Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CIE4180: Plates and Slabs
Wednesday 12 April 2017, 13:30 – 16:30

This exam consists of 6 questions.
The respective weights are 1.6, 1.6, 1.5, 2.1, 1.6 and 1.6.

Write your name and student number **on every page of your work**.

In this exam the notation and sign conventions of the reader (part 1 and 2) and the book (Plates and FEM) have been adopted. **You are supposed to use the same notation and sign conventions**.

Question 1

Consider the plane stress (or plate membrane) problem below. The measurements of the quadrilateral plate with a snipped corner are in mm. The length of the plate is 300 mm. The height is 300 mm. The plate has an uniform thickness t = 100 mm.

The plate is loaded by an equally distributed line load f ("force per length"), acting at the top of the plate. f = 100 N/mm.

At the bottom of the plate, both vertical and horizontal displacements are prohibited.

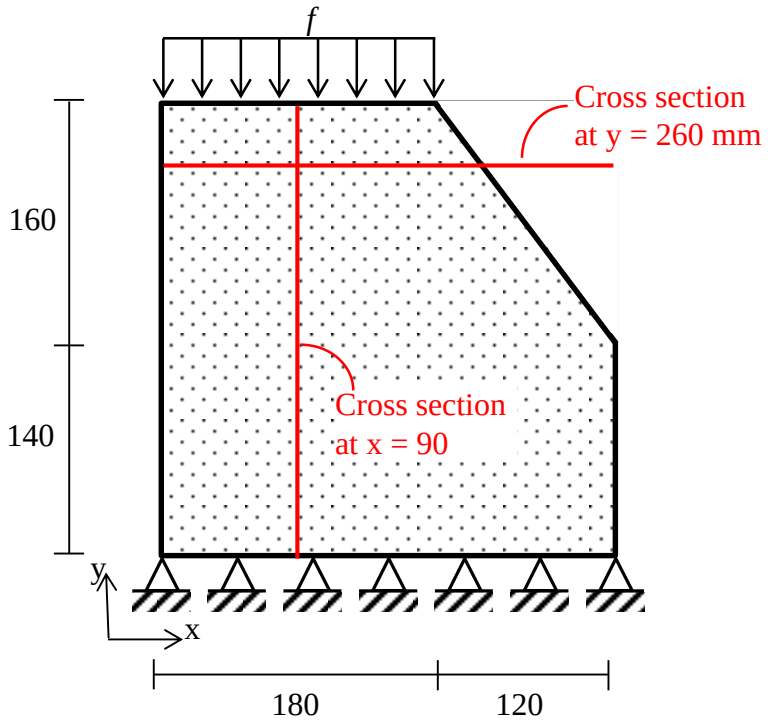
An adequate finite element analysis has been made using a mesh with 8-noded plane stress elements of 5×5 mm^2.

E = 30000 N/mm^2 , ν = 0.2

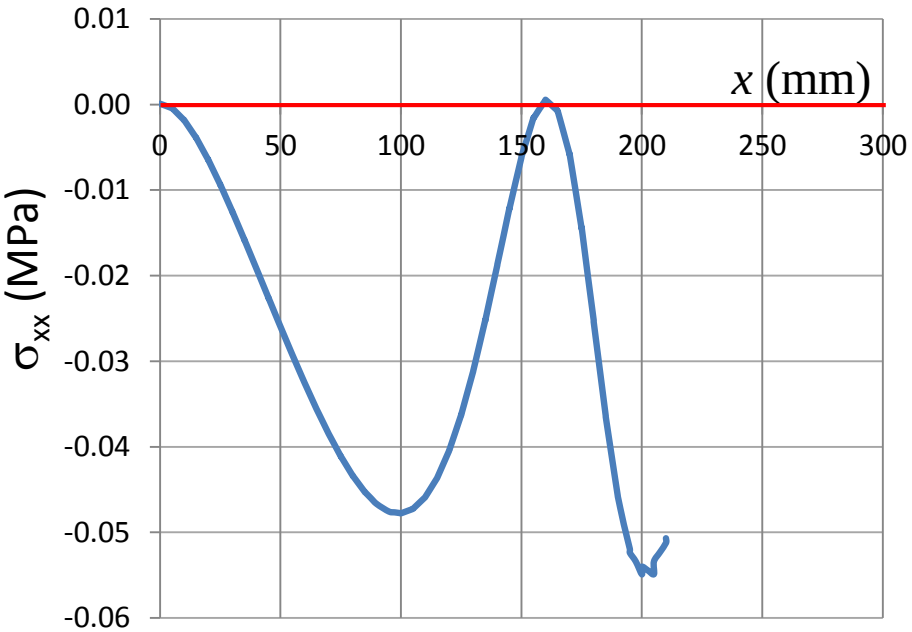
Along the two red lines (indicated as "Cross section") results are plotted (next page):

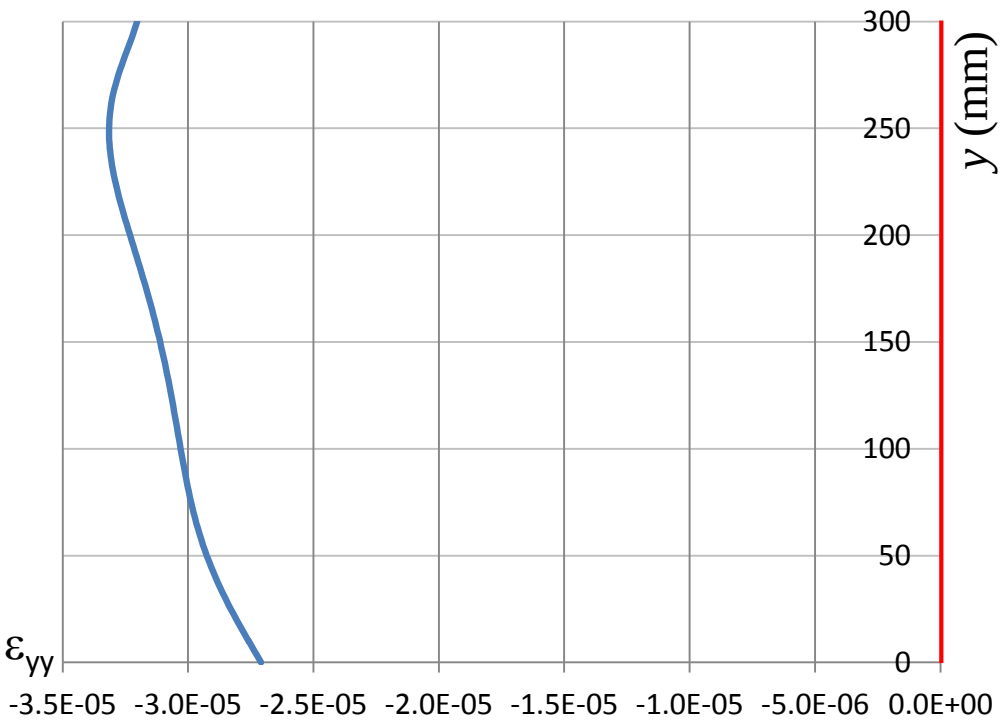
- Along the horizontal line the stress σ_{xx} is plotted (in MPa).
- Along the vertical line the strain ε_{yy} is plotted.

Values can be measured from the diagrams.



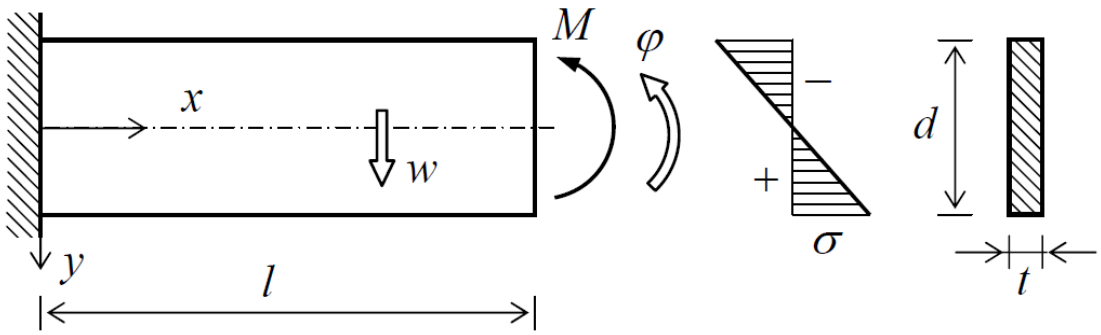
a) Could the diagram with the stress σ_{xx} be a diagram for this case? Explain your answer.
b) Calculate the stress σ_{yy} at x = 90, y=300.
c) Calculate the stress σ_{yy} at x=90, y=0.
d) Now, consider that we change the y-axis to its opposite direction. Which of the following quantities change sign? σ_{xx}, σ_{yy}, σ_{xy}


0.01
0.00
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
σxx (MPa)
x (mm)
0
50
100
150
200
250
300


300
250
200
150
100
50
0
y (mm)
εyy
-3.5E-05
-3.0E-05
-2.5E-05
-2.0E-05
-1.5E-05
-1.0E-05
-5.0E-06
0.0E+00

Question 2

Consider the classic case in the Euler beam theory of a cantilever beam, which is loaded by a positive moment *M* at the free end:



Now we apply (plane stress) plate theory. As a trial solution for the *x*-displacements we consider:

$$u_x(x, y) = a_1 + a_2 x + a_3 y + a_4 x^2 + a_5 xy + a_6 y^2$$

An adequate trial solution for the *y*-displacements is used.

a) Assume the Poisson's ratio $\nu = 0$. Express the coefficient a_i in terms of *M*, *d*, *t* and the Young's modulus *E*, by comparing plate theory with Euler beam theory.

b) Assume the Poisson's ratio $\nu \neq 0$. Consider the cross-section at $x = \frac{1}{2}l$. Sketch qualitatively: a graph of displacements u_y versus coordinate y. Explain your graph in words.

Question 3

a) What is the most important supposition in the derivation of the theory for plates loaded perpendicularly to their plane? Choose answer A, B, C or D.

A The thickness of the plate remains unchanged.
B The edges of the plates are simply supported.
C The strain energy is distributed evenly over the plate.
D Straight needles remains straight.

b) Where in plates occur singularities?
Choose A, B, C, D. You can choose more than one answer.

A At point loads
B At point supports
C In free edges
D In re-entrant corners

c) What is a possible consequence of choosing very very small finite elements to compute a linear elastic plate?
Choose A, B, C, D. You can choose more than one answer.

A The computed stresses in singularities are very very large.
B The computation time is small.
C The computation results are not accurate.
D The computation stops due to insufficient hard disk space.

d) What is the formula for calculating the normal stress σ_{xx} in the top of a plate? ($z = t / 2$) Choose A, B, C or D.

A $n_{xx}\, t^2 + m_{xx}\, t$
B $n_{xx} / t + 6\, m_{xx} / t^2$
C $n_{xx}\, t / m_{xx}$
D $3\, n_{xx} / t^2 + m_{xx} / t^3$

e) Which scientist was the first able to calculate correctly the mode shapes of vibrating thin plates? Choose A, B, C or D.

A Napoleon Bonaparte in 1804
B Sophie Germain in 1806
C Gustav Kirchhoff in 1850
D Raymond Mindlin in 1940

Question 4

A plate is loaded perpendicularly. It is fixed at one edge and simply supported at another edge (Fig. 1). There is one point support. The plate length is 10 m and the width is 5 m. The plate thickness is 200 mm. A distributed load of -5 kN/m^2 is applied. This distributed load includes self-weight. A linear elastic finite element model is made. E = 27100 N/mm^2, ν = 0.2. Applied are four node plate elements (Kirchhoff theory). The element size is 0.2 m. Results are shown in Figs. 2-7.

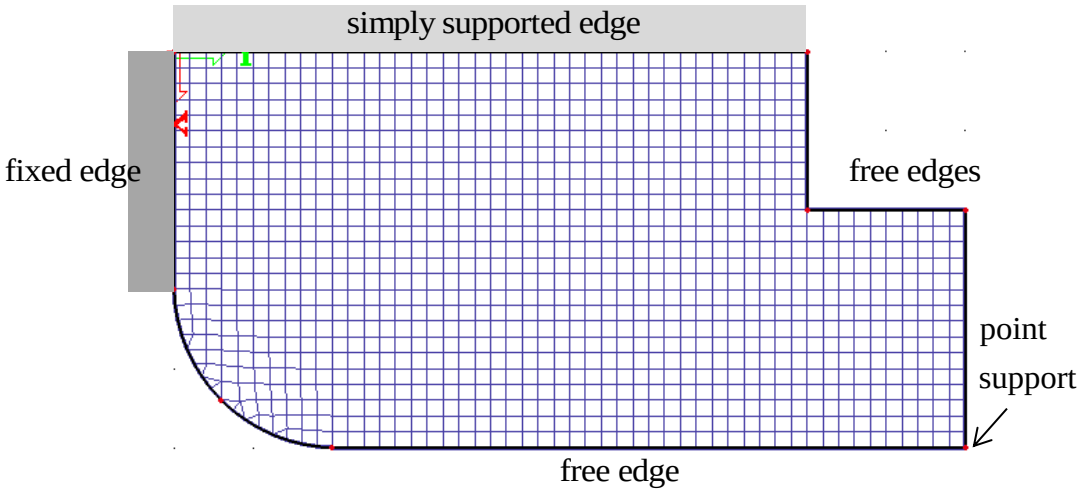


Figure 1. Plate with supports and finite element mesh

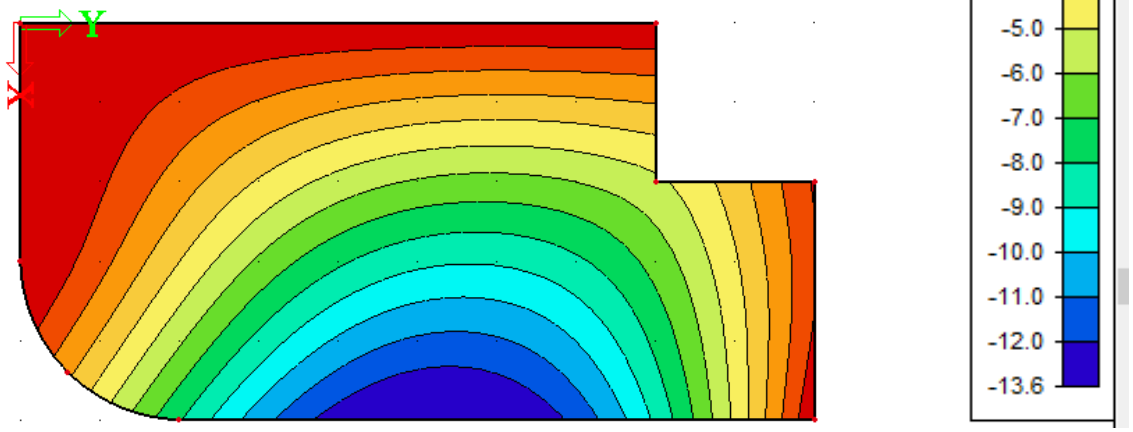


Figure 2. Contour plot of the plate deformation u_z.

a) How much is the plate deflection?
Does the deflection agree with the imposed boundary conditions?

b) Do the moments agree with the specified supports? Explain your answer.

c) What is the largest value of the bending moment m_{xx} in the plate? Where does it occur?

d) The bending moment m_{xx} on the curved free edge is not equal to zero everywhere. What causes this?

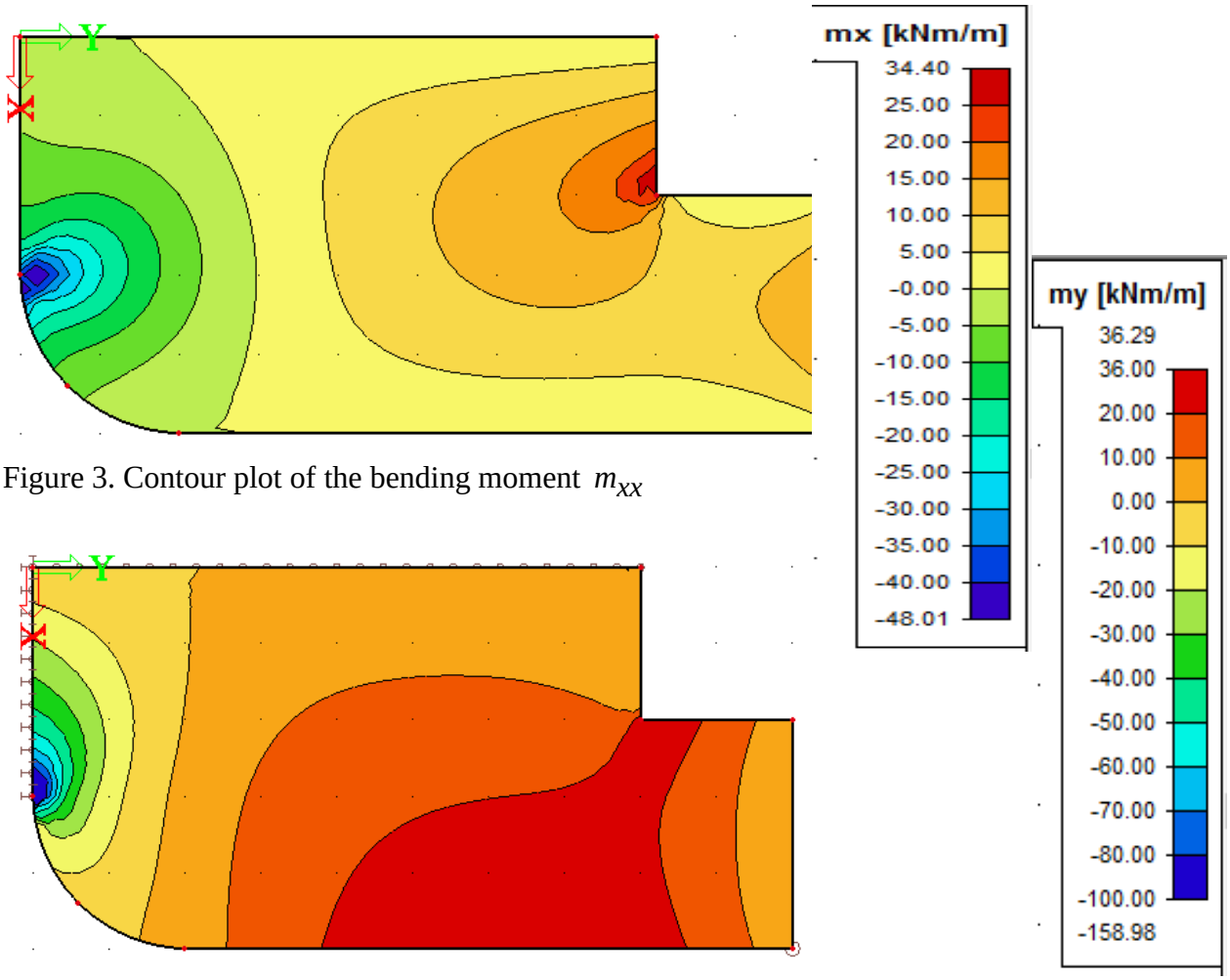


Figure 3. Contour plot of the bending moment m_{xx}

Figure 4. Contour plot of bending moment m_{yy}

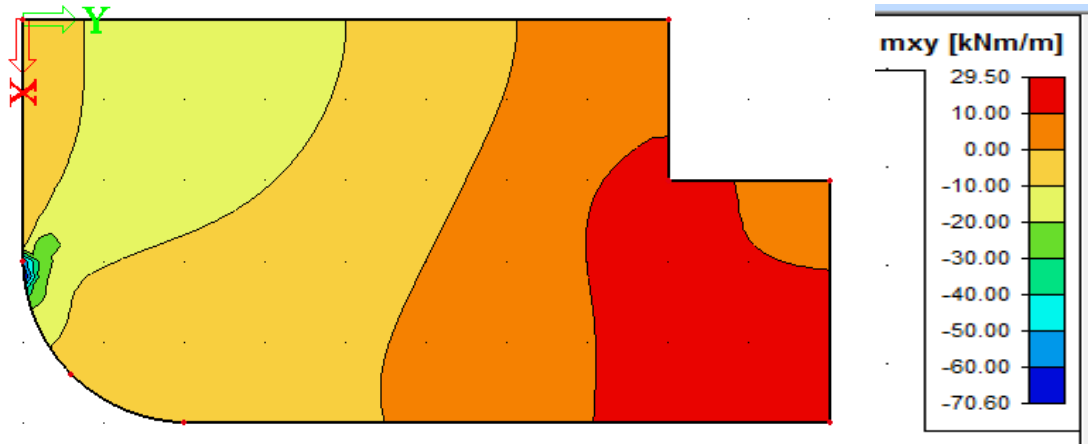


Figure 5. Contour plot of torsion moment m_{xy}

e) On the right hand edges, the torsion moment m_{xy} is not zero. What causes this?

f) What is the largest value of v_x in this plate? How do we interpret this largest value?

g) What is the value of the support reaction in the point support?

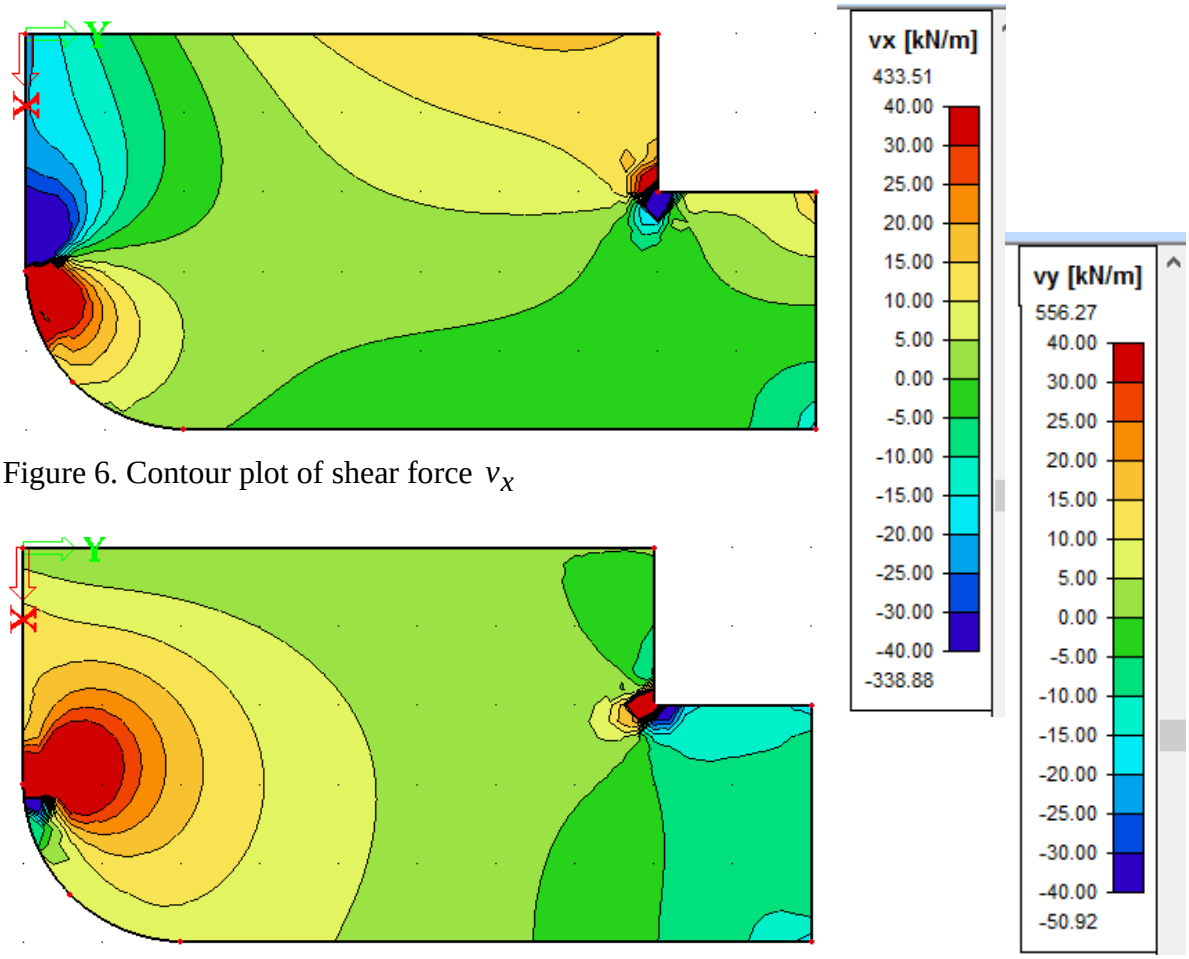


Figure 6. Contour plot of shear force v_x

Figure 7. Contour plot of shear force v_y

Question 5

A reinforced concrete plate is loaded in plane. Two load cases are considered. At some location the membrane forces are

Load case 1: $n_{xx} = 240$ kN/m, $n_{yy} = 140$ kN/m, $n_{xy} = -100$ kN/m.

Load case 2: $n_{xx} = 320$ kN/m, $n_{yy} = -60$ kN/m, $n_{xy} = 220$ kN/m.

The yield stress of the reinforcement is 500 N/mm^2.
Partial safety factors are included in the values.

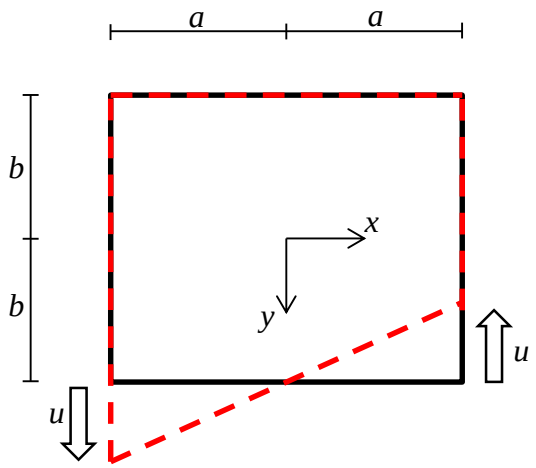
a) Calculate the reinforcement force [kN/m] that is minimally required for Load case 1 in the *x* and *y* directions. Is this reinforcement sufficient for Load case 2? Explain your answer.

b) Calculate the reinforcement force [kN/m] that is minimally required for Load case 2 in the *x* and *y* directions. Is this reinforcement sufficient for Load case 1? Explain your answer.

c) Somebody proposes two layers of bars in x-direction and two layers of bars in y-direction. The diameters in the two directions are the same: 12 mm. The spacing is the two directions is the same.

What is the required spacing for the bars that satisfies both load cases?

Question 6



Consider a standard rectangular membrane element with four nodes in the corners. See the figure above.

All nodes have a zero displacement in *x*-direction.

One node has a displacement *u* in y-direction.
One node has a displacement -*u* in *y*-direction.
Two nodes have a zero displacement in *y*-direction.

The displacement field is illustrated with red dashed lines above.

a) Give the displacement field in *x*- and *y*-directions. Express in *a*, *b*, *u*, *x* and *y*.

b) Give expressions for all strain components. Again, express in *a*, *b*, *u*, *x* and *y*.

Now consider that it is a constant shear element, and we repeat the questions:

c) Give the displacement field in *x*- and *y*-directions.

d) Give expressions for all strains components.

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Exam CIE4180: Plates and Slabs (brief answers)
Thursday 12 April 2017, 13:30 – 16:30

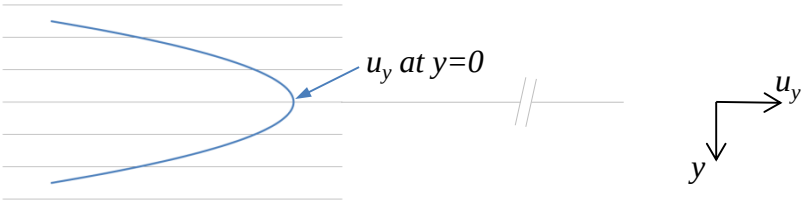
Question 1

a) Yes. $\sigma_{xx} = 0$ at *x*=0 satisfies the boundary conditions. σ_{xx} is not necessary zero at the other (inclined) edge.
b) $\sigma_{yy} = -f/t = -1.00$ MPa.
c) $\sigma_{yy} = \frac{E}{1-\nu^2}(\nu\varepsilon_{xx} + \varepsilon_{yy})$, $\varepsilon_{xx} = 0$ and $\varepsilon_{yy} = -2.7\ 10^{-5}$ gives -0.84 MPa.
d) σ_{xy}

Question 2

a) $a_5 = \frac{M}{EI}$ with $I = \frac{1}{12}d^3t$. All other $a_i = 0$

b)



Poisson effect will lead to additional (negative) displacements for $y \neq 0$.

Question 3

a) D
b) ABD
c) ACD =100%, AD =100%, A = 50%
d) B
e) C

Question 4

a) 13.6 mm
Yes. The deflection is and only is zero at the fixe edge, at the simply supported edge and at the point support
b) Yes. At the fixed edge the computed myy is not equal to zero, which is correct. On the other edges the computed bending moments are approximately zero, which is correct. (only "approximately" due to the finite element approximation.)
c) Infinitely large because there are singularities. In the re-entrant corner, at the edge of the fixed support, perhaps also at the point support.
d) It is not supposed to be zero. The x-axis is not perpendicular to the edge.

e) Concentrated shear force
f) Infinitely large because it is in a singularity. In reality this value will be smaller due to yielding or cracking of the plate or the fixed support. Therefore, we integrate the shear force value [kN/m] over some width [m] and make sure that the reinforcement (hairpins) in this width can carry this force [kN].
g) 2 x 29.50 kNm/m = 59.00 kN in the z-direction.

Question 5

a) nsx1=nxx1+|nxy1|=340 kN/m
nsy1=nyy1+|nxy1|=240 kN/m
(nsx1-nxx2)(nsy1-nyy2) >? nxy2^2
6000>?48400 : no
b) nsx2=nxx2+|nxy2|=540 kN/m
nsy2=nyy2+|nxy2|=160 kN/m
(nsx2-nxx1)(nsy2-nyy1)>?nxy1^2
6000>?10000 : no
c) assume: ns3=420.8 kN/m = 420.8 N/mm
(ns3-nxx1)(ns3-nyy1)>?nxy1^2
50768>?10000 : yes
(ns3-nxx2)(ns3-nyy2)>?nxy2^2
48465>?48400 : yes
so assumption is just OK
ns3=2*1/4*3.1415*d^2*fy/s : s=269 mm

Question 6

a) $u_x(x,y) = 0, \quad u_y(x,y) = \left(-\frac{xy}{2ab} - \frac{x}{2a}\right)u$
b) $\varepsilon_{xx} = 0, \quad \varepsilon_{yy}(x,y) = -\frac{x}{2ab}u, \quad \varepsilon_{xy}(x,y) = -(\frac{y}{2ab} + \frac{1}{2a})u$
c) see a)
d) as b) except $\varepsilon_{xy}(x,y) = -\frac{1}{2a}u$