Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CIE4180: Plate Analysis
Friday 8 April 2022, 9:00 – 12:00

This exam consists of 6 questions.
The respective weights are 1.4, 1.4, 2.1, 2.0, 1.6 and 1.5.

Use **separate sheets of paper** **for each question**!

Write your name and student number **on every page of your work.**

In this exam the notation and sign conventions of the reader (part 1 and 2) and the book (Plates and FEM) have been adopted. **You are supposed to use the same notation and sign convention.**

Question 1

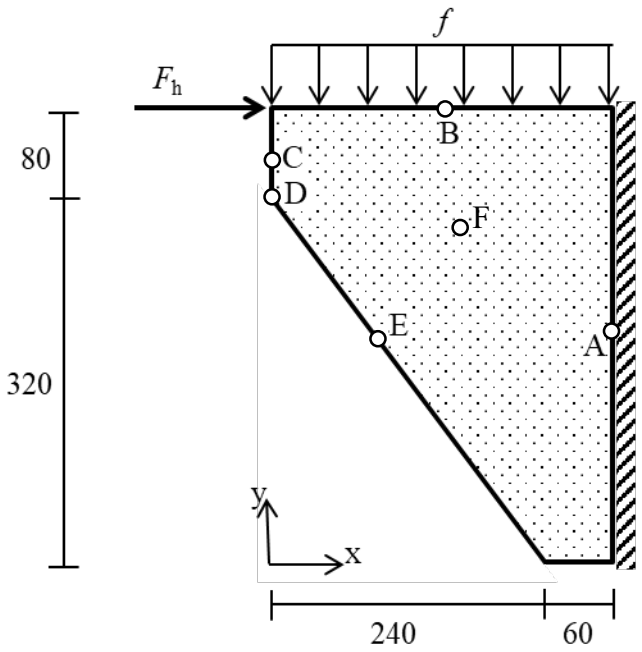
Use **separate sheets of paper for each question**!

Consider the plane stress (or plate membrane) problem below. The measurements of the quadrilateral plate with a snipped corner are in mm. The length of the plate is 300 mm. The height is 400 mm. The plate has an uniform thickness t = 150 mm.

The plate is loaded by an equally distributed line load f ("force per length"), acting at the top edge of the plate in downwards direction. The plate is also loaded by a horizontal force F_h.

At the right hand side of the plate, both vertical and horizontal displacements are prohibited.

In order to verify the outcomes of a finite element analysis you want to calculate some strain and stress field variables a priori.

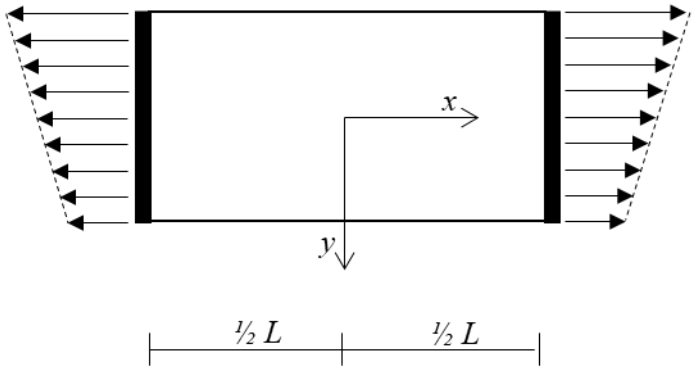


a) Could you calculate ε_{xx} in point A?
b) Could you calculate σ_{yy} in point B?
c) Could you calculate ε_{xx} in point C?
d) Could you calculate σ_{yy} in point D?
e) Could you calculate τ_{xy} in point E?
f) Could you calculate ε_{zz} in point E?
g) Could you calculate $\int_{y=0}^{y=400} \tau_{xy}(x = 300, y)dy$?
h) Could you calculate $\int_{y=0}^{y=400} \sigma_{yy}(x = 300, y)dy$?

For the above questions answer either Yes and indicate how or No and explain why not.

Question 2

Use **separate sheets of paper** **for each question**!



Consider a rectangular plates of unit thickness and length L, see above. Assume a plane stress (or plate membrane) situation.

The plate is free to deform along the two horizontal edges.

Along both vertical edges, deformations in x-directions are applied which vary linearly in y:

$$u_x\left(x = \tfrac{1}{2}L, y\right) = u - \varphi y$$

$$u_x\left(x = -\tfrac{1}{2}L, y\right) = -u + \varphi y$$

where u and φ are positive constants. The vertical displacements along these vertical edges are zero:

$$u_y\left(x = -\tfrac{1}{2}L, y\right) = u_y(x = \tfrac{1}{2}L, y) = 0$$

The Poisson's ratio ν = 0.

a) Could the displacement field $u_x(x, y)$ of the plate be described with a polynomial with constant and linear terms? Explain your answer.
b) Could the displacement field $u_y(x, y)$ of the plate be described with a polynomial with constant and linear terms? Explain your answer.
c) Give the polynomial for the displacement field $u_x(x, y)$ and express the non-zero coefficients in u, φ and L.
d) In a finite element model of this plate, how many standard rectangular plate elements (4-noded) are needed to exactly capture the $\varepsilon_{xx}(x, y)$ strain field?

Question 3

Use **separate sheets of paper** **for each question**!

a) In which direction is the z axis? Choose A, B, C or D.

A Perpendicular to the plate.
B In the direction of the load.
C Vertically down.
D In any direction.

b) How many boundary conditions exist on a thick plate edge?
How many boundary conditions exist on a thin plate edge?

c) What is the value of η in $v_x = \frac{Gt}{\eta}\gamma_x$ in the theory for perpendicularly loaded plates?

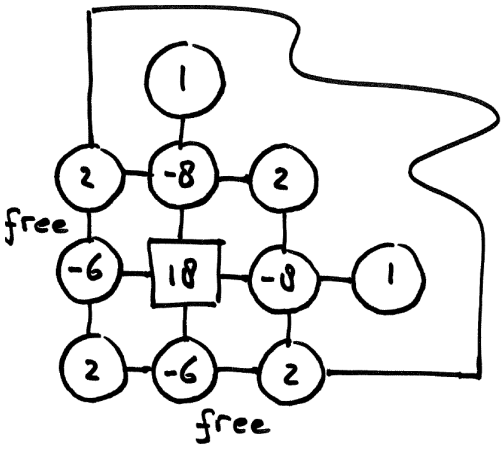
Choose A, B, C or D.

A $\eta = \frac{1}{2}$
B $\eta = \frac{1}{\pi^2}$
C $\eta = \frac{5}{2\sqrt{10}}$
D $\eta = \frac{6}{5}$

d) Consider a free edge of a thin plate. Which are the boundary conditions?
Choose A, B, C or D.

A $w = 0,\ \varphi_y = 0$
B $m_{yy} = 0,\ v_y + \frac{\partial m_{xy}}{\partial x} = 0$
C $w = 0,\ m_{yy} = 0$
D $m_{yy} = 0,\ m_{xy} = 0$

e) Consider the following molecule of the finite difference method of thin plates. What is the number of discrete elements in this molecule?



f) Are singularities real? Choose A, B, C or D.

A No, infinite stresses are not possible in reality.
B Yes, they are a side effect of the finite element method.
C Yes, large stresses can lead to fatigue or fracture.
D Yes, they have never been measured but most engineers believe they do exist.

g) When was the thin plate theory complete? Choose A, B, C or D.

A In 1811 when Sophie Germain submitted her derivation.
B In 1816 when Sophie Germain won a prize from the Paris Academy of Sciences.
C In 1850 when Gustav Kirchhoff derived the correct boundary conditions.
D In 1944 when Eric Reissner formulated the correct differential equations.

Question 4

Use **separate sheets of paper** **for each question**!

A plate is loaded perpendicularly. Most of the edges are free except for a fixed part and a simply supported part (Fig. 4.1). The plate length is 12 m and the width is 7 m. The plate thickness is 250 mm. A distributed load of -5 kN/m^2 is applied. This distributed load includes self-weight. A linear elastic finite element model is made. E = 30500 N/mm^2, ν = 0.2. Applied are four node plate elements (Kirchhoff theory). The element size is 500 mm. The results are shown in Figures 4.2 to 4.7.

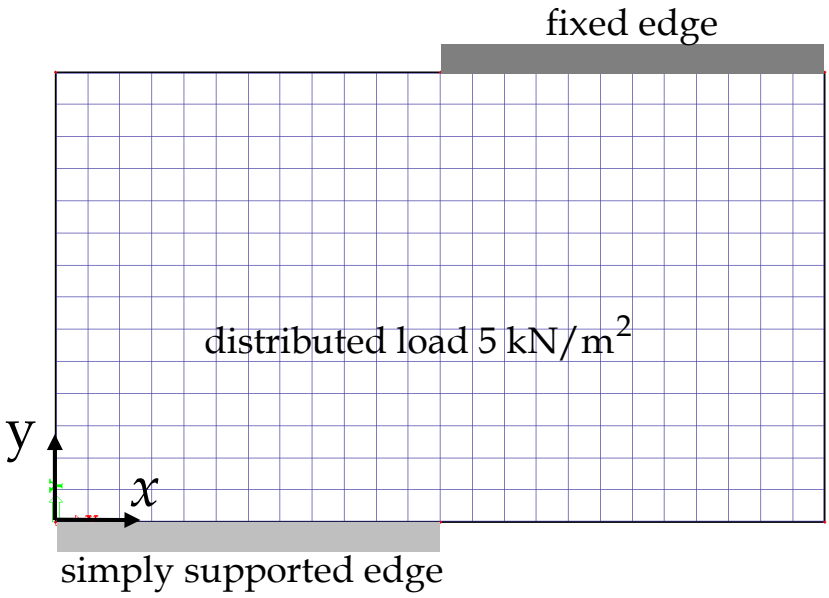


Figure 4.1. Plate with supports, load and finite element mesh

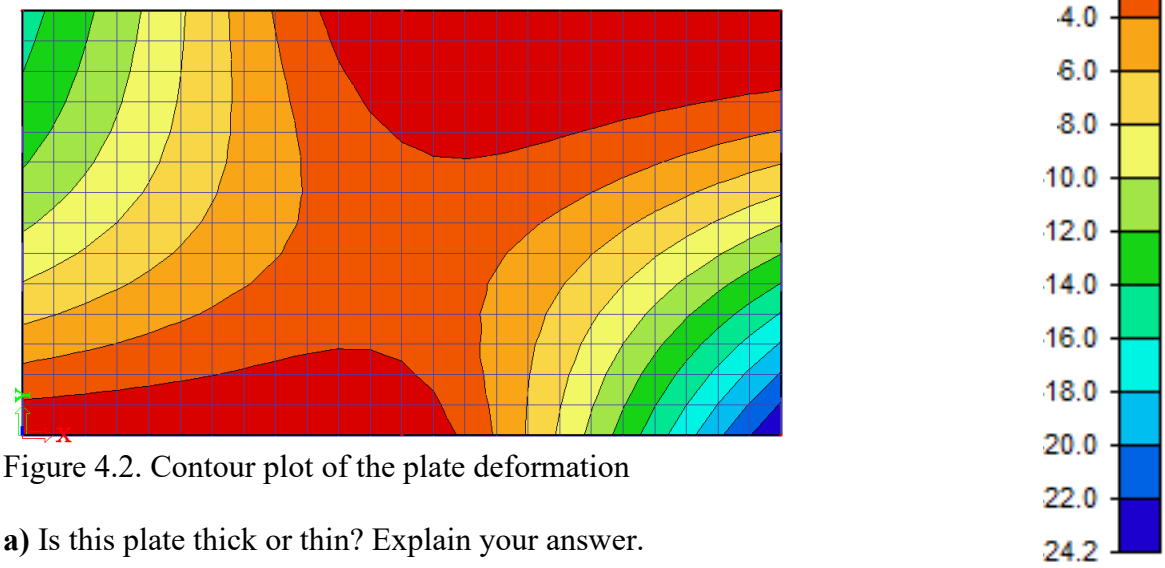


Figure 4.2. Contour plot of the plate deformation

a) Is this plate thick or thin? Explain your answer.

b) The deflection at the supports is somewhere between 0 and 2 mm.
How can we compute this more accurately?

c) Where can we expect singularities in this plate? Specify for each whether it actually shows.

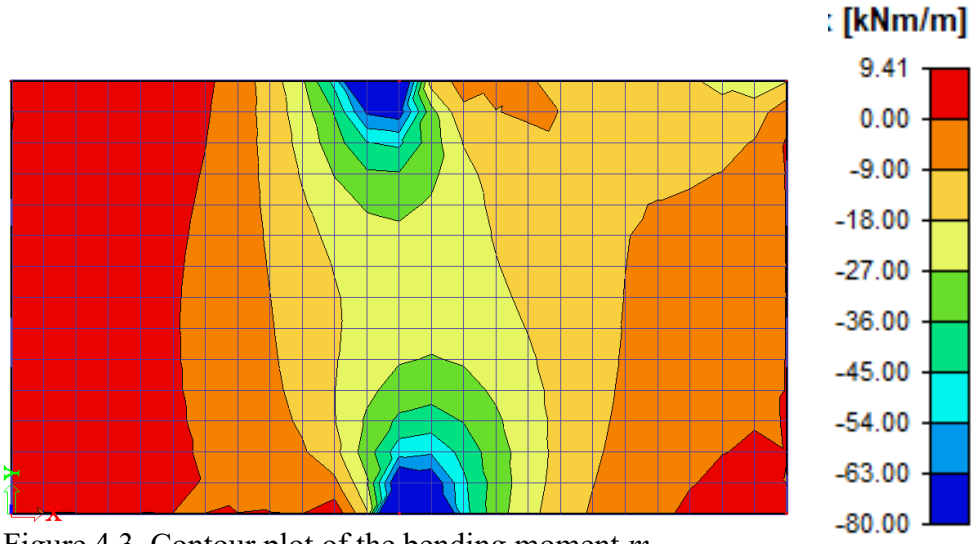


Figure 4.3. Contour plot of the bending moment m_{xx}

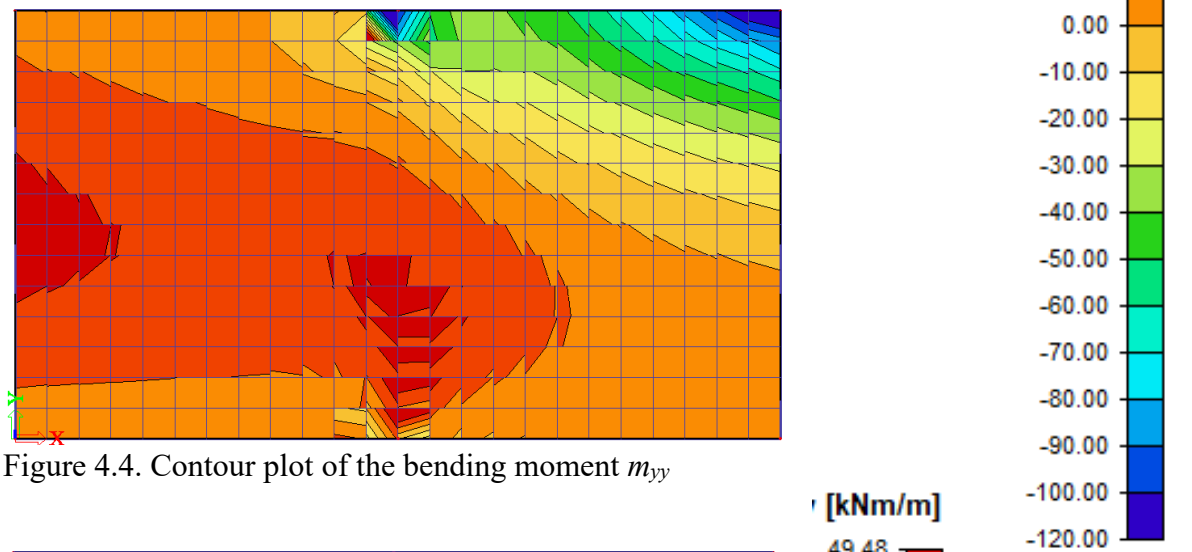


Figure 4.4. Contour plot of the bending moment m_{yy}

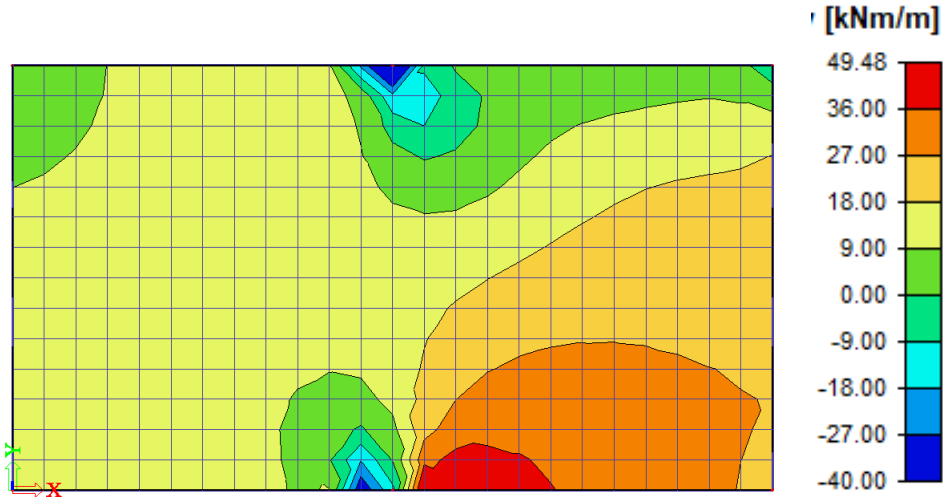


Figure 4.5. Contour plot of the torsion moment m_{xy}

d) Do the moments agree with the boundary conditions? Explain your answer.

e) Some contour lines are not smooth. Does this have a physical cause or a computational cause? Explain your answer.

f) Where is the concentrated shear force largest?

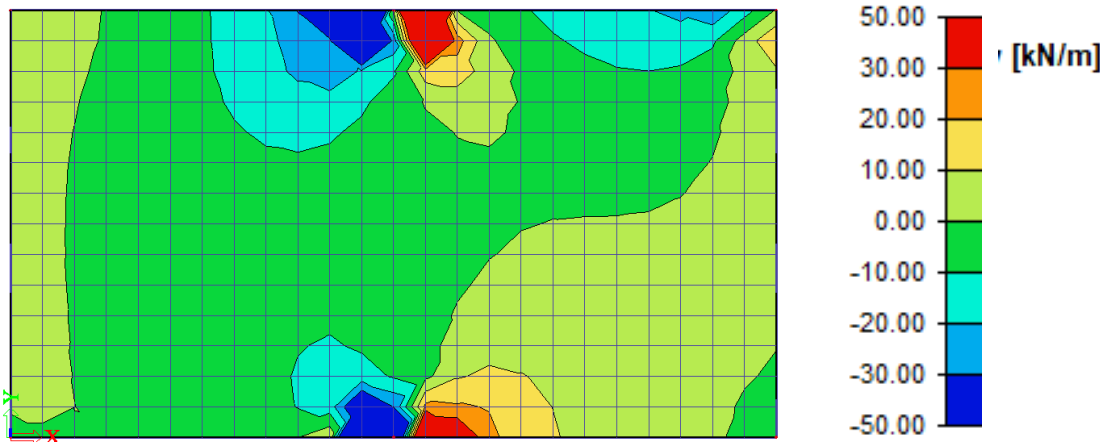
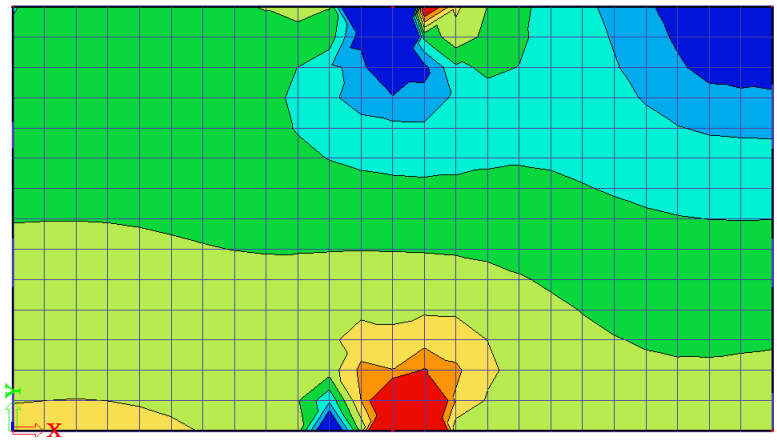


Figure 4.6. Contour plot of the shear force v_x


Figure 4.7. Contour plot of the shear force v_y

g) Both v_x and v_y vary between exactly -50 and 50 kN/m. Explain this phenomenon?

h) What is the shear stress σ_{yz} in the edge at x = 12 m, y = 3.5 m

Question 5

Use **separate sheets of paper** **for each question**!

A reinforced concrete floor has a thickness of 190 mm. The bending moments at a particular location are

m_{xx} = 91 kNm/m
m_{yy} = -23 kNm/m
m_{xy} = 87 kNm/m

The compressive strength of the concrete is 20 N/mm^2 (partial factor included). The reinforcement consists of two meshes. As usual, one mesh is close to the top surface the other mesh is close to the bottom surface. Each mesh has bars in the x direction and bars in the y direction. All bars have a diameter of 21 mm and are spaced 180 mm (centre to centre). The yield strength of the reinforcing steel is 350 N/mm^2 (partial factor included). The bending moment capacities due to the bars are

m_{xt} = 96.4 kN/m m_{yt} = 96.4 kN/m
m_{xb} = 89.1 kN/m m_{yb} = 89.1 kN/m

a) Suppose that the floor is linear elastic. What are the normal stresses and in plane shear stress in the bottom of the plate? (The plate bottom is on the positive side of the z axis.)

b) Is the reinforcement sufficient? Explain your answer.

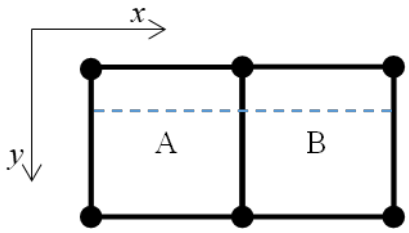
c) Calculate the reinforcement capacities that that are minimally required in the x and y directions.

d) Calculate the reinforcement capacities that that are minimally required in the principal directions.

Useful equations
nsx = nxx+|nxy| nsy = nyy+|nxy|
mxt = -mxx+|mxy| myt = -myy+|mxy|
mxb = mxx+|mxy| myb = myy+|mxy|
m1 = (mxx+myy)/2+sqrt(((mxx-myy)/2)^2+mxy^2)
m2 = (mxx+myy)/2 -sqrt(((mxx-myy)/2)^2+mxy^2)
theta = 1/2*arctan(2*mxy/(mxx-myy))
(nsx-nxx)*(nsy-nyy) >= nxy^2
(-mxt-mxx)*(-myt-myy) >= mxy^2
(mxb-mxx)*(myb-myy) >= mxy^2

Question 6

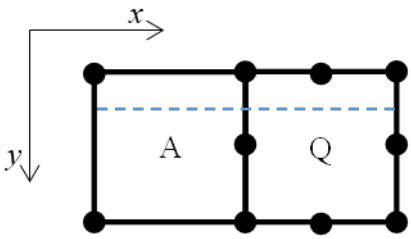
Use **separate sheets of paper for each question**!



The connected elements A and B are a part of a larger plane stress finite element model. See the figure above. Both A and B are standard rectangular membrane elements with four nodes.

a) Assume that element A has a Poisson's ratio $\nu = 0$, whereas B has a Poisson's ratio $\nu \neq 0$. Is compatibility of displacements preserved between the elements? Explain your answer in maximum 50 words.

b) Assume now that elements A and B have the same material and geometrical properties. Consider that we evaluate the results along the horizontal dashed line. Will ε_{xx} be continuous? And ε_{yy}? And γ_{xy}?



The connected element A and Q are a part of a larger plane stress finite element model. See the figure above. Element A is standard rectangular membrane elements with four nodes element Q is a quadratic rectangular membrane element with eight nodes

c) Assume that elements A and Q have the same material and geometrical properties. Is compatibility of displacements preserved between the elements? Explain your answer in maximum 50 words.

d) Assume that elements A and Q have the same material and geometrical properties. Consider that we evaluate the results along the horizontal dashed line. Will ε_{xx} be continuous? And ε_{yy}? And γ_{xy}?

Brief answers

Question 1

a) No, $u_x = 0$ but $\frac{du_x}{dx}$ is not necessarily zero. (only the strain $\varepsilon_{yy} = 0$ is known from the kinematic BCs since $\frac{du_y}{dy} = 0$).

b) Yes, as $-f/t$.

c) No, only in case of a zero Poisson's ratio ε_{xx} would be zero.

d) No, only the stresses $\sigma_{xx} = 0$ and $\tau_{xy} = 0$ are known from the dynamic BCs

e) No, only the shear stress in a rotated axes system will be zero.

f) No, only in case of a zero Poisson's ratio ε_{zz} would be zero.

g) Yes, as $300f/t$.

h) No, only in case of a zero Poisson's ratio $\sigma_{yy}(300, y)$ would be zero. The integral also cannot be determined on the basis of equilibrium.

Question 2

a) No, an *xy*-term will be needed.

b) Yes, and all the three terms will have a zero coefficient.

c) $u_x(x, y) = \frac{2u}{L}x - \frac{2\varphi}{L}xy$

d) One single element will suffice. The displacement field of **c)** can be captured by a standard rectangular plate element, and consequently also the $\varepsilon_{xx}(x, y)$ field.

Question 3

a) A

b) 3, 2

c) D

d) B

e) 8

f) A

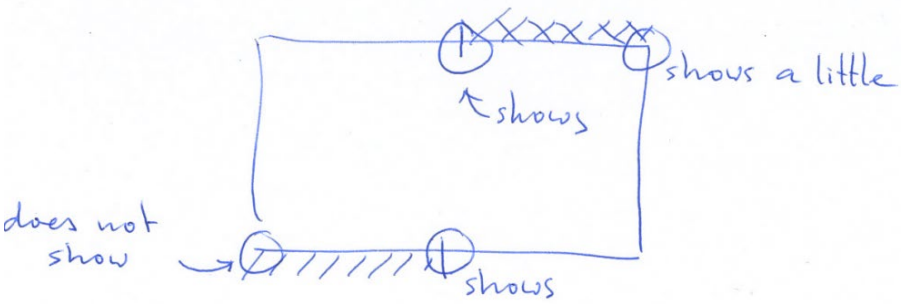
g) C

Question 4

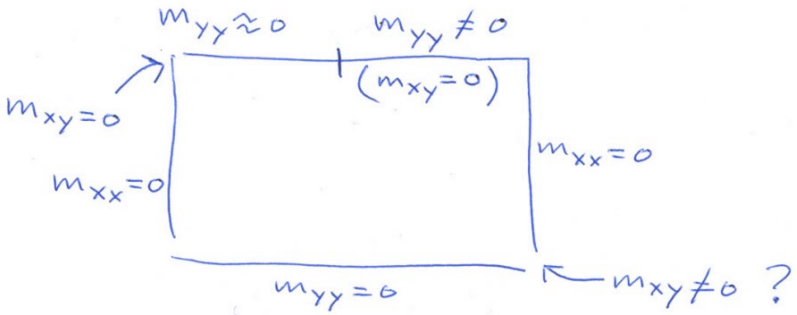
a) Thin: 7/0.25 = 28 >> 10

b) Finer division of the colour bar.

c) At the support ends

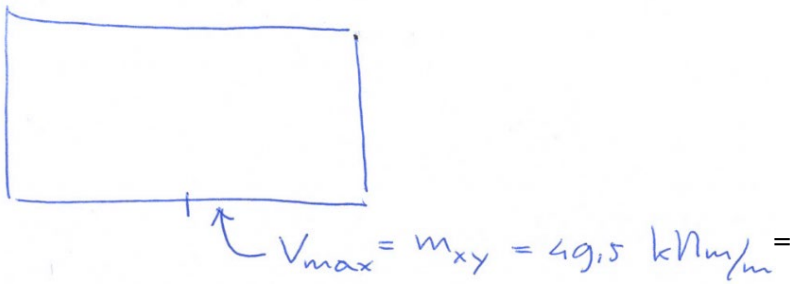


d) Yes, except that mxy in a free corner is not zero.



e) Computational cause. Moment myy was not averaged in nodes, apparently. For even smoother contour lines we need to use smaller elements.

f)



= singularity

g) The colour bar has been truncated by specifying the min and max value.

h)

$$\sigma_{yz} = -\frac{3}{2}\frac{v_y}{t} + \frac{3}{2}\sqrt{10}\frac{V}{t^2} = -\frac{3}{2}\frac{-10\ \mathrm{N/mm}}{250\ \mathrm{mm}} + \frac{3}{2}\sqrt{10}\frac{22000\ \mathrm{Nmm/mm}}{(250\ \mathrm{mm})^2} = 0.06 + 1.67 = 1.73\ \mathrm{N/mm^2}$$

Question 5

```
> h := 190.·mm : mxx := 91·kN : myy := -23·kN : mxy := 87.·kN :
> fc := 20·N/mm^2 : d := 21·mm : s := 180·mm : fs := 350·N/mm^2 :
> kN := 1000·N : assume(N > 0) :
> msx_capacity := 1/4·3.1415·d^2·fs·1/s·0.9·(h - 25·mm - 6·mm);
                        msx_capacity := 96371.99184 N~
> msy_capacity := 1/4·3.1415·d^2·fs·1/s·0.9·(h - 25·mm - 12·mm - 6·mm);
                        msy_capacity := 89098.63397 N~
> mxt := msx_capacity : myt := msy_capacity :
> mxb := msx_capacity : myb := msy_capacity :
> # ---------- 5a ----------
> sigmaxx := mxx/(1/6·h^2); sigmayy := myy/(1/6·h^2); sigmaxy := mxy/(1/6·h^2);
                        sigmaxx := 15.12465374 N~/mm^2
                        sigmayy := -3.822714682 N~/mm^2
                        sigmaxy := 14.45983379 N~/mm^2
```

```
> # ---------- 5b ----------
> (-mxt-mxx) * (-myt-myy) ≥ mxy^2; # top reinforcement is sufficient,
                                   7.569000000 10^9 N~^2 ≤ 1.238503270 10^10 N~^2
> (mxb-mxx) * (myb-myy) ≥ mxy^2; # bottom reinforcement is not.
                                   7.569000000 10^9 N~^2 ≤ 6.021929471 10^8 N~^2
> # ---------- 5c ----------
> mxt2 := -mxx + |mxy| : mxt2 := 0; myt2 := -myy + |mxy|;
                                                  mxt2 := 0
                                                 myt2 := 110000. N~
> mxb2 := mxx + |mxy|;                    myb2 := myy + |mxy|;
                                                  mxb2 := 178000. N~
                                                  myb2 := 64000. N~
> # ---------- 5d ----------
> m1 := (mxx + myy)/2 + sqrt(((mxx-myy)/2)^2 + mxy^2);
                                                 m1 := 138009.6149 N~
> m2 := (mxx + myy)/2 - sqrt(((mxx-myy)/2)^2 + mxy^2);
                                                 m2 := -70009.6149 N~
> m1t := -m1 + |0| : m1t := 0;   m2t := -m2 + |0|;
                                                  m1t := 0
                                                 m2t := 70009.6149 N~
> m1b := m1 + |0|;                        m2b := m2 + |0| : m2b := 0;
                                                 m1b := 138009.6149 N~
                                                  m2b := 0
```

Question 6

a) Yes. For both elements the displacements along the common edge depend only on the displacements of the two common nodes. Both elements interpolate these displacements in the same way.

b) Only ε_{yy} will be necessarily continuous, since ε_{yy} only depends on the (u_y-) displacements along the common edge which are the same due to the compatibility of displacements (see **a)**). Note that nodal averaging will make that the ε_{xx} and γ_{xy}strains *appear* continuous.

c) No. Element A will *linearly* interpolate the (u_x- and u_y-) displacements along the common edge between the common corner nodes. Element Q will *quadratically* interpolate the displacements along the common edge between the common corner nodes. Gapping (or overlapping) as well as sliding between the common corner nodes can occur.

d) None of these strains will be necessarily continuous. Specifically for ε_{yy}: the (u_y-) displacements along the common edge will no longer be necessarily the same due to the incompatibility of displacements (see **c)**)