Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CIE4180: Plate Analysis
Thursday 1 February 2018, 13:30 – 16:30

This exam consists of 6 questions.
The respective weights are 1.7, 1.7, 1.5, 1.9, 1.7, and 1.5.

Write your name and student number **on every page of your work.**

In this exam the notation and sign conventions of the reader (part 1 and 2) and the book (Plates and FEM) have been adopted. **You are supposed to use the same notation and sign convention.**

Question 1

Consider the plane stress (or plate membrane) problem below. The measurements of the quadrilateral plate with a snipped corner are in mm. The length of the plate is 300 mm. The height is 300 mm. The plate has an uniform thickness t = 100 mm.

The plate is loaded by an equally distributed line load f ("force per edge length"), acting at the snipped corner of the plate in vertical direction, f = 100 N/mm.

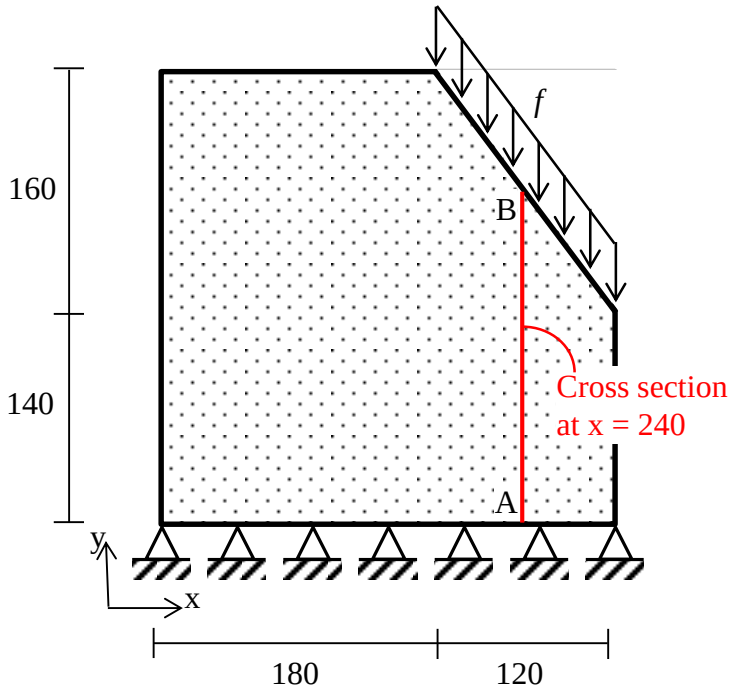
At the bottom of the plate, both vertical and horizontal displacements are prohibited.

An adequate finite element analysis has been made using a mesh with 8-noded plane stress elements of 5×5 mm^2.

E = 30000 N/mm^2 , ν = 0.2

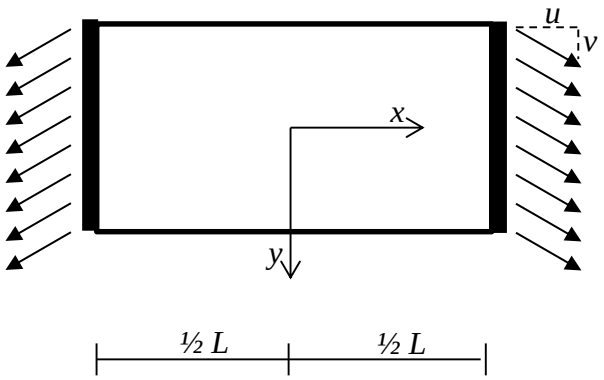
Along the vertical line A-B (indicated as "Cross section") the stress σ_{yy} is obtained from the analysis. The stresses in the points A and B are:

- Point A: σ_{yy} = -1.33 MPa
- Point B: σ_{yy} = -1.67 MPa



a) Calculate the strain ε_{yy} in point A.
b) Calculate the shear stress σ_{xy} in point B
c) Calculate the strain ε_{yy} in point B

Question 2



Consider a rectangular plates of unit thickness and length L, see above. Assume a plane stress (or plate membrane) situation.

The plate is free to deform along the two horizontal edges.

Along both vertical edges uniform deformations in x- and y-directions are applied:

$$u_x(x = \tfrac{1}{2}L, y) = u$$

$$u_x\left(x = -\tfrac{1}{2}L, y\right) = -u$$

$$u_y\left(x = -\tfrac{1}{2}L, y\right) = u_y(x = \tfrac{1}{2}L, y) = v$$

where u and v are positive constants.

As a trial solution for the displacement field of the plate the following polynomials terms are considered:

$$u_x(x,y) = a_1 + a_2x + a_3y + a_4x^2 + a_5xy + a_6y^2 + a_7x^3 + a_8x^2y + a_9xy^2 + a_{10}y^3 + a_{11}x^3y + a_{12}xy^3$$

$$u_y(x,y) = b_1 + b_2x + b_3y + b_4x^2 + b_5xy + b_6y^2 + b_7x^3 + b_8x^2y + b_9xy^2 + b_{10}y^3 + b_{11}x^3y + b_{12}xy^3$$

a) Assume that the Poisson's ratio ν = 0. Which terms would you select for u_x and u_y?
b) Now assume that ν ≠ 0. The strain ε_{yy} will now vary as a function of x. Which terms would you select for u_y such that you can capture this varying strain ε_{yy}?
c) Consider the coefficients b_i's of the terms you selected in answer **b)**. Give two expressions for these b_i's in which you relate them to u, v, L or to each other.

Note: only mention relevant, non-zero, terms.

Question 3

a) Order the following words from small to large: infinite; infinitesimal; finite.

b) What is the correct equation for calculating the concentrated shear force *V* in a plate edge? Choose A, B, C or D.

A $V = \frac{\partial m_{xy}}{\partial x} + v_y$
B $V = 2m_{xy}$
C $V = m_{xy}$
D $V = \frac{\partial n_{xy}}{\partial x} + \frac{\partial n_{xy}}{\partial y}$

c) A rectangular deep beam is modelled with the finite difference method.
The beam does not have openings.
How many interior molecules are needed?
How many edge molecules are needed?
How many corner molecules are needed?
Choose A, B, C or D.

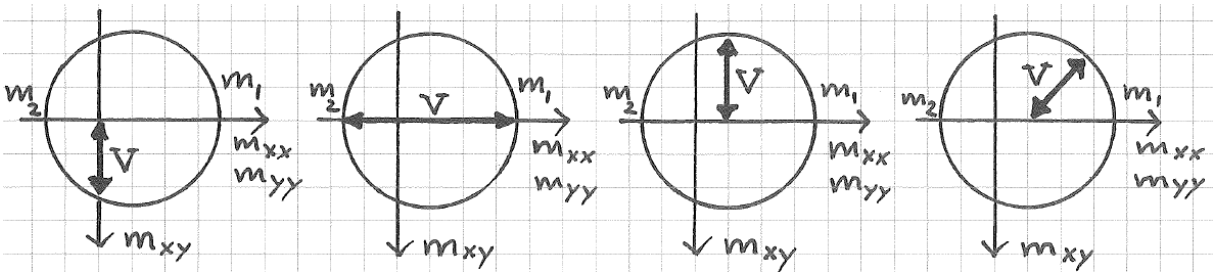
A 1, 2, 4
B 1, 4, 4
C 2, 4, 4
D 2, 8, 8

d) What is the number of discrete elements in a deep beam interior molecule?

e) Plate molecules can correctly include Poisson's ratio.
How are these molecules obtained? Choose A, B, C or D.

A They can be derived with the discrete element method.
B They can be copied from the course readers or course handouts.
C They can be derived from the differential equations and boundary conditions.
D They can be derived with the virtual work equation and appropriate shape functions.

f) Which of the following figures shows correctly the concentrated shear force *V* ?



A B C D

Question 4

A plate is loaded perpendicularly. Most of the edges are free except for a fixed part and a simply supported part (Fig. 4). Also one of the corners is simply supported. The plate length is 12 m and the width is 7 m. The plate thickness is 250 mm. A distributed load of -5 kN/m^2 is applied. This distributed load includes self-weight. A point load of -50 kN is applied in one of the corners. A linear elastic finite element model is made. E = 30500 N/mm^2, ν = 0.2. Applied are four node plate elements (Kirchhoff theory). The element size is 500 mm. The results are shown in Figures 5 to 10.

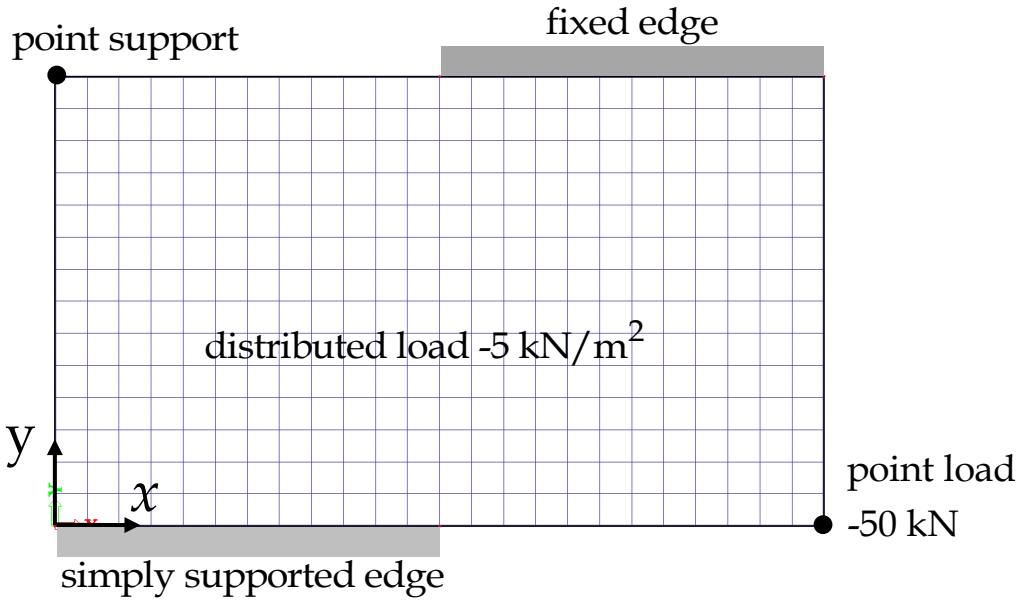


Figure 4. Plate with supports, load and finite element mesh

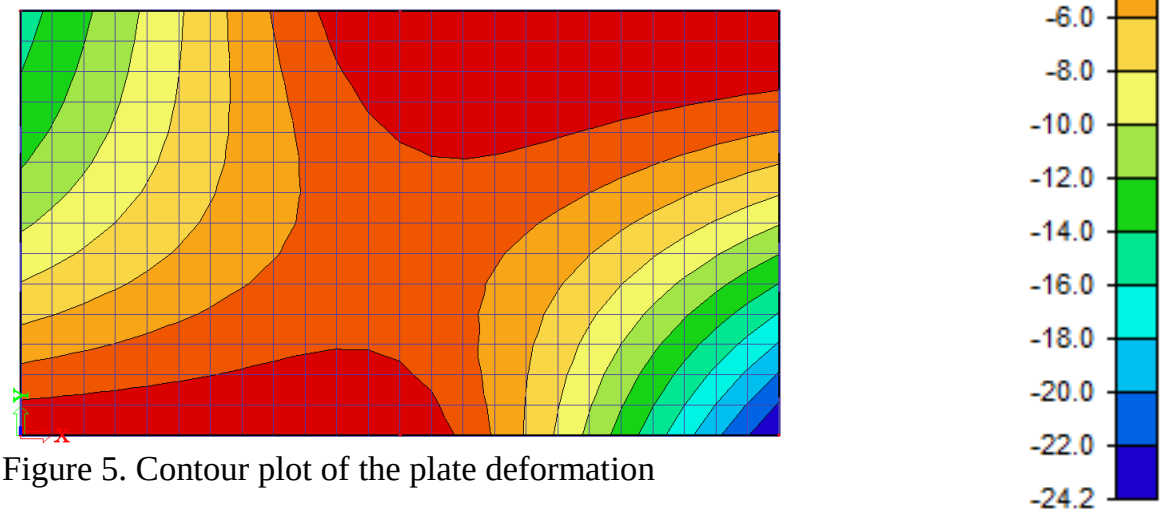


Figure 5. Contour plot of the plate deformation

a) How large is the plate deflection?
Does the deformation agree with the boundary conditions? Explain your answer.

b) Do the moments agree with the boundary conditions? Explain your answer.

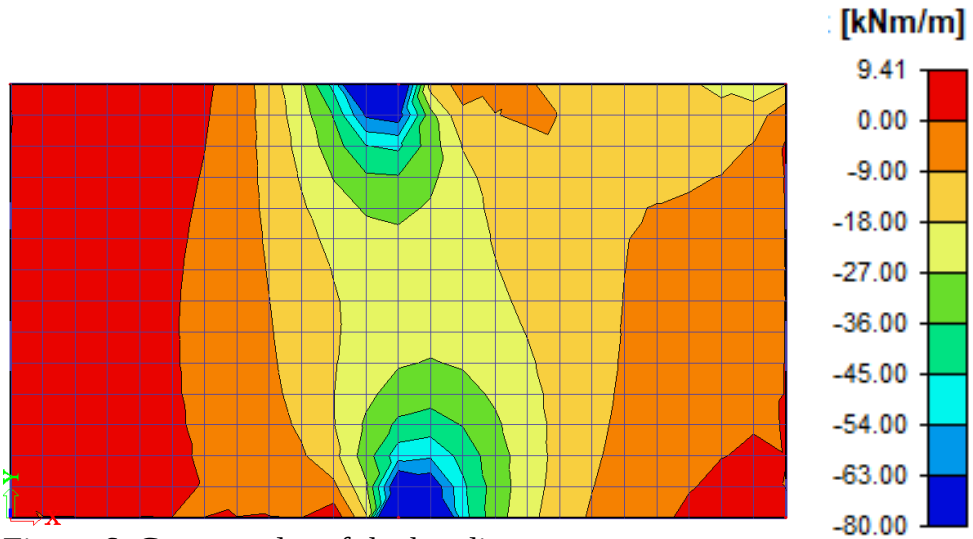


Figure 6. Contour plot of the bending moment m_{xx}

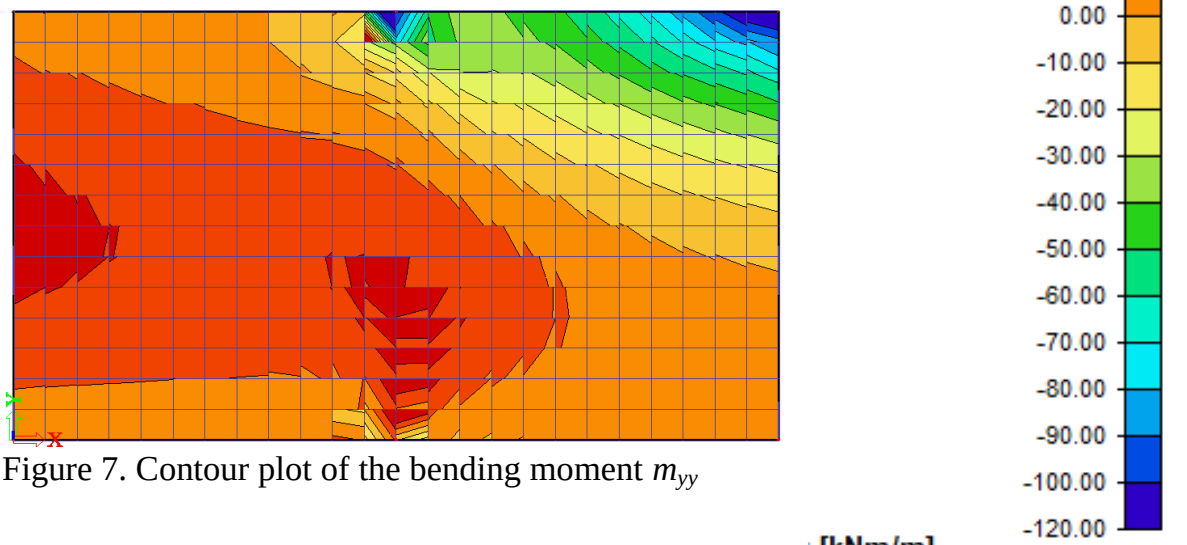


Figure 7. Contour plot of the bending moment m_{yy}

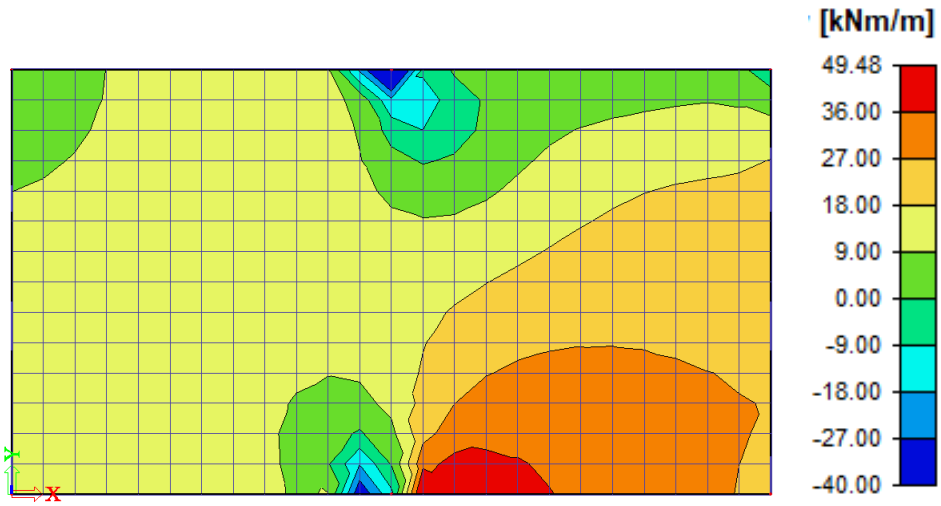


Figure 8. Contour plot of the torsion moment m_{xy}

c) What is the largest value of the bending moment m_{xx} in the plate? (ignoring the sign)

d) All contour plots are averaged in the nodes except for one.
Which contour plot is not averaged in nodes?

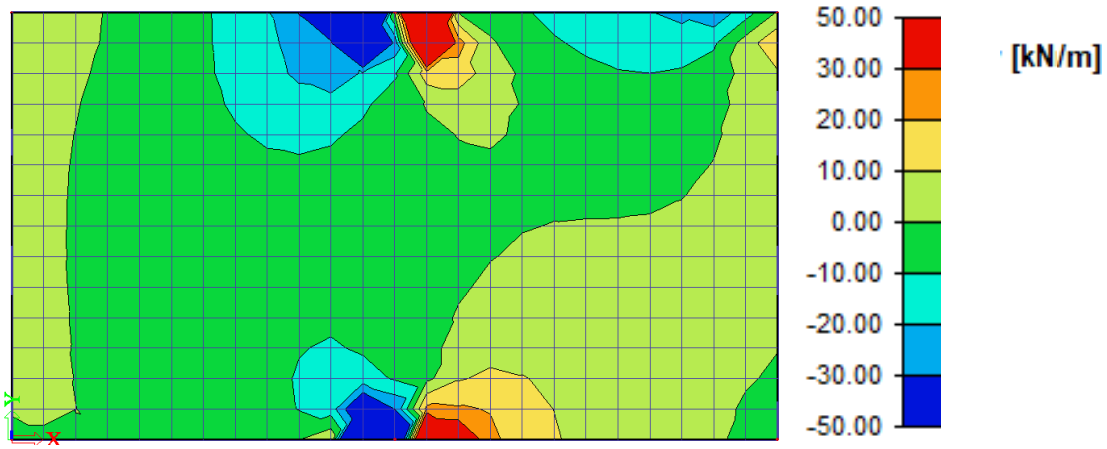


Figure 9. Contour plot of the shear force v_x

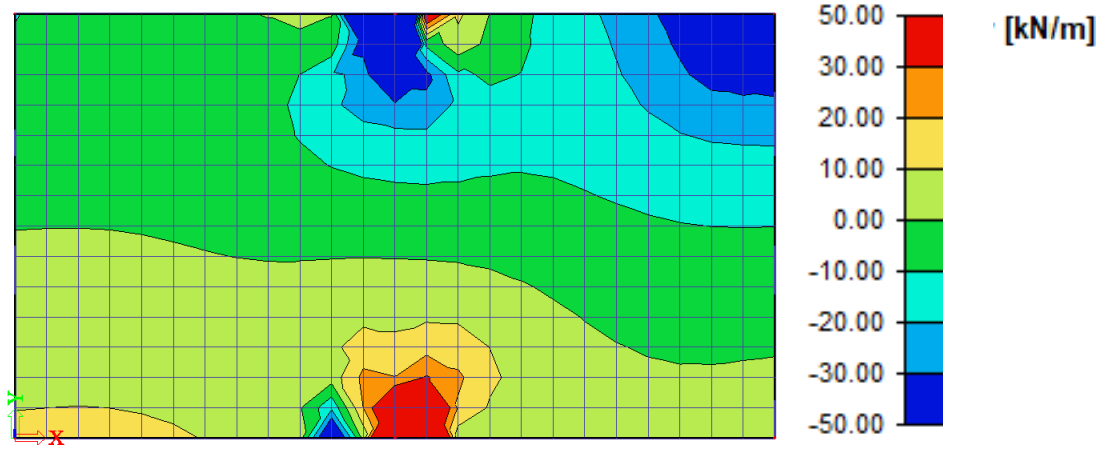


Figure 10. Contour plot of the shear force v_y

e) On the right hand edge, the torsion moment m_{xy} is not zero. Give a correct explanation of this result.

f) At location (x = 6.5 m, y = 0) there is a large positive value of v_y . However, this location is a free edge and on a free edge there cannot be a shear force. Give a correct explanation of the large value of v_y .

g) Is the point load carried by v_x, v_y or by V ? Explain your answer.

Question 5

A reinforced concrete plate has a thickness of 100 mm.
At a particular location the membrane forces are
n_{xx} = 1200 kN/m,
n_{yy} = -200 kN/m,
n_{xy} = -400 kN/m.

The reinforcement consists of two meshes. Each mesh has bars in the *x* direction and bars in the *y* direction. The bars have a diameter of 16 mm and are spaced 150 mm
The yield strength of the reinforcing steel is 500 N/mm^2 (partial factor included).

a) Is the reinforcement sufficient? Explain your answer.

b) Calculate the reinforcement strength [kN/m] that that is minimally required in the *x* and *y* directions.

The concrete strength is 30 N/mm^2 (partial factor included).

c) Is the concrete strength sufficient for answer 5b? Explain your answer.

We change the design and direct the reinforcing bars in the principal directions.
The principal membrane forces are n_1 = 1306 kN/m, n_2 = -306 kN/m.

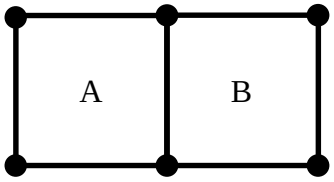
d) What reinforcement strength [kN/m] is now minimally required?

The specific mass of the reinforcement is 7850 kg/m^3.

e) Calculate the reinforcement mass per cubic meter of reinforced concrete of question 5a, 5b and 5d.

Useful equations
nsx = nxx+|nxy|
nsy = nyy+|nxy|
nc= -2*|nxy|
(nsx-nxx)*(nsy-nyy) >= nxy^2

Question 6

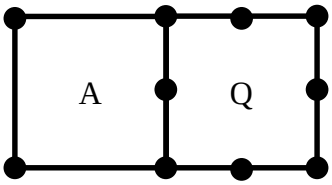


Consider a finite element model with two finite elements, A and B. See the figure above.

a) Assume that both A and B are standard rectangular membrane elements with four nodes. Both elements have the same properties. Is compatibility of displacements preserved between the elements? Explain your answer in maximum 50 words.

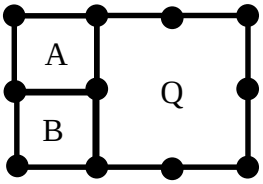
b) Again assume that both A and B are standard rectangular membrane elements with four nodes. Now, element A and B have a different thickness. Is compatibility of displacements preserved between the elements?

c) Now assume that A is a standard rectangular membrane element and B is a constant shear element. Is compatibility of displacements preserved between the elements?



Consider a finite element model with two finite elements, A and Q. See the figure above.

d) Element A is a standard rectangular membrane element with four nodes. Element Q is a quadratic rectangular membrane element with eight nodes. Is compatibility of displacements preserved between the elements? Explain your answer in maximum 50 words.



Consider a finite element model with three finite elements, A, B and Q. See the figure above.

e) Elements A and B are standard rectangular membrane elements with four nodes. Element Q is a quadratic rectangular membrane elements with eight nodes. Is compatibility of displacements preserved between the elements? Explain you answer in maximum 50 words.

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Brief answers, exam CIE4180: Plate Analysis
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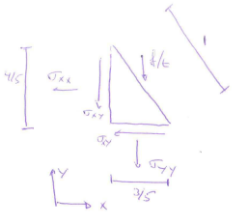
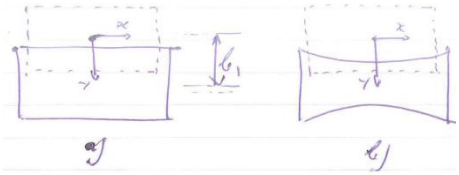
Question 1



a) $(1-\nu^2)\,\sigma_{yy} = E\,(\nu\varepsilon_{xx} + \varepsilon_{yy})$ with $\varepsilon_{xx} = 0$ gives $\varepsilon_{yy} = -0.000043$
b) From horizontal and vertical equilibrium at the snipped corner:
$(4/5)\,\sigma_{xx} + (3/5)\,\sigma_{xy} = 0$ and $(4/5)\,\sigma_{xy} + (3/5)\,\sigma_{yy} + (1)\,f/t = 0$.
The second equation gives $\sigma_{xy} = 0.0025$ MPa
c) The first equation then gives $\sigma_{xx} = -0.0019$ MPa
and $E\,\varepsilon_{yy} = -\nu\sigma_{xx} + \sigma_{yy}$ gives $\varepsilon_{yy} = -0.000056$

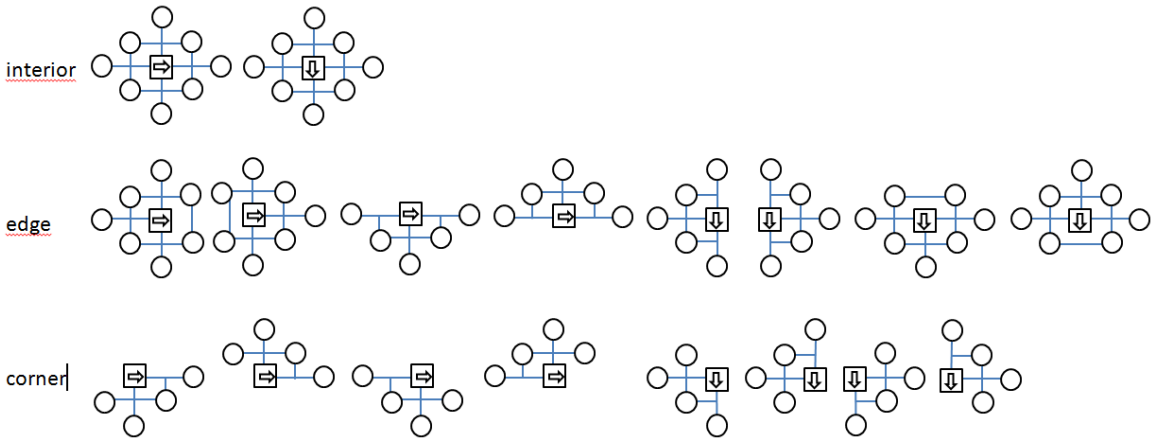
Question 2



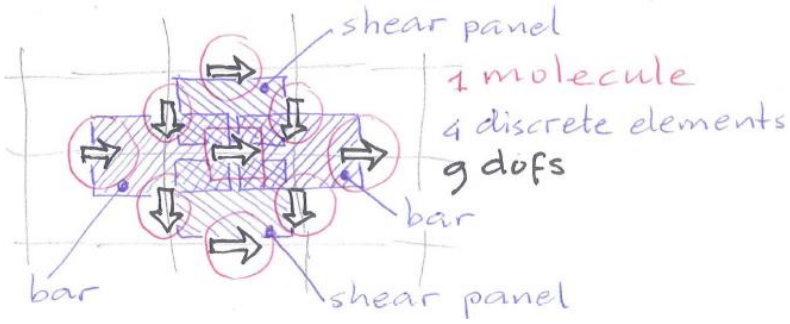
a) $u_x = a_2 x$ and $u_y = b_1$
b) $u_y = b_1 + b_3 y + b_8 x^2 y$
Note that we need even powers of x (since $u_y(x) = u_y(-x)$ for all x) and odd powers of y (since $u_y(y) = -u_y(-y)$ for all y and neglecting the rigid body term b_1)
c) $b_1 = v$
Noting that $\varepsilon_{yy} = b_3 + b_8 x^2$ and that $\varepsilon_{yy} = 0$ for $x = \pm\frac{1}{2}L$ gives $4b_3 + b_8 L^2 = 0$.
Alternatively, noting that $\forall y,\ v = b_1 + b_3 y + b_8(\pm\frac{1}{2}L)^2 y$ or $\forall y,\ v = b_1 + (b_3 + \frac{1}{4}b_8 L^2)y$ also gives $b_1 = v$ and $4b_3 + b_8 L^2 = 0$.

Question 3

a) infinitesimal; finite; infinite.
b) C
c) D



d) 4



e) C
f) A

Question 4
a)
24.2 mm
No, there is a deflection at the point support. It seems that the point support has not been modelled. This is a mistake. The deflection at the clamped part is correctly zero. The deflection at the simply supported part is correctly zero.
b)
Yes, m_{xx} is zero at the edges $x = 0$ and $x = 12$ m. m_{yy} is zero at the edges $y = 0$ and $y = 7$, except for the part that is fixed. This is correct.
c)
Infinity … The largest values are in singularities.
d)
m_{yy}
e)
m_{xy} on this edge needs to be interpreted as a concentrated shear force in the edge.
f)
The boundary condition of the free edge is not $v_y = 0$. The correct boundary condition is

$$\frac{\partial m_{xy}}{\partial x} + v_y = 0$$

m_{xy} changes strongly at this location (Fig. 8), therefore, v_y is not zero.
g)
By V. The results show that 18 kN < V < 27 kN and 0 < $|v_x|$, v_y < 10 kN/m. The rule that 50 kN = 2V seems confirmed in the finite element results.

Question 5
a)
nsxa = nsya = 2*(1/4*3.14*16^2 * 1000/150)*fy = 1340 kN/m
(nsxa-nxx)*(nsya-nyy) >=? nxy^2
216 227 >=? 160 000 OK
b)
nsxb = nxx+|nxy| = 1600 kN/m
nsyb = nyy+|nxy| = 200 kN/m
c)
nc= -2*|nxy|

sigma = nc/t = 8 N/mm^2 <? 30 N/mm^2 OK

d)

nsxd = n1= 1306 kN/m

nsyd = 0

e)

(nsxa+nsya)*rho/(t*fy) = 421 kg/m^3

(nsxb+nsyb)*rho/(t*fy) = 283 kg/m^3

(nsxd+nsyd)*rho/(t*fy) = 205 kg/m^3

Question 6

a) Yes. For both elements the displacements along the common edge depend only on the two common nodes. Both elements interpolate these displacements in the same way.

b) Yes. (see **a)**)

c) Yes. (see **a)**)

d) No. Element A will *linearly* interpolate the displacements along the common edge between the common corner nodes. Element Q will *quadratically* interpolate the displacements along the common edge between the common corner nodes. Gapping or overlapping can occur.

e) No. Consider the conjugated edge between elements A&B and Q and the three common nodes. The elements A&B will *piecewise linearly* interpolate the displacements of this edge. Element Q will *quadratically* interpolate these displacements. Gapping or overlapping can occur.