Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CIE4180: Plate Analysis
Thursday 2 February 2023

This exam consists of 6 questions.
The respective weights are 1.5, 1.5, 1.6, 2.4, 1.5 and 1.5.

Use a **separate sheet of paper** for each question.

Write your name and student number **on every page of your work.**

In this exam the notation and sign conventions of the reader (part 1 and 2) and the book (Plates and FEM) have been adopted. **You are supposed to use the same notation and sign convention.**

Question 1 Use a **separate sheet of paper** for this question.

Consider the plane stress (or plate membrane) problem in Figure 1.1. The plate is quadrilateral with a snipped corner. The length of the plate is 300 mm. The height is 300 mm. The plate has a uniform thickness $t = 13$ mm. The plate top is loaded by an equally distributed line load $f = 13$ N/mm. At the bottom of the plate, both vertical and horizontal displacements are prohibited. $E = 30000$ N/mm^2 , $\nu = 0.2$. An adequate finite element analysis has been performed using a mesh with 8-node plane stress elements of 5×5 mm^2.

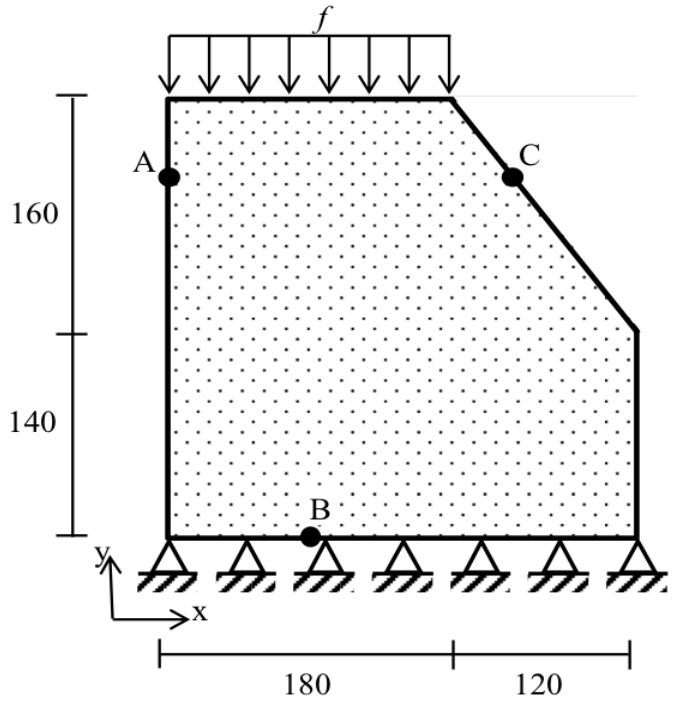


Figure 1.1: Plane stress (or plate membrane) problem

a) In point A is a node. For this node, the finite element analysis gives $\sigma_{xx} > 0$, with nodal averaging. Can this be a valid result? Explain your answer.

b) In point B is a node. For this node the finite element analysis gives $\varepsilon_{yy} = -2.6 \cdot 10^{-5}$, with nodal averaging. Calculate σ_{yy}.

c) In point C is a node. For this node the finite element analysis gives $\sigma_{xx} = -0.05$ MPa, with nodal averaging. Can this be a valid result? Explain your answer.

d) The analyst wants to verify the vertical stress distribution along the horizontal line AC and uses as check: $\int_{x=x_A}^{x_C} \sigma_{yy}\, dx \approx 180 f$. Does this verification make sense? If yes, explain why? If no, what should be modified?

e) Now, consider that we change the x axis in its opposite direction. Will σ_{xx} change sign? Explain your answer.

Question 2 Use a **separate sheet of paper** for this question.

Consider the plane stress (or plate membrane) problem in Figure 2.1. The plate consists of two materials with a different stiffness and a different thickness. The edge load f is 24 kN per inclined m.

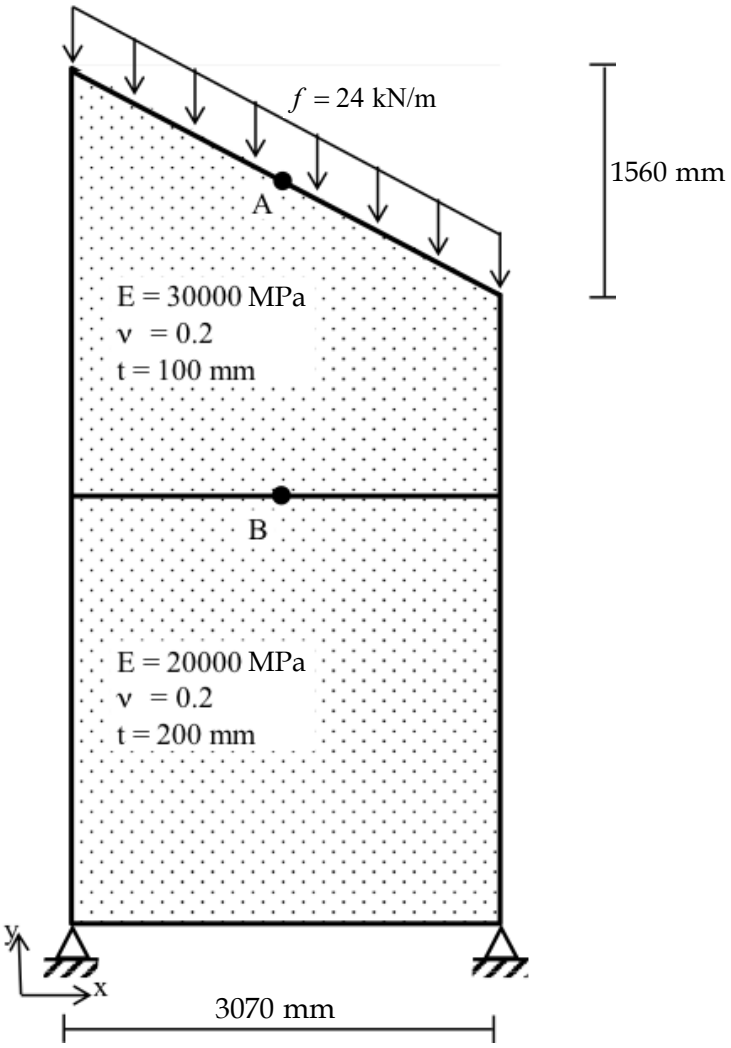


Figure 1.2: Plane stress (or plate membrane) problem

a) Can we calculate σ_{yy} in point A? If yes, give the numerical value and unit. If no, explain in words why not.

b) What is the σ_{yy} stress ratio of a point immediately above point B to a point immediately below point B.

c) What is the ε_{xx} strain ratio of a point immediately above point B to a point immediately below point B.

Question 3 Use a **separate sheet of paper** for this question.

a) Consider a plate edge that goes in the x direction or in the y direction. The magnitude of the concentrated shear force in this edge is equal to m_{xy}. Is this always true?

Choose A, B, C or D.

A Yes
B Not in fixed plate edges
C Not in free plate edges
D Not in corners

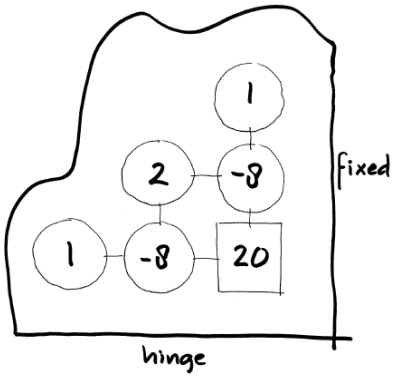
b) How do we deal with singularities? Choose A, B, C or D.

A Very carefully.
B Refine a model with yielding, cracking, round corners, distributed supports et cetera.
C Ignore or replace by hand calculations.
D Choose a mesh with very small elements to compute the stress concentration accurately.

c) What is the finite difference method? Choose A, B, C or D.

A Computational way of solving differential equations.
B Systematic way of formulating the plate differential equations.
C Method for avoiding infinitesimal singularities.
D Method for analysing discrete elements like torsion panels.

d) Shown is a molecule of the finite difference method.
How many discrete elements are in this molecule?



e) In the Kirchhoff plate theory, the element size should be larger than the thickness.
Why is this true? Choose A, B, C or D.

A The question says this is true. The teacher wrote the question. Teachers know these things.

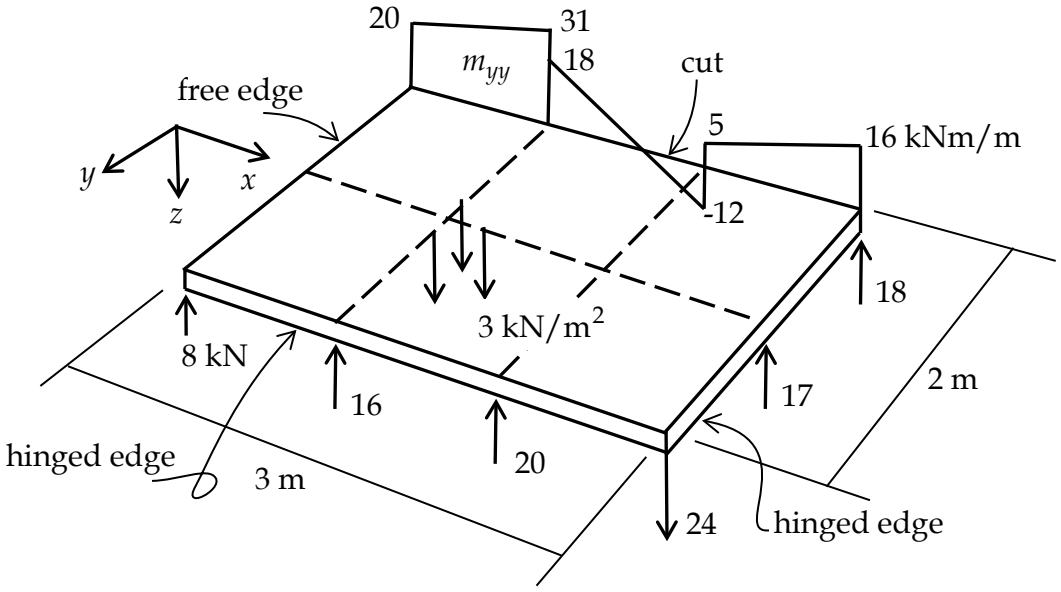
B This is not always true. For example, at a point load much smaller elements may be required to compute the bending moments and the torsion moment with scientific accuracy.

C Somebody checked that even smaller elements only show larger singularities.

D This is stated in the reader (Blaauwendraad, Plates and FEM, p. 286).

f) A plate has been analysed by the finite element method. Part of this plate is cut free. The figure below shows the free part with distributed load and support reactions. The figure also shows the bending moment m_{yy} in the cut. The torsion moment m_{xy} and the shear force v_y occur also in this cut but these are not displayed.

Is this plate part in moment equilibrium around the *x* axis? Explain your answer.



g) A square plate is simply supported (hinged) at all four edges.
The perpendicular load is distributed in a two-way sine shape.

Young's modulus	$E = 30\,000$ N/mm^2
Poisson's ratio	$\nu = 0.15$
thickness	$t = 200$ mm
length and width	$a = 10$ m
load amplitude	$\hat{p} = 4$ kN/m^2

Deflection formula $\hat{w} = \dfrac{\hat{p}\,a^4}{4\pi^4 D}$ (Blaauwendraad p. 108)

How large is the deflection of the plate?

h) Consider the shear forces v_x and v_y in a plate loaded by a distributed load.
Do they depend on Young's modulus E ?
Do they depend on Poisson's ratio ν ? Choose A, B, C or D.

A yes, yes
B yes, no
C no, yes
D no, no

Question 4 Use a **separate sheet of paper** for this question.

A plate is loaded out of plane. Most of the edges are free except for a fixed part and a simply supported part (Fig. 4). The plate length is 12 m and the width is 7 m. The plate thickness is 250 mm. A distributed load of -5 kN/m^2 is applied. This distributed load includes self-weight. A linear elastic finite element model is made. E = 30500 N/mm^2, ν = 0.2. Applied are four node plate elements (Kirchhoff theory). The element size is 500 mm. The results are shown in Figures 4.2 to 4.7.

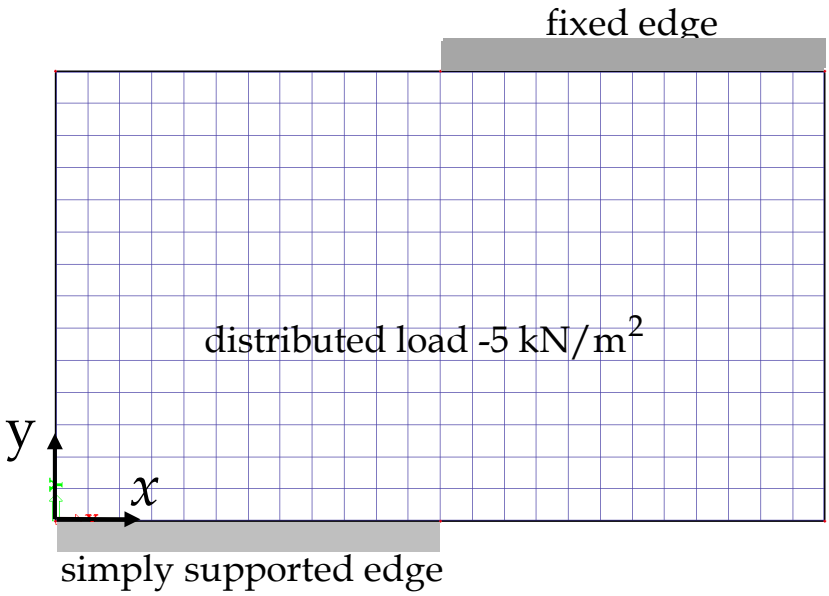


Figure 4.1. Plate with supports, load and finite element mesh

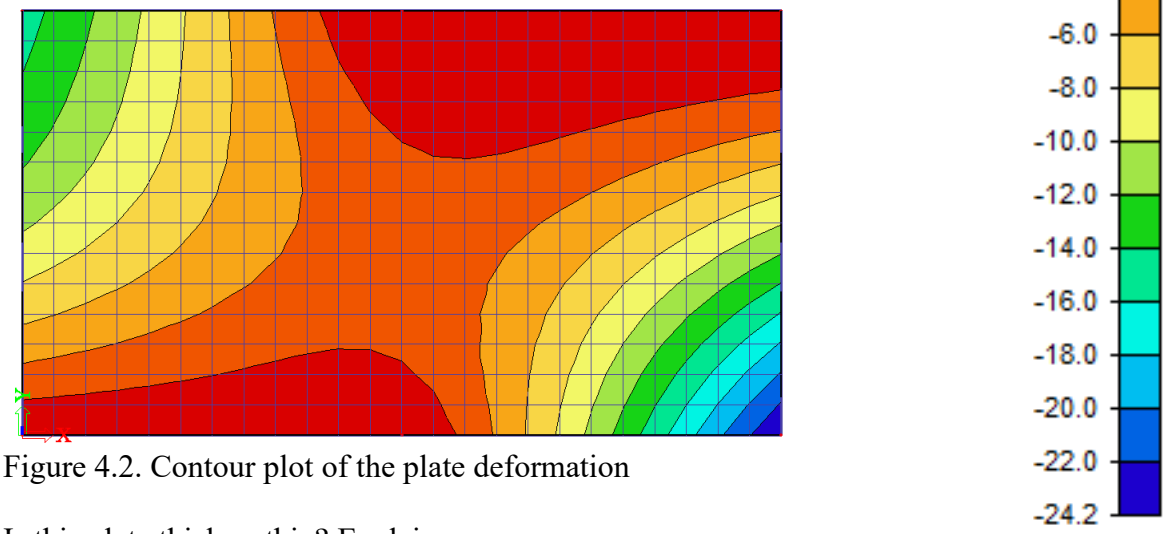


Figure 4.2. Contour plot of the plate deformation

a) Is this plate thick or thin? Explain your answer.

b) The deflection at the supports is somewhere between 0 and -2 mm. How can we compute this more accurately?

c) Where can we expect singularities in this plate? Specify for each whether it actually shows.

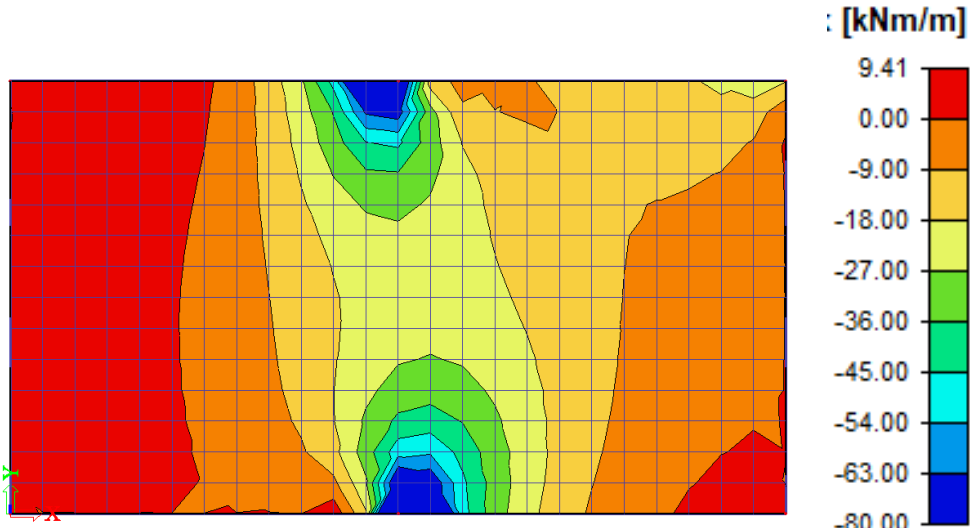


Figure 4.3. Contour plot of the bending moment m_{xx}

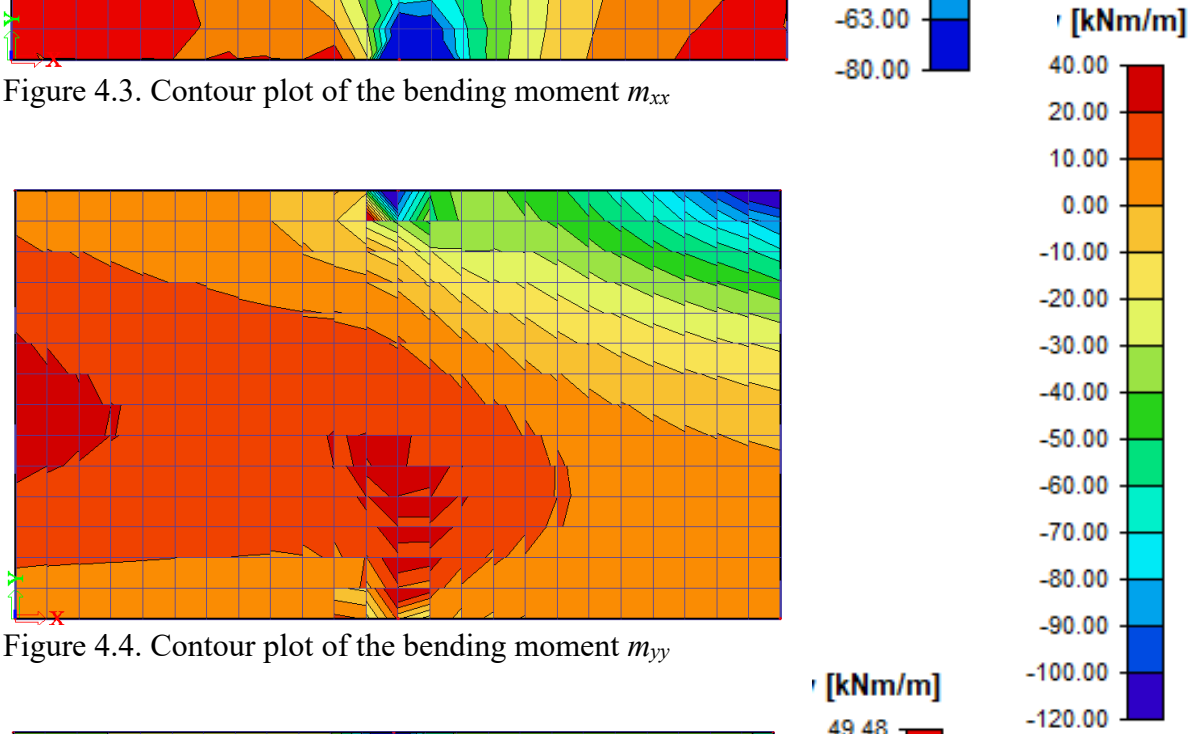


Figure 4.4. Contour plot of the bending moment m_{yy}

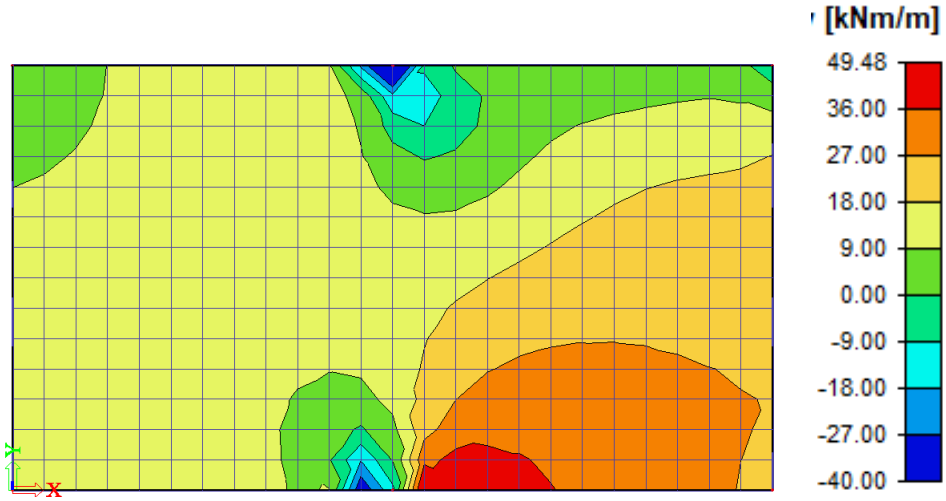


Figure 4.5. Contour plot of the torsion moment m_{xy}

d) Do the moments agree with the boundary conditions? Explain your answer.

e) The contour lines are not smooth. Does this have a physical cause or a computational cause? Explain your answer.

f) Where is the concentrated shear force largest?

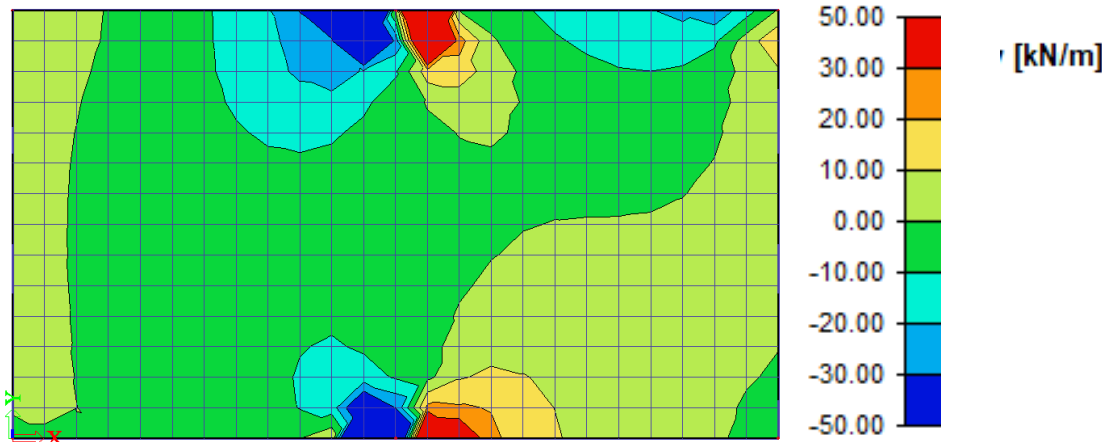


Figure 4.6. Contour plot of the shear force v_x

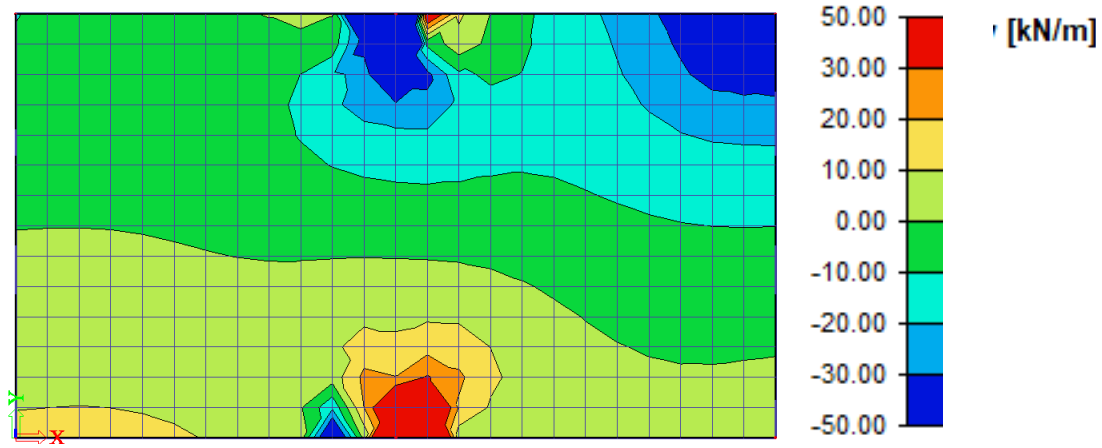


Figure 4.7. Contour plot of the shear force v_y

g) Both v_x and v_y vary between exactly -50 and 50 kN/m. Explain this phenomenon?

h) What is the shear stress σ_{yz} in the edge at x = 12 m, y = 3.5 m .
(If you are colour blind, ask the teacher for help.)

Question 5 Use a **separate sheet of paper** for this question.

A reinforced concrete cantilever balcony has a thickness of 180 mm. The plate carries 3 kN/m^2 (self-weight plus live load; partial factors included). At a particular location the moments are

m_{xx} = -19 kNm/m
m_{yy} = -5 kNm/m
m_{xy} = 9 kNm/m

The reinforcement consists of bars in the *x* direction and bars in the *y* direction. The plate does not have out of plane shear reinforcement. All bars have a diameter of 12 mm. The yield strength of the reinforcing steel is 430 N/mm^2 (partial factor included).

a) A computer calculates the following required moment capacities of the reinforcement. Is this the least amount of reinforcement that can carry the moments? Explain your answer.

m_{xt} = 28 kNm/m m_{yt} = 14 kNm/m
m_{xb} = 0 m_{yb} = 4 kNm/m

b) An engineer selects reinforcement that provides the following moment capacities. Did the engineer make a mistake? Explain your answer.

m_{xt} = 30 kNm/m m_{yt} = 15 kNm/m
m_{xb} = 5 kNm/m m_{yb} = 0 kNm/m

c) Do we actually need bottom reinforcement at the considered location of the balcony? Explain your answer.

d) Is the reinforcement sufficient for an extra wind load of –1 kN/m^2 ? (upwards) Explain your answer.

e) Will the balcony collapse at a load of 15/14 × 3 kN/m^2 ? Explain your answer.

Useful equations
top $m_{xt} = -m_{xx}+|m_{xy}|$ $m_{yt} = -m_{yy}+|m_{xy}|$
bottom $m_{xb} = m_{xx}+|m_{xy}|$ $m_{yb} = m_{yy}+|m_{xy}|$
$m_1=(m_{xx}+m_{yy})/2+\text{sqrt}(((m_{xx}-m_{yy})/2)^2+m_{xy}^2)$
$m_2=(m_{xx}+m_{yy})/2 -\text{sqrt}(((m_{xx}-m_{yy})/2)^2+m_{xy}^2)$
top $(m_{xt}+m_{xx})*(m_{yt}+m_{yy}) >= m_{xy}^2$
bottom $(m_{xb}-m_{xx})*(m_{yb}-m_{yy}) >= m_{xy}^2$

Question 6 Use a **separate sheet of paper** for this question.

a) What is the advantage of using a constant shear element instead of a standard rectangular membrane element?

b) Does it make sense to use both constant shear elements and standard rectangular membrane elements in different areas of one single finite element model? Explain your answer.

c) Suppose you made a very large model with standard rectangular membrane elements? In an attempt to reduce the calculation time, you keep the mesh the same but use constant shear elements instead. Does this help in reducing the calculation time? Explain your answer

Brief answers

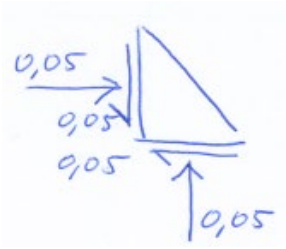
Question 1

a) σ_{xx} should be zero. However, dynamic boundary conditions are possibly not exactly satisfied in the (displacement-based) finite element method. A "small" σ_{xx} is acceptable.

b) $E := 30000 \cdot \frac{N}{mm^2}$: nu := 0.2 : $eyy := -2.6 \cdot 10^{-5}$: $exx := 0$:

$syy := \frac{E}{1 - nu^2} \cdot (eyy + nu \cdot exx)$;

$$syy := -\frac{0.8125000000\ N}{mm^2}$$



c) It can. No inconsistency in equilibrium (see figure).

d) No, a minus sign and the plate thickness are missing.

e) No, following the sign conventions it will only affect the sign of the shear stress.

Question 2

a) No; the dynamic boundary conditions give two (linear) expressions in σ_{xx}, σ_{yy} and σ_{xy}. We cannot solve σ_{yy} from these.
No; for drawing Mohr circle we need the stresses on two faces. We know the stresses on one face only.

b) $n_{yy}^{-} = n_{yy}^{+}$ equilibrium

$\sigma_{yy}^{-} t^{-} = \sigma_{yy}^{+} t^{+}$

$\frac{\sigma_{yy}^{+}}{\sigma_{yy}^{-}} = \frac{t^{-}}{t^{+}} = \frac{200}{100} = 2$

c) $\varepsilon_{xx}^{-} = \varepsilon_{xx}^{+}$ connected

$\frac{\varepsilon_{xx}^{+}}{\varepsilon_{xx}^{-}} = 1$

Question 3

a) A

b) C

c) A

d) 9 = 4 torsion panels + 4 complete bending elements + 1 halve bending element

e) C

f) Yes moment in the section
(20+31)/2*1 + (18-12)/2*1 + (5+16)/2*1 = 39 kNm
moment due to the support reactions and the load
8*2 + 16*2 + 20*2 - 24*2 + 17*1 - 3*2*3*1 = 39 kNm

g) $E := 30000 \cdot \frac{N}{mm^2}$: nu := 0.15 : $t := 200 \cdot mm$: $a := 10 \cdot m$: $pt := 4 \cdot \frac{kN}{m^2}$: $m := 1000 \cdot mm$: $kN := 1000 \cdot N$:

$Dt := \frac{E \cdot t^3}{12(1 - \nu^2)}$: $wt := \frac{pt \cdot a^4}{4 \cdot 3.1415^4 \cdot Dt}$;

$$wt := 5.018090785\ mm$$

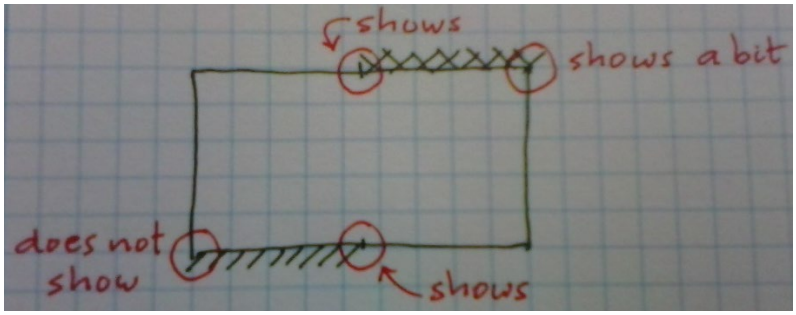
h) D … Not easy to proof. We can see it in the example of a two-way sine load on a square plate.

Question 4

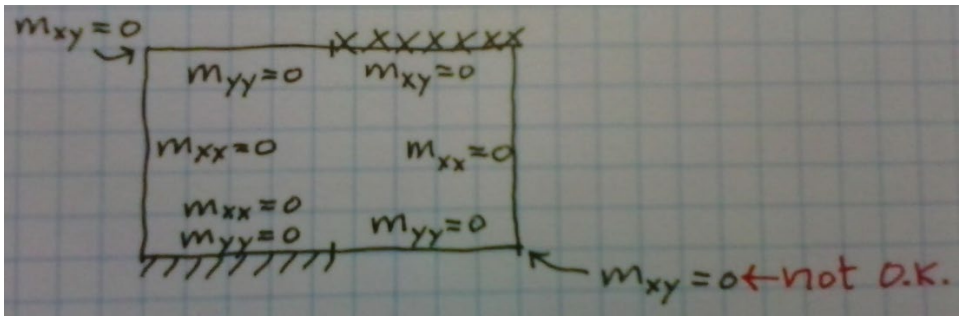
a) Thin. The ratio span / thickness = 7000 / 250 = 28 > 10

b) It is already computed accurately. (Kinematic boundary conditions are not approximated in the finite element method.) It can be displayed more accurately, for example by choosing more colours in the displacement contour plot.

c)



d) No



e) Computational cause. The solution to the differential equation is smooth, except singularities.

f) V is mxy is infinite, in the singularities.

g) The colour scale is truncated between -50 and 50 kN/m. Otherwise most colours would occur in the singularities. (The word “phenomenon” is too strong for this simple adjustment.)

h)

```
> t := 250·mm :
> vy := -10·kN/m :
> V := (18 + 27)/2 · kN·m/m :
> m := 1000·mm : kN := 1000·N :
> sigma_yz := -3/2 · vy/t + 3/2 · sqrt(10.) · V/t^2 :
> simplify(sigma_yz);
```

$$\frac{1.767629937\,N}{mm^2}$$

Question 5

a) No For example mxt = 28; myt = 14; mxb = 0; myb = 3 , which is less …

(mxt+mxx)*(myt+myy) >=? mxy^2

(28-19)*(14-5) >= ? 9^2

81 >=? 81 OK

(mxb-mxx)*(myb-myy) >=? mxy^2

(0+19)*(3+5) >=? 9^2

152 >=? 81 OK

… and can carry the load too.

b) No (30-19)*(15-5) >=? 9^2
110 >=? 81 OK
(5+19)*(0+5) >=? 9^2
120 >=? 81 OK

c) No (28-19)*(14-5) >=? 9^2
81 >=? 81 OK
(0+19)*(0+5) >=? 9^2
95 >=? 81 OK

d) Yes (30-19*2/3)*(15-5*2/3) >=? (9*2/3)^2
202 >=? 36 OK
(5+19*2/3)*(0+5*2/3) >=? (9*2/3)^2
59 >=? 36 OK

e) No (30-19*15/14)*(15-5*15/14) >=? (9*15/14)^2
93 >=? 93 OK
(5+19*15/14)*(0+5*15/14) >=? (9*15/14)^2
136 >=? 93 OK
The top reinforcement yields at this load and does not carry extra load but very likely other locations of the balcony can carry more load before yielding.

Question 6

a) The constant shear element has advantages when used for beams that are loaded in bending. When using only one element over the depth of the beam, the standard element is far too stiff.

b) Yes. If the mesh deforms in an hourglass mode (see figure) due to constant shear elements everywhere. (Hourglass modes are very rare and can be also removed by making changes to the mesh.)
No. The constant shear element compared to the standard element is better in bending and the same in shear.

c) No: The total number of degrees of freedom stays the same. The structural stiffness matrix is just as large. Solving the stiffness matrix takes much more time than the other structural analysis steps. The computation time stays almost the same.

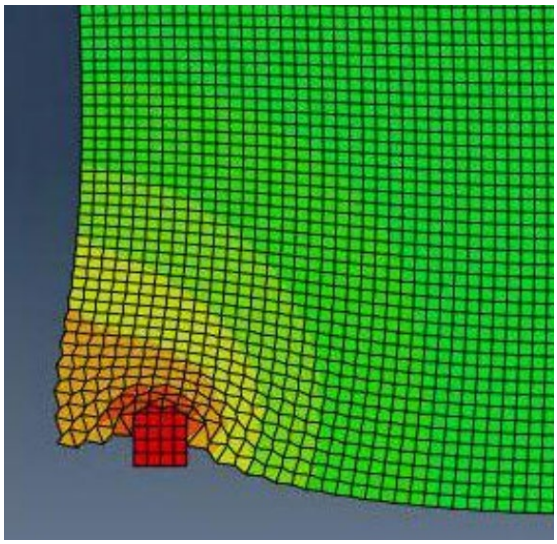


Figure: Hourglass mode at the support of a deep beam finite element model (Abaqus, 4 node constant shear elements)
[Curtesy of A. Al-Sharqi, CIE4180 course, assignment 1, November 2023]