Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CIE4180: Plate Analysis
Friday, 21 April 2023, 9:00 – 12:00 hours, room 2.98

This exam consists of 6 questions.
The respective weights are 1.6, 1.5, 1.6, 2.2, 1.5 and 1.6.

Write your name and student number **on every page of your work.**

In this exam the notation and sign conventions of the reader (part 1 and 2) and the book (Plates and FEM) have been adopted. **You are supposed to use the same notation and sign convention.**

Question 1 (copied from April 2017)

Consider the plane stress (or plate membrane) problem below. The measurements of the quadrilateral plate with a snipped corner are in mm. The length of the plate is 300 mm. The height is 300 mm. The plate has an uniform thickness $t = 100$ mm.

The plate is loaded by an equally distributed line load $f = 100$ N/mm, acting at the top of the plate. At the bottom of the plate, both vertical and horizontal displacements are prohibited.

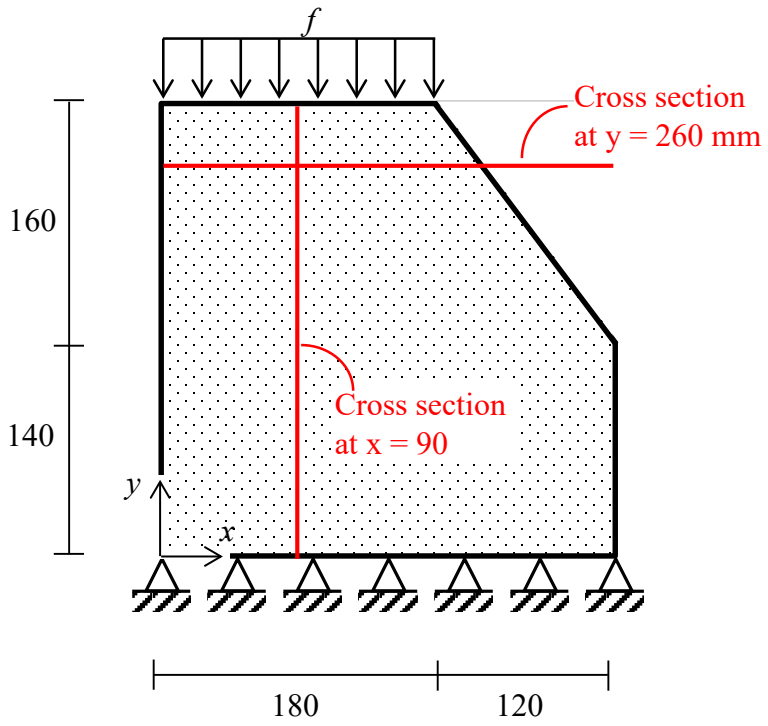
An adequate finite element analysis has been made using a mesh with 8-noded plane stress elements of 5×5 mm^2.

$E = 30000$ N/mm^2 , $\nu = 0.2$

Along the two red lines (indicated as "Cross section") results are plotted (next page):

- Along the horizontal line the stress σ_{xx} is plotted (in MPa).
- Along the vertical line the strain ε_{yy} is plotted.

Values can be measured in the diagrams.

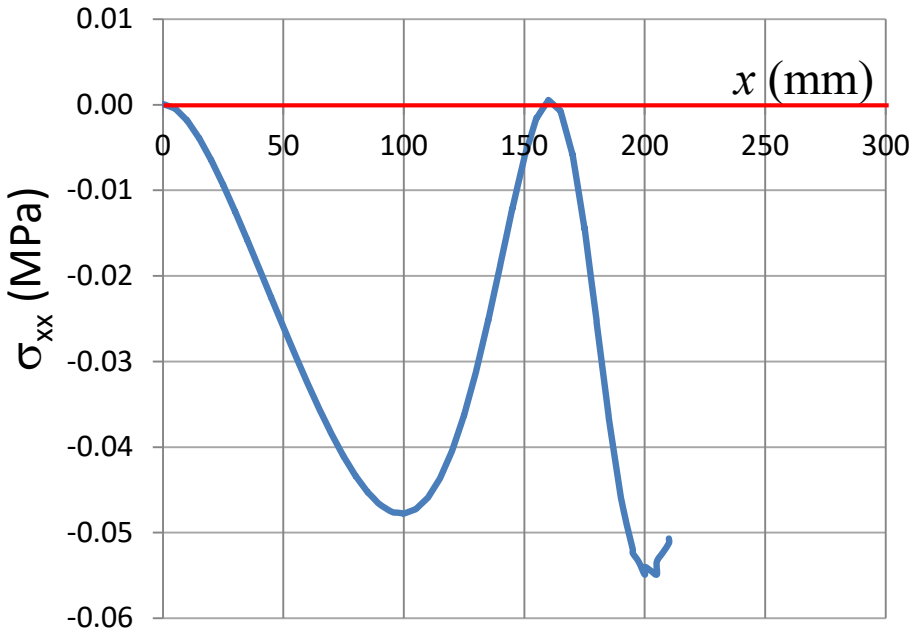


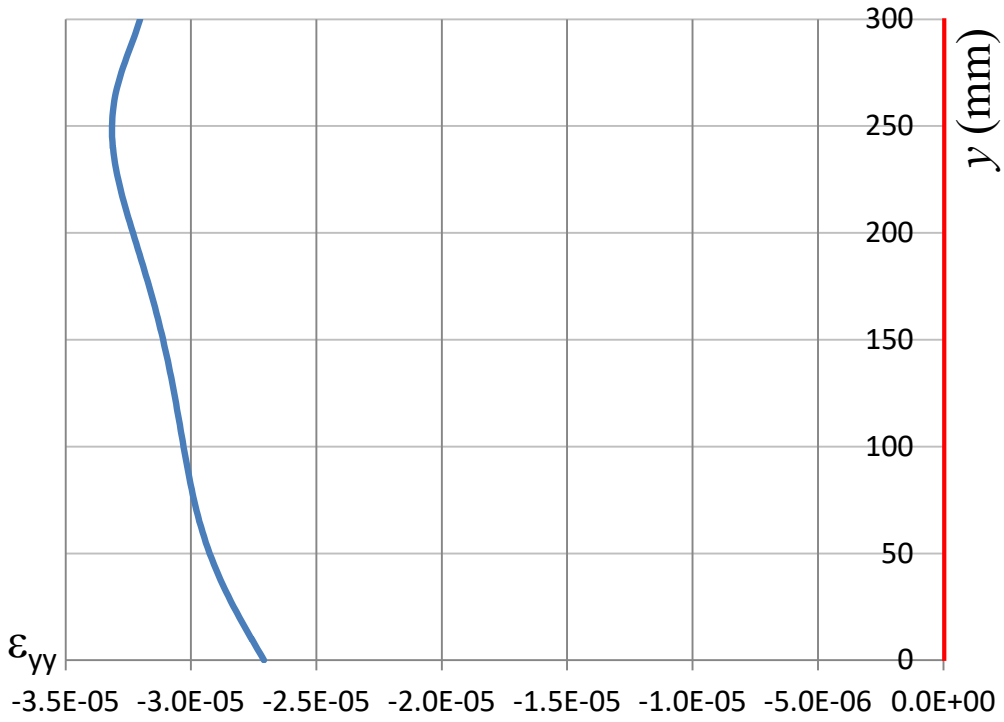
a) Could the diagram with the stress σ_{xx} be a diagram for this case? Explain your answer.

b) Calculate the stress σ_{yy} at $x = 90$, $y = 300$.

c) Calculate the stress σ_{yy} at $x = 90$, $y = 0$.

d) Now, consider that we change the *y*-axis to its opposite direction. Which of the following quantities change sign? σ_{xx}, σ_{yy}, σ_{xy}





Question 2 (copied from April 2021)

A plate consists of two parts, A and B. See Figure 2.1 below.

Note that these parts are made of the two materials, with the same stiffness *E*, different Poisson's ratio ν, and a different thickness (½*t* and *t*), as indicated in the figure.

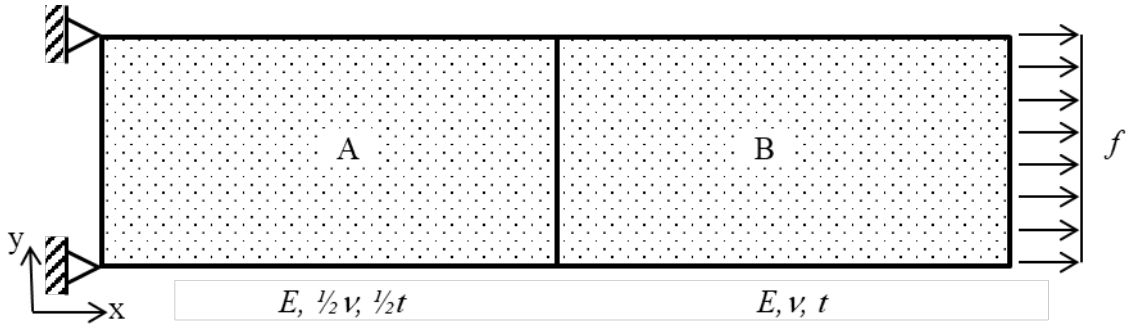


Figure 2.1: A plate consists of two parts, A and B.

a) Give the condition for ε_{xx} for the transition zone between the plate parts A and B. In other words, how does the strain ε_{xx}^{A} immediately left from the transition zone relate to the strain ε_{xx}^{B} immediately right from the transition zone.

b) Same question, but now for the strain ε_{yy}.

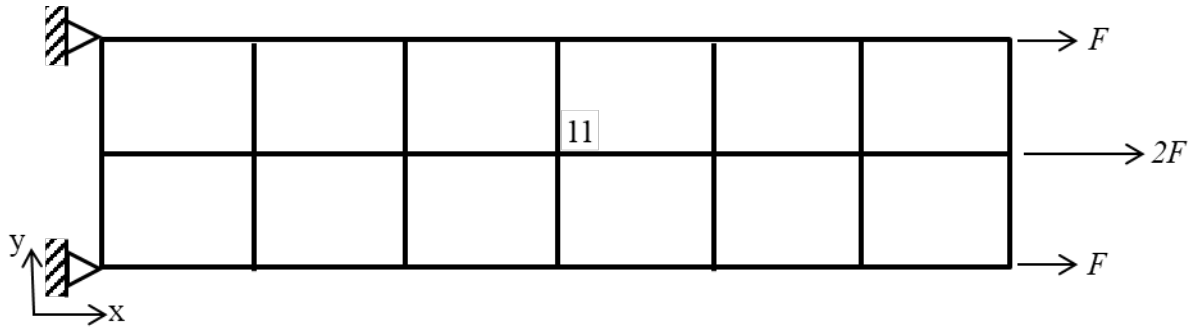


Figure 2.2: Finite element model of this plate. The node number 11 is indicated.

c) Figure 2.2 shows a finite element model for this plate consisting of 12 elements and 21 nodes. Node number 11 is indicated.
Will nodal averaging influence the result for ε_{xx} in node 11?
Will nodal averaging influence the result for ε_{yy} in node 11?
Will nodal averaging influence the result for u_x in node 11?
Three subsequent Yes/No answers will suffice.

Question 3 (copied from February 2023)

a) Consider a plate edge that goes in the x direction or in the y direction. The magnitude of the concentrated shear force in this edge is equal to m_{xy}. Is this always true?

Choose A, B, C or D.

A Yes
B Not in fixed plate edges
C Not in free plate edges
D Not in corners

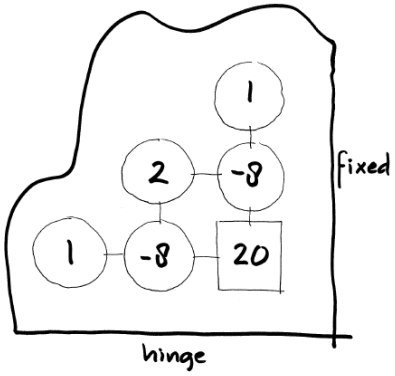
b) How do we deal with singularities? Choose A, B, C or D.

A Very carefully.
B Refine a model with yielding, cracking, round corners, distributed supports et cetera.
C Ignore or replace by hand calculations.
D Choose a mesh with very small elements to compute the stress concentration accurately.

c) What is the finite difference method? Choose A, B, C or D.

A Computational way of solving differential equations.
B Systematic way of formulating the plate differential equations.
C Method for avoiding infinitesimal singularities.
D Method for analysing discrete elements like torsion panels.

d) Shown is a molecule of the finite difference method.
How many discrete elements are in this molecule?



e) In the Kirchhoff plate theory, the element size should be larger than the thickness.
Why is this true? Choose A, B, C or D.

A The question says this is true. The teacher wrote the question. Teachers know these things.

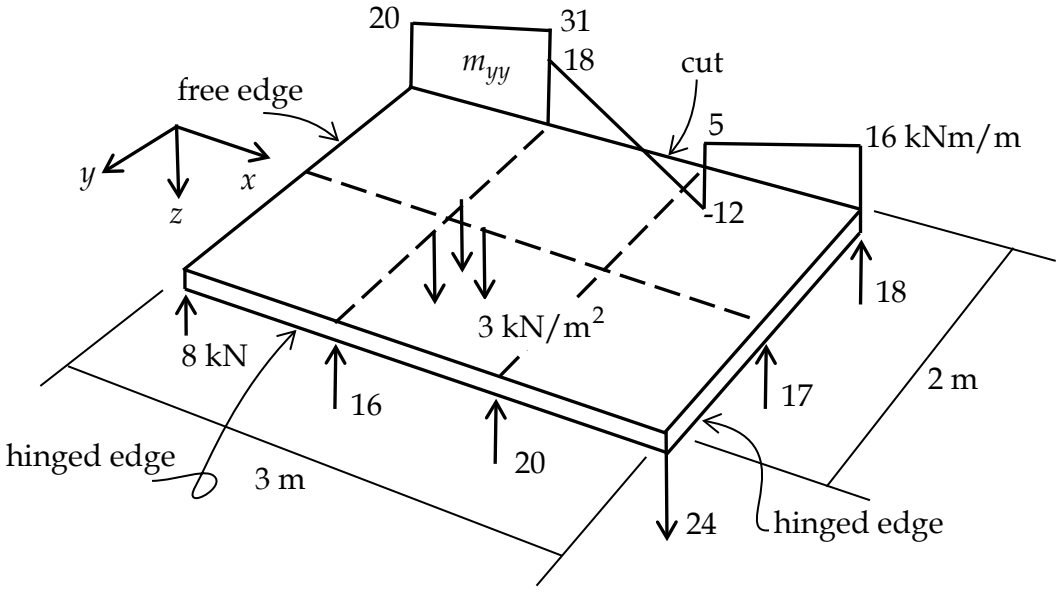
B This is not always true. For example, at a point load much smaller elements may be required to compute the bending moments and the torsion moment with scientific accuracy.

C Somebody checked that even smaller elements only show larger singularities.

D This is stated in the reader (Blaauwendraad, Plates and FEM, p. 286).

f) A plate has been analysed by the finite element method. Part of this plate is cut free. The figure below shows the free part with distributed load and support reactions. The figure also shows the bending moment m_{yy} in the cut. The torsion moment m_{xy} and the shear force v_y occur also in this cut but these are not displayed.

Is this plate part in moment equilibrium around the *x* axis? Explain your answer.



g) A square plate is simply supported (hinged) at all four edges.
The perpendicular load is distributed in a two-way sine shape.

Young's modulus	$E = 30\,000$ N/mm^2
Poisson's ratio	$\nu = 0.15$
thickness	$t = 200$ mm
length and width	$a = 10$ m
load amplitude	$\hat{p} = 4$ kN/m^2

Deflection formula $\hat{w} = \dfrac{\hat{p}\,a^4}{4\pi^4 D}$ (Blaauwendraad p. 108)

How large is the deflection of the plate?

h) Consider the shear forces v_x and v_y in a plate loaded by a distributed load.
Do they depend on Young's modulus E ?
Do they depend on Poisson's ratio ν ? Choose A, B, C or D.

A yes, yes
B yes, no
C no, yes
D no, no

Question 4 (copied and adapted from January 2021)

A plate is loaded perpendicularly (Fig. 4.1). The plate length is 16 m and the width is 8 m. The plate thickness is 250 mm. A distributed load of 9 kN/m^2 is applied. This distributed load includes self-weight. A linear elastic finite element model is made. E = 34100 N/mm^2, ν = 0.2. Applied are four node plate elements (Kirchhoff theory). The element size is 500 mm. Results are shown in Figures 4.2 to 4.5.

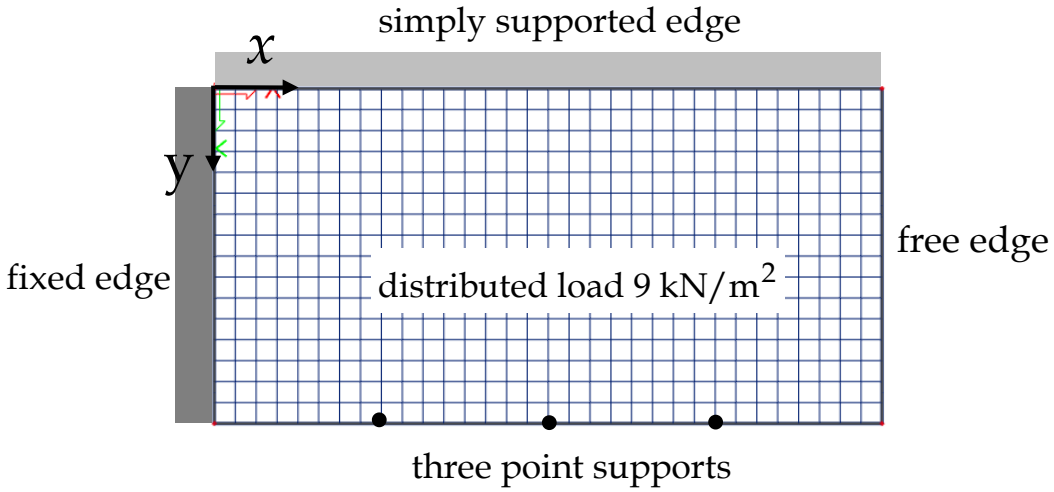


Figure 4.1. Plate with supports, load and finite element mesh

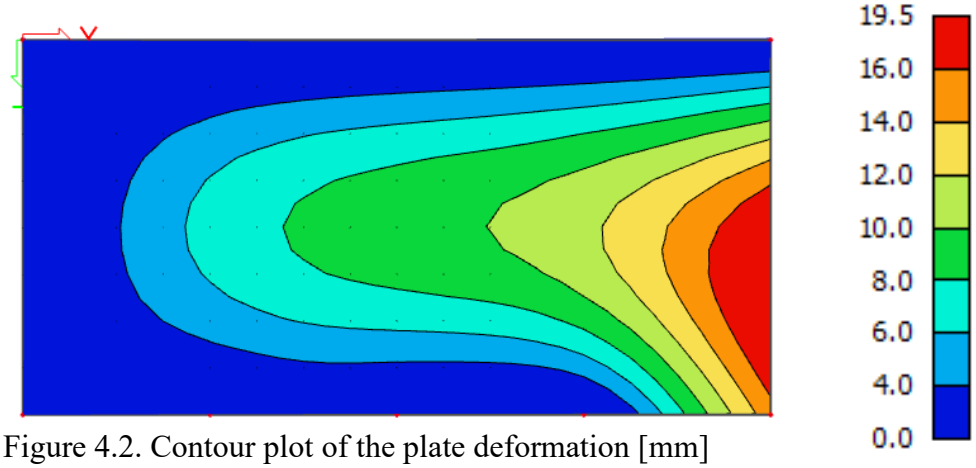


Figure 4.2. Contour plot of the plate deformation [mm]

a) Does the plate deform in the correct direction? Explain your answer.

b) Do the moments agree with the boundary conditions? Explain your answer.

c) Where in this plate can we expect a singularity? Please, answer with a drawing. Which of these potential singularities actually show up in the result plots?

d) How large is the largest moment in the plate? (in absolute value)

e) The torsion moment is positive left of the point supports and negative right of the point supports. Explain this.

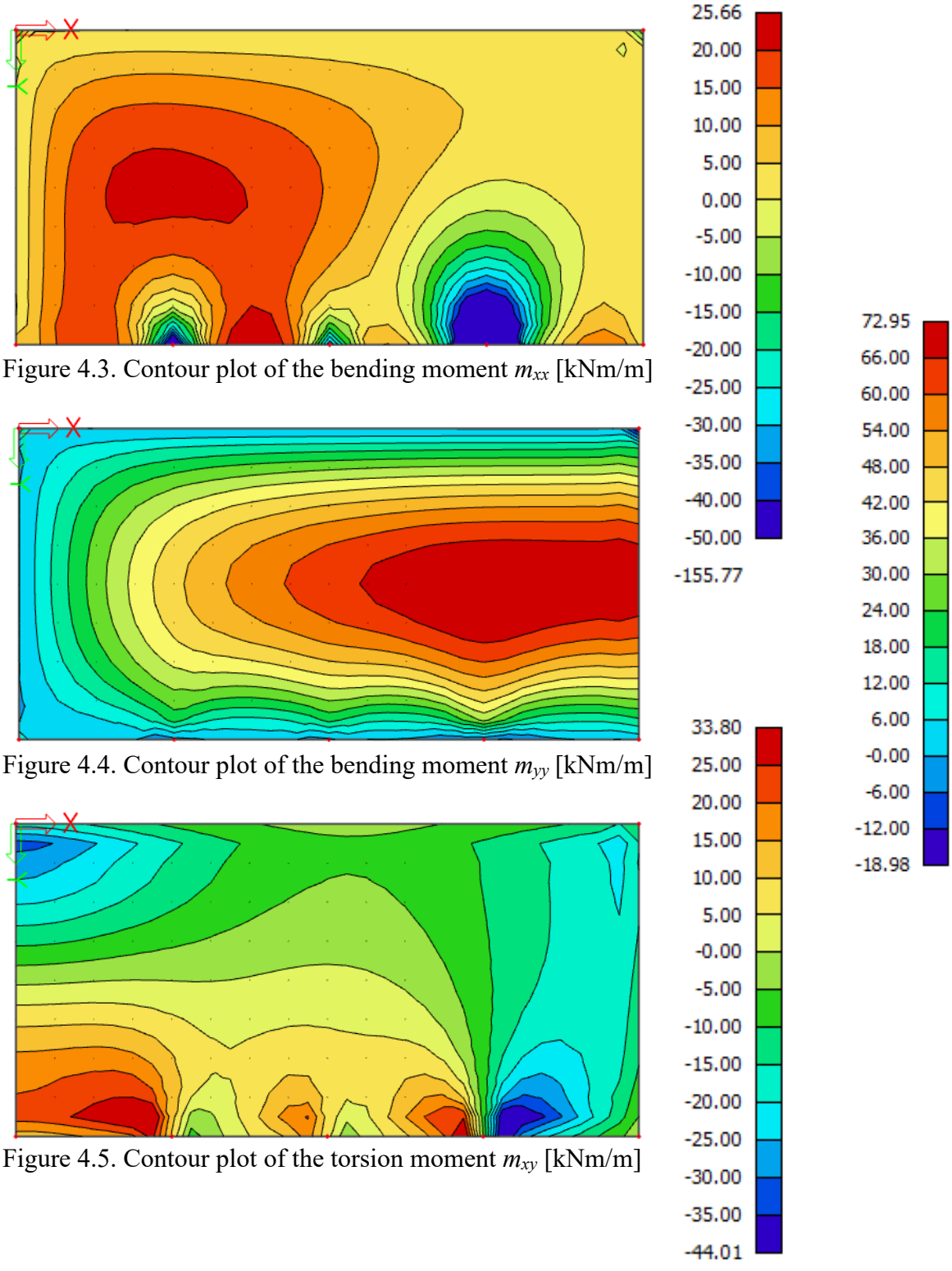


Figure 4.3. Contour plot of the bending moment m_{xx} [kNm/m]

Figure 4.4. Contour plot of the bending moment m_{yy} [kNm/m]

Figure 4.5. Contour plot of the torsion moment m_{xy} [kNm/m]

Question 5 (copied from January 2019)

A reinforced concrete plate has a thickness of 310 mm. The plate carries 11 kN/m^2 (self-weight plus live load; partial factors included). At a particular location the moments are
m_{xx} =111 kNm/m,
m_{yy} = -42 kNm/m,
m_{xy} = -59 kNm/m.

The reinforcement consists of two meshes. As usual, one mesh is close to the bottom surface the other mesh is close to the top surface. Each mesh has bars in the *x* direction and bars in the *y* direction. All bars have a diameter of 16 mm and are spaced 120 mm. The yield strength of the reinforcing steel is 380 N/mm^2 (partial factor included). The moment capacity due to the bars in the *x* direction is 141 kNm/m. The moment capacity due to the bars in the *y* direction is 131 kNm/m.

a) Are the reinforcing bars in the principal moment directions? How do we know?

b) Suppose that the plate is linear elastic. What are the normal stresses and shear stresses?

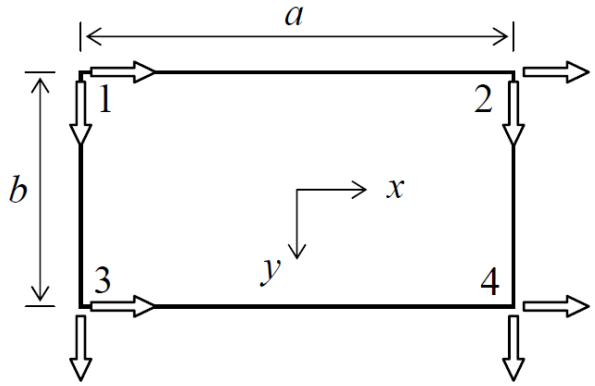
c) Calculate the reinforcement moment capacities that are minimally required in the *x* and *y* directions.

d) Is the applied reinforcement sufficient? Explain your answer.

e) Suppose that there are also membrane forces and out of plane shear forces in the plate. Which model can be used to check or design the reinforcement?

Useful equations
top $m_{sxt} = -m_{xx}+|m_{xy}|$ $m_{syt} = -m_{yy}+|m_{xy}|$
bottom $m_{sxb} = m_{xx}+|m_{xy}|$ $m_{syb} = m_{yy}+|m_{xy}|$
$m_1=(m_{xx}+m_{yy})/2+\text{sqrt}(((m_{xx}-m_{yy})/2)^2+m_{xy}^2)$
$m_2=(m_{xx}+m_{yy})/2 -\text{sqrt}(((m_{xx}-m_{yy})/2)^2+m_{xy}^2)$
top $(m_{sxt}+m_{xx})*(m_{syt}+m_{yy}) >= m_{xy}^2$
bottom $(m_{sxb}-m_{xx})*(m_{syb}-m_{yy}) >= m_{xy}^2$

Question 6 (copied and adapted from April 2019)



Consider a rectangular membrane element (a by b) with four nodes in the corners. The displacement field is written as follows:

$$u_x(x,y) = n_1(x,y)\, u_{x1} + n_2(x,y)\, u_{x2} + n_3(x,y)\, u_{x3} + n_4(x,y)\, u_{x4}$$

$$u_y(x,y) = n_1(x,y)\, u_{y1} + n_2(x,y)\, u_{y2} + n_3(x,y)\, u_{y3} + n_4(x,y)\, u_{y4}$$

Additional functions are defined as:

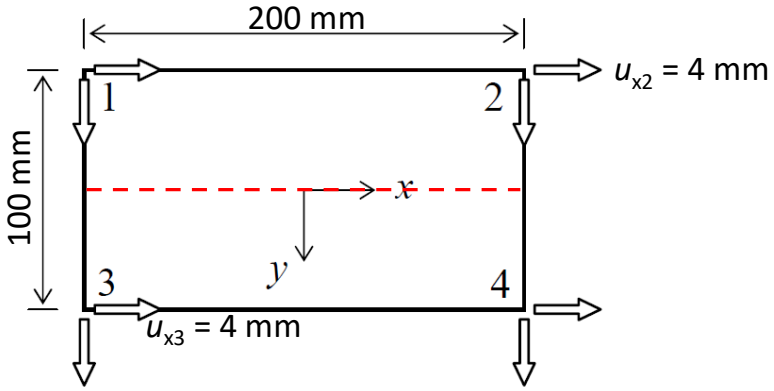
$$f^-(x) = ½ + x/a$$

$$g^-(y) = ½ + y/b$$

$$f^+(x) = ½ - x/a$$

$$g^+(y) = ½ - y/b$$

a) Express the shape functions $n_2(x,y)$ and $n_3(x,y)$ in $f^-(x)$, $g^-(y)$, $f^+(x)$ or $g^+(y)$.



Now consider the dimensions of the plate as indicated in the figure above. Also, consider that all nodal displacements are zero, except for $u_{x2} = u_{x3} = 4$ mm.

b) Sketch a graph of the strain ε_{xx} along the dashed line. Indicate values along the axes.

c) Similar for the strain ε_{yy}.

d) Similar for the strain γ_{xy}.