Ultra High Performance Concrete in Large Span Shell Structures

Calculations Appendix

R.N. ter Maten

Calculations

Calc. 8.1

Sagitta to span ratio

8.1.1. Model

The model is set up as a monolith dome. For these calculations the next parameters hold:

Parameters:	
Span:	d = 150m
Thickness:	t = 100, 200, 300, 400
Supports:	All hinged
Loads:	According to Chapter 5 Load combinations and general vertical load

Variables:	
Sagitta to Span - ratio:	1/2 to 1/10
Design limitations:	
Buckling:	Linear elastic calculation
Stress:	Design capacities UHPC

A figure of a model, with d/s = 4, is presented below:

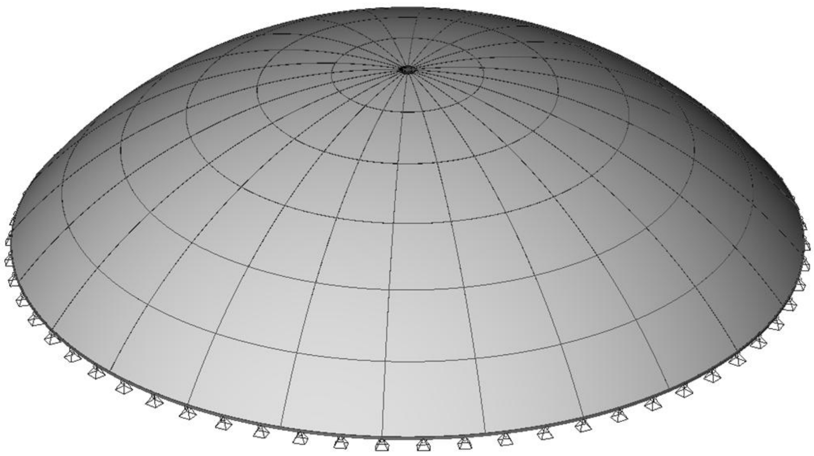
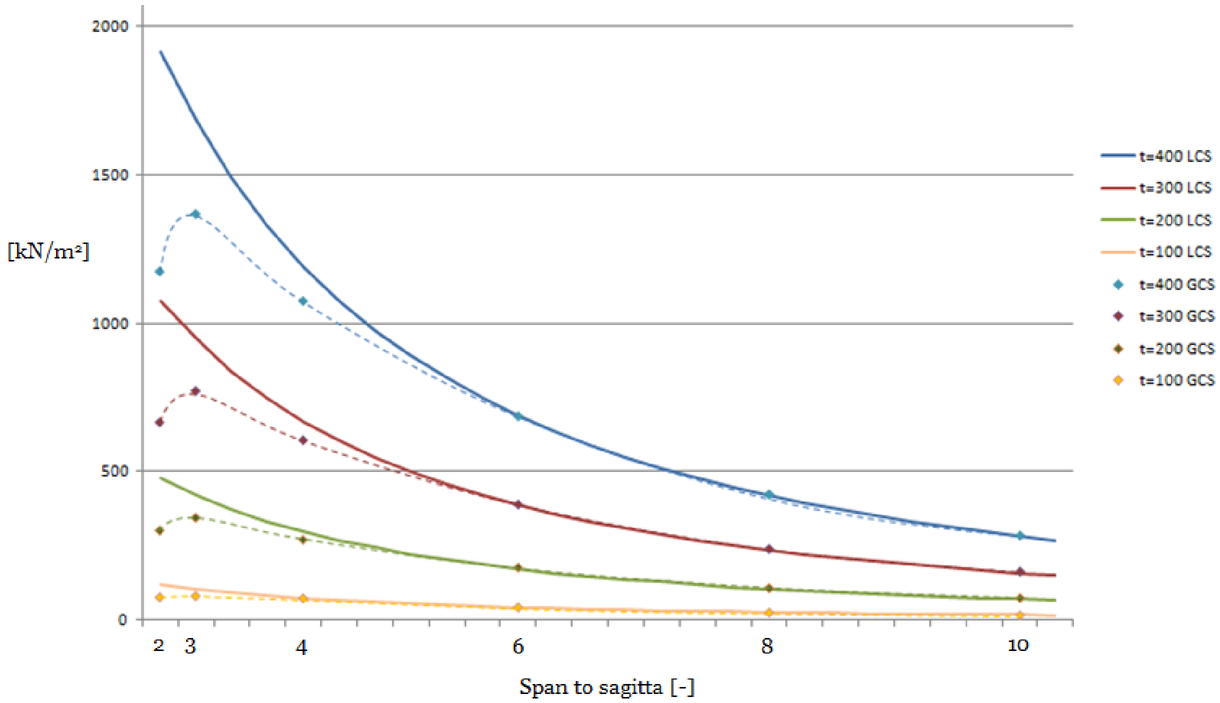


Fig.Calc.7.1.1 FEM-model

8.1.2. Buckling

	Critical perpendicular load [kN/m²]				FEM-result vertical load [kN/m²]				γ –factor
Thickness:	100	200	300	400	100	200	300	400	
Span to sagitta:									
2	120	478	1076	1914	78	300	664	1176	0.627
3	102	408	917	1631	82	336	745	1334	0.814
4	77	306	689	1225	75	276	611	1125	0.919
6	43	172	388	689	45	175	388	688	1.00
8	26	106	238	424	26	106	238	424	1.00
10	18	71	159	283	19	72	162	286	1.00



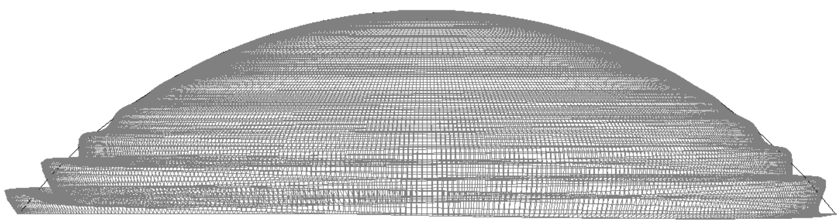
Formula test:

For this calculation the next parameters are chosen:
d/s = 4
d = 150m
R = 93,75m
t =

$$t^2 = (1,2 \cdot 2.75 + 1,5 \cdot 0.45) \cdot \frac{93.75^2 \cdot 6}{58 \cdot 10^6 \cdot 0.919} \cdot \frac{\sqrt{3(1-0^2)}}{2}$$

$$t \geq 58mm$$

t = 58mm



d / s = 4
t = 58mm
P_{vert} = 3,98kN/m²
λ ≈5,88

8.1.3. Calculation of shell thickness

<u>Span to sagitta: 04</u>

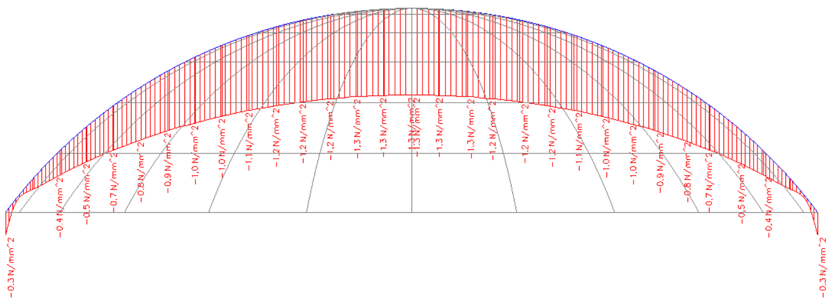
For this calculation the next parameters hold:
Diameter d: 150m
Sagitta: 150 / 4 = 37,5m
Material: Ductal
Supports: All hinged

The stresses (in N/mm²) due to dead load of the shell are independent of the shell thickness. Other effect are given independent of the thickness when charted in (N/mm).
All maximum and minimum values are found on the x-axis as a result of wind loading in x-direction. This does not hold for meridional stress in case of wind loading. The maximum tension is found at an angle of 50° of the x-axis.

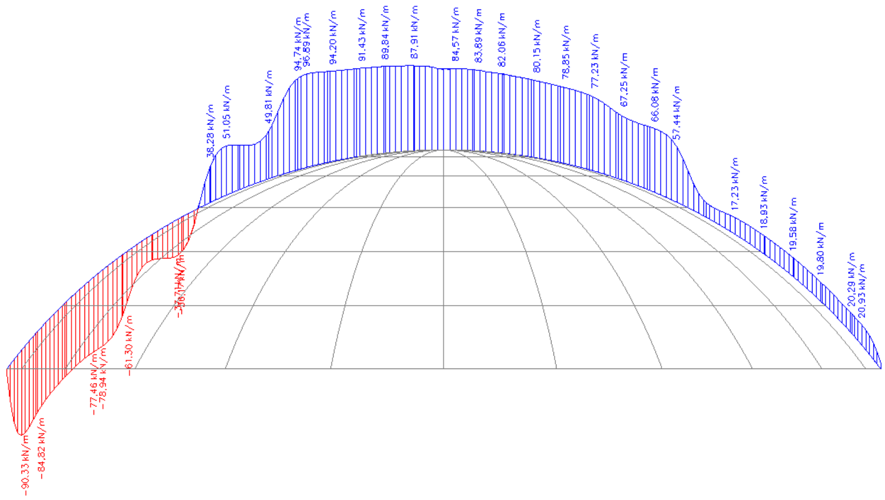
<u>Internal Force distribution</u>
Circumferential stress Nx

	LC Dead load (N/mm2)	LC Wind x (N/mm)	LC Snow. Evenly (N/mm)	LC Snow. Redistr. (N/mm)
Top	-1.3	84.6	-21.0	-16.5
Bottom	-0.3	-90.3 / 20.9	-5.3	-3.1
Max	-	**96.9**	-	6.1
Min	-	-90.3	-	-20.4

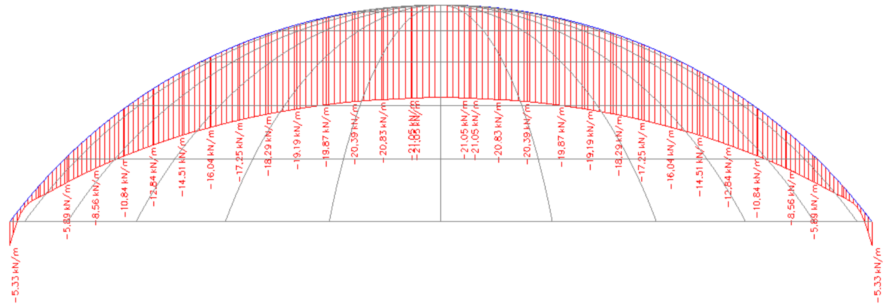
<u>SLS Permanent Load. Stress [N/mm²]</u>



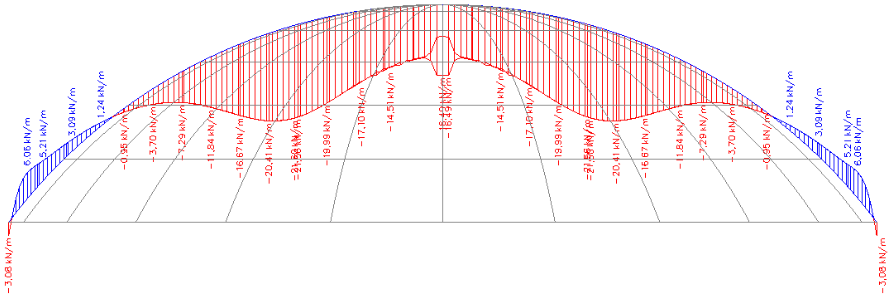
<u>Load cases:</u>
Load case 3: wind x. Force per unit length [N/mm]



Load case 4: evenly distributed snow. Force per unit length [N/mm]

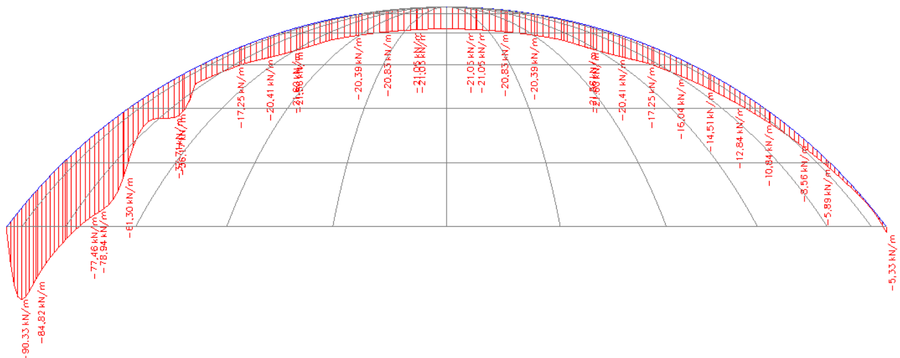


Load case 5: redistributed snow. Force per unit length [N/mm]

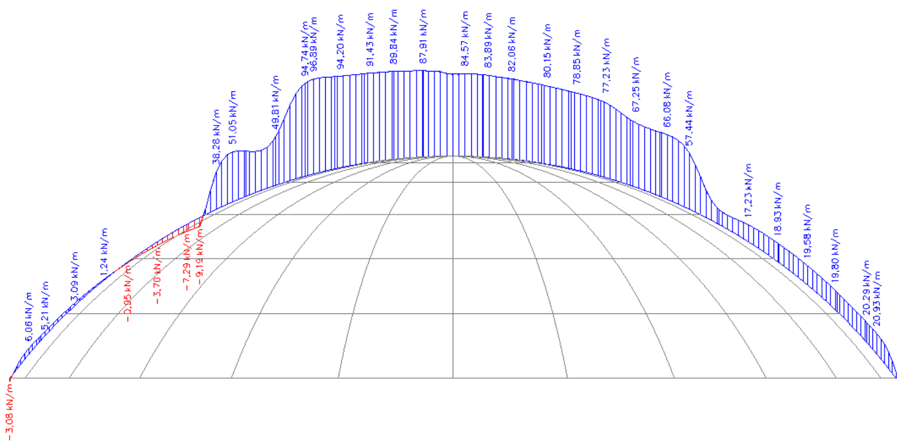


<u>Envelop Circumferential stress Nx for load cases</u>

Minimum:



Maximum:



Required thickness

The required thickness is calculated with the maximum occurring tensile and compression due to the independent load cases. The capacity is lowered by the estimated value of the stress due to permanent load at the location of the peak value.

Compression:

$$\frac{f_{d,loadcases}\left[N/mm\right]}{f'_b\left[N/mm^2\right]-f_{d,permanent}\left[N/mm^2\right]}=t\left[mm\right]$$

Min. required concrete thickness: $\frac{1{,}5(-90.3)}{-117{,}7-1.2(-0.10)}=1.2mm$

Tension:

$$\frac{f_{d,loadcases}\left[N/mm\right]}{f_b\left[N/mm^2\right]-f_{d,permanent}\left[N/mm^2\right]}=t\left[mm\right]$$

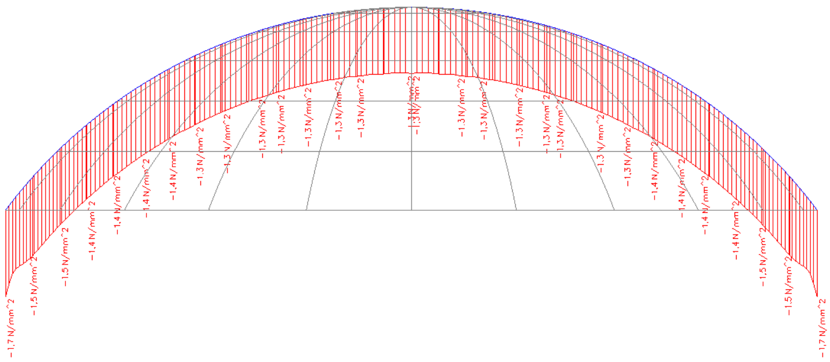
Min. required concrete thickness: $\frac{1{,}5(96.89)}{8-0{,}9(-1.20)}=27mm$

<u>Internal Force distribution</u>
Meridional stress Ny

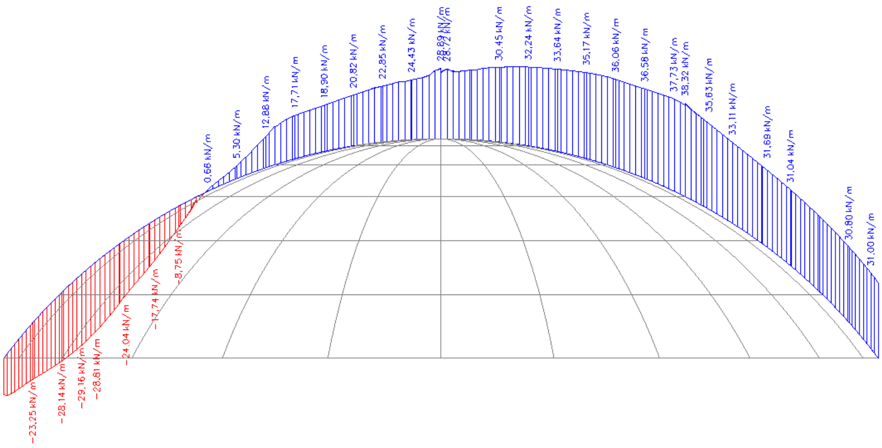
	LC Dead load (N/mm2)	LC Wind x (N/mm)	LC Snow. Evenly (N/mm)	LC Snow. Redistr. (N/mm)
Top	-1.3	64.13*	-21.1	-17.2
Bottom	-1.7	-23.3 / 48.9*	-26.5	-15.4
Max	-	99.8	-	-
Min	-	-29.2	-	-1.2

*Not found in wind-direction

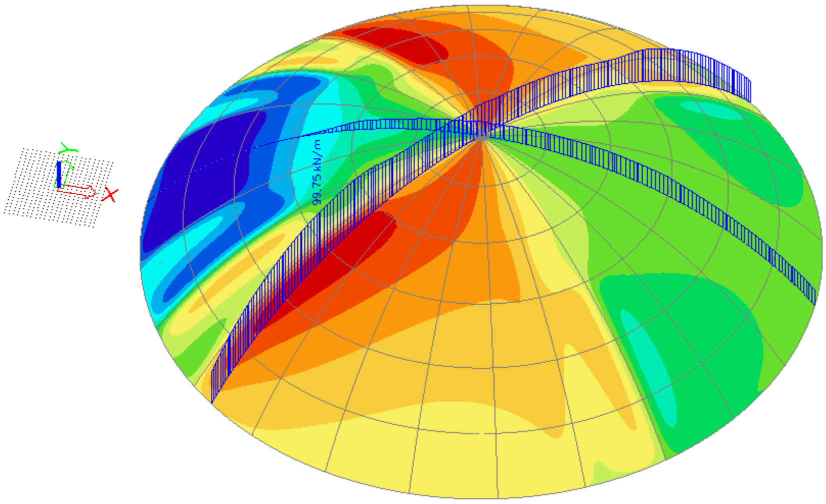
SLS Permanent Load. Stress in N/mm²



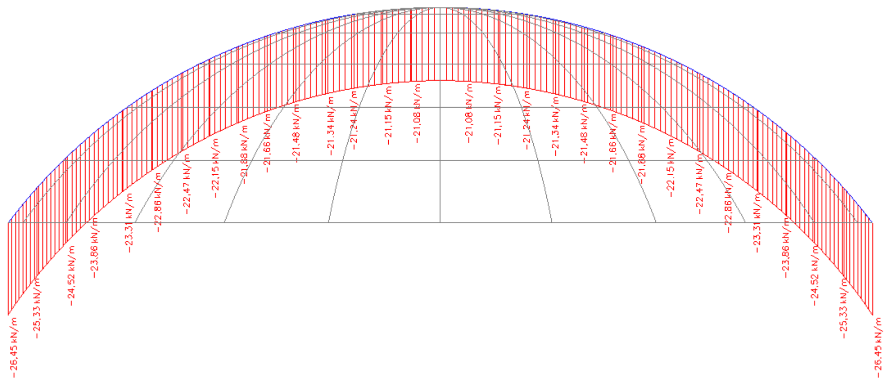
Load case 3: wind x. Force per unit length [N/mm]
The maximum compression is found by:



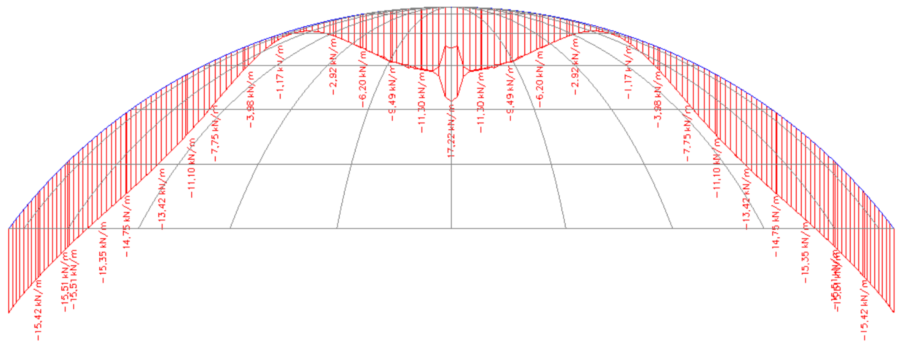
The maximum tension is found by:



Load case 4: evenly distributed snow. Force per unit length [N/mm]

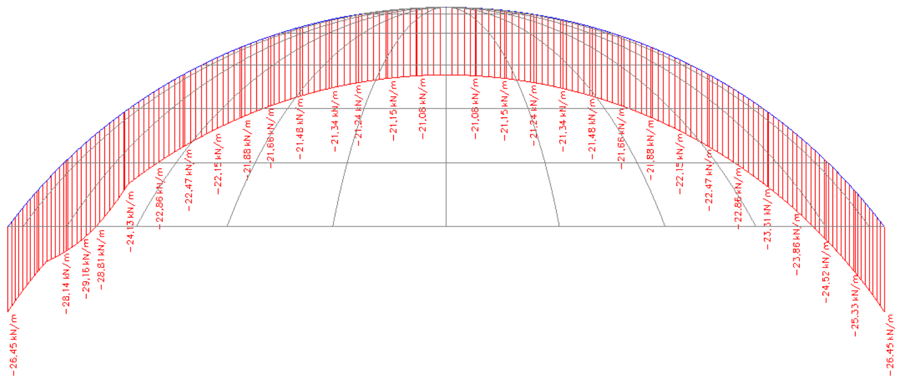


Load case 5: redistributed snow. Force per unit length [N/mm]

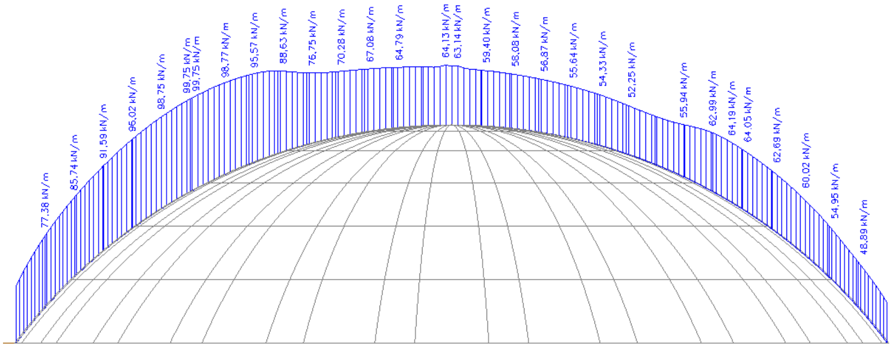


Envelop Circumferential stress Ny for load cases

Minimum:



Maximum:
*Not in wind-direction



Required thickness

The required thickness is calculated with the maximum occurring tensile and compression due to the independent load cases. The capacity is lowered by the estimated value of the stress due to permanent load at the location of the peak value.

Compression:

$$\frac{f_{d,loadcases}\left[N/mm\right]}{f_b\left[N/mm^2\right]-f_{d,permanent}\left[N/mm^2\right]}=t\left[mm\right]$$

Min. required concrete thickness: $\frac{1{,}5(-29.16)}{-117.7-0.9(-1.40)}=0.4mm$

Tension:

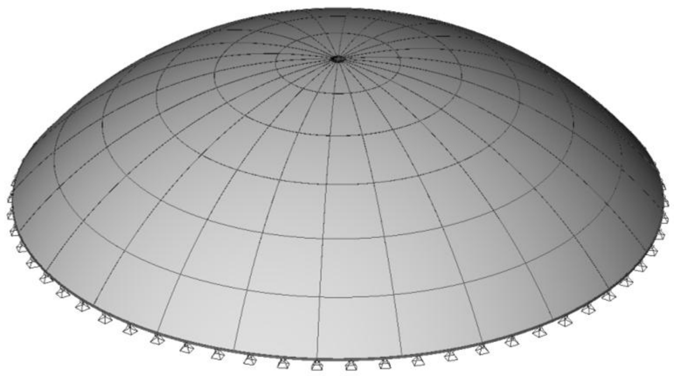
$$\frac{f_{d,loadcases}\left[N/mm\right]}{f_b\left[N/mm^2\right]-f_{d,permanent}\left[N/mm^2\right]}=t\left[mm\right]$$

Min. required concrete thickness: $\frac{1{,}5(99.75)}{4.3-0{,}9(-1.40)}=27mm$

Calc. 8.2.

Edge ring

8.2.1. Model



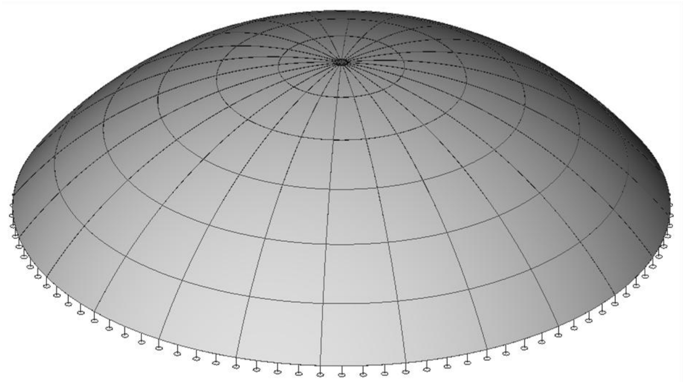


Fig.Calc.7.1.1 FEM-model

Parameters:	
Span:	d = 150m
Sagitta:	= ¼ *150 = 37.5m
Overall thickness:	t = 60mm
Supports:	All hinged & All rolled plus edge ring
Loads:	General vertical load

Variables:	
Edge ring	Dimensions Material Prestress
Design limitations:	
Buckling:	Linear elastic calculation

8.2.2. Results

With the model with hinged supports the horizontal reaction forces for the SLS permanent load are determined.

Horizontal reaction (pressure):	Permanent load (SLS)	61.0 kN/m
	Normative combination Snow total (SLS)	76.9 kN/m

It is seen that, for a shell with a thickness of 60mm, the contribution to the ring pressure of the dead load is approximately 80%.
The required prestress force is determined by $N = Q \cdot r_o = 61.0 \cdot 75 = 4575 kN$

It is reasoned that this indication is the required prestress force which causes the displacements due to permanent load in SLS to be restricted to virtually zero. The choice for this prestress force is based on this restriction.

Now the effect on the effect on the buckling load of the shell depends on the stiffness of the edge ring. The results are presented below.

Material	Cross-section [h x w]	Buckling load factor (β)	Buckling load
C90/105	500 x 500	0,54	14.8
C90/105	600 x 600	0.67	18.4
C90/105	750 x 750	0.82	22.5
C90/105	850 x 850	0.86	23.6
C90/105	1000 x 1000	0.94	25.7
C90/105	1500 x 1500	1.00	27.4
C90/105	2000 x 2000	1.00	27.4

Calc. 8.3.

Ribs & Stiffeners

8.3.1. Model

The model is set up as a ribbed dome, as described in chapter 7.5. For these calculations the next parameters hold:

Parameters:	
Span:	d = 150m
Sagitta:	= ¼ *150 = 37.5m
Overall thickness:	t = 60mm
Supports:	All hinged
Loads:	Buckling load
Width ribs & stiffeners:	60mm

Variables:	
Thickness distribution:	According to table Dx.
Design limitations:	
Buckling:	Linear elastic calculation

Figure of the model is presented below:

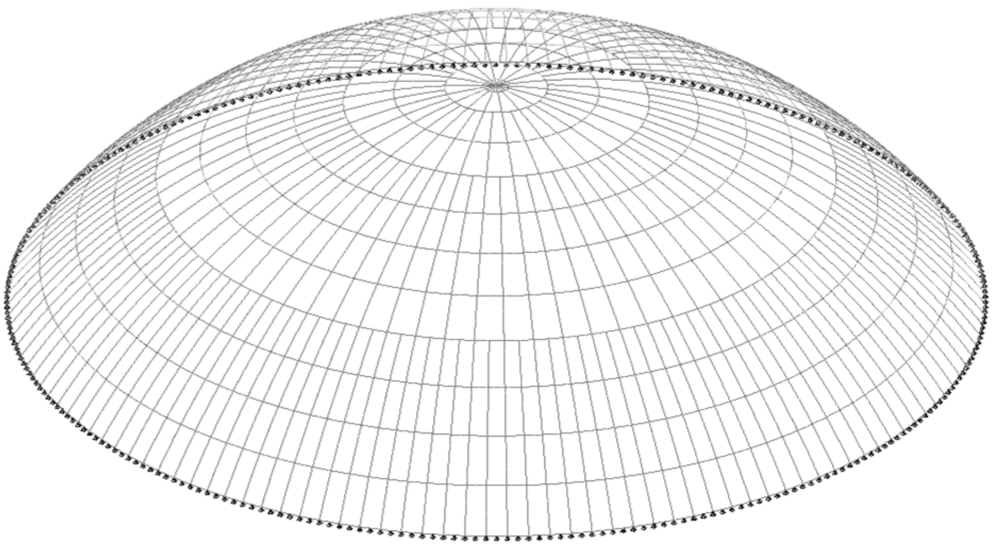
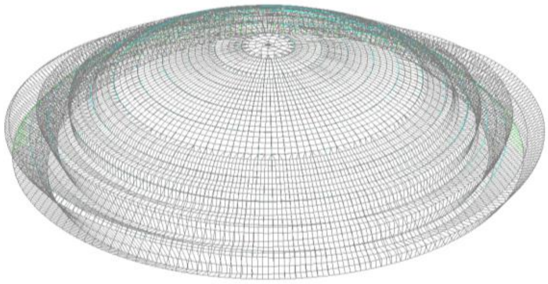
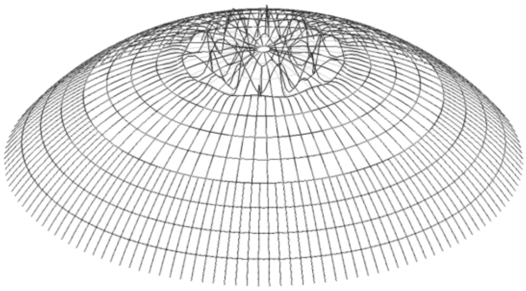
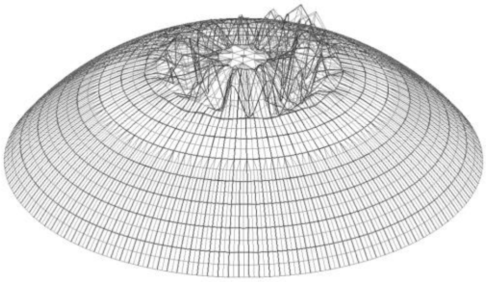
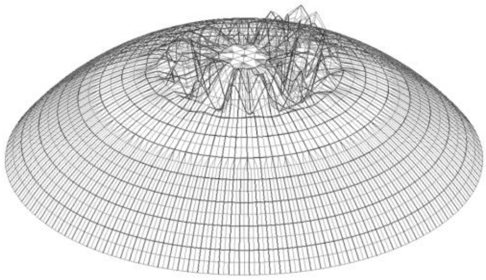
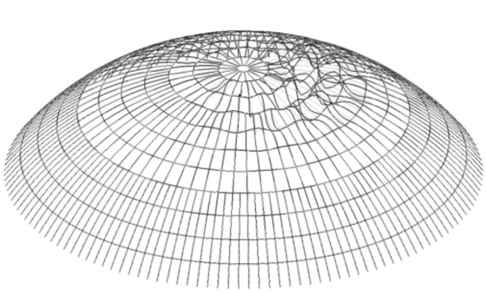
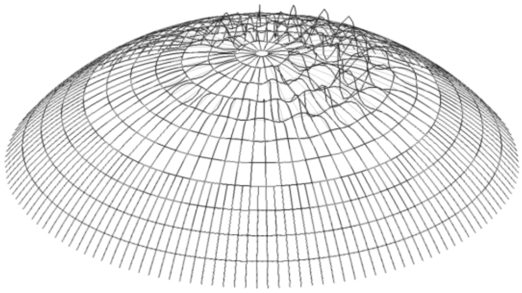
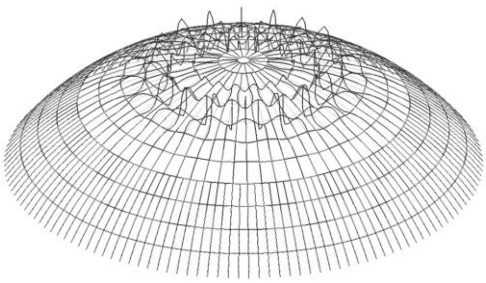
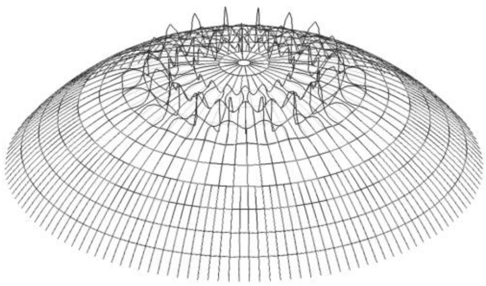
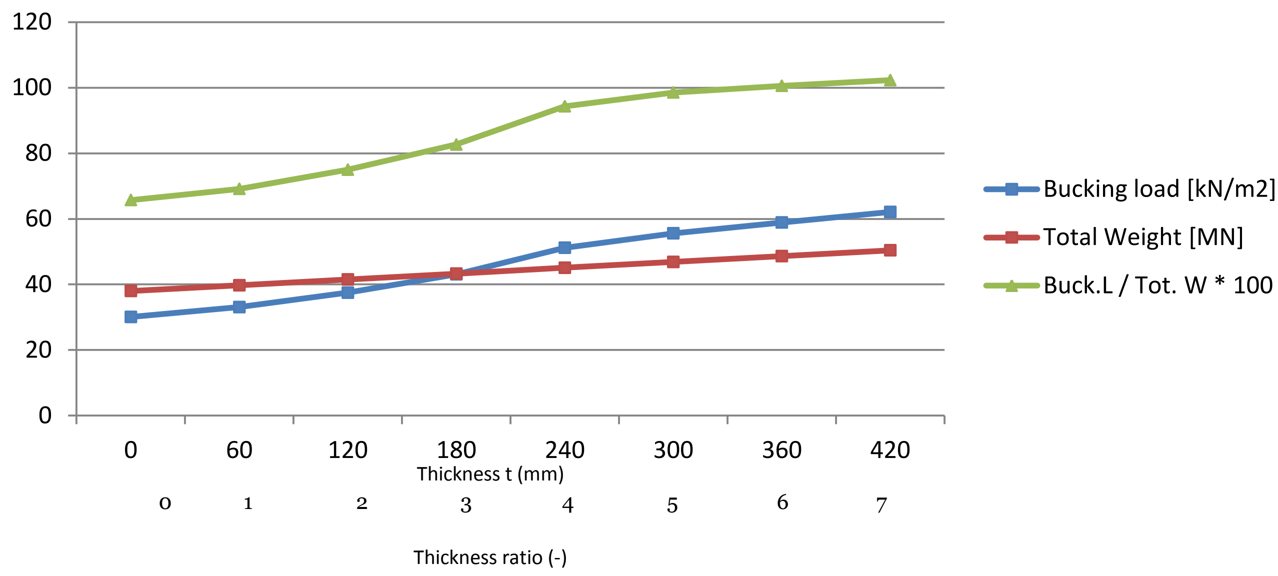


Fig.Calc.7.3.1 FEM-model for calculations [Elements & Ribs]

8.3.2. Results

Case 0. t = 60mm. h = 0mm P = 25 kN/m^2	Case 1. t = 60mm. h = 60mm P = 27.5 kN/m^2
	
Case 2. t = 60mm. h = 120mm P = 31.1 kN/m^2	Case 3. t = 60mm. h = 180mm P = 35.8 kN/m^2
	
Case 4. t = 60mm. h = 240mm P = 42.5 kN/m^2	Case 5. t = 60mm. h = 300mm P = 46.2 kN/m^2
	
Case 6. t = 60mm. h = 360mm P = 48.9 kN/m^2	Case 7. t = 60mm. h = 420mm P = 51.6 kN/m^2
	

Results; Comparison to material use



Shell thickness [mm]	Ribs & Stiffeners [mm]	Buckling load [kN/m²]	Weight [MN]	Ratio [Buck.L / W * 100]
60	0	25	38,0	65,8
	60	27,5	39,8	69,2
	120	31,1	41,5	75,0
	180	35,8	43,3	82,7
	240	42,5	45,1	94,3
	300	46,2	46,9	98,5
	360	48,9	48,6	100,6

To see the effect of the ratio for other shell thickness multiple configurations are iteratively tested:

Shell thickness [mm]	Ribs & Stiffeners [mm]	Buckling load [kN/m²]	Safety factor LC5	Weight [MN]
30	240	14.5	5.9	28.8
30	360	16.6	6.8	32.4
35	150	16.1	5.7	28.9
35	180	17.5	6.2	29.8
40	100	17.1	6.1	30.1
40	120	18.1	6.3	30.7
50	120	26.8	8.5	36.1
50	150	29.2	8.9	37.0

The chosen configuration is compared to a shell with constant thickness to illustrate the positive effect of rib stiffening

Shell thickness [mm]	Ribs & Stiffeners [mm]	Buckling load [kN/m²]	Safety factor LC5	Weight [MN]
35	180	17.5	6.2	29.8
44	0	13.8	4.3	29.4
		27% increase	44% increase	

Calc. 8.4

Edge thickness

8.4.1. Model

The model is set up as a monolith dome. For these calculations the next parameters hold:

Parameters:	
Span:	d = 150m
Sagitta:	= ¼ *150 = 37.5m
Overall thickness:	t = 60mm
Loads:	According to Chapter 5

Variables:	
Thickness distribution:	According to tables appendix Calculations 8.4
Design limitations:	
Buckling:	Linear elastic calculation

A figure of the model is presented below:

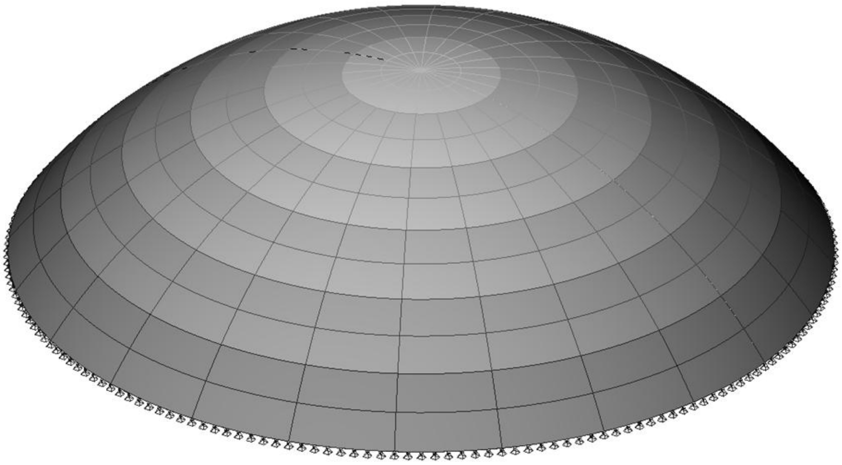


Fig. Calc4.1 FEM-model

8.4.2. Results

The increase of material use is first set to an extra of 60mm, which is equally spread over multiple variants. This implies an overall material increase of approximately 9,1%.

	Case 0	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8
t1	60	120	90	80	75	72	70	68,6	67,5
t2	60	60	90	80	75	72	70	68,6	67,5
t3	60	60	60	80	75	72	70	68,6	67,5
t4	60	60	60	60	75	72	70	68,6	67,5
t5	60	60	60	60	60	72	70	68,6	67,5
t6	60	60	60	60	60	60	70	68,6	67,5
t7	60	60	60	60	60	60	60	68,6	67,5
t8	60	60	60	60	60	60	60	60	67,5
t9	60	60	60	60	60	60	60	60	60
t10	60	60	60	60	60	60	60	60	60
t11	60	60	60	60	60	60	60	60	60

	Case0	Case1	Case2	Case3	Case4	Case5	Case6	Case7	Case8
Buck.load [kN/m²]	28.6	31.3	32.0	32.6	32.9	32.9	32.9	32.9	32.9
Relative increase [%]		9,4	11,9	14,0	15,0	15,0	15,0	15,0	15,0

Tab.Calc4.1 Results for material increase of 60mm

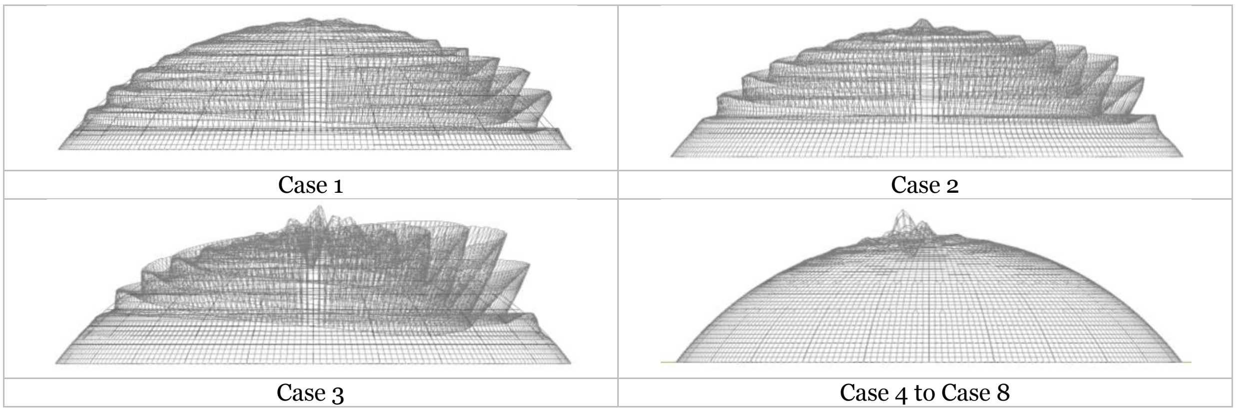


Fig. Calc4.2 Buckling mode for Case 1 to Case 8

Optimization

The conclusion, stating that which was a result of the calculation above is tested for other configurations. Again the principle of a material increase is applied and its held constant, meaning the extra material is spread out over a number of elements.

	Case 0	Case 3x1	Case 4x1	Case 5x1		Case 3x2	Case 4x2	Case 5x2		Case 3x3	Case 4x3	Case 5x3
t1	60	70	67,5	66		67,5	65,6	64,5		65	63,8	63
t2	60	70	67,5	66		67,5	65,6	64,5		65	63,8	63
t3	60	70	67,5	66		67,5	65,6	64,5		65	63,8	63
t4	60	60	67,5	66		60	65,6	64,5		60	63,8	63
t5	60	60	60	66		60	60	64,5		60	60	63
t6	60	60	60	60		60	60	60		60	60	60
t7	60	60	60	60		60	60	60		60	60	60
t8	60	60	60	60		60	60	60		60	60	60
t9	60	60	60	60		60	60	60		60	60	60
t10	60	60	60	60		60	60	60		60	60	60
t11	60	60	60	60		60	60	60		60	60	60

	Case 0	Case 3x1	Case 4x1	Case 5x1		Case 3x2	**Case 4x2**	Case 5x2		Case 3x3	Case 4x3	Case 5x3
Buck.load [kN/m²]	28.6	32,4	32,9	32,9		32,3	**32,9**	32,9		32,2	32,0	31,3
Relative increase [%]		13,3	15,0	15,0		12,9	**15,0**	15,0		12,6	11, 9	09,4

Tab.Calc4.2 Results for material optimization

Calc. 8.5.

Connection requirements

8.5.1. Model

The model is set up as a ribbed dome, as described in chapter 7.5. For these calculations the next parameters hold:

Parameters:		Variables:
Span:	d = 150m	-
Sagitta to Span - ratio:	1/4	
Loads:	According to Chapter 5	

Figures of the model are presented below:

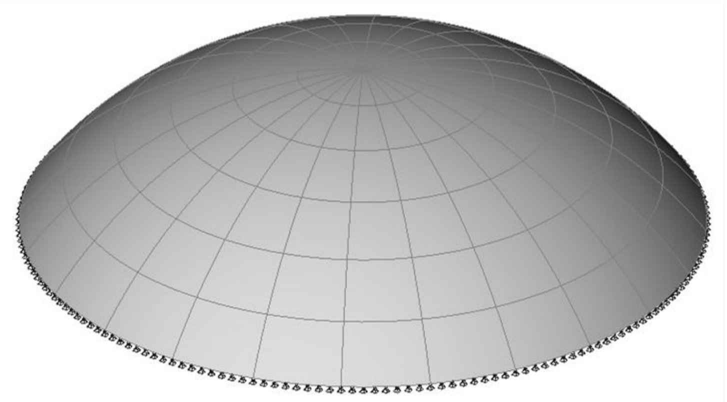
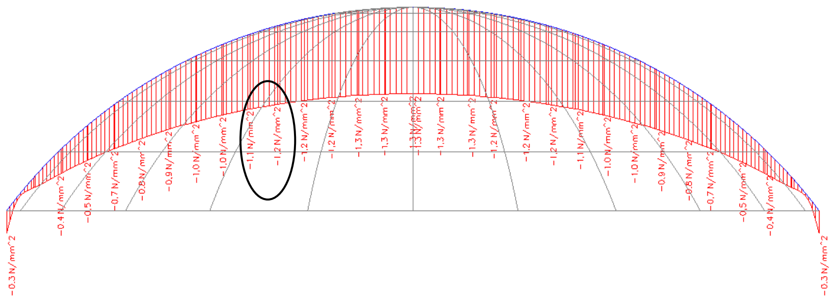


Fig. Calc.5.1. FEM-model for joint calculations

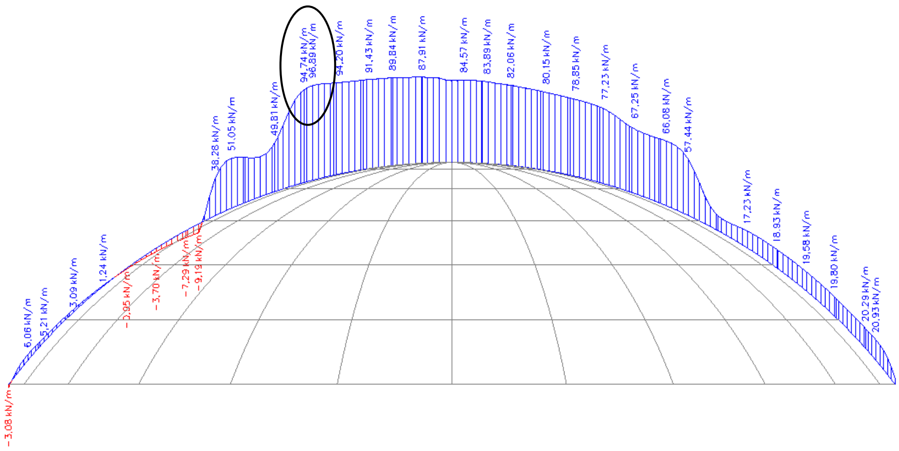
8.5.2. Results

The maximum tensile stresses are found by the calculations of paragraph 7.1.

<u>Circumferential direction:</u>
SLS Permanent Load. Stress [N/mm²]



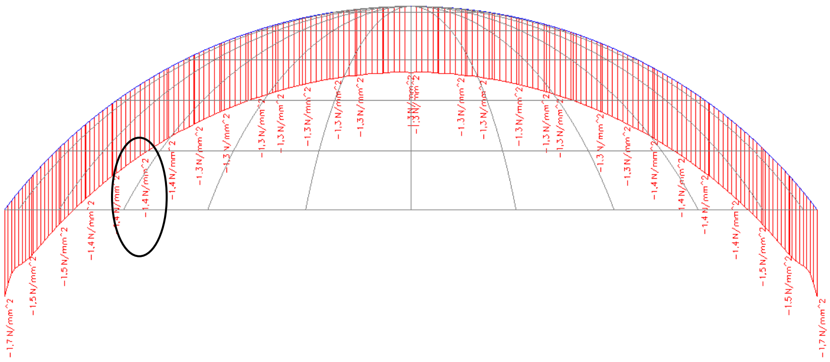
Maximum:



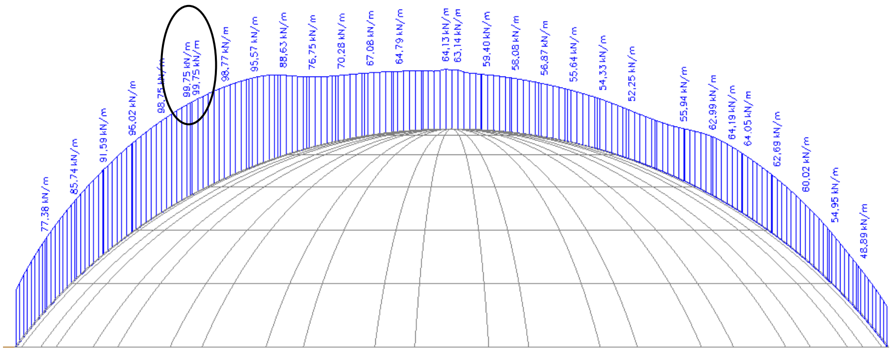
Max. tensile stress:	96.9 N/mm
Stress due to dead load:	1.2 N/mm²

$$t_{req}: \quad \frac{1{,}5 \cdot 96{,}9}{0{,}9 \cdot 1{,}2} = 135\text{mm}$$

<u>Meridional direction:</u>
SLS Permanent Load. Stress in N/mm²



Maximum:
*Not in x-direction



Max. tensile stress: 99.8 N/mm
Stress due to dead load: 1.3 N/mm²

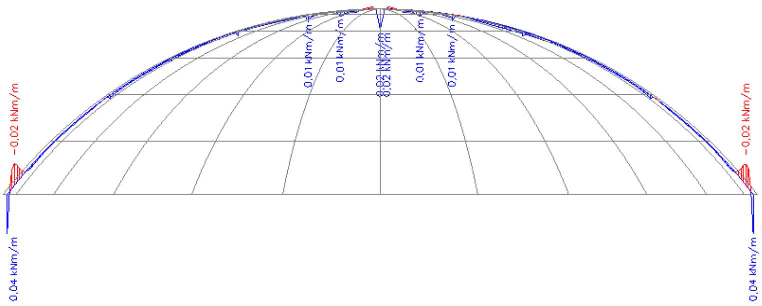
t_{req}: $$\frac{1{,}5 \cdot 99{,}8}{0{,}9 \cdot 1{,}3} = 128\text{mm}$$

If the shell thickness is not increased, for a solid shell of 60mm it holds that the tensile stress are within the order of magnitude:

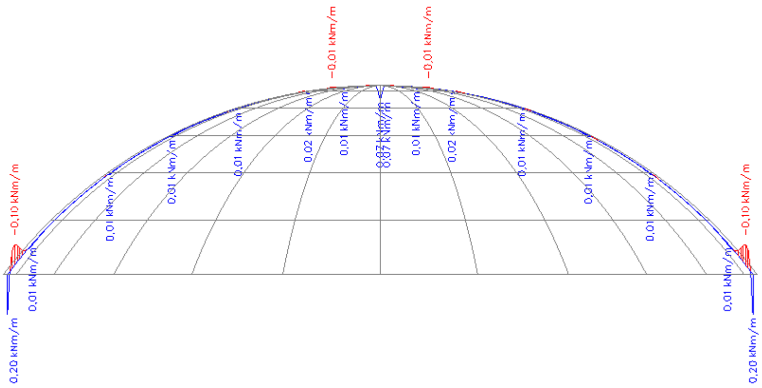
$$\sigma_{t,meridional} = \frac{1{,}5 \cdot 99{,}8}{60} - 0{,}9 \cdot 1{,}3 = 1{,}33\ \text{N/mm}^2$$

$$\sigma_{t,circumferential} = \frac{1{,}5 \cdot 96{,}9}{60} - 0{,}9 \cdot 1{,}2 = 1{,}34\ \text{N/mm}^2$$

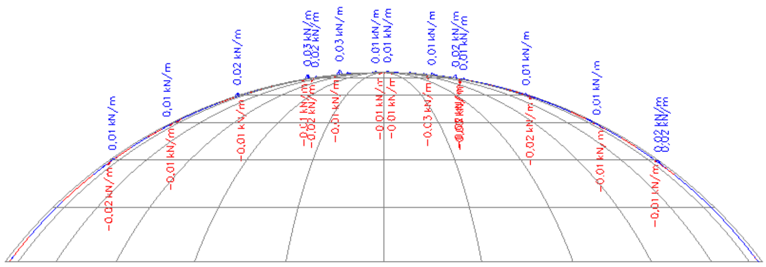
Moments ; mx
SLS Permanent Load. [kNm/m]



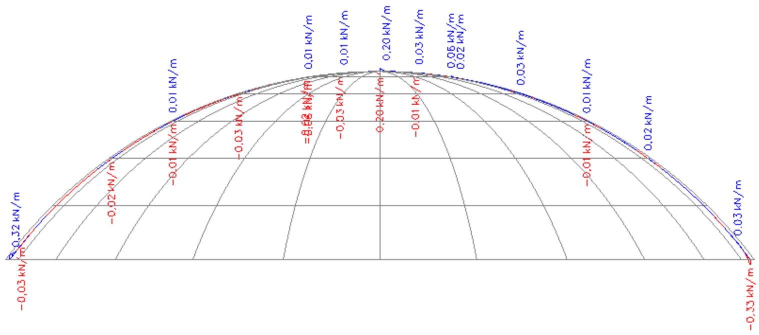
Moments; my
SLS Permanent Load. [kNm/m]



Shear; vx
SLS Permanent Load. [kNm/m]

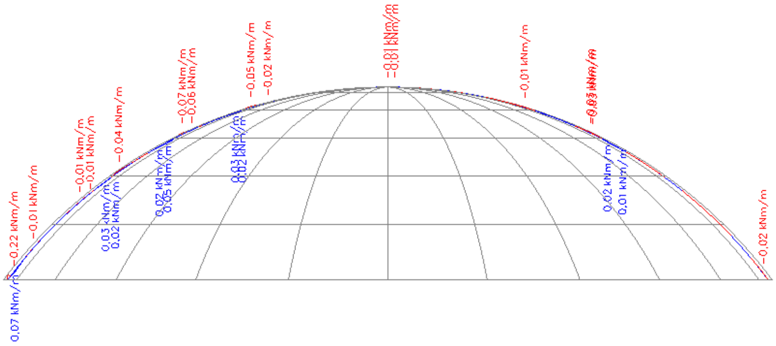


Shear; vx
SLS Permanent Load. [kNm/m]



Moments ; mx

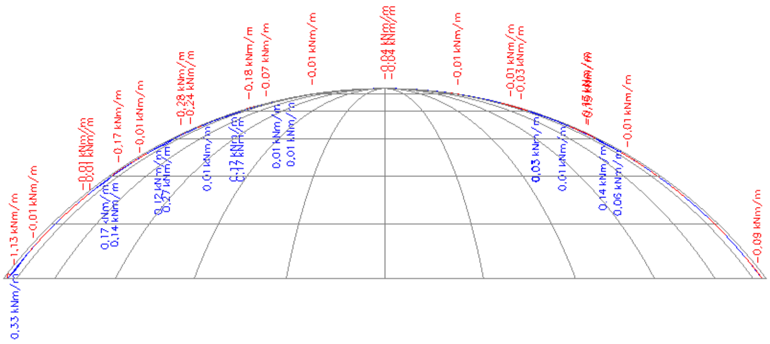
Normative Load case: Load case 3: wind x. Force per unit length [N/mm]



*Not in wind-direction

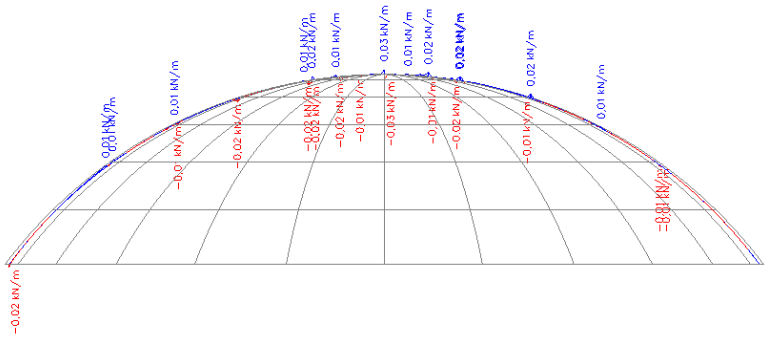
Moments; my

Normative Load case: Load case 3: wind x. Force per unit length [N/mm]



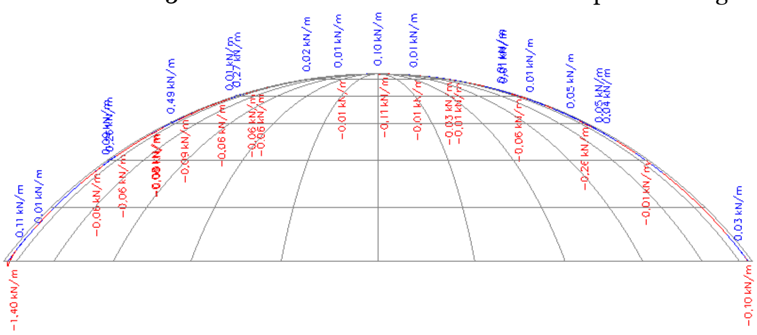
Shear; vx

Normative Load case: Load case 3: wind x. Force per unit length [N/mm]



Shear; vx

Normative Load case: Load case 3: wind x. Force per unit length [N/mm]



Calc. 8.6.

Dynamic response

8.6.1. Model

The model is set up as a monolith dome. For these calculations the next parameters hold:

Parameters:	
Span:	d = 150m
Thickness:	t = 60 mm
Sagitta to Span - ratio:	1/4
Support:	All hinged

Variables:	-
Design limitations:	Eigenfrequencies

A figure of the model is presented below:

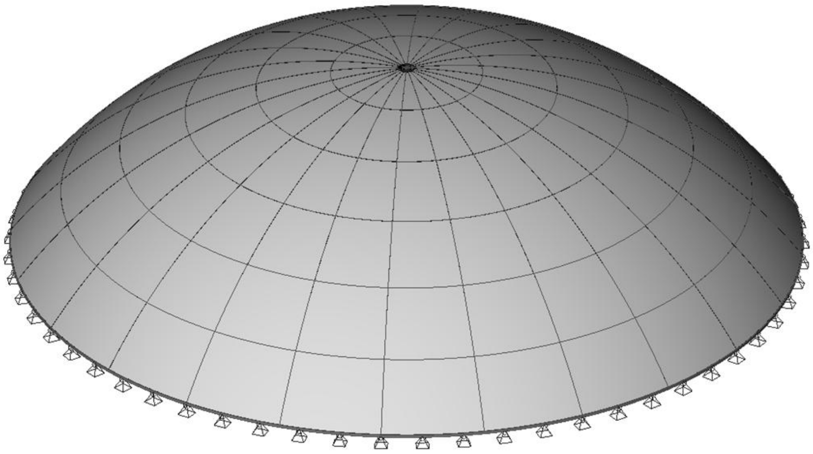


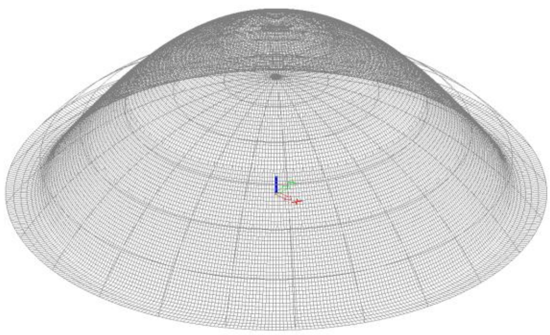
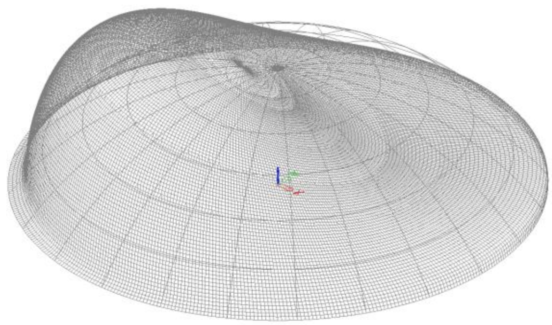
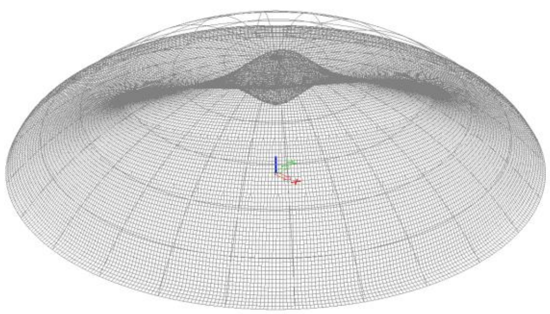
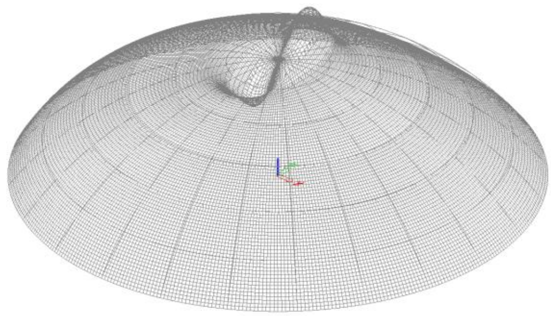
Fig.Calc.6.1 FEM-model for dynamic calculations

8.6.2. Results

<u>Eigenfrequencies Dead weight</u>

Natural frequency	f [Hz]	T [sec]
1	7,54	0.13
2	7,62	0.13
3	7,63	0.13
4	8,09	0.12

Vibration shapes

1	2
	
3	4
	

Calc. 8.7

Thermal response

8.7.1. Model

The model is set up as a monolith dome. For these calculations the next parameters hold:

Parameters:	
Span:	d = 150m
Thickness:	t = 60 mm
Sagitta to Span - ratio:	1/4
Thermal expansion:	11,8*10-6 m/m/°C
Loads:	Temperature loads NEN-EN 1991-1-5 Based on figure D29.

Variables:	Temperature distribution
Design limitations:	Stresses, deformations

A figure of the model is presented below:

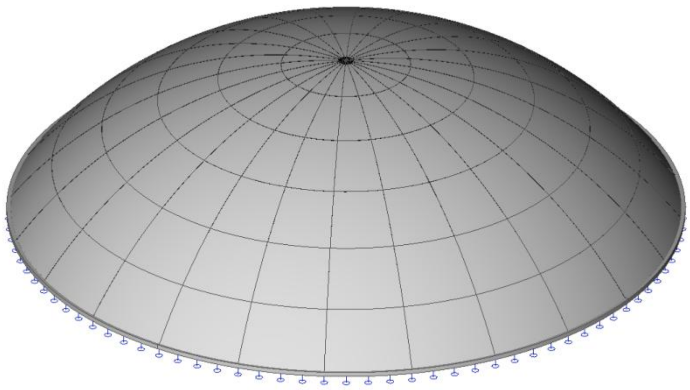
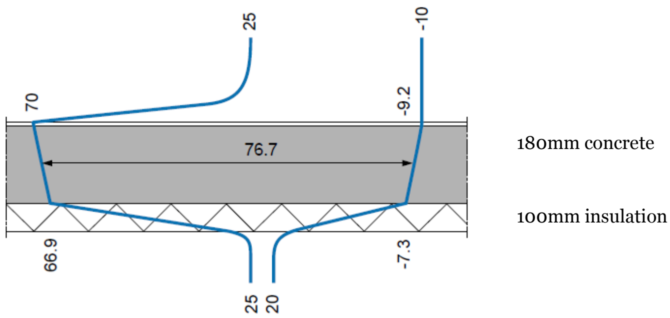


Fig. Calc7.1 FEM-model for calculations on thermal effects

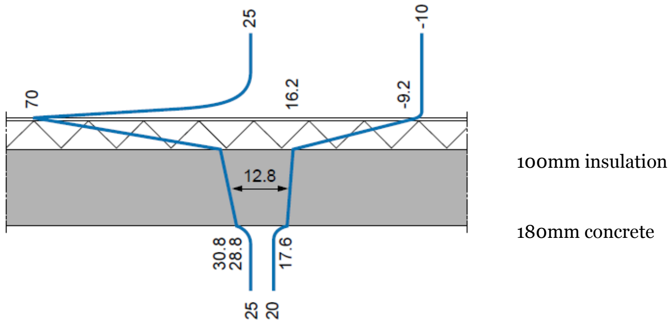
Season	Temperature	
	Indicative value [°C]	Extreme value [°C]
Summer – outside Indirect radiation	17	30
Direct radiation Bright color[a] Light color[b] Dark color[c]	 17 17 17	 50 60 75
Summer – inside	17	25
Winter – outside	4	-25
Winter – inside	17	20

a	White, yellow
b	Green, light-blue
c	Black, blue, red

Tab. Calc7.1. Temperatures for calculations on thermal effects [based on Eurocode 1991-1-5]



Concrete roof, insulation on top side



Concrete roof, insulation on bottom side

Fig. Calc7.2. Insulated concrete roof temperature progression
[Building physics, A.C. van der Linden]

Now, to present the effect of temperature on the shell five extreme load cases are examined, for both internal as external insulation. The temperature gradient over the shell cross section is modeled as 5 °C, temperature values are based on figure Calc7.2.

Interior insulation:

Case 1: Summer; extreme. Temperature distribution constant over shell surface
(T_o = 75 °C, T_i = 70 °C)

Case 2: Winter; extreme. Temperature distribution constant over shell surface
(T_o = -25 °C, T_i = -20 °C)

Case 3: Summer; extreme. Estimated realistic temperature distribution over shell surface by figure Calc7.2

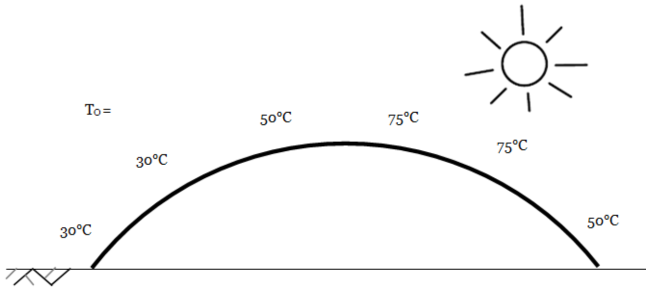


Fig. Calc7.3. Shell surface subjected to estimated temperature distribution

Exterior insulation:

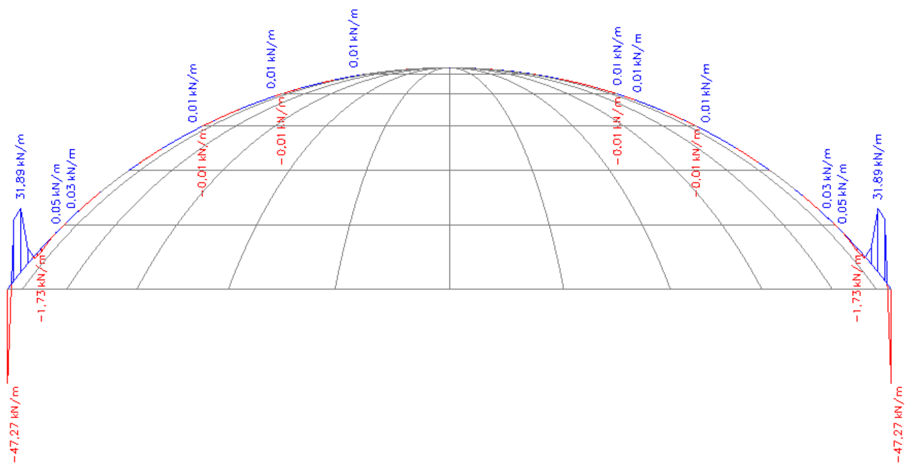
Case 4: Summer; extreme. Temperature distribution constant over shell surface
(T_o = 35°C, T_i = 30 °C)

Case 5: Winter; extreme. Temperature distribution constant over shell surface
(T_o = 15°C, T_i = 20 °C)

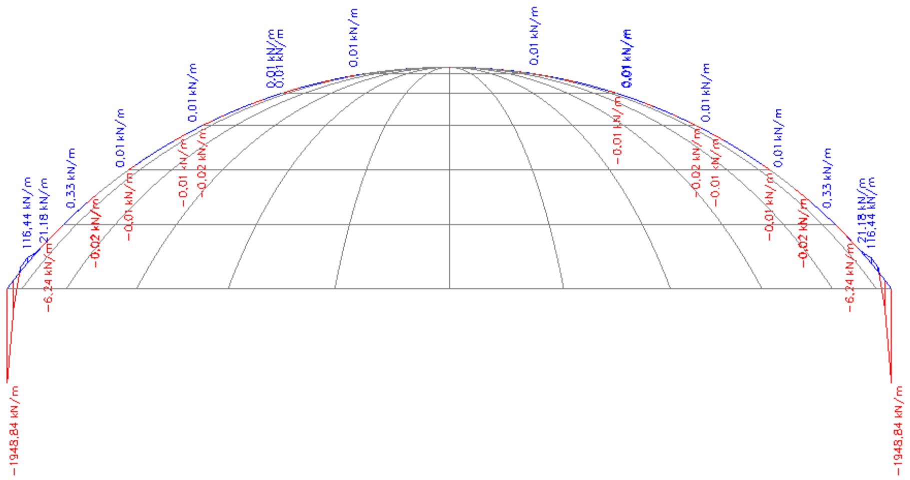
8.7.2. Results

Case 1. Interior insulation: Summer; extreme. Temperature distribution constant over shell surface

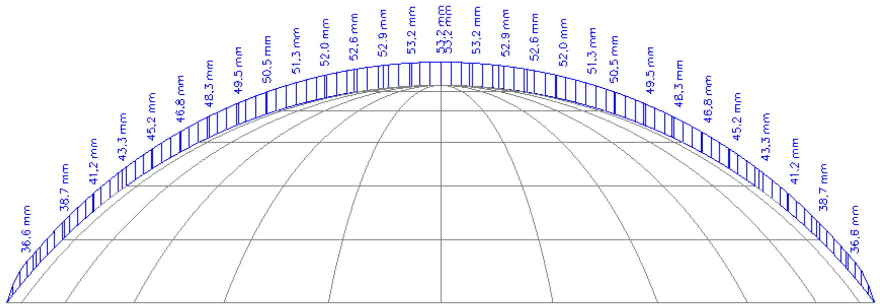
Meridional force (ny):



Circumferential force (nx):

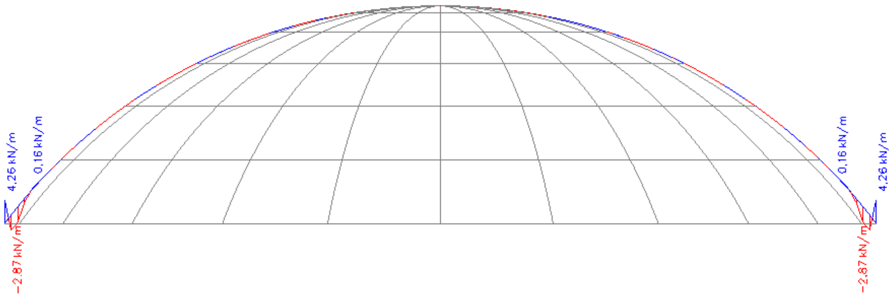


Deformations (uz):

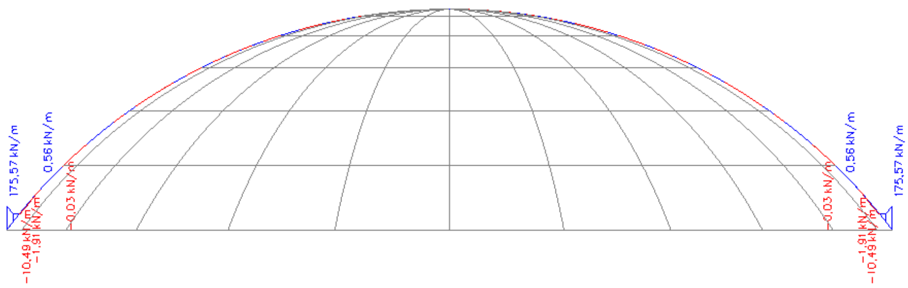


Case 2. Interior insulation: Winter; extreme. Temperature distribution constant over shell surface

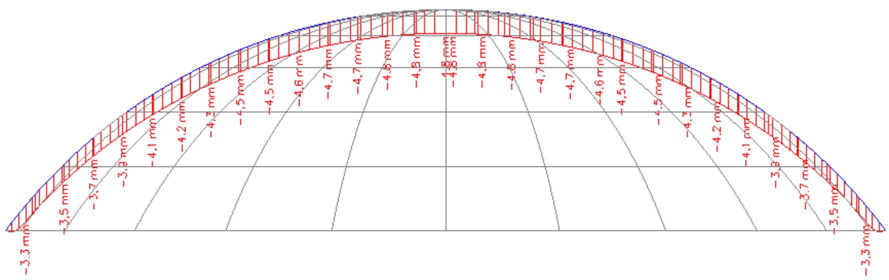
Meridional force (ny):



Circumferential force (nx):

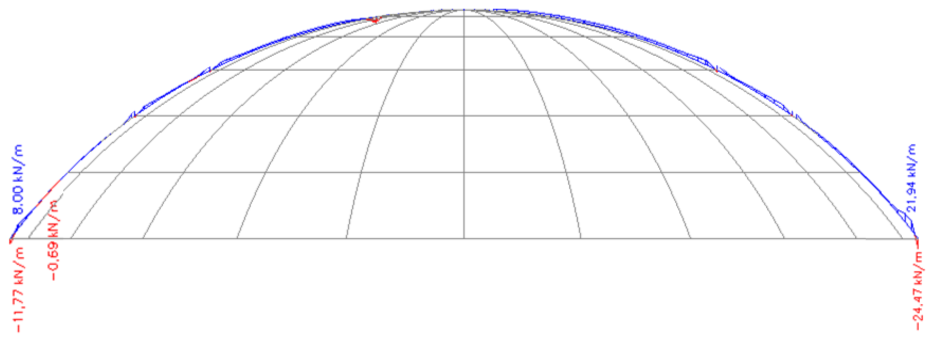


Deformations (uz):

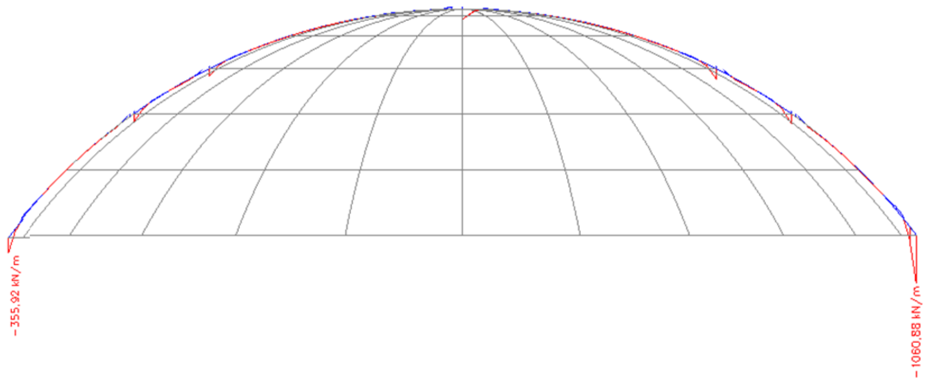


Case 3. Interior insulation: Summer; extreme. Estimated realistic temperature distribution

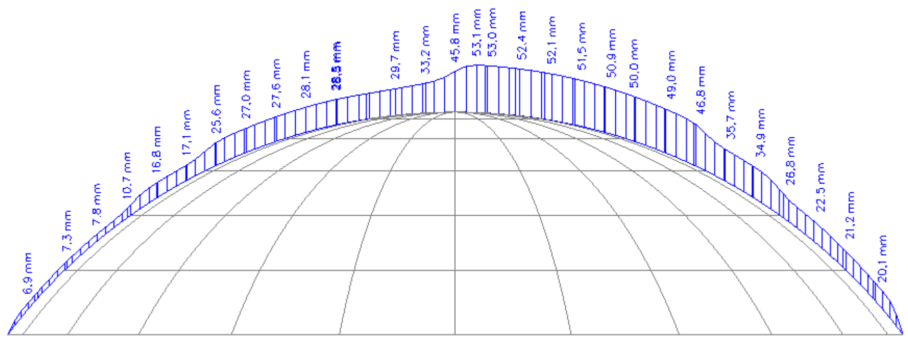
Meridional force (ny):



Circumferential force (nx):

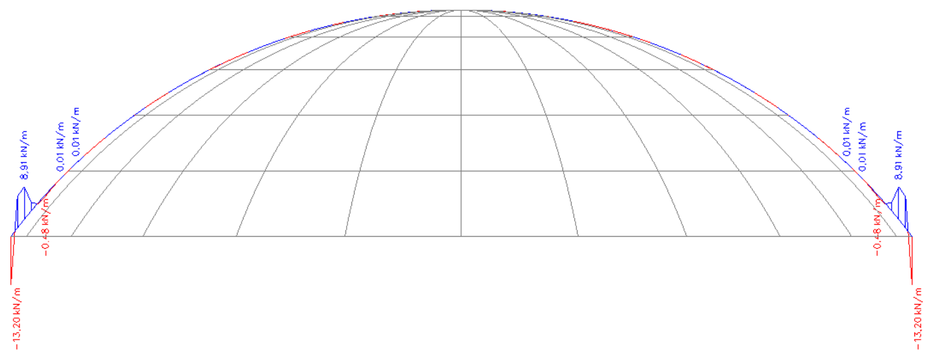


Deformations (uz):

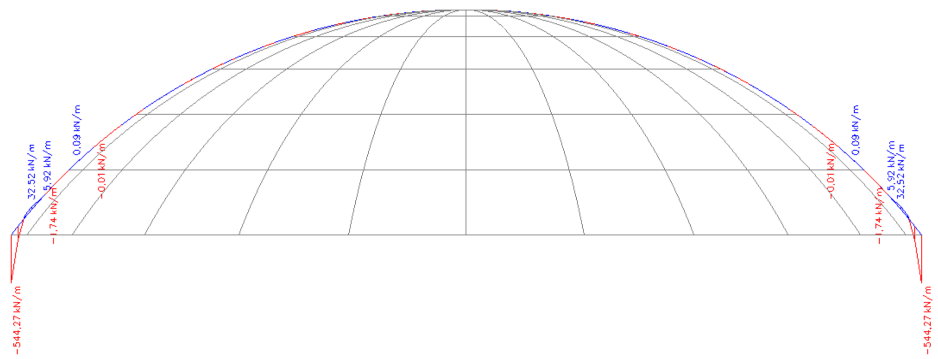


Case 4. Exterior insulation: Summer; extreme. Temperature distribution constant over shell surface

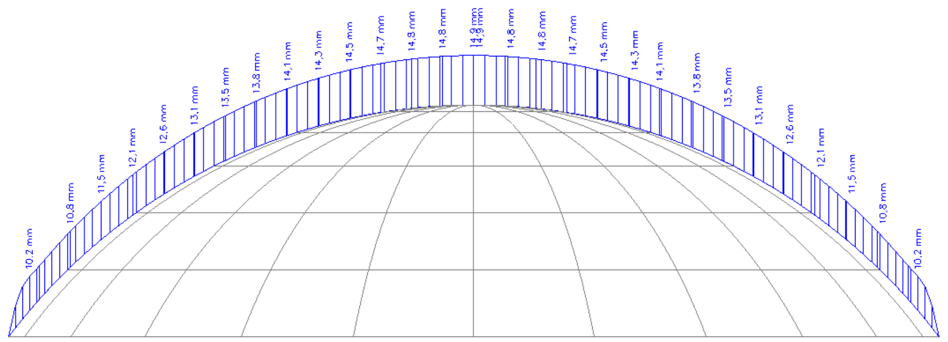
Meridional force (ny):



Circumferential force (nx):

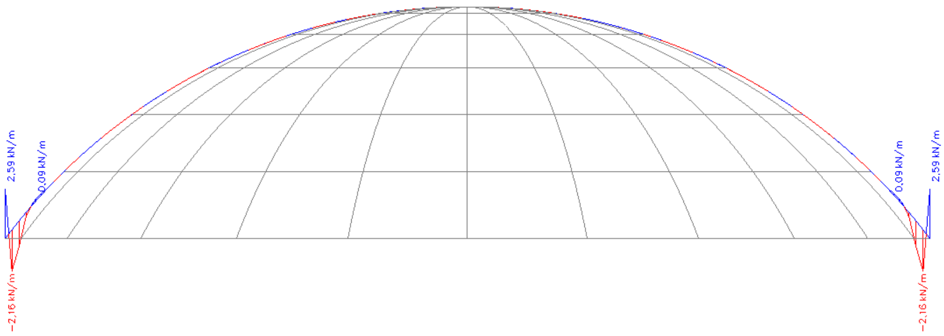


Deformations (uz):

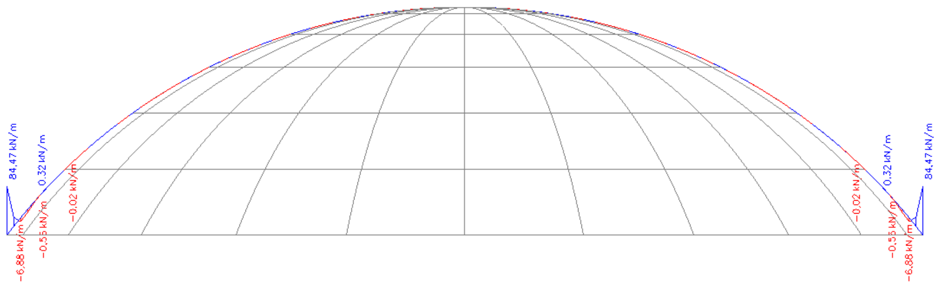


<u>Case 5.</u> Exterior insulation: Winter; extreme. Temperature distribution constant over shell surface

Meridional force (ny):



Circumferential force (nx):



Deformations (uz):