Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CIE4180: Plate Analysis
Thursday 11 April 2018, 13:30 – 16:30

This exam consists of 6 questions.
The respective weights are 1.8, 1.6, 1.7, 1.7, 1.6, and 1.6.

Write your name and student number **on every page of your work.**

In this exam the notation and sign conventions of the reader (part 1 and 2) and the book (Plates and FEM) have been adopted. **You are supposed to use the same notation and sign convention.**

Question 1

Consider the plane stress (or plate membrane) problem below. The measurements of the quadrilateral plate with a snipped corner are in mm. The length of the plate is 300 mm. The height is 400 mm. The plate has an uniform thickness

t = 150 mm.

The plate is loaded by an equally distributed line load f ("force per length"), acting at the top edge of the plate in vertical direction.

The plate is also loaded by an horizontal force F_h.

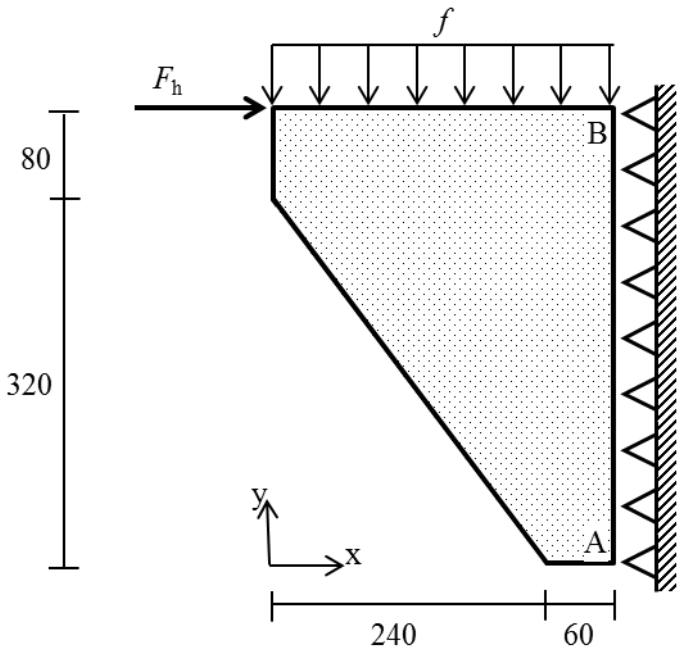
f = ? N/mm, F_h = ? kN.

At the right hand side of the plate, both vertical and horizontal displacements are prohibited.

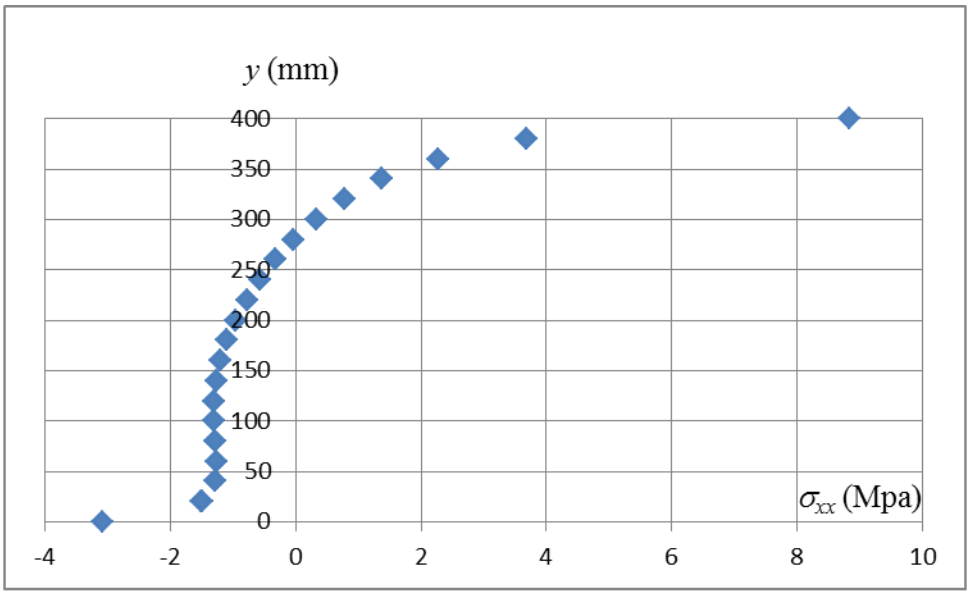
An adequate finite element analysis has been made using a mesh with 4-noded plane stress elements of approximately 20×20 mm^2.

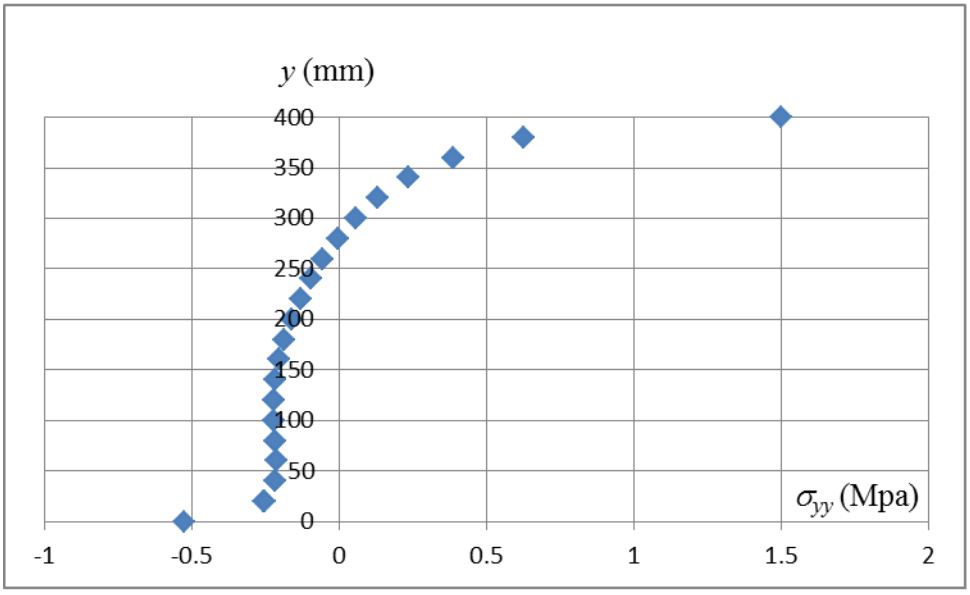
E = 30000 N/mm^2 , ν = ?

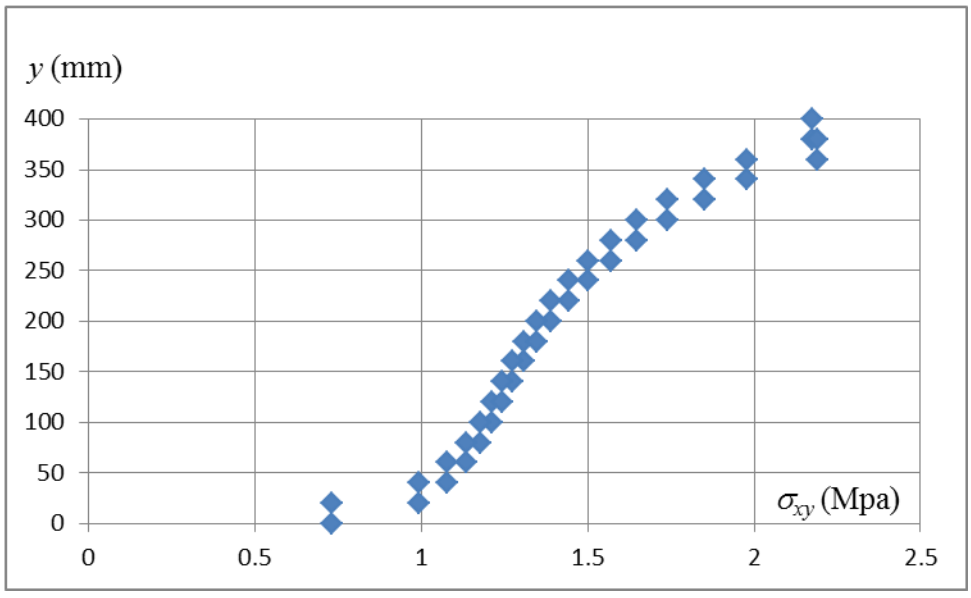
Along the vertical line A-B the stress distributions σ_{xx}, σ_{yy} and σ_{xy} are obtained from the analysis. These are presented on the next page.



a) Calculate F_h.
b) Calculate the Poisson's ratio ν
c) Calculate f.


y (mm)
400
350
300
250
200
150
100
50
0
-4
-2
0
2
4
6
8
10
σxx (Mpa)


y (mm)
400
350
300
250
200
150
100
50
0
-1
-0.5
0
0.5
1
1.5
2
σyy (Mpa)


y (mm)
400
350
300
250
200
150
100
50
0
0
0.5
1
1.5
2
2.5
σxy (Mpa)

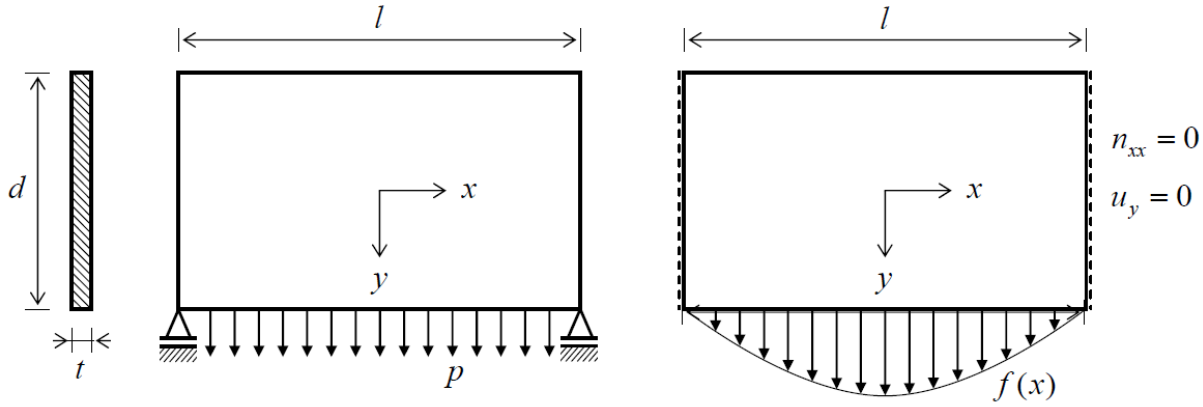
Question 2

a) Consider a rectangular plate with the following field of displacements:

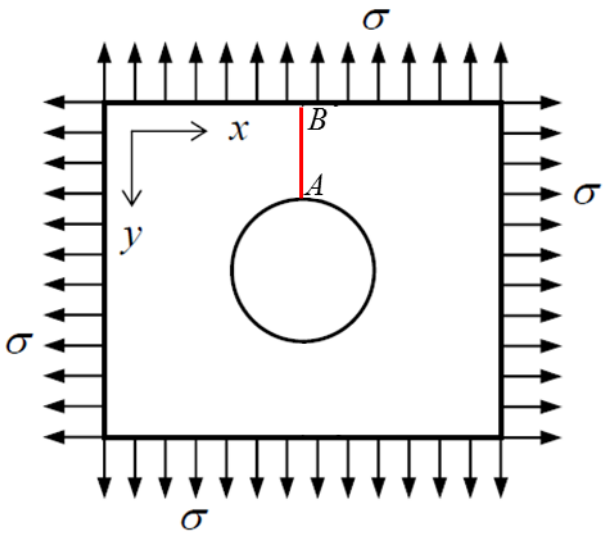
$$u_x(x,y) = 0.007 - \quad 0.003y$$
$$u_y(x,y) = 0.005 + 0.003x$$

Describe the displacement field, the deformation field and the stress field in words. Use no more than 50 words in total.

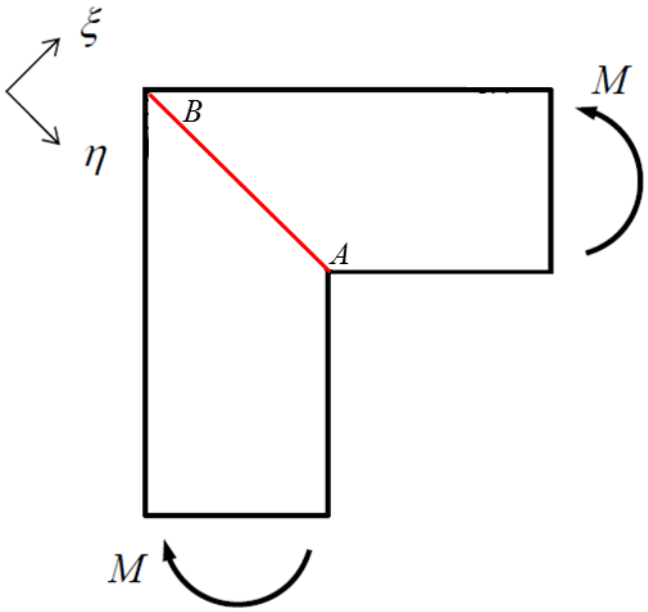
b) Consider a deep beam on two simple supports, which is loaded along its bottom edge by a homogeneously distributed load p, as is shown in the left-hand part of the figure below. We want to determine an analytical solution of the bending stress σ_{xx} in the vertical axis $x = 0$. We replace the structure and load by the problem stated in the right-hand part of the figure. Describe the changes in the two problems and explain *why* these changes have been made. Use no more than 50 words in total.



c) Consider a circular hole in a plate subjected to a homogeneously distributed stress state in which the (normal) stress σ is equal in all directions. Consider cross section AB in the figure below. Sketch the distributions σ_{xx} and σ_{yy} along AB. Indicate extreme values and provide expressions for these extreme values.



d) Consider a beam-column connection with no rounding in the corner, loaded by a moment *M*. Consider cross section AB in the figure below. Sketch the distributions $\sigma_{\xi\xi}$ along AB, where ξ is the direction perpendicular to AB.



Question 3

a) What is a limitation of the plate theories studied in this course.
Choose from A, B, C, D. You can select more than one answer.

A They are valid for elastic materials only.
B They are valid for small displacements only.
C They are valid for small loads only.
D They are valid for a uniformly distributed load only. (Jumps in the load are allowed at internal boundaries.)

b) What is the correct definition of the torsion moment in a thin plate?
Choose A, B, C or D.

A $m_{xy} = V$
B $m_{xy} = \int_{-t/2}^{t/2} z\sigma_{xy} dz$
C $m_{xy} = \frac{1}{2}(1-v)D\rho_{xy}$
D $m_{xy} = \frac{\partial n_{xy}}{\partial x} + \frac{\partial n_{yx}}{\partial y}$

c) Many engineers and software developers think that we do not really need the Kirchhoff plate theory because the Mindlin–Reissner plate theory can be used for thin plates too. Why is this not true?

d) Consider an interior molecule of a plate that is loaded perpendicularly. What is the number of discrete elements in this molecule?

e) What is a limitation of the discrete element method? Choose A, B, C or D.

A The method does not give understanding in the ways that plates carry load.
B The method cannot be used in dynamic analyses.
C The grid needs to consist of squares and not of rectangles or other shapes.
D Poisson's ratio needs to be zero; a nonzero value cannot be included correctly.

f) The elastic plate theory and boundary conditions are correct because …
Choose A, B, C or D.

A … eminent professors say so.
B … they are included in the course readers and the course book.
C … they can be mathematically derived from the three dimensional elasticity theory and a few approximations (suppositions).
D … they have been confirmed by many experiments (Rhoorda 1930, Timoshenko 1936, De Vries and Wasblom 1986, Kirchhoff 2001 *etc*.).

Question 4

A plate is loaded perpendicularly. Most of the edges are free except for a fixed part and a simply supported part (Fig. 6). Also one of the corners is simply supported. The plate length is 10 m and the width is 7 m. The plate thickness is 210 mm. A distributed load of 6 kN/m^2 is applied. This distributed load includes self-weight. A point load of 40 kN is applied in one of the corners.
A linear elastic finite element model is made. E = 32800 N/mm^2, ν = 0.2. Applied are four node plate elements (Kirchhoff theory). The element size is 500 mm. The results are shown in Figures 7 to 12.

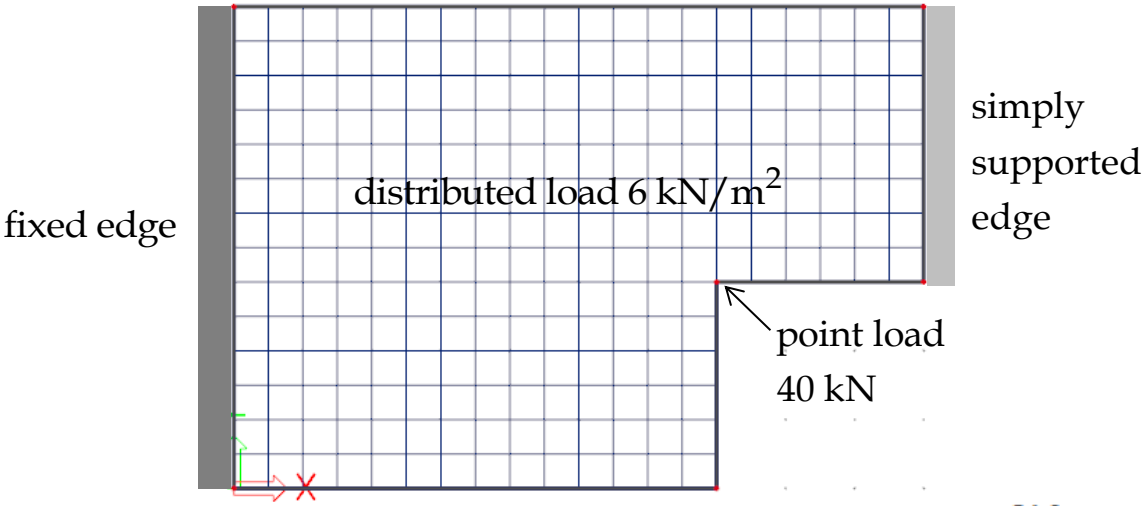


Figure 6. Plate with supports, load and finite element mesh

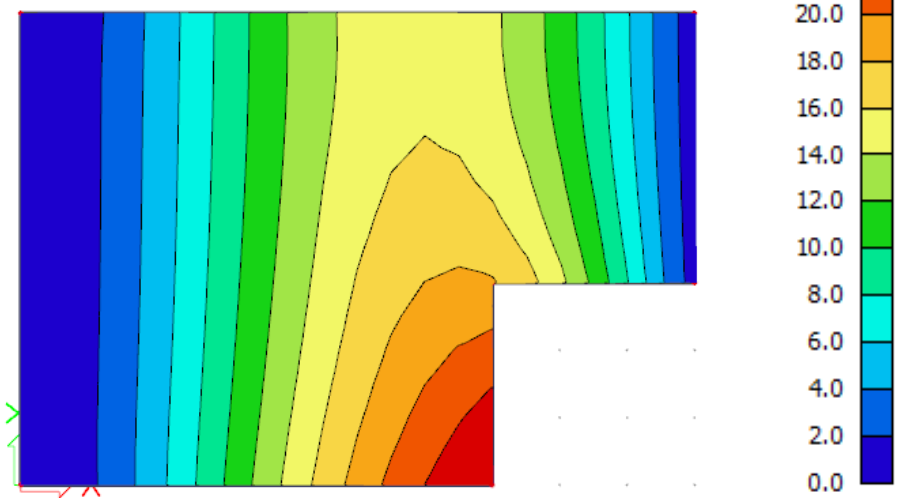


Figure 7. Contour plot of the plate deformation [mm]

a) Is the element size OK? Explain your answer.

b) How large is the plate deflection?
Does the deformation agree with the boundary conditions? Explain your answer.

c) Where in this plate can we expect singularities?

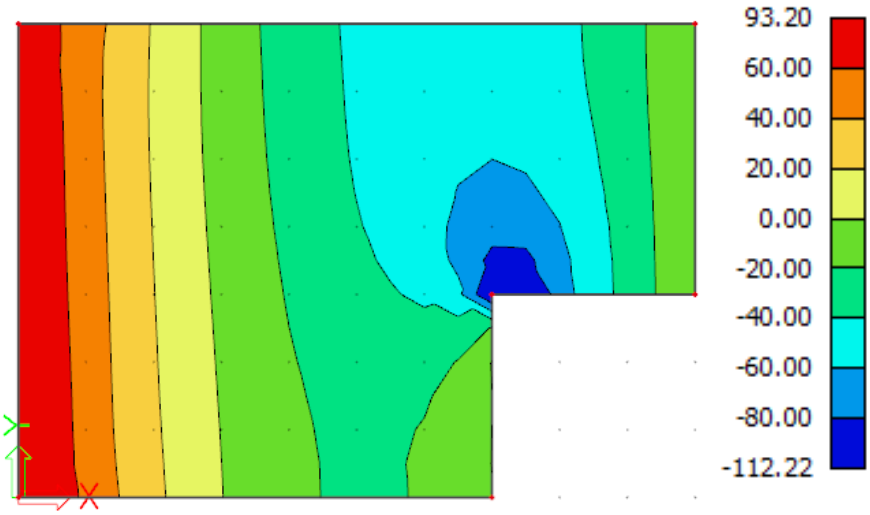


Figure 8. Contour plot of the bending moment m_{xx} [kNm/m]

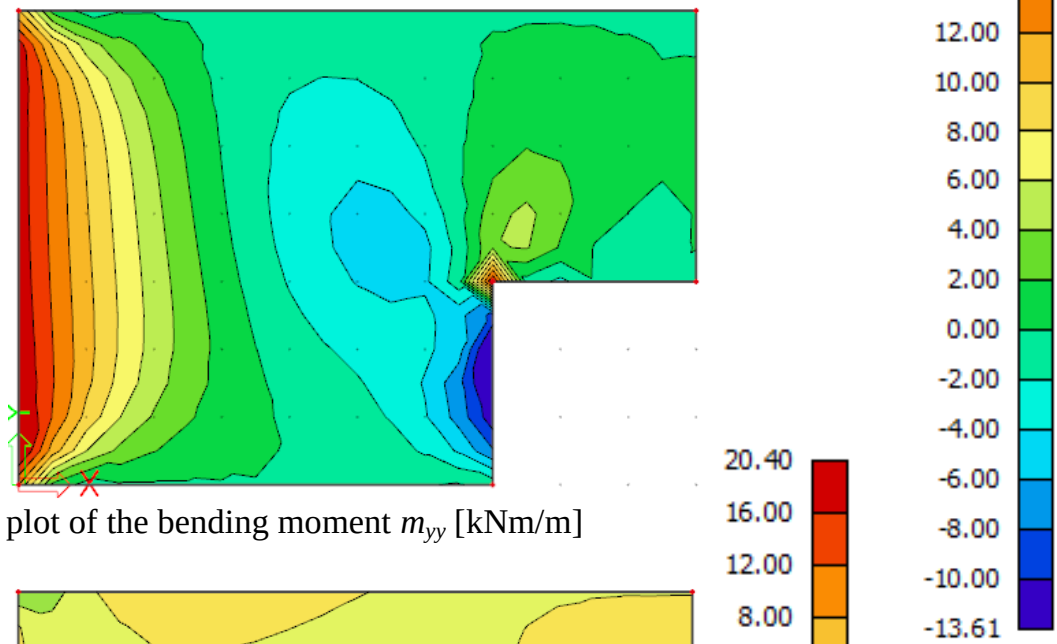


Figure 9. Contour plot of the bending moment m_{yy} [kNm/m]

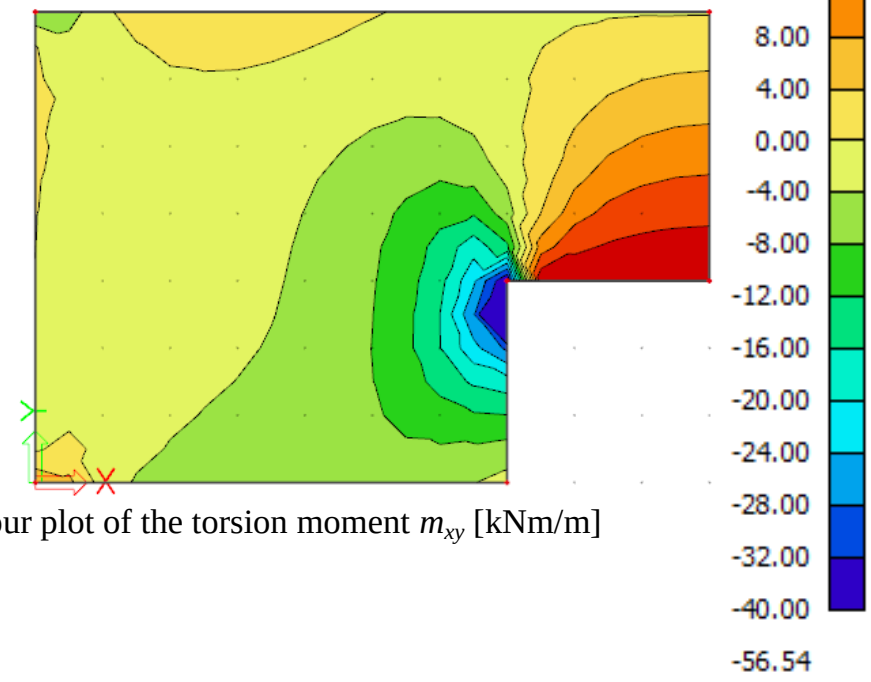


Figure 10. Contour plot of the torsion moment m_{xy} [kNm/m]

d) Do the moments agree with the boundary conditions? Explain your answer.

e) There is a torsion moment m_{xy} in the right hand edge. Apparently, the teacher has applied a distributed torsion moment load up to 20.40 kNm/m to this edge and forgot to specify this load in this question. Do you agree with this conclusion? Is there a better explanation for this computation result? If so, give this explanation.

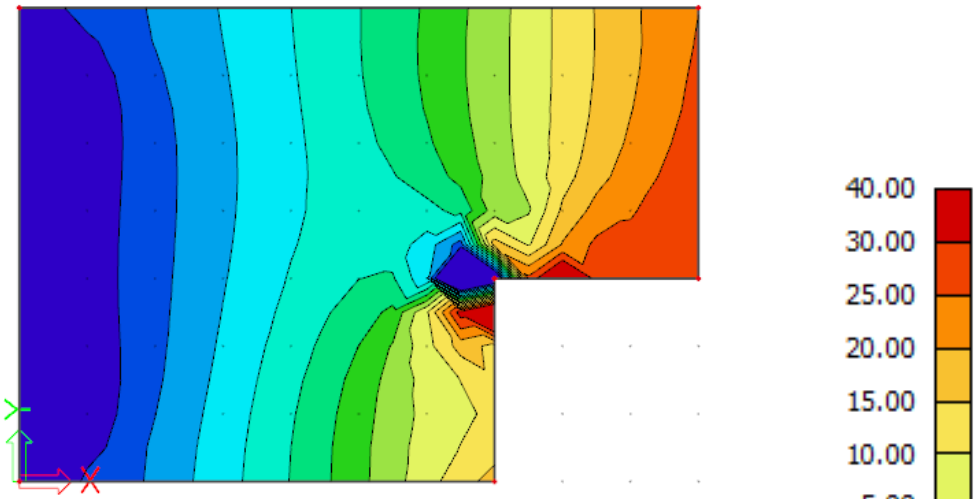


Figure 11. Contour plot of the shear force v_x

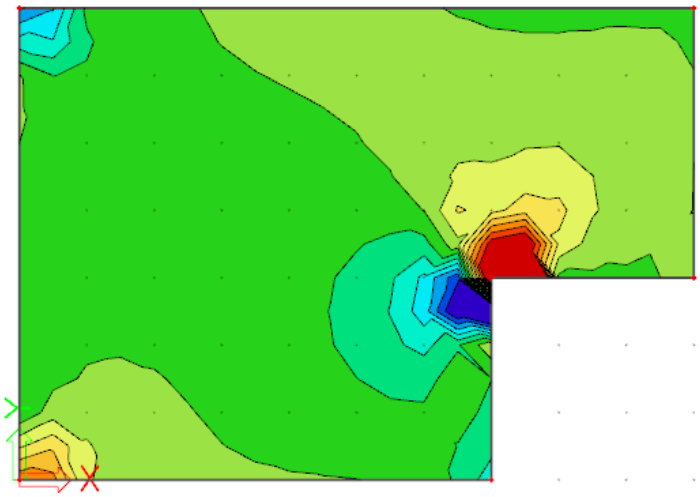


Figure 12. Contour plot of the shear force v_y

f) What is the largest value of v_y in the finite element model of the plate? Where does it occur?

The computation is repeated for an element size of 0.1 m (Fig. 13).

g) Does this improve the accuracy? Explain your answer.

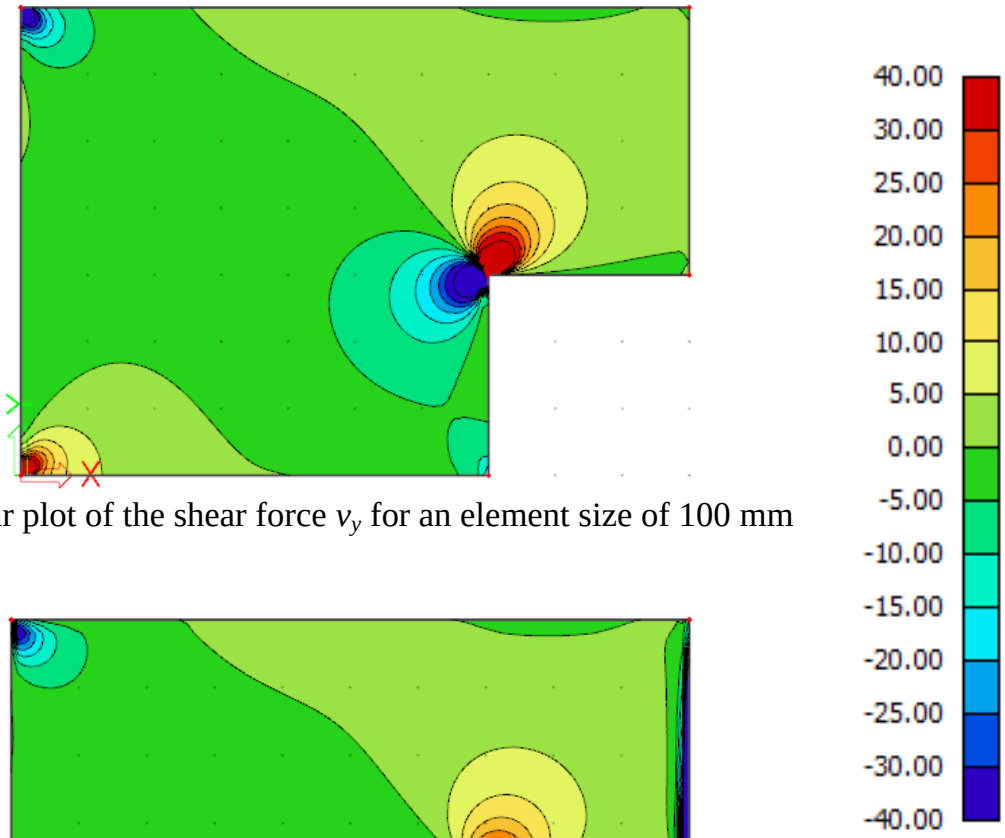


Figure 13. Contour plot of the shear force v_y for an element size of 100 mm

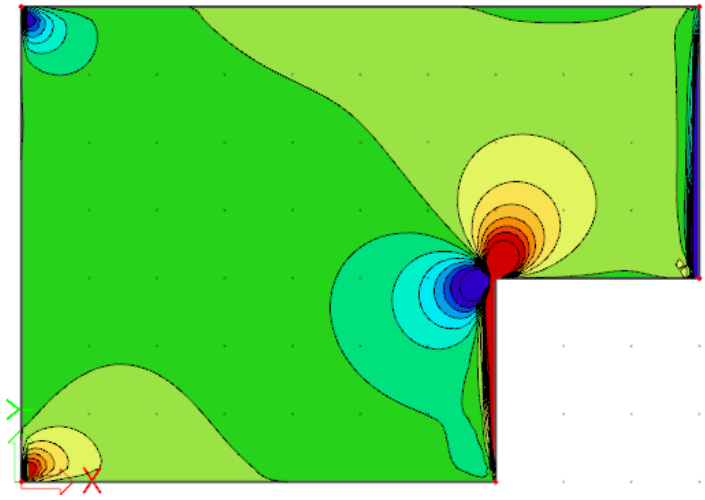


Figure 14. Contour plot of the shear force v_y for an element size of 100 mm and for the Mindlin-Reissner theory

The computation is repeated for the Mindlin-Reissner plate theory (Fig. 14).

h) Does this improve the accuracy? Explain your answer.

Question 5

A reinforced concrete plate has a thickness of 180 mm.
At a particular location the moments are
m_{xx} = 9 kNm/m,
m_{yy} = 28 kNm/m,
m_{xy} = -24 kNm/m
(partial factor included).

The reinforcement consists of two meshes. As usual, one mesh is close to the top surface the other mesh is close to the bottom surface. Each mesh has bars in the *x* direction and bars in the *y* direction. All bars have a diameter of 12 mm and are spaced 150 mm. The yield strength of the reinforcing steel is 500 N/mm^2 (partial factor included). The moment capacity due to the bars in the *x* direction is 53 kNm/m. The moment capacity due to the bars in the *y* direction is 48 kNm/m.

a) The moment capacities are different in the *x* and *y* direction.
Give a reasonable explanation for this.

b) Is the reinforcement sufficient? Explain your answer.

c) Calculate the moment capacities that that are minimally required in the *x* and *y* directions.

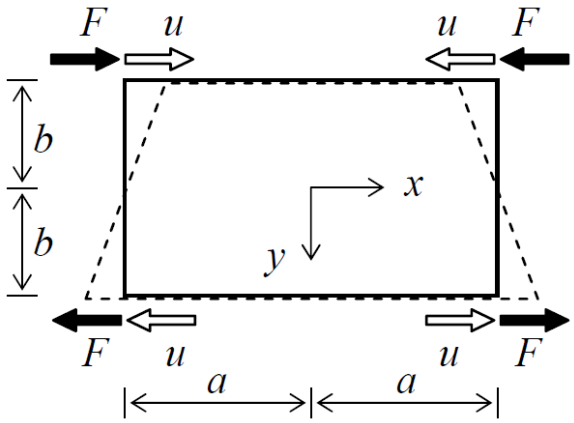
We change the design and direct the reinforcing bars in the principal directions.

d) What moment capacities are now minimally required?

e) Which design; 5b, 5c or 5d requires the least amount of reinforcement?
Explain your answer.

Useful equations
bottom msxb = mxx+|mxy| msyb = myy+|mxy|
top msxt = -mxx+|mxy| msyt = -myy+|mxy|
m1=(mxx+myy)/2+sqrt(((mxx-myy)/2)^2+mxy^2)
m2=(mxx+myy)/2 -sqrt(((mxx-myy)/2)^2+mxy^2)
bottom (msxb-mxx)*(msyb-myy) >= mxy^2
top (msxt+mxx)*(msyt+myy) >= mxy^2

Question 6



Consider the above standard rectangular membrane element with four nodes. The u_x displacements of the four nodes are $+u$ or $-u$, as indicated in the figure. The u_y displacements of the four nodes are all zero.

The Poisson's ratio $\nu = 0$.

a) Express the displacement fields $u_x(x, y)$ and $u_y(x, y)$ in x, y, u , a and b.

b) Similar for the strain fields $\varepsilon_{xx}(x, y)$, $\varepsilon_{yy}(x, y)$ and $\gamma_{xy}(x, y)$.

c) Similar for the internal strain energy density field (i.e. energy stored per unit of volume).

d) Explain in words why a standard rectangular membrane element is too stiff to produce states of constant moment. Make possibly use of your answer **c)**. Use no more than 50 words.

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Brief answers, exam CIE4180: Plate Analysis
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Question 1

a) $F_{\mathrm{h}} = t\int_0^{400} \sigma_{xx}\, dy \approx 10000\ \mathrm{N} = 10\ \mathrm{kN}$

b) $\varepsilon_{\mathrm{yy}} = \frac{1}{E}(\sigma_{yy} - \nu\sigma_{xx})$ with $\varepsilon_{\mathrm{yy}} = 0$ along AB gives $\nu = \frac{\sigma_{yy}}{\sigma_{xx}} \approx 0.17$ for an arbitrary point along AB.

c) $300\ f = t\int_0^{400} \sigma_{xy}\, dy \approx 600\ t$ gives $f \approx 300\ \mathrm{N/mm}$

Question 2

a) This field consists of only rigid body displacements. All three strains and all three stresses are zero.

b) The supports have been replaced by boundary conditions for both vertical edges: vertical displacements are prohibited. The homogeneously distributed load *p* is replaced by a varying load, which has a cosine distribution. These changes have been made to facility the derivation of an analytical solution.

c) From A to B σ_{xx} varies from 2σ to σ and σ_{yy} varies from 0 to σ.

d) From A to B $\sigma_{\xi\xi}$ varies from +infinity to zero, becomes negative and again zero in B.

Question 3

a) A, B

b) B

c) In theory, Mindlin–Reissner can be used for thin plates too. However, Mindlin–Reissner requires very small finite elements at the edges in order to obtain accurate results. In practice this is inconvenient and takes much time. (So, when you meet a software developer, tell him or her that we do need the Kirchhoff plate theory. One day they will include it in our software.)

d) 10

e) D

f) C

Question 4

a) The element size is OK. It needs to be approximately 2 times the thickness.

b) 25 mm. The fixed and simply supported edges have zero displacement. This is OK.

c) At point loads and point supports, in re-entrant corners, where a support stops.

d) All OK.
Fixed edge: Bending moments in both directions (the same sign) and no torsion moment.
Simply supported edge: No bending moments but a torsion moment.
Free edge: No bending moment perpendicular to the edge.
Along the edge a bending moment and a torsion moment.

e) A torsion moment m_{xy} on an edge needs to be interpreted as a concentrated shear force *V* in the edge.

f) Infinite (singularity)

g) More accurate. But from a practical point of view there is no need for more accuracy.

h) Less accurate. The concentrated shear force V is visible in the v_y plot. (Not shown is that V goes to zero in the m_{xy} plot.) But to compute this accurately the elements need to be even smaller than 100 mm.

Question 5

a) Practically, the *x* bars and the *y* bars cannot cross trough each other. The *y* bars provide less capacity while they have the same strength, diameter and spacing as the *x* bars. Therefore, they must have a smaller internal lever arm, so they must have more cover.

b) (msxb-mxx)*(msyb-myy) >= mxy^2
(53-9)*(48-28)>=?(-24)^2
880>=?576
(msxt+mxx)*(msyt+myy) >= mxy^2
(53+9)*(48+28)>=?(-24)^2
4712>=?576
more than sufficient capacity

c) bottom msxb = mxx+|mxy| = 9+24=33 kNm/m msyb = myy+|mxy| = 28+24=52 kNm/m
top msxt = -mxx+|mxy| =-9+24=15 kNm/m msyt =-myy+|mxy| =-28+24=0 kNm/m

d) m1=(mxx+myy)/2+sqrt(((mxx-myy)/2)^2+mxy^2)
=(9+28)/2+sqrt((9-28)^2/4+(-24)^2
=18.5+25.8
=44.3 kNm/m
m2=18.5-25.8
=-7.3 kNm/m
In the first principal direction we need a moment capacity of 44.3 kNm/m (bottom bars)
In the second principal direction we need a moment capacity of 7.3 kNm/m (top bars)

e) 5b) 53+48+53+48=202 kNm/m
5c) 33+52+15+0=100 kNm/m
5d) 44.3+0+0+7.3=51.6 kNm/m
Design 5d requires the least amount of reinforcement.
(However, bending the bars in the principal directions is very labour intensive.)

Question 6

a) $u_x(x,y) = \frac{x}{a}\frac{y}{b}u$ and $u_y(x,y) = 0$

b) $\varepsilon_{xx}(x,y) = \frac{1}{a}\frac{y}{b}u$, $\varepsilon_{yy}(x,y) = 0$ and $\gamma_{xy}(x,y) = \frac{x}{a}\frac{1}{b}u$

c) $\sigma_{xx}(x,y) = E\frac{1}{a}\frac{y}{b}u$, $\sigma_{yy}(x,y) = 0$ and $\sigma_{xy}(x,y) = \frac{E}{2}\frac{x}{a}\frac{1}{b}u$ gives

$U(x,y) = \frac{1}{2}\varepsilon_{xx}\sigma_{xx} + 0 + \frac{1}{2}\gamma_{xy}\sigma_{xy}$

d) Although $\sigma_{xx}(x,y)$ shows that the bending stresses vary linear over the height of the element, the last term of $u(x,y)$ shows that the element simultaneously produces shear strains, stresses and internal energy.