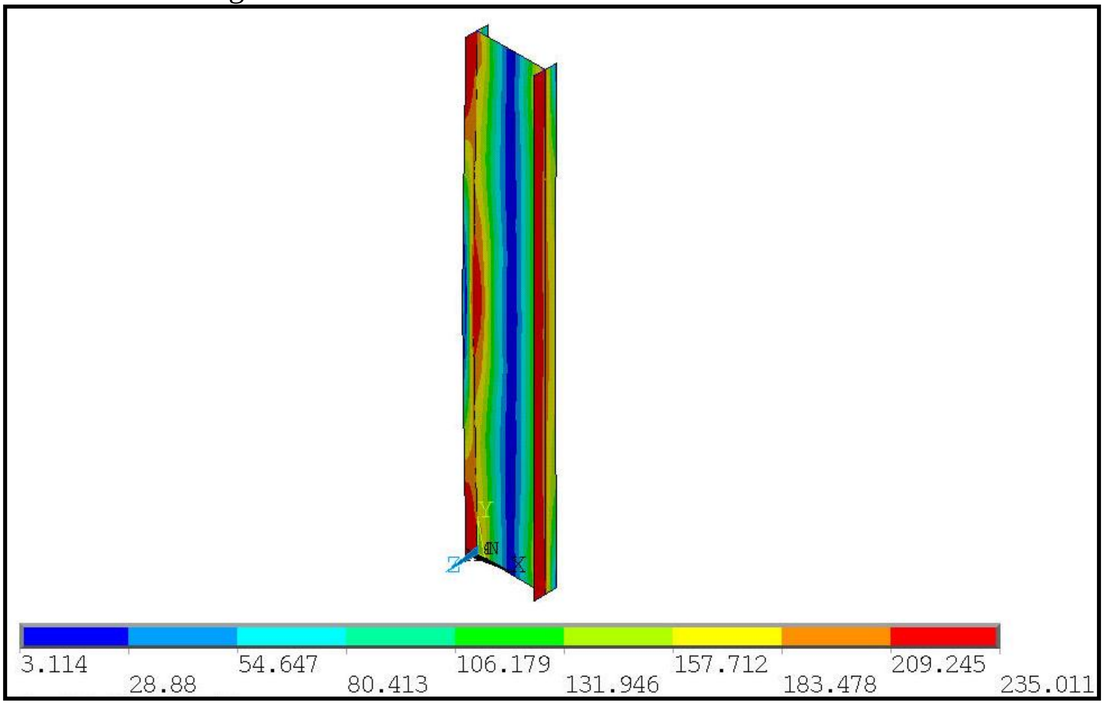


Figure 110: Von Mises stress at maximum loading

It is clear that the Von Mises stress is equal to the yield stress in the flanges where the bending moment is largest. If the stress is slightly over the yield stress, this can be explained by the very small increase of the Young's modulus after yielding and by the extrapolation of the stresses from the integration points in the elements to the edges of the elements.

Another very easy check is to check whether the total reactions are zero at the large displacement used above. This is given in Table 9. The results are indeed very small so this is correct.

Reaction force	Value [N]
F_x	$-0{,}74 * 10^{-10}$
F_y	0
F_z	0

Table 10: Total reactions for all nodes

Results

All results are here provided. All dimensions are given in mm and N. The first column is the length of the column. The second column is the result in Ansys using the imperfections in the NEN-EN1993-1-5 and a single half sine wave in the web. (For many cases this column is omitted.) The third column is the result in Ansys using the imperfections according to the NEN-EN1993-1-5.

Flanges 200x9, web 200x5

a	m=?
1000	93000000
1500	91000000
2000	89000000
2500	87000000
3000	84000000
3500	82000000
4000	80000000
4500	77000000
5000	75000000
5500	72000000
6000	69000000
6500	67000000
7000	64000000
7500	62000000
8000	59000000
8500	57000000
9000	56000000
9500	54000000
10000	52000000
10500	48000000
11000	48000000
11500	45000000
12000	45000000
12500	43000000
13000	41000000
13500	40000000
14000	38000000
14500	36000000
15000	33000000
15500	38000000

Flanges 200x9, web 200x10

a	m=?
1000	104000000
1500	102000000
2000	100000000
2500	97000000
3000	94000000
3500	91000000
4000	88000000
4500	85000000
5000	83000000
5500	79000000
6000	77000000
6500	74000000
7000	71000000
7500	68000000
8000	66000000
8500	64000000
9000	61000000
9500	60000000
10000	58000000
10500	56000000
11000	55000000
11500	53000000
12000	51000000

12500	48000000
13000	48000000
13500	46000000
14000	44000000
14500	43000000
15000	41000000
15500	39000000

Flanges 150x15, web 1500x15

a	m=?
1500	1920000000
1750	1815000000
2000	1710000000
2250	1590000000
2500	1575000000
2750	1455000000
3000	1335000000
3250	1230000000
3500	1080000000
3750	990000000
4000	930000000
4250	855000000
4500	795000000
4750	735000000
5000	690000000
5250	645000000
5500	615000000
5750	585000000
6000	555000000
6250	525000000
6500	495000000
6750	480000000
7000	450000000
7250	420000000
7500	405000000
7750	390000000
8000	375000000
8250	360000000
8500	345000000
8750	330000000

Flanges 400x40, web 500x8

a	m=?
2000	1935000000
4000	1860000000
6000	1770000000
8000	1680000000
10000	1590000000
12000	1515000000
14000	1425000000
16000	1365000000
18000	1290000000
20000	1245000000
22000	1185000000
24000	1140000000
26000	1080000000
28000	1035000000
30000	1005000000
32000	960000000
34000	945000000
36000	900000000
38000	870000000
40000	810000000

Flanges 400x40, web 500x20

a	m=?
2000	2100000000
4000	2025000000
6000	1935000000
8000	1845000000
10000	1770000000
12000	1680000000
14000	1590000000
16000	1515000000
18000	1440000000
20000	1365000000
22000	1290000000
24000	1230000000
26000	1170000000
28000	1125000000
30000	1080000000
32000	1035000000
34000	990000000
36000	960000000
38000	915000000
40000	870000000

Flanges 500x25, web 1000x10

a	m=?
1500	3375000000
2500	3325000000
3500	3275000000
4500	3225000000
5500	3150000000
6500	3075000000
7500	3000000000
8500	2900000000
9500	2800000000

10500	2700000000
11500	2600000000
12500	2475000000
13500	2375000000
14500	2250000000
15500	2150000000
16500	2050000000
17500	1950000000
18500	1850000000
19500	1775000000
20500	1675000000
21500	1600000000
22500	1525000000
23500	1450000000
24500	1400000000
25500	1325000000
26500	1275000000
27500	1225000000
28500	1175000000
29500	1125000000
30500	1075000000

Flanges 500x25, web 1500x10

a	m=1	m=?
1500	5070000000	5250000000
2500	5220000000	5250000000
3500	5130000000	5070000000
4500	5070000000	5010000000
5500	4950000000	4890000000
6500	4800000000	4800000000
7500	4650000000	4680000000
8500	4500000000	4500000000
9500	4320000000	4320000000
10500	4140000000	4140000000
11500	3930000000	3960000000
12500	3720000000	3780000000
13500	3540000000	3600000000
14500	3330000000	3390000000
15500	3150000000	3210000000
16500	3000000000	3030000000
17500	2820000000	2880000000
18500	2670000000	2730000000
19500	2520000000	2580000000
20500	2370000000	2430000000
21500	2250000000	2280000000
22500	2130000000	2160000000
23500	2010000000	2070000000
24500	1920000000	1950000000
25500	1830000000	1860000000
26500	1740000000	1770000000
27500	1650000000	1680000000
28500	1590000000	1620000000
29500	1500000000	1530000000
30500	1440000000	1470000000

Flanges 500x25, web 1500x15

a	m=?
1500	5910000000
2500	5940000000
3500	5700000000
4500	5610000000
5500	5460000000
6500	5310000000
7500	5160000000
8500	4950000000
9500	4740000000
10500	4530000000
11500	4290000000
12500	4050000000
13500	3840000000
14500	3600000000
15500	3390000000
16500	3210000000
17500	3030000000
18500	2850000000
19500	2670000000
20500	2520000000
21500	2400000000
22500	2250000000
23500	2130000000
24500	2040000000
25500	1920000000
26500	1830000000
27500	1740000000
28500	1680000000
29500	1590000000
30500	1530000000

Flanges 500x25, web 1500x20

a	m=?
1500	6600000000
2500	6640000000
3500	6360000000
4500	6240000000
5500	6040000000
6500	5840000000
7500	5640000000
8500	5400000000
9500	5120000000

10500	4880000000
11500	4600000000
12500	4320000000
13500	4040000000
14500	3800000000
15500	3560000000
16500	3360000000
17500	3160000000
18500	2960000000
19500	2800000000
20500	2640000000
21500	2480000000
22500	2360000000
23500	2200000000
24500	2120000000
25500	2000000000
26500	1920000000
27500	1800000000
28500	1720000000
29500	1640000000
30500	1600000000

Flanges 500x25, web 1500x25

a	m=?
1500	7065000000
2500	7290000000
3500	7020000000
4500	6840000000
5500	6615000000
6500	6390000000
7500	6120000000
8500	5805000000
9500	5535000000
10500	5175000000
11500	4860000000
12500	4545000000
13500	4275000000
14500	4005000000
15500	3735000000
16500	3510000000
17500	3285000000
18500	3105000000
19500	2925000000
20500	2745000000
21500	2565000000
22500	2430000000
23500	2295000000
24500	2205000000
25500	2070000000
26500	1980000000

27500	1890000000
28500	1800000000
29500	1755000000
30500	1665000000

Flanges 500x25, web 1500x30

a	m=?
1500	7920000000
2500	7965000000
3500	7650000000
4500	7425000000
5500	7200000000
6500	6930000000
7500	6615000000
8500	6255000000
9500	5895000000
10500	5490000000
11500	5175000000
12500	4815000000
13500	4500000000
14500	4185000000
15500	3915000000
16500	3690000000
17500	3465000000
18500	3240000000
19500	3060000000
20500	2880000000
21500	2700000000
22500	2565000000
23500	2430000000
24500	2340000000
25500	2205000000
26500	2115000000
27500	2025000000
28500	1935000000
29500	1845000000
30500	1800000000

Flanges 500x25, web 1500x35

a	m=?
1500	8700000000
2500	8600000000
3500	8300000000
4500	8050000000
5500	7750000000
6500	7450000000
7500	7100000000
8500	6650000000
9500	6250000000

10500	5800000000
11500	5450000000
12500	5050000000
13500	4750000000
14500	4400000000
15500	4150000000
16500	3850000000
17500	3650000000
18500	3400000000
19500	3200000000
20500	3050000000
21500	2900000000
22500	2750000000
23500	2600000000
24500	2450000000
25500	2350000000
26500	2250000000
27500	2150000000
28500	2050000000
29500	2000000000
30500	1900000000

Flanges 500x25, web 1500x40

a	m=?
1500	9350000000
2500	9240000000
3500	8965000000
4500	8690000000
5500	8305000000
6500	7975000000
7500	7590000000
8500	7040000000
9500	6600000000
10500	6160000000
11500	5720000000
12500	5335000000
13500	5005000000
14500	4675000000
15500	4345000000
16500	4070000000
17500	3850000000
18500	3630000000
19500	3410000000
20500	3245000000
21500	3080000000
22500	2915000000
23500	2805000000
24500	2640000000
25500	2530000000
26500	2420000000
27500	2310000000
28500	2255000000
29500	2145000000
30500	2090000000

Flanges 500x25, web 1500x45

a	m=?
1500	10010000000
2500	9900000000
3500	9570000000
4500	9295000000
5500	8910000000
6500	8470000000
7500	8030000000
8500	7480000000
9500	6930000000
10500	6490000000
11500	6050000000
12500	5610000000
13500	5280000000
14500	4950000000
15500	4620000000
16500	4345000000
17500	4125000000
18500	3850000000
19500	3685000000
20500	3465000000
21500	3300000000
22500	3135000000
23500	3025000000
24500	2860000000
25500	2750000000
26500	2640000000
27500	2530000000
28500	2475000000
29500	2365000000
30500	2310000000

Flanges 500x25, web 1500x50

a	m=1	m=?
1500	10620000000	10680000000
2500	10380000000	10560000000
3500	10080000000	10200000000
4500	9720000000	9840000000
5500	9240000000	9420000000
6500	8760000000	9000000000
7500	8220000000	8520000000
8500	7620000000	7920000000
9500	7080000000	7320000000

10500	6600000000	6840000000
11500	6180000000	6420000000
12500	5760000000	5940000000
13500	5400000000	5580000000
14500	5100000000	5220000000
15500	4800000000	4920000000
16500	4500000000	4620000000
17500	4260000000	4380000000
18500	4020000000	4140000000
19500	3840000000	3960000000
20500	3660000000	3780000000
21500	3480000000	3600000000
22500	3360000000	3420000000
23500	3240000000	3300000000
24500	3060000000	3120000000
25500	2940000000	3000000000
26500	2820000000	2880000000
27500	2760000000	2820000000
28500	2640000000	2700000000
29500	2580000000	2580000000
30500	2460000000	2520000000

Flanges 500x25, web 2000x10

a	m=?
1500	7290000000
2500	7110000000
3500	7110000000
4500	7020000000
5500	6705000000
6500	6480000000
7500	6300000000
8500	6075000000
9500	5895000000
10500	5625000000
11500	5355000000
12500	5085000000
13500	4815000000
14500	4545000000
15500	4320000000
16500	4050000000
17500	3825000000
18500	3600000000
19500	3375000000
20500	3195000000
21500	3015000000
22500	2835000000
23500	2700000000
24500	2520000000
25500	2430000000
26500	2295000000

27500	2160000000
28500	2070000000
29500	1980000000
30500	1890000000

Flanges 500x40, web 1000x10

a	m=?
1500	5130000000
3000	5010000000
4500	4890000000
6000	4740000000
7500	4560000000
9000	4380000000
10500	4170000000
12000	3930000000
13500	3720000000
15000	3510000000
16500	3300000000
18000	3090000000
19500	2910000000
21000	2730000000
22500	2550000000
24000	2430000000
25500	2280000000
27000	2160000000
28500	2070000000
30000	1950000000
31500	1890000000
33000	1800000000
34500	1710000000
36000	1650000000
37500	1590000000
39000	1560000000
40500	1500000000
42000	1410000000
43500	1380000000
45000	1290000000

Flanges 500x50, web 1500x20

a	m=?
1500	10985000000
3000	10920000000
4500	10400000000
6000	10010000000
7500	9620000000
9000	9100000000
10500	8580000000
12000	8060000000
13500	7475000000

15000	6955000000
16500	6435000000
18000	5980000000
19500	5590000000
21000	5200000000
22500	4810000000
24000	4485000000
25500	4225000000
27000	3965000000
28500	3705000000
30000	3510000000
31500	3380000000
33000	3185000000
34500	3055000000
36000	2925000000
37500	2795000000
39000	2665000000
40500	2535000000
42000	2470000000
43500	2275000000
45000	2210000000

Annex J: Calculation tool

Plooicontrole hoofdliggerlijf

Aannames

-Buigend moment met druk in bovenflens! (Geometrie evt spiegelen als druk in onderflens is)
-Voor SLS worden de boven- en onderflens en de verstijvingen als volledig effectief aangemerkt en dus alleen de deelplaten van het lijf op plooi getoetst.
-Voor SLS wordt de spanning niet getoetst in de flenzen.
-Geometrie en belasting voor patch loading dient apart te worden ingevoerd.
-De spanning en plooi in de fictieve pla(a)t(en) wordt niet gecontroleerd
-Verstijvingen zijn als dunwandige strips uitgerekend. Zware verstijvingen zijn dus niet toegestaan.
-Eventuele troggen worden alleen op de individuele platen gereduceerd en het globale knikgedrag van de trogplaat als geheel wordt niet meegenomen omdat dit afhangt van de specifieke geometrie.

Algemene factoren

Rekenwaarde vloeigrens (let op maximale plaatdikte)

Vloeisterkte materiaal $f_{yd} := 235 \cdot \frac{N}{mm^2}$

Afschuifsterkte materiaal $\tau_{yd} := \frac{f_{yd}}{\sqrt{3}}$ $\tau_{yd} = 136 \cdot \frac{N}{mm^2}$

Elasticiteitsmodulus $E_d := 210000 \cdot \frac{N}{mm^2}$

Poisson factor $\nu := 0.3$

$$\varepsilon := \sqrt{\frac{235 \frac{N}{mm^2}}{f_{yd}}}$$ $\varepsilon = 1.00$

$$\eta := \begin{cases} 1.0 & \text{if } f_{yd} > 460 \frac{N}{mm^2} \\ 1.2 & \text{if } f_{yd} \le 460 \frac{N}{mm^2} \end{cases}$$ $\eta = 1.20$

$$G := \frac{E_d}{2 \cdot (1 + \nu)}$$ $G = 80769.23 \cdot \frac{N}{mm^2}$

Partiele factoren

Conform EN 1993-2 Steel bridges
Tabel 6.1 (NB)

weerstand van doorsneden met betrekking tot het overschrijden van de vloeigrens met inbegrip van lokaal plooien $\gamma_{M0} := 1.0$

Weerstand van doorsneden in trek tot aan breuk $\gamma_{M1} := 1.0$

Weerstand van verbindingen $\gamma_{M2} := 1.25$

Invoer geometrie

totale hoogte (bovenkant dekplaat tot onderkant onderflens)
$h_t := 1500 \cdot mm$

Lijfdikte
$t_w := 8 \cdot mm$

Breedte onderflens links: $b_{foL} := 400 \cdot mm$ Breedte onderflens rechts: $b_{foR} := 400mm$

Dikte onderflens links: $t_{foL} := 30 \cdot mm$ Dikte onderflens rechts: $t_{foR} := 30 \cdot mm$

Breedte van de bovenflens (links): $b_{fbL} := 500 \cdot mm$ Breedte van de bovenflens (rechts): $b_{fbR} := 500 \cdot mm$

Dikte van de bovenflens (links): $t_{fbL} := 40 \cdot mm$ Dikte van de bovenflens (rechts): $t_{fbR} := 40 \cdot mm$

Breedte verstijving 1: $b_{v1} := 140 \cdot mm$ Breedte verstijving 2: $b_{v2} := 100 \cdot mm$

Dikte verstijving 1: $t_{v1} := 14 \cdot mm$ Dikte verstijving 2: $t_{v2} := 10 \cdot mm$

Locatie verstijving 1 (t.o.v. bovenkant dekplaat): $h_{v1} := 400 \cdot mm$ Locatie verstijving 2 (t.o.v. bovenkant dekplaat): $h_{v2} := 700 \cdot mm$

Altijd: $h_{v1} \le h_{v2}$ **!**

Bij 1 verstijving, gebruik verstijving 1!

Fictieve bovenplaat:
Oppervlak bovenplaat: $A_{fic.1} := 0mm^2$
Eigen traagheidsmoment bovenplaat: $I_{fic.1} := 0mm^4$
Locatie hart bovenplaat (t.o.v. bovenkant dekplaat): $z_{fic.1} := 0mm$
Reductiefactor axiale druk bovenplaat: $\rho_{fic.1.a} := 1$
Reductiefactor moment bovenplaat: $\rho_{fic.1.m} := 1$

Fictieve onderplaat:
Oppervlak onderplaat: $A_{fic.2} := 0mm^2$
Eigen traagheidsmoment onderplaat: $I_{fic.2} := 0mm^4$
Locatie hart onderplaat (t.o.v. bovenkant dekplaat): $z_{fic.2} := 0mm$
Reductiefactor axiale druk onderplaat: $\rho_{fic.2.a} := 1$

fictief1 := ☐ Aanwezig? fictief2 := ☐ Aanwezig? verstijving1 := ☑ Check Box verstijving2 := ☐ Check Box

trog1 := ☐ Aanwezig? trog2 := ☐ Aanwezig? trog3 := ☐ Aanwezig? trog4 := ☐ Aanwezig?

$$A_{fic.1} := \begin{cases} A_{fic.1} & \text{if fictief1} = 1 \\ 0 & \text{otherwise} \end{cases} \quad I_{fic.1} := \begin{cases} I_{fic.1} & \text{if fictief1} = 1 \\ 0 & \text{otherwise} \end{cases}$$

$A_{fic.1} = 0.00 \cdot mm^2$ $I_{fic.1} = 0.00 \cdot mm^4$

$$A_{fic.2} := \begin{cases} A_{fic.2} & \text{if fictief2} = 1 \\ 0 & \text{otherwise} \end{cases} \quad I_{fic.2} := \begin{cases} I_{fic.2} & \text{if fictief2} = 1 \\ 0 & \text{otherwise} \end{cases}$$

$A_{fic.2} = 0.00 \cdot mm^2$ $I_{fic.2} = 0.00 \cdot mm^4$

$$b_{v1} := \begin{cases} b_{v1} & \text{if verstijving1} = 1 \\ 0.001mm & \text{otherwise} \end{cases} \quad t_{v1} := \begin{cases} t_{v1} & \text{if verstijving1} = 1 \\ 0.001mm & \text{otherwise} \end{cases}$$

$b_{v1} = 140.00 \cdot mm$ $t_{v1} = 14.00 \cdot mm$

$$b_{v2} := \begin{cases} b_{v2} & \text{if verstijving2} = 1 \\ 0.001mm & \text{otherwise} \end{cases} \quad t_{v2} := \begin{cases} t_{v2} & \text{if verstijving2} = 1 \\ 0.001mm & \text{otherwise} \end{cases}$$

$b_{v2} = 0.00 \cdot mm$ $t_{v2} = 0.00 \cdot mm$

Trog 1

Trog 1 boven externe breedte:

$b_{t1.e} := 300mm$

Trog 1 boven binnen breedte:

$b_{t1.i} := 300mm$

Trog 1 boven onderflens trog breedte:

$b_{t1.o} := 250mm$

Trog 1 boven bovenplaat dikte:

$t_{t1.b} := 14mm$

Trog 1 boven lijf trog dikte:

$t_{t1.t} := 8mm$

Trog 1 boven onderflens trog dikte:

$t_{t1.o} := 8mm$

Trog 1 boven hoogte:

$h_{t1.t} := 250mm$

Trog 1 hoogte t.o.v. bovenkant plaat

$z_{t1.bk} := 100mm$

Trog 1 aantal mee te nemen:

$n_{t1.t} := 3$

Bruto oppervlakte trog:

$$A_{t1} := \left[\left(b_{t1.e} + b_{t1.i}\right)\cdot t_{t1.b} + b_{t1.o}\cdot t_{t1.o} + 2\cdot h_{t1.t}\cdot t_{t1.t}\right]\cdot n_{t1.t} \qquad A_{t1} = 43200.00\cdot mm^2$$

Bruto statisch moment:

$$S_{t1} := \left[t_{t1.b}\cdot\left(b_{t1.e} + b_{t1.i}\right)\cdot\frac{t_{t1.b}}{2} + b_{t1.o}\cdot t_{t1.o}\cdot\left(t_{t1.b} + h_{t1.t} + \frac{t_{t1.o}}{2}\right) + 2\cdot h_{t1.t}\cdot t_{t1.t}\cdot\left(\frac{h_{t1.t}}{2} + t_{t1.b}\right)\right]\cdot n_{t1.t} \qquad S_{t1} = 3452400.00\cdot mm^3$$

Bruto zwaartepunt:

$$z_{t1} := \frac{S_{t1}}{A_{t1}} \qquad z_{t1} = 79.92\cdot mm$$

$$I_{t1.eigen} := \frac{1}{12}\cdot\left[\left(b_{t1.e} + b_{t1.i}\right)\cdot t_{t1.b}^3 + 2\cdot t_{t1.t}\cdot h_{t1.t}^3 + b_{t1.o}\cdot t_{t1.o}^3\right]\cdot n_{t1.t}$$

$$I_{t1.steiner} := \left[t_{t1.b}\cdot\left(b_{t1.e} + b_{t1.i}\right)\cdot\left(\frac{t_{t1.b}}{2} - z_{t1}\right)^2 + b_{t1.o}\cdot t_{t1.o}\cdot\left(t_{t1.b} + h_{t1.t} + \frac{t_{t1.o}}{2} - z_{t1}\right)^2 + 2\cdot h_{t1.t}\cdot t_{t1.t}\cdot\left(\frac{h_{t1.t}}{2} + t_{t1.b} - z_{t1}\right)^2\right]\cdot n_{t1.t}$$

Bruto traagheidsmoment:

$$I_{t1} := I_{t1.eigen} + I_{t1.steiner} \qquad I_{t1} = 451070100.00\cdot mm^4$$

Trog 1 reduceren:

$k_\sigma := 4$

$\psi := 1$

$$\lambda_{t1.e} := \sqrt{\frac{f_{yd}}{k_\sigma\cdot\frac{\pi^2\cdot E_d}{12\cdot\left(1-\nu^2\right)}\cdot\left(\frac{t_{t1.b}}{b_{t1.e}}\right)^2}}$$

$$\rho_{t1.e} := \begin{cases} \frac{\lambda_{t1.e} - 0.055\cdot(3+\psi)}{\lambda_{t1.e}^2} & \text{if } \frac{\lambda_{t1.e} - 0.055\cdot(3+\psi)}{\lambda_{t1.e}^2} \le 1 \\ 1 & \text{otherwise} \end{cases} \qquad \rho_{t1.e} = 1.00$$

$$b_{t1.e.eff} := \rho_{t1.e}\cdot b_{t1.e} \qquad b_{t1.e.eff} = 300.00\cdot mm$$

$$\lambda_{t1.i} := \sqrt{\frac{f_{yd}}{k_\sigma\cdot\frac{\pi^2\cdot E_d}{12\cdot\left(1-\nu^2\right)}\cdot\left(\frac{t_{t1.b}}{b_{t1.i}}\right)^2}}$$

$$\rho_{t1.i} := \begin{cases} \frac{\lambda_{t1.i} - 0.055\cdot(3+\psi)}{\lambda_{t1.i}^2} & \text{if } \frac{\lambda_{t1.i} - 0.055\cdot(3+\psi)}{\lambda_{t1.i}^2} \le 1 \\ 1 & \text{otherwise} \end{cases} \qquad \rho_{t1.i} = 1.00$$

$$b_{t1.i.eff} := \rho_{t1.i}\cdot b_{t1.i} \qquad b_{t1.i.eff} = 300.00\cdot mm$$

$$\lambda_{t1.t} := \sqrt{\frac{f_{yd}}{k_\sigma\cdot\frac{\pi^2\cdot E_d}{12\cdot\left(1-\nu^2\right)}\cdot\left(\frac{t_{t1.t}}{h_{t1.t}}\right)^2}}$$

$$\rho_{t1.t} := \begin{cases} \frac{\lambda_{t1.t} - 0.055\cdot(3+\psi)}{\lambda_{t1.t}^2} & \text{if } \frac{\lambda_{t1.t} - 0.055\cdot(3+\psi)}{\lambda_{t1.t}^2} \le 1 \\ 1 & \text{otherwise} \end{cases} \qquad \rho_{t1.t} = 1.00$$

$$h_{t1.t.eff} := \rho_{t1.t}\cdot h_{t1.t} \qquad h_{t1.t.eff} = 250.00\cdot mm$$

$$\lambda_{t1.o} := \sqrt{\frac{f_{yd}}{k_\sigma\cdot\frac{\pi^2\cdot E_d}{12\cdot\left(1-\nu^2\right)}\cdot\left(\frac{t_{t1.o}}{b_{t1.o}}\right)^2}}$$

$$\rho_{t1.o} := \begin{cases} \frac{\lambda_{t1.o} - 0.055\cdot(3+\psi)}{\lambda_{t1.o}^2} & \text{if } \frac{\lambda_{t1.o} - 0.055\cdot(3+\psi)}{\lambda_{t1.o}^2} \le 1 \\ 1 & \text{otherwise} \end{cases} \qquad \rho_{t1.o} = 1.00$$

$$b_{t1.o.eff} := \rho_{t1.o}\cdot b_{t1.o} \qquad b_{t1.o.eff} = 250.00\cdot mm$$

Effectieve oppervlakte trog:

$$A_{t1.eff} := \left[\left(b_{t1.e.eff} + b_{t1.i.eff}\right)\cdot t_{t1.b} + b_{t1.o.eff}\cdot t_{t1.o} + 2\cdot h_{t1.t.eff}\cdot t_{t1.t}\right]\cdot n_{t1.t} \qquad A_{t1.eff} = 43200.00\cdot mm^2$$

Effectief statisch moment:

$$S_{t1.eff} := \left[t_{t1.b}\cdot\left(b_{t1.e.eff} + b_{t1.i.eff}\right)\cdot\frac{t_{t1.b}}{2} + b_{t1.o.eff}\cdot t_{t1.o}\cdot\left(t_{t1.b} + h_{t1.t} + \frac{t_{t1.o}}{2}\right) + 2\cdot h_{t1.t.eff}\cdot t_{t1.t}\cdot\left(\frac{h_{t1.t}}{2} + t_{t1.b}\right)\right]\cdot n_{t1.t}$$

Effectief zwaartepunt:

$$z_{t1.eff} := \frac{S_{t1.eff}}{A_{t1.eff}} \qquad S_{t1.eff} = 3452400.00\cdot mm^3 \qquad z_{t1.eff} = 79.92\cdot mm$$

$$I_{t1.eigen.eff} := \frac{1}{12}\cdot\left[\left(b_{t1.e.eff} + b_{t1.i.eff}\right)\cdot t_{t1.b}^3 + 2\cdot 2\cdot t_{t1.t}\cdot\left(\frac{h_{t1.t.eff}}{2}\right)^3 + b_{t1.o.eff}\cdot t_{t1.o}^3\right]\cdot n_{t1.t}$$

$$I_{t1.steiner.eff} := \left[t_{t1.b} \cdot (b_{t1.e.eff} + b_{t1.i.eff}) \cdot \left(\frac{t_{t1.b}}{2} - z_{t1.eff} \right)^2 + b_{t1.o.eff} \cdot t_{t1.o} \cdot \left(t_{t1.b} + h_{t1.t} + \frac{t_{t1.o}}{2} - z_{t1.eff} \right)^2 \cdots \right.$$
$$\left. + 2 \cdot \frac{h_{t1.t.eff}}{2} \cdot t_{t1.t} \cdot \left(\frac{h_{t1.t.eff}}{4} + t_{t1.b} - z_{t1.eff} \right)^2 + 2 \cdot \frac{h_{t1.t.eff}}{2} \cdot t_{t1.t} \cdot \left(h_{t1.t} - \frac{h_{t1.t.eff}}{4} + t_{t1.b} - z_{t1.eff} \right)^2 \right] \cdot n_{t1.t}$$

Effectief traagheidsmoment:

$I_{t1.eff} := I_{t1.eigen.eff} + I_{t1.steiner.eff}$ $\quad I_{t1.eff} = 451070100.00 \cdot mm^4$

$A_{t1} := \begin{cases} A_{t1} & \text{if } trog1 = 1 \\ 0 & \text{otherwise} \end{cases}$ $\quad A_{t1} = 0.00 \cdot mm^2$

$z_{t1} := \begin{cases} z_{t1} & \text{if } trog1 = 1 \\ 0 & \text{otherwise} \end{cases}$ $\quad z_{t1} = 0.00 \cdot mm$

$I_{t1} := \begin{cases} I_{t1} & \text{if } trog1 = 1 \\ 0 & \text{otherwise} \end{cases}$ $\quad I_{t1} = 0.00 \cdot mm^4$

$A_{t1.eff} := \begin{cases} A_{t1.eff} & \text{if } trog1 = 1 \\ 0 & \text{otherwise} \end{cases}$ $\quad A_{t1.eff} = 0.00 \cdot mm^2$

$z_{t1.eff} := \begin{cases} z_{t1.eff} & \text{if } trog1 = 1 \\ 0 & \text{otherwise} \end{cases}$ $\quad z_{t1.eff} = 0.00 \cdot mm$

$I_{t1.eff} := \begin{cases} I_{t1.eff} & \text{if } trog1 = 1 \\ 0 & \text{otherwise} \end{cases}$ $\quad I_{t1.eff} = 0.00 \cdot mm^4$

Trog 1

Trog 2

Trog 3

Trog 4

berekening shear lag effect

Nieuw waardes t.g.v. shear lag (wanneer gewenst):

$b_{fbL} := \blacksquare \, mm$ $\quad b_{fbR} := \blacksquare \, mm$

$b_{foL} := \blacksquare \, mm$ $\quad b_{foR} := \blacksquare \, mm$

Bruto doorsnede eigenschappen

Berekening Bruto doorsnede eigenschappen

$t_{fbmax} := \begin{cases} t_{fbL} & \text{if } t_{fbL} > t_{fbR} \\ t_{fbR} & \text{otherwise} \end{cases}$ $\qquad t_{fbmax} = 40.00 \cdot mm$

$t_{fomax} := \begin{cases} t_{foL} & \text{if } t_{foL} > t_{foR} \\ t_{foR} & \text{otherwise} \end{cases}$ $\qquad t_{fomax} = 30.00 \cdot mm$

Lijfhoogte

$h_w := h_t - t_{fbmax} - t_{fomax}$ $\qquad h_w = 1430.00 \cdot mm$

Deel lijf 1:	$h_1 := h_{v1} - t_{fbmax}$	$h_1 = 360.00 \cdot mm$
Deel lijf 2:	$h_2 := h_{v2} - t_{fbmax} - h_1$	$h_2 = 300.00 \cdot mm$
Deel lijf 3:	$h_3 := h_w - h_1 - h_2$	$h_3 = 770.00 \cdot mm$
Oppervlakte bovenflens (links)	$A_{fbL} := b_{fbL} \cdot t_{fbL}$	$A_{fbL} = 20000 \cdot mm^2$
Oppervlakte bovenflens (rechts)	$A_{fbR} := b_{fbR} \cdot t_{fbR}$	$A_{fbR} = 20000 \cdot mm^2$
Oppervlakte lijf	$A_w := h_w \cdot t_w$	$A_w = 11440 \cdot mm^2$
Oppervlakte flens onder (links)	$A_{foL} := b_{foL} \cdot t_{foL}$	$A_{foL} = 12000 \cdot mm^2$
Oppervlakte flens onder (rechts)	$A_{foR} := b_{foR} \cdot t_{foR}$	$A_{foR} = 12000 \cdot mm^2$
Oppervlakte lijf langsverstijver 1 (lijf)	$A_{v1} := b_{v1} \cdot t_{v1}$	$A_{v1} = 1960.00 \cdot mm^2$
Oppervlakte lijf langsverstijver 2 (lijf)	$A_{v2} := b_{v2} \cdot t_{v2}$	$A_{v2} = 0.00 \cdot mm^2$

Oppervlakte totaal:

$A_{tot} := A_{fbL} + A_{fbR} + A_w + A_{foL} + A_{foR} + A_{v1} + A_{v2} + A_{fic.1} + A_{fic.2} + A_{t1} + A_{t2} + A_{t3} + A_{t4}$ $\qquad \boxed{A_{tot} = 77400 \cdot mm^2}$

Statisch moment t.o.v. bovenkant profiel:

$$S := A_{fbL} \cdot \frac{t_{fbL}}{2} + A_{fbR} \cdot \frac{t_{fbR}}{2} + A_w \cdot \left(t_{fbmax} + \frac{h_w}{2}\right) + A_{foL} \cdot \left(h_t - \frac{t_{foL}}{2}\right) + A_{foR} \cdot \left(h_t - \frac{t_{foR}}{2}\right) \ldots$$
$$+ A_{v1} \cdot h_{v1} + A_{v2} \cdot h_{v2} + A_{fic.1} \cdot z_{fic.1} + A_{fic.2} \cdot z_{fic.2} + A_{t1} \cdot (z_{t1.bk} + z_{t1}) + A_{t2} \cdot (z_{t2.bk} + z_{t2}) + A_{t3} \cdot (z_{t3.bk} + z_{t3}) + A_{t4} \cdot (z_{t4.bk} + z_{t4})$$

$\boxed{S = 4.586 \times 10^7 \cdot mm^3}$

Afstand zwaartepunt e t.o.v. bk bovenflens	$e_b := \frac{S}{A_{tot}}$	$\boxed{e_b = 593 \cdot mm}$
Afstand zwaartepunt e t.o.v. ok onderflens	$e_o := h_t - e_b$	$\boxed{e_o = 907 \cdot mm}$
Excentriciteit van de onderdelen t.o.v. z	$e_{fbL} := e_b - \frac{t_{fbL}}{2}$	$e_{fbL} = 573 \cdot mm$
	$e_{fbR} := e_b - \frac{t_{fbR}}{2}$	$e_{fbR} = 573 \cdot mm$
	$e_w := e_b - t_{fbmax} - \frac{h_w}{2}$	$e_w = -162 \cdot mm$
	$e_{foL} := e_b - h_t + \frac{t_{foL}}{2}$	$e_{foL} = -892 \cdot mm$
	$e_{foR} := e_b - h_t + \frac{t_{foR}}{2}$	$e_{foR} = -892 \cdot mm$
	$e_{v1} := e_b - h_{v1}$	$e_{v1} = 193 \cdot mm$
	$e_{v2} := e_b - h_{v2}$	$e_{v2} = -107 \cdot mm$
	$e_{fic.1} := e_b - z_{fic.1}$	$e_{fic.1} = 592.52 \cdot mm$
	$e_{fic.2} := e_b - z_{fic.2}$	$e_{fic.2} = 592.52 \cdot mm$

$e_1 := e_b \qquad e_2 := e_b - t_{fbmax} \qquad e_3 := 0 \cdot mm \qquad e_4 := -e_o + t_{fomax} \qquad e_5 := -e_o$

$\boxed{e_1 = 593 \cdot mm} \quad \boxed{e_2 = 553 \cdot mm} \quad \boxed{e_3 = 0 \cdot mm} \quad \boxed{e_4 = -877 \cdot mm} \quad \boxed{e_5 = -907 \cdot mm}$

Traagheidsmoment y

$$I_{y.0} := \frac{1}{12} \cdot \left(b_{fbL} \cdot t_{fbL}^3 + b_{fbR} \cdot t_{fbR}^3 + t_w \cdot h_w^3 + b_{foL} \cdot t_{foL}^3 + b_{foR} \cdot t_{foR}^3 \ldots + b_{v1} \cdot t_{v1}^3 + b_{v2} \cdot t_{v2}^3\right) + I_{fic.1} + I_{fic.2} + I_{t1} + I_{t2} + I_{t3} + I_{t4}$$

$$I_{y.c} := A_{fbL} \cdot e_{fbL}^2 + A_{fbR} \cdot e_{fbR}^2 + A_w \cdot e_w^2 + A_{foL} \cdot e_{foL}^2 + A_{foR} \cdot e_{foR}^2 + A_{v1} \cdot e_{v1}^2 + A_{v2} \cdot e_{v2}^2 + A_{fic.1} \cdot e_{fic.1}^2 + A_{fic.2} \cdot e_{fic.2}^2 \ldots$$
$$+ A_{t1} \cdot (z_{t1.bk} + z_{t1} - e_b)^2 + A_{t2} \cdot (z_{t2.bk} + z_{t2} - e_b)^2 + A_{t3} \cdot (z_{t3.bk} + z_{t3} - e_b)^2 + A_{t4} \cdot (z_{t4.bk} + z_{t4} - e_b)^2$$

$I_y := I_{y.0} + I_{y.c}$ $\qquad \boxed{I_y = 3.4559 \times 10^{10} \cdot mm^4}$

Statisch moment bovenflens:

$S_{fb} := A_{fbL} \cdot e_{fbL} + A_{fbR} \cdot e_{fbR} + A_{fic.1} \cdot |e_{fic.1}| + A_{t1} \cdot |z_{t1.bk} + z_{t1} - e_b| + A_{t2} \cdot |z_{t2.bk} + z_{t2} - e_b|$ $\qquad S_{fb} = 2.2901 \times 10^7 \cdot mm^3$

Statisch moment onderflens:

$S_{fo} := A_{foL} \cdot |e_{foL}| + A_{foR} \cdot |e_{foR}| + A_{fic.2} \cdot |e_{fic.2}| + A_{t3} \cdot |z_{t3.bk} + z_{t3} - e_b| + A_{t4} \cdot |z_{t4.bk} + z_{t4} - e_b|$ $\qquad S_{fo} = 2.1419 \times 10^7 \cdot mm^3$

Maximaal statisch moment

Aandeel verstijvers:

$$(S_o) := \begin{cases} S_{fo} + 0.5 \cdot (e_o - t_{fomax})^2 \cdot t_w & \text{if } e_{v1} > 0 \text{ if } e_{v2} > 0 \\ S_{fo} + 0.5 \cdot (e_o - t_{fomax})^2 \cdot t_w + -e_{v2} \cdot A_{v2} & \text{if } e_{v2} < 0 \text{ if } e_{v1} > 0 \\ S_{fo} + 0.5 \cdot (e_o - t_{fomax})^2 \cdot t_w + -e_{v1} \cdot A_{v1} & \text{if } e_{v2} > 0 \text{ if } e_{v1} < 0 \\ S_{fo} + 0.5 \cdot (e_o - t_{fomax})^2 \cdot t_w + -e_{v1} \cdot A_{v1} + -e_{v2} \cdot A_{v2} & \text{if } e_{v2} < 0 \text{ if } e_{v1} < 0 \end{cases}$$

$$(S_b) := \begin{cases} S_{fb} + 0.5 \cdot (e_b - t_{fbmax})^2 \cdot t_w & \text{if } e_{v2} < 0 \text{ if } e_{v1} < 0 \\ S_{fb} + 0.5 \cdot (e_b - t_{fbmax})^2 \cdot t_w + e_{v2} \cdot A_{v2} & \text{if } e_{v2} > 0 \text{ if } e_{v1} < 0 \\ S_{fb} + 0.5 \cdot (e_b - t_{fbmax})^2 \cdot t_w + e_{v1} \cdot A_{v1} & \text{if } e_{v2} < 0 \text{ if } e_{v1} > 0 \\ S_{fb} + 0.5 \cdot (e_b - t_{fbmax})^2 \cdot t_w + e_{v1} \cdot A_{v1} + e_{v2} \cdot A_{v2} & \text{if } e_{v2} > 0 \text{ if } e_{v1} > 0 \end{cases}$$

$S_o = 24499343.68 \cdot mm^3$

$S_b = 24499343.68 \cdot mm^3$

$S_{max} := \begin{cases} S_o & \text{if } S_o > S_b \\ S_b & \text{otherwise} \end{cases}$ $\qquad \boxed{S_{max} = 2.45 \times 10^7 \cdot mm^3}$

Controleer of S.o = S.b!

Als dit niet correct is, dan is de geometrie niet goed ingevoerd.

Bruto doorsnede eigenschappen

Berekening gereduceerde doorsnede eigenschappen t.g.v. axiale druk:

$a := 15000mm$

Plate buckling effects due to direct stresses at the ultimate limit state (4)

Reduceren doorsnede flenzen t.g.v. axiale druk

$\psi := 1.0$

Bovenflens links:

$k_{\sigma.fbL.a} := 0.43$

$$\sigma_{cr.p.fbL.a} := k_{\sigma.fbL.a} \cdot \frac{\pi^2 \cdot E_d}{12 \cdot (1-\nu^2)} \cdot \left(\frac{t_{fbL}}{b_{fbL}}\right)^2$$

$$\lambda_{p.fbL.a} := \sqrt{\frac{f_{yd}}{\sigma_{cr.p.fbL.a}}}$$

$$\rho_{loc.fbL.a} := \begin{cases} 1.0 & \text{if } \lambda_{p.fbL.a} \le 0.748 \\ \dfrac{\lambda_{p.fbL.a} - 0.188}{\lambda_{p.fbL.a}^2} & \text{if } \lambda_{p.fbL.a} > 0.748 \end{cases}$$

$$\sigma_{cr.c.fbL.a} := \frac{\pi^2 \cdot E_d \cdot t_{fbL}^2}{12 \cdot (1-\nu^2) \cdot a^2}$$

$$\lambda_{c.fbL.a} := \sqrt{\frac{f_{yd}}{\sigma_{cr.c.fbL.a}}}$$

$$\Phi_{c.fbL.a} := 0.5 \cdot \left[1 + 0.21 \cdot (\lambda_{c.fbL.a} - 0.2) + \lambda_{c.fbL.a}^2\right]$$

$$\chi_{c.fbL.a} := \frac{1}{\Phi_{c.fbL.a} + \sqrt{\Phi_{c.fbL.a}^2 - \lambda_{c.fbL.a}^2}}$$

$$\xi_{c.fbL.a} := \begin{cases} 1 & \text{if } \dfrac{\sigma_{cr.p.fbL.a}}{\sigma_{cr.c.fbL.a}} > 1 \\ 0 & \text{if } 0 > \dfrac{\sigma_{cr.p.fbL.a}}{\sigma_{cr.c.fbL.a}} \\ \dfrac{\sigma_{cr.p.fbL.a}}{\sigma_{cr.c.fbL.a}} & \text{if } 0 \le \dfrac{\sigma_{cr.p.fbL.a}}{\sigma_{cr.c.fbL.a}} \le 1 \end{cases}$$

$\xi_{c.fbL.a} = 1.00$

$\rho_{c.fbL.a} := (\rho_{loc.fbL.a} - \chi_{c.fbL.a}) \cdot \xi_{c.fbL.a} \cdot (2 - \xi_{c.fbL.a}) + \chi_{c.fbL.a}$ $\quad \rho_{c.fbL.a} = 1.00$

$b_{fbL.a.eff} := \rho_{c.fbL.a} \cdot b_{fbL}$ $\quad b_{fbL.a.eff} = 500.00 \cdot mm$

Bovenflens rechts:

$k_{\sigma.fbR.a} := 0.43$

$$\sigma_{cr.p.fbR.a} := k_{\sigma.fbR.a} \cdot \frac{\pi^2 \cdot E_d}{12 \cdot (1-\nu^2)} \cdot \left(\frac{t_{fbR}}{b_{fbR}}\right)^2$$

$$\lambda_{p.fbR.a} := \sqrt{\frac{f_{yd}}{\sigma_{cr.p.fbR.a}}}$$

$$\rho_{loc.fbR.a} := \begin{cases} 1.0 & \text{if } \lambda_{p.fbR.a} \le 0.748 \\ \dfrac{\lambda_{p.fbR.a} - 0.188}{\lambda_{p.fbR.a}^2} & \text{if } \lambda_{p.fbR.a} > 0.748 \end{cases}$$

$$\sigma_{cr.c.fbR.a} := \frac{\pi^2 \cdot E_d \cdot t_{fbR}^2}{12 \cdot (1-\nu^2) \cdot a^2}$$

$$\lambda_{c.fbR.a} := \sqrt{\frac{f_{yd}}{\sigma_{cr.c.fbR.a}}}$$

$$\Phi_{c.fbR.a} := 0.5 \cdot \left[1 + 0.21 \cdot (\lambda_{c.fbR.a} - 0.2) + \lambda_{c.fbR.a}^2\right]$$

$$\chi_{c.fbR.a} := \frac{1}{\Phi_{c.fbR.a} + \sqrt{\Phi_{c.fbR.a}^2 - \lambda_{c.fbR.a}^2}}$$

$$\xi_{c.fbR.a} := \begin{cases} 1 & \text{if } \dfrac{\sigma_{cr.p.fbR.a}}{\sigma_{cr.c.fbR.a}} > 1 \\ 0 & \text{if } 0 > \dfrac{\sigma_{cr.p.fbR.a}}{\sigma_{cr.c.fbR.a}} \\ \dfrac{\sigma_{cr.p.fbR.a}}{\sigma_{cr.c.fbR.a}} & \text{if } 0 \le \dfrac{\sigma_{cr.p.fbR.a}}{\sigma_{cr.c.fbR.a}} \le 1 \end{cases}$$

$\xi_{c.fbR.a} = 1.00$

$\rho_{c.fbR.a} := (\rho_{loc.fbR.a} - \chi_{c.fbR.a}) \cdot \xi_{c.fbR.a} \cdot (2 - \xi_{c.fbR.a}) + \chi_{c.fbR.a}$ $\quad \rho_{c.fbR.a} = 1.00$

$b_{fbR.a.eff} := \rho_{c.fbR.a} \cdot b_{fbR}$ $\quad b_{fbR.a.eff} = 500.00 \cdot mm$

Onderflens links:

$k_{\sigma.foL.a} := 0.43$

$$\sigma_{cr.p.foL.a} := k_{\sigma.foL.a} \cdot \frac{\pi^2 \cdot E_d}{12 \cdot (1-\nu^2)} \cdot \left(\frac{t_{foL}}{b_{foL}}\right)^2$$

$$\lambda_{p.foL.a} := \sqrt{\frac{f_{yd}}{\sigma_{cr.p.foL.a}}}$$

$$\rho_{loc.foL.a} := \begin{cases} 1.0 & \text{if } \lambda_{p.foL.a} \le 0.748 \\ \dfrac{\lambda_{p.foL.a} - 0.188}{\lambda_{p.foL.a}^2} & \text{if } \lambda_{p.foL.a} > 0.748 \end{cases}$$

$$\sigma_{cr.c.foL.a} := \frac{\pi^2 \cdot E_d \cdot t_{foL}^2}{12 \cdot (1-\nu^2) \cdot a^2}$$

$$\lambda_{c.foL.a} := \sqrt{\frac{f_{yd}}{\sigma_{cr.c.foL.a}}}$$

$$\Phi_{c.foL.a} := 0.5 \cdot \left[1 + 0.21 \cdot (\lambda_{c.foL.a} - 0.2) + \lambda_{c.foL.a}^2\right]$$

$$\chi_{c.foL.a} := \frac{1}{\Phi_{c.foL.a} + \sqrt{\Phi_{c.foL.a}^2 - \lambda_{c.foL.a}^2}}$$

$$\xi_{c.foL.a} := \begin{cases} 1 & \text{if } \dfrac{\sigma_{cr.p.foL.a}}{\sigma_{cr.c.foL.a}} > 1 \\ 0 & \text{if } 0 > \dfrac{\sigma_{cr.p.foL.a}}{\sigma_{cr.c.foL.a}} \\ \dfrac{\sigma_{cr.p.foL.a}}{\sigma_{cr.c.foL.a}} & \text{if } 0 \le \dfrac{\sigma_{cr.p.foL.a}}{\sigma_{cr.c.foL.a}} \le 1 \end{cases}$$

$\xi_{c.foL.a} = 1.00$

$\rho_{c.foL.a} := (\rho_{loc.foL.a} - \chi_{c.foL.a}) \cdot \xi_{c.foL.a} \cdot (2 - \xi_{c.foL.a}) + \chi_{c.foL.a}$ $\quad \rho_{c.foL.a} = 1.00$

$b_{foL.a.eff} := \rho_{c.foL.a} \cdot b_{foL}$ $\quad b_{foL.a.eff} = 400.00 \cdot mm$

Onderflens rechts:

$$k_{\sigma.foR.a} := 0.43$$

$$\sigma_{cr.p.foR.a} := k_{\sigma.foR.a} \cdot \frac{\pi^2 \cdot E_d}{12 \cdot \left(1 - \nu^2\right)} \cdot \left(\frac{t_{foR}}{b_{foR}}\right)^2$$

$$\lambda_{p.foR.a} := \sqrt{\frac{f_{yd}}{\sigma_{cr.p.foR.a}}}$$

$$\rho_{loc.foR.a} := \begin{cases} 1.0 & \text{if } \lambda_{p.foR.a} \le 0.748 \\ \dfrac{\lambda_{p.foR.a} - 0.188}{\lambda_{p.foR.a}^2} & \text{if } \lambda_{p.foR.a} > 0.748 \end{cases}$$

$$\sigma_{cr.c.foR.a} := \frac{\pi^2 \cdot E_d \cdot t_{foR}^2}{12 \cdot \left(1 - \nu^2\right) \cdot a^2}$$

$$\lambda_{c.foR.a} := \sqrt{\frac{f_{yd}}{\sigma_{cr.c.foR.a}}}$$

$$\Phi_{c.foR.a} := 0.5 \cdot \left[1 + 0.21 \cdot \left(\lambda_{c.foR.a} - 0.2\right) + \lambda_{c.foR.a}^2\right]$$

$$\chi_{c.foR.a} := \frac{1}{\Phi_{c.foR.a} + \sqrt{\Phi_{c.foR.a}^2 - \lambda_{c.foR.a}^2}}$$

$$\xi_{c.foR.a} := \begin{cases} 1 & \text{if } \dfrac{\sigma_{cr.p.foR.a}}{\sigma_{cr.c.foR.a}} > 1 \\ 0 & \text{if } 0 > \dfrac{\sigma_{cr.p.foR.a}}{\sigma_{cr.c.foR.a}} \\ \dfrac{\sigma_{cr.p.foR.a}}{\sigma_{cr.c.foR.a}} & \text{if } 0 \le \dfrac{\sigma_{cr.p.foR.a}}{\sigma_{cr.c.foR.a}} \le 1 \end{cases} \qquad \xi_{c.foR.a} = 1.00$$

$$\rho_{c.foR.a} := \left(\rho_{loc.foR.a} - \chi_{c.foR.a}\right) \cdot \xi_{c.foR.a} \cdot \left(2 - \xi_{c.foR.a}\right) + \chi_{c.foR.a} \qquad \rho_{c.foR.a} = 1.00$$

$$b_{foR.a.eff} := \rho_{c.foR.a} \cdot b_{foR} \qquad b_{foR.a.eff} = 400.00 \cdot mm$$

Reduceren doorsnede flenzen t.g.v. axiale druk

Lijf zonder verstijver (DRUK)

$$k_{\sigma.w.a} := 4$$

$$\sigma_{cr.p.w.a} := k_{\sigma.w.a} \cdot \frac{\pi^2 \cdot E_d}{12 \cdot \left(1 - \nu^2\right)} \cdot \left(\frac{t_w}{h_w}\right)^2$$

$$\lambda_{p.w.a} := \sqrt{\frac{f_{yd}}{\sigma_{cr.p.w.a}}}$$

$$\rho_{loc.w.a} := \begin{cases} 1.0 & \text{if } \lambda_{p.w.a} \le 0.5 + \sqrt{0.085 - 0.055\psi} \\ \dfrac{\lambda_{p.w.a} - 0.055 \cdot (3 + \psi)}{\lambda_{p.w.a}^2} & \text{if } \lambda_{p.w.a} > 0.5 + \sqrt{0.085 - 0.055\psi} \end{cases} \qquad \rho_{loc.w.a} = 0.30$$

$$\sigma_{cr.c.w.a} := \frac{\pi^2 \cdot E_d \cdot t_w^2}{12 \cdot \left(1 - \nu^2\right) \cdot a^2}$$

$$\lambda_{c.w.a} := \sqrt{\frac{f_{yd}}{\sigma_{cr.c.w.a}}}$$

$$\Phi_{c.w.a} := 0.5 \cdot \left[1 + 0.21 \cdot \left(\lambda_{c.w.a} - 0.2\right) + \lambda_{c.w.a}^2\right]$$

$$\chi_{c.w.a} := \frac{1}{\Phi_{c.w.a} + \sqrt{\Phi_{c.w.a}^2 - \lambda_{c.w.a}^2}}$$

$$\xi_{c.w.a} := \begin{cases} 1 & \text{if } \dfrac{\sigma_{cr.p.w.a}}{\sigma_{cr.c.w.a}} > 1 \\ 0 & \text{if } 0 > \dfrac{\sigma_{cr.p.w.a}}{\sigma_{cr.c.w.a}} \\ \dfrac{\sigma_{cr.p.w.a}}{\sigma_{cr.c.w.a}} & \text{if } 0 \le \dfrac{\sigma_{cr.p.w.a}}{\sigma_{cr.c.w.a}} \le 1 \end{cases} \qquad \xi_{c.w.a} = 1.00$$

$$\rho_{c.w.a} := \left(\rho_{loc.w.a} - \chi_{c.w.a}\right) \cdot \xi_{c.w.a} \cdot \left(2 - \xi_{c.w.a}\right) + \chi_{c.w.a} \qquad \rho_{c.w.a} = 0.30$$

$$h_{w.a0.eff} := \rho_{c.w.a} \cdot h_w \qquad h_{w.a0.eff} = 422.90 \cdot mm$$

Oppervlak bruto met 0 verstijvingen:

$$A_{a0} := b_{fbL} \cdot t_{fbL} + b_{fbR} \cdot t_{fbR} + b_{foL} \cdot t_{foL} + b_{foR} \cdot t_{foR} + h_w \cdot t_w + A_{fic.1} + A_{fic.2} + A_{t1} + A_{t2} + A_{t3} + A_{t4} \qquad A_{a0} = 75440.00 \cdot mm^2$$

Statisch moment bruto met 0 verstijvingen:

$$S_{a0} := b_{fbL} \cdot t_{fbL} \cdot \frac{t_{fbL}}{2} + b_{fbR} \cdot t_{fbR} \cdot \frac{t_{fbR}}{2} + h_w \cdot t_w \cdot \left(t_{fbmax} + \frac{h_w}{2}\right) + b_{foL} \cdot t_{foL} \cdot \left(h_t - \frac{t_{foL}}{2}\right) + b_{foR} \cdot t_{foR} \cdot \left(h_t - \frac{t_{foR}}{2}\right) \dots$$
$$+ A_{fic.1} \cdot z_{fic.1} + A_{fic.2} \cdot z_{fic.2} + A_{t1} \cdot \left(z_{t1.bk} + z_{t1}\right) + A_{t2} \cdot \left(z_{t2.bk} + z_{t2}\right) + A_{t3} \cdot \left(z_{t3.bk} + z_{t3}\right) + A_{t4} \cdot \left(z_{t4.bk} + z_{t4}\right)$$

$$S_{a0} = 45077200.00 \cdot mm^3$$

Zwaartepunt bruto met 0 verstijvingen:

$$e_{b.a0} := \frac{S_{a0}}{A_{a0}} \qquad e_{b.a0} = 597.52 \cdot mm$$

Oppervlak effectief met 0 verstijvingen:

$A_{a0.eff} := b_{fbL.a.eff} \cdot t_{fbL} + b_{fbR.a.eff} \cdot t_{fbR} + b_{foL.a.eff} \cdot t_{foL} + b_{foR.a.eff} \cdot t_{foR} + h_{w.a0.eff} \cdot t_w + \rho_{fic.1.a} \cdot A_{fic.1} + \rho_{fic.2.a} \cdot A_{fic.2} \ldots$
$+ A_{t1.eff} + A_{t2.eff} + A_{t3.eff} + A_{t4.eff}$

$A_{a0.eff} = 67383.20 \cdot mm^2$

Statisch moment netto met 0 verstijvingen:

$S_{a0.eff} := b_{fbL.a.eff} \cdot t_{fbL} \cdot \frac{t_{fbL}}{2} + b_{fbR.a.eff} \cdot t_{fbR} \cdot \frac{t_{fbR}}{2} + h_{w.a0.eff} \cdot t_w \cdot \left(t_{fbmax} + \frac{h_w}{2}\right) + b_{foL.a.eff} \cdot t_{foL} \cdot \left(h_t - \frac{t_{foL}}{2}\right) \ldots$
$+ b_{foR.a.eff} \cdot t_{foR} \cdot \left(h_t - \frac{t_{foR}}{2}\right) + \rho_{fic.1.a} \cdot A_{fic.1} \cdot z_{fic.1} + \rho_{fic.2.a} \cdot A_{fic.2} \cdot z_{fic.2} + A_{t1.eff} \cdot (z_{t1.bk} + z_{t1.eff}) + A_{t2.eff} \cdot (z_{t2.bk} + z_{t2.eff}) \ldots$
$+ A_{t3.eff} \cdot (z_{t3.bk} + z_{t3.eff}) + A_{t4.eff} \cdot (z_{t4.bk} + z_{t4.eff})$

$S_{a0.eff} = 38994318.17 \cdot mm^3$

Zwaartepunt netto met 0 verstijvingen:

$e_{b.a0.eff} := \frac{S_{a0.eff}}{A_{a0.eff}}$ $e_{b.a0.eff} = 578.69 \cdot mm$

Lijf zonder verstijver (DRUK)

Lijf met 1 verstijver (DRUK)

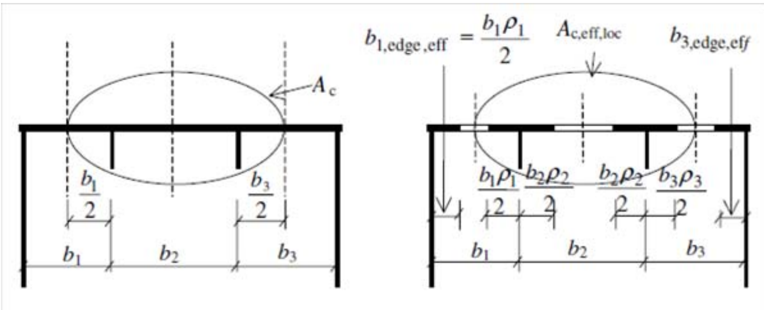


Figure 4.4: Stiffened plate under uniform compression (uit NEN-EN1993-1-5 pagina 18)

$b_1 := h_1$ $b_1 = 360.00 \cdot mm$

$b_2 := h_2 + h_3$ $b_2 = 1070.00 \cdot mm$

$b_{lum} := b_1 + b_2$ $b_{lum} = 1430.00 \cdot mm$

$A_{sl} := \frac{b_{lum}}{2} \cdot t_w + b_{v1} \cdot t_{v1}$ $A_{sl} = 7680.00 \cdot mm^2$

$e_{sti} := \frac{b_{v1} \cdot t_{v1} \cdot \left(\frac{b_{v1}}{2} + \frac{t_w}{2}\right)}{A_{sl}}$ $e_{sti} = 18.89 \cdot mm$

$I_{sl} := \frac{1}{12} \cdot \frac{b_{lum}}{2} \cdot t_w^3 + \frac{1}{12} \cdot t_{v1} \cdot b_{v1}^3 + \frac{b_{lum}}{2} \cdot t_w \cdot e_{sti}^2 + b_{v1} \cdot t_{v1} \cdot \left(e_{sti} - \frac{b_{v1}}{2} - \frac{t_w}{2}\right)^2$ $I_{sl} = 11225659.17 \cdot mm^4$

$a_{c.1} := 4.33 \cdot \sqrt[4]{\frac{I_{sl} \cdot (b_1)^2 \cdot (b_2)^2}{t_w^3 \cdot b_{lum}}}$ $a_{c.1} = 5317.81 \cdot mm$

$\sigma_{cr.sl.1} := \begin{cases} \frac{1.05 \cdot E_d}{A_{sl}} \cdot \frac{\sqrt{I_{sl} \cdot t_w^3 \cdot b_{lum}}}{b_1 \cdot b_2} & \text{if } a \geq a_{c.1} \\ \frac{\pi^2 \cdot E_d \cdot I_{sl}}{A_{sl} \cdot a^2} + \frac{E_d \cdot t_w^3 \cdot b_{lum} \cdot a^2}{4 \cdot \pi^2 (1 - \nu^2) \cdot A_{sl} \cdot (b_1)^2 \cdot (b_2)^2} & \text{if } a < a_{c.1} \end{cases}$ $\sigma_{cr.sl.1} = 213.68 \cdot \frac{N}{mm^2}$

Subpanelen reduceren:

Lijf h.1 zonder verstijvingen:

$k_{\sigma.h1.a} := 4$

$\sigma_{cr.p.h1.a} := k_{\sigma.h1.a} \cdot \frac{\pi^2 \cdot E_d}{12 \cdot (1 - \nu^2)} \cdot \left(\frac{t_w}{h_1}\right)^2$

$\lambda_{p.h1.a} := \sqrt{\frac{f_{yd}}{\sigma_{cr.p.h1.a}}}$

$\rho_{loc.h1.a} := \begin{cases} 1.0 & \text{if } \lambda_{p.h1.a} \leq 0.5 + \sqrt{0.085 - 0.055\psi} \\ \frac{\lambda_{p.h1.a} - 0.055 \cdot (3 + \psi)}{\lambda_{p.h1.a}^2} & \text{if } \lambda_{p.h1.a} > 0.5 + \sqrt{0.085 - 0.055\psi} \end{cases}$

$\sigma_{cr.c.h1.a} := \frac{\pi^2 \cdot E_d \cdot t_w^2}{12 (1 - \nu^2) \cdot a^2}$

$\lambda_{c.h1.a} := \sqrt{\frac{f_{yd}}{\sigma_{cr.c.h1.a}}}$

$\Phi_{c.h1.a} := 0.5 \cdot \left[1 + 0.21 \cdot (\lambda_{c.h1.a} - 0.2) + \lambda_{c.h1.a}^2\right]$

$\chi_{c.h1.a} := \frac{1}{\Phi_{c.h1.a} + \sqrt{\Phi_{c.h1.a}^2 - \lambda_{c.h1.a}^2}}$

$\xi_{c.h1.a} := \begin{cases} 1 & \text{if } \frac{\sigma_{cr.p.h1.a}}{\sigma_{cr.c.h1.a}} > 1 \\ 0 & \text{if } 0 > \frac{\sigma_{cr.p.h1.a}}{\sigma_{cr.c.h1.a}} \\ \frac{\sigma_{cr.p.h1.a}}{\sigma_{cr.c.h1.a}} & \text{if } 0 \leq \frac{\sigma_{cr.p.h1.a}}{\sigma_{cr.c.h1.a}} \leq 1 \end{cases}$ $\xi_{c.h1.a} = 1.00$

$\rho_{c.h1.a} := (\rho_{loc.h1.a} - \chi_{c.h1.a}) \cdot \xi_{c.h1.a} \cdot (2 - \xi_{c.h1.a}) + \chi_{c.h1.a}$ $\rho_{c.h1.a} = 0.91$

$h_{1.a1.eff} := \rho_{c.h1.a} \cdot h_1$ $h_{1.a1.eff} = 328.36 \cdot mm$

Lijf h.23 zonder verstijvingen:

$k_{\sigma.h23.a} := 4$

$\sigma_{cr.p.h23.a} := k_{\sigma.h23.a} \cdot \frac{\pi^2 \cdot E_d}{12 \cdot (1 - \nu^2)} \cdot \left(\frac{t_w}{h_2 + h_3}\right)^2$

$\lambda_{p.h23.a} := \sqrt{\frac{f_{yd}}{\sigma_{cr.p.h23.a}}}$

$\rho_{loc.h23.a} := \begin{cases} 1.0 & \text{if } \lambda_{p.h23.a} \le 0.5 + \sqrt{0.085 - 0.055\psi} \\ \dfrac{\lambda_{p.h23.a} - 0.055 \cdot (3 + \psi)}{\lambda_{p.h23.a}^2} & \text{if } \lambda_{p.h23.a} > 0.5 + \sqrt{0.085 - 0.055\psi} \end{cases}$

$\sigma_{cr.c.h23.a} := \dfrac{\pi^2 \cdot E_d \cdot t_w^2}{12 \cdot (1 - \nu^2) \cdot a^2}$

$\lambda_{c.h23.a} := \sqrt{\dfrac{f_{yd}}{\sigma_{cr.c.h23.a}}}$

$\Phi_{c.h23.a} := 0.5 \cdot \left[1 + 0.21 \cdot (\lambda_{c.h23.a} - 0.2) + \lambda_{c.h23.a}^2\right]$

$\chi_{c.h23.a} := \dfrac{1}{\Phi_{c.h23.a} + \sqrt{\Phi_{c.h23.a}^2 - \lambda_{c.h23.a}^2}}$

$\xi_{c.h23.a} := \begin{cases} 1 & \text{if } \dfrac{\sigma_{cr.p.h23.a}}{\sigma_{cr.c.h23.a}} > 1 \\ 0 & \text{if } 0 > \dfrac{\sigma_{cr.p.h23.a}}{\sigma_{cr.c.h23.a}} \\ \dfrac{\sigma_{cr.p.h23.a}}{\sigma_{cr.c.h23.a}} & \text{if } 0 \le \dfrac{\sigma_{cr.p.h23.a}}{\sigma_{cr.c.h23.a}} \le 1 \end{cases}$ $\qquad \xi_{c.h23.a} = 1.00$

$\rho_{c.h23.a} := (\rho_{loc.h23.a} - \chi_{c.h23.a}) \cdot \xi_{c.h23.a} \cdot (2 - \xi_{c.h23.a}) + \chi_{c.h23.a}$ $\qquad \rho_{c.h23.a} = 0.39$

$h_{23.a1.eff} := \rho_{c.h23.a} \cdot (h_2 + h_3)$ $\qquad h_{23.a1.eff} = 412.20 \cdot mm$

Verstijver 1 zonder verstijvingen:

$k_{\sigma.v1.a} := 0.43$

$\sigma_{cr.p.v1.a} := k_{\sigma.v1.a} \cdot \dfrac{\pi^2 \cdot E_d}{12 \cdot (1 - \nu^2)} \cdot \left(\dfrac{t_{v1}}{b_{v1}}\right)^2$

$\lambda_{p.v1.a} := \sqrt{\dfrac{f_{yd}}{\sigma_{cr.p.v1.a}}}$

$\rho_{loc.v1.a} := \begin{cases} 1.0 & \text{if } \lambda_{p.v1.a} \le 0.748 \\ \dfrac{\lambda_{p.v1.a} - 0.188}{\lambda_{p.v1.a}^2} & \text{if } \lambda_{p.v1.a} > 0.748 \end{cases}$

$\sigma_{cr.c.v1.a} := \dfrac{\pi^2 \cdot E_d \cdot t_{v1}^2}{12 \cdot (1 - \nu^2) \cdot a^2}$

$\lambda_{c.v1.a} := \sqrt{\dfrac{f_{yd}}{\sigma_{cr.c.v1.a}}}$

$\Phi_{c.v1.a} := 0.5 \cdot \left[1 + 0.21 \cdot (\lambda_{c.v1.a} - 0.2) + \lambda_{c.v1.a}^2\right]$

$\chi_{c.v1.a} := \dfrac{1}{\Phi_{c.v1.a} + \sqrt{\Phi_{c.v1.a}^2 - \lambda_{c.v1.a}^2}}$

$\xi_{c.v1.a} := \begin{cases} 1 & \text{if } \dfrac{\sigma_{cr.p.v1.a}}{\sigma_{cr.c.v1.a}} > 1 \\ 0 & \text{if } 0 > \dfrac{\sigma_{cr.p.v1.a}}{\sigma_{cr.c.v1.a}} \\ \dfrac{\sigma_{cr.p.v1.a}}{\sigma_{cr.c.v1.a}} & \text{if } 0 \le \dfrac{\sigma_{cr.p.v1.a}}{\sigma_{cr.c.v1.a}} \le 1 \end{cases}$ $\qquad \xi_{c.v1.a} = 1.00$

$\rho_{c.v1.a} := (\rho_{loc.v1.a} - \chi_{c.v1.a}) \cdot \xi_{c.v1.a} \cdot (2 - \xi_{c.v1.a}) + \chi_{c.v1.a}$ $\qquad \rho_{c.v1.a} = 1.00$

$b_{v1.a1.eff} := \rho_{c.v1.a} \cdot b_{v1}$ $\qquad b_{v1.a1.eff} = 140.00 \cdot mm$

Global plate like behaviour:

$A_c := \left(\dfrac{h_1}{2} + \dfrac{h_2 + h_3}{2}\right) \cdot t_w + b_{v1} \cdot t_{v1}$

$A_{c.eff.loc} := \left(\dfrac{h_{1.a1.eff}}{2} + \dfrac{h_{23.a1.eff}}{2}\right) \cdot t_w + b_{v1.a1.eff} \cdot t_{v1}$

$\beta_{A.c} := \dfrac{A_{c.eff.loc}}{A_c}$ $\qquad \beta_{A.c} = 0.64$

$\lambda_{p.gl.a} := \sqrt{\dfrac{\beta_{A.c} \cdot f_{yd}}{\sigma_{cr.sl.1}}}$

$\psi := 1$

$\rho_{loc.gl.a} := \begin{cases} 1.0 & \text{if } \lambda_{p.gl.a} \le 0.5 + \sqrt{0.085 - 0.055\psi} \\ \dfrac{\lambda_{p.gl.a} - 0.055 \cdot (3 + \psi)}{\lambda_{p.gl.a}^2} & \text{if } \lambda_{p.gl.a} > 0.5 + \sqrt{0.085 - 0.055\psi} \end{cases}$ $\qquad \rho_{loc.gl.a} = 0.88$

$\sigma_{cr.c.gl.a} := \dfrac{\pi^2 \cdot E_d \cdot I_{sl}}{A_{sl} \cdot a^2}$

$\lambda_{c.gl.a} := \sqrt{\dfrac{\beta_{A.c} \cdot f_{yd}}{\sigma_{cr.c.gl.a}}}$

$i_{c.gl} := \sqrt{\dfrac{I_{sl}}{A_{sl}}}$

$e_{c.gl} := \begin{cases} e_{sti} & \text{if } e_{sti} \ge \dfrac{b_{v1}}{2} + \dfrac{t_w}{2} - e_{sti} \\ \dfrac{b_{v1}}{2} + \dfrac{t_w}{2} - e_{sti} & \text{if } \dfrac{b_{v1}}{2} + \dfrac{t_w}{2} - e_{sti} \ge e_{sti} \end{cases}$ $\qquad e_{c.gl} = 55.11 \cdot mm$

$\alpha_e := 0.49 + 0.09 \cdot \dfrac{i_{c.gl}}{e_{c.gl}}$

$\Phi_{c.gl.a} := 0.5 \cdot \left[1 + \alpha_e \cdot (\lambda_{c.gl.a} - 0.2) + \lambda_{c.gl.a}^2\right]$

$\chi_{c.gl.a} := \dfrac{1}{\Phi_{c.gl.a} + \sqrt{\Phi_{c.gl.a}^2 - \lambda_{c.gl.a}^2}}$

$\xi_{c.gl.a} := \begin{cases} 1 & \text{if } \dfrac{\sigma_{cr.sl.1}}{\sigma_{cr.c.gl.a}} > 1 \\ 0 & \text{if } 0 > \dfrac{\sigma_{cr.sl.1}}{\sigma_{cr.c.gl.a}} \\ \dfrac{\sigma_{cr.sl.1}}{\sigma_{cr.c.gl.a}} & \text{if } 0 \le \dfrac{\sigma_{cr.sl.1}}{\sigma_{cr.c.gl.a}} \le 1 \end{cases}$ $\qquad \xi_{c.gl.a} = 1.00$

$\rho_{c.gl.a} := (\rho_{loc.gl.a} - \chi_{c.gl.a}) \cdot \xi_{c.gl.a} \cdot (2 - \xi_{c.gl.a}) + \chi_{c.gl.a}$ $\qquad \rho_{c.gl.a} = 0.88$

$t_{w.a1.eff} := \rho_{c.gl.a} \cdot t_w$ $\qquad t_{w.a1.eff} = 7.03 \cdot mm$

$t_{v1.a1.eff} := \rho_{c.gl.a} \cdot t_{v1}$ $\qquad t_{v1.a1.eff} = 12.31 \cdot mm$

Oppervlak bruto met 1 verstijvingen:

$$A_{a1} := b_{fbL} \cdot t_{fbL} + b_{fbR} \cdot t_{fbR} + b_{foL} \cdot t_{foL} + b_{foR} \cdot t_{foR} + h_w \cdot t_w + t_{v1} \cdot b_{v1} + A_{fic.1} + A_{fic.2} + A_{t1} + A_{t2} + A_{t3} + A_{t4}$$

$A_{a1} = 77400.00 \cdot mm^2$

Statisch moment bruto met 1 verstijvingen:

$$S_{a1} := b_{fbL} \cdot t_{fbL} \cdot \frac{t_{fbL}}{2} + b_{fbR} \cdot t_{fbR} \cdot \frac{t_{fbR}}{2} + h_w \cdot t_w \cdot \left(t_{fbmax} + \frac{h_w}{2}\right) + b_{foL} \cdot t_{foL} \cdot \left(h_t - \frac{t_{foL}}{2}\right) + b_{foR} \cdot t_{foR} \cdot \left(h_t - \frac{t_{foR}}{2}\right) \ldots$$
$$+ t_{v1} \cdot b_{v1} \cdot h_{v1} + A_{fic.1} \cdot z_{fic.1} + A_{fic.2} \cdot z_{fic.2} + A_{t1} \cdot (z_{t1.bk} + z_{t1}) + A_{t2} \cdot (z_{t2.bk} + z_{t2}) + A_{t3} \cdot (z_{t3.bk} + z_{t3}) + A_{t4} \cdot (z_{t4.bk} + z_{t4})$$

$S_{a1} = 45861200.00 \cdot mm^3$

Zwaartepunt bruto met 1 verstijvingen:

$e_{b.a1} := \frac{S_{a1}}{A_{a1}}$ $\qquad e_{b.a1} = 592.52 \cdot mm$

Oppervlak netto met 1 verstijvingen:

$$A_{a1.eff} := b_{fbL.a.eff} \cdot t_{fbL} + b_{fbR.a.eff} \cdot t_{fbR} + b_{foL.a.eff} \cdot t_{foL} + b_{foR.a.eff} \cdot t_{foR} + \frac{h_{1.a1.eff}}{2} \cdot t_w + \frac{h_{1.a1.eff}}{2} \cdot t_{w.a1.eff} + \frac{h_{23.a1.eff}}{2} \cdot t_{w.a1.eff} \ldots$$
$$+ \frac{h_{23.a1.eff}}{2} \cdot t_w + b_{v1.a1.eff} \cdot t_{v1.a1.eff} + \rho_{fic.1.a} \cdot A_{fic.1} + \rho_{fic.2.a} \cdot A_{fic.2} + A_{t1.eff} + A_{t2.eff} + A_{t3.eff} + A_{t4.eff}$$

$A_{a1.eff} = 71288.77 \cdot mm^2$

Statisch moment netto met 1 verstijvingen:

$$S_{a1.eff} := b_{fbL.a.eff} \cdot t_{fbL} \cdot \frac{t_{fbL}}{2} + b_{fbR.a.eff} \cdot t_{fbR} \cdot \frac{t_{fbR}}{2} + b_{foL.a.eff} \cdot t_{foL} \cdot \left(h_t - \frac{t_{foL}}{2}\right) + b_{foR.a.eff} \cdot t_{foR} \cdot \left(h_t - \frac{t_{foR}}{2}\right) \ldots$$
$$+ \frac{h_{1.a1.eff}}{2} \cdot t_w \cdot \left(\frac{h_{1.a1.eff}}{4} + t_{fbmax}\right) + \frac{h_{1.a1.eff}}{2} \cdot t_{w.a1.eff} \cdot \left(h_1 - \frac{h_{1.a1.eff}}{4} + t_{fbmax}\right) \ldots$$
$$+ \frac{h_{23.a1.eff}}{2} \cdot t_{w.a1.eff} \cdot \left(h_1 + \frac{h_{23.a1.eff}}{4} + t_{fbmax}\right) + \frac{h_{23.a1.eff}}{2} \cdot t_w \cdot \left(h_w - \frac{h_{23.a1.eff}}{4} + t_{fbmax}\right) + b_{v1.a1.eff} \cdot t_{v1.a1.eff} \cdot h_{v1} \ldots$$
$$+ \rho_{fic.1.a} \cdot A_{fic.1} \cdot z_{fic.1} + \rho_{fic.2.a} \cdot A_{fic.2} \cdot z_{fic.2} + A_{t1.eff} \cdot (z_{t1.bk} + z_{t1.eff}) + A_{t2.eff} \cdot (z_{t2.bk} + z_{t2.eff}) \ldots$$
$$+ A_{t3.eff} \cdot (z_{t3.bk} + z_{t3.eff}) + A_{t4.eff} \cdot (z_{t4.bk} + z_{t4.eff})$$

$S_{a1.eff} = 40639371.53 \cdot mm^3$

Zwaartepunt netto met 1 verstijvingen:

$e_{b.a1.eff} := \frac{S_{a1.eff}}{A_{a1.eff}}$ $\qquad e_{b.a1.eff} = 570.07 \cdot mm$

Lijf met 1 verstijver (DRUK)

Lijf met 2 verstijvers (DRUK)

Lijf met 2 verstijvers:

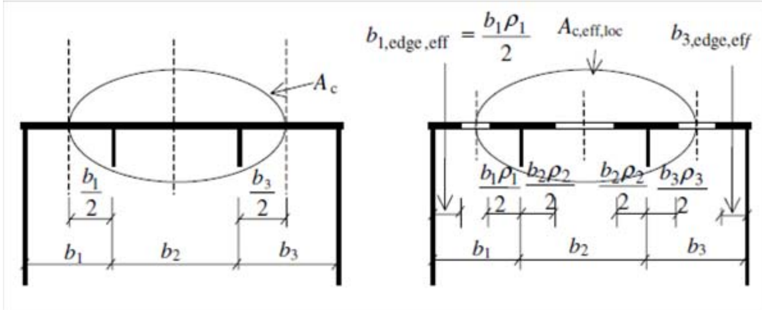


Figure 4.4: Stiffened plate under uniform compression (uit NEN-EN1993-1-5 pagina 18)

$b_1 := h_1$ $\qquad b_1 = 360.00 \cdot mm$

$b_2 := h_2$ $\qquad b_2 = 300.00 \cdot mm$

$b_3 := h_3$ $\qquad b_3 = 770.00 \cdot mm$

1 stiffener lumped:

$b_{lum.1} := b_1 + b_2$ $\qquad b_{lum.1} = 660.00 \cdot mm$

$A_{sl.1} := \frac{b_{lum.1}}{2} \cdot t_w + b_{v1} \cdot t_{v1}$ $\qquad A_{sl.1} = 4600.00 \cdot mm^2$

$e_{sti.1} := \frac{b_{v1} \cdot t_{v1} \cdot \left(\frac{b_{v1}}{2} + \frac{t_w}{2}\right)}{A_{sl.1}}$ $\qquad e_{sti.1} = 31.53 \cdot mm$

$I_{sl.1} := \frac{1}{12} \cdot \frac{b_{lum.1}}{2} \cdot t_w^3 + \frac{1}{12} \cdot t_{v1} \cdot b_{v1}^3 + \frac{b_{lum.1}}{2} \cdot t_w \cdot e_{sti.1}^2 + b_{v1} \cdot t_{v1} \cdot \left(e_{sti.1} - \frac{b_{v1}}{2} - \frac{t_w}{2}\right)^2$ $\qquad I_{sl.1} = 9375199.07 \cdot mm^4$

$a_{c.1.1} := 4.33 \cdot \sqrt[4]{\frac{I_{sl.1} \cdot b_1^2 \cdot b_2^2}{t_w^3 \cdot b_{lum.1}}}$ $\qquad a_{c.1.1} = 3265.81 \cdot mm$

$$\sigma_{cr.sl.1} := \begin{cases} \frac{1.05 \cdot E_d}{A_{sl.1}} \cdot \frac{\sqrt{I_{sl.1} \cdot t_w^3 \cdot b_{lum.1}}}{b_1 \cdot b_2} & \text{if } a \geq a_{c.1.1} \\ \frac{\pi^2 \cdot E_d \cdot I_{sl.1}}{A_{sl.1} \cdot a^2} + \frac{E_d \cdot t_w^3 \cdot b_{lum.1} \cdot a^2}{4 \cdot \pi^2 (1 - \nu^2) \cdot A_{sl.1} \cdot b_1^2 \cdot b_2^2} & \text{if } a < a_{c.1.1} \end{cases}$$

$\sigma_{cr.sl.1} = 789.99 \cdot \frac{N}{mm^2}$

2 stiffener lumped:

$b_{lum.2} := b_2 + b_3$ $\qquad b_{lum.2} = 1070.00 \cdot mm$

$A_{sl.2} := \frac{b_{lum.2}}{2} \cdot t_w + b_{v2} \cdot t_{v2}$ $\qquad A_{sl.2} = 4280.00 \cdot mm^2$

$e_{sti.2} := \frac{b_{v2} \cdot t_{v2} \cdot \left(\frac{b_{v2}}{2} + \frac{t_w}{2}\right)}{A_{sl.2}}$ $\qquad e_{sti.2} = 9.35 \times 10^{-10} \cdot mm$

$I_{sl.2} := \frac{1}{12} \cdot \frac{b_{lum.2}}{2} \cdot t_w^3 + \frac{1}{12} \cdot t_{v2} \cdot b_{v2}^3 + \frac{b_{lum.2}}{2} \cdot t_w \cdot e_{sti.2}^2 + b_{v2} \cdot t_{v2} \cdot \left(e_{sti.2} - \frac{b_{v2}}{2} - \frac{t_w}{2}\right)^2$ $\qquad I_{sl.2} = 22826.67 \cdot mm^4$

$a_{c.1.2} := 4.33 \cdot \sqrt[4]{\frac{I_{sl.2} \cdot b_2^2 \cdot b_3^2}{t_w^3 \cdot b_{lum.2}}}$ $\qquad a_{c.1.2} = 940.25 \cdot mm$

$$\sigma_{cr.sl.2} := \begin{cases} \dfrac{1.05\cdot E_d}{A_{sl.2}}\cdot\dfrac{\sqrt{I_{sl.2}\cdot t_w^{3}\cdot b_{lum.2}}}{b_2\cdot b_3} & \text{if } a \ge a_{c.1.2} \\ \dfrac{\pi^2\cdot E_d\cdot I_{sl.2}}{A_{sl.2}\cdot a^2} + \dfrac{E_d\cdot t_w^{3}\, b_{lum.2}\cdot a^2}{4\cdot\pi^2\left(1-\nu^2\right)\cdot A_{sl.2}\cdot b_2^{2}\cdot b_3^{2}} & \text{if } a < a_{c.1.2} \end{cases}$$

$\sigma_{cr.sl.2} = 24.94\cdot\dfrac{N}{mm^2}$

1 en 2 stiffeners lumped:

$b_{lum.12} := h_1 + h_2 + h_3$ $\qquad b_{lum.12} = 1430.00\cdot mm$

$b_{1.12} := \dfrac{A_{sl.2}}{A_{sl.1} + A_{sl.1}}\cdot h_2 + h_1$ $\qquad b_{1.12} = 499.57\cdot mm$

$b_{2.12} := h_w - b_{1.12}$ $\qquad b_{2.12} = 930.43\cdot mm$

$A_{sl.12} := A_{sl.1} + A_{sl.2}$ $\qquad A_{sl.12} = 8880.00\cdot mm^2$

$I_{sl.12} := I_{sl.1} + I_{sl.2}$ $\qquad I_{sl.12} = 9398025.74\cdot mm^4$

$a_{c.1.12} := 4.33\cdot\sqrt[4]{\dfrac{I_{sl.12}\cdot b_{1.12}^{2}\cdot b_{2.12}^{2}}{t_w^{3}\, b_{lum.12}}}$ $\qquad a_{c.1.12} = 5587.72\cdot mm$

$$\sigma_{cr.sl.12} := \begin{cases} \dfrac{1.05\cdot E_d}{A_{sl.12}}\cdot\dfrac{\sqrt{I_{sl.12}\cdot t_w^{3}\, b_{lum.12}}}{b_{1.12}\cdot b_{2.12}} & \text{if } a \ge a_{c.1.12} \\ \dfrac{\pi^2\cdot E_d\cdot I_{sl.12}}{A_{sl.12}\cdot a^2} + \dfrac{E_d\cdot t_w^{3}\cdot b_{lum.12}\cdot a^2}{4\cdot\pi^2\left(1-\nu^2\right)\cdot A_{sl.12}\cdot b_{1.12}^{2}\cdot b_{2.12}^{2}} & \text{if } a < a_{c.1.12} \end{cases}$$

$\sigma_{cr.sl.12} = 140.13\cdot\dfrac{N}{mm^2}$

Subpanelen reduceren:

Lijf h.1 zonder verstijvingen:

$k_{\sigma.h1.a} := 4$

$\sigma_{cr.p.h1.a} := k_{\sigma.h1.a}\cdot\dfrac{\pi^2\cdot E_d}{12\cdot\left(1-\nu^2\right)}\cdot\left(\dfrac{t_w}{h_1}\right)^2$

$\lambda_{p.h1.a} := \sqrt{\dfrac{f_{yd}}{\sigma_{cr.p.h1.a}}}$

$$\rho_{loc.h1.a} := \begin{cases} 1.0 & \text{if } \lambda_{p.h1.a} \le 0.5 + \sqrt{0.085 - 0.055\psi} \\ \dfrac{\lambda_{p.h1.a} - 0.055\cdot(3+\psi)}{\lambda_{p.h1.a}^{2}} & \text{if } \lambda_{p.h1.a} > 0.5 + \sqrt{0.085 - 0.055\psi} \end{cases}$$

$\sigma_{cr.c.h1.a} := \dfrac{\pi^2\cdot E_d\cdot t_w^{2}}{12\cdot\left(1-\nu^2\right)\cdot a^2}$

$\lambda_{c.h1.a} := \sqrt{\dfrac{f_{yd}}{\sigma_{cr.c.h1.a}}}$

$\Phi_{c.h1.a} := 0.5\cdot\left[1 + 0.21\cdot\left(\lambda_{c.h1.a} - 0.2\right) + \lambda_{c.h1.a}^{2}\right]$

$\chi_{c.h1.a} := \dfrac{1}{\Phi_{c.h1.a} + \sqrt{\Phi_{c.h1.a}^{2} - \lambda_{c.h1.a}^{2}}}$

$$\xi_{c.h1.a} := \begin{cases} 1 & \text{if } \dfrac{\sigma_{cr.p.h1.a}}{\sigma_{cr.c.h1.a}} > 1 \\ 0 & \text{if } 0 > \dfrac{\sigma_{cr.p.h1.a}}{\sigma_{cr.c.h1.a}} \\ \dfrac{\sigma_{cr.p.h1.a}}{\sigma_{cr.c.h1.a}} & \text{if } 0 \le \dfrac{\sigma_{cr.p.h1.a}}{\sigma_{cr.c.h1.a}} \le 1 \end{cases}$$

$\rho_{c.h1.a} := \left(\rho_{loc.h1.a} - \chi_{c.h1.a}\right)\cdot\xi_{c.h1.a}\cdot\left(2 - \xi_{c.h1.a}\right) + \chi_{c.h1.a}$ $\qquad \rho_{c.h1.a} = 0.91$

$h_{1.a2.eff} := \rho_{c.h1.a}\cdot h_1$ $\qquad h_{1.a2.eff} = 328.36\cdot mm$

Lijf h.2 zonder verstijvingen:

$k_{\sigma.h2.a} := 4$

$\sigma_{cr.p.h2.a} := k_{\sigma.h2.a}\cdot\dfrac{\pi^2\cdot E_d}{12\cdot\left(1-\nu^2\right)}\cdot\left(\dfrac{t_w}{h_2}\right)^2$

$\lambda_{p.h2.a} := \sqrt{\dfrac{f_{yd}}{\sigma_{cr.p.h2.a}}}$

$$\rho_{loc.h2.a} := \begin{cases} 1.0 & \text{if } \lambda_{p.h2.a} \le 0.5 + \sqrt{0.085 - 0.055\psi} \\ \dfrac{\lambda_{p.h2.a} - 0.055\cdot(3+\psi)}{\lambda_{p.h2.a}^{2}} & \text{if } \lambda_{p.h2.a} > 0.5 + \sqrt{0.085 - 0.055\psi} \end{cases}$$

$\sigma_{cr.c.h2.a} := \dfrac{\pi^2\cdot E_d\cdot t_w^{2}}{12\cdot\left(1-\nu^2\right)\cdot a^2}$

$\lambda_{c.h2.a} := \sqrt{\dfrac{f_{yd}}{\sigma_{cr.c.h2.a}}}$

$\Phi_{c.h2.a} := 0.5\cdot\left[1 + 0.21\cdot\left(\lambda_{c.h2.a} - 0.2\right) + \lambda_{c.h2.a}^{2}\right]$

$\chi_{c.h2.a} := \dfrac{1}{\Phi_{c.h2.a} + \sqrt{\Phi_{c.h2.a}^{2} - \lambda_{c.h2.a}^{2}}}$

$$\xi_{c.h2.a} := \begin{cases} 1 & \text{if } \dfrac{\sigma_{cr.p.h2.a}}{\sigma_{cr.c.h2.a}} > 1 \\ 0 & \text{if } 0 > \dfrac{\sigma_{cr.p.h2.a}}{\sigma_{cr.c.h2.a}} \\ \dfrac{\sigma_{cr.p.h2.a}}{\sigma_{cr.c.h2.a}} & \text{if } 0 \le \dfrac{\sigma_{cr.p.h2.a}}{\sigma_{cr.c.h2.a}} \le 1 \end{cases}$$

$\rho_{c.h2.a} := \left(\rho_{loc.h2.a} - \chi_{c.h2.a}\right)\cdot\xi_{c.h2.a}\cdot\left(2 - \xi_{c.h2.a}\right) + \chi_{c.h2.a}$ $\qquad \rho_{c.h2.a} = 1.00$

$h_{2.a2.eff} := \rho_{c.h2.a}\cdot h_2$ $\qquad h_{2.a2.eff} = 300.00\cdot mm$

Lijf h.3 zonder verstijvingen:

$k_{\sigma.h3.a} := 4$

$\sigma_{cr.p.h3.a} := k_{\sigma.h3.a}\cdot\dfrac{\pi^2\cdot E_d}{12\cdot\left(1-\nu^2\right)}\cdot\left(\dfrac{t_w}{h_3}\right)^2$

$\lambda_{p.h3.a} := \sqrt{\dfrac{f_{yd}}{\sigma_{cr.p.h3.a}}}$

$$\rho_{loc.h3.a} := \begin{cases} 1.0 & \text{if } \lambda_{p.h3.a} \le 0.5 + \sqrt{0.085 - 0.055\psi} \\ \dfrac{\lambda_{p.h3.a} - 0.055\cdot(3+\psi)}{\lambda_{p.h3.a}^{2}} & \text{if } \lambda_{p.h3.a} > 0.5 + \sqrt{0.085 - 0.055\psi} \end{cases}$$

$$\sigma_{cr.c.h3.a} := \frac{\pi^2 \cdot E_d \cdot t_w^2}{12 \cdot (1-\nu^2) \cdot a^2}$$

$$\lambda_{c.h3.a} := \sqrt{\frac{f_{yd}}{\sigma_{cr.c.h3.a}}}$$

$$\Phi_{c.h3.a} := 0.5 \cdot \left[1 + 0.21 \cdot (\lambda_{c.h3.a} - 0.2) + \lambda_{c.h3.a}^2\right]$$

$$\chi_{c.h3.a} := \frac{1}{\Phi_{c.h3.a} + \sqrt{\Phi_{c.h3.a}^2 - \lambda_{c.h3.a}^2}}$$

$$\xi_{c.h3.a} := \begin{cases} 1 & \text{if } \frac{\sigma_{cr.p.h3.a}}{\sigma_{cr.c.h3.a}} > 1 \\ 0 & \text{if } 0 > \frac{\sigma_{cr.p.h3.a}}{\sigma_{cr.c.h3.a}} \\ \frac{\sigma_{cr.p.h3.a}}{\sigma_{cr.c.h3.a}} & \text{if } 0 \le \frac{\sigma_{cr.p.h3.a}}{\sigma_{cr.c.h3.a}} \le 1 \end{cases}$$

$$\rho_{c.h3.a} := (\rho_{loc.h3.a} - \chi_{c.h3.a}) \cdot \xi_{c.h3.a} \cdot (2 - \xi_{c.h3.a}) + \chi_{c.h3.a}$$ $\rho_{c.h3.a} = 0.51$

$$h_{3.a2.eff} := \rho_{c.h3.a} \cdot h_3$$ $h_{3.a2.eff} = 395.64 \cdot mm$

Verstijver 1 zonder verstijvingen:

$$k_{\sigma.v1.a} := 0.43$$

$$\sigma_{cr.p.v1.a} := k_{\sigma.v1.a} \cdot \frac{\pi^2 \cdot E_d}{12 \cdot (1-\nu^2)} \cdot \left(\frac{t_{v1}}{b_{v1}}\right)^2$$

$$\lambda_{p.v1.a} := \sqrt{\frac{f_{yd}}{\sigma_{cr.p.v1.a}}}$$

$$\rho_{loc.v1.a} := \begin{cases} 1.0 & \text{if } \lambda_{p.v1.a} \le 0.748 \\ \frac{\lambda_{p.v1.a} - 0.188}{\lambda_{p.v1.a}^2} & \text{if } \lambda_{p.v1.a} > 0.748 \end{cases}$$

$$\sigma_{cr.c.v1.a} := \frac{\pi^2 \cdot E_d \cdot t_{v1}^2}{12 \cdot (1-\nu^2) \cdot a^2}$$

$$\lambda_{c.v1.a} := \sqrt{\frac{f_{yd}}{\sigma_{cr.c.v1.a}}}$$

$$\Phi_{c.v1.a} := 0.5 \cdot \left[1 + 0.21 \cdot (\lambda_{c.v1.a} - 0.2) + \lambda_{c.v1.a}^2\right]$$

$$\chi_{c.v1.a} := \frac{1}{\Phi_{c.v1.a} + \sqrt{\Phi_{c.v1.a}^2 - \lambda_{c.v1.a}^2}}$$

$$\xi_{c.v1.a} := \begin{cases} 1 & \text{if } \frac{\sigma_{cr.p.v1.a}}{\sigma_{cr.c.v1.a}} > 1 \\ 0 & \text{if } 0 > \frac{\sigma_{cr.p.v1.a}}{\sigma_{cr.c.v1.a}} \\ \frac{\sigma_{cr.p.v1.a}}{\sigma_{cr.c.v1.a}} & \text{if } 0 \le \frac{\sigma_{cr.p.v1.a}}{\sigma_{cr.c.v1.a}} \le 1 \end{cases}$$

$$\rho_{c.v1.a} := (\rho_{loc.v1.a} - \chi_{c.v1.a}) \cdot \xi_{c.v1.a} \cdot (2 - \xi_{c.v1.a}) + \chi_{c.v1.a}$$ $\rho_{c.v1.a} = 1.00$

$$b_{v1.a2.eff} := \rho_{c.v1.a} \cdot b_{v1}$$ $b_{v1.a2.eff} = 140.00 \cdot mm$

Verstijver 2 zonder verstijvingen:

$$k_{\sigma.v2.a} := 0.43$$

$$\sigma_{cr.p.v2.a} := k_{\sigma.v2.a} \cdot \frac{\pi^2 \cdot E_d}{12 \cdot (1-\nu^2)} \cdot \left(\frac{t_{v2}}{b_{v2}}\right)^2$$

$$\lambda_{p.v2.a} := \sqrt{\frac{f_{yd}}{\sigma_{cr.p.v2.a}}}$$

$$\rho_{loc.v2.a} := \begin{cases} 1.0 & \text{if } \lambda_{p.v2.a} \le 0.748 \\ \frac{\lambda_{p.v2.a} - 0.188}{\lambda_{p.v2.a}^2} & \text{if } \lambda_{p.v2.a} > 0.748 \end{cases}$$

$$\sigma_{cr.c.v2.a} := \frac{\pi^2 \cdot E_d \cdot t_{v2}^2}{12 \cdot (1-\nu^2) \cdot a^2}$$

$$\lambda_{c.v2.a} := \sqrt{\frac{f_{yd}}{\sigma_{cr.c.v2.a}}}$$

$$\Phi_{c.v2.a} := 0.5 \cdot \left[1 + 0.21 \cdot (\lambda_{c.v2.a} - 0.2) + \lambda_{c.v2.a}^2\right]$$

$$\chi_{c.v2.a} := \frac{1}{\Phi_{c.v2.a} + \sqrt{\Phi_{c.v2.a}^2 - \lambda_{c.v2.a}^2}}$$

$$\xi_{c.v2.a} := \begin{cases} 1 & \text{if } \frac{\sigma_{cr.p.v2.a}}{\sigma_{cr.c.v2.a}} > 1 \\ 0 & \text{if } 0 > \frac{\sigma_{cr.p.v2.a}}{\sigma_{cr.c.v2.a}} \\ \frac{\sigma_{cr.p.v2.a}}{\sigma_{cr.c.v2.a}} & \text{if } 0 \le \frac{\sigma_{cr.p.v2.a}}{\sigma_{cr.c.v2.a}} \le 1 \end{cases}$$

$$\rho_{c.v2.a} := (\rho_{loc.v2.a} - \chi_{c.v2.a}) \cdot \xi_{c.v2.a} \cdot (2 - \xi_{c.v2.a}) + \chi_{c.v2.a}$$ $\rho_{c.v2.a} = 1.00$

$$b_{v2.a2.eff} := \rho_{c.v2.a} \cdot b_{v2}$$ $b_{v2.a2.eff} = 0.00 \cdot mm$

Global plate like behaviour:

$$A_c := \left(\frac{h_1}{2} + h_2 + \frac{h_3}{2}\right) \cdot t_w + b_{v1} \cdot t_{v1} + b_{v2} \cdot t_{v2}$$

$$A_{c.eff.loc} := \left(\frac{h_{1.a2.eff}}{2} + h_{2.a2.eff} + \frac{h_{3.a2.eff}}{2}\right) \cdot t_w + b_{v1.a2.eff} \cdot t_{v1} + b_{v2.a2.eff} \cdot t_{v2}$$

$$\beta_{A.c} := \frac{A_{c.eff.loc}}{A_c}$$ $\beta_{A.c} = 0.82$

$$\sigma_{cr.sl.lum} := \begin{cases} \sigma_{cr.sl.1} & \text{if } \sigma_{cr.sl.2} \ge \sigma_{cr.sl.1} \text{ if } \sigma_{cr.sl.12} \ge \sigma_{cr.sl.1} \\ \sigma_{cr.sl.2} & \text{if } \sigma_{cr.sl.1} \ge \sigma_{cr.sl.2} \text{ if } \sigma_{cr.sl.12} \ge \sigma_{cr.sl.2} \\ \sigma_{cr.sl.12} & \text{if } \sigma_{cr.sl.1} \ge \sigma_{cr.sl.12} \text{ if } \sigma_{cr.sl.2} \ge \sigma_{cr.sl.12} \end{cases}$$ $\sigma_{cr.sl.lum} = 24.94 \cdot \frac{N}{mm^2}$

$$\lambda_{p.gl.a} := \sqrt{\frac{\beta_{A.c} \cdot f_{yd}}{\sigma_{cr.sl.lum}}}$$

$$\rho_{loc.gl.a} := \begin{cases} 1.0 & \text{if } \lambda_{p.gl.a} \le 0.5 + \sqrt{0.085 - 0.055\psi} \\ \frac{\lambda_{p.gl.a} - 0.055 \cdot (3+\psi)}{\lambda_{p.gl.a}^2} & \text{if } \lambda_{p.gl.a} > 0.5 + \sqrt{0.085 - 0.055\psi} \end{cases}$$ $\rho_{loc.gl.a} = 0.33$

$$\sigma_{cr.c.gl.a} := \frac{\pi^2 \cdot E_d \cdot I_{sl.1}}{A_{sl.1} \cdot a^2}$$

$\lambda_{c.gl.a} := \sqrt{\dfrac{\beta_{A.c} \cdot f_{yd}}{\sigma_{cr.c.gl.a}}}$

$i_{c.gl} := \sqrt{\dfrac{I_{sl.1}}{A_{sl.1}}}$

$e_{c.gl} := \begin{cases} e_{sti.1} & \text{if } e_{sti.1} \ge \dfrac{b_{v1}}{2} + \dfrac{t_w}{2} - e_{sti.1} \\ \dfrac{b_{v1}}{2} + \dfrac{t_w}{2} - e_{sti.1} & \text{if } \dfrac{b_{v1}}{2} + \dfrac{t_w}{2} - e_{sti.1} \ge e_{sti.1} \end{cases}$ $e_{c.gl} = 42.47 \cdot mm$

$\alpha_e := 0.49 + 0.09 \cdot \dfrac{i_{c.gl}}{e_{c.gl}}$

$\Phi_{c.gl.a} := 0.5 \cdot \left[1 + \alpha_e \cdot (\lambda_{c.gl.a} - 0.2) + \lambda_{c.gl.a}^2\right]$

$\chi_{c.gl.a} := \dfrac{1}{\Phi_{c.gl.a} + \sqrt{\Phi_{c.gl.a}^2 - \lambda_{c.gl.a}^2}}$ $\chi_{c.gl.a} = 0.08$

$\xi_{c.gl.a} := \begin{cases} 1 & \text{if } \dfrac{\sigma_{cr.sl.lum}}{\sigma_{cr.c.gl.a}} > 1 \\ 0 & \text{if } 0 > \dfrac{\sigma_{cr.sl.lum}}{\sigma_{cr.c.gl.a}} \\ \dfrac{\sigma_{cr.sl.lum}}{\sigma_{cr.c.gl.a}} & \text{if } 0 \le \dfrac{\sigma_{cr.sl.lum}}{\sigma_{cr.c.gl.a}} \le 1 \end{cases}$ $\xi_{c.gl.a} = 1.00$

$\rho_{c.gl.a} := (\rho_{loc.gl.a} - \chi_{c.gl.a}) \cdot \xi_{c.gl.a} \cdot (2 - \xi_{c.gl.a}) + \chi_{c.gl.a}$ $\rho_{c.gl.a} = 0.33$

$t_{w.a2.eff} := \rho_{c.gl.a} \cdot t_w$ $t_{w.a2.eff} = 2.65 \cdot mm$

$t_{v1.a2.eff} := \rho_{c.gl.a} \cdot t_{v1}$ $t_{v1.a2.eff} = 4.65 \cdot mm$

$t_{v2.a2.eff} := \rho_{c.gl.a} \cdot t_{v2}$ $t_{v2.a2.eff} = 0.00 \cdot mm$

Oppervlak bruto met 2 verstijvingen:

$A_{a2} := b_{fbL} \cdot t_{fbL} + b_{fbR} \cdot t_{fbR} + b_{foL} \cdot t_{foL} + b_{foR} \cdot t_{foR} + h_w \cdot t_w + t_{v1} \cdot b_{v1} + t_{v2} \cdot b_{v2} + A_{fic.1} + A_{fic.2} + A_{t1} + A_{t2} + A_{t3} + A_{t4}$

$A_{a2} = 77400.00 \cdot mm^2$

Statisch moment bruto met 2 verstijvingen:

$S_{a2} := b_{fbL} \cdot t_{fbL} \cdot \dfrac{t_{fbL}}{2} + b_{fbR} \cdot t_{fbR} \cdot \dfrac{t_{fbR}}{2} + h_w \cdot t_w \cdot \left(t_{fbmax} + \dfrac{h_w}{2}\right) + b_{foL} \cdot t_{foL} \cdot \left(h_t - \dfrac{t_{foL}}{2}\right) + b_{foR} \cdot t_{foR} \cdot \left(h_t - \dfrac{t_{foR}}{2}\right) \ldots$
$+ t_{v1} \cdot b_{v1} \cdot h_{v1} + t_{v2} \cdot b_{v2} \cdot h_{v2} + A_{fic.1} \cdot z_{fic.1} + A_{fic.2} \cdot z_{fic.2} + A_{t1} \cdot (z_{t1.bk} + z_{t1}) + A_{t2} \cdot (z_{t2.bk} + z_{t2}) + A_{t3} \cdot (z_{t3.bk} + z_{t3}) + A_{t4} \cdot (z_{t4.bk} + z_{t4})$

Zwaartepunt bruto met 2 verstijvingen: $S_{a2} = 45861200.00 \cdot mm^3$

$e_{b.a2} := \dfrac{S_{a2}}{A_{a2}}$ $e_{b.a2} = 592.52 \cdot mm$

Oppervlak netto met 2 verstijvingen:

$A_{a2.eff} := b_{fbL.a.eff} \cdot t_{fbL} + b_{fbR.a.eff} \cdot t_{fbR} + b_{foL.a.eff} \cdot t_{foL} + b_{foR.a.eff} \cdot t_{foR} + \dfrac{h_{1.a2.eff}}{2} \cdot t_w + \dfrac{h_{1.a2.eff}}{2} \cdot t_{w.a2.eff} \ldots$
$+ h_{2.a2.eff} \cdot t_{w.a2.eff} + \dfrac{h_{3.a2.eff}}{2} \cdot t_{w.a2.eff} + \dfrac{h_{3.a2.eff}}{2} \cdot t_w + b_{v1.a2.eff} \cdot t_{v1.a2.eff} + b_{v2.a2.eff} \cdot t_{v2.a2.eff} \ldots$
$+ \rho_{fic.1.a} \cdot A_{fic.1} + \rho_{fic.2.a} \cdot A_{fic.2} + A_{t1.eff} + A_{t2.eff} + A_{t3.eff} + A_{t4.eff}$

$A_{a2.eff} = 69303.62 \cdot mm^2$

Statisch moment netto met 2 verstijvingen:

$S_{a2.eff} := b_{fbL.a.eff} \cdot t_{fbL} \cdot \dfrac{t_{fbL}}{2} + b_{fbR.a.eff} \cdot t_{fbR} \cdot \dfrac{t_{fbR}}{2} + b_{foL.a.eff} \cdot t_{foL} \cdot \left(h_t - \dfrac{t_{foL}}{2}\right) + b_{foR.a.eff} \cdot t_{foR} \cdot \left(h_t - \dfrac{t_{foR}}{2}\right) \ldots$
$+ \dfrac{h_{1.a2.eff}}{2} \cdot t_w \cdot \left(\dfrac{h_{1.a2.eff}}{4} + t_{fbmax}\right) + \dfrac{h_{1.a2.eff}}{2} \cdot t_{w.a2.eff} \cdot \left(h_1 - \dfrac{h_{1.a2.eff}}{4} + t_{fbmax}\right) + h_{2.a2.eff} \cdot t_{w.a2.eff} \cdot \left(h_1 + t_{fbmax} + \dfrac{h_2}{2}\right) \ldots$
$+ \dfrac{h_{3.a2.eff}}{2} \cdot t_{w.a2.eff} \cdot \left(h_1 + h_2 + t_{fbmax} + \dfrac{h_{3.a2.eff}}{4}\right) + \dfrac{h_{3.a2.eff}}{2} \cdot t_w \cdot \left(h_w + t_{fbmax} - \dfrac{h_{3.a2.eff}}{4}\right) + b_{v1.a2.eff} \cdot t_{v1.a2.eff} \cdot h_{v1} \ldots$
$+ b_{v2.a2.eff} \cdot t_{v2.a2.eff} \cdot h_{v2} + \rho_{fic.1.a} \cdot A_{fic.1} \cdot z_{fic.1} + \rho_{fic.2.a} \cdot A_{fic.2} \cdot z_{fic.2} + A_{t1.eff} \cdot (z_{t1.bk} + z_{t1.eff}) \ldots$
$+ A_{t2.eff} \cdot (z_{t2.bk} + z_{t2.eff}) + A_{t3.eff} \cdot (z_{t3.bk} + z_{t3.eff}) + A_{t4.eff} \cdot (z_{t4.bk} + z_{t4.eff})$

$S_{a2.eff} = 40026378.19 \cdot mm^3$

Zwaartepunt netto met 2 verstijvingen:

$e_{b.a2.eff} := \dfrac{S_{a2.eff}}{A_{a2.eff}}$ $e_{b.a2.eff} = 577.55 \cdot mm$

Lijf met 2 verstijvers (DRUK)

Berekening gereduceerde doorsnede eigenschappen t.g.v. buigend moment:

Plate buckling effects due to direct stresses at the ultimate limit state (4)

Reduceren doorsnede flenzen t.g.v. buigend moment

Bovenflens reduceren:

Bruto oppervlakte:

$A_{bi} := b_{fbL} \cdot t_{fbL} + b_{fbR} \cdot t_{fbR} + b_{foL} \cdot t_{foL} + b_{foR} \cdot t_{foR} + h_w \cdot t_w + t_{v1} \cdot b_{v1} + t_{v2} \cdot b_{v2} \ldots + A_{fic.1} + A_{fic.2} + A_{t1} + A_{t2} + A_{t3} + A_{t4}$

$A_{bi} = 77400.00 \cdot mm^2$

Bruto statisch moment:

$S_{bi} := b_{fbL} \cdot t_{fbL} \cdot \frac{t_{fbL}}{2} + b_{fbR} \cdot t_{fbR} \cdot \frac{t_{fbR}}{2} + h_w \cdot t_w \cdot \left(t_{fbmax} + \frac{h_w}{2}\right) + b_{foL} \cdot t_{foL} \cdot \left(h_t - \frac{t_{foL}}{2}\right) + b_{foR} \cdot t_{foR} \cdot \left(h_t - \frac{t_{foR}}{2}\right) \ldots + t_{v1} \cdot b_{v1} \cdot h_{v1} + t_{v2} \cdot b_{v2} \cdot h_{v2} + A_{fic.1} \cdot z_{fic.1} + A_{fic.2} \cdot z_{fic.2} + A_{t1} \cdot (z_{t1.bk} + z_{t1}) + A_{t2} \cdot (z_{t2.bk} + z_{t2}) + A_{t3} \cdot (z_{t3.bk} + z_{t3}) + A_{t4} \cdot (z_{t4.bk} + z_{t4})$

Bruto zwaartepunt:

$S_{bi} = 45861200.00 \cdot mm^3$

$e_{b.bi} := \frac{S_{bi}}{A_{bi}}$

$e_{b.bi} = 592.52 \cdot mm$

Effectieve oppervlakte (gereduceerde bovenflenzen):

$A_{bi.eff} := b_{fbL.a.eff} \cdot t_{fbL} + b_{fbR.a.eff} \cdot t_{fbR} + b_{foL} \cdot t_{foL} + b_{foR} \cdot t_{foR} + h_w \cdot t_w + t_{v1} \cdot b_{v1} + t_{v2} \cdot b_{v2} \ldots + \rho_{fic.1.m} \cdot A_{fic.1} + A_{fic.2} + A_{t1.eff} + A_{t2.eff} + A_{t3} + A_{t4}$

$A_{bi.eff} = 77400.00 \cdot mm^2$

Effectief statisch moment (gereduceerde bovenflenzen):

$S_{bi.eff} := b_{fbL.a.eff} \cdot t_{fbL} \cdot \frac{t_{fbL}}{2} + b_{fbR.a.eff} \cdot t_{fbR} \cdot \frac{t_{fbR}}{2} + h_w \cdot t_w \cdot \left(t_{fbmax} + \frac{h_w}{2}\right) + b_{foL} \cdot t_{foL} \cdot \left(h_t - \frac{t_{foL}}{2}\right) + b_{foR} \cdot t_{foR} \cdot \left(h_t - \frac{t_{foR}}{2}\right) \ldots + t_{v1} \cdot b_{v1} \cdot h_{v1} + t_{v2} \cdot b_{v2} \cdot h_{v2} + \rho_{fic.1.m} \cdot A_{fic.1} \cdot z_{fic.1} + A_{fic.2} \cdot z_{fic.2} + A_{t1.eff} \cdot (z_{t1.bk} + z_{t1.eff}) + A_{t2.eff} \cdot (z_{t2.bk} + z_{t2.eff}) \ldots + A_{t3} \cdot (z_{t3.bk} + z_{t3}) + A_{t4} \cdot (z_{t4.bk} + z_{t4})$

Effectief zwaartepunt (gereduceerde bovenflenzen):

$S_{bi.eff} = 45861200.00 \cdot mm^3$

$e_{b.bi.eff} := \frac{S_{bi.eff}}{A_{bi.eff}}$

$e_{b.bi.eff} = 592.52 \cdot mm$

$I_{y.0.bi.eff} := \frac{1}{12} \cdot \left(b_{fbL.a.eff} \cdot t_{fbL}^3 + b_{fbR.a.eff} \cdot t_{fbR}^3 + t_w \cdot h_w^3 + b_{foL} \cdot t_{foL}^3 + b_{foR} \cdot t_{foR}^3 \ldots + b_{v1} \cdot t_{v1}^3 + b_{v2} \cdot t_{v2}^3\right) + \rho_{fic.1.m} \cdot I_{fic.1} + I_{fic.2} + I_{t1.eff} + I_{t2.eff} + I_{t3} + I_{t4}$

$I_{y.c.bi.eff} := b_{fbL.a.eff} \cdot t_{fbL} \cdot \left(e_{b.bi.eff} - \frac{t_{fbL}}{2}\right)^2 + b_{fbR.a.eff} \cdot t_{fbR} \cdot \left(e_{b.bi.eff} - \frac{t_{fbR}}{2}\right)^2 + A_w \cdot \left(e_{b.bi.eff} - t_{fbmax} - \frac{h_w}{2}\right)^2 \ldots + A_{foL} \cdot \left(h_t - \frac{t_{foL}}{2} - e_{b.bi.eff}\right)^2 + A_{foR} \cdot \left(h_t - \frac{t_{foR}}{2} - e_{b.bi.eff}\right)^2 + A_{v1} \cdot (h_{v1} - e_{b.bi.eff})^2 + A_{v2} \cdot (h_{v2} - e_{b.bi.eff})^2 \ldots + \rho_{fic.1.m} \cdot A_{fic.1} \cdot (z_{fic.1} - e_{b.bi.eff})^2 + A_{fic.2} \cdot (z_{fic.2} - e_{b.bi.eff})^2 + A_{t1.eff} \cdot (z_{t1.bk} + z_{t1.eff} - e_{b.bi.eff})^2 \ldots + A_{t2.eff} \cdot (z_{t2.bk} + z_{t2.eff} - e_{b.bi.eff})^2 + A_{t3} \cdot (z_{t3.bk} + z_{t3} - e_{b.bi.eff})^2 + A_{t4} \cdot (z_{t4.bk} + z_{t4} - e_{b.bi.eff})^2$

Effectief traagheidsmoment (gereduceerde bovenflenzen):

$I_{y.bi.eff} := I_{y.0.bi.eff} + I_{y.c.bi.eff}$

$I_{y.bi.eff} = 3.4559 \times 10^{10} \cdot mm^4$

$e_{1.bi.eff} := e_{b.bi.eff}$

$e_{2.bi.eff} := e_{b.bi.eff} - t_{fbmax}$

$e_{3.bi.eff} := 0$

$e_{4.bi.eff} := e_{b.bi.eff} - h_t + t_{fomax}$

$e_{5.bi.eff} := e_{b.bi.eff} - h_t$

$e_{1.bi.eff} = 593 \cdot mm$ | $e_{2.bi.eff} = 553 \cdot mm$ | $e_{3.bi.eff} = 0 \cdot mm$ | $e_{4.bi.eff} = -877 \cdot mm$ | $e_{5.bi.eff} = -907 \cdot mm$

Reduceren doorsnede flenzen t.g.v. buigend moment

Lijf zonder verstijver (MOMENT)

$\psi := \frac{e_{4.bi.eff}}{e_{2.bi.eff}}$

$\psi = -1.59$

$b_{c.m0} := \begin{cases} h_w & \text{if } \psi \ge 0 \\ \frac{h_w}{(1-\psi)} & \text{if } \psi < 0 \end{cases}$

$b_{c.m0} = 552.52 \cdot mm$

$b_{t.m0} := h_w - b_{c.m0}$

$b_{t.m0} = 877.48 \cdot mm$

$b_{c.m0.sup} := \begin{cases} b_{c.m0} \cdot \frac{2}{5-\psi} & \text{if } \psi \ge 0 \\ 0.4 \cdot b_{c.m0} & \text{if } \psi < 0 \end{cases}$

$b_{c.m0.sup} = 221.01 \cdot mm$

$b_{c.m0.inf} := \begin{cases} b_{c.m0} \cdot \frac{3-\psi}{5-\psi} & \text{if } \psi \ge 0 \\ 0.6 \cdot b_{c.m0} & \text{if } \psi < 0 \end{cases}$

$b_{c.m0.inf} = 331.51 \cdot mm$

$k_{\sigma.w.m} := \begin{cases} 4 & \text{if } \psi = 1 \\ \frac{8.2}{1.05+\psi} & \text{if } 0 < \psi < 1 \\ 7.8 & \text{if } \psi = 0 \\ 7.81 - 6.29 \cdot \psi + 9.78 \cdot \psi^2 & \text{if } 0 > \psi > -1 \\ 23.9 & \text{if } \psi = -1 \\ 5.98 \cdot (1-\psi)^2 & \text{if } -1 > \psi \ge -3 \\ 5.98 \cdot (1 - -3)^2 & \text{if } \psi < -3 \end{cases}$

$k_{\sigma.w.m} = 40.06$

$\sigma_{cr.p.w.m} := k_{\sigma.w.m} \cdot \frac{\pi^2 \cdot E_d}{12 \cdot (1-\nu^2)} \cdot \left(\frac{t_w}{h_w}\right)^2$

$\lambda_{p.w.m} := \sqrt{\frac{f_{yd}}{\sigma_{cr.p.w.m}}}$

$\lambda_{p.w.m} = 0.99$

$\rho_{loc.w.m} := \begin{cases} 1.0 & \text{if } \lambda_{p.w.m} \le 0.5 + \sqrt{0.085 - 0.055\psi} \\ \frac{\lambda_{p.w.m} - 0.055 \cdot (3+\psi)}{\lambda_{p.w.m}^2} & \text{if } \lambda_{p.w.m} > 0.5 + \sqrt{0.085 - 0.055\psi} \end{cases}$

$\sigma_{cr.c.w.m} := \frac{\pi^2 \cdot E_d \cdot t_w^2}{12 \cdot (1-\nu^2) \cdot a^2}$

$\lambda_{c.w.m} := \sqrt{\frac{f_{yd}}{\sigma_{cr.c.w.m}}}$

$\Phi_{c.w.m} := 0.5 \cdot \left[1 + 0.21 \cdot (\lambda_{c.w.m} - 0.2) + \lambda_{c.w.m}^2\right]$

$\chi_{c.w.m} := \frac{1}{\Phi_{c.w.m} + \sqrt{\Phi_{c.w.m}^2 - \lambda_{c.w.m}^2}}$

$$\xi_{c.w.m} := \begin{cases} 1 & \text{if } \dfrac{\sigma_{cr.p.w.m}}{\sigma_{cr.c.w.m}} > 1 \\ 0 & \text{if } 0 > \dfrac{\sigma_{cr.p.w.m}}{\sigma_{cr.c.w.m}} \\ \dfrac{\sigma_{cr.p.w.m}}{\sigma_{cr.c.w.m}} & \text{if } 0 \le \dfrac{\sigma_{cr.p.w.m}}{\sigma_{cr.c.w.m}} \le 1 \end{cases}$$

$$\rho_{c.w.m} := \left(\rho_{loc.w.m} - \chi_{c.w.m}\right)\cdot\xi_{c.w.m}\cdot\left(2 - \xi_{c.w.m}\right) + \chi_{c.w.m}$$ $\rho_{c.w.m} = 0.93$

$b_{c.m0.sup.eff} := \rho_{c.w.m}\cdot b_{c.m0.sup}$ $b_{c.m0.sup.eff} = 205.01\cdot mm$

$b_{c.m0.inf.eff} := \rho_{c.w.m}\cdot b_{c.m0.inf}$ $b_{c.m0.inf.eff} = 307.52\cdot mm$

Bruto oppervlakte:

$$A_{m0} := b_{fbL}\cdot t_{fbL} + b_{fbR}\cdot t_{fbR} + b_{foL}\cdot t_{foL} + b_{foR}\cdot t_{foR} + h_w\cdot t_w + A_{fic.1} + A_{fic.2} + A_{t1} + A_{t2} + A_{t3} + A_{t4} + b_{v1}\cdot t_{v1} + b_{v2}\cdot t_{v2}$$

$A_{m0} = 77400.00\cdot mm^2$

Bruto statisch moment:

$$S_{m0} := b_{fbL}\cdot t_{fbL}\cdot\frac{t_{fbL}}{2} + b_{fbR}\cdot t_{fbR}\cdot\frac{t_{fbR}}{2} + h_w\cdot t_w\cdot\left(t_{fbmax} + \frac{h_w}{2}\right) + b_{foL}\cdot t_{foL}\cdot\left(h_t - \frac{t_{foL}}{2}\right) + b_{foR}\cdot t_{foR}\cdot\left(h_t - \frac{t_{foR}}{2}\right) + A_{fic.1}\cdot z_{fic.1} \ldots$$
$$+ A_{fic.2}\cdot z_{fic.2} + A_{t1}\cdot\left(z_{t1.bk} + z_{t1}\right) + A_{t2}\cdot\left(z_{t2.bk} + z_{t2}\right) + A_{t3}\cdot\left(z_{t3.bk} + z_{t3}\right) + A_{t4}\cdot\left(z_{t4.bk} + z_{t4}\right) + b_{v1}\cdot t_{v1}\cdot h_{v1} + b_{v2}\cdot t_{v2}\cdot h_{v2}$$

$S_{m0} = 45861200.00\cdot mm^3$

Bruto zwaartepunt:

$$e_{b.m0} := \frac{S_{m0}}{A_{m0}}$$ $e_{b.m0} = 592.52\cdot mm$

Effectieve oppervlakte:

$$A_{m0.eff} := b_{fbL.a.eff}\cdot t_{fbL} + b_{fbR.a.eff}\cdot t_{fbR} + b_{foL}\cdot t_{foL} + b_{foR}\cdot t_{foR} + \left(b_{c.m0.sup.eff} + b_{c.m0.inf.eff}\right)\cdot t_w + b_{t.m0}\cdot t_w \ldots$$
$$+ \rho_{fic.1.m}\cdot A_{fic.1} + A_{fic.2} + A_{t1.eff} + A_{t2.eff} + A_{t3} + A_{t4} + b_{v1}\cdot t_{v1} + b_{v2}\cdot t_{v2}$$

$A_{m0.eff} = 77080.08\cdot mm^2$

Effectief statisch moment:

$$S_{m0.eff} := b_{fbL.a.eff}\cdot t_{fbL}\cdot\frac{t_{fbL}}{2} + b_{fbR.a.eff}\cdot t_{fbR}\cdot\frac{t_{fbR}}{2} + b_{foL}\cdot t_{foL}\cdot\left(h_t - \frac{t_{foL}}{2}\right) + b_{foR}\cdot t_{foR}\cdot\left(h_t - \frac{t_{foR}}{2}\right) \ldots$$
$$+ b_{t.m0}\cdot t_w\cdot\left(\frac{b_{t.m0}}{2} + b_{c.m0.sup} + b_{c.m0.inf} + t_{fbmax}\right) + b_{c.m0.sup.eff}\cdot t_w\cdot\left(\frac{b_{c.m0.sup.eff}}{2} + t_{fbmax}\right) \ldots$$
$$+ b_{c.m0.inf.eff}\cdot t_w\cdot\left(b_{c.m0.sup} + b_{c.m0.inf} + t_{fbmax} - \frac{b_{c.m0.inf.eff}}{2}\right) + \rho_{fic.1.m}\cdot A_{fic.1}\cdot z_{fic.1} + A_{fic.2}\cdot z_{fic.2} \ldots$$
$$+ A_{t1.eff}\cdot\left(z_{t1.bk} + z_{t1.eff}\right) + A_{t2.eff}\cdot\left(z_{t2.bk} + z_{t2.eff}\right) + A_{t3}\cdot\left(z_{t3.bk} + z_{t3}\right) + A_{t4}\cdot\left(z_{t4.bk} + z_{t4}\right) + b_{v1}\cdot t_{v1}\cdot h_{v1} + b_{v2}\cdot t_{v2}\cdot h_{v2}$$

$S_{m0.eff} = 45776418.29\cdot mm^3$

Effectief zwaartepunt:

$$e_{b.m0.eff} := \frac{S_{m0.eff}}{A_{m0.eff}}$$ $e_{b.m0.eff} = 593.88\cdot mm$

$$I_{y.0.m0.eff} := \frac{1}{12}\cdot\left(\begin{array}{l} b_{fbL.a.eff}\cdot t_{fbL}^3 + b_{fbR.a.eff}\cdot t_{fbR}^3 + b_{foL}\cdot t_{foL}^3 + b_{foR}\cdot t_{foR}^3 \ldots \\ + t_w\cdot b_{c.m0.sup.eff}^3 + t_w\cdot b_{c.m0.inf.eff}^3 + t_w\cdot b_{t.m0}^3 + b_{v1}\cdot t_{v1}^3 + b_{v2}\cdot t_{v2}^3 \end{array}\right) \ldots$$
$$+ \rho_{fic.1.m}\cdot I_{fic.1} + I_{fic.2} + I_{t1.eff} + I_{t2.eff} + I_{t3} + I_{t4}$$

$$I_{y.c.m0.eff} := b_{fbL.a.eff}\cdot t_{fbL}\cdot\left(e_{b.m0.eff} - \frac{t_{fbL}}{2}\right)^2 + b_{fbR.a.eff}\cdot t_{fbR}\cdot\left(e_{b.m0.eff} - \frac{t_{fbR}}{2}\right)^2 \ldots$$
$$+ b_{c.m0.sup.eff}\cdot t_w\cdot\left(e_{b.m0.eff} - t_{fbmax} - \frac{b_{c.m0.sup.eff}}{2}\right)^2 \ldots$$
$$+ b_{c.m0.inf.eff}\cdot t_w\cdot\left(e_{b.m0.eff} - t_{fbmax} - b_{c.m0.inf} - b_{c.m0.sup} + -\frac{b_{c.m0.inf.eff}}{2}\right)^2 \ldots$$
$$+ b_{t.m0}\cdot t_w\cdot\left(e_{b.m0.eff} - t_{fbmax} - h_w + \frac{b_{t.m0}}{2}\right)^2 \ldots$$
$$+ b_{foL}\cdot t_{foL}\cdot\left(h_t - \frac{t_{foL}}{2} - e_{b.m0.eff}\right)^2 + b_{foR}\cdot t_{foR}\cdot\left(h_t - \frac{t_{foR}}{2} - e_{b.m0.eff}\right)^2 \ldots$$
$$+ \rho_{fic.1.m}\cdot A_{fic.1}\cdot\left(z_{fic.1} - e_{b.m0.eff}\right)^2 + A_{fic.2}\cdot\left(z_{fic.2} - e_{b.m0.eff}\right)^2 + A_{t1.eff}\cdot\left(z_{t1.bk} + z_{t1.eff} - e_{b.m0.eff}\right)^2 \ldots$$
$$+ A_{t2.eff}\cdot\left(z_{t2.bk} + z_{t2.eff} - e_{b.m0.eff}\right)^2 + A_{t3}\cdot\left(z_{t3.bk} + z_{t3} - e_{b.m0.eff}\right)^2 + A_{t4}\cdot\left(z_{t4.bk} + z_{t4} - e_{b.m0.eff}\right)^2 \ldots$$
$$+ b_{v1}\cdot t_{v1}\cdot\left(h_{v1} - e_{b.m0.eff}\right)^2 + b_{v2}\cdot t_{v2}\cdot\left(h_{v2} - e_{b.m0.eff}\right)^2$$

Effectief traagheidsmoment:

$I_{y.m0.eff} := I_{y.0.m0.eff} + I_{y.c.m0.eff}$ $I_{y.m0.eff} = 3.4522\times 10^{10}\cdot mm^4$

Lijf zonder verstijver (MOMENT)

Lijf met 1 verstijver in drukzone (MOMENT)

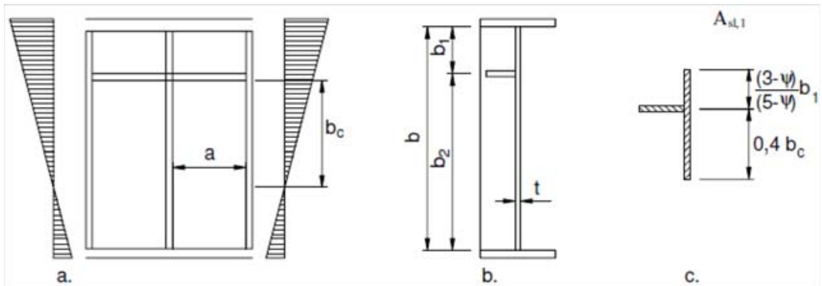


Figure A.1: Notations for longitudinally stiffened plates (uit NEN-EN1993-1-5 pagina 40)

$b_1 := h_1$ $b_1 = 360.00\cdot mm$

$b_2 := h_2 + h_3$ $b_2 = 1070.00\cdot mm$

$$\psi_{m1.b1} := \frac{e_{b.bi.eff} - \left(b_1 + t_{fbmax}\right)}{e_{2.bi.eff}}$$ $\psi_{m1.b1} = 0.35$

$$\psi_{m1.b2} := \begin{cases} 1 & \text{if } \dfrac{e_{b.bi.eff} - \left(b_1 + b_2 + t_{fbmax}\right)}{e_{b.bi.eff} - \left(b_1 + t_{fbmax}\right)} > 1 \\ \dfrac{e_{b.bi.eff} - \left(b_1 + b_2 + t_{fbmax}\right)}{e_{b.bi.eff} - \left(b_1 + t_{fbmax}\right)} & \text{otherwise} \end{cases}$$ $\psi_{m1.b2} = -4.56$

$$b_{1.m1.edge} := \frac{2}{5 - \psi_{m1.b1}}\cdot b_1$$ $b_{1.m1.edge} = 154.79\cdot mm$

$b_{1.m1.inf} := \frac{3 - \psi_{m1.b1}}{5 - \psi_{m1.b1}} \cdot b_1$ $\qquad b_{1.m1.inf} = 205.21 \cdot mm$

$$b_{2.m1.sup} := \begin{cases} \frac{2}{5 - \psi_{m1.b2}} \cdot b_2 & \text{if } \psi_{m1.b2} \geq 0 \\ \frac{0.4 b_2}{(1 - \psi_{m1.b2})} & \text{if } \psi_{m1.b2} < 0 \end{cases}$$ $b_{2.m1.sup} = 77.01 \cdot mm$

$$b_{2.m1.inf} := \begin{cases} \frac{3 - \psi_{m1.b2}}{5 - \psi_{m1.b2}} \cdot b_2 & \text{if } \psi_{m1.b2} \geq 0 \\ \frac{0.6 b_2}{(1 - \psi_{m1.b2})} & \text{if } \psi_{m1.b2} < 0 \end{cases}$$ $b_{2.m1.inf} = 115.51 \cdot mm$

$$b_{t.m1} := \begin{cases} |e_{4.bi.eff}| & \text{if } e_{4.bi.eff} < 0 \\ 0 & \text{if } e_{4.bi.eff} \geq 0 \end{cases}$$ $b_{t.m1} = 877.48 \cdot mm$

$b_{lum} := b_1 + b_2$ $\qquad b_{lum} = 1430.00 \cdot mm$

$A_{sl} := (b_{1.m1.inf} + b_{2.m1.sup}) \cdot t_w + b_{v1} \cdot t_{v1}$ $\qquad A_{sl} = 4217.78 \cdot mm^2$

$e_{sti} := \frac{b_{v1} \cdot t_{v1} \cdot \left(\frac{b_{v1}}{2} + \frac{t_w}{2}\right)}{A_{sl}}$ $\qquad e_{sti} = 34.39 \cdot mm$

$$I_{sl} := \frac{1}{12} \cdot (b_{1.m1.inf} + b_{2.m1.sup}) \cdot t_w^3 + \frac{1}{12} \cdot t_{v1} \cdot b_{v1}^3 + (b_{1.m1.inf} + b_{2.m1.sup}) \cdot t_w \cdot e_{sti}^2 + b_{v1} \cdot t_{v1} \cdot \left(e_{sti} - \frac{b_{v1}}{2} - \frac{t_w}{2}\right)^2$$

$I_{sl} = 8958728.93 \cdot mm^4$

$a_{c.1} := 4.33 \cdot \sqrt[4]{\frac{I_{sl} \cdot (b_1)^2 \cdot (b_2)^2}{t_w^3 \cdot b_{lum}}}$ $\qquad a_{c.1} = 5026.22 \cdot mm$

$$\sigma_{cr.sl.1.1} := \begin{cases} \frac{1.05 \cdot E_d}{A_{sl}} \cdot \frac{\sqrt{I_{sl} \cdot t_w^3 \cdot b_{lum}}}{b_1 \cdot b_2} & \text{if } a \geq a_{c.1} \\ \frac{\pi^2 \cdot E_d \cdot I_{sl}}{A_{sl} \cdot a^2} + \frac{E_d \cdot t_w^3 \cdot b_{lum} \cdot a^2}{4 \cdot \pi^2 (1 - \nu^2) \cdot A_{sl} \cdot (b_1)^2 \cdot (b_2)^2} & \text{if } a < a_{c.1} \end{cases}$$ $\sigma_{cr.sl.1.1} = 347.59 \cdot \frac{N}{mm^2}$

$\sigma_{cr.sl.1} := \frac{e_{2.bi.eff}}{e_{2.bi.eff} - h_1} \cdot \sigma_{cr.sl.1.1}$ $\qquad \sigma_{cr.sl.1} = 997.55 \cdot \frac{N}{mm^2}$

Subpanelen reduceren:
Lijf h.1 zonder verstijvingen:
$\psi_{m1.b1} = 0.35$

$$k_{\sigma.h1.m1} := \begin{cases} 4 & \text{if } \psi_{m1.b1} = 1 \\ \frac{8.2}{1.05 + \psi_{m1.b1}} & \text{if } 0 < \psi_{m1.b1} < 1 \\ 7.8 & \text{if } \psi_{m1.b1} = 0 \\ 7.81 - 6.29 \cdot \psi_{m1.b1} + 9.78 \cdot \psi_{m1.b1}^2 & \text{if } 0 > \psi_{m1.b1} > -1 \\ 23.9 & \text{if } \psi_{m1.b1} = -1 \\ 5.98 \cdot (1 - \psi_{m1.b1})^2 & \text{if } -1 > \psi_{m1.b1} \geq -3 \end{cases}$$

$$\sigma_{cr.p.h1.m1} := k_{\sigma.h1.m1} \cdot \frac{\pi^2 \cdot E_d}{12 \cdot (1 - \nu^2)} \cdot \left(\frac{t_w}{h_1}\right)^2$$

$$\lambda_{p.h1.m1} := \sqrt{\frac{f_{yd}}{\sigma_{cr.p.h1.m1}}}$$

$$\rho_{loc.h1.m1} := \begin{cases} 1.0 & \text{if } \lambda_{p.h1.m1} \leq 0.5 + \sqrt{0.085 - 0.055 \psi_{m1.b1}} \\ \frac{\lambda_{p.h1.m1} - 0.055 \cdot (3 + \psi_{m1.b1})}{\lambda_{p.h1.m1}^2} & \text{if } \lambda_{p.h1.m1} > 0.5 + \sqrt{0.085 - 0.055 \psi_{m1.b1}} \end{cases}$$

$$\sigma_{cr.c.h1.m1} := \frac{\pi^2 \cdot E_d \cdot t_w^2}{12 \cdot (1 - \nu^2) \cdot a^2}$$

$$\lambda_{c.h1.m1} := \sqrt{\frac{f_{yd}}{\sigma_{cr.c.h1.m1}}}$$

$$\Phi_{c.h1.m1} := 0.5 \cdot \left[1 + 0.21 \cdot (\lambda_{c.h1.m1} - 0.2) + \lambda_{c.h1.m1}^2\right]$$

$$\chi_{c.h1.m1} := \frac{1}{\Phi_{c.h1.m1} + \sqrt{\Phi_{c.h1.m1}^2 - \lambda_{c.h1.m1}^2}}$$

$$\xi_{c.h1.m1} := \begin{cases} 1 & \text{if } \frac{\sigma_{cr.p.h1.m1}}{\sigma_{cr.c.h1.m1}} > 1 \\ 0 & \text{if } 0 > \frac{\sigma_{cr.p.h1.m1}}{\sigma_{cr.c.h1.m1}} \\ \frac{\sigma_{cr.p.h1.m1}}{\sigma_{cr.c.h1.m1}} & \text{if } 0 \leq \frac{\sigma_{cr.p.h1.m1}}{\sigma_{cr.c.h1.m1}} \leq 1 \end{cases}$$

$\rho_{c.h1.m1} := (\rho_{loc.h1.m1} - \chi_{c.h1.m1}) \cdot \xi_{c.h1.m1} \cdot (2 - \xi_{c.h1.m1}) + \chi_{c.h1.m1}$ $\qquad \rho_{c.h1.m1} = 1.00$

$b_{1.m1.edge.eff} := \rho_{c.h1.m1} \cdot b_{1.m1.edge}$ $\qquad b_{1.m1.edge.eff} = 154.79 \cdot mm$
$b_{1.m1.inf.eff} := \rho_{c.h1.m1} \cdot b_{1.m1.inf}$ $\qquad b_{1.m1.inf.eff} = 205.21 \cdot mm$

Lijf h.23 zonder verstijvingen:
$\psi_{m1.b2} = -4.56$

$$k_{\sigma.h23.m1} := \begin{cases} 4 & \text{if } \psi_{m1.b2} = 1 \\ \frac{8.2}{1.05 + \psi_{m1.b2}} & \text{if } 0 < \psi_{m1.b2} < 1 \\ 7.8 & \text{if } \psi_{m1.b2} = 0 \\ 7.81 - 6.29 \cdot \psi_{m1.b2} + 9.78 \cdot \psi_{m1.b2}^2 & \text{if } 0 > \psi_{m1.b2} > -1 \\ 23.9 & \text{if } \psi_{m1.b2} = -1 \\ 5.98 \cdot (1 - \psi_{m1.b2})^2 & \text{if } -1 > \psi_{m1.b2} \geq -3 \\ 95.68 & \text{if } \psi_{m1.b2} < -3 \end{cases}$$

$$\sigma_{cr.p.h23.m1} := k_{\sigma.h23.m1} \cdot \frac{\pi^2 \cdot E_d}{12 \cdot (1 - \nu^2)} \cdot \left(\frac{t_w}{h_2 + h_3}\right)^2$$

$$\lambda_{p.h23.m1} := \sqrt{\frac{f_{yd}}{\sigma_{cr.p.h23.m1}}}$$

$$\rho_{loc.h23.m1} := \begin{cases} 1.0 & \text{if } \lambda_{p.h23.m1} \le 0.5 + \sqrt{0.085 - 0.055\psi_{m1.b2}} \\ \dfrac{\lambda_{p.h23.m1} - 0.055\cdot(3 + \psi_{m1.b2})}{\lambda_{p.h23.m1}^2} & \text{if } \lambda_{p.h23.m1} > 0.5 + \sqrt{0.085 - 0.055\psi_{m1.b2}} \end{cases}$$

$$\sigma_{cr.c.h23.m1} := \frac{\pi^2\cdot E_d\cdot t_w^2}{12\cdot(1-\nu^2)\cdot a^2}$$

$$\lambda_{c.h23.m1} := \sqrt{\frac{f_{yd}}{\sigma_{cr.c.h23.m1}}}$$

$$\Phi_{c.h23.m1} := 0.5\cdot\left[1 + 0.21\cdot(\lambda_{c.h23.m1} - 0.2) + \lambda_{c.h23.m1}^2\right]$$

$$\chi_{c.h23.m1} := \frac{1}{\Phi_{c.h23.m1} + \sqrt{\Phi_{c.h23.m1}^2 - \lambda_{c.h23.m1}^2}}$$

$$\xi_{c.h23.m1} := \begin{cases} 1 & \text{if } \dfrac{\sigma_{cr.p.h23.m1}}{\sigma_{cr.c.h23.m1}} > 1 \\ 0 & \text{if } 0 > \dfrac{\sigma_{cr.p.h23.m1}}{\sigma_{cr.c.h23.m1}} \\ \dfrac{\sigma_{cr.p.h23.m1}}{\sigma_{cr.c.h23.m1}} & \text{if } 0 \le \dfrac{\sigma_{cr.p.h23.m1}}{\sigma_{cr.c.h23.m1}} \le 1 \end{cases}$$

$$\rho_{c.h23.m1} := (\rho_{loc.h23.m1} - \chi_{c.h23.m1})\cdot\xi_{c.h23.m1}\cdot(2 - \xi_{c.h23.m1}) + \chi_{c.h23.m1} \qquad \rho_{c.h23.m1} = 1.00$$

$$b_{2.m1.sup.eff} := \rho_{c.h23.m1}\cdot b_{2.m1.sup} \qquad b_{2.m1.sup.eff} = 77.01\cdot mm$$

$$b_{2.m1.inf.eff} := \rho_{c.h23.m1}\cdot b_{2.m1.inf} \qquad b_{2.m1.inf.eff} = 115.51\cdot mm$$

$$b_{t.m1} = 877.48\cdot mm$$

Verstijver 1 zonder verstijvingen:

$$\psi := 1$$

$$k_{\sigma.v1.m1} := 0.43$$

$$\sigma_{cr.p.v1.m1} := k_{\sigma.v1.m1}\cdot\frac{\pi^2\cdot E_d}{12\cdot(1-\nu^2)}\cdot\left(\frac{t_{v1}}{b_{v1}}\right)^2$$

$$\lambda_{p.v1.m1} := \sqrt{\frac{f_{yd}}{\sigma_{cr.p.v1.m1}}}$$

$$\rho_{loc.v1.m1} := \begin{cases} 1.0 & \text{if } \lambda_{p.v1.m1} \le 0.748 \\ \dfrac{\lambda_{p.v1.m1} - 0.188}{\lambda_{p.v1.m1}^2} & \text{if } \lambda_{p.v1.m1} > 0.748 \end{cases}$$

$$\sigma_{cr.c.v1.m1} := \frac{\pi^2\cdot E_d\cdot t_{v1}^2}{12\cdot(1-\nu^2)\cdot a^2}$$

$$\lambda_{c.v1.m1} := \sqrt{\frac{f_{yd}}{\sigma_{cr.c.v1.m1}}}$$

$$\Phi_{c.v1.m1} := 0.5\cdot\left[1 + 0.21\cdot(\lambda_{c.v1.m1} - 0.2) + \lambda_{c.v1.m1}^2\right]$$

$$\chi_{c.v1.m1} := \frac{1}{\Phi_{c.v1.m1} + \sqrt{\Phi_{c.v1.m1}^2 - \lambda_{c.v1.m1}^2}}$$

$$\xi_{c.v1.m1} := \begin{cases} 1 & \text{if } \dfrac{\sigma_{cr.p.v1.m1}}{\sigma_{cr.c.v1.m1}} > 1 \\ 0 & \text{if } 0 > \dfrac{\sigma_{cr.p.v1.m1}}{\sigma_{cr.c.v1.m1}} \\ \dfrac{\sigma_{cr.p.v1.m1}}{\sigma_{cr.c.v1.m1}} & \text{if } 0 \le \dfrac{\sigma_{cr.p.v1.m1}}{\sigma_{cr.c.v1.m1}} \le 1 \end{cases}$$

$$\rho_{c.v1.m1} := (\rho_{loc.v1.m1} - \chi_{c.v1.m1})\cdot\xi_{c.v1.m1}\cdot(2 - \xi_{c.v1.m1}) + \chi_{c.v1.m1} \qquad \rho_{c.v1.m1} = 1.00$$

$$b_{v1.m1.eff} := \rho_{c.v1.m1}\cdot b_{v1} \qquad b_{v1.m1.eff} = 140.00\cdot mm$$

Global plate like behaviour:

$$\psi := \frac{e_{4.bi.eff}}{e_{2.bi.eff}} \qquad \psi = -1.59$$

$$A_c := (b_{1.m1.inf} + b_{2.m1.sup})\cdot t_w + b_{v1}\cdot t_{v1}$$

$$A_{c.eff.loc} := (b_{1.m1.inf.eff} + b_{2.m1.sup.eff})\cdot t_w + b_{v1.m1.eff}\cdot t_{v1}$$

$$\beta_{A.c} := \frac{A_{c.eff.loc}}{A_c} \qquad \beta_{A.c} = 1.00$$

$$\lambda_{p.gl.m} := \sqrt{\frac{\beta_{A.c}\cdot f_{yd}}{|\sigma_{cr.sl.1}|}} \qquad \lambda_{p.gl.m} = 0.49$$

$$\rho_{loc.gl.m} := \begin{cases} 1.0 & \text{if } \lambda_{p.gl.m} \le 0.5 + \sqrt{0.085 - 0.055\psi} \\ \dfrac{\lambda_{p.gl.m} - 0.055\cdot(3 + \psi)}{\lambda_{p.gl.m}^2} & \text{if } \lambda_{p.gl.m} > 0.5 + \sqrt{0.085 - 0.055\psi} \end{cases} \qquad \rho_{loc.gl.m} = 1.00$$

$$\sigma_{cr.c.gl.m.1} := \frac{\pi^2\cdot E_d\cdot I_{sl}}{A_{sl}\cdot a^2}$$

$$\sigma_{cr.c.gl.m} := \frac{e_{2.bi.eff}}{e_{2.bi.eff} - h_1}\cdot\sigma_{cr.c.gl.m.1} \qquad \sigma_{cr.c.gl.m} = 56.15\cdot\frac{N}{mm^2}$$

$$\lambda_{c.gl.m} := \sqrt{\frac{\beta_{A.c}\cdot f_{yd}}{\sigma_{cr.c.gl.m}}}$$

$$i_{c.gl} := \sqrt{\frac{I_{sl}}{A_{sl}}}$$

$$e_{c.gl} := \begin{cases} e_{sti} & \text{if } e_{sti} \ge \dfrac{b_{v1}}{2} + \dfrac{t_w}{2} - e_{sti} \\ \dfrac{b_{v1}}{2} + \dfrac{t_w}{2} - e_{sti} & \text{if } \dfrac{b_{v1}}{2} + \dfrac{t_w}{2} - e_{sti} \ge e_{sti} \end{cases} \qquad e_{c.gl} = 39.61\cdot mm$$

$$\alpha_e := 0.49 + 0.09\cdot\frac{i_{c.gl}}{e_{c.gl}}$$

$$\Phi_{c.gl.m} := 0.5\cdot\left[1 + \alpha_e\cdot(\lambda_{c.gl.m} - 0.2) + \lambda_{c.gl.m}^2\right]$$

$$\chi_{c.gl.m} := \frac{1}{\Phi_{c.gl.m} + \sqrt{\Phi_{c.gl.m}^2 - \lambda_{c.gl.m}^2}}$$

$$\xi_{c.gl.m} := \begin{cases} 1 & \text{if } \frac{\sigma_{cr.sl.1}}{\sigma_{cr.c.gl.m}} > 1 \\ 0 & \text{if } 0 > \frac{\sigma_{cr.sl.1}}{\sigma_{cr.c.gl.m}} \\ \frac{\sigma_{cr.sl.1}}{\sigma_{cr.c.gl.m}} & \text{if } 0 \le \frac{\sigma_{cr.sl.1}}{\sigma_{cr.c.gl.m}} \le 1 \end{cases}$$

$\xi_{c.gl.m} = 1.00$

$\rho_{c.gl.m} := (\rho_{loc.gl.m} - \chi_{c.gl.m}) \cdot \xi_{c.gl.m} \cdot (2 - \xi_{c.gl.m}) + \chi_{c.gl.m}$

$\rho_{c.gl.m} = 1.00$

$t_{w.m1.eff} := \rho_{c.gl.m} \cdot t_w$

$t_{w.m1.eff} = 8.00 \cdot mm$

$t_{v1.m1.eff} := \rho_{c.gl.m} \cdot t_{v1}$

$t_{v1.m1.eff} = 14.00 \cdot mm$

Bruto oppervlakte:

$A_{m1} := b_{fbL} \cdot t_{fbL} + b_{fbR} \cdot t_{fbR} + b_{foL} \cdot t_{foL} + b_{foR} \cdot t_{foR} + h_w \cdot t_w + t_{v1} \cdot b_{v1} + t_{v2} \cdot b_{v2} + A_{fic.1} + A_{fic.2} + A_{t1} + A_{t2} + A_{t3} + A_{t4}$

$A_{m1} = 77400.00 \cdot mm^2$

Bruto statisch moment:

$S_{m1} := b_{fbL} \cdot t_{fbL} \cdot \frac{t_{fbL}}{2} + b_{fbR} \cdot t_{fbR} \cdot \frac{t_{fbR}}{2} + h_w \cdot t_w \cdot \left(t_{fbmax} + \frac{h_w}{2}\right) + b_{foL} \cdot t_{foL} \cdot \left(h_t - \frac{t_{foL}}{2}\right) + b_{foR} \cdot t_{foR} \cdot \left(h_t - \frac{t_{foR}}{2}\right) \dots$
$+ t_{v1} \cdot b_{v1} \cdot h_{v1} + t_{v2} \cdot b_{v2} \cdot h_{v2} + A_{fic.1} \cdot z_{fic.1} + A_{fic.2} \cdot z_{fic.2} + A_{t1} \cdot (z_{t1.bk} + z_{t1}) + A_{t2} \cdot (z_{t2.bk} + z_{t2}) + A_{t3} \cdot (z_{t3.bk} + z_{t3}) + A_{t4} \cdot (z_{t4.bk} + z_{t4})$

$S_{m1} = 45861200.00 \cdot mm^3$

Bruto zwaartepunt:

$e_{b.m1} := \frac{S_{m1}}{A_{m1}}$

$e_{b.m1} = 592.52 \cdot mm$

Effectieve oppervlakte:

$A_{m1.eff} := b_{fbL.a.eff} \cdot t_{fbL} + b_{fbR.a.eff} \cdot t_{fbR} + b_{foL} \cdot t_{foL} + b_{foR} \cdot t_{foR} \dots$
$+ b_{1.m1.edge.eff} \cdot t_w + b_{1.m1.inf.eff} \cdot t_{w.m1.eff} + b_{2.m1.sup.eff} \cdot t_{w.m1.eff} \dots$
$+ b_{2.m1.inf.eff} \cdot t_w + b_{t.m1} \cdot t_w + b_{v1.m1.eff} \cdot t_{v1.m1.eff} + \rho_{fic.1.m} \cdot A_{fic.1} + A_{fic.2} + A_{t1.eff} + A_{t2.eff} + A_{t3} + A_{t4} + t_{v2} \cdot b_{v2}$

$A_{m1.eff} = 77400.00 \cdot mm^2$

Effectief statisch moment:

$S_{m1.eff} := b_{fbL.a.eff} \cdot t_{fbL} \cdot \frac{t_{fbL}}{2} + b_{fbR.a.eff} \cdot t_{fbR} \cdot \frac{t_{fbR}}{2} + b_{foL} \cdot t_{foL} \cdot \left(h_t - \frac{t_{foL}}{2}\right) + b_{foR} \cdot t_{foR} \cdot \left(h_t - \frac{t_{foR}}{2}\right) \dots$
$+ b_{1.m1.edge.eff} \cdot t_w \cdot \left(\frac{b_{1.m1.edge.eff}}{2} + t_{fbmax}\right) + b_{1.m1.inf.eff} \cdot t_{w.m1.eff} \cdot \left(b_1 + t_{fbmax} - \frac{b_{1.m1.inf.eff}}{2}\right) \dots$
$+ b_{2.m1.sup.eff} \cdot t_{w.m1.eff} \cdot \left(b_1 + t_{fbmax} + \frac{b_{2.m1.sup.eff}}{2}\right) + b_{2.m1.inf.eff} \cdot t_w \cdot \left(b_1 + b_2 + t_{fbmax} - b_{t.m1} - \frac{b_{2.m1.inf.eff}}{2}\right) \dots$
$+ b_{t.m1} \cdot t_w \cdot \left(b_1 + b_2 + t_{fbmax} - \frac{b_{t.m1}}{2}\right) + b_{v1.m1.eff} \cdot t_{v1.m1.eff} \cdot h_{v1} + \rho_{fic.1.m} \cdot A_{fic.1} \cdot z_{fic.1} + A_{fic.2} \cdot z_{fic.2} \dots$
$+ A_{t1.eff} \cdot (z_{t1.bk} + z_{t1.eff}) + A_{t2.eff} \cdot (z_{t2.bk} + z_{t2.eff}) + A_{t3} \cdot (z_{t3.bk} + z_{t3}) + A_{t4} \cdot (z_{t4.bk} + z_{t4}) + b_{v2} \cdot t_{v2} \cdot h_{v2}$

$S_{m1.eff} = 45861200.00 \cdot mm^3$

Effectief zwaartepunt:

$e_{b.m1.eff} := \frac{S_{m1.eff}}{A_{m1.eff}}$

$e_{b.m1.eff} = 592.52 \cdot mm$

$$I_{y.0.m1.eff} := \frac{1}{12} \cdot \left(b_{fbL.a.eff} \cdot t_{fbL}^3 + b_{fbR.a.eff} \cdot t_{fbR}^3 + b_{foL} \cdot t_{foL}^3 + b_{foR} \cdot t_{foR}^3 \dots + b_{v1.m1.eff} \cdot t_{v1.m1.eff}^3 + t_w \cdot b_{1.m1.edge.eff}^3 \dots + t_{w.m1.eff} \cdot b_{1.m1.inf.eff}^3 + t_{w.m1.eff} \cdot b_{2.m1.sup.eff}^3 + t_w \cdot b_{2.m1.inf.eff}^3 + t_w \cdot b_{t.m1}^3 + b_{v2} \cdot t_{v2}^3 \right) \dots + \rho_{fic.1.m} \cdot I_{fic.1} + I_{fic.2} + I_{t1.eff} + I_{t2.eff} + I_{t3} + I_{t4}$$

$I_{y.c.m1.eff} := b_{fbL.a.eff} \cdot t_{fbL} \cdot \left(e_{b.m1.eff} - \frac{t_{fbL}}{2}\right)^2 + b_{fbR.a.eff} \cdot t_{fbR} \cdot \left(e_{b.m1.eff} - \frac{t_{fbR}}{2}\right)^2 \dots$
$+ b_{1.m1.edge.eff} \cdot t_w \cdot \left(e_{b.m1.eff} - t_{fbmax} - \frac{b_{1.m1.edge.eff}}{2}\right)^2 \dots$
$+ b_{1.m1.inf.eff} \cdot t_{w.m1.eff} \cdot \left(e_{b.m1.eff} - t_{fbmax} - b_1 + \frac{b_{1.m1.inf.eff}}{2}\right)^2 \dots$
$+ b_{2.m1.sup.eff} \cdot t_{w.m1.eff} \cdot \left(e_{b.m1.eff} - t_{fbmax} - b_1 - \frac{b_{2.m1.sup.eff}}{2}\right)^2 \dots$
$+ b_{2.m1.inf.eff} \cdot t_w \cdot \left(e_{b.m1.eff} - h_w - t_{fbmax} + b_{t.m1} + \frac{b_{2.m1.inf.eff}}{2}\right)^2 \dots$
$+ b_{t.m1} \cdot t_w \cdot \left(e_{b.m1.eff} - t_{fbmax} - h_w + \frac{b_{t.m1}}{2}\right)^2 + b_{foL} \cdot t_{foL} \cdot \left(h_t - \frac{t_{foL}}{2} - e_{b.m1.eff}\right)^2 \dots$
$+ b_{foR} \cdot t_{foR} \cdot \left(h_t - \frac{t_{foR}}{2} - e_{b.m1.eff}\right)^2 + b_{v1.m1.eff} \cdot t_{v1.m1.eff} \cdot (h_{v1} - e_{b.m1.eff})^2 + b_{v2} \cdot t_{v2} \cdot (h_{v2} - e_{b.m1.eff})^2 \dots$
$+ \rho_{fic.1.m} \cdot A_{fic.1} \cdot (z_{fic.1} - e_{b.m1.eff})^2 + A_{fic.2} \cdot (z_{fic.2} - e_{b.m1.eff})^2 + A_{t1.eff} \cdot (z_{t1.bk} + z_{t1.eff} - e_{b.m1.eff})^2 \dots$
$+ A_{t2.eff} \cdot (z_{t2.bk} + z_{t2.eff} - e_{b.m1.eff})^2 + A_{t3} \cdot (z_{t3.bk} + z_{t3} - e_{b.m1.eff})^2 + A_{t4} \cdot (z_{t4.bk} + z_{t4} - e_{b.m1.eff})^2$

Effectief traagheidsmoment:

$I_{y.m1.eff} := I_{y.0.m1.eff} + I_{y.c.m1.eff}$

$I_{y.m1.eff} = 3.4559 \times 10^{10} \cdot mm^4$

Lijf met 1 verstijver in drukzone (MOMENT)

Lijf met 2 verstijvers (MOMENT)

Lijf met 2 verstijvers:

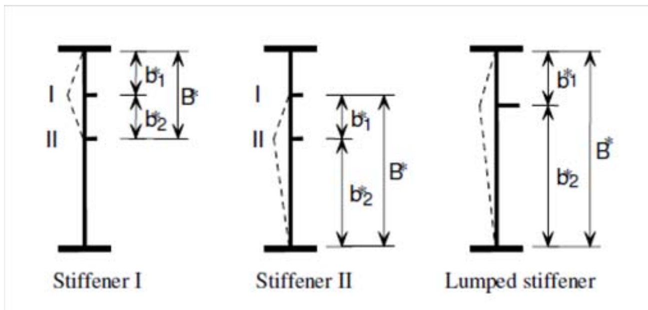


Figure A.3: Notations for plate with two stiffeners in the compression zone (uit NEN-EN1993-1-5 pagina 41)

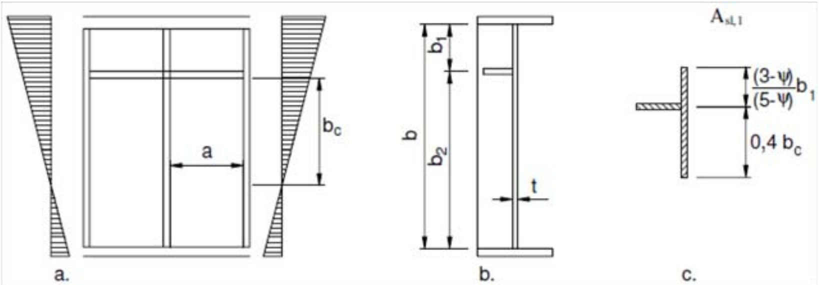


Figure A.1: Notations for longitudinally stiffened plates (uit NEN-EN1993-1-5 pagina 40)

$b_1 := h_1$ $\quad b_1 = 360.00 \cdot mm$

$b_2 := h_2$ $\quad b_2 = 300.00 \cdot mm$

$b_3 := h_3$ $\quad b_3 = 770.00 \cdot mm$

$$\psi_{m2.b1} := \frac{e_{b.bi.eff} - (b_1 + t_{fbmax})}{e_{2.bi.eff}} \qquad \psi_{m2.b1} = 0.35$$

$$\psi_{m2.b2} := \begin{cases} 1 & \text{if } \dfrac{e_{b.bi.eff} - (b_1 + b_2 + t_{fbmax})}{e_{b.bi.eff} - (b_1 + t_{fbmax})} > 1 \\ \dfrac{e_{b.bi.eff} - (b_1 + b_2 + t_{fbmax})}{e_{b.bi.eff} - (b_1 + t_{fbmax})} & \text{otherwise} \end{cases} \qquad \psi_{m2.b2} = -0.56$$

$$\psi_{m2.b3} := \begin{cases} 1 & \text{if } \dfrac{e_{b.bi.eff} - (b_1 + b_2 + b_3 + t_{fbmax})}{e_{b.bi.eff} - (b_1 + b_2 + t_{fbmax})} > 1 \\ \dfrac{e_{b.bi.eff} - (b_1 + b_2 + b_3 + t_{fbmax})}{e_{b.bi.eff} - (b_1 + b_2 + t_{fbmax})} & \text{otherwise} \end{cases} \qquad \psi_{m2.b3} = 1.00$$

$$b_{1.m2.edge} := \frac{2}{5 - \psi_{m2.b1}} \cdot b_1 \qquad b_{1.m2.edge} = 154.79 \cdot mm$$

$$b_{1.m2.inf} := \frac{3 - \psi_{m2.b1}}{5 - \psi_{m2.b1}} \cdot b_1 \qquad b_{1.m2.inf} = 205.21 \cdot mm$$

$$b_{2.m2.sup} := \frac{2}{5 - \psi_{m2.b2}} \cdot b_2 \qquad b_{2.m2.sup} = 107.95 \cdot mm$$

$$b_{2.m2.inf} := \frac{3 - \psi_{m2.b2}}{5 - \psi_{m2.b2}} \cdot b_2 \qquad b_{2.m2.inf} = 192.05 \cdot mm$$

$$b_{3.m2.sup} := \begin{cases} \dfrac{2}{5 - \psi_{m2.b3}} \cdot b_3 & \text{if } \psi_{m2.b3} \ge 0 \\ \dfrac{0.4 b_3}{(1 - \psi_{m2.b3})} & \text{if } \psi_{m2.b3} < 0 \end{cases} \qquad b_{3.m2.sup} = 385.00 \cdot mm$$

$$b_{3.m2.inf} := \begin{cases} \dfrac{3 - \psi_{m2.b3}}{5 - \psi_{m2.b3}} \cdot b_3 & \text{if } \psi_{m2.b3} \ge 0 \\ \dfrac{0.6 b_3}{(1 - \psi_{m2.b3})} & \text{if } \psi_{m2.b3} < 0 \end{cases} \qquad b_{3.m2.inf} = 385.00 \cdot mm$$

$$b_{t.m2} := \begin{cases} |e_{4.bi.eff}| & \text{if } e_{4.bi.eff} < 0 \\ 0 & \text{if } e_{4.bi.eff} \ge 0 \end{cases} \qquad b_{t.m2} = 877.48 \cdot mm$$

1 stiffener lumped:

$b_{lum.1} := b_1 + b_2$ $\quad b_{lum.1} = 660.00 \cdot mm$

$$A_{sl.1} := (b_{1.m2.inf} + b_{2.m2.sup}) t_w + b_{v1} \cdot t_{v1} \qquad A_{sl.1} = 4465.28 \cdot mm^2$$

$$e_{sti.1} := \frac{b_{v1} \cdot t_{v1} \cdot \left(\frac{b_{v1}}{2} + \frac{t_w}{2}\right)}{A_{sl.1}} \qquad e_{sti.1} = 32.48 \cdot mm$$

$$I_{sl.1} := \frac{1}{12} \cdot (b_{1.m2.inf} + b_{2.m2.sup}) \cdot t_w^3 + \frac{1}{12} \cdot t_{v1} \cdot b_{v1}^3 + (b_{1.m2.inf} + b_{2.m2.sup}) \cdot t_w \cdot e_{sti.1}^2 + b_{v1} \cdot t_{v1} \cdot \left(e_{sti.1} - \frac{b_{v1}}{2} - \frac{t_w}{2}\right)^2$$

$$I_{sl.1} = 9236509.78 \cdot mm^4$$

$$a_{c.1.1} := 4.33 \cdot \sqrt[4]{\frac{I_{sl.1} \cdot b_1^2 \cdot b_2^2}{t_w^3 \cdot b_{lum.1}}} \qquad a_{c.1.1} = 3253.67 \cdot mm$$

$$\sigma_{cr.sl.1.1} := \begin{cases} \dfrac{1.05 \cdot E_d}{A_{sl.1}} \cdot \dfrac{\sqrt{I_{sl.1} \cdot t_w^3 \cdot b_{lum.1}}}{b_1 \cdot b_2} & \text{if } a \ge a_{c.1.1} \\ \dfrac{\pi^2 \cdot E_d \cdot I_{sl.1}}{A_{sl.1} \cdot a^2} + \dfrac{E_d \cdot t_w^3 \cdot b_{lum.1} \cdot a^2}{4 \cdot \pi^2 (1 - \nu^2) \cdot A_{sl.1} \cdot b_1^2 \cdot b_2^2} & \text{if } a < a_{c.1.1} \end{cases} \qquad \sigma_{cr.sl.1.1} = 807.79 \cdot \frac{N}{mm^2}$$

$$\sigma_{cr.sl.1} := \frac{e_{2.bi.eff}}{e_{2.bi.eff} - h_1} \cdot \sigma_{cr.sl.1.1} \qquad \sigma_{cr.sl.1} = 2318.28 \cdot \frac{N}{mm^2}$$

2 stiffener lumped:

$b_{lum.2} := b_2 + b_3$ $\quad b_{lum.2} = 1070.00 \cdot mm$

$$A_{sl.2} := (b_{2.m2.inf} + b_{3.m2.sup}) \cdot t_w + b_{v2} \cdot t_{v2} \qquad A_{sl.2} = 4616.42 \cdot mm^2$$

$$e_{sti.2} := \frac{b_{v2} \cdot t_{v2} \cdot \left(\frac{b_{v2}}{2} + \frac{t_w}{2}\right)}{A_{sl.2}} \qquad e_{sti.2} = 8.67 \times 10^{-10} \cdot mm$$

$$I_{sl.2} := \frac{1}{12} \cdot (b_{2.m2.inf} + b_{3.m2.sup}) \cdot t_w^3 + \frac{1}{12} \cdot t_{v2} \cdot b_{v2}^3 + (b_{2.m2.inf} + b_{3.m2.sup}) \cdot t_w \cdot e_{sti.2}^2 + b_{v2} \cdot t_{v2} \cdot \left(e_{sti.2} - \frac{b_{v2}}{2} - \frac{t_w}{2}\right)^2$$

$$I_{sl.2} = 24620.91 \cdot mm^4$$

$$a_{c.1.2} := 4.33 \cdot \sqrt[4]{\frac{I_{sl.2} \cdot b_2^2 \cdot b_3^2}{t_w^3 b_{lum.2}}} \qquad a_{c.1.2} = 958.20 \cdot mm$$

$$\sigma_{cr.sl.2.1} := \begin{cases} \dfrac{1.05 \cdot E_d}{A_{sl.2}} \cdot \dfrac{\sqrt{I_{sl.2} \cdot t_w^3 \cdot b_{lum.2}}}{b_2 \cdot b_3} & \text{if } a \ge a_{c.1.2} \\ \dfrac{\pi^2 \cdot E_d \cdot I_{sl.2}}{A_{sl.2} \cdot a^2} + \dfrac{E_d \cdot t_w^3 \cdot 2 b_{lum.2} \cdot a^2}{4 \cdot \pi^2 (1 - \nu^2) \cdot A_{sl.2} \cdot b_2^2 \cdot b_3^2} & \text{if } a < a_{c.1.2} \end{cases} \qquad \sigma_{cr.sl.2.1} = 24.01 \cdot \frac{N}{mm^2}$$

$$\sigma_{cr.sl.2} := \frac{e_{2.bi.eff}}{e_{2.bi.eff} - h_1 - h_2} \cdot \sigma_{cr.sl.2.1} \qquad \sigma_{cr.sl.2} = -123.45 \cdot \frac{N}{mm^2}$$

1 en 2 stiffeners lumped:

$b_{lum.12} := h_1 + h_2 + h_3$ $\quad b_{lum.12} = 1430.00 \cdot mm$

$$b_{1.12} := \frac{A_{sl.2} \cdot (e_{1.bi.eff} - h_1 - h_2)}{A_{sl.1} \cdot (e_{1.bi.eff} - h_1) + A_{sl.2} \cdot (e_{1.bi.eff} - h_1 - h_2)} \cdot h_2 + h_1 \qquad b_{1.12} = 231.41 \cdot mm$$

$b_{2.12} := h_w - b_{1.12}$ $\quad b_{2.12} = 1198.59 \cdot mm$

$A_{sl.12} := A_{sl.1} + A_{sl.2}$ $\quad A_{sl.12} = 9081.71 \cdot mm^2$

$I_{sl.12} := I_{sl.1} + I_{sl.2}$ $\qquad I_{sl.12} = 9261130.69\cdot mm^4$

$$a_{c.1.12} := 4.33\cdot\sqrt[4]{\frac{I_{sl.12}\cdot b_{1.12}^2\cdot b_{2.12}^2}{t_w^3 b_{lum.12}}} \qquad a_{c.1.12} = 4300.63\cdot mm$$

$$\sigma_{cr.sl.12.1} := \begin{cases} \frac{1.05\cdot E_d}{A_{sl.12}}\cdot\frac{\sqrt{I_{sl.12}\cdot t_w^3 b_{lum.12}}}{b_{1.12}\cdot b_{2.12}} & \text{if } a \ge a_{c.1.12} \\ \frac{\pi^2\cdot E_d\cdot I_{sl.12}}{A_{sl.12}\cdot a^2} + \frac{E_d\cdot t_w^3\cdot b_{lum.12}\cdot a^2}{4\cdot\pi^2\left(1-\nu^2\right)\cdot A_{sl.12}\cdot b_{1.12}^2\cdot b_{2.12}^2} & \text{if } a < a_{c.1.12} \end{cases} \qquad \sigma_{cr.sl.12.1} = 227.94\cdot\frac{N}{mm^2}$$

$$\sigma_{cr.sl.12} := \frac{e_{2.bi.eff}}{e_{2.bi.eff} - b_{1.12}}\cdot\sigma_{cr.sl.12.1} \qquad \sigma_{cr.sl.12} = 392.21\cdot\frac{N}{mm^2}$$

Subpanelen reduceren:
Lijf h.1 zonder verstijvingen:
$\psi_{m2.b1} = 0.35$

$$k_{\sigma.h1.m2} := \begin{cases} 4 & \text{if } \psi_{m2.b1} = 1 \\ \frac{8.2}{1.05+\psi_{m2.b1}} & \text{if } 0 < \psi_{m2.b1} < 1 \\ 7.8 & \text{if } \psi_{m2.b1} = 0 \\ 7.81 - 6.29\cdot\psi_{m2.b1} + 9.78\cdot\psi_{m2.b1}^2 & \text{if } 0 > \psi_{m2.b1} > -1 \\ 23.9 & \text{if } \psi_{m2.b1} = -1 \\ 5.98\cdot\left(1-\psi_{m2.b1}\right)^2 & \text{if } -1 > \psi_{m2.b1} \ge -3 \end{cases}$$

$$\sigma_{cr.p.h1.m2} := k_{\sigma.h1.m2}\cdot\frac{\pi^2\cdot E_d}{12\cdot\left(1-\nu^2\right)}\cdot\left(\frac{t_w}{h_1}\right)^2$$

$$\lambda_{p.h1.m2} := \sqrt{\frac{f_{yd}}{\sigma_{cr.p.h1.m2}}}$$

$$\rho_{loc.h1.m2} := \begin{cases} 1.0 & \text{if } \lambda_{p.h1.m2} \le 0.5 + \sqrt{0.085 - 0.055\psi_{m2.b1}} \\ \frac{\lambda_{p.h1.m2} - 0.055\cdot\left(3+\psi_{m2.b1}\right)}{\lambda_{p.h1.m2}^2} & \text{if } \lambda_{p.h1.m2} > 0.5 + \sqrt{0.085 - 0.055\psi_{m2.b1}} \end{cases}$$

$$\sigma_{cr.c.h1.m2} := \frac{\pi^2\cdot E_d\cdot t_w^2}{12\cdot\left(1-\nu^2\right)\cdot a^2}$$

$$\lambda_{c.h1.m2} := \sqrt{\frac{f_{yd}}{\sigma_{cr.c.h1.m2}}}$$

$$\Phi_{c.h1.m2} := 0.5\cdot\left[1 + 0.21\cdot\left(\lambda_{c.h1.m2} - 0.2\right) + \lambda_{c.h1.m2}^2\right]$$

$$\chi_{c.h1.m2} := \frac{1}{\Phi_{c.h1.m2} + \sqrt{\Phi_{c.h1.m2}^2 - \lambda_{c.h1.m2}^2}}$$

$$\xi_{c.h1.m2} := \begin{cases} 1 & \text{if } \frac{\sigma_{cr.p.h1.m2}}{\sigma_{cr.c.h1.m2}} > 1 \\ 0 & \text{if } 0 > \frac{\sigma_{cr.p.h1.m2}}{\sigma_{cr.c.h1.m2}} \\ \frac{\sigma_{cr.p.h1.m2}}{\sigma_{cr.c.h1.m2}} & \text{if } 0 \le \frac{\sigma_{cr.p.h1.m2}}{\sigma_{cr.c.h1.m2}} \le 1 \end{cases} \qquad \xi_{c.h1.m2} = 1.00$$

$\rho_{c.h1.m2} := \left(\rho_{loc.h1.m2} - \chi_{c.h1.m2}\right)\cdot\xi_{c.h1.m2}\cdot\left(2-\xi_{c.h1.m2}\right) + \chi_{c.h1.m2}$ $\qquad \rho_{c.h1.m2} = 1.00$
$b_{1.m2.edge.eff} := \rho_{c.h1.m2}\cdot b_{1.m2.edge}$ $\qquad b_{1.m2.edge.eff} = 154.79\cdot mm$
$b_{1.m2.inf.eff} := \rho_{c.h1.m2}\cdot b_{1.m2.inf}$ $\qquad b_{1.m2.inf.eff} = 205.21\cdot mm$

Lijf h.2 zonder verstijvingen:
$\psi_{m2.b2} = -0.56$

$$k_{\sigma.h2.m2} := \begin{cases} 4 & \text{if } \psi_{m2.b2} = 1 \\ \frac{8.2}{1.05+\psi_{m2.b2}} & \text{if } 0 < \psi_{m2.b2} < 1 \\ 7.8 & \text{if } \psi_{m2.b2} = 0 \\ 7.81 - 6.29\cdot\psi_{m2.b2} + 9.78\cdot\psi_{m2.b2}^2 & \text{if } 0 > \psi_{m2.b2} > -1 \\ 23.9 & \text{if } \psi_{m2.b2} = -1 \\ 5.98\cdot\left(1-\psi_{m2.b2}\right)^2 & \text{if } -1 > \psi_{m2.b2} \ge -3 \\ 5.98\cdot(1 - -3)^2 & \text{if } \psi_{m2.b2} < -3 \end{cases}$$

$$\sigma_{cr.p.h2.m2} := k_{\sigma.h2.m2}\cdot\frac{\pi^2\cdot E_d}{12\cdot\left(1-\nu^2\right)}\cdot\left(\frac{t_w}{h_2}\right)^2$$

$$\lambda_{p.h2.m2} := \sqrt{\frac{f_{yd}}{\sigma_{cr.p.h2.m2}}}$$

$$\rho_{loc.h2.m2} := \begin{cases} 1.0 & \text{if } \lambda_{p.h2.m2} \le 0.5 + \sqrt{0.085 - 0.055\psi_{m2.b2}} \\ \frac{\lambda_{p.h2.m2} - 0.055\cdot\left(3+\psi_{m2.b2}\right)}{\lambda_{p.h2.m2}^2} & \text{if } \lambda_{p.h2.m2} > 0.5 + \sqrt{0.085 - 0.055\psi_{m2.b2}} \end{cases}$$

$$\sigma_{cr.c.h2.m2} := \frac{\pi^2\cdot E_d\cdot t_w^2}{12\cdot\left(1-\nu^2\right)\cdot a^2}$$

$$\lambda_{c.h2.m2} := \sqrt{\frac{f_{yd}}{\sigma_{cr.c.h2.m2}}}$$

$$\Phi_{c.h2.m2} := 0.5\cdot\left[1 + 0.21\cdot\left(\lambda_{c.h2.m2} - 0.2\right) + \lambda_{c.h2.m2}^2\right]$$

$$\chi_{c.h2.m2} := \frac{1}{\Phi_{c.h2.m2} + \sqrt{\Phi_{c.h2.m2}^2 - \lambda_{c.h2.m2}^2}}$$

$$\xi_{c.h2.m2} := \begin{cases} 1 & \text{if } \frac{\sigma_{cr.p.h2.m2}}{\sigma_{cr.c.h2.m2}} > 1 \\ 0 & \text{if } 0 > \frac{\sigma_{cr.p.h2.m2}}{\sigma_{cr.c.h2.m2}} \\ \frac{\sigma_{cr.p.h2.m2}}{\sigma_{cr.c.h2.m2}} & \text{if } 0 \le \frac{\sigma_{cr.p.h2.m2}}{\sigma_{cr.c.h2.m2}} \le 1 \end{cases} \qquad \xi_{c.h2.m2} = 1.00$$

$\rho_{c.h2.m2} := \left(\rho_{loc.h2.m2} - \chi_{c.h2.m2}\right)\cdot\xi_{c.h2.m2}\cdot\left(2-\xi_{c.h2.m2}\right) + \chi_{c.h2.m2}$ $\qquad \rho_{c.h2.m2} = 1.00$
$b_{2.m2.sup.eff} := \rho_{c.h2.m2}\cdot b_{2.m2.sup}$ $\qquad b_{2.m2.sup.eff} = 107.95\cdot mm$
$b_{2.m2.inf.eff} := \rho_{c.h2.m2}\cdot b_{2.m2.inf}$ $\qquad b_{2.m2.inf.eff} = 192.05\cdot mm$

Lijf h.3 zonder verstijvingen:
$\psi_{m2.b3} = 1.00$

$k_{\sigma.h3.m2} :=$ $\begin{cases} 4 & \text{if } \psi_{m2.b3} = 1 \\ \dfrac{8.2}{1.05 + \psi_{m2.b3}} & \text{if } 0 < \psi_{m2.b3} < 1 \\ 7.8 & \text{if } \psi_{m2.b3} = 0 \\ 7.81 - 6.29\cdot\psi_{m2.b3} + 9.78\cdot\psi_{m2.b3}^2 & \text{if } 0 > \psi_{m2.b3} > -1 \\ 23.9 & \text{if } \psi_{m2.b3} = -1 \\ 5.98\cdot(1 - \psi_{m2.b3})^2 & \text{if } -1 > \psi_{m2.b3} \geq -3 \\ 5.98\cdot(1 - -3)^2 & \text{if } \psi_{m2.b3} < -3 \end{cases}$

$\sigma_{cr.p.h3.m2} := k_{\sigma.h3.m2}\cdot\dfrac{\pi^2\cdot E_d}{12\cdot(1-\nu^2)}\cdot\left(\dfrac{t_w}{h_3}\right)^2$

$\lambda_{p.h3.m2} := \sqrt{\dfrac{f_{yd}}{\sigma_{cr.p.h3.m2}}}$

$\rho_{loc.h3.m2} :=$ $\begin{cases} 1.0 & \text{if } \lambda_{p.h3.m2} \leq 0.5 + \sqrt{0.085 - 0.055\psi_{m2.b3}} \\ \dfrac{\lambda_{p.h3.m2} - 0.055\cdot(3+\psi_{m2.b3})}{\lambda_{p.h3.m2}^2} & \text{if } \lambda_{p.h3.m2} > 0.5 + \sqrt{0.085 - 0.055\psi_{m2.b3}} \end{cases}$

$\sigma_{cr.c.h3.m2} := \dfrac{\pi^2\cdot E_d\cdot t_w^2}{12\cdot(1-\nu^2)\cdot a^2}$

$\lambda_{c.h3.m2} := \sqrt{\dfrac{f_{yd}}{\sigma_{cr.c.h3.m2}}}$

$\Phi_{c.h3.m2} := 0.5\cdot\left[1 + 0.21\cdot(\lambda_{c.h3.m2} - 0.2) + \lambda_{c.h3.m2}^2\right]$

$\chi_{c.h3.m2} := \dfrac{1}{\Phi_{c.h3.m2} + \sqrt{\Phi_{c.h3.m2}^2 - \lambda_{c.h3.m2}^2}}$

$\xi_{c.h3.m2} :=$ $\begin{cases} 1 & \text{if } \dfrac{\sigma_{cr.p.h3.m2}}{\sigma_{cr.c.h3.m2}} > 1 \\ 0 & \text{if } 0 > \dfrac{\sigma_{cr.p.h3.m2}}{\sigma_{cr.c.h3.m2}} \\ \dfrac{\sigma_{cr.p.h3.m2}}{\sigma_{cr.c.h3.m2}} & \text{if } 0 \leq \dfrac{\sigma_{cr.p.h3.m2}}{\sigma_{cr.c.h3.m2}} \leq 1 \end{cases}$ $\xi_{c.h3.m2} = 1.00$

$\rho_{c.h3.m2} := (\rho_{loc.h3.m2} - \chi_{c.h3.m2})\cdot\xi_{c.h3.m2}\cdot(2 - \xi_{c.h3.m2}) + \chi_{c.h3.m2}$ $\rho_{c.h3.m2} = 0.51$

$b_{3.m2.sup.eff} := \rho_{c.h3.m2}\cdot b_{3.m2.sup}$ $b_{3.m2.sup.eff} = 197.82\cdot mm$

$b_{3.m2.inf.eff} := \rho_{c.h3.m2}\cdot b_{3.m2.inf}$ $b_{3.m2.inf.eff} = 197.82\cdot mm$

$b_{t.m2} = 877.48\cdot mm$

Verstijver 1 zonder verstijvingen:

$\psi := 1$

$k_{\sigma.v1.m2} := 0.43$

$\sigma_{cr.p.v1.m2} := k_{\sigma.v1.m2}\cdot\dfrac{\pi^2\cdot E_d}{12\cdot(1-\nu^2)}\cdot\left(\dfrac{t_{v1}}{b_{v1}}\right)^2$

$\lambda_{p.v1.m2} := \sqrt{\dfrac{f_{yd}}{\sigma_{cr.p.v1.m2}}}$

$\rho_{loc.v1.m2} :=$ $\begin{cases} 1.0 & \text{if } \lambda_{p.v1.m2} \leq 0.748 \\ \dfrac{\lambda_{p.v1.m2} - 0.188}{\lambda_{p.v1.m2}^2} & \text{if } \lambda_{p.v1.m2} > 0.748 \end{cases}$

$\sigma_{cr.c.v1.m2} := \dfrac{\pi^2\cdot E_d\cdot t_{v1}^2}{12\cdot(1-\nu^2)\cdot a^2}$

$\lambda_{c.v1.m2} := \sqrt{\dfrac{f_{yd}}{\sigma_{cr.c.v1.m2}}}$

$\Phi_{c.v1.m2} := 0.5\cdot\left[1 + 0.21\cdot(\lambda_{c.v1.m2} - 0.2) + \lambda_{c.v1.m2}^2\right]$

$\chi_{c.v1.m2} := \dfrac{1}{\Phi_{c.v1.m2} + \sqrt{\Phi_{c.v1.m2}^2 - \lambda_{c.v1.m2}^2}}$

$\xi_{c.v1.m2} :=$ $\begin{cases} 1 & \text{if } \dfrac{\sigma_{cr.p.v1.m2}}{\sigma_{cr.c.v1.m2}} > 1 \\ 0 & \text{if } 0 > \dfrac{\sigma_{cr.p.v1.m2}}{\sigma_{cr.c.v1.m2}} \\ \dfrac{\sigma_{cr.p.v1.m2}}{\sigma_{cr.c.v1.m2}} & \text{if } 0 \leq \dfrac{\sigma_{cr.p.v1.m2}}{\sigma_{cr.c.v1.m2}} \leq 1 \end{cases}$ $\xi_{c.v1.m2} = 1.00$

$\rho_{c.v1.m2} := (\rho_{loc.v1.m2} - \chi_{c.v1.m2})\cdot\xi_{c.v1.m2}\cdot(2 - \xi_{c.v1.m2}) + \chi_{c.v1.m2}$ $\rho_{c.v1.m2} = 1.00$

$b_{v1.m2.eff} := \rho_{c.v1.m2}\cdot b_{v1}$ $b_{v1.m2.eff} = 140.00\cdot mm$

Verstijver 2 zonder verstijvingen:

$\psi := 1$

$k_{\sigma.v2.m2} := 0.43$

$\sigma_{cr.p.v2.m2} := k_{\sigma.v2.m2}\cdot\dfrac{\pi^2\cdot E_d}{12\cdot(1-\nu^2)}\cdot\left(\dfrac{t_{v2}}{b_{v2}}\right)^2$

$\lambda_{p.v2.m2} := \sqrt{\dfrac{f_{yd}}{\sigma_{cr.p.v2.m2}}}$

$\rho_{loc.v2.m2} :=$ $\begin{cases} 1.0 & \text{if } \lambda_{p.v2.m2} \leq 0.748 \\ \dfrac{\lambda_{p.v2.m2} - 0.188}{\lambda_{p.v2.m2}^2} & \text{if } \lambda_{p.v2.m2} > 0.748 \end{cases}$

$\sigma_{cr.c.v2.m2} := \dfrac{\pi^2\cdot E_d\cdot t_{v2}^2}{12\cdot(1-\nu^2)\cdot a^2}$

$\lambda_{c.v2.m2} := \sqrt{\dfrac{f_{yd}}{\sigma_{cr.c.v2.m2}}}$

$\Phi_{c.v2.m2} := 0.5\cdot\left[1 + 0.21\cdot(\lambda_{c.v2.m2} - 0.2) + \lambda_{c.v2.m2}^2\right]$

$\chi_{c.v2.m2} := \dfrac{1}{\Phi_{c.v2.m2} + \sqrt{\Phi_{c.v2.m2}^2 - \lambda_{c.v2.m2}^2}}$

$\xi_{c.v2.m2} :=$ $\begin{cases} 1 & \text{if } \dfrac{\sigma_{cr.p.v2.m2}}{\sigma_{cr.c.v2.m2}} > 1 \\ 0 & \text{if } 0 > \dfrac{\sigma_{cr.p.v2.m2}}{\sigma_{cr.c.v2.m2}} \\ \dfrac{\sigma_{cr.p.v2.m2}}{\sigma_{cr.c.v2.m2}} & \text{if } 0 \leq \dfrac{\sigma_{cr.p.v2.m2}}{\sigma_{cr.c.v2.m2}} \leq 1 \end{cases}$ $\xi_{c.v2.m2} = 1.00$

$\rho_{c.v2.m2} := (\rho_{loc.v2.m2} - \chi_{c.v2.m2})\cdot\xi_{c.v2.m2}\cdot(2 - \xi_{c.v2.m2}) + \chi_{c.v2.m2}$ $\rho_{c.v2.m2} = 1.00$

$b_{v2.m2.eff} := \rho_{c.v2.m2}\cdot b_{v2}$ $b_{v2.m2.eff} = 0.00\cdot mm$

Global plate like behaviour:

$\psi := \dfrac{e_{4.bi.eff}}{e_{2.bi.eff}}$ $\psi = -1.59$

$A_c := (b_{1.m2.inf} + b_{2.m2.sup} + b_{2.m2.inf} + b_{3.m2.sup})\cdot t_w + b_{v1}\cdot t_{v1} + b_{v2}\cdot t_{v2}$

$$A_{c.eff.loc} := \left(b_{1.m2.inf.eff} + b_{2.m2.sup.eff} + b_{2.m2.inf.eff} + b_{3.m2.sup.eff}\right)\cdot t_w + b_{v1.m2.eff}\cdot t_{v1} + b_{v2.m2.eff}\cdot t_{v2}$$

$$\beta_{A.c} := \frac{A_{c.eff.loc}}{A_c}$$

$\beta_{A.c} = 0.84$

$$\sigma_{cr.sl.lum} := \begin{cases} \sigma_{cr.sl.1} & \text{if } \sigma_{cr.sl.2} \geq \sigma_{cr.sl.1} \text{ if } \sigma_{cr.sl.12} \geq \sigma_{cr.sl.1} \\ \sigma_{cr.sl.2} & \text{if } \sigma_{cr.sl.1} \geq \sigma_{cr.sl.2} \text{ if } \sigma_{cr.sl.12} \geq \sigma_{cr.sl.2} \\ \sigma_{cr.sl.12} & \text{if } \sigma_{cr.sl.1} \geq \sigma_{cr.sl.12} \text{ if } \sigma_{cr.sl.2} \geq \sigma_{cr.sl.12} \end{cases}$$

$\sigma_{cr.sl.lum} = -123.45\cdot\frac{N}{mm^2}$

$$\lambda_{p.gl.m2} := \sqrt{\frac{\beta_{A.c}\cdot f_{yd}}{\left|\sigma_{cr.sl.lum}\right|}}$$

$\lambda_{p.gl.m2} = 1.26$

$$\rho_{loc.gl.m2} := \begin{cases} 1.0 & \text{if } \lambda_{p.gl.m2} \leq 0.5 + \sqrt{0.085 - 0.055\psi} \\ \dfrac{\lambda_{p.gl.m2} - 0.055\cdot(3+\psi)}{\lambda_{p.gl.m2}^2} & \text{if } \lambda_{p.gl.m2} > 0.5 + \sqrt{0.085 - 0.055\psi} \end{cases}$$

$\rho_{loc.gl.m2} = 0.74$

$$\sigma_{cr.c.gl.m.2} := \frac{\pi^2\cdot E_d\cdot I_{sl.1}}{A_{sl.1}\cdot a^2}$$

$$\sigma_{cr.c.gl.m2} := \frac{e_{2.bi.eff}}{e_{2.bi.eff} - h_1}\cdot\sigma_{cr.c.gl.m.2}$$

$\sigma_{cr.c.gl.m2} = 54.68\cdot\frac{N}{mm^2}$

$$\lambda_{c.gl.m2} := \sqrt{\frac{\beta_{A.c}\cdot f_{yd}}{\sigma_{cr.c.gl.m2}}}$$

$$i_{c.gl} := \sqrt{\frac{I_{sl.1}}{A_{sl.1}}}$$

$$e_{c.gl} := \begin{cases} e_{sti.1} & \text{if } e_{sti.1} \geq \dfrac{b_{v1}}{2} + \dfrac{t_w}{2} - e_{sti.1} \\ \dfrac{b_{v1}}{2} + \dfrac{t_w}{2} - e_{sti.1} & \text{if } \dfrac{b_{v1}}{2} + \dfrac{t_w}{2} - e_{sti.1} \geq e_{sti.1} \end{cases}$$

$e_{c.gl} = 41.52\cdot mm$

$$\alpha_e := 0.49 + 0.09\cdot\frac{i_{c.gl}}{e_{c.gl}}$$

$$\Phi_{c.gl.m2} := 0.5\cdot\left[1 + \alpha_e\cdot\left(\lambda_{c.gl.m2} - 0.2\right) + \lambda_{c.gl.m2}^2\right]$$

$$\chi_{c.gl.m2} := \frac{1}{\Phi_{c.gl.m2} + \sqrt{\Phi_{c.gl.m2}^2 - \lambda_{c.gl.m2}^2}}$$

$$\xi_{c.gl.m2} := \begin{cases} 1 & \text{if } \dfrac{\sigma_{cr.sl.lum}}{\sigma_{cr.c.gl.m2}} > 1 \\ 0 & \text{if } 0 > \dfrac{\sigma_{cr.sl.lum}}{\sigma_{cr.c.gl.m2}} \\ \dfrac{\sigma_{cr.sl.lum}}{\sigma_{cr.c.gl.m2}} & \text{if } 0 \leq \dfrac{\sigma_{cr.sl.lum}}{\sigma_{cr.c.gl.m2}} \leq 1 \end{cases}$$

$\xi_{c.gl.m2} = 0.00$

$$\rho_{c.gl.m2} := \left(\rho_{loc.gl.m2} - \chi_{c.gl.m2}\right)\cdot\xi_{c.gl.m2}\cdot\left(2 - \xi_{c.gl.m2}\right) + \chi_{c.gl.m2}$$

$t_{w.m2.eff} := \rho_{c.gl.m2}\cdot t_w$

$t_{v1.m2.eff} := \rho_{c.gl.m2}\cdot t_{v1}$

$t_{v2.m2.eff} := \rho_{c.gl.m2}\cdot t_{v2}$

$\rho_{c.gl.m2} = 0.21$

$t_{w.m2.eff} = 1.65\cdot mm$

$t_{v1.m2.eff} = 2.89\cdot mm$

$t_{v2.m2.eff} = 0.00\cdot mm$

Bruto oppervlakte:

$$A_{m2} := b_{fbL}\cdot t_{fbL} + b_{fbR}\cdot t_{fbR} + b_{foL}\cdot t_{foL} + b_{foR}\cdot t_{foR} + h_w\cdot t_w + t_{v1}\cdot b_{v1} + t_{v2}\cdot b_{v2} + A_{fic.1} + A_{fic.2} + A_{t1} + A_{t2} + A_{t3} + A_{t4}$$

$A_{m2} = 77400.00\cdot mm^2$

Bruto statisch moment:

$$S_{m2} := b_{fbL}\cdot t_{fbL}\cdot\frac{t_{fbL}}{2} + b_{fbR}\cdot t_{fbR}\cdot\frac{t_{fbR}}{2} + h_w\cdot t_w\cdot\left(t_{fbmax} + \frac{h_w}{2}\right) + b_{foL}\cdot t_{foL}\cdot\left(h_t - \frac{t_{foL}}{2}\right) + b_{foR}\cdot t_{foR}\cdot\left(h_t - \frac{t_{foR}}{2}\right) \ldots$$
$$+ t_{v1}\cdot b_{v1}\cdot h_{v1} + t_{v2}\cdot b_{v2}\cdot h_{v2} + A_{fic.1}\cdot z_{fic.1} + A_{fic.2}\cdot z_{fic.2} + A_{t1}\cdot\left(z_{t1.bk} + z_{t1}\right) + A_{t2}\cdot\left(z_{t2.bk} + z_{t2}\right) + A_{t3}\cdot\left(z_{t3.bk} + z_{t3}\right) + A_{t4}\cdot\left(z_{t4.bk} + z_{t4}\right)$$

$S_{m2} = 45861200.00\cdot mm^3$

Bruto zwaartepunt:

$$e_{b.m2} := \frac{S_{m2}}{A_{m2}}$$

$e_{b.m2} = 592.52\cdot mm$

Effectieve oppervlakte:

$$A_{m2.eff} := b_{fbL.a.eff}\cdot t_{fbL} + b_{fbR.a.eff}\cdot t_{fbR} + b_{foL}\cdot t_{foL} + b_{foR}\cdot t_{foR} \ldots$$
$$+ b_{v1.m2.eff}\cdot t_{v1.m2.eff} + b_{v2.m2.eff}\cdot t_{v2.m2.eff} \ldots$$
$$+ b_{1.m2.edge.eff}\cdot t_w + b_{1.m2.inf.eff}\cdot t_{w.m2.eff} + b_{2.m2.sup.eff}\cdot t_{w.m2.eff} \ldots$$
$$+ b_{2.m2.inf.eff}\cdot t_{w.m2.eff} + b_{3.m2.sup.eff}\cdot t_{w.m2.eff} + b_{3.m2.inf.eff}\cdot t_w + b_{t.m2}\cdot t_w \ldots$$
$$+ \rho_{fic.1.m}\cdot A_{fic.1} + A_{fic.2} + A_{t1.eff} + A_{t2.eff} + A_{t3} + A_{t4}$$

$A_{m2.eff} = 75405.89\cdot mm^2$

Effectief statisch moment:

$$S_{m2.eff} := b_{fbL.a.eff}\cdot t_{fbL}\cdot\frac{t_{fbL}}{2} + b_{fbR.a.eff}\cdot t_{fbR}\cdot\frac{t_{fbR}}{2} + b_{foL}\cdot t_{foL}\cdot\left(h_t - \frac{t_{foL}}{2}\right) + b_{foR}\cdot t_{foR}\cdot\left(h_t - \frac{t_{foR}}{2}\right) \ldots$$
$$+ b_{v1.m2.eff}\cdot t_{v1.m2.eff}\cdot h_{v1} + b_{v2.m2.eff}\cdot t_{v2.m2.eff}\cdot h_{v2} + b_{1.m2.edge.eff}\cdot t_w\cdot\left(t_{fbmax} + \frac{b_{1.m2.edge.eff}}{2}\right) \ldots$$
$$+ b_{1.m2.inf.eff}\cdot t_{w.m2.eff}\cdot\left(t_{fbmax} + b_1 - \frac{b_{1.m2.inf.eff}}{2}\right) + b_{2.m2.sup.eff}\cdot t_{w.m2.eff}\cdot\left(t_{fbmax} + b_1 + \frac{b_{2.m2.sup.eff}}{2}\right) \ldots$$
$$+ b_{2.m2.inf.eff}\cdot t_{w.m2.eff}\cdot\left(t_{fbmax} + b_1 + b_2 - \frac{b_{2.m2.inf.eff}}{2}\right) + b_{3.m2.sup.eff}\cdot t_{w.m2.eff}\cdot\left(t_{fbmax} + b_1 + b_2 + \frac{b_{3.m2.sup.eff}}{2}\right) \ldots$$
$$+ b_{3.m2.inf.eff}\cdot t_w\cdot\left(t_{fbmax} + b_1 + b_2 + b_{3.m2.sup} + b_{3.m2.inf} - \frac{b_{3.m2.inf.eff}}{2}\right) + b_{t.m2}\cdot t_w\cdot\left(h_t - t_{fomax} - \frac{b_{t.m2}}{2}\right) \ldots$$
$$+ \rho_{fic.1.m}\cdot A_{fic.1}\cdot z_{fic.1} + A_{fic.2}\cdot z_{fic.2} + A_{t1.eff}\cdot\left(z_{t1.bk} + z_{t1.eff}\right) + A_{t2.eff}\cdot\left(z_{t2.bk} + z_{t2.eff}\right) + A_{t3}\cdot\left(z_{t3.bk} + z_{t3}\right) + A_{t4}\cdot\left(z_{t4.bk} + z_{t4}\right)$$

$S_{m2.eff} = 46790354.79\cdot mm^3$

Effectief zwaartepunt:

$$e_{b.m2.eff} := \frac{S_{m2.eff}}{A_{m2.eff}}$$

$e_{b.m2.eff} = 620.51\cdot mm$

$$I_{y.0.m2.eff} := \frac{1}{12}\cdot\left(b_{fbL.a.eff}\cdot t_{fbL}^3 + b_{fbR.a.eff}\cdot t_{fbR}^3 + b_{foL}\cdot t_{foL}^3 + b_{foR}\cdot t_{foR}^3 + b_{v1.m2.eff}\cdot t_{v1.m2.eff}^3 \ldots + b_{v2.m2.eff}\cdot t_{v2.m2.eff}^3 + t_w\cdot b_{1.m2.edge.eff}^3 + t_{w.m2.eff} b_{1.m2.inf.eff}^3 \ldots + t_{w.m2.eff}\cdot b_{2.m2.sup.eff}^3 + t_{w.m2.eff}\cdot b_{2.m2.inf.eff}^3 \ldots + t_{w.m2.eff}\cdot b_{3.m2.sup.eff}^3 + t_w\cdot b_{3.m2.inf.eff}^3 + t_w\cdot b_{t.m2}^3\right) \ldots$$
$$+ \rho_{fic.1.m}\cdot I_{fic.1} + I_{fic.2} + I_{t1.eff} + I_{t2.eff} + I_{t3} + I_{t4}$$

$$
\begin{aligned}
I_{y.c.m2.eff} := \; & b_{fbL.a.eff} \cdot t_{fbL} \cdot \left(e_{b.m2.eff} - \frac{t_{fbL}}{2}\right)^2 + b_{fbR.a.eff} \cdot t_{fbR} \cdot \left(e_{b.m2.eff} - \frac{t_{fbR}}{2}\right)^2 \ldots \\
& + b_{1.m2.edge.eff} \cdot t_w \cdot \left(e_{b.m2.eff} - t_{fbmax} - \frac{b_{1.m2.edge.eff}}{2}\right)^2 \ldots \\
& + b_{1.m2.inf.eff} \cdot t_{w.m2.eff} \cdot \left(e_{b.m2.eff} - t_{fbmax} - b_1 + \frac{b_{1.m2.inf.eff}}{2}\right)^2 \ldots \\
& + b_{2.m2.sup.eff} \cdot t_{w.m2.eff} \cdot \left(e_{b.m2.eff} - t_{fbmax} - b_1 - \frac{b_{2.m2.sup.eff}}{2}\right)^2 \ldots \\
& + b_{2.m2.inf.eff} \cdot t_{w.m2.eff} \cdot \left(e_{b.m2.eff} - t_{fbmax} - b_1 - b_2 + \frac{b_{2.m2.inf.eff}}{2}\right)^2 \ldots \\
& + b_{3.m2.sup.eff} \cdot t_{w.m2.eff} \cdot \left(e_{b.m2.eff} - t_{fbmax} - b_1 - b_2 - \frac{b_{3.m2.sup.eff}}{2}\right)^2 \ldots \\
& + b_{3.m2.inf.eff} \cdot t_w \cdot \left(e_{b.m2.eff} - t_{fbmax} - b_1 - b_2 - b_{3.m2.sup} - b_{3.m2.inf} + \frac{b_{3.m2.inf.eff}}{2}\right)^2 \ldots \\
& + b_{t.m2} \cdot t_w \cdot \left(e_{b.m2.eff} - t_{fbmax} - h_w + \frac{b_{t.m2}}{2}\right)^2 + b_{foL} \cdot t_{foL} \cdot \left(h_t - \frac{t_{foL}}{2} - e_{b.m2.eff}\right)^2 \ldots \\
& + b_{foR} \cdot t_{foR} \left(h_t - \frac{t_{foR}}{2} - e_{b.m2.eff}\right)^2 + b_{v1.m2.eff} \cdot t_{v1.m2.eff} \cdot \left(h_{v1} - e_{b.m2.eff}\right)^2 \ldots \\
& + b_{v2.m2.eff} \cdot t_{v2.m2.eff} \cdot \left(h_{v2} - e_{b.m2.eff}\right)^2 + \rho_{fic.1.m} \cdot A_{fic.1} \cdot \left(z_{fic.1} - e_{b.m2.eff}\right)^2 + A_{fic.2} \cdot \left(z_{fic.2} - e_{b.m2.eff}\right)^2 \ldots \\
& + A_{t1.eff} \left(z_{t1.bk} + z_{t1.eff} - e_{b.m2.eff}\right)^2 + A_{t2.eff} \left(z_{t2.bk} + z_{t2.eff} - e_{b.m2.eff}\right)^2 + A_{t3} \cdot \left(z_{t3.bk} + z_{t3} - e_{b.m2.eff}\right)^2 \ldots \\
& + A_{t4} \cdot \left(z_{t4.bk} + z_{t4} - e_{b.m2.eff}\right)^2
\end{aligned}
$$

Effectief traagheidsmoment:

$I_{y.m2.eff} := I_{y.0.m2.eff} + I_{y.c.m2.eff}$

$I_{y.m2.eff} = 3.5289 \times 10^{10} \cdot mm^4$

Lijf met 2 verstijvers (MOMENT)

rigidend := ☐ Check Box

rigidend = 0.00

Schuifkracht zonder verstijver

Restance to shear (5)

$$
Toetsnodig := \begin{cases} 1 & \text{if } \frac{h_w}{t_w} > 72 \cdot \frac{\varepsilon}{\eta} \\ 0 & \text{if } \frac{h_w}{t_w} \le 72 \cdot \frac{\varepsilon}{\eta} \end{cases}
$$

Toetsnodig = 1.00

$$
k_{\tau.s0} := \begin{cases} 4 + 5.34 \cdot \left(\frac{h_w}{a}\right)^2 & \text{if } \frac{a}{h_w} < 1.0 \\ 5.34 + 4 \cdot \left(\frac{h_w}{a}\right)^2 & \text{if } \frac{a}{h_w} \ge 1.0 \end{cases}
$$

$k_{\tau.s0} = 5.38$

$$
\sigma_E := \frac{\pi^2 \cdot E_d}{12 \cdot \left(1 - \nu^2\right)} \cdot \left(\frac{t_w}{h_w}\right)^2
$$

$$
\lambda_{\tau.s0} := \sqrt{\frac{\frac{f_{yd}}{\sqrt{3}}}{\sigma_E \cdot k_{\tau.s0}}}
$$

$\lambda_{\tau.s0} = 2.06$

$$
\chi_{w.s0.rigid} := \begin{cases} \eta & \text{if } \lambda_{\tau.s0} < \frac{0.83}{\eta} \\ \frac{0.83}{\lambda_{\tau.s0}} & \text{if } \frac{0.83}{\eta} \le \lambda_{\tau.s0} < 1.08 \\ \frac{1.37}{0.7 + \lambda_{\tau.s0}} & \text{if } \lambda_{\tau.s0} \ge 1.08 \end{cases}
$$

$\chi_{w.s0.rigid} = 0.50$

$$
\chi_{w.s0.nonrigid} := \begin{cases} \eta & \text{if } \lambda_{\tau.s0} < \frac{0.83}{\eta} \\ \frac{0.83}{\lambda_{\tau.s0}} & \text{if } \lambda_{\tau.s0} \ge \frac{0.83}{\eta} \end{cases}
$$

$\chi_{w.s0.nonrigid} = 0.40$

$$
\chi_{w.s0} := \begin{cases} \chi_{w.s0.rigid} & \text{if } rigidend = 1 \\ \chi_{w.s0.nonrigid} & \text{if } rigidend = 0 \end{cases}
$$

$\chi_{w.s0} = 0.40$

$$
V_{b.rd} := h_w \cdot t_w \cdot \frac{\eta \cdot f_{yd}}{\sqrt{3}}
$$

Maximale toelaatbare dwarskracht:

$$
V_{bw.rd.s0} := \begin{cases} V_{b.rd} & \text{if } V_{b.rd} < \chi_{w.s0} \cdot h_w \cdot t_w \cdot \frac{f_{yd}}{\sqrt{3}} \\ \chi_{w.s0} \cdot h_w \cdot t_w \cdot \frac{f_{yd}}{\sqrt{3}} & \text{if } \chi_{w.s0} \cdot h_w \cdot t_w \cdot \frac{f_{yd}}{\sqrt{3}} \le V_{b.rd} \end{cases}
$$

$V_{bw.rd.s0} = 625.03 \cdot kN$

Bijdrage van flenzen verwaarlozen vanwege momenten

Schuifkracht zonder verstijver

Schuifkracht met 1 verstijver

Restance to shear (5)

$b_1 := h_1$ $\qquad b_1 = 360.00 \cdot mm$

$b_2 := h_2 + h_3$ $\qquad b_2 = 1070.00 \cdot mm$

$$b_{eff.1} := \begin{cases} 15 \cdot \varepsilon \cdot t_w & \text{if } 15 \cdot \varepsilon \cdot t_w \le \frac{1}{2} \cdot b_1 \\ \frac{1}{2} \cdot b_1 & \text{if } 15 \cdot \varepsilon \cdot t_w > \frac{1}{2} \cdot b_1 \end{cases} \qquad b_{eff.1} = 120.00 \cdot mm$$

$$b_{eff.2} := \begin{cases} 15 \cdot \varepsilon \cdot t_w & \text{if } 15 \cdot \varepsilon \cdot t_w \le \frac{1}{2} \cdot b_2 \\ \frac{1}{2} \cdot b_2 & \text{if } 15 \cdot \varepsilon \cdot t_w > \frac{1}{2} \cdot b_2 \end{cases} \qquad b_{eff.2} = 120.00 \cdot mm$$

$$A_{sl.s1} := (b_{eff.1} + b_{eff.2}) \cdot t_w + b_{v1} \cdot t_{v1} \qquad A_{sl.s1} = 3880.00 \cdot mm^2$$

$$e_{sti.s1} := \frac{b_{v1} \cdot t_{v1} \cdot \left(\frac{b_{v1}}{2} + \frac{t_w}{2}\right)}{A_{sl.s1}} \qquad e_{sti.s1} = 37.38 \cdot mm$$

$$I_{sl.s1} := \frac{1}{12} \cdot \left[(b_{eff.1} + b_{eff.2}) \cdot t_w^3 + b_{v1}^3 \cdot t_{v1}\right] + (b_{eff.1} + b_{eff.2}) \cdot t_w \cdot e_{sti.s1}^2 + b_{v1} \cdot t_{v1} \cdot \left(e_{sti.s1} - \frac{b_{v1}}{2} - \frac{t_w}{2}\right)^2$$

$I_{sl.s1} = 8522728.80 \cdot mm^4$

$$k_{\tau sl.s1} := \begin{cases} \frac{2.1}{t_w} \cdot \sqrt[3]{\frac{I_{sl.s1}}{h_w}} & \text{if } \frac{2.1}{t_w} \cdot \sqrt[3]{\frac{I_{sl.s1}}{h_w}} > 9 \cdot \left(\frac{h_w}{a}\right)^2 \cdot \sqrt[4]{\left(\frac{I_{sl.s1}}{t_w^3 \cdot h_w}\right)^3} \\ 9 \cdot \left(\frac{h_w}{a}\right)^2 \cdot \sqrt[4]{\left(\frac{I_{sl.s1}}{t_w^3 \cdot h_w}\right)^3} & \text{if } 9 \cdot \left(\frac{h_w}{a}\right)^2 \cdot \sqrt[4]{\left(\frac{I_{sl.s1}}{t_w^3 \cdot h_w}\right)^3} \ge \frac{2.1}{t_w} \cdot \sqrt[3]{\frac{I_{sl.s1}}{h_w}} \end{cases} \qquad k_{\tau sl.s1} = 4.76$$

$$k_{\tau 1.1.s1} := \begin{cases} 4 + 5.34 \cdot \left(\frac{h_w}{a}\right)^2 + k_{\tau sl.s1} & \text{if } \frac{a}{h_w} < 1 \\ 5.34 + 4 \cdot \left(\frac{h_w}{a}\right)^2 + k_{\tau sl.s1} & \text{if } \frac{a}{h_w} \ge 1 \end{cases} \qquad k_{\tau 1.1.s1} = 10.14$$

$$k_{\tau 1.2.s1} := 4.1 + \frac{6.3 + 0.18 \cdot \frac{I_{sl.s1}}{t_w^3 \cdot h_w}}{\left(\frac{a}{h_w}\right)^2} + 2.2 \cdot \sqrt[3]{\frac{I_{sl.s1}}{t_w^3 \cdot h_w}} \qquad k_{\tau 1.2.s1} = 9.16$$

$$k_{\tau 1.s1} := \begin{cases} k_{\tau 1.1.s1} & \text{if } \frac{a}{h_w} \ge 3 \\ k_{\tau 1.2.s1} & \text{if } \frac{a}{h_w} < 3 \end{cases} \qquad k_{\tau 1.s1} = 10.14$$

$$\sigma_E := \frac{\pi^2 \cdot E_d}{12 \cdot (1 - \nu^2)} \cdot \left(\frac{t_w}{h_w}\right)^2$$

$$\lambda_{\tau 1.s1} := \sqrt{\frac{\frac{f_{yd}}{\sqrt{3}}}{\sigma_E \cdot k_{\tau 1.s1}}} \qquad \lambda_{\tau 1.s1} = 1.50$$

Suppanelen berekenen:

Lijf h.1 zonder verstijvingen:

$$k_{\tau 2.s1} := \begin{cases} 4 + 5.34 \cdot \left(\frac{b_1}{a}\right)^2 & \text{if } \frac{a}{b_1} < 1.0 \\ 5.34 + 4 \cdot \left(\frac{b_1}{a}\right)^2 & \text{if } \frac{a}{b_1} \ge 1.0 \end{cases} \qquad k_{\tau 2.s1} = 5.34$$

$$\sigma_E := \frac{\pi^2 \cdot E_d}{12 \cdot (1 - \nu^2)} \cdot \left(\frac{t_w}{b_1}\right)^2$$

$$\lambda_{\tau 2.s1} := \sqrt{\frac{\frac{f_{yd}}{\sqrt{3}}}{\sigma_E \cdot k_{\tau 2.s1}}} \qquad \lambda_{\tau 2.s1} = 0.52$$

Lijf h.2 zonder verstijvingen:

$$k_{\tau 3.s1} := \begin{cases} 4 + 5.34 \cdot \left(\frac{b_2}{a}\right)^2 & \text{if } \frac{a}{b_2} < 1.0 \\ 5.34 + 4 \cdot \left(\frac{b_2}{a}\right)^2 & \text{if } \frac{a}{b_2} \ge 1.0 \end{cases} \qquad k_{\tau 3.s1} = 5.36$$

$$\sigma_E := \frac{\pi^2 \cdot E_d}{12 \cdot (1 - \nu^2)} \cdot \left(\frac{t_w}{b_2}\right)^2$$

$$\lambda_{\tau 3.s1} := \sqrt{\frac{\frac{f_{yd}}{\sqrt{3}}}{\sigma_E \cdot k_{\tau 3.s1}}} \qquad \lambda_{\tau 3.s1} = 1.54$$

$$\lambda_{\tau .s1} := \begin{cases} \lambda_{\tau 1.s1} & \text{if } \lambda_{\tau 2.s1} \le \lambda_{\tau 1.s1} \text{ if } \lambda_{\tau 3.s1} \le \lambda_{\tau 1.s1} \\ \lambda_{\tau 2.s1} & \text{if } \lambda_{\tau 1.s1} \le \lambda_{\tau 2.s1} \text{ if } \lambda_{\tau 3.s1} \le \lambda_{\tau 2.s1} \\ \lambda_{\tau 3.s1} & \text{if } \lambda_{\tau 1.s1} \le \lambda_{\tau 3.s1} \text{ if } \lambda_{\tau 2.s1} \le \lambda_{\tau 3.s1} \end{cases} \qquad \lambda_{\tau .s1} = 1.54$$

$$\chi_{w.s1.rigid} := \begin{cases} \eta & \text{if } \lambda_{\tau .s1} < \frac{0.83}{\eta} \\ \frac{0.83}{\lambda_{\tau .s1}} & \text{if } \frac{0.83}{\eta} \le \lambda_{\tau .s1} < 1.08 \\ \frac{1.37}{0.7 + \lambda_{\tau .s1}} & \text{if } \lambda_{\tau .s1} \ge 1.08 \end{cases} \qquad \chi_{w.s1.rigid} = 0.61$$

$$\chi_{w.s1.nonrigid} := \begin{cases} \eta & \text{if } \lambda_{\tau .s1} < \frac{0.83}{\eta} \\ \frac{0.83}{\lambda_{\tau .s1}} & \text{if } \lambda_{\tau .s1} \ge \frac{0.83}{\eta} \end{cases} \qquad \chi_{w.s1.nonrigid} = 0.54$$

$$\chi_{w.s1} := \begin{cases} \chi_{w.s1.rigid} & \text{if } rigidend = 1 \\ \chi_{w.s1.nonrigid} & \text{if } rigidend = 0 \end{cases} \qquad \chi_{w.s1} = 0.54$$

$V_{b.rd} := h_w \cdot t_w \cdot \frac{\eta \cdot f_{yd}}{\sqrt{3}}$

Maximale toelaatbare dwarskracht:

$V_{bw.rd.s1} := \begin{cases} V_{b.rd} & \text{if } V_{b.rd} < \chi_{w.s1} \cdot h_w \cdot t_w \cdot \frac{f_{yd}}{\sqrt{3}} \\ \chi_{w.s1} \cdot h_w \cdot t_w \cdot \frac{f_{yd}}{\sqrt{3}} & \text{if } \chi_{w.s1} \cdot h_w \cdot t_w \cdot \frac{f_{yd}}{\sqrt{3}} \le V_{b.rd} \end{cases}$ $\qquad V_{bw.rd.s1} = 834.08 \cdot kN$

Bijdrage van flenzen verwaarlozen vanwege momenten

Schuifkracht met 1 verstijver

Schuifkracht met 2 verstijvers

Restance to shear (5)

$b_1 := h_1$ $\qquad b_1 = 360.00 \cdot mm$

$b_2 := h_2$ $\qquad b_2 = 300.00 \cdot mm$

$b_3 := h_3$ $\qquad b_3 = 770.00 \cdot mm$

$b_{eff.1} := \begin{cases} 15 \cdot \varepsilon \cdot t_w & \text{if } 15 \cdot \varepsilon \cdot t_w \le \frac{1}{2} \cdot b_1 \\ \frac{1}{2} \cdot b_1 & \text{if } 15 \cdot \varepsilon \cdot t_w > \frac{1}{2} \cdot b_1 \end{cases}$ $\qquad b_{eff.1} = 120.00 \cdot mm$

$b_{eff.2} := \begin{cases} 15 \cdot \varepsilon \cdot t_w & \text{if } 15 \cdot \varepsilon \cdot t_w \le \frac{1}{2} \cdot b_2 \\ \frac{1}{2} \cdot b_2 & \text{if } 15 \cdot \varepsilon \cdot t_w > \frac{1}{2} \cdot b_2 \end{cases}$ $\qquad b_{eff.2} = 120.00 \cdot mm$

$b_{eff.3} := \begin{cases} 15 \cdot \varepsilon \cdot t_w & \text{if } 15 \cdot \varepsilon \cdot t_w \le \frac{1}{2} \cdot b_3 \\ \frac{1}{2} \cdot b_3 & \text{if } 15 \cdot \varepsilon \cdot t_w > \frac{1}{2} \cdot b_3 \end{cases}$ $\qquad b_{eff.3} = 120.00 \cdot mm$

$A_{sl.s2} := (b_{eff.1} + b_{eff.2}) \cdot t_w + (b_{eff.2} + b_{eff.3}) \cdot t_w + b_{v1} \cdot t_{v1} + b_{v2} \cdot t_{v2}$ $\qquad A_{sl.s2} = 5800.00 \cdot mm^2$

$e_{sti.s2} := \frac{b_{v1} \cdot t_{v1} \cdot \left(\frac{b_{v1}}{2} + \frac{t_w}{2}\right) + b_{v2} \cdot t_{v2} \cdot \left(\frac{b_{v2}}{2} + \frac{t_w}{2}\right)}{A_{sl.s2}}$ $\qquad e_{sti.s2} = 25.01 \cdot mm$

$I_{sl.s2} := \frac{1}{12} \cdot \left[(b_{eff.1} + b_{eff.2}) \cdot t_w^3 + (b_{eff.2} + b_{eff.3}) \cdot t_w^3 + b_{v1}^3 \cdot t_{v1} + b_{v2}^3 \cdot t_{v2}\right] + (b_{eff.1} + b_{eff.2}) \cdot t_w \cdot e_{sti.s2}^2 \ldots$
$\quad + (b_{eff.2} + b_{eff.3}) \cdot t_w \cdot e_{sti.s2}^2 + b_{v1} \cdot t_{v1} \cdot \left(e_{sti.s2} - \frac{b_{v1}}{2} - \frac{t_w}{2}\right)^2 + b_{v2} \cdot t_{v2} \cdot \left(e_{sti.s2} - \frac{b_{v2}}{2} - \frac{t_w}{2}\right)^2$ $\qquad I_{sl.s2} = 10327773.06 \cdot mm^4$

$k_{\tau sl.s2} := \begin{cases} \frac{2.1}{t_w} \cdot \sqrt[3]{\frac{I_{sl.s2}}{h_w}} & \text{if } \frac{2.1}{t_w} \cdot \sqrt[3]{\frac{I_{sl.s2}}{h_w}} > 9 \cdot \left(\frac{h_w}{a}\right)^2 \cdot \sqrt[4]{\left(\frac{I_{sl.s2}}{t_w^3 \cdot h_w}\right)^3} \\ 9 \cdot \left(\frac{h_w}{a}\right)^2 \cdot \sqrt[4]{\left(\frac{I_{sl.s2}}{t_w^3 \cdot h_w}\right)^3} & \text{if } 9 \cdot \left(\frac{h_w}{a}\right)^2 \cdot \sqrt[4]{\left(\frac{I_{sl.s2}}{t_w^3 \cdot h_w}\right)^3} \ge \frac{2.1}{t_w} \cdot \sqrt[3]{\frac{I_{sl.s2}}{h_w}} \end{cases}$ $\qquad k_{\tau sl.s2} = 5.07$

$k_{\tau 1.1.s2} := \begin{cases} 4 + 5.34 \cdot \left(\frac{h_w}{a}\right)^2 + k_{\tau sl.s2} & \text{if } \frac{a}{h_w} < 1 \\ 5.34 + 4 \cdot \left(\frac{h_w}{a}\right)^2 + k_{\tau sl.s2} & \text{if } \frac{a}{h_w} \ge 1 \end{cases}$ $\qquad k_{\tau 1.1.s2} = 10.45$

$k_{\tau 1.2.s2} := 4.1 + \frac{6.3 + 0.18 \cdot \frac{I_{sl.s2}}{t_w^3 \cdot h_w}}{\left(\frac{a}{h_w}\right)^2} + 2.2 \cdot \sqrt[3]{\frac{I_{sl.s2}}{t_w^3 \cdot h_w}}$ $\qquad k_{\tau 1.2.s2} = 9.50$

$k_{\tau 1.s2} := \begin{cases} k_{\tau 1.1.s2} & \text{if } \frac{a}{h_w} \ge 3 \\ k_{\tau 1.2.s2} & \text{if } \frac{a}{h_w} < 3 \end{cases}$ $\qquad k_{\tau 1.s2} = 10.45$

$\sigma_E := \frac{\pi^2 \cdot E_d}{12 \cdot (1 - \nu^2)} \cdot \left(\frac{t_w}{h_w}\right)^2$

$\lambda_{\tau 1.s2} := \sqrt{\frac{\frac{f_{yd}}{\sqrt{3}}}{\sigma_E \cdot k_{\tau 1.s2}}}$ $\qquad \lambda_{\tau 1.s2} = 1.48$

Subpanelen berekenen:

Lijf h.1 zonder verstijvingen:

$k_{\tau 2.s2} := \begin{cases} 4 + 5.34 \cdot \left(\frac{b_1}{a}\right)^2 & \text{if } \frac{a}{b_1} < 1.0 \\ 5.34 + 4 \cdot \left(\frac{b_1}{a}\right)^2 & \text{if } \frac{a}{b_1} \ge 1.0 \end{cases}$ $\qquad k_{\tau 2.s2} = 5.34$

$\sigma_E := \frac{\pi^2 \cdot E_d}{12 \cdot (1 - \nu^2)} \cdot \left(\frac{t_w}{b_1}\right)^2$

$\lambda_{\tau 2.s2} := \sqrt{\frac{\frac{f_{yd}}{\sqrt{3}}}{\sigma_E \cdot k_{\tau 2.s2}}}$ $\qquad \lambda_{\tau 2.s2} = 0.52$

Lijf h.2 zonder verstijvingen:

$k_{\tau 3.s2} := \begin{cases} 4 + 5.34 \cdot \left(\frac{b_2}{a}\right)^2 & \text{if } \frac{a}{b_2} < 1.0 \\ 5.34 + 4 \cdot \left(\frac{b_2}{a}\right)^2 & \text{if } \frac{a}{b_2} \ge 1.0 \end{cases}$ $\qquad k_{\tau 3.s2} = 5.34$

$\sigma_E := \frac{\pi^2 \cdot E_d}{12 \cdot (1 - \nu^2)} \cdot \left(\frac{t_w}{b_2}\right)^2$

$\lambda_{\tau 3.s2} := \sqrt{\frac{\frac{f_{yd}}{\sqrt{3}}}{\sigma_E \cdot k_{\tau 3.s2}}}$ $\qquad \lambda_{\tau 3.s2} = 0.43$

Lijf h.3 zonder verstijvingen:

$$k_{\tau4.s2} := \begin{cases} 4 + 5.34\cdot\left(\frac{b_3}{a}\right)^2 & \text{if } \frac{a}{b_3} < 1.0 \\ 5.34 + 4\cdot\left(\frac{b_3}{a}\right)^2 & \text{if } \frac{a}{b_3} \geq 1.0 \end{cases}$$

$k_{\tau4.s2} = 5.35$

$$\sigma_E := \frac{\pi^2\cdot E_d}{12\cdot\left(1-\nu^2\right)}\cdot\left(\frac{t_w}{b_3}\right)^2$$

$$\lambda_{\tau4.s2} := \sqrt{\frac{\frac{f_{yd}}{\sqrt{3}}}{\sigma_E\cdot k_{\tau4.s2}}}$$

$\lambda_{\tau4.s2} = 1.11$

$$\lambda_{\tau.s2} := \begin{cases} \lambda_{\tau1.s2} & \text{if } \lambda_{\tau2.s2} \leq \lambda_{\tau1.s2} \text{ if } \lambda_{\tau3.s2} \leq \lambda_{\tau1.s2} \text{ if } \lambda_{\tau4.s2} \leq \lambda_{\tau1.s2} \\ \lambda_{\tau2.s2} & \text{if } \lambda_{\tau1.s2} \leq \lambda_{\tau2.s2} \text{ if } \lambda_{\tau3.s2} \leq \lambda_{\tau2.s2} \text{ if } \lambda_{\tau4.s2} \leq \lambda_{\tau2.s2} \\ \lambda_{\tau3.s2} & \text{if } \lambda_{\tau1.s2} \leq \lambda_{\tau3.s2} \text{ if } \lambda_{\tau2.s2} \leq \lambda_{\tau3.s2} \text{ if } \lambda_{\tau4.s2} \leq \lambda_{\tau3.s2} \\ \lambda_{\tau4.s2} & \text{if } \lambda_{\tau1.s2} \leq \lambda_{\tau4.s2} \text{ if } \lambda_{\tau2.s2} \leq \lambda_{\tau4.s2} \text{ if } \lambda_{\tau3.s2} \leq \lambda_{\tau4.s2} \end{cases}$$

$\lambda_{\tau.s2} = 1.48$

$$\chi_{w.s2.rigid} := \begin{cases} \eta & \text{if } \lambda_{\tau.s2} < \frac{0.83}{\eta} \\ \frac{0.83}{\lambda_{\tau.s2}} & \text{if } \frac{0.83}{\eta} \leq \lambda_{\tau.s2} < 1.08 \\ \frac{1.37}{0.7+\lambda_{\tau.s2}} & \text{if } \lambda_{\tau.s2} \geq 1.08 \end{cases}$$

$\chi_{w.s2.rigid} = 0.63$

$$\chi_{w.s2.nonrigid} := \begin{cases} \eta & \text{if } \lambda_{\tau.s2} < \frac{0.83}{\eta} \\ \frac{0.83}{\lambda_{\tau.s2}} & \text{if } \lambda_{\tau.s2} \geq \frac{0.83}{\eta} \end{cases}$$

$\chi_{w.s2.nonrigid} = 0.56$

$$\chi_{w.s2} := \begin{cases} \chi_{w.s2.rigid} & \text{if rigidend} = 1 \\ \chi_{w.s2.nonrigid} & \text{if rigidend} = 0 \end{cases}$$

$\chi_{w.s2} = 0.56$

$$V_{b.rd} := h_w\cdot t_w\cdot\frac{\eta\cdot f_{yd}}{\sqrt{3}}$$

Maximale toelaatbare dwarskracht:

$$V_{bw.rd.s2} := \begin{cases} V_{b.rd} & \text{if } V_{b.rd} < \chi_{w.s2}\cdot h_w\cdot t_w\cdot\frac{f_{yd}}{\sqrt{3}} \\ \chi_{w.s2}\cdot h_w\cdot t_w\cdot\frac{f_{yd}}{\sqrt{3}} & \text{if } \chi_{w.s2}\cdot h_w\cdot t_w\cdot\frac{f_{yd}}{\sqrt{3}} \leq V_{b.rd} \end{cases}$$

$V_{bw.rd.s2} = 871.42\cdot kN$

Bijdrage van flenzen verwaarlozen vanwege momenten

Schuifkracht met 2 verstijvers

Patch loading

Resistance to transverse forces (6)

Langsverstijvers zijn verwaarloosd

Type krachtsinleiding — Kiest Type "a","b" of "c" uit figuur 6.1

type := "a"

Puntlast	$F_{Ed1} := 100\cdot kN$
Breedte flens	$b_f := 500\cdot mm$
Dikte flens	$t_f := 20\cdot mm$
Lastbreedte	$s_s := 400\cdot mm$
Zijn er dwarsverstijvers aanwezig (conform afbeelding 6.1)? kies "ja" of "nee"	verstijvers := "ja"
Afstand tussen dwarsverstijvers (kies 0 indien afwezig)	$a_s := 15000\cdot mm$
Afstand c (kies 0 indien n.v.t.)	$c := 0\cdot mm$
Vloeisterkte lijf	$f_{yw} := f_{yd} = 235.00\cdot\frac{N}{mm^2}$
Vloeisterkte flens	$f_{yf} := f_{yd} = 235.00\cdot\frac{N}{mm^2}$

$$a_s := \begin{cases} 9999999999999999999999999\cdot mm & \text{if verstijvers} = \text{"nee"} \\ a_s & \text{if verstijvers} = \text{"ja"} \end{cases}$$

$a_s = 15000.00\cdot mm$

Bepalen factor k.F

$$k_F := \begin{cases} 6 + 2\cdot\left(\frac{h_w}{a_s}\right)^2 & \text{if type} = \text{"a"} \\ 3.5 + 2\cdot\left(\frac{h_w}{a_s}\right)^2 & \text{if type} = \text{"b"} \\ 2 + 6\cdot\left(\frac{s_s+c}{h_w}\right)^2 & \text{if (type} = \text{"c") if } 2 + 6\cdot\left(\frac{s_s+c}{h_w}\right)^2 \leq 6 \\ 6 & \text{if (type} = \text{"c") if } 6 \leq 2 + 6\cdot\left(\frac{s_s+c}{h_w}\right)^2 \end{cases}$$

$k_F = 6.02$

Bepalen F.cr: $F_{cr} := 0.9\cdot k_F\cdot E_d\cdot\frac{t_w^3}{h_w}$ — $F_{cr} = 407.25\cdot kN$

Bepaling m1: $m_1 := \frac{f_{yf}\cdot b_f}{f_{yw}\cdot t_w}$ — $m_1 = 62.50$

Bepaling m2: $m_2 := 0$ — $m_2 = 0.00$

Effectieve belaste lengte: $l_{y.ab} := s_s + 2\cdot t_f\cdot\left(1+\sqrt{m_1+m_2}\right)$ — $l_{y.ab} = 756.23\cdot mm$

$$\lambda_f := \sqrt{\frac{l_{y.ab}\cdot t_w\cdot f_{yw}}{F_{cr}}}$$

$\lambda_f = 1.87$

Verificatie m2 $m_2 := \begin{cases} 0 & \text{if } \lambda_f \le 0.5 \\ 0.02 \cdot \left(\frac{h_w}{t_f}\right)^2 & \text{if } \lambda_f > 0.5 \end{cases}$ $m_2 = 102.25$

Effectieve belaste lengte type a of b $l_{y.ab} := s_s + 2 \cdot t_f \cdot \left(1 + \sqrt{m_1 + m_2}\right)$ $l_{y.ab} = 953.41 \cdot mm$

$$l_e := \begin{cases} \frac{k_F \cdot E_d \cdot t_w^2}{2 \cdot f_{yw} \cdot h_w} & \text{if } \frac{k_F \cdot E_d \cdot t_w^2}{2 \cdot f_{yw} \cdot h_w} \le (s_s + c) \\ (s_s + c) & \text{if } (s_s + c) \le \frac{k_F \cdot E_d \cdot t_w^2}{2 \cdot f_{yw} \cdot h_w} \end{cases}$$ $l_e = 120.35 \cdot mm$

Bepaling m1 $m_1 := \frac{f_{yf} \cdot b_f}{f_{yw} \cdot t_w}$ $m_1 = 62.50$

Bepaling m2 $m_2 := 0$ $m_2 = 0.00$

Effectieve belaste lengte $l_{y.c1} := l_e + t_f \cdot \sqrt{\frac{m_1}{2} + \left(\frac{l_e}{t_f}\right)^2 + m_2}$ $l_{y.c1} = 284.61 \cdot mm$

$\lambda_f := \sqrt{\frac{l_{y.c1} \cdot t_w \cdot f_{yw}}{F_{cr}}}$ $\lambda_f = 1.15$

Verificatie m2 $m_2 := \begin{cases} 0 & \text{if } \lambda_f \le 0.5 \\ 0.02 \cdot \left(\frac{h_w}{t_f}\right)^2 & \text{if } \lambda_f > 0.5 \end{cases}$ $m_2 = 102.25$

Effectieve belaste lengte type c1 $l_{y.c1} := l_e + t_f \cdot \sqrt{\frac{m_1}{2} + \left(\frac{l_e}{t_f}\right)^2 + m_2}$ $l_{y.c1} = 380.89 \cdot mm$

Bepaling m1 $m_1 := \frac{f_{yf} \cdot b_f}{f_{yw} \cdot t_w}$ $m_1 = 62.50$

Bepaling m2 $m_2 := 0$ $m_2 = 0.00$

Effectieve belaste lengte $l_{y.c2} := l_e + t_f \cdot \sqrt{m_1 + m_2}$ $l_{y.c2} = 278.46 \cdot mm$

$\lambda_f := \sqrt{\frac{l_{y.c2} \cdot t_w \cdot f_{yw}}{F_{cr}}}$ $\lambda_f = 1.13$

Verificatie m2 $m_2 := \begin{cases} 0 & \text{if } \lambda_f \le 0.5 \\ 0.02 \cdot \left(\frac{h_w}{t_f}\right)^2 & \text{if } \lambda_f > 0.5 \end{cases}$ $m_2 = 102.25$

Effectieve belaste lengte type c2 $l_{y.c2} := l_e + t_f \cdot \sqrt{m_1 + m_2}$ $l_{y.c2} = 377.05 \cdot mm$

$$l_y := \begin{cases} l_{y.ab} & \text{if type = "a"} \\ l_{y.ab} & \text{if type = "b"} \\ l_{y.ab} & \text{if type = "c" if } l_{y.ab} \le l_{y.c1} \text{ if } l_{y.ab} \le l_{y.c2} \\ l_{y.c1} & \text{if type = "c" if } l_{y.c1} \le l_{y.ab} \text{ if } l_{y.c1} \le l_{y.c2} \\ l_{y.c2} & \text{if type = "c" if } l_{y.c2} \le l_{y.c1} \text{ if } l_{y.c2} \le l_{y.ab} \end{cases}$$

$l_y = 953.41 \cdot mm$

$\lambda_f := \sqrt{\frac{l_y \cdot t_w \cdot f_{yw}}{F_{cr}}}$ $\lambda_f = 2.10$

Reductiefactor voor lokaal plooien $\chi_F := \begin{cases} \frac{0.5}{\lambda_f} & \text{if } \frac{0.5}{\lambda_f} \le 1 \\ 1 & \text{otherwise} \end{cases}$ $\chi_F = 0.24$

Effectieve lengte $L_{eff} := \chi_F \cdot l_y$ $L_{eff} = 227.23 \cdot mm$

Ontwerpsterkte voor lokaal plooien $F_{Rd} := \frac{(f_{yw} \cdot L_{eff} \cdot t_w)}{\gamma_{M1}}$ $F_{Rd} = 427.19 \cdot kN$

Verficatie η_2 $uc := \frac{F_{Ed1}}{\left(\frac{f_{yw} \cdot L_{eff} \cdot t_w}{\gamma_{M1}}\right)}$ $uc = 0.23$

$Controle := \begin{cases} \text{"ok"} & \text{if } uc \le 1 \\ \text{"niet ok"} & \text{otherwise} \end{cases}$ $Controle = \text{"ok"}$

Patch loading

ULS Toets:

Rekenwaarde dwarsbelasting:	$F_{ed} := 100 \cdot kN$	Dwarsbelasting positief!
Rekenwaarde dwarskracht:	$V_{ed} := 200 \cdot kN$	Dwarskracht positief!
Rekenwaarde axiale druk:	$N_{ed} := 9000 \cdot kN$	Druk positief !
Rekenwaarde buigend moment:	$M_{ed} := 3000 \cdot kN \cdot m$	Altijd positief invullen! Druk boven! (zo niet, profiel spiegelen)

NEN6771 toetsing lijf (0 verstijvingen)

NEN6771 toetsing lijf (1 verstijvingen)

NEN6771 toetsing lijf (2 verstijvingen)

ULS Toets

Kiezen correcte oppervlakte, zwaartepunt, traagheidsmoment

Effectieve oppervlakte voor axiale druk:

$$A_{a.eff} := \begin{cases} A_{a0.eff} & \text{if } verstijving1 = 0 \\ A_{a1.eff} & \text{if } verstijving1 = 1 \\ A_{a2.eff} & \text{if } verstijving2 = 1 \end{cases}$$ $A_{a.eff} = 71288.77 \cdot mm^2$

$$A_{a.eff} := \begin{cases} A_{tot} & \text{if } N_{ed} < 0 \\ A_{a.eff} & \text{if } N_{ed} \geq 0 \end{cases}$$ $A_{a.eff} = 71288.77 \cdot mm^2$

Effectief zwaartepunt voor axiale druk:

$$e_{b.a.eff} := \begin{cases} e_{b.a0.eff} & \text{if } verstijving1 = 0 \\ e_{b.a1.eff} & \text{if } verstijving1 = 1 \\ e_{b.a2.eff} & \text{if } verstijving2 = 1 \end{cases}$$ $e_{b.a.eff} = 570.07 \cdot mm$

Bruto zwaartepunt voor axiale druk:

$$e_{b.a} := \begin{cases} e_{b.a0} & \text{if } verstijving1 = 0 \\ e_{b.a1} & \text{if } verstijving1 = 1 \\ e_{b.a2} & \text{if } verstijving2 = 1 \end{cases}$$ $e_{b.a} = 592.52 \cdot mm$

Verschuiving zwaartepunt voor axiale druk:

$\Delta e_a := e_{b.a.eff} - e_{b.a}$ $\Delta e_a = -22.46 \cdot mm$

$$\Delta e_a := \begin{cases} 0 & \text{if } \Delta e_a < 0 \text{ if } N_{ed} \geq 0 \\ 0 & \text{if } \Delta e_a \geq 0 \text{ if } N_{ed} < 0 \\ \Delta e_a & \text{if } \Delta e_a \geq 0 \text{ if } N_{ed} \geq 0 \\ \Delta e_a & \text{if } \Delta e_a < 0 \text{ if } N_{ed} < 0 \end{cases}$$ $\Delta e_a = 0.00 \cdot mm$

Verstijving 1 in trekzone voor moment:

$$trek1 := \begin{cases} 0 & \text{if } e_b \geq h_{v1} \\ 1 & \text{otherwise} \end{cases}$$ $trek1 = 0.00$

Verstijving 2 in trekzone voor moment:

$$trek2 := \begin{cases} 0 & \text{if } e_b \geq h_{v2} \\ 1 & \text{otherwise} \end{cases}$$ $trek2 = 1.00$

$I_{y.m0.eff} = 3.45 \times 10^{10} \cdot mm^4$

$$I_{y.m1.eff} := \begin{cases} I_{y.m0.eff} & \text{if } trek1 = 1 \\ I_{y.m1.eff} & \text{otherwise} \end{cases}$$ $I_{y.m1.eff} = 3.46 \times 10^{10} \cdot mm^4$

$$I_{y.m2.eff} := \begin{cases} I_{y.m1.eff} & \text{if } trek2 = 1 \\ I_{y.m2.eff} & \text{otherwise} \end{cases}$$ $I_{y.m2.eff} = 3.46 \times 10^{10} \cdot mm^4$

Effectief traagheidsmoment voor moment:

$$I_{y.m.eff} := \begin{cases} I_{y.m0.eff} & \text{if } verstijving1 = 0 \\ I_{y.m1.eff} & \text{if } verstijving1 = 1 \\ I_{y.m2.eff} & \text{if } verstijving2 = 1 \end{cases}$$ $I_{y.m.eff} = 3.46 \times 10^{10} \cdot mm^4$

$e_{b.m0.eff} = 593.88 \cdot mm$

$$e_{b.m1.eff} := \begin{cases} e_{b.m0.eff} & \text{if } trek1 = 1 \\ e_{b.m1.eff} & \text{otherwise} \end{cases}$$ $e_{b.m1.eff} = 592.52 \cdot mm$

$$e_{b.m2.eff} := \begin{cases} e_{b.m1.eff} & \text{if } trek2 = 1 \\ e_{b.m2.eff} & \text{otherwise} \end{cases}$$ $e_{b.m2.eff} = 592.52 \cdot mm$

Effectief zwaartepunt voor moment:

$$e_{b.m.eff} := \begin{cases} e_{b.m0.eff} & \text{if } verstijving1 = 0 \\ e_{b.m1.eff} & \text{if } verstijving1 = 1 \\ e_{b.m2.eff} & \text{if } verstijving2 = 1 \end{cases}$$

$e_{b.m.eff} = 592.52 \cdot mm$

Effectieve afstand tot uiterste vezel voor moment:

$$z_{max} := \begin{cases} e_{b.m.eff} & \text{if } e_{b.m.eff} \geq h_t - e_{b.m.eff} \\ (h_t - e_{b.m.eff}) & \text{otherwise} \end{cases}$$

$z_{max} = 907.48 \cdot mm$

Maximale dwarskracht voor plooi:

$$V_{bw.rd} := \begin{cases} V_{bw.rd.s0} & \text{if } verstijving1 = 0 \\ V_{bw.rd.s1} & \text{if } verstijving1 = 1 \\ V_{bw.rd.s2} & \text{if } verstijving2 = 1 \end{cases}$$

$V_{bw.rd} = 834.08 \cdot kN$

Toetsing axiale druk en moment:

$$\eta_{1.1} := \frac{N_{ed}}{\left(\frac{f_{yd} \cdot A_{a.eff}}{\gamma_{M0}}\right)} + \frac{M_{ed} + N_{ed} \cdot \Delta e_a}{\left(\frac{f_{yd} \cdot \frac{I_{y.m.eff}}{e_{b.m.eff}}}{\gamma_{M0}}\right)}$$

$\eta_{1.1} = 0.76$

$$\eta_{1.2} := \left| \frac{N_{ed}}{\left(\frac{f_{yd} \cdot A_{a.eff}}{\gamma_{M0}}\right)} + \frac{-M_{ed} - N_{ed} \cdot \Delta e_a}{\left(\frac{f_{yd} \cdot \frac{I_{y.m.eff}}{h_t - e_{b.m.eff}}}{\gamma_{M0}}\right)} \right|$$

$\eta_{1.2} = 0.20$

Toetsing axiale spanning:

$$\eta_1 := \begin{cases} \eta_{1.1} & \text{if } \eta_{1.1} \geq \eta_{1.2} \\ \eta_{1.2} & \text{if } \eta_{1.2} > \eta_{1.1} \end{cases}$$

$\eta_1 = 0.76$

Toetsing maximale dwarsbelasting

$$\eta_2 := \frac{F_{ed}}{F_{Rd}}$$

$\eta_2 = 0.23$

Toetsing maximale dwarskracht:

$$\eta_3 := \frac{V_{ed}}{V_{bw.rd}}$$

$\eta_3 = 0.24$

ULS Toets

Individuele toetsing:

$\eta_1 = 0.76$ Toets <1.0

$\eta_2 = 0.23$ Toets <1.0

$\eta_3 = 0.24$ Toets <1.0

Interactie toetsing:

Formule 7.1 $\eta_1 + (2 \cdot \eta_3 - 1)^2 = 1.03$ Toets <1.0 Alleen toepassen als $\eta_3 > 0.5$

Formule 7.2 $\eta_2 + 0.8 \cdot \eta_1 = 0.84$ Toets <1.4

Formule 2.94 volgens Beg et. al $\left[\eta_3 \cdot \left(1 - \frac{F_{ed}}{2 \cdot V_{ed}}\right)\right]^{1.6} + \eta_2 = 0.30$ Toets <1.0

Toetsing van het profiel op basis van reductie van het lijf op basis van gereduceerde spanningen methode:

$M_k := M_{ed}$

$V_k := V_{ed}$

$N_k := N_{ed}$

SLS Zonder verstijver

Reduced stress method for web (10)

$$\sigma_{x1.k} := \frac{M_k}{I_y} \cdot e_2 + \frac{N_k}{A_{tot}}$$

$\sigma_{x1.k} = 164.24 \cdot \frac{N}{mm^2}$

$$\sigma_{x2.k} := \frac{M_k}{I_y} \cdot e_4 + \frac{N_k}{A_{tot}}$$

$\sigma_{x2.k} = 40.11 \cdot \frac{N}{mm^2}$

$$\tau_k := \frac{V_k}{h_w \cdot t_w}$$

$\tau_k = 17.48 \cdot \frac{N}{mm^2}$

$$\Psi_x := \frac{\sigma_{x2.k}}{\sigma_{x1.k}}$$

$\Psi_x = 0.24$

$$k_x := \begin{cases} 4 & \text{if } \Psi_x = 1 \\ \frac{8.2}{(1.05 + \Psi_x)} & \text{if } 0 < \Psi_x < 1 \\ 7.8 & \text{if } \Psi_x = 0 \\ \left(7.81 - 6.29 \cdot \Psi_x + 9.78 \cdot \Psi_x^2\right) & \text{if } 0 > \Psi_x > -1 \\ 23.9 & \text{if } \Psi_x = -1 \\ 5.98 \cdot (1 - \Psi_x)^2 & \text{if } -1 > \Psi_x > -3 \\ 95.68 & \text{if } \Psi_x \leq -3 \end{cases}$$

$k_x = 6.34$

$$\sigma_{x.cr} := k_x \frac{\pi^2 \cdot E_d}{12 \cdot (1 - \nu^2)} \cdot \left(\frac{t_w}{h_w}\right)^2$$

$\sigma_{x.cr} = 37.64 \cdot \frac{N}{mm^2}$

$$\alpha_{cr.x} := \frac{\sigma_{x.cr}}{\sigma_{x1.k}}$$

$\alpha_{cr.x} = 0.23$

$$k_\tau := \begin{cases} \left[5.34 + 4 \cdot \left(\frac{h_w}{a}\right)^2\right] & \text{if } \frac{a}{h_w} \geq 1 \\ \left[4 + 5.34 \cdot \left(\frac{h_w}{a}\right)^2\right] & \text{if } \frac{a}{h_w} < 1 \end{cases}$$

$k_\tau = 5.38$

$$\tau_{cr} := k_\tau \cdot \frac{\pi^2 \cdot E_d}{12 \cdot (1 - \nu^2)} \cdot \left(\frac{t_w}{h_w}\right)^2$$

$\tau_{cr} = 31.94 \cdot \frac{N}{mm^2}$

$$\alpha_{cr.\tau} := \frac{\tau_{cr}}{\tau_k}$$

$\alpha_{cr.\tau} = 1.83$

$$\sigma_{eq.k} := \sqrt{\sigma_{x1.k}^2 + 3 \cdot \tau_k^2}$$

$\sigma_{eq.k} = 167.01 \cdot \frac{N}{mm^2}$

$\alpha_{ult.k} := \frac{f_{yd}}{\sigma_{eq.k}}$ $\alpha_{ult.k} = 1.41$

$\alpha_{cr} := \frac{1}{\frac{1+\Psi_x}{4\cdot\alpha_{cr.x}} + \sqrt{\left(\frac{1+\Psi_x}{4\cdot\alpha_{cr.x}}\right)^2 + \frac{1-\Psi_x}{2\cdot\alpha_{cr.x}^2} + \frac{1}{\alpha_{cr.\tau}^2}}}$ $\alpha_{cr} = 0.23$

$\lambda_p := \sqrt{\frac{\alpha_{ult.k}}{\alpha_{cr}}}$ $\lambda_p = 2.49$

x richting:

$\sigma_{x.cb} := \frac{\pi^2\cdot E_d\cdot t_w^2}{12\cdot(1-\nu^2)\cdot a^2}$

$\lambda_{cx} := \sqrt{\frac{\sigma_{x1.k}}{\sigma_{x.cb}}}$

$\Phi_{cx} := 0.5\cdot\left[1 + 0.21\cdot(\lambda_{cx} - 0.2) + \lambda_{cx}^2\right]$

$\chi_{cx} := \begin{cases} 1 & \text{if } 1 < \frac{1}{\Phi_{cx} + \sqrt{\Phi_{cx}^2 - \lambda_{cx}^2}} \\ \frac{1}{\Phi_{cx} + \sqrt{\Phi_{cx}^2 - \lambda_{cx}^2}} & \text{otherwise} \end{cases}$ $\chi_{cx} = 0.00$

$\alpha_{c.crit.x} := \frac{\sigma_{x.cb}}{\sigma_{x1.k}}$

$\rho_{x.p} := \begin{cases} 1 & \text{if } \lambda_p \le 0.5 + \sqrt{0.085 - 0.055\cdot\Psi_x} \\ \frac{\lambda_p - 0.055\cdot(3+\Psi_x)}{\lambda_p^2} & \text{otherwise} \end{cases}$ $\rho_{x.p} = 0.37$

$\xi_x := \begin{cases} 1 & \text{if } \frac{\alpha_{cr}}{\alpha_{c.crit.x}} - 1 > 1 \\ 0 & \text{if } 0 > \frac{\alpha_{cr}}{\alpha_{c.crit.x}} - 1 \\ \frac{\alpha_{cr}}{\alpha_{c.crit.x}} - 1 & \text{otherwise} \end{cases}$ $\xi_x = 1.00$

$\rho_x := (\rho_{x.p} - \chi_{cx})\cdot\xi_x\cdot(2 - \xi_x) + \chi_{cx}$ $\rho_x = 0.37$

Schuifkracht :

$\chi_{w.rigid} := \begin{cases} \eta & \text{if } \lambda_p < \frac{0.83}{\eta} \\ \frac{0.83}{\lambda_p} & \text{otherwise} \\ \frac{1.37}{0.7+\lambda_p} & \text{if } \lambda_p \ge 1.08 \end{cases}$ $\chi_{w.rigid} = 0.43$

$\chi_{w.nonrigid} := \begin{cases} \eta & \text{if } \lambda_p < \frac{0.83}{\eta} \\ \frac{0.83}{\lambda_p} & \text{otherwise} \end{cases}$ $\chi_{w.nonrigid} = 0.33$

$\chi_w := \begin{cases} \chi_{w.nonrigid} & \text{if rigidend} = 0 \\ \chi_{w.rigid} & \text{otherwise} \end{cases}$ $\chi_w = 0.33$

Toets :

$u_{sls.0} := \sqrt{\left(\frac{\sigma_{x1.k}}{\rho_x\cdot f_{yd}}\right)^2 + 3\left(\frac{\tau_k}{\chi_w\cdot f_{yd}}\right)^2}$ $u_{sls.0} = 1.92$

SLS Zonder verstijver

SLS met 1 verstijver

Reduced stress method for web (10)

Bovenkant tot verstijver 1:

$h_1 = 360.00\cdot mm$

$h_2 = 300.00\cdot mm$

$h_3 = 770.00\cdot mm$

$\sigma_{x1.k} := \frac{M_k}{I_y}\cdot e_2 + \frac{N_k}{A_{tot}}$ $\sigma_{x1.k} = 164.24\cdot\frac{N}{mm^2}$

$\sigma_{x2.k} := \frac{M_k}{I_y}\cdot e_{v1} + \frac{N_k}{A_{tot}}$ $\sigma_{x2.k} = 132.99\cdot\frac{N}{mm^2}$

$\tau_k := \frac{V_k}{h_w\cdot t_w}$ $\tau_k = 17.48\cdot\frac{N}{mm^2}$

$\Psi_x := \frac{\sigma_{x2.k}}{\sigma_{x1.k}}$ $\Psi_x = 0.81$

$k_x := \begin{cases} 4 & \text{if } \Psi_x = 1 \\ \frac{8.2}{(1.05+\Psi_x)} & \text{if } 0 < \Psi_x < 1 \\ 7.8 & \text{if } \Psi_x = 0 \\ (7.81 - 6.29\cdot\Psi_x + 9.78\cdot\Psi_x^2) & \text{if } 0 > \Psi_x > -1 \\ 23.9 & \text{if } \Psi_x = -1 \\ 5.98\cdot(1-\Psi_x)^2 & \text{if } -1 > \Psi_x > -3 \\ 95.68 & \text{if } \Psi_x \le -3 \end{cases}$ $k_x = 4.41$

$\sigma_{x.cr} := k_x\cdot\frac{\pi^2\cdot E_d}{12\cdot(1-\nu^2)}\cdot\left(\frac{t_w}{h_1}\right)^2$ $\sigma_{x.cr} = 413.27\cdot\frac{N}{mm^2}$

$\alpha_{cr.x} := \frac{\sigma_{x.cr}}{\sigma_{x1.k}}$ $\alpha_{cr.x} = 2.52$

$k_\tau := \begin{cases} \left[5.34 + 4\cdot\left(\frac{h_1}{a}\right)^2\right] & \text{if } \frac{a}{h_1} \ge 1 \\ \left[4 + 5.34\cdot\left(\frac{h_1}{a}\right)^2\right] & \text{if } \frac{a}{h_1} < 1 \end{cases}$ $k_\tau = 5.34$

$\tau_{cr} := k_\tau \cdot \frac{\pi^2 \cdot E_d}{12\cdot(1-\nu^2)} \cdot \left(\frac{t_w}{h_1}\right)^2$ $\tau_{cr} = 500.73 \cdot \frac{N}{mm^2}$

$\alpha_{cr.\tau} := \frac{\tau_{cr}}{\tau_k}$ $\alpha_{cr.\tau} = 28.64$

$\sigma_{eq.k} := \sqrt{\sigma_{x1.k}^2 + 3\cdot\tau_k^2}$ $\sigma_{eq.k} = 167.01 \cdot \frac{N}{mm^2}$

$\alpha_{ult.k} := \frac{f_{yd}}{\sigma_{eq.k}}$ $\alpha_{ult.k} = 1.41$

$\alpha_{cr} := \frac{1}{\frac{1+\Psi_x}{4\cdot\alpha_{cr.x}} + \sqrt{\left(\frac{1+\Psi_x}{4\cdot\alpha_{cr.x}}\right)^2 + \frac{1-\Psi_x}{2\cdot\alpha_{cr.x}^2} + \frac{1}{\alpha_{cr.\tau}^2}}}$ $\alpha_{cr} = 2.50$

$\lambda_p := \sqrt{\frac{\alpha_{ult.k}}{\alpha_{cr}}}$ $\lambda_p = 0.75$

x richting:

$\sigma_{x.cb} := \frac{\pi^2 \cdot E_d \cdot t_w^2}{12\cdot(1-\nu^2)\cdot a^2}$

$\lambda_{cx} := \sqrt{\frac{\sigma_{x1.k}}{\sigma_{x.cb}}}$

$\Phi_{cx} := 0.5\cdot\left[1 + 0.21\cdot(\lambda_{cx} - 0.2) + \lambda_{cx}^2\right]$

$\chi_{cx} := \begin{cases} 1 & \text{if } 1 < \frac{1}{\Phi_{cx} + \sqrt{\Phi_{cx}^2 - \lambda_{cx}^2}} \\ \frac{1}{\Phi_{cx} + \sqrt{\Phi_{cx}^2 - \lambda_{cx}^2}} & \text{otherwise} \end{cases}$ $\chi_{cx} = 0.00$

$\alpha_{c.crit.x} := \frac{\sigma_{x.cb}}{\sigma_{x1.k}}$

$\rho_{x.p} := \begin{cases} 1 & \text{if } \lambda_p \le 0.5 + \sqrt{0.085 - 0.055\cdot\Psi_x} \\ \frac{\lambda_p - 0.055\cdot(3+\Psi_x)}{\lambda_p^2} & \text{otherwise} \end{cases}$ $\rho_{x.p} = 0.96$

$\xi_x := \begin{cases} 1 & \text{if } \frac{\alpha_{cr}}{\alpha_{c.crit.x}} - 1 > 1 \\ 0 & \text{if } 0 > \frac{\alpha_{cr}}{\alpha_{c.crit.x}} - 1 \\ \frac{\alpha_{cr}}{\alpha_{c.crit.x}} - 1 & \text{otherwise} \end{cases}$ $\xi_x = 1.00$

$\rho_x := (\rho_{x.p} - \chi_{cx})\cdot\xi_x\cdot(2 - \xi_x) + \chi_{cx}$ $\rho_x = 0.96$

Schuifkracht :

$\chi_{w.rigid} := \begin{cases} \eta & \text{if } \lambda_p < \frac{0.83}{\eta} \\ \frac{0.83}{\lambda_p} & \text{otherwise} \\ \frac{1.37}{0.7+\lambda_p} & \text{if } \lambda_p \ge 1.08 \end{cases}$ $\chi_{w.rigid} = 1.11$

$\chi_{w.nonrigid} := \begin{cases} \eta & \text{if } \lambda_p < \frac{0.83}{\eta} \\ \frac{0.83}{\lambda_p} & \text{otherwise} \end{cases}$ $\chi_{w.nonrigid} = 1.11$

$\chi_w := \begin{cases} \chi_{w.nonrigid} & \text{if rigidend} = 0 \\ \chi_{w.rigid} & \text{otherwise} \end{cases}$ $\chi_w = 1.11$

Toets :

$u_{sls.1.1} := \sqrt{\left(\frac{\sigma_{x1.k}}{\rho_x \cdot f_{yd}}\right)^2 + 3\left(\frac{\tau_k}{\chi_w \cdot f_{yd}}\right)^2}$ $u_{sls.1.1} = 0.74$

Verstijver 1 tot onderkant:

$h_1 = 360.00 \cdot mm$

$h_2 = 300.00 \cdot mm$

$h_3 = 770.00 \cdot mm$

$\sigma_{x1.k} := \frac{M_k}{I_y}\cdot e_{v1} + \frac{N_k}{A_{tot}}$ $\sigma_{x1.k} = 132.99 \cdot \frac{N}{mm^2}$

$\sigma_{x2.k} := \frac{M_k}{I_y}\cdot e_4 + \frac{N_k}{A_{tot}}$ $\sigma_{x2.k} = 40.11 \cdot \frac{N}{mm^2}$

$\tau_k := \frac{V_k}{h_w \cdot t_w}$ $\tau_k = 17.48 \cdot \frac{N}{mm^2}$

$\Psi_x := \begin{cases} 1 & \text{if } \frac{\sigma_{x2.k}}{\sigma_{x1.k}} > 1 \\ \frac{\sigma_{x2.k}}{\sigma_{x1.k}} & \text{otherwise} \end{cases}$ $\Psi_x = 0.30$

$$k_x := \begin{cases} 4 & \text{if } \Psi_x = 1 \\ \dfrac{8.2}{(1.05 + \Psi_x)} & \text{if } 0 < \Psi_x < 1 \\ 7.8 & \text{if } \Psi_x = 0 \\ \left(7.81 - 6.29 \cdot \Psi_x + 9.78 \cdot \Psi_x^2\right) & \text{if } 0 > \Psi_x > -1 \\ 23.9 & \text{if } \Psi_x = -1 \\ 5.98 \cdot (1 - \Psi_x)^2 & \text{if } -1 > \Psi_x > -3 \\ 95.68 & \text{if } \Psi_x \le -3 \end{cases} \qquad k_x = 6.07$$

$$\sigma_{x.cr} := k_x \frac{\pi^2 \cdot E_d}{12 \cdot (1 - \nu^2)} \cdot \left(\frac{t_w}{h_2 + h_3}\right)^2 \qquad \sigma_{x.cr} = 64.37 \cdot \frac{N}{mm^2}$$

$$\alpha_{cr.x} := \frac{\sigma_{x.cr}}{\sigma_{x1.k}} \qquad \alpha_{cr.x} = 0.48$$

$$k_\tau := \begin{cases} \left[5.34 + 4 \cdot \left(\dfrac{h_2 + h_3}{a}\right)^2\right] & \text{if } \dfrac{a}{h_2 + h_3} \ge 1 \\ \left[4 + 5.34 \cdot \left(\dfrac{h_2 + h_3}{a}\right)^2\right] & \text{if } \dfrac{a}{h_2 + h_3} < 1 \end{cases} \qquad k_\tau = 5.36$$

$$\tau_{cr} := k_\tau \cdot \frac{\pi^2 \cdot E_d}{12 \cdot (1 - \nu^2)} \cdot \left(\frac{t_w}{h_2 + h_3}\right)^2 \qquad \tau_{cr} = 56.87 \cdot \frac{N}{mm^2}$$

$$\alpha_{cr.\tau} := \frac{\tau_{cr}}{\tau_k} \qquad \alpha_{cr.\tau} = 3.25$$

$$\sigma_{eq.k} := \sqrt{\sigma_{x1.k}^2 + 3 \cdot \tau_k^2} \qquad \sigma_{eq.k} = 136.40 \cdot \frac{N}{mm^2}$$

$$\alpha_{ult.k} := \frac{f_{yd}}{\sigma_{eq.k}} \qquad \alpha_{ult.k} = 1.72$$

$$\alpha_{cr} := \frac{1}{\dfrac{1 + \Psi_x}{4 \cdot \alpha_{cr.x}} + \sqrt{\left(\dfrac{1 + \Psi_x}{4 \cdot \alpha_{cr.x}}\right)^2 + \dfrac{1 - \Psi_x}{2 \cdot \alpha_{cr.x}^2} + \dfrac{1}{\alpha_{cr.\tau}^2}}} \qquad \alpha_{cr} = 0.48$$

$$\lambda_p := \sqrt{\frac{\alpha_{ult.k}}{\alpha_{cr}}} \qquad \lambda_p = 1.90$$

x richting:

$$\sigma_{x.cb} := \frac{\pi^2 \cdot E_d \cdot t_w^2}{12 \cdot (1 - \nu^2) \cdot a^2}$$

$$\lambda_{cx} := \sqrt{\frac{|\sigma_{x1.k}|}{\sigma_{x.cb}}}$$

$$\Phi_{cx} := 0.5 \cdot \left[1 + 0.21 \cdot (\lambda_{cx} - 0.2) + \lambda_{cx}^2\right]$$

$$\chi_{cx} := \begin{cases} 1 & \text{if } 1 < \dfrac{1}{\Phi_{cx} + \sqrt{\Phi_{cx}^2 - \lambda_{cx}^2}} \\ \dfrac{1}{\Phi_{cx} + \sqrt{\Phi_{cx}^2 - \lambda_{cx}^2}} & \text{otherwise} \end{cases} \qquad \chi_{cx} = 0.00$$

$$\alpha_{c.crit.x} := \frac{\sigma_{x.cb}}{\sigma_{x1.k}}$$

$$\rho_{x.p} := \begin{cases} 1 & \text{if } \lambda_p \le 0.5 + \sqrt{0.085 - 0.055 \cdot \Psi_x} \\ \dfrac{\lambda_p - 0.055 \cdot (3 + \Psi_x)}{\lambda_p^2} & \text{otherwise} \end{cases} \qquad \rho_{x.p} = 0.48$$

$$\xi_x := \begin{cases} 1 & \text{if } \dfrac{\alpha_{cr}}{\alpha_{c.crit.x}} - 1 > 1 \\ 0 & \text{if } 0 > \dfrac{\alpha_{cr}}{\alpha_{c.crit.x}} - 1 \\ \dfrac{\alpha_{cr}}{\alpha_{c.crit.x}} - 1 & \text{otherwise} \end{cases} \qquad \xi_x = 1.00$$

$$\rho_x := (\rho_{x.p} - \chi_{cx}) \cdot \xi_x \cdot (2 - \xi_x) + \chi_{cx} \qquad \rho_x = 0.48$$

Schuifkracht :

$$\chi_{w.rigid} := \begin{cases} \eta & \text{if } \lambda_p < \dfrac{0.83}{\eta} \\ \dfrac{0.83}{\lambda_p} & \text{otherwise} \\ \dfrac{1.37}{0.7 + \lambda_p} & \text{if } \lambda_p \ge 1.08 \end{cases} \qquad \chi_{w.rigid} = 0.53$$

$$\chi_{w.nonrigid} := \begin{cases} \eta & \text{if } \lambda_p < \dfrac{0.83}{\eta} \\ \dfrac{0.83}{\lambda_p} & \text{otherwise} \end{cases} \qquad \chi_{w.nonrigid} = 0.44$$

$$\chi_w := \begin{cases} \chi_{w.nonrigid} & \text{if } rigidend = 0 \\ \chi_{w.rigid} & \text{otherwise} \end{cases} \qquad \chi_w = 0.44$$

Toets :

$$u_{sls.1.2} := \sqrt{\left(\frac{\sigma_{x1.k}}{\rho_x \cdot f_{yd}}\right)^2 + 3\left(\frac{\tau_k}{\chi_w \cdot f_{yd}}\right)^2} \qquad u_{sls.1.2} = 1.23$$

$$u_{sls.1} := \begin{cases} u_{sls.1.1} & \text{if } u_{sls.1.2} \le u_{sls.1.1} \\ u_{sls.1.2} & \text{if } u_{sls.1.1} \le u_{sls.1.2} \end{cases} \qquad u_{sls.1} = 1.23$$

SLS met 1 verstijver

SLS met 2 verstijvers

Reduced stress method for web (10)

Bovenkant tot verstijver 1:

$h_1 = 360.00 \cdot mm$
$h_2 = 300.00 \cdot mm$
$h_3 = 770.00 \cdot mm$

$\sigma_{x1.k} := \frac{M_k}{I_y} \cdot e_2 + \frac{N_k}{A_{tot}}$ $\quad \sigma_{x1.k} = 164.24 \cdot \frac{N}{mm^2}$

$\sigma_{x2.k} := \frac{M_k}{I_y} \cdot e_{v1} + \frac{N_k}{A_{tot}}$ $\quad \sigma_{x2.k} = 132.99 \cdot \frac{N}{mm^2}$

$\tau_k := \frac{V_k}{h_w \cdot t_w}$ $\quad \tau_k = 17.48 \cdot \frac{N}{mm^2}$

$\Psi_x := \frac{\sigma_{x2.k}}{\sigma_{x1.k}}$ $\quad \Psi_x = 0.81$

$$k_x := \begin{cases} 4 & \text{if } \Psi_x = 1 \\ \frac{8.2}{(1.05 + \Psi_x)} & \text{if } 0 < \Psi_x < 1 \\ 7.8 & \text{if } \Psi_x = 0 \\ \left(7.81 - 6.29 \cdot \Psi_x + 9.78 \cdot \Psi_x^2\right) & \text{if } 0 > \Psi_x > -1 \\ 23.9 & \text{if } \Psi_x = -1 \\ 5.98 \cdot (1 - \Psi_x)^2 & \text{if } -1 > \Psi_x > -3 \\ 95.68 & \text{if } \Psi_x \le -3 \end{cases}$$ $\quad k_x = 4.41$

$\sigma_{x.cr} := k_x \frac{\pi^2 \cdot E_d}{12 \cdot (1 - \nu^2)} \cdot \left(\frac{t_w}{h_1}\right)^2$ $\quad \sigma_{x.cr} = 413.27 \cdot \frac{N}{mm^2}$

$\alpha_{cr.x} := \frac{\sigma_{x.cr}}{\sigma_{x1.k}}$ $\quad \alpha_{cr.x} = 2.52$

$$k_\tau := \begin{cases} \left[5.34 + 4 \cdot \left(\frac{h_1}{a}\right)^2\right] & \text{if } \frac{a}{h_1} \ge 1 \\ \left[4 + 5.34 \cdot \left(\frac{h_1}{a}\right)^2\right] & \text{if } \frac{a}{h_1} < 1 \end{cases}$$ $\quad k_\tau = 5.34$

$\tau_{cr} := k_\tau \cdot \frac{\pi^2 \cdot E_d}{12 \cdot (1 - \nu^2)} \cdot \left(\frac{t_w}{h_1}\right)^2$ $\quad \tau_{cr} = 500.73 \cdot \frac{N}{mm^2}$

$\alpha_{cr.\tau} := \frac{\tau_{cr}}{\tau_k}$ $\quad \alpha_{cr.\tau} = 28.64$

$\sigma_{eq.k} := \sqrt{\sigma_{x1.k}^2 + 3 \cdot \tau_k^2}$ $\quad \sigma_{eq.k} = 167.01 \cdot \frac{N}{mm^2}$

$\alpha_{ult.k} := \frac{f_{yd}}{\sigma_{eq.k}}$ $\quad \alpha_{ult.k} = 1.41$

$$\alpha_{cr} := \frac{1}{\frac{1 + \Psi_x}{4 \cdot \alpha_{cr.x}} + \sqrt{\left(\frac{1 + \Psi_x}{4 \cdot \alpha_{cr.x}}\right)^2 + \frac{1 - \Psi_x}{2 \cdot \alpha_{cr.x}^2} + \frac{1}{\alpha_{cr.\tau}^2}}}$$ $\quad \alpha_{cr} = 2.50$

$\lambda_p := \sqrt{\frac{\alpha_{ult.k}}{\alpha_{cr}}}$ $\quad \lambda_p = 0.75$

x richting:

$\sigma_{x.cb} := \frac{\pi^2 \cdot E_d \cdot t_w^2}{12 \cdot (1 - \nu^2) \cdot a^2}$

$\lambda_{cx} := \sqrt{\frac{\sigma_{x1.k}}{\sigma_{x.cb}}}$

$\Phi_{cx} := 0.5 \cdot \left[1 + 0.21 \cdot (\lambda_{cx} - 0.2) + \lambda_{cx}^2\right]$

$$\chi_{cx} := \begin{cases} 1 & \text{if } 1 < \frac{1}{\Phi_{cx} + \sqrt{\Phi_{cx}^2 - \lambda_{cx}^2}} \\ \frac{1}{\Phi_{cx} + \sqrt{\Phi_{cx}^2 - \lambda_{cx}^2}} & \text{otherwise} \end{cases}$$ $\quad \chi_{cx} = 0.00$

$\alpha_{c.crit.x} := \frac{\sigma_{x.cb}}{\sigma_{x1.k}}$

$$\rho_{x.p} := \begin{cases} 1 & \text{if } \lambda_p \le 0.5 + \sqrt{0.085 - 0.055 \cdot \Psi_x} \\ \frac{\lambda_p - 0.055 \cdot (3 + \Psi_x)}{\lambda_p^2} & \text{otherwise} \end{cases}$$ $\quad \rho_{x.p} = 0.96$

$$\xi_x := \begin{cases} 1 & \text{if } \frac{\alpha_{cr}}{\alpha_{c.crit.x}} - 1 > 1 \\ 0 & \text{if } 0 > \frac{\alpha_{cr}}{\alpha_{c.crit.x}} - 1 \\ \frac{\alpha_{cr}}{\alpha_{c.crit.x}} - 1 & \text{otherwise} \end{cases}$$ $\quad \xi_x = 1.00$

$\rho_x := (\rho_{x.p} - \chi_{cx}) \cdot \xi_x \cdot (2 - \xi_x) + \chi_{cx}$ $\quad \rho_x = 0.96$

Schuifkracht :

$$\chi_{w.rigid} := \begin{cases} \eta & \text{if } \lambda_p < \frac{0.83}{\eta} \\ \frac{0.83}{\lambda_p} & \text{otherwise} \\ \frac{1.37}{0.7 + \lambda_p} & \text{if } \lambda_p \ge 1.08 \end{cases}$$ $\quad \chi_{w.rigid} = 1.11$

$$\chi_{w.nonrigid} := \begin{cases} \eta & \text{if } \lambda_p < \frac{0.83}{\eta} \\ \frac{0.83}{\lambda_p} & \text{otherwise} \end{cases}$$ $\quad \chi_{w.nonrigid} = 1.11$

$$\chi_w := \begin{cases} \chi_{w.nonrigid} & \text{if rigidend} = 0 \\ \chi_{w.rigid} & \text{otherwise} \end{cases}$$ $\quad \chi_w = 1.11$

Toets :

$u_{sls.2.1} := \sqrt{\left(\frac{\sigma_{x1.k}}{\rho_x \cdot f_{yd}}\right)^2 + 3\left(\frac{\tau_k}{\chi_w \cdot f_{yd}}\right)^2}$ $\quad u_{sls.2.1} = 0.74$

Verstijver 1 tot verstijver 2:

$h_1 = 360.00 \cdot mm$
$h_2 = 300.00 \cdot mm$
$h_3 = 770.00 \cdot mm$

$\sigma_{x1.k} := \frac{M_k}{I_y} \cdot e_{v1} + \frac{N_k}{A_{tot}}$ $\quad \sigma_{x1.k} = 132.99 \cdot \frac{N}{mm^2}$

$$\sigma_{x2.k} := \frac{M_k}{I_y} \cdot e_{v2} + \frac{N_k}{A_{tot}} \qquad \sigma_{x2.k} = 106.95 \cdot \frac{N}{mm^2}$$

$$\tau_k := \frac{V_k}{h_w \cdot t_w} \qquad \tau_k = 17.48 \cdot \frac{N}{mm^2}$$

$$\Psi_x := \begin{cases} 1 & \text{if } \frac{\sigma_{x2.k}}{\sigma_{x1.k}} > 1 \\ \frac{\sigma_{x2.k}}{\sigma_{x1.k}} & \text{otherwise} \end{cases} \qquad \Psi_x = 0.80$$

$$k_x := \begin{cases} 4 & \text{if } \Psi_x = 1 \\ \frac{8.2}{(1.05 + \Psi_x)} & \text{if } 0 < \Psi_x < 1 \\ 7.8 & \text{if } \Psi_x = 0 \\ \left(7.81 - 6.29 \cdot \Psi_x + 9.78 \cdot \Psi_x^2\right) & \text{if } 0 > \Psi_x > -1 \\ 23.9 & \text{if } \Psi_x = -1 \\ 5.98 \cdot (1 - \Psi_x)^2 & \text{if } -1 > \Psi_x > -3 \\ 95.68 & \text{if } \Psi_x \le -3 \end{cases} \qquad k_x = 4.42$$

$$\sigma_{x.cr} := k_x \frac{\pi^2 \cdot E_d}{12 \cdot \left(1 - \nu^2\right)} \cdot \left(\frac{t_w}{h_2}\right)^2 \qquad \sigma_{x.cr} = 596.89 \cdot \frac{N}{mm^2}$$

$$\alpha_{cr.x} := \frac{\sigma_{x.cr}}{\sigma_{x1.k}} \qquad \alpha_{cr.x} = 4.49$$

$$k_\tau := \begin{cases} \left[5.34 + 4 \cdot \left(\frac{h_2}{a}\right)^2\right] & \text{if } \frac{a}{h_2} \ge 1 \\ \left[4 + 5.34 \cdot \left(\frac{h_2}{a}\right)^2\right] & \text{if } \frac{a}{h_2} < 1 \end{cases} \qquad k_\tau = 5.34$$

$$\tau_{cr} := k_\tau \cdot \frac{\pi^2 \cdot E_d}{12 \cdot \left(1 - \nu^2\right)} \cdot \left(\frac{t_w}{h_2}\right)^2 \qquad \tau_{cr} = 720.95 \cdot \frac{N}{mm^2}$$

$$\alpha_{cr.\tau} := \frac{\tau_{cr}}{\tau_k} \qquad \alpha_{cr.\tau} = 41.24$$

$$\sigma_{eq.k} := \sqrt{\sigma_{x1.k}^2 + 3 \cdot \tau_k^2} \qquad \sigma_{eq.k} = 136.40 \cdot \frac{N}{mm^2}$$

$$\alpha_{ult.k} := \frac{f_{yd}}{\sigma_{eq.k}} \qquad \alpha_{ult.k} = 1.72$$

$$\alpha_{cr} := \frac{1}{\frac{1 + \Psi_x}{4 \cdot \alpha_{cr.x}} + \sqrt{\left(\frac{1 + \Psi_x}{4 \cdot \alpha_{cr.x}}\right)^2 + \frac{1 - \Psi_x}{2 \cdot \alpha_{cr.x}^2} + \frac{1}{\alpha_{cr.\tau}^2}}} \qquad \alpha_{cr} = 4.44$$

$$\lambda_p := \sqrt{\frac{\alpha_{ult.k}}{\alpha_{cr}}} \qquad \lambda_p = 0.62$$

x richting:

$$\sigma_{x.cb} := \frac{\pi^2 \cdot E_d \cdot t_w^2}{12 \cdot \left(1 - \nu^2\right) \cdot a^2}$$

$$\lambda_{cx} := \sqrt{\frac{|\sigma_{x1.k}|}{\sigma_{x.cb}}}$$

$$\Phi_{cx} := 0.5 \cdot \left[1 + 0.21 \cdot \left(\lambda_{cx} - 0.2\right) + \lambda_{cx}^2\right]$$

$$\chi_{cx} := \begin{cases} 1 & \text{if } 1 < \frac{1}{\Phi_{cx} + \sqrt{\Phi_{cx}^2 - \lambda_{cx}^2}} \\ \frac{1}{\Phi_{cx} + \sqrt{\Phi_{cx}^2 - \lambda_{cx}^2}} & \text{otherwise} \end{cases} \qquad \chi_{cx} = 0.00$$

$$\alpha_{c.crit.x} := \frac{\sigma_{x.cb}}{\sigma_{x1.k}}$$

$$\rho_{x.p} := \begin{cases} 1 & \text{if } \lambda_p \le 0.5 + \sqrt{0.085 - 0.055 \cdot \Psi_x} \\ \frac{\lambda_p - 0.055 \cdot (3 + \Psi_x)}{\lambda_p^2} & \text{otherwise} \end{cases} \qquad \rho_{x.p} = 1.00$$

$$\xi_x := \begin{cases} 1 & \text{if } \frac{\alpha_{cr}}{\alpha_{c.crit.x}} - 1 > 1 \\ 0 & \text{if } 0 > \frac{\alpha_{cr}}{\alpha_{c.crit.x}} - 1 \\ \frac{\alpha_{cr}}{\alpha_{c.crit.x}} - 1 & \text{otherwise} \end{cases} \qquad \xi_x = 1.00$$

$$\rho_x := \left(\rho_{x.p} - \chi_{cx}\right) \cdot \xi_x \cdot \left(2 - \xi_x\right) + \chi_{cx} \qquad \rho_x = 1.00$$

Schuifkracht :

$$\chi_{w.rigid} := \begin{cases} \eta & \text{if } \lambda_p < \frac{0.83}{\eta} \\ \frac{0.83}{\lambda_p} & \text{otherwise} \\ \frac{1.37}{0.7 + \lambda_p} & \text{if } \lambda_p \ge 1.08 \end{cases} \qquad \chi_{w.rigid} = 1.20$$

$$\chi_{w.nonrigid} := \begin{cases} \eta & \text{if } \lambda_p < \frac{0.83}{\eta} \\ \frac{0.83}{\lambda_p} & \text{otherwise} \end{cases} \qquad \chi_{w.nonrigid} = 1.20$$

$$\chi_w := \begin{cases} \chi_{w.nonrigid} & \text{if rigidend} = 0 \\ \chi_{w.rigid} & \text{otherwise} \end{cases} \qquad \chi_w = 1.20$$

Toets :

$$u_{sls.2.2} := \sqrt{\left(\frac{\sigma_{x1.k}}{\rho_x \cdot f_{yd}}\right)^2 + 3\left(\frac{\tau_k}{\chi_w \cdot f_{yd}}\right)^2} \qquad u_{sls.2.2} = 0.58$$

Verstijver 2 tot onderkant:

$h_1 = 360.00 \cdot mm$

$h_2 = 300.00 \cdot mm$

$h_3 = 770.00 \cdot mm$

$$\sigma_{x1.k} := \frac{M_k}{I_y} \cdot e_{v2} + \frac{N_k}{A_{tot}} \qquad \sigma_{x1.k} = 106.95 \cdot \frac{N}{mm^2}$$

$$\sigma_{x2.k} := \frac{M_k}{I_y} \cdot e_4 + \frac{N_k}{A_{tot}} \qquad \sigma_{x2.k} = 40.11 \cdot \frac{N}{mm^2}$$

$$\tau_k := \frac{V_k}{h_w \cdot t_w} \qquad \tau_k = 17.48 \cdot \frac{N}{mm^2}$$

$$\Psi_x := \begin{cases} 1 & \text{if } \dfrac{\sigma_{x2.k}}{\sigma_{x1.k}} > 1 \\ \dfrac{\sigma_{x2.k}}{\sigma_{x1.k}} & \text{otherwise} \end{cases} \qquad \Psi_x = 0.38$$

$$k_x := \begin{cases} 4 & \text{if } \Psi_x = 1 \\ \dfrac{8.2}{(1.05 + \Psi_x)} & \text{if } 0 < \Psi_x < 1 \\ 7.8 & \text{if } \Psi_x = 0 \\ \left(7.81 - 6.29\cdot\Psi_x + 9.78\cdot\Psi_x^2\right) & \text{if } 0 > \Psi_x > -1 \\ 23.9 & \text{if } \Psi_x = -1 \\ 5.98\cdot(1 - \Psi_x)^2 & \text{if } -1 > \Psi_x > -3 \\ 95.68 & \text{if } \Psi_x \le -3 \end{cases} \qquad k_x = 5.75$$

$$\sigma_{x.cr} := k_x \frac{\pi^2\cdot E_d}{12\cdot(1-\nu^2)}\cdot\left(\frac{t_w}{h_3}\right)^2 \qquad \sigma_{x.cr} = 117.89\cdot\frac{N}{mm^2}$$

$$\alpha_{cr.x} := \frac{\sigma_{x.cr}}{\sigma_{x1.k}} \qquad \alpha_{cr.x} = 1.10$$

$$k_\tau := \begin{cases} \left[5.34 + 4\cdot\left(\dfrac{h_3}{a}\right)^2\right] & \text{if } \dfrac{a}{h_3} \ge 1 \\ \left[4 + 5.34\cdot\left(\dfrac{h_3}{a}\right)^2\right] & \text{if } \dfrac{a}{h_3} < 1 \end{cases} \qquad k_\tau = 5.35$$

$$\tau_{cr} := k_\tau\cdot\frac{\pi^2\cdot E_d}{12\cdot(1-\nu^2)}\cdot\left(\frac{t_w}{h_3}\right)^2 \qquad \tau_{cr} = 109.62\cdot\frac{N}{mm^2}$$

$$\alpha_{cr.\tau} := \frac{\tau_{cr}}{\tau_k} \qquad \alpha_{cr.\tau} = 6.27$$

$$\sigma_{eq.k} := \sqrt{\sigma_{x1.k}^2 + 3\cdot\tau_k^2} \qquad \sigma_{eq.k} = 111.15\cdot\frac{N}{mm^2}$$

$$\alpha_{ult.k} := \frac{f_{yd}}{\sigma_{eq.k}} \qquad \alpha_{ult.k} = 2.11$$

$$\alpha_{cr} := \frac{1}{\dfrac{1+\Psi_x}{4\cdot\alpha_{cr.x}} + \sqrt{\left(\dfrac{1+\Psi_x}{4\cdot\alpha_{cr.x}}\right)^2 + \dfrac{1-\Psi_x}{2\cdot\alpha_{cr.x}^2} + \dfrac{1}{\alpha_{cr.\tau}^2}}} \qquad \alpha_{cr} = 1.08$$

$$\lambda_p := \sqrt{\frac{\alpha_{ult.k}}{\alpha_{cr}}} \qquad \lambda_p = 1.40$$

x richting:

$$\sigma_{x.cb} := \frac{\pi^2\cdot E_d\cdot t_w^2}{12\cdot(1-\nu^2)\cdot a^2}$$

$$\lambda_{cx} := \sqrt{\frac{|\sigma_{x1.k}|}{\sigma_{x.cb}}}$$

$$\Phi_{cx} := 0.5\left[1 + 0.21\cdot(\lambda_{cx} - 0.2) + \lambda_{cx}^2\right]$$

$$\chi_{cx} := \begin{cases} 1 & \text{if } 1 < \dfrac{1}{\Phi_{cx} + \sqrt{\Phi_{cx}^2 - \lambda_{cx}^2}} \\ \dfrac{1}{\Phi_{cx} + \sqrt{\Phi_{cx}^2 - \lambda_{cx}^2}} & \text{otherwise} \end{cases} \qquad \chi_{cx} = 0.00$$

$$\alpha_{c.crit.x} := \frac{\sigma_{x.cb}}{\sigma_{x1.k}}$$

$$\rho_{x.p} := \begin{cases} 1 & \text{if } \lambda_p \le 0.5 + \sqrt{0.085 - 0.055\cdot\Psi_x} \\ \dfrac{\lambda_p - 0.055\cdot(3+\Psi_x)}{\lambda_p^2} & \text{otherwise} \end{cases} \qquad \rho_{x.p} = 0.62$$

$$\xi_x := \begin{cases} 1 & \text{if } \dfrac{\alpha_{cr}}{\alpha_{c.crit.x}} - 1 > 1 \\ 0 & \text{if } 0 > \dfrac{\alpha_{cr}}{\alpha_{c.crit.x}} - 1 \\ \dfrac{\alpha_{cr}}{\alpha_{c.crit.x}} - 1 & \text{otherwise} \end{cases} \qquad \xi_x = 1.00$$

$$\rho_x := (\rho_{x.p} - \chi_{cx})\cdot\xi_x\cdot(2 - \xi_x) + \chi_{cx} \qquad \rho_x = 0.62$$

Schuifkracht :

$$\chi_{w.rigid} := \begin{cases} \eta & \text{if } \lambda_p < \dfrac{0.83}{\eta} \\ \dfrac{0.83}{\lambda_p} & \text{otherwise} \\ \dfrac{1.37}{0.7 + \lambda_p} & \text{if } \lambda_p \ge 1.08 \end{cases} \qquad \chi_{w.rigid} = 0.65$$

$$\chi_{w.nonrigid} := \begin{cases} \eta & \text{if } \lambda_p < \dfrac{0.83}{\eta} \\ \dfrac{0.83}{\lambda_p} & \text{otherwise} \end{cases} \qquad \chi_{w.nonrigid} = 0.59$$

$$\chi_w := \begin{cases} \chi_{w.nonrigid} & \text{if rigidend} = 0 \\ \chi_{w.rigid} & \text{otherwise} \end{cases} \qquad \chi_w = 0.59$$

Toets :

$$u_{sls.2.3} := \sqrt{\left(\frac{\sigma_{x1.k}}{\rho_x\cdot f_{yd}}\right)^2 + 3\left(\frac{\tau_k}{\chi_w\cdot f_{yd}}\right)^2} \qquad u_{sls.2.3} = 0.77$$

$$u_{sls.2} := \begin{cases} u_{sls.2.1} & \text{if } u_{sls.2.2} \le u_{sls.2.1} \text{ if } u_{sls.2.3} \le u_{sls.2.1} \\ u_{sls.2.2} & \text{if } u_{sls.2.1} \le u_{sls.2.2} \text{ if } u_{sls.2.3} \le u_{sls.2.2} \\ u_{sls.2.3} & \text{if } u_{sls.2.1} \le u_{sls.2.3} \text{ if } u_{sls.2.2} \le u_{sls.2.3} \end{cases} \qquad u_{sls.2} = 0.77$$

SLS met 2 verstijvers

$$u_{sls.1} := \begin{cases} u_{sls.0} & \text{if trek1} = 1 \\ u_{sls.1} & \text{otherwise} \end{cases}$$

$$u_{sls.2} := \begin{cases} u_{sls.1} & \text{if trek2} = 1 \\ u_{sls.2} & \text{otherwise} \end{cases}$$

Indien 0 verstijvers: $u_{sls.0} = 1.92$
Indien 1 verstijver: $u_{sls.1} = 1.23$
Indien 2 verstijvers: $u_{sls.2} = 1.23$

$$u_{sls} := \begin{cases} u_{sls.0} & \text{if verstijving1} = 0 \\ u_{sls.1} & \text{if verstijving1} = 1 \\ u_{sls.2} & \text{if verstijving2} = 1 \end{cases}$$

Dus hier: $u_{sls} = 1.23$

Toetsing Von Mises spanning:

$$u_{sls.sp} := \begin{cases} \sqrt{\dfrac{\left(\dfrac{N_k}{A_{tot}} + \dfrac{M_k}{I_y}\cdot e_1\right)^2 + 3\cdot\left(\dfrac{V_k}{h_w\cdot t_w}\right)^2}{f_{yd}^{\,2}}} & \text{if } \left(\dfrac{N_k}{A_{tot}} + \dfrac{M_k}{I_y}\cdot e_1\right)^2 + 3\cdot\left(\dfrac{V_k}{h_w\cdot t_w}\right)^2 \geq \left(\dfrac{N_k}{A_{tot}} + \dfrac{M_k}{I_y}\cdot e_5\right)^2 + 3\cdot\left(\dfrac{V_k}{h_w\cdot t_w}\right)^2 \\ \sqrt{\dfrac{\left(\dfrac{N_k}{A_{tot}} + \dfrac{M_k}{I_y}\cdot e_5\right)^2 + 3\cdot\left(\dfrac{V_k}{h_w\cdot t_w}\right)^2}{f_{yd}^{\,2}}} & \text{if } \left(\dfrac{N_k}{A_{tot}} + \dfrac{M_k}{I_y}\cdot e_5\right)^2 + 3\cdot\left(\dfrac{V_k}{h_w\cdot t_w}\right)^2 > \left(\dfrac{N_k}{A_{tot}} + \dfrac{M_k}{I_y}\cdot e_1\right)^2 + 3\cdot\left(\dfrac{V_k}{h_w\cdot t_w}\right)^2 \end{cases}$$

$u_{sls.sp} = 0.73$

Plooicontrole hoofdliggerlijf conform Eurocode
Gemeentewerken
Gemeente Rotterdam
Ingenieursbureau

Flange induced buckling

$k := 0.55$

Flange induced buckling (8)

$$A_f := \begin{cases} \left(b_{foL}\cdot t_{foL} + b_{foR}\cdot t_{foR}\right) & \text{if } b_{foL}\cdot t_{foL} + b_{foR}\cdot t_{foR} \geq b_{fbL}\cdot t_{fbL} + b_{fbR}\cdot t_{fbR} \\ \left(b_{fbL}\cdot t_{fbL} + b_{fbR}\cdot t_{fbR}\right) & \text{if } b_{fbL}\cdot t_{fbL} + b_{fbR}\cdot t_{fbR} \geq b_{foL}\cdot t_{foL} + b_{foR}\cdot t_{foR} \end{cases}$$

$$UC := \frac{\dfrac{h_t - \dfrac{t_{fbmax}}{2} - \dfrac{t_{fomax}}{2}}{t_w}}{\left[k\cdot\dfrac{E_d}{f_{yd}}\cdot\sqrt{\dfrac{\left(h_t - \dfrac{t_{fbmax}}{2} - \dfrac{t_{fomax}}{2}\right)\cdot t_w}{A_f}}\right]}$$

$UC = 0.69$ Toets <1,0

Flange induced buckling

Torsional stiffness (Stiffener 1)

Minimum requirements for longitudinal stiffeners (9.2.2)

$I_{T.v1} := \frac{1}{3} \cdot t_{v1}^3 \cdot b_{v1}$ $\quad I_{T.v1} = 128053.33 \cdot mm^4$

$I_{y.v1} := \frac{1}{12} \cdot t_{v1} \cdot b_{v1}^3$ $\quad I_{y.v1} = 3201333.33 \cdot mm^4$

$I_{z.v1} := \frac{1}{12} \cdot t_{v1}^3 \cdot b_{v1}$ $\quad I_{z.v1} = 32013.33 \cdot mm^4$

$I_{p.v1} := I_{y.v1} + I_{z.v1} + t_{v1} \cdot b_{v1} \cdot \left(\frac{b_{v1}}{2}\right)^2$ $\quad I_{p.v1} = 12837346.67 \cdot mm^4$

$UC := 5.3 \cdot \frac{f_{yd}}{E_d} \cdot \frac{I_{p.v1}}{I_{T.v1}}$ $\quad UC = 0.59$ Toets <1,0. als het niet voldoet kan met de actuele spanning worden gerekend ipv de vloeispanning volgens Beg, et al. Design of Plated Structures (p139)

$\sigma_{st1} := \frac{N_{ed}}{A_{a.eff}} + \frac{M_{ed}}{I_{y.m.eff}} \cdot (e_{b.m.eff} - h_{v1})$ $\quad \sigma_{st1} = 142.96 \cdot \frac{N}{mm^2}$

$UC := 5.3 \cdot \frac{\sigma_{st1}}{E_d} \cdot \frac{I_{p.v1}}{I_{T.v1}}$ $\quad UC = 0.36$ Toets <1,0

Torsional stiffness (Stiffener 1)

Torsional stiffness (Stiffener 2)

Minimum requirements for longitudinal stiffeners (9.2.2)

$I_{T.v2} := \frac{1}{3} \cdot t_{v2}^3 \cdot b_{v2}$ $\quad I_{T.v2} = 3.33 \times 10^{-13} \cdot mm^4$

$I_{y.v2} := \frac{1}{12} \cdot t_{v2} \cdot b_{v2}^3$ $\quad I_{y.v2} = 8.33 \times 10^{-14} \cdot mm^4$

$I_{z.v2} := \frac{1}{12} \cdot t_{v2}^3 \cdot b_{v2}$ $\quad I_{z.v2} = 8.33 \times 10^{-14} \cdot mm^4$

$I_{p.v2} := I_{y.v2} + I_{z.v2} + t_{v2} \cdot b_{v2} \cdot \left(\frac{b_{v2}}{2}\right)^2$ $\quad I_{p.v2} = 4.17 \times 10^{-13} \cdot mm^4$

$UC := 5.3 \cdot \frac{f_{yd}}{E_d} \cdot \frac{I_{p.v2}}{I_{T.v2}}$ $\quad UC = 0.01$ Toets <1,0. als het niet voldoet kan met de actuele spanning worden gerekend ipv de vloeispanning volgens Beg, et al. Design of Plated Structures (p139)

$\sigma_{st2} := \frac{N_{ed}}{A_{a.eff}} + \frac{M_{ed}}{I_{y.m.eff}} \cdot (e_{b.m.eff} - h_{v2})$ $\quad \sigma_{st2} = 116.92 \cdot \frac{N}{mm^2}$

$UC := 5.3 \cdot \frac{\sigma_{st2}}{E_d} \cdot \frac{I_{p.v2}}{I_{T.v2}}$ $\quad UC = 0.00$ Toets <1,0

Torsional stiffness (Stiffener 2)

Controle langsverstijving volgens NEN6771