Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Write your name and study number at the top right-hand of your work.

Exam CT5143 Stresses in Shells
Monday 25 August 2008, 9:00 – 12:00 hours

Problem 1 (2.5 point, 45 minutes)

Explain shortly the meaning of the following eight words in relation to structural shells.

Edge disturbance
Imperfection sensitivity
Edge beam
Normal mode
Knock down factor
Hypar
Membrane action
Middle surface

Problem 2 (2.5 points, 45 minutes)

a Figure 1 shows a shell part with forces and moments in the positive directions. Add to this figure the variables for the directions, forces and moments (for example x, n_{xx} , ... Make clear which variable belongs to which arrow.)

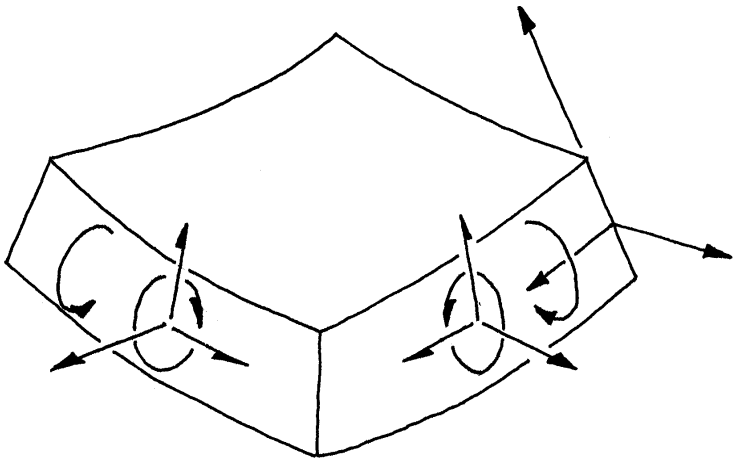

Figure 1. Representative shell part

b Which forces or moments are important for buckling?

c Which formulas are used to compute the stresses in location $(x,y,z) = (0,0,t/2)$? (t Is the shell thickness.)

d Suppose an aluminium shell is analysed. How can we check whether the stresses in Problem 2c would cause yielding?

e Buckling in plates is resisted by the bending stiffness. In general, buckling in shells is resisted by the bending stiffness and something extra. What is the extra stiffness?

Problem 3 (2.5 points, 45 minutes)

A shell buckles when the normal force is

$$n = -0.1\frac{E\,t^2}{a}.$$

Where E is Young's modulus, t is the thickness and a is the radius in a section perpendicular to the normal force. (Knockdown factor is included.)
A shell yields or crushes when the normal force is

$$n = -f\,t\ ,$$

Where f is the material compressive strength. (Moments are neglected.)

a Use these equations to derive a formula for the ratio a/t at which buckling and breaking (or crushing) would happen at the same time.

b Calculate this ratio a/t for the materials in Table 1. Use a material safety factor of 1.5.

c Which of the following conclusions can we draw from the results? In most shell structures buckling is decisive; or in most shell structures breaking or crushing is decisive. Explain your answer.

Table 1. Material properties

	Young's modulus E	Compressive strength f
Steel	210000 N/mm²	350 N/mm²
Concrete	35000	40
Aluminum	70000	110
Wood (Pine)	13000	40
Glass	70000	50
Plastic (Acrilic)	2300	70

Problem 4 (2.5 points, 45 minutes)

A hypar shell is supported by two columns (Fig. 2). The columns are connected by a horizontal beam. The horizontal beam does not touch the hypar. The shell thickness is t = 3 mm. The dead weight of the shell plus snow loading is p = 1.2 kN/m².

a Calculate the membrane forces and the largest bending moment in the shell.

b Calculate the forces in the edge beams, columns and connection beam.

c The shell is made out of 3 mm thick aluminium plates that are riveted together (Fig. 3). A flat aluminium plate can be deformed into a hypar shape provided that one of its corners is displaced less than 16.8 t out of the flat plane. Determine the aluminium plate dimensions.

d Check the buckling stability of this shell.

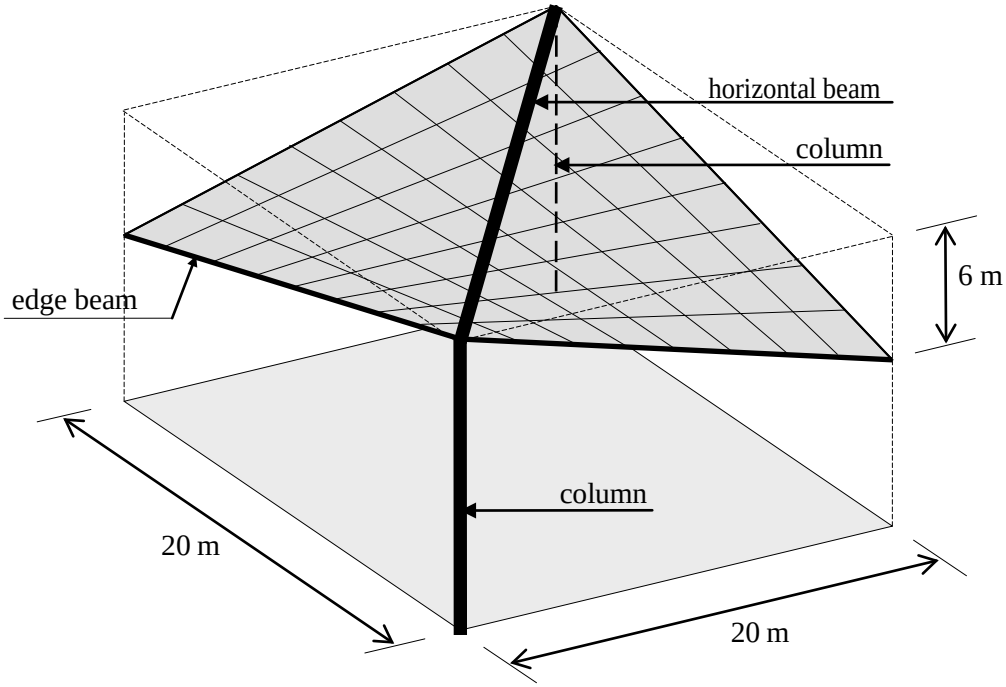

Figure 2. Hypar shell

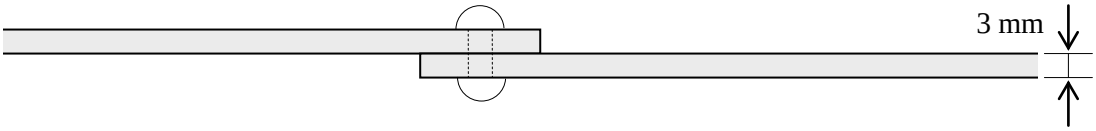

Figure 3. Aluminium plate connection

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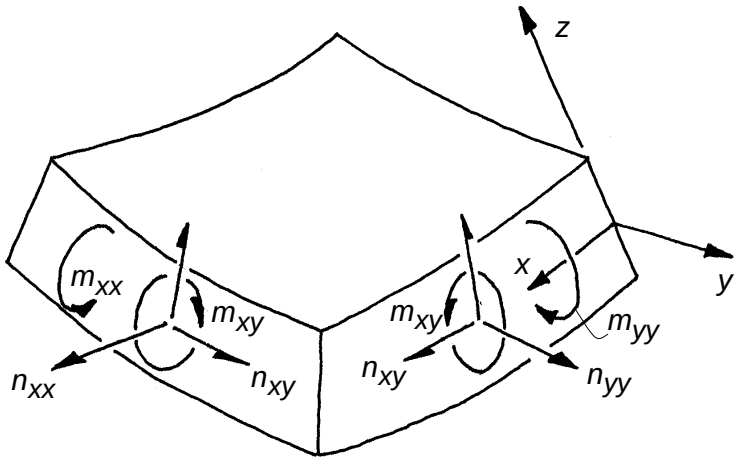
Solution to Problem 1

Edge disturbance ……..	A local increase in the bending moment close to the edge of a shell
Imperfection sensitivity …….	The strong influence of disturbances in geometry, material stiffness and boundary stiffness on the ultimate load of a shell
Edge beam ……..	A beam connected to the edge of a thin shell
Normal mode ……..	A shape in which a shell can move when vibrating freely
Knock down factor ……..	The reduction factor of the critical load (linear buckling) to account for imperfection sensitivity
Hypar …….....	Hyperbolical paraboloid
Membrane action ……..	A way to carry the loading by n_{xx} , n_{yy} and n_{xy} only
Middle surface ……..	The imaginary surface half way between the inner surface and outer surface of a shell

Solution to Problem 2

a Variables



b Buckling
The smallest principal normal force n_2

c Stresses

$$\sigma_{xx} = \frac{n_{xx}}{t} + \frac{m_{xx}}{\frac{1}{6}t^2}$$

$$\sigma_{yy} = \frac{n_{yy}}{t} + \frac{m_{yy}}{\frac{1}{6}t^2}$$

$$\sigma_{zz} \approx 0$$

$$\sigma_{xy} = \frac{n_{xy}}{t} + \frac{m_{xy}}{\frac{1}{6}t^2}$$

$$\sigma_{xz} = 0$$

$$\sigma_{yz} = 0$$

d Yielding

Aluminium yields if the Von Mises stress would become larger than the uniaxial yield stress.

$$\sigma_{VM} = \sqrt{\frac{1}{2}\left((\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right)}$$

e Membrane stiffness, or extensional deformation, or the changing Gaussian curvature.

Solution to Problem 3

a $$-0.1\frac{E\,t^2}{a} = -f\,t \quad \Rightarrow \quad \frac{a}{t} = 0.1\frac{E}{f}$$

b $$\frac{a}{t} = 0.1\frac{210000}{350/1.5} = \frac{90}{1}$$

	a/*t*
Steel	90/1
Concrete	130/1
Aluminium	95/1
Wood (Pine)	49/1
Glass	210/1
Plastic (Acrylic)	5/1

c A higher ratio than above means that the structure is slender and will fail due to buckling. A lower ration than above means that the structure will fail due to yielding or crushing. The calculated ratios are small compared to those of modern shell structures (Jena planetarium 200/1 – Kanehoe shopping centre 1000/1). Therefore, for almost all shell structures loaded in compression, buckling will be decisive for the ultimate limit state.

Solution to Problem 4

a Stresses

The radius of the hypar is

$$a = \frac{b^2}{h} = \frac{10^2}{3} = 33.33 \text{ m.}$$

The direction of the dead load is approximately perpendicular to the shell surface. Therefore the distributed membrane force is

$$n_{xy} = \tfrac{1}{2}a\,p_z = \tfrac{1}{2}33.33 \times 1.2 = 20 \text{ kN/m.}$$

The largest moment in the shell occurs at the edge beam. We assume that the edge beam will not rotate. The moment is

$$m_{xx} = 0.511\frac{p}{l^2}\sqrt[3]{(a\,t\,l)^4} = 0.511\frac{1.2}{20.88^2}\sqrt[3]{(33.33 \times 0.003 \times 20.88)^4} = 0.0038 \text{ kNm/m.}$$

b Force flow

The length of the edge beams is $l = \sqrt{20^2 + 6^2} = 20.88$ m.

The normal force in the top of the edge beams is

$N = 20.88 \times 20 = 418$ kN tension.

The horizontal and vertical component of *N* are

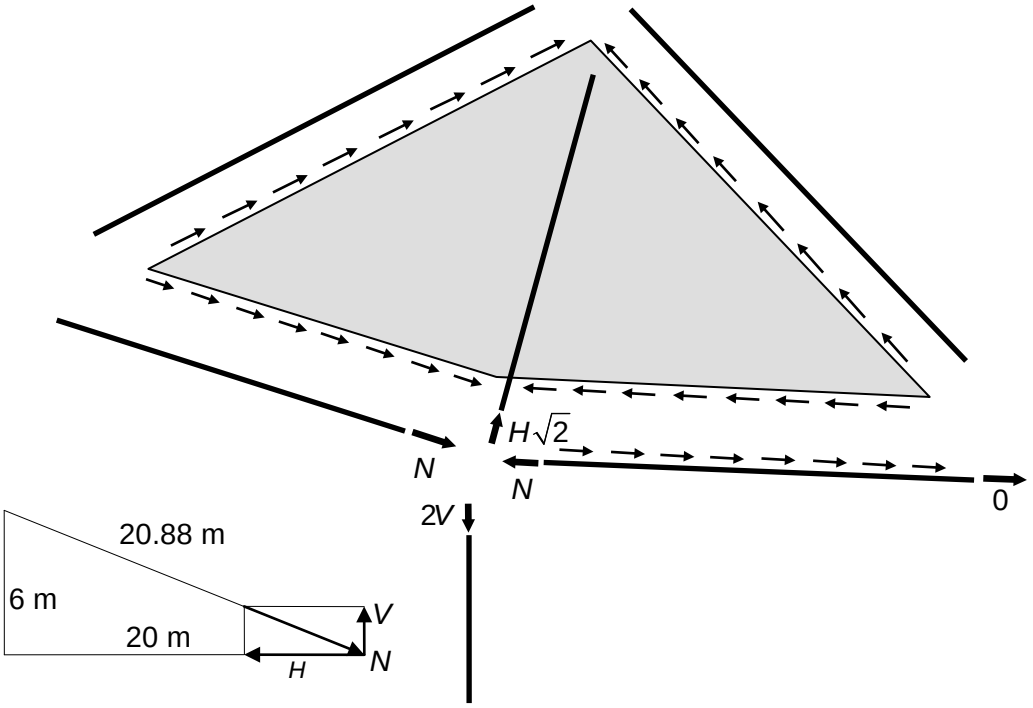
$H = 418\dfrac{20}{20.88} = 400$ kN, $\qquad V = 418\dfrac{6}{20.88} = 120$ kN.

The normal force in the connection beam is

$H\sqrt{2} = 566$ kN compression.

The force in the columns is $2V = 2 \times 120 = 240$ kN compression.

As a check we calculate the total load $G = 20 \times 20 \times 1.2 = 480$ kN which is equal to the sum of both column forces.



c Plate size

$a = \dfrac{b^2}{16.8 \times t} = 33.33 \quad \Rightarrow \quad b = \sqrt{33.33 \times 16.8 \times 0.003} = 1.30$ m

d Buckling

$n_{cr} = -0.6\dfrac{E\,t^2}{a} = -0.6\dfrac{70000 \times 0.003^2}{33.33} = -11.34$ kN/m.

The shear force n_{xy} = 20 kN/m gives principal forces n_1 = 20 kN/m and n_2 = -20 kN/m.

The loading is larger (in absolute sense) than the critical loading and therefore this design will buckle.

(A shell with a plate thickness of 4 mm would not buckle.)