Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CIE4180: Plate Analysis
Friday 9 April 2021, 09:00 – 10:30

This exam consists of 6 questions, each with sub-questions.
All sub-questions will have the same weight in the total evaluation.

In this exam the notation and sign conventions of the reader (part 1 and 2) and the book (Plates and FEM) have been adopted. **You are supposed to use the same notation and sign convention.**

Question 1

The following equations may be useful for this question:

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x}$$
$$\varepsilon_{yy} = \frac{\partial u_y}{\partial y}$$
$$\gamma_{xy} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}$$

$$-\left(\frac{\partial n_{xx}}{\partial x} + \frac{\partial n_{yx}}{\partial y}\right) = p_x$$
$$-\left(\frac{\partial n_{yy}}{\partial y} + \frac{\partial n_{xy}}{\partial x}\right) = p_y$$

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \frac{1}{E\,t} \begin{bmatrix} 1 & -\upsilon & 0 \\ -\upsilon & 1 & 0 \\ 0 & 0 & 2(1+\upsilon) \end{bmatrix} \begin{Bmatrix} n_{xx} \\ n_{yy} \\ n_{xy} \end{Bmatrix} \quad (\textit{constitutive equations in flexibility formulation})$$

$$\begin{Bmatrix} n_{xx} \\ n_{yy} \\ n_{xy} \end{Bmatrix} = \frac{E\,t}{1-\upsilon^2} \begin{bmatrix} 1 & \upsilon & 0 \\ \upsilon & 1 & 0 \\ 0 & 0 & (1-\upsilon)/2 \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} \quad (\textit{constitutive equations in stiffness formulation})$$

Consider the plane stress (or plate membrane) problem of a square plate, see Figure 1.1 below.

The length l and depth d of the plate is 1100 mm.

The thickness t is 100 mm.

The Young's modulus is 30000 MPa and the Poisson's ratio is 0.20.

The plate is supported at 2 points.

The plate is loaded, in-plane, by a uniformly distributed load f = 2100 N/mm at the top of the plate. There is no dead load.

The x- and y-axes are indicated in the figure. The origin of the axes system is at the centre of the plate.

Two finite element analyses are performed, both with linear (4-noded) plane stress elements, but with different element sizes:

Coarse mesh: the element size is 100 mm.

Fine mesh: the element size is 50 mm.

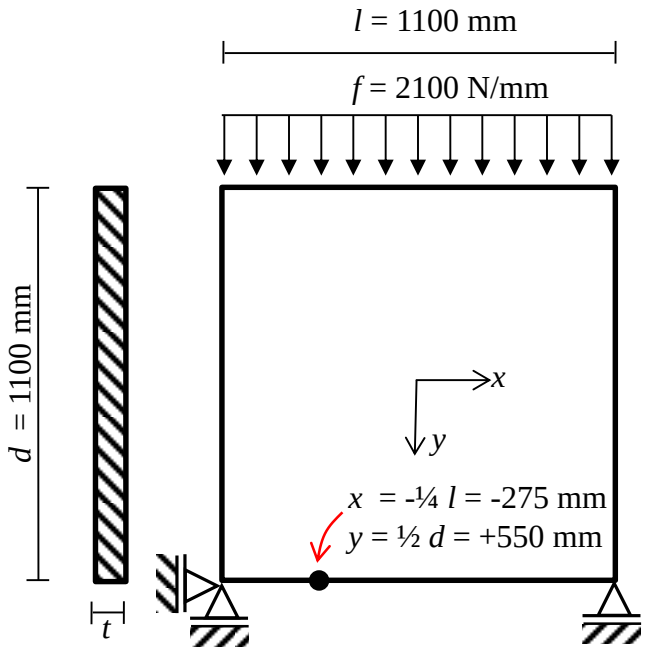


Figure 1.1: plane stress (or plate membrane) problem

The provided results from the finite element models are the vertical displacements (u_y) along the bottom line y = ½ d = + 550 mm, for x = –550 to +550 mm. See the Figures 1.2 and 1.3 below. Figure 1.3 zooms in to the results by changing the vertical axes.

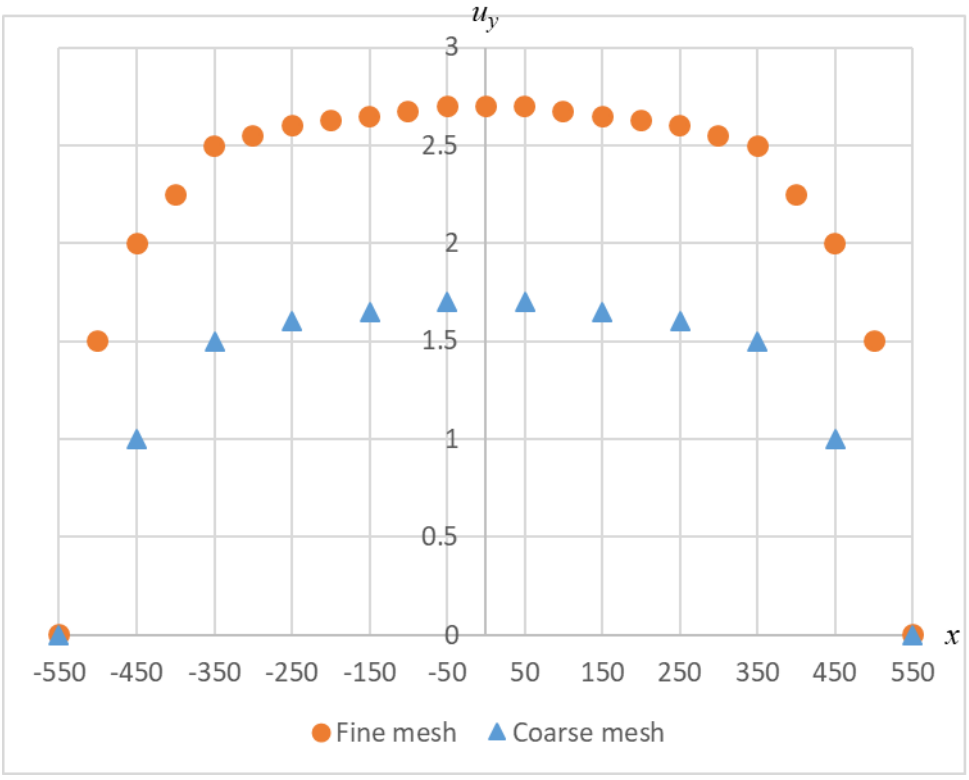


Figure 1.2: vertical displacement (u_y)
along the bottom line y = + 550 mm, for x = –550 to +550 mm.

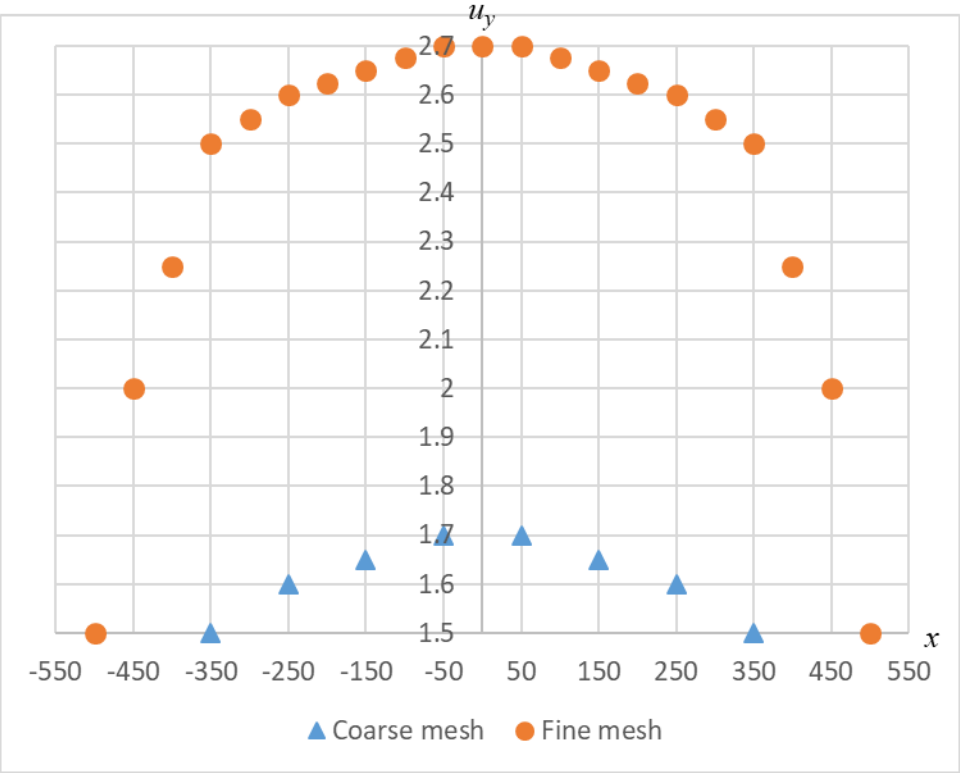


Figure 1.3: vertical displacement (u_y), zoomed in, along the bottom line y = + 550 mm, for x = –550 to +550 mm.

a) Estimate, using the results shown in Figures 1.2 and 1.3, the value of $\frac{\partial u_x}{\partial y}$ in the point at the bottom line with coordinates x = -275 mm and y = +550 mm. This point is marked in Figure 1.1. Provide estimates both based on the coarse and fine mesh results. Explain your estimates in maximum 75 words.

b) Explain the difference between the coarse mesh and fine mesh results of the vertical displacement (u_y) in Figure 1.2, in maximum 75 words.

c) Would it be possible to estimate ε_{yy} in the marked point at the bottom line with coordinates x = -275 mm and y = +550 mm with the provided results? Explain your answer in maximum 75 words.

Question 2

A plate consists of two parts, A and B. See Figure 2.1 below.

Note that these parts are made of the two materials, with the same stiffness *E*, a different Poisson's ratio ν, and a different thickness (½*t* and *t*), as indicated in the figure.

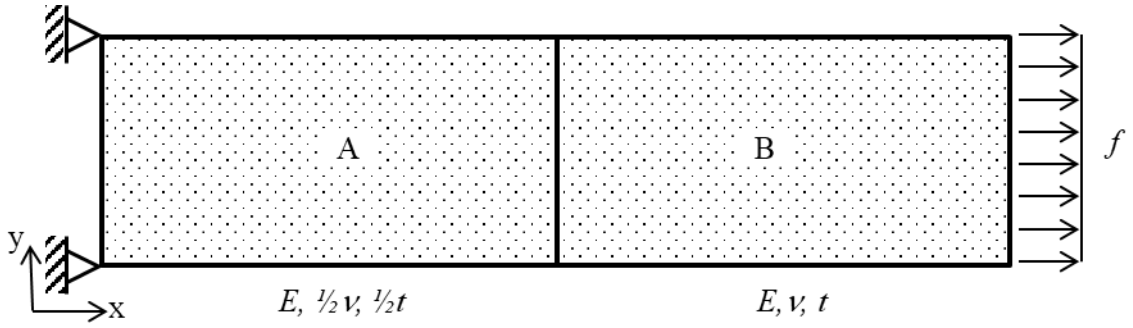


Figure 2.1: a plate consists of two parts, A and B.

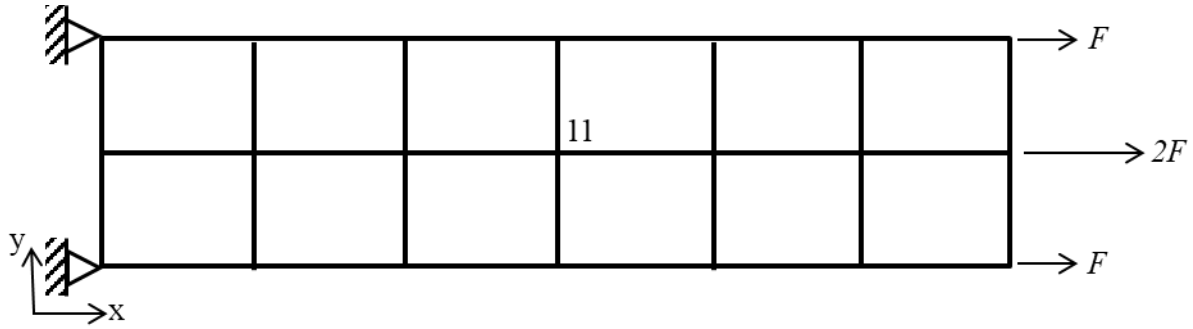


Figure 2.2: a finite element model of this plate. The node number 11 is indicated.

a) Give the condition for ε_{xx} for the transition zone between the plate parts A and B. That is, how does the strain ε_{xx}^{A} immediately left from the transition zone relate to the strain ε_{xx}^{B} immediately right from the transition zone.

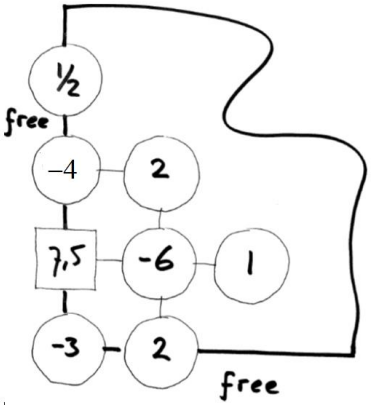
b) Same question, but now for the strain ε_{yy}.

c) Figure 2.2 shows a finite element model for this plate consisting of 12 elements and 21 nodes. Node number 11 is indicated.
Will nodal averaging influence the result for ε_{xx} in node 11?
Will nodal averaging influence the result for ε_{yy} in node 11?
Will nodal averaging influence the result for u_x in node 11?
Three subsequent Yes/No answers will suffice.

Question 3

a) Consider the edge of a thin plate loaded out of plane.
How many boundary conditions does it have?

b) Shown is a molecule of the finite difference method for thin plates loaded out of plane.
How many discrete elements are in this molecule?



c) What is the important difference between the thin plate theory and the thick plate theory?

d) Consider a free edge in the *y* direction. So, the edge is not supported and not loaded.
Sometimes the shear force v_x on this edge is not zero. What causes this?

Question 4

A plate is loaded perpendicularly (Fig. 4.1). The plate length is 16 m and the width is 8 m. The plate thickness is 250 mm. A distributed load of 9 kN/m^2 is applied. This distributed load includes self-weight. A linear elastic finite element model is made. E = 34100 N/mm^2, ν = 0.2. Applied are four node plate elements (Kirchhoff theory). The element size is 500 mm. Results are shown in Figures 4.2 to 4.5.

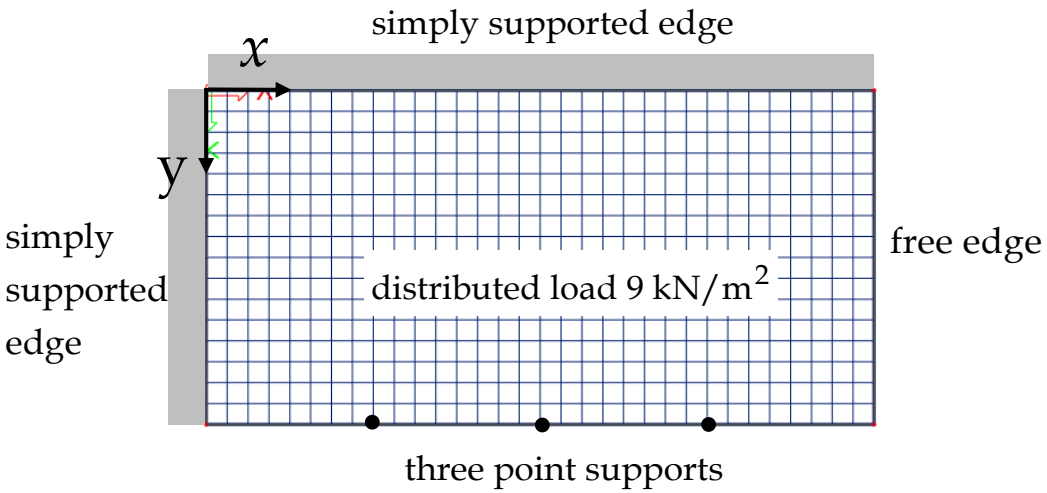


Figure 4.1. Plate with supports, load and finite element mesh

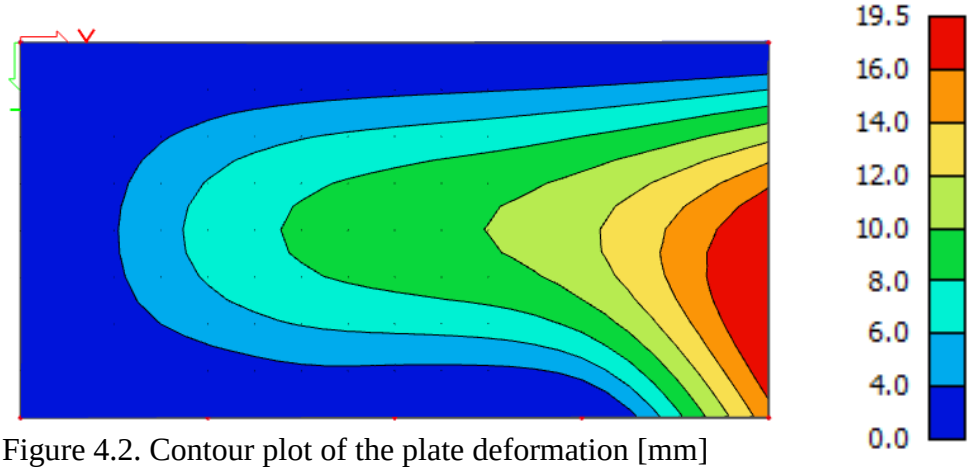


Figure 4.2. Contour plot of the plate deformation [mm]

a) Does the deformation agree with the boundary conditions? Explain your answer.

b) The torsion moment m_{xy} is not zero on the plate edges. How do we interpret this?

c) Are the moments averaged in the nodes? Explain your answer.

d) The free corner (bottom-right) is special. There, all moments are zero. Explain this.

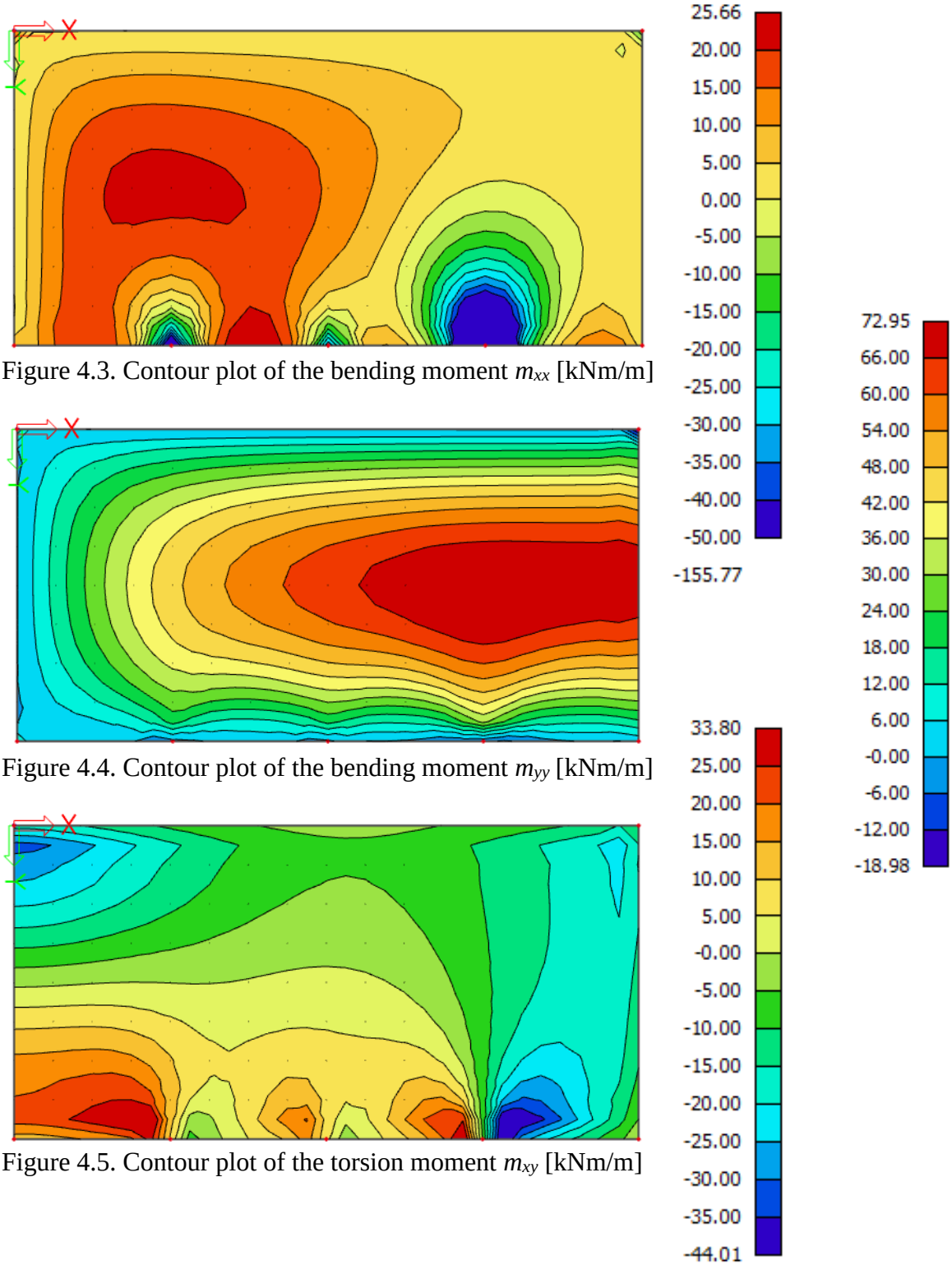


Figure 4.3. Contour plot of the bending moment m_{xx} [kNm/m]

Figure 4.4. Contour plot of the bending moment m_{yy} [kNm/m]

Figure 4.5. Contour plot of the torsion moment m_{xy} [kNm/m]

Question 5

A reinforced concrete thin plate has a thickness of 180 mm.
The plate carries 5 kN/m^2 (self-weight plus live load; partial factors included).
At a particular location the moments are

m_{xx} = 13 kNm/m
m_{yy} = -37 kNm/m
m_{xy} = -53 kNm/m

The reinforcement consists of bars in the *x* direction and bars in the *y* direction. The plate does not have out of plane shear reinforcement. All bars have a diameter of 12 mm. The yield strength of the reinforcing steel is 430 N/mm^2 (partial factor included).

Somebody proposes to space the bars 100 mm. Is this sufficient? Explain your answer.

Useful equations

top $m_{xt} = -m_{xx} + |m_{xy}|$ $m_{yt} = -m_{yy} + |m_{xy}|$
bottom $m_{xb} = m_{xx} + |m_{xy}|$ $m_{yb} = m_{yy} + |m_{xy}|$
$m_1 = (m_{xx} + m_{yy})/2 + \text{sqrt}(((m_{xx} - m_{yy})/2)^2 + m_{xy}^2)$
$m_2 = (m_{xx} + m_{yy})/2 - \text{sqrt}(((m_{xx} - m_{yy})/2)^2 + m_{xy}^2)$
top $(m_{xt} + m_{xx}) \times (m_{yt} + m_{yy}) >= m_{xy}^2$
bottom $(m_{xb} - m_{xx}) \times (m_{yb} - m_{yy}) >= m_{xy}^2$

Question 6

Consider a standard rectangular membrane element with four nodes in the corners. See Figure 6.1. The dimensions of the element are given in the figure. The nodes are indicated as 1, 2 ,3 and 4. The origin of the axes system is in the middle of the element. The *x*- and *y*-axes are parallel to the edges of the element.

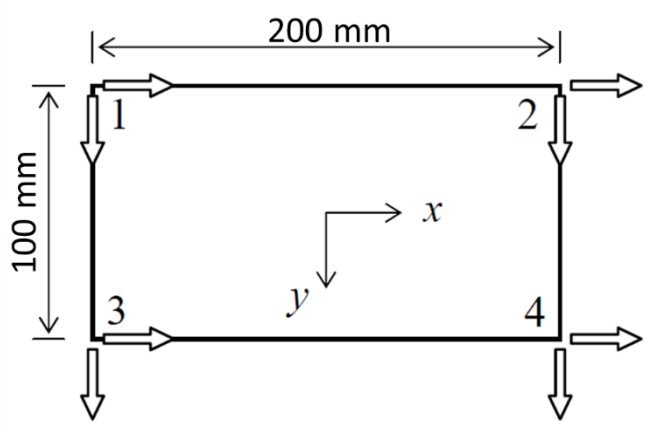


Figure 6.1: a standard rectangular membrane element

The displacements in *x*- and *y*-directions of the 4 nodes are known:

$$u_x^1 = u_x(-100, -50) = -2$$
$$u_x^2 = u_x(+100, -50) = +2$$
$$u_x^3 = u_x(-100, +50) = +2$$
$$u_x^4 = u_x(+100, +50) = -2$$

$$u_y^1 = u_y^2 = u_y^3 = u_y^4 = +3.14$$

a) What is the rigid body displacement for translation in *x*- and *y*-direction?

b) Give the displacement field for displacements in x-direction.

c) Specifically for this displacement field, would a constant shear element lead to other shear strains than the standard rectangular membrane element? Explain your answer in maximum 75 words.

Brief answers

Question 1

a) From Figure 1.3 we estimate $\frac{\partial u_y}{\partial x} \approx 0.001$ at x = -275 mm, both from the fine mesh and the coarse mesh results. From the boundary condition we know that n_{xy} should be zero. Consequently, γ_{xy} should be zero. Consequently, $\frac{\partial u_x}{\partial y} \approx$ - 0.001.

b) There are excessive deformations near the two supports, which is a result of modelling the actual supports as points supports. Relatively far away from these supports the displacement gradients, like $\frac{\partial u_y}{\partial x}$, are similar. Here the strain results are not mesh dependent.

c) No, with $n_{yy} = n_{xy} = 0$ (boundary conditions), ε_{yy} will depend on n_{xx} (constitutive equations). And n_{xx} could not be estimated from the results provided.

Question 2

a) $\frac{\varepsilon_{xx}^{A}}{\varepsilon_{xx}^{B}} = 2$ **b)** $\frac{\varepsilon_{yy}^{A}}{\varepsilon_{yy}^{B}} = 1$ **c)** Yes, No, No

Question 3

a) 2

b) 5 = 2 torsion panels + 1 horizontal bending element + 2 vertical bending elements

c) Thin has no shear deformation; thick has shear deformation.

d) This shear force is equal to the change in the concentrated shear force m_{xy}.

Question 4

a) Yes, at the supports the displacements are blue (0 to 4 mm). It should be 0. This is consistent.

b) Torsion moment m_{xy} on an edge is in fact a concentrated shear force V in the edge.

c) Yes, the moments are mostly smooth and do not jump at element edges.

d) On the x edge $m_{yy} = 0$. On the y edge $m_{xx} = 0$. There is no point load, so $2m_{xy} = 0$ too.

Question 5

No

$mxx := 13 \cdot kN : myy := -37 \cdot kN : mxy := -53 \cdot kN :$

$fy := 430 \cdot \frac{N}{mm^2} : t := 180 \cdot mm : d := 12 \cdot mm : s := 100 \cdot mm :$

$N := \frac{kN}{1000} : mm := \frac{m}{1000} :$

$F := \frac{3.14}{4} \cdot d^2 \cdot fy :$

$$mxt := \frac{F \cdot 0.9 \cdot \left(t - 20 \cdot mm - \frac{3}{2} \cdot d \right)}{s};$$

$mxt := 62.12000160\ kN$

$$myt := \frac{F \cdot 0.9 \cdot \left(t - 20 \cdot mm - \frac{d}{2} \right)}{s};$$

$myt := 67.36957920\ kN$

$mxb := myt :$

$myb := mxt :$

$(mxt + mxx) \cdot (myt + myy); \# >=? mxy^2$

$2281.362838\ kN^2$

$(mxb - mxx) \cdot (myb - myy); \# >=? mxy^2$

$5389.112777\ kN^2$

$mxy^2;$

$2809\ kN^2$

Question 6

a) 0 mm respectively +3.14 mm

b) $u_x = \frac{-xy}{2500}$

c) Yes, in a standard rectangular membrane element this displacement field would lead to varying shear strains, which will no longer be the case within a constant shear element.