Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

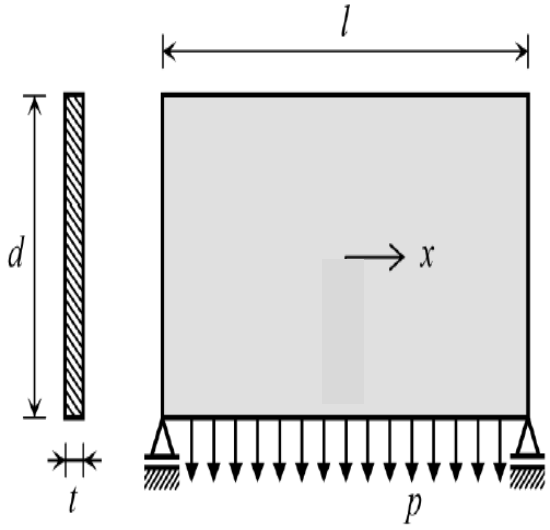
Exam CIE4180: Plates and Slabs
Thursday 28 January 2016, 13:30 – 16:30

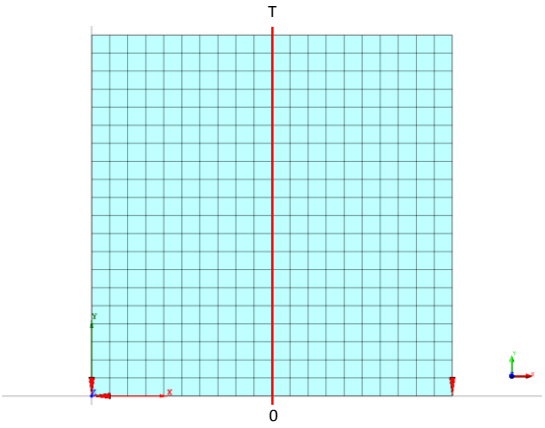
This exam consists of **6** questions (on 13 pages).
The respective weights are 1.6, 1.6, 1.6, 1.6, 1.6 and 2.0.

Write your name and student number **on every page of your work**.

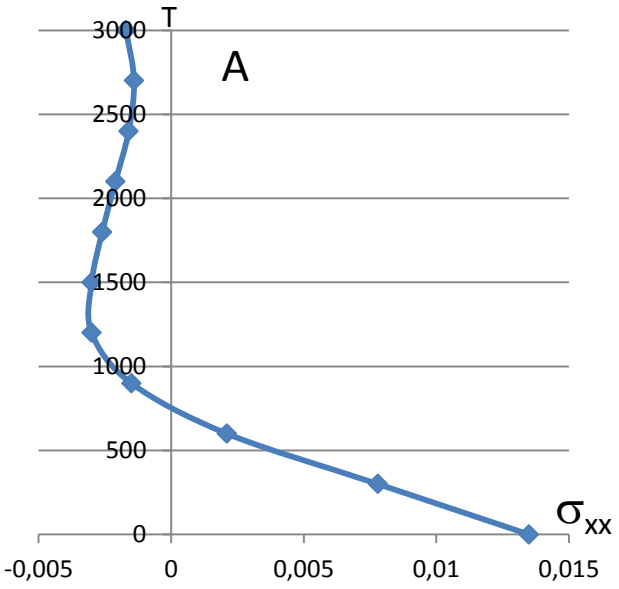
In this exam the notation and sign conventions of the reader (part 1 and 2) and the book (Plates and FEM) have been adopted. **You are supposed to use the same notation and sign conventions**.

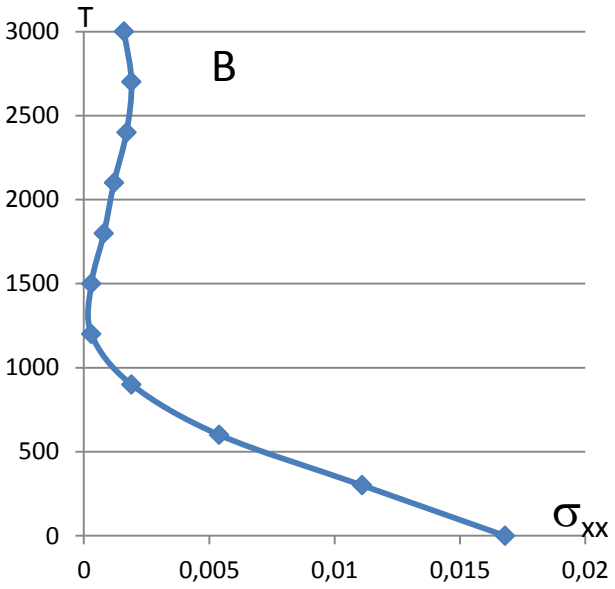
Question 1

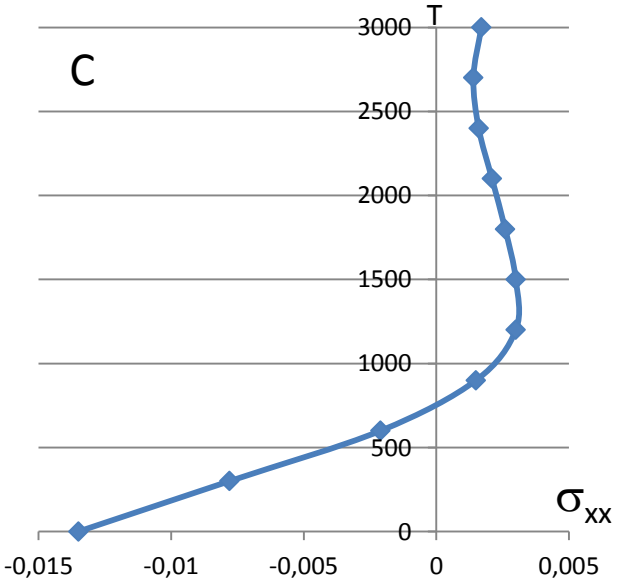


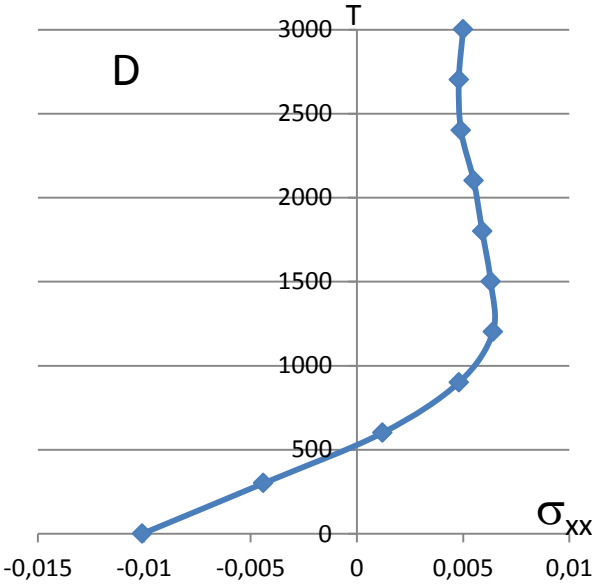


Consider the plane stress (or plate membrane) problem and the finite element model above, with the following parameters: E = 30000 N/mm^2, ν = 0.2, l = d = 3000 mm, p = 1.0 kN/m. Along the red line (0-T) the horizontal stress σ_{xx} is plotted (in MPa). Below are four possible diagrams (A, B, C, D) of σ_{xx} along that line.







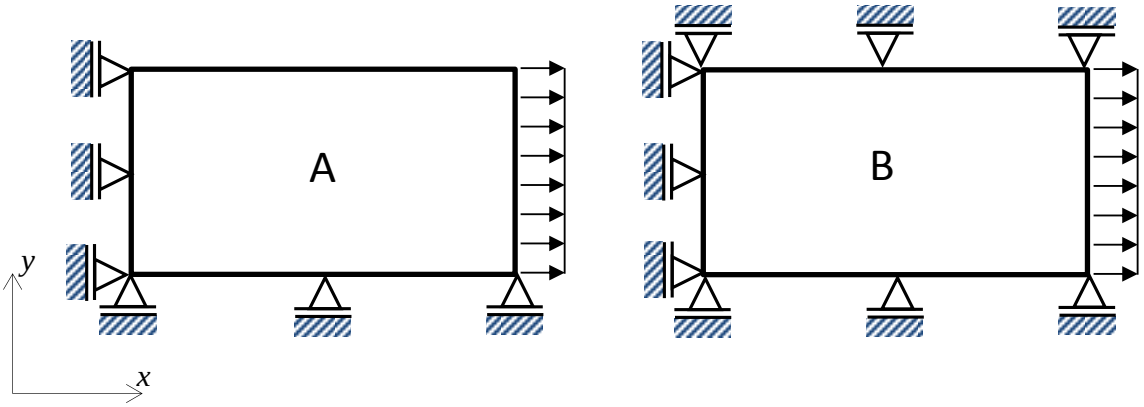


Values can be measured from the diagrams.

a) Which of these diagrams can be correct? Multiple diagrams can be correct. Explain your answer by commenting each diagram. Be concise; do not use more than 50 words per diagram.

b) Select a diagram that can be correct. For this diagram, calculate the thickness of the plate.

Question 2



Consider two identical rectangular plates A and B of unit thickness, see above. Assume a plane stress (or plate membrane) situation. Plate A is supported along two edges as indicated in the figure. Plate B is supported along three edges. Both plates are loaded by the same uniform edge load along the right edge.

a) Answer qualitatively: How does the strain ε_{xx} in plate A compare to the strain ε_{xx} in plate B?

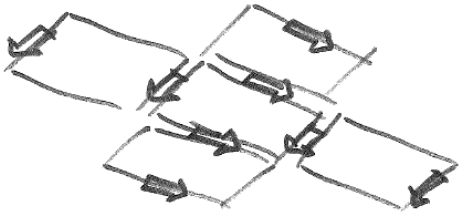
b) Give an expression of the ratio of the strain ε_{xx} in plate A to the strain ε_{xx} in plate B.

Question 3

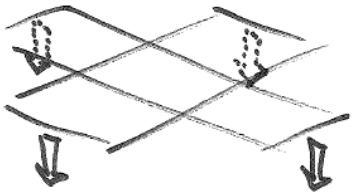
This question consists of five multiple choice questions (**No** explanation needed).

Question 3a

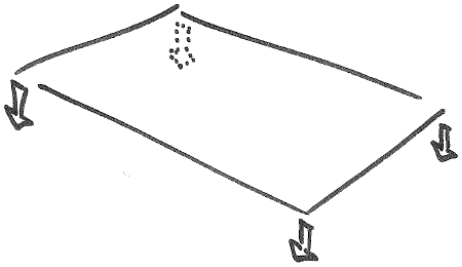
Consider the finite difference method. Which is the main molecule for plates loaded perpendicularly to their plane? Choose A, B, C or D.



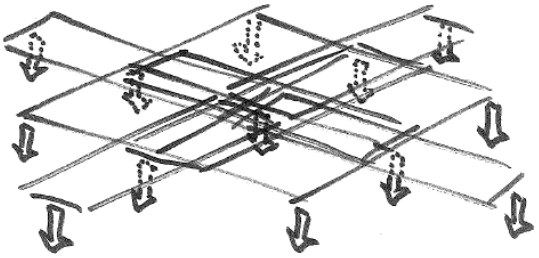
A



B



C



D

Why is the finite difference method not suitable for plates with curved edges? Choose A, B, C or D.

Question 3b

A It is difficult to draw an orthogonal grid along a curved edge.

B The concentrated shear force cannot be determined if the principal directions change along an edge.

C There are no edge molecules for curved edges.

D The shape functions need straight edges.

Question 3c

The engineering interpretation of the finite difference method is called discrete element method. How can the latter method be described for plates loaded in their plane? Choose A, B, C or D.

A A plate is modelled by a lot of springs and small shear panels.

B The differences are not infinitely small, they are approximately equal to the plate thickness.

C The shape functions need to be integrated over the area of the element.

D The interaction of the plate molecules can be solved in Excel.

Question 3d

The Mindlin-Reissner theory has an advantage over the Kirchhoff theory. What is this advantage? Choose A, B, C or D.

A A torsion moment on an edge can be interpreted as a concentrated shear force.

B The elements can be rather large while still obtaining converged results.

C It is also valid for thick plates.

D It is definitely correct because Mindlin and Reissner were brilliant engineers.

Question 3e

Which method has been invented by engineers and improved by mathematicians? Choose A, B, C or D.

A Discrete Element Method

B Finite Element Method

C Potential Energy Method

D Finite Difference Method

Question 4

A square plate is simply supported at all edges. Figure 4.1 shows a finite element model of this plate. The plate length and width are 5000 mm. The plate thickness is 250 mm. Self-weight is applied (2400 kg/m^3). In the middle, a point load of 300 000 N acts perpendicular to the plate. E = 30 000 N/mm², ν = 0.16. The finite element model of the plate uses eight node shell elements with shear deformation. Element size is 500 mm.

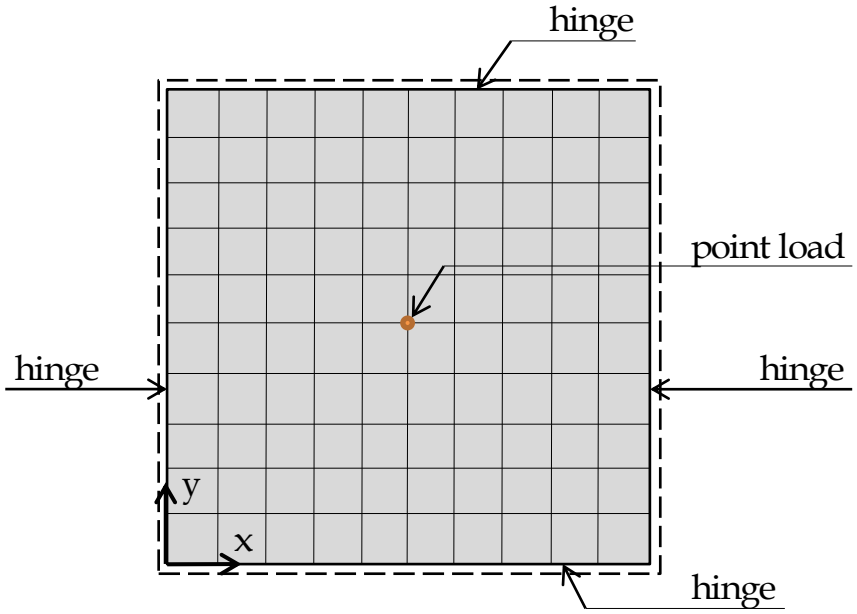


Figure 4.1. Simply supported plate with a point load and finite element mesh

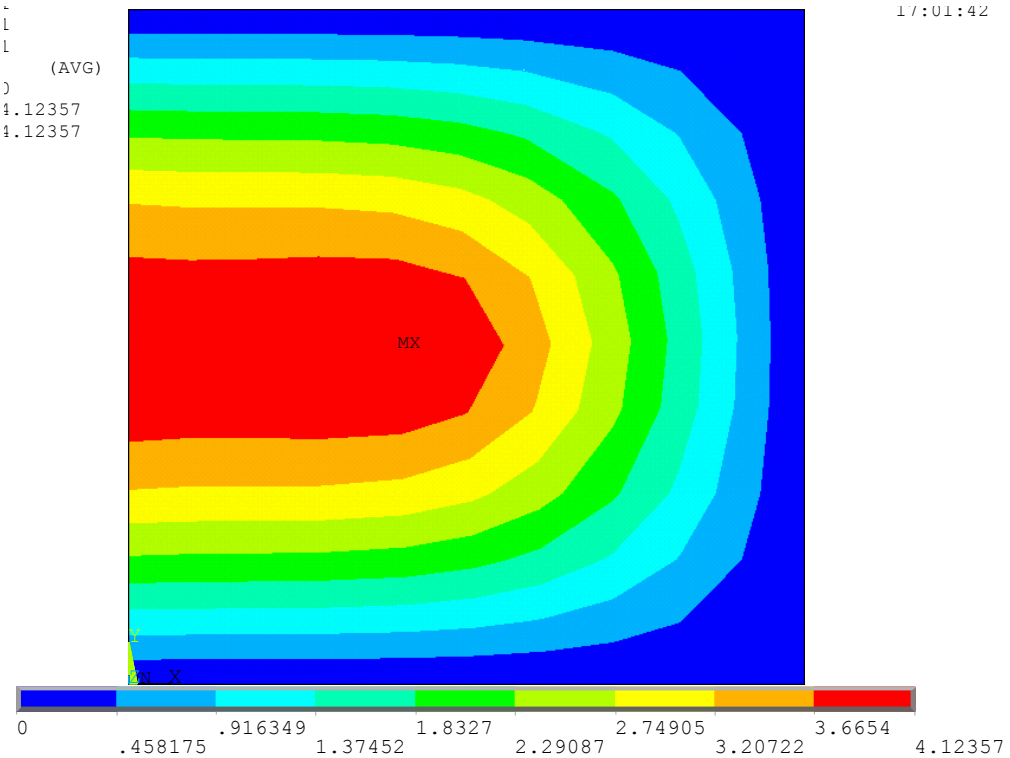


Figure 4.2. Contour plot of the plate deformation u_z

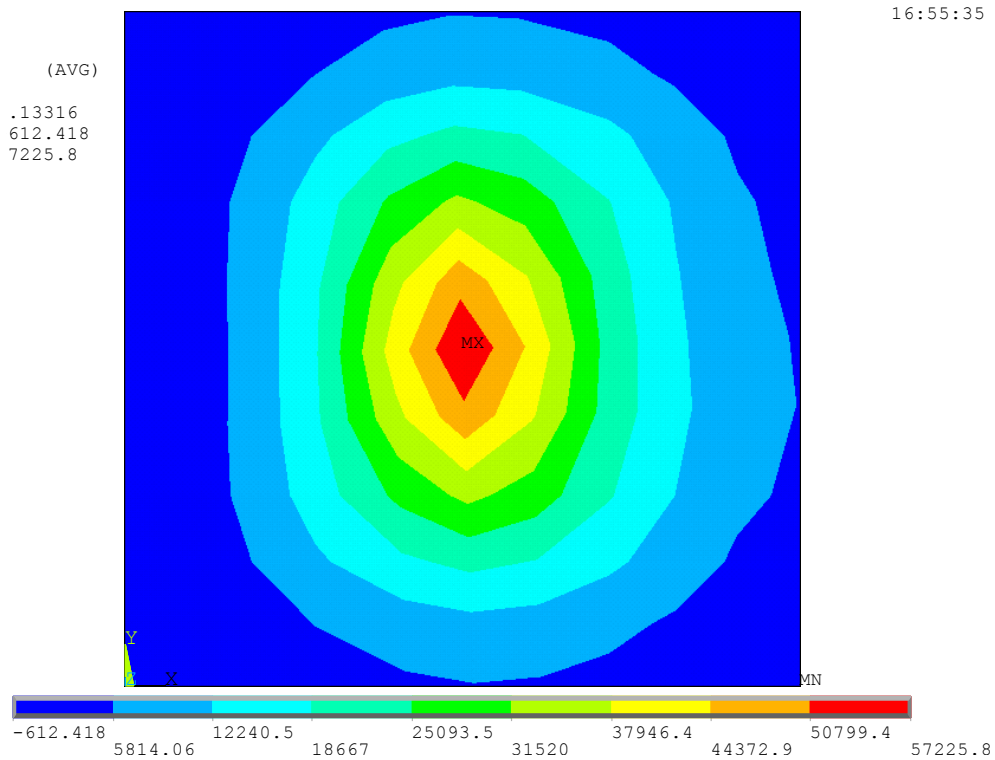


Figure 4.3. Contour plots of the bending moment m_{xx}

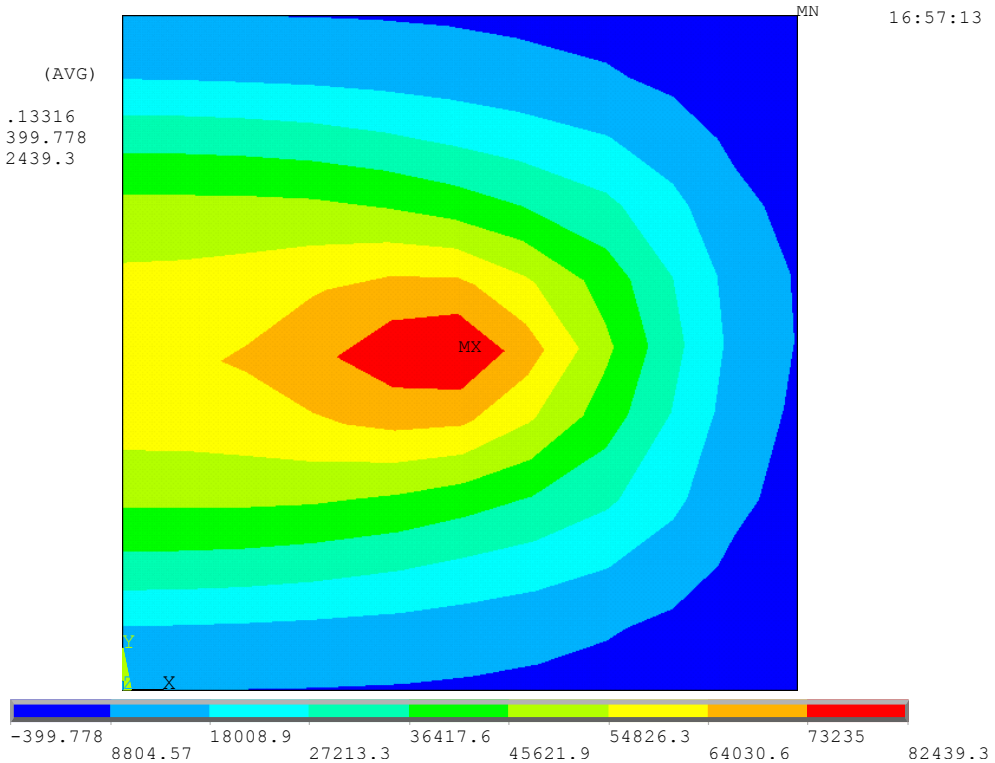


Figure 4.4. Contour plots of the bending moment m_{yy}

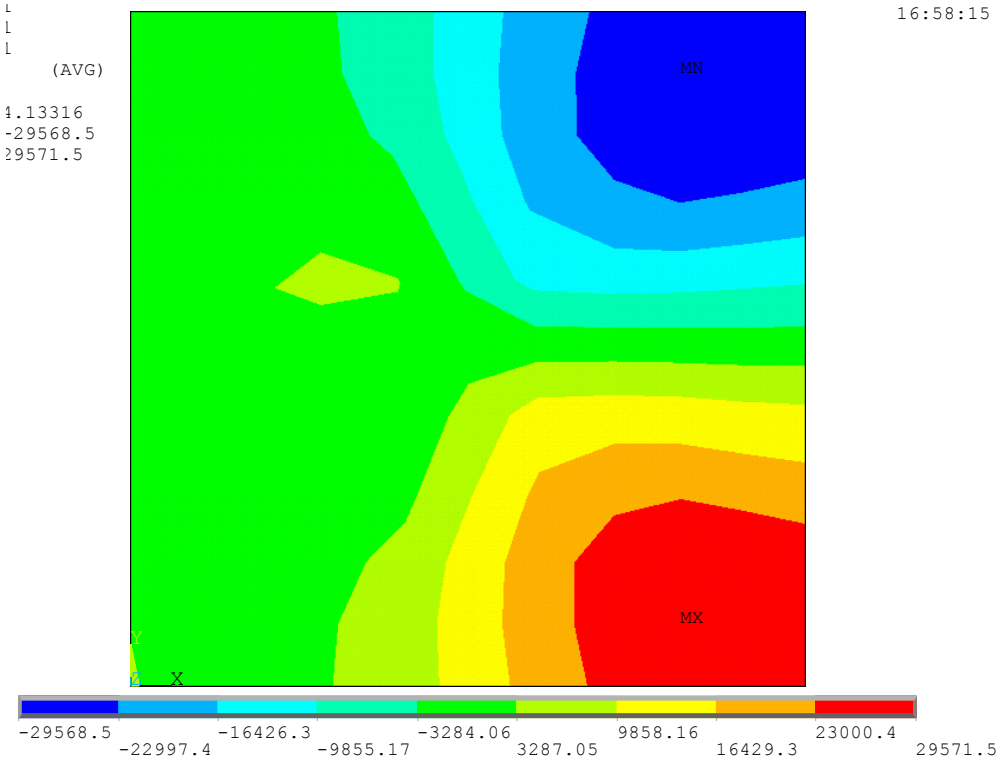


Figure 4.5. Contour plots of the bending moment m_{xy}

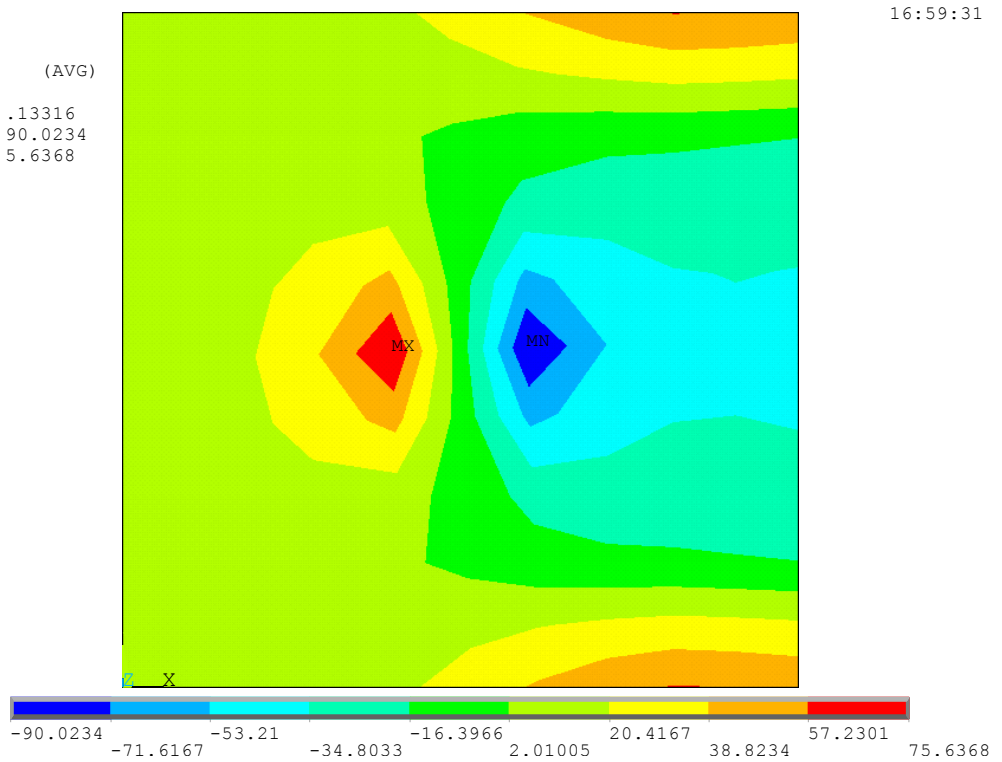


Figure 4.6. Contour plot of the shear force v_x

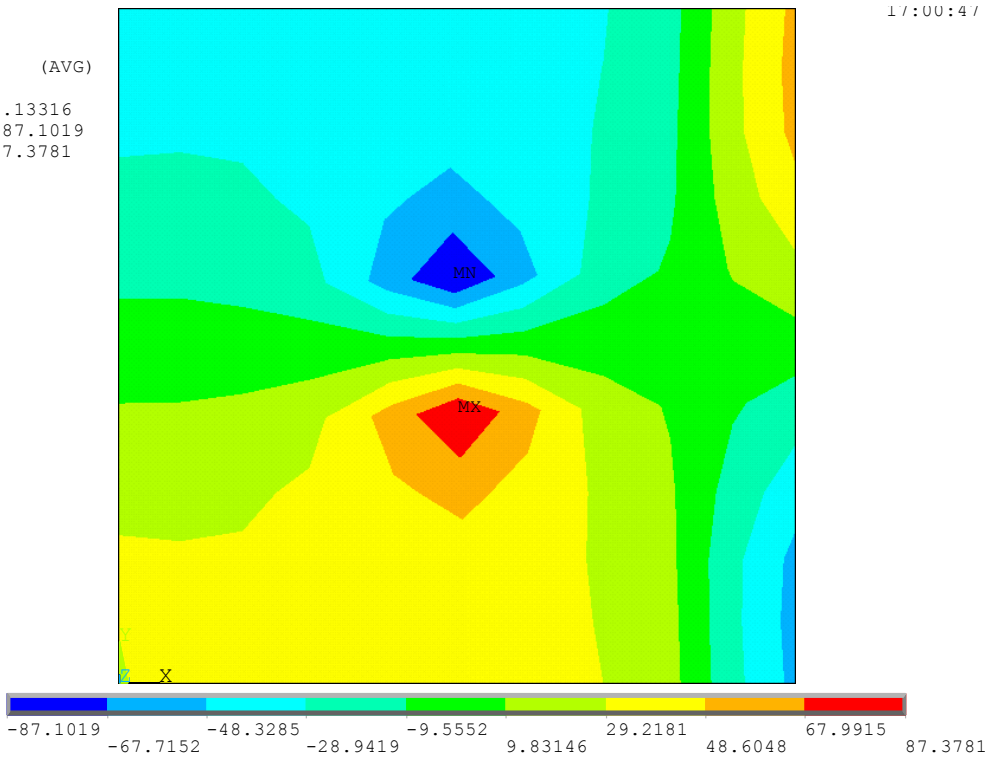


Figure 4.7. Contour plot of the shear force v_y

a) How much is the plate deflection? What is the unit of the deflection?

b) Does the deformation agree with the imposed boundary conditions?

c) What is the largest value of the bending moment m_{xx} in the plate? Which unit? Where does it occur?

d) Why is the bending moment m_{yy} on the edge $y = 0$ not equal to zero everywhere?

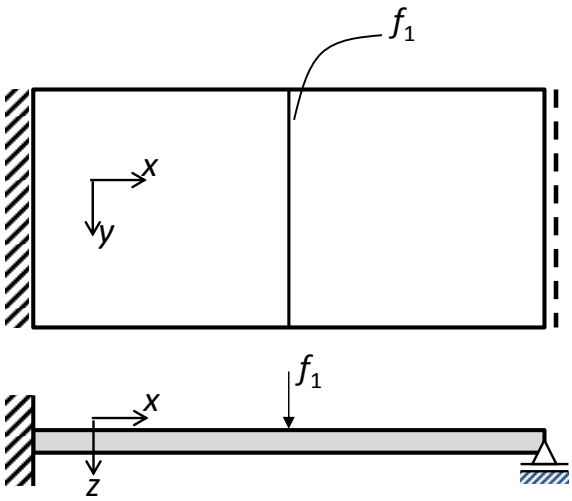
e) Why is the torsion moment m_{xy} at the edges not equal to zero?

f) What is the largest value of the torsion moment m_{xy} in this plate? Which unit? Where does it occur?

g) What is the largest value of v_x in this plate? Which unit? Where does it occur?

h) Should v_x be equal to zero on the edge $x = 5000$ mm? Explain your answer.

Question 5

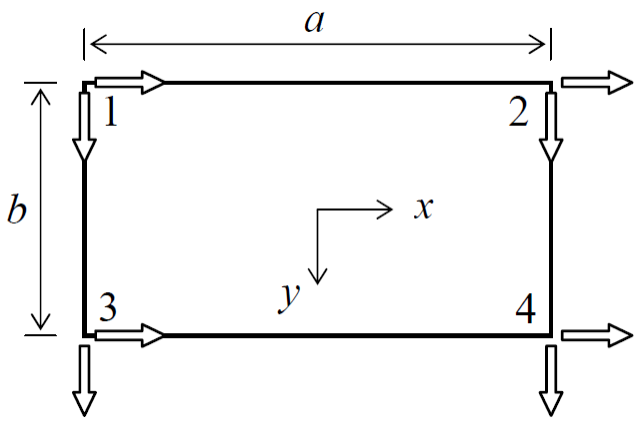


Consider a rectangular thin plate, see the figure above. Along the left-hand side the plate is clamped. Along the right-hand side the plate is simply supported. A distributed line load, f_1 , is applied at mid-span.

a) Give <u>all</u> (kinematic and/or dynamic) boundary conditions for each of the four edges of the plate. Use symbols w, φ, f (for a distributed line load) and t (for a distributed line torque).

b) Give the conditions for the transition zone between the two plate parts.

Question 6



Consider a rectangular membrane element with four nodes in the corners. The displacement field is written as follows:

$$u_x(x,y) = n_1(x,y)\, u_{x1} + n_2(x,y)\, u_{x2} + n_3(x,y)\, u_{x3} + n_4(x,y)\, u_{x4}$$
$$u_y(x,y) = n_1(x,y)\, u_{y1} + n_2(x,y)\, u_{y2} + n_3(x,y)\, u_{y3} + n_4(x,y)\, u_{y4}$$

Additional functions are defined as:

$$f^-(x) = ½ + x/a$$
$$g^-(y) = ½ + y/b$$
$$f^+(x) = ½ - x/a$$
$$g^+(y) = ½ - y/b$$

With positive constants a and b. Consider the following sets of shape functions:

Set 1:
$n_1(x,y) = f^+(x)\, g^+(y)$
$n_2(x,y) = f^-(x)\, g^+(y)$
$n_3(x,y) = f^+(x)\, g^-(y)$
$n_4(x,y) = f^-(x)\, g^-(y)$

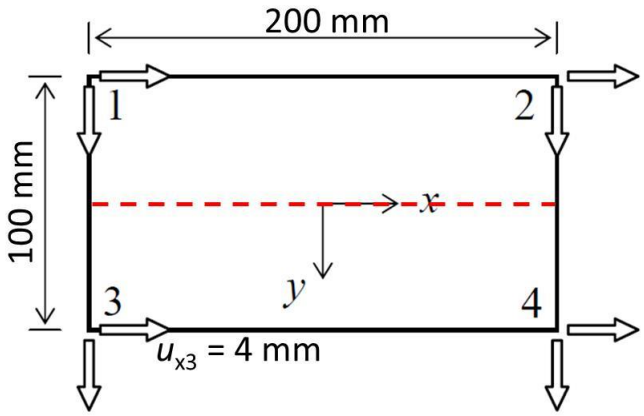
Set 2:
$n_1(x,y) = f^-(x)\, g^-(y)$
$n_2(x,y) = f^+(x)\, g^-(y)$
$n_3(x,y) = f^-(x)\, g^+(y)$
$n_4(x,y) = f^+(x)\, g^+(y)$

Set 3:
$n_1(x,y) = f^+(x) + g^+(y) - 1$
$n_2(x,y) = f^-(x) + g^+(y) - 1$
$n_3(x,y) = f^+(x) + g^-(y) - 1$
$n_4(x,y) = f^-(x) + g^-(y) - 1$

a) Which set comprises suitable shape functions? Multiple answers possible. Explain.

Proceed with a suitable set of shape functions. The kinematic relation is written in matrix notation as $\boldsymbol{\varepsilon} = \boldsymbol{B}\, \boldsymbol{u}$

b) Define the columns $\boldsymbol{\varepsilon}$ and $\boldsymbol{u}$ and derive closed-form expressions for all elements of the kinematic $\boldsymbol{B}$ matrix which are needed for determining the strain ε_{xx}.



Now consider the dimensions of the plate as indicated in the figure above. Also, consider that all nodal displacements are zero, except for u_{x3} = 4 mm.

c) Sketch a graph of the strain ε_{xx} along the dashed line. Indicate values along the axes.
d) Similar for the strain ε_{yy}.
e) Similar for the strain γ_{xy}.

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Question 1

a) "A" can be correct. Integrating the stresses over the depth gives a zero normal force and a positive bending moment. "B" and "D" cannot be correct. The resulting mormal force is non-zero. "C" cannot be correct. The resulting bending moment is negative.

b) Roughly integrating $\int_0^{3000} r\,\sigma_{xx}\,dx$ of diagram "A" as $\frac{1}{2} \times 0.013 \times 750 \times \frac{2}{3} \times 750 + 0.002 \times 2250 \times \frac{1}{2} \times 2250$ yields 7500 N. The internal moment is thus 7500t. The external moment is $\frac{1}{8}pl^2$ yielding 1,125,000 Nmm. Equilibrium leads to t = 1,125,000/7500 = 150 mm.

Question 2

a) $\varepsilon_{xx}^B < \varepsilon_{xx}^A$ for $\nu \neq 0$.

b) $\varepsilon_{xx} = \frac{1}{Et}(n_{xx} - \nu n_{yy})$ gives $\varepsilon_{xx}^A = \frac{1}{Et} n_{xx}$, whereas $n_{xx} = \frac{Et}{(1-\nu^2)}(\varepsilon_{xx} + \nu\varepsilon_{yy})$ gives $\varepsilon_{xx}^B = \frac{(1-\nu^2)}{Et} n_{xx}$. Consequently, $\varepsilon_{xx}^A / \varepsilon_{xx}^B = \frac{1}{(1-\nu^2)}$.

Question 3

a) D
b) A
c) A
d) C
e) B

Question 4

a) 4.12 mm
b) No; An input mistake has been made.
c) Infinitely large (singularity); Nmm/mm or Nm/m; Under the point load
d) Element accuracy; For very small elements m_{yy} would be zero on that edge.
e) Concentrated shear force; Torsion moment on a plate edge needs to be interpreted as a concentrated shear force in the edge.
f) 29.6 kNm/m; Close to the right-hand corners but not in the corners. (Ansys: MX = maximum, MN = minimum)
g) Infinitely large (singularity); N/mm; Under the point load.
h) No; Equilibrium exist between v_x , the support reaction and the change of the concentrated shear force in the y direction.

Question 5

a) For the left-hand side $w = \varphi = 0$. For the right-hand side $w = t = 0$. For the two other sides $f = t = 0$.

b) $v_x^{right} = v_x^{left} - f_1$. The moments m_{xx} , m_{xy} and m_{yy} and v_y are continuous across the transition line.

Question 6

a) Set 1: In node 1, $n_1 = 1$. Along the opposite edges 2-4 and 4-3, $n_1 = 0$. Similar for n_2, n_3 and n_4. Set 1 is suitable.

Set 2: In node 1, $n_1 = 0$. Set 2 is not suitable.
Set 3: In node 4, $n_1 = -1$. Set 3 is not suitable.

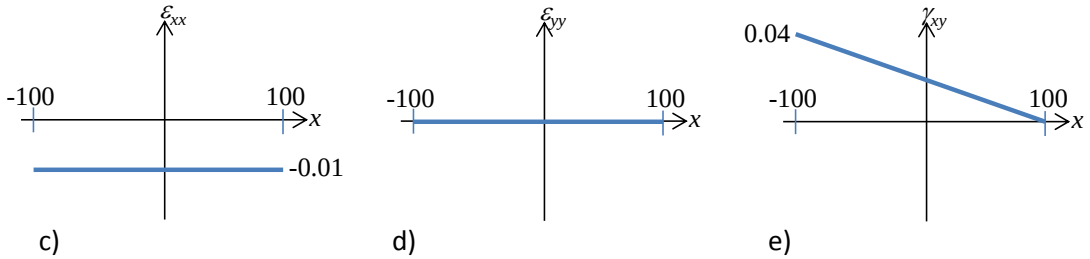
b) $\varepsilon_{xx} = \frac{\partial u_x}{\partial x} = \frac{\partial n_1}{\partial x}u_{x1} + \frac{\partial n_2}{\partial x}u_{x2} + \frac{\partial n_3}{\partial x}u_{x3} + \frac{\partial n_4}{\partial x}u_{x4}$. Using Set 1 gives $\frac{\partial n_1}{\partial x} = \frac{-g^+}{a}$, $\frac{\partial n_2}{\partial x} = \frac{g^+}{a}$, $\frac{\partial n_3}{\partial x} = \frac{-g^-}{a}$, and $\frac{\partial n_4}{\partial x} = \frac{g^-}{a}$.We now write $\boldsymbol{\varepsilon} = \boldsymbol{Bu}$ as

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \left[\begin{array}{c|c|c|c|c|c|c|c} -g^+/a & 0 & g^+/a & 0 & -g^-/a & 0 & g^-/a & 0 \\ \hline \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \hline \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \end{array}\right] \begin{Bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \end{Bmatrix}$$

c) Now $\varepsilon_{xx} = \frac{-g^-}{a}u_{x3} = \frac{-1}{200}\left(\frac{1}{2} + \frac{y}{100}\right)4 = -\frac{1}{100} + \frac{1}{5000}y$ and $\varepsilon_{xx}(x, y = 0) = -\frac{1}{100}$

d) A similar derivation as in b) will show that ε_{yy} does depend on the nodal *y*-displacements only. Now $\varepsilon_{yy} = 0$ throughout the element.

e) Now $\gamma_{xy} = \frac{\partial n_3}{\partial y}u_{x3} = \frac{f^+}{b}u_{x3} = \frac{1}{100}\left(\frac{1}{2} - \frac{x}{200}\right)4 = \frac{2}{100} - \frac{2}{10000}x$



c) d) e)