Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CIE4180: Plate Analysis
Thursday 28 January 2021, 13:30 – 15:00

This exam consists of 6 questions, each with sub-questions.
All sub-questions will have the same weight in the total evaluation.

In this exam the notation and sign conventions of the reader (part 1 and 2) and the book (Plates and FEM) have been adopted. **You are supposed to use the same notation and sign convention.**

Question 1

Consider the plane stress (or plate membrane) problem below: a square plate. See the figure below.

The length *l* and depth *d* of the plate is 1100 mm.

The thickness *t* is not given.

The plate is supported at 2 points.

The plate is loaded, in-plane, by a uniformly distributed load $f = 2100$ N/m at the top of the plate. There is no dead load.

The *x*- and *y*-axes are indicated in the figure. The origin of the axes system is at the centre of the plate.

A finite element analysis is performed with linear (4-noded) plane stress elements. The element size is 100 mm.

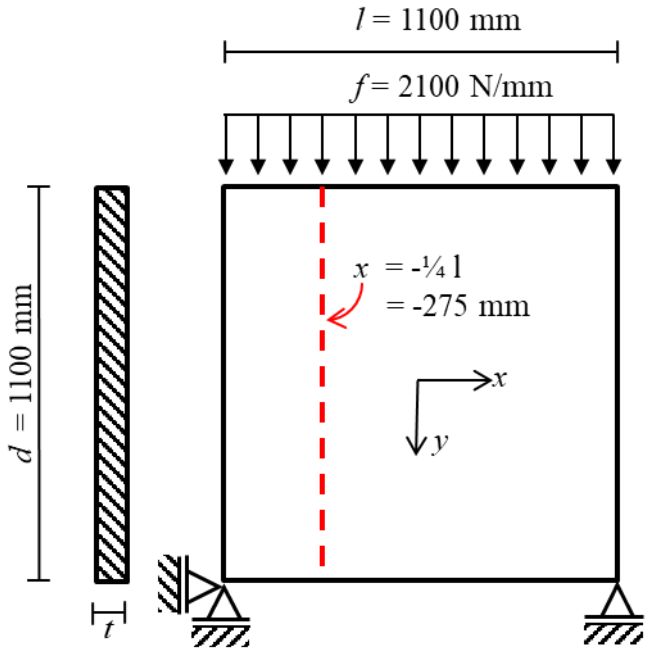


Figure 1.1: plane stress (or plate membrane) problem

The results from the finite element model include the shear stress σ_{xy} distribution along the line $x = -¼\ l = -275$ mm, for $y = -550$ to $+550$ mm. See the Figure 1.2 below:

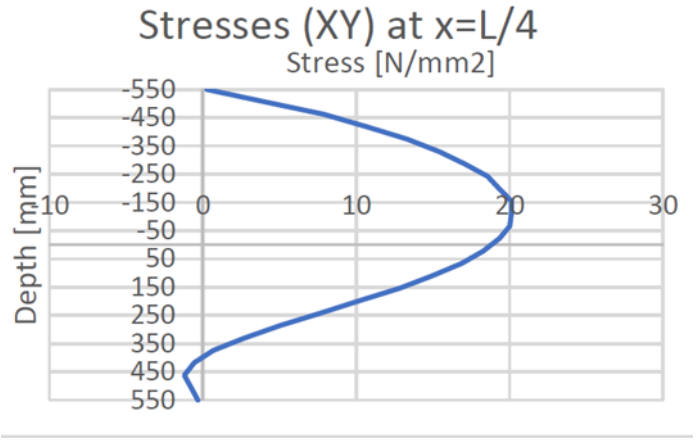


Figure 1.2: Shear stress σ_{xy} *along the vertical line* $x = -275$ mm, *for* $y = -550$ *to* $+550$ *mm*

a) Could the shear stress distribution presented in the figure above be correct? Motivate your answer, in maximum 75 words

b) Estimate the thickness t of the plate.

c) In reality the plate is supported by two support plates, rather than point supports. Discuss the accuracy of the obtained u_y, ε_{xx} and ε_{yy} in point $(x,y) = (0, 550$ mm), in maximum 75 words.

Question 2

A plate consists of four parts, A, B, C and D. See the figure below.

Note that these parts are made of different materials, with a different stiffness (E and $2E$) and a different thickness ($\frac{1}{2}t$ and t), as indicated in the figure.

Between the parts B and C a distributed spring is applied in x-direction, with spring constant k. The spring constant relates the line load on the distributed spring ("force per length") to the extension of the spring in x-direction, Δx.

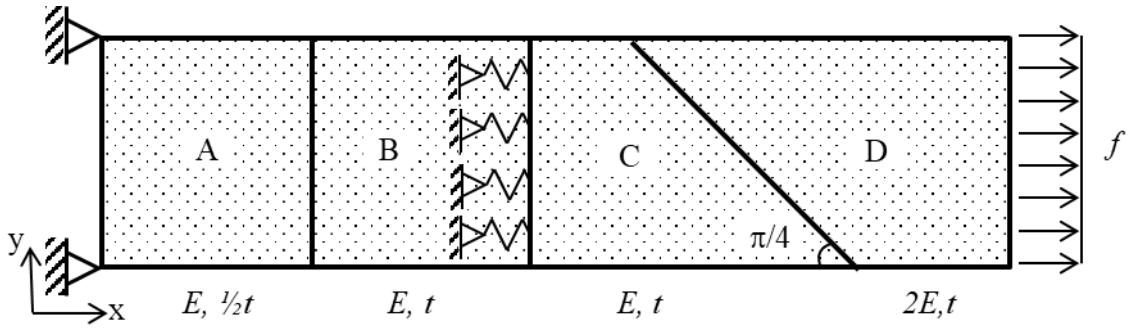


Figure 2.1: a plate consists of four parts, A, B, C and D.

a) Give the condition for $\sigma_{xx}$for the transition zone between the plate parts A and B. That is, how does the stress σ_{xx}^{A} immediately left from the transition zone relate to the stress σ_{xx}^{B} immediately right from the transition zone.

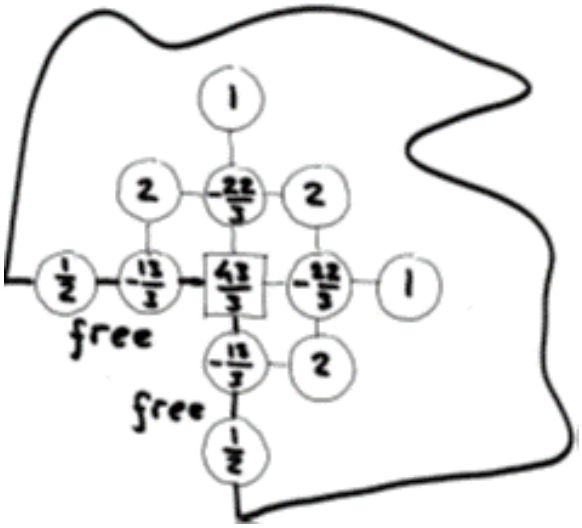
b) Same question, but now for the transition zone between the plate parts B and C.

c) Same question, but now for the transition zone between the plate parts C and D.

Question 3

a) We calculate the concentrated shear force with a formula. Give this formula.

b) Shown is a molecule of the finite difference method.
How many discrete elements are in this molecule?



c) What is the recommended finite element size for thin plates loaded out of plane?

d) What is the problem if we use Mindlin-Reissner for a thin plate?

Question 4

A plate is loaded perpendicularly (Fig. 4.1). The plate length is 16 m and the width is 8 m. The plate thickness is 250 mm. A distributed load of 9 kN/m^2 is applied. This distributed load includes self-weight. A linear elastic finite element model is made. E = 34100 N/mm^2, ν = 0.2. Applied are four node plate elements (Kirchhoff theory). The element size is 500 mm. Results are shown in Figures 4.2 to 4.5.

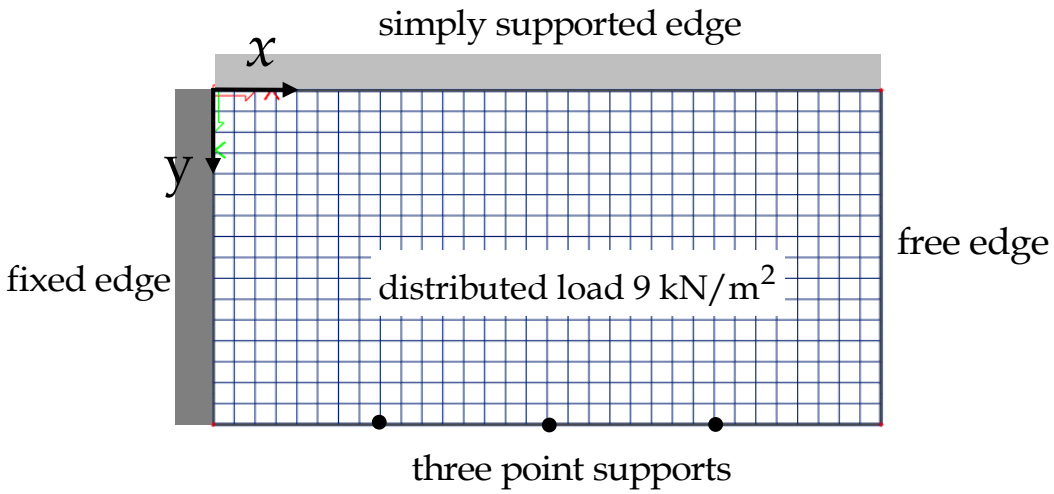


Figure 4.1. Plate with supports, load and finite element mesh

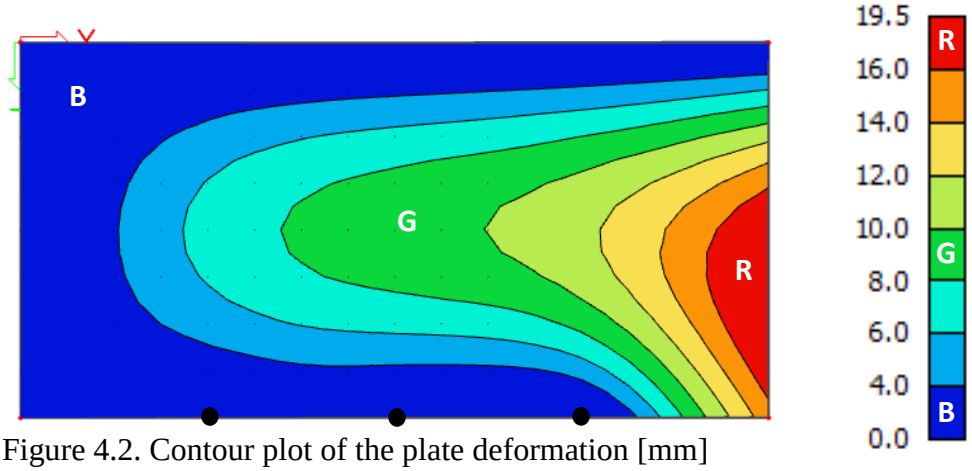


Figure 4.2. Contour plot of the plate deformation [mm]

a) Does the plate deform in the correct direction? Explain your answer.

b) Do the moments agree with the boundary conditions? Explain your answer.

c) How large is the largest moment in the plate? (in absolute value)

d) The torsion moment is positive at the left of the point supports and negative at the right of the point supports. Explain this.

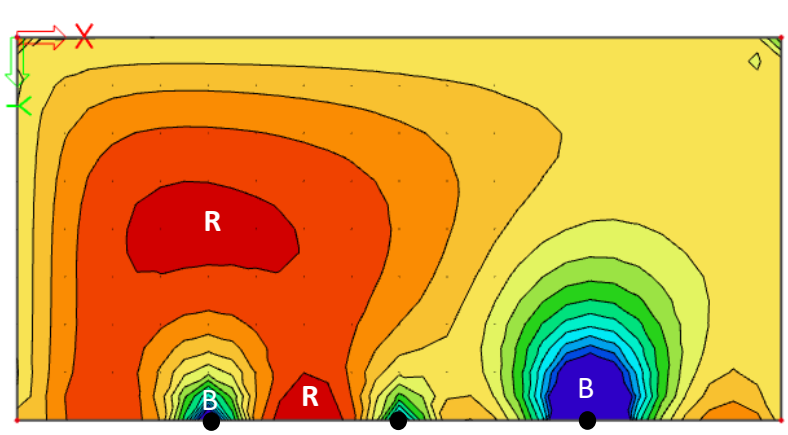


Figure 4.3. Contour plot of the bending moment m_{xx} [kNm/m]

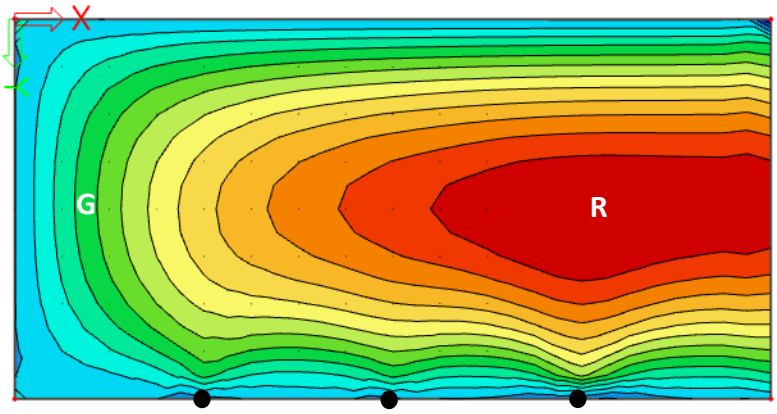


Figure 4.4. Contour plot of the bending moment m_{yy} [kNm/m]

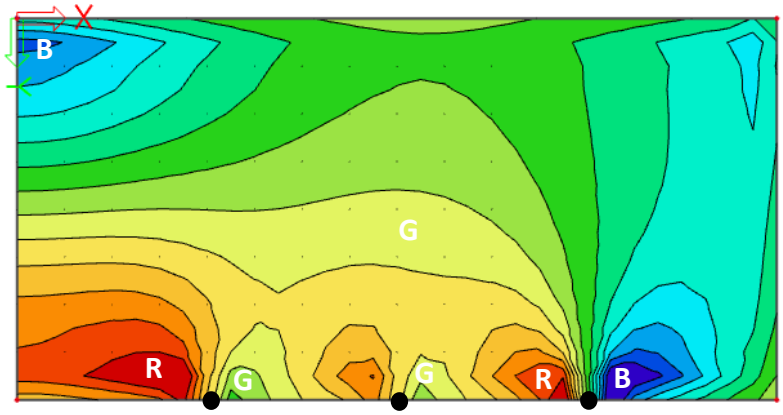


Figure 4.5. Contour plot of the torsion moment m_{xy} [kNm/m]

Question 5

A reinforced concrete thick plate has a thickness of 300 mm.
The plate carries 7 kN/m^2 (self-weight plus live load; partial factors included).
At a particular location the moments are

m_{xx} = 68 kNm/m
m_{yy} = -187 kNm/m
m_{xy} = 108 kNm/m

The reinforcement consists of bars in the *x* direction and bars in the *y* direction. The plate does not have out of plane shear reinforcement. All bars have a diameter of 32 mm. The yield strength of the reinforcing steel is 450 N/mm^2 (partial factor included).

a) Propose reinforcement in kNm/m for the plate location. (Sufficient but not way too much)

b) How many bars are needed in the most loaded direction?

Useful equations

top $m_{xt} = -m_{xx} + |m_{xy}|$ $m_{yt} = -m_{yy} + |m_{xy}|$
bottom $m_{xb} = m_{xx} + |m_{xy}|$ $m_{yb} = m_{yy} + |m_{xy}|$
$m_1 = (m_{xx} + m_{yy})/2 + \text{sqrt}(((m_{xx} - m_{yy})/2)^2 + m_{xy}^2)$
$m_2 = (m_{xx} + m_{yy})/2 - \text{sqrt}(((m_{xx} - m_{yy})/2)^2 + m_{xy}^2)$
top $(m_{xt} + m_{xx}) \times (m_{yt} + m_{yy}) >= m_{xy}^2$
bottom $(m_{xb} - m_{xx}) \times (m_{yb} - m_{yy}) >= m_{xy}^2$

Question 6

Consider a rectangular membrane plate. The dimensions are given in the figure below. The corners are indicated as 1, 2 ,3 and 4. The origin of the axes system is in the middle of the plate. The *x*- and *y*-axes are parallel to the edges of the plate.

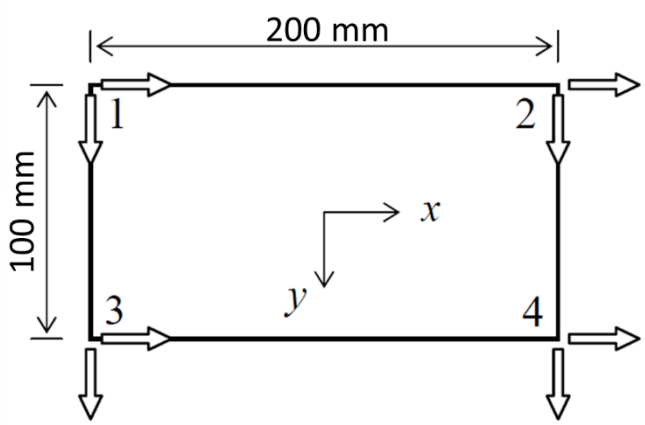


Figure 6.1: a rectangular membrane plate

The displacement field is known:

$$u_x(x, y) = 0.0031$$
$$u_y(x, y) = 0.0027|x| + 0.0016y$$

where $|x|$ denotes the absolute value of x.

a) What is the rigid body displacement for translation in *y*-direction?

b) What is the shear strain γ_{xy} in point $(x, y) = (0, -50)$?

c) What is the minimum number of rectangular 4-noded membrane elements needed to adequately describe the displacement field? Explain your answer in maximum 75 words.

Brief answers

Question 1

a) Yes, could be. The shear stress is close to zero at the top and bottom, which fulfils the boundary conditions. According to the sign convention, the shear stress must be predominantly positive. This is the case.

b) From symmetry of the structure and equilibrium along the line $x = -275$ mm we should have $\int_{-550}^{550} \sigma_{xy} t dy = \frac{1}{4} fl$ or $11000t \approx 577500$ or $t \approx 52$ mm.

d) The obtained u_y, will be substantially higher than the real value, this is a result of excessive deformations near the two point supports. Relatively far away from these supports, the obtained ε_{xx} and ε_{yy} values will not suffer from this and can be considered as reasonable estimates (provided that other modelling assumptions, such as linear material behaviour are accurate and assuming that the span has not changed).

Question 2

a) $\frac{\sigma_{xx}^{A}}{\sigma_{xx}^{B}} = 2$ **b)** $\sigma_{xx}^{B} + k\Delta u = \sigma_{xx}^{C}$ **c)** $\frac{\sigma_{xx}^{C}}{\sigma_{xx}^{D}} = 1$

Question 3

a) V = mxy … or … concentrated shear force = mxy

b) 9 = 3 torsion panels + 3 horizontal bending elements + 3 vertical bending elements

c) plate thickness … or … two times the plate thickness

d) We should use very small element on the plate edges.

Question 4

a) Yes, the load is positive and the deflection is positive.

b) No, there is a torsion moment on the fixed edge.
The fixed edge seems to be simply supported instead of fixed.

c) -155 kNm/m is computed, however, it is in a singularity so it needs to be interpreted.

d) The torsion moment on the edge is in fact a concentrated shear force in the edge. The plate edge is like a beam over four supports. The shear force diagram in the beam would be negative and positive in the same way as the torsion moment in the plate edge.

Question 5

a)

top $m_{xt} = -m_{xx}+|m_{xy}| = -68+108 = 40$ kNm/m $m_{yt} = -m_{yy}+|m_{xy}| = 187+108 = 295$ kNm/m

bottom $m_{xb} = m_{xx}+|m_{xy}| = 68+108 = 176$ kNm/m $m_{yb} = m_{yy}+|m_{xy}| = -187+108 = 30$ kNm/m

(30 kNm/m because I choose some practical reinforcement instead of nothing.)

b)

bar force	$\pi/4 \times 32^2 \times 450 = 362\,000$ N = 362 kN
lever arm	$0.7 \times (300 - 35 - 16) = 174$ mm = 0.174 m
5 bars per meter	m = 5 × 362 × 0.174 = 314 > 295 kNm/m OK

Question 6

a) 0.0

b) It will depend on how we will approach point (0,-50). $\gamma_{xy} = -0.0027$ if we approach the point from the left and $\gamma_{xy} = +0.0027$ if we approach the point from the right.

c) 2 elements, with a common edge along $x = 0$. The element on the left side will describe the $u_x(x,y) = 0.0031, u_y(x,y) = -0.0027x + 0.0016y$ displacement field, whereas the element on the right side will describe the $u_x(x,y) = 0.0031, u_y(x,y) = +0.0027x + 0.0016y$ displacement field.