Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CIE4180: Plate Analysis
Thursday 27 January 2022, 13:30 – 16:30

This exam consists of 6 questions.
The respective weights are 1.4, 1.4, 2.1, 2.0, 1.6 and 1.5.

Use **separate sheets of paper for each question**!

Write your name and student number **on every page of your work.**

In this exam the notation and sign conventions of the reader (part 1 and 2) and the book (Plates and FEM) have been adopted. **You are supposed to use the same notation and sign convention.**

Question 1

Use **separate sheets of paper for each question**!

A linear finite element analysis is made of a plate with dimensions l_1+l_2 by d, see Figure 1.1 below. The plate has an uniform thickness t. Young's modulus is E and Poisson's ratio is ν.

The plate is supported at 3 points. The plate is loaded, in-plane, by a uniformly distributed load p at the top of the plate over a length l_1. There is no dead load.

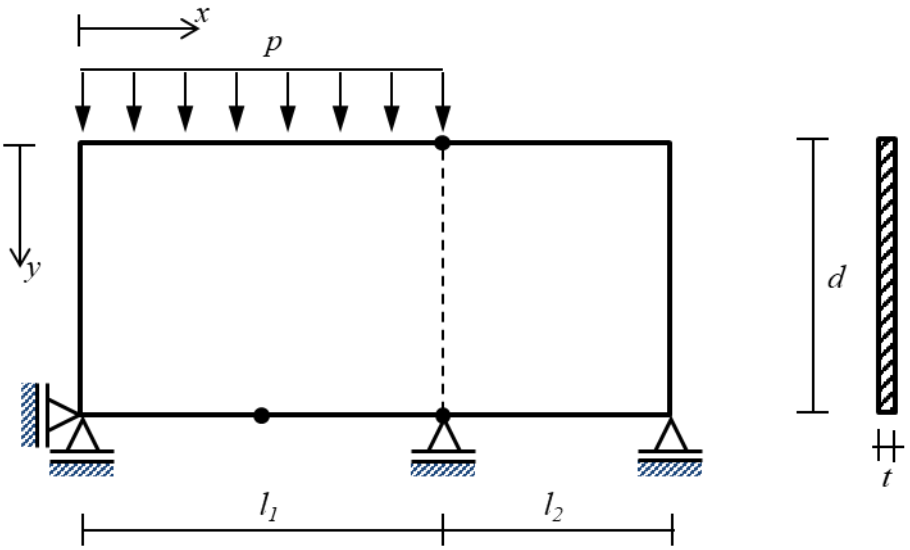


Figure 1.1: a plate with dimensions l_1+l_2 by d

l_1 = 2400 mm, l_2 = 1200 mm, d =1600 mm, t = 110 mm, E = 32200 MPa, ν = 0.15, p = 2000 N/mm. In the FE analysis 4-noded (i.e. linear) plane stress elements are used with element size h = 100 mm.

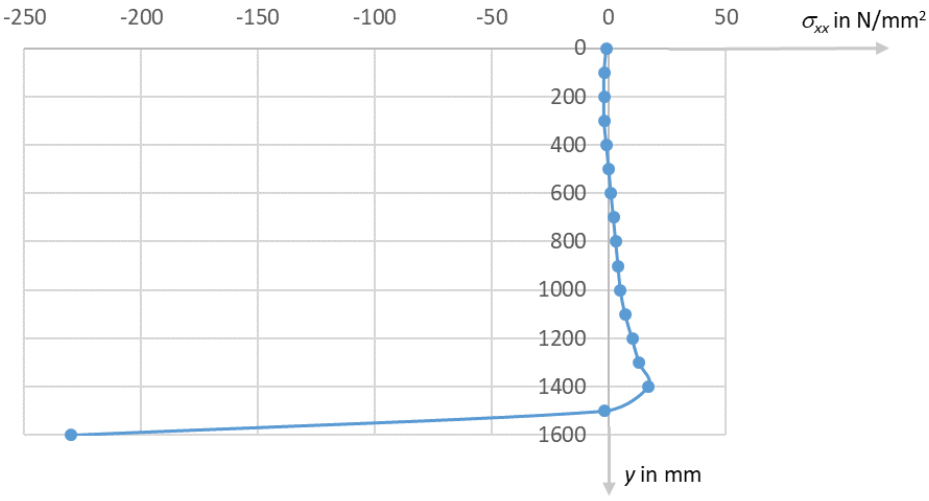


Figure 1.2: σ_{xx} as function of y at x = l_1

Figure 1.2 is a graph of σ_{xx} as function of y (with y along the vertical axis) at the vertical cut at x = l_1.

a) Considering Figure 1.2, how do you interpret the value σ_{xx} = -230 MPa at y = 1600 mm?

b) Considering Figure 1.2, provide two integral equations that could be used to verify equilibrium. It is sufficient to provide the two integral equations and clarify the symbols, where necessary; you do not have to verify equilibrium.

c) How would you obtain a realistic stress σ_{yy} in the plate at the intermediate support from the FE analysis results?

Now, for a completely different plate loaded in plane, an inclined uniformly distributed load acts on an inclined edge of this plate. Figure 1.3 shows the part of the plate including the inclined edge and the inclined distributed load p as well. The edge has an angle of 30° with the x-axis. The load has an angle of 30° with the edge.

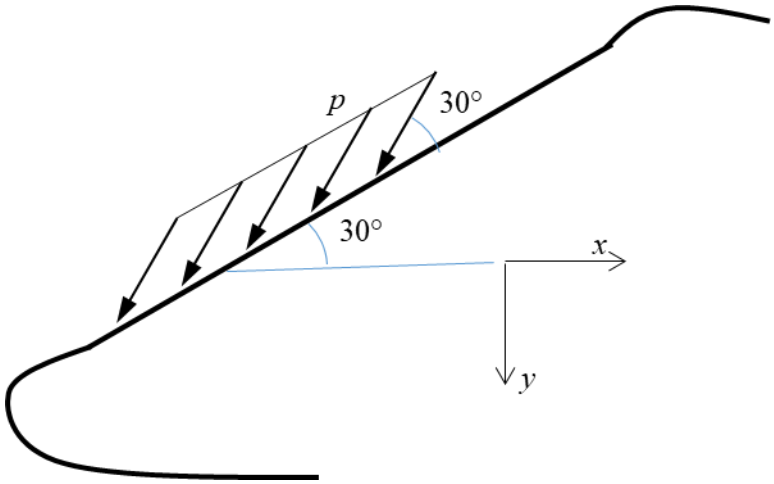


Figure 1.3: a part of a plate with an inclined edge and an inclined uniformly distributed load

d) The FE analyst finds at this edge a shear membrane force n_{xy} of zero and she/he doubts whether this can be correct. Can this be correct? Motivate your answer.

Question 2

Use **separate sheets of paper** **for each question**!

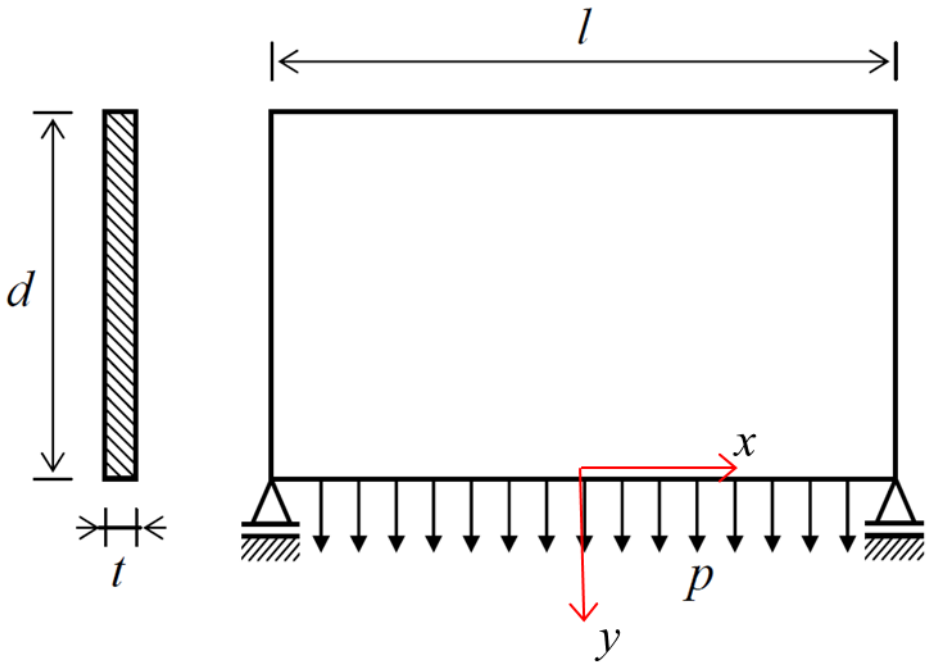


Figure 2.1: Deep beam

Consider the plane stress problem of a deep beam in Figure 2.1 with a homogeneously distributed load p (force per length). The Poisson's ratio is zero. The origin of the axis system is located at midspan at the bottom of the beam.

A young researcher is exploring the suitability of a third order polynomial for the vertical displacements:

$$u_y(x,y) = b_1 + b_2x + b_3y + b_4x^2 + b_5xy + b_6y^2 + b_7x^3 + b_8x^2y + b_9xy^2 + b_{10}y^3$$

a) Based on the symmetry of the displacement field, which terms could be omitted?

b) Based on a positive value for the load p, what is the sign of b_1?

c) Based on the kinematic boundary condition at the bottom of the beam, derive an expression for one of the coefficients b_i.

d) Will the young researcher be successful by using this displacement field? Explain.

Question 3

Use **separate sheets of paper** **for each question**!

a) In Delft we write beam moments with a capital *M* and plate moments with a lower case *m*, for example m_{xx} . Why this difference? Choose A, B, C or D.

A *M* is in kNm and *m* is in kNm/m.
B *M* and *m* are both second order tensors.
C *M* is often much larger than *m*.
D *M* is real and *m* is imaginary.

b) Is the plate theory true? Choose A, B, C, D, E or F and explain your answer.
A Yes, it is in the bible.
B Yes, it is based on experiments.
C Yes, it follows from 3D elasticity.
D Yes, important people use it.
E No, it is just an opinion.
F No, in the future it might be disproved.

c) Which equation is used for calculating the shear force v_x in thin plates? Choose A, B, C or D.

A $v_x = \dfrac{\partial^2 w}{\partial x \partial y}$

B $v_x = \int_{z=-\frac{1}{2}t}^{\frac{1}{2}t} \sigma_{xz} dz$

C $v_x = \dfrac{Gt}{\eta}\gamma_x$

D $v_x = \dfrac{\partial m_{xx}}{\partial x} + \dfrac{\partial m_{xy}}{\partial y}$

d) Consider a free edge of a thin plate. Which is the boundary condition? Choose A, B, C or D.

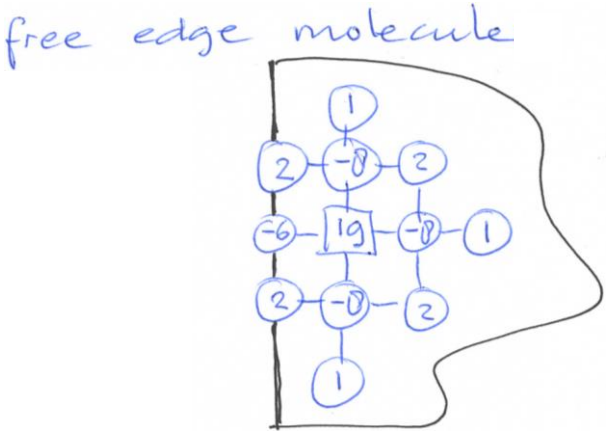
A $w = 0,\ \varphi_y = 0$

B $m_{yy} = 0,\ v_y + \dfrac{\partial m_{xy}}{\partial x} = 0$

C $w = 0,\ m_{yy} = 0$

D $m_{yy} = 0,\ v_y = 0$

e) Consider the following molecule of the finite difference method of thin plates. What is the number of discrete elements in this molecule?



f) A linear elastic model can have infinite stresses in particular locations. What is the name of such a location? Choose A, B, C or D.

A Stress intensity
B Singularity
C Stress concentration
D Supposition

g) Who was Stephen Timoshenko? Choose A, B, C or D.

A Somebody who gave money to science and in particular to the development of the thin plate theory.
B A French mathematician who started deriving the plate theory.
C A German physicist who derived the boundary conditions of thin plates.
D An engineer who wrote structural mechanics books.

Question 4

Use **separate sheets of paper** **for each question**!

A plate is loaded perpendicularly to its plane. One edge is simply supported, one is fixed and the others are free (Fig. 6). The plate length is 15 m and the width is 10 m. The plate thickness is 190 mm. A distributed load of 10 kN/m^2 is applied in the z-direction. This distributed load includes self-weight.
A linear elastic finite element model is made. The following material properties are used, E = 31500 N/mm^2, ν = 0.2. Four-node plate elements are used (Kirchhoff theory). The element size is 400 mm. The results are shown in Figures 4.1 to 4.6.

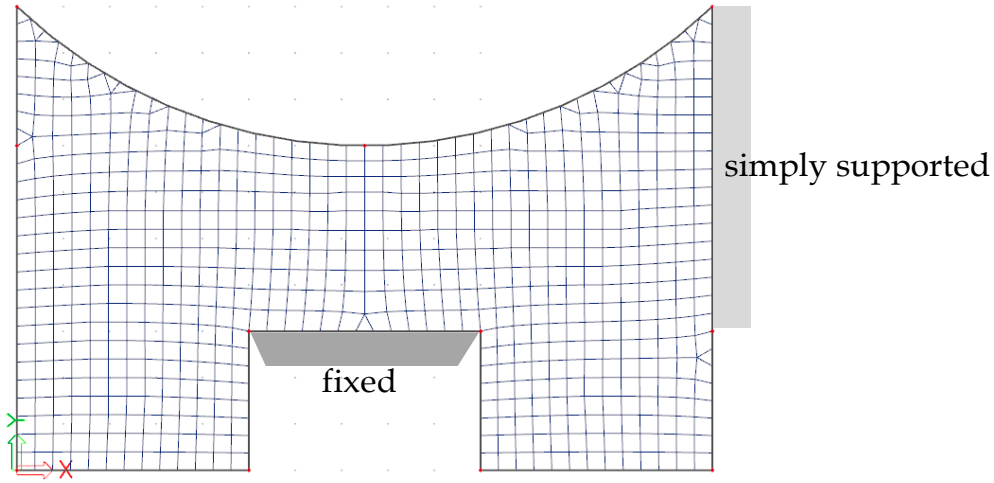


Figure 4.1. Plate with supports and finite element mesh

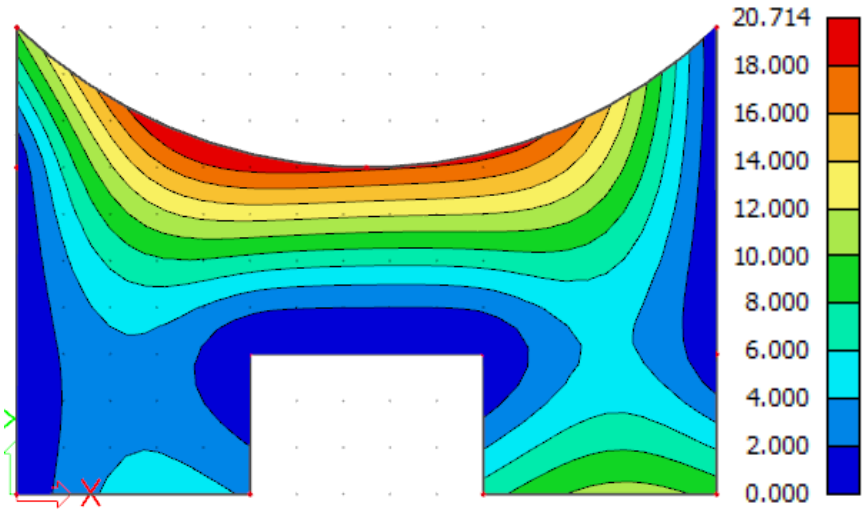


Figure 4.2. Contour plot of the plate deformation [mm]

a) Is the deformation in the correct direction? Explain your answer.

b) Is the element size sufficiently small? Explain your answer.

c) Does the deformation fulfil the boundary conditions? Explain your answer.

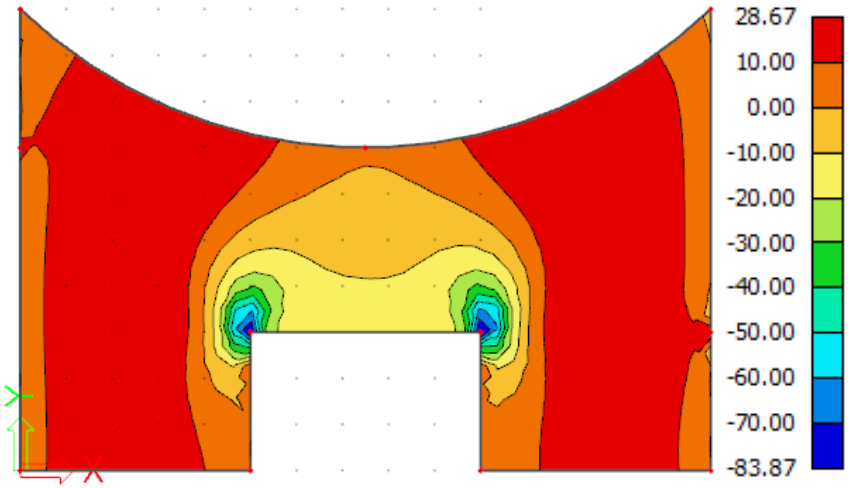


Figure 4.3. Contour plot of the bending moment m_{xx} [kNm/m]

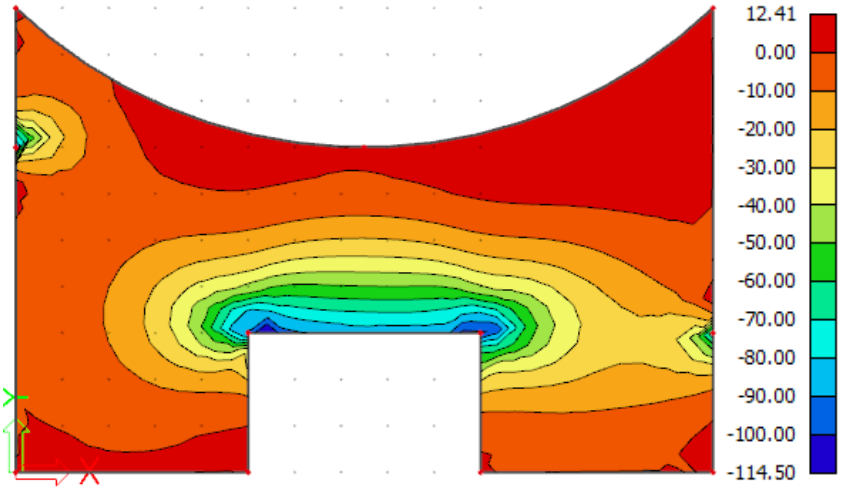


Figure 4.4. Contour plot of the bending moment m_{yy} [kNm/m]

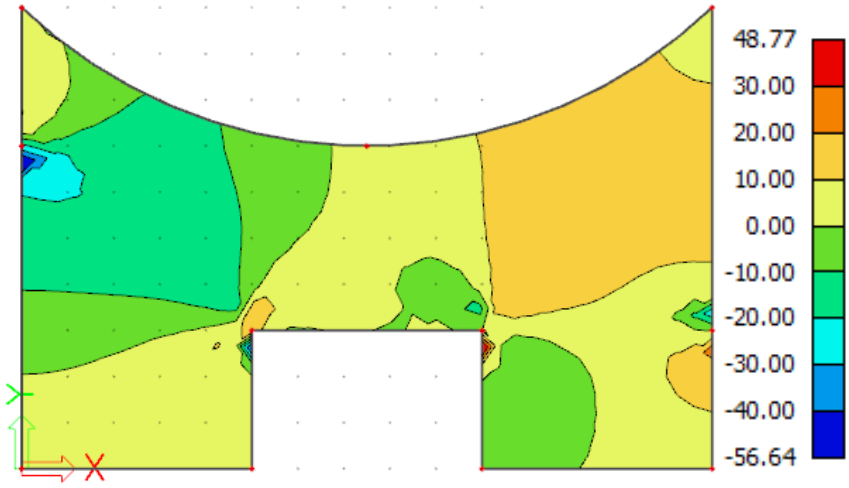


Figure 4.5. Contour plot of the torsion moment m_{xy} [kNm/m]

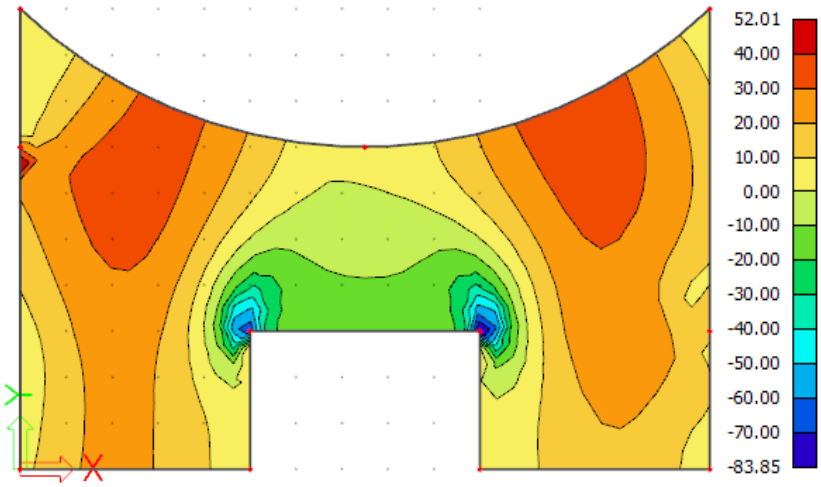


Figure 4.6. Contour plot of the largest principal moment m_1 [kNm/m]

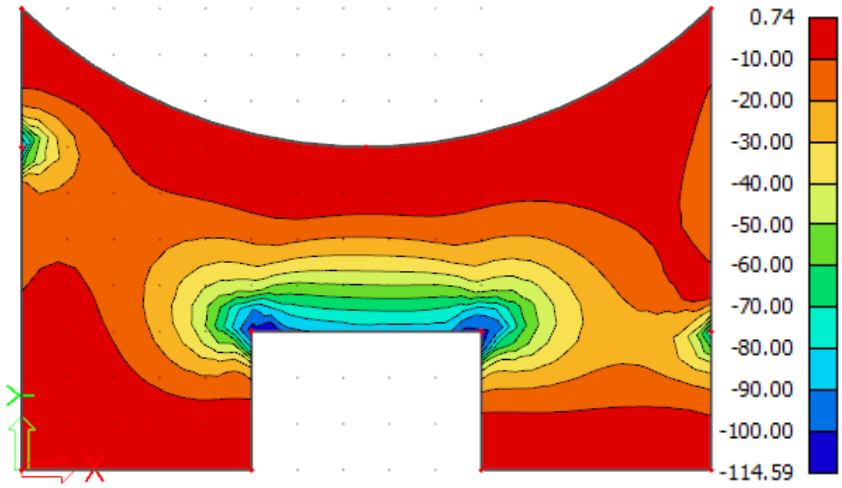


Figure 4.7. Contour plot of the smallest principal moment m_2 [kNm/m]

d) What is the value of the most negative moment in the plate?
Where does it occur?

e) The contour plot of m_{xy} shows almost no red and almost no blue.
What do we need to do in order to display more red and more blue?

f) The following statement can be derived from the computation results.
"The concentrated support reaction in the right top corner cannot be larger than 20 kN."
Explain this.

g) On the curved edge, m_2 is between -10.00 and 0.74 kNm/m. It is tempting to think that the exact value is zero. However, the exact value of m_2 is not perfectly zero. Logically explain this.

h) The program uses a different sign convention for bending moments than the plate course.
The teacher has done something to obtain the correct signs for this exam. What?

Question 5

Use **separate sheets of paper** **for each question**!

A reinforced concrete wall has a thickness of 190 mm. The membrane forces at a particular location are

n_{xx} = 109 kN/m
n_{yy} = -320 kN/m
n_{xy} = 478 kN/m

The compressive strength of the concrete is 32 N/mm^2 (partial factor included). The reinforcement consists of two meshes. As usual, one mesh is close to the front surface the other mesh is close to the back surface. Each mesh has bars in the *x* direction and bars in the *y* direction. All bars have a diameter of 12 mm and are spaced 210 mm. The yield strength of the reinforcing steel is 350 N/mm^2 (partial factor included). The membrane force capacity due to the bars is

n_{sx} = 377 kN/m
n_{sy} = 377 kN/m

a) Suppose that the wall is linear elastic. What are the normal stresses and shear stress?

b) Is the reinforcement sufficient? Explain your answer.

c) Calculate the reinforcement capacities that that are minimally required in the *x* and *y* directions.

d) Calculate the reinforcement capacities that that are minimally required in the principal directions.

Useful equations
nsx = nxx+|nxy| nsy = nyy+|nxy|
n1 = (nxx+nyy)/2+sqrt(((nxx-nyy)/2)^2+nxy^2)
n2 = (nxx+nyy)/2 -sqrt(((nxx-nyy)/2)^2+nxy^2)
theta = 1/2*arctan(2*nxy/(nxx-nyy))
(nsx-nxx)*(nsy-nyy) >= nxy^2

Question 6

Use **separate sheets of paper** **for each question**!

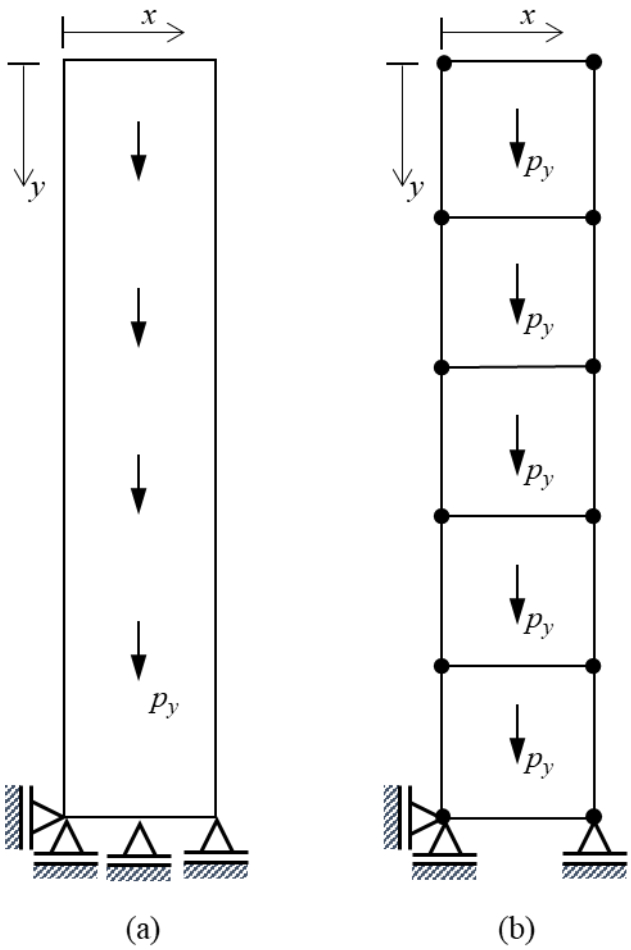


Figure 6.1: a rectangular flat plate with an uniformly distributed load p_y per unit of area (a), a finite element model for this plate consisting of 5 rectangular (standard) plate elements (b)

Figure 6.1(a) shows a rectangular flat plate with uniform thickness and with a downwards uniformly distributed in-plane load p_y per unit of area (think e.g. of the load exerted by the self-weight of the plate). At the bottom of the plate the boundary conditions are such that the plate is free to extend in horizontal direction.

a) Consider writing the stress field of σ_{yy} as a linear field $\sigma_{yy}(x, y) = a + bx + cy$ with three coefficients a, b and c. Is a linear field adequate? For each of these coefficients indicate whether they are negative, zero or positive.

b) Similar question for the stress field of σ_{xx}

c) Similar question for the strain field of ε_{yy}

d) Similar question for the strain field of ε_{xx}

Figure 6.1(b) shows a finite element model for this plate with 5 rectangular (standard) plate elements.

e) To which extend does this finite element model capture the displacement field $u_y(x, y)$ of the plate shown in Figure 6.1(a)?

f) Will the finite element result of the strain field $\varepsilon_{yy}(x, y)$ show as continuous?

g) Similar question for the strain field of ε_{xx}

Brief answers

Question 1

a) At this position the stress σ_{yy} is an approximation of negative infinity. The stress σ_{xx} is negative infinity too due to Poisson's ratio.

b) $\int_{y=0}^{1600} \sigma_{xx}\, dy = 0$ and $\int_{y=0}^{1600} y\, \sigma_{xx}\, dy = l_2 R_3$ where R_3 is the support reaction in the third support

c) A realistic stress is the negative support reaction over support area

d) It could be correct. The (kinematic) boundary conditions provide (only) 2 linear equations in 3 unknown membrane forces, n_{xx}, n_{yy} and n_{xy}. $n_{xy} = 0$ is possible. (more specific for this case, these 2 equations read: $\sqrt{3}n_{xy} + n_{xx} - p = 0$ from horizontal equilibrium and $\sqrt{3}n_{yy} + n_{xy} + \sqrt{3}p = 0$ from vertical equilibrium)

Question 2

a) The terms with an odd power of x: b_2, b_5, b_7 and b_9

b) Positive

c) $\varepsilon_{yy}(x, y) = b_3 + 2b_6 y + b_8 x^2 + 3b_{10} y^2$ and thus $\varepsilon_{yy}(x, 0) = b_3 + b_8 x^2$. With zero Poisson's ratio $n_{yy}(x, 0) = Et(b_3 + b_8 x^2)$. Since $n_{yy}(x, 0) = p$ for all x, we obtain $b_3 = p/Et$.

d) No. We can e.g. note this from the results off c). $n_{yy}\left(\pm\frac{1}{2}l, 0\right) = p > 0$ will not satisfy equilibrium.

Question 3

a) A

b) C … plus the assumption that a straight needle remains straight.
F … This is called Popper's falsification principle. Indeed, more plate theories may be developed. Several nonlinear plate theories are already available. We (teachers) do not think any will replace the Kirchhoff or the Mindlin-Reissner plate theory but this cannot be proofed of course.

c) D

d) B

e) 9

f) B

g) D

Question 4

a) Yes, the load is upwards and the deformation is upwards.

b) Yes, 400 mm is approximately two times the thickness of 190 mm.

c) No, one expects some displacement of the left edge and there is none. A mistake was made.

d) -115 kNm/m, however, this occurs in singularities, so the exact value is negative infinity.

e) Red and blue are in the singularities. Adjust the colour bar. (User defined smallest and largest value, for example -30 and 20 kNm/m).

f) m_{xy} is light green in the corner, between 0 and 10 kNm/m. $R = 2\, m_{xy}$.

g) The exact bending moment on the free edge is perfectly zero … but principal directions are defined as the directions in which the torsion moment is zero and free edges have in general a large concentrated shear force (torsion moment). So, the edge direction is not the principal direction and the principal moment is not perfectly zero.

h) The teacher entered the load negatively for the bending moment plots.

Question 5

> $nxx := 109 \cdot \frac{kN}{m} : nyy := -320 \cdot \frac{kN}{m} : nxy := 478 \cdot \frac{kN}{m} : nsx := 377 \cdot \frac{kN}{m} : nsy := 377 \cdot \frac{kN}{m} : nsx + nsy;$

$$\frac{754\ kN}{m}$$

> $t := 190 \cdot mm : kN := 1000. \cdot N : m := 1000. \cdot mm : assume(N > 0) : assume(mm > 0) :$

> $sxx := \frac{nxx}{t}; syy := \frac{nyy}{t}; sxy := \frac{nxy}{t};$ *# a)*

$$sxx := \frac{0.5736842105\ N\sim}{mm\sim^2}$$

$$syy := -\frac{1.684210526\ N\sim}{mm\sim^2}$$

$$sxy := \frac{2.515789474\ N\sim}{mm\sim^2}$$

> $(nsx - nxx) \cdot (nsy - nyy) > nxy^2;$ *# b) Not sufficient*

$$\frac{228484.0000\ N\sim^2}{mm\sim^2} < \frac{186796.0000\ N\sim^2}{mm\sim^2}$$

> $nsx2 := nxx + \mathrm{abs}(nxy); nsy2 := nyy + \mathrm{abs}(nxy);\ nsx2 + nsy2;$ *# c) Minimal in x and y:*

$$nsx2 := \frac{587.0000000\ N\sim}{mm\sim}$$

$$nsy2 := \frac{158.0000000\ N\sim}{mm\sim}$$

$$\frac{745.0000000\ N\sim}{mm\sim}$$

> $n1 := \frac{nxx + nyy}{2} + \mathrm{sqrt}\left(\left(\frac{nxx - nyy}{2} \right)^2 + nxy^2 \right); n2 := \frac{nxx + nyy}{2} - \mathrm{sqrt}\left(\left(\frac{nxx - nyy}{2} \right)^2 + nxy^2 \right);$
 # d) Minimal in principal directions

$$n1 := \frac{418.4219885\ N\sim}{mm\sim}$$

$$n2 := -\frac{629.4219885\ N\sim}{mm\sim}$$

> $nsx3 := n1; nsy3 := 0; nsx3 + nsy3;$

$$nsx3 := \frac{418.4219885\ N\sim}{mm\sim}$$

$$nsy3 := 0$$

$$\frac{418.4219885\ N\sim}{mm\sim}$$

Question 6

a) Yes, with c < 0 and a = b = 0 (see equilibrium equation)
b) Yes, with a = b = c = 0 (see equilibrium equation)
c) Yes, with c < 0 and a = b = 0 (see constitutive equation in flexibility formulation)
d) Yes, with c > 0 and a = b = 0 (see constitutive equation in flexibility formulation)
e) $u_y(x, y)$ is quadratic in y (see kinematic equation). In the finite element result $u_y(x, y)$ is continuous piecewise linear (with five "pieces").
f) No, ε_{yy} will be piecewise constant in y-direction. Nodal averaging will make it look like continuous piecewise linear though.
g) Yes, ε_{xx} can vary linearly over y within an element. At the element transitions it will attain the same value for both elements.