Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Write your name and study number at the top right-hand of your work.

Exam CT5143 Stresses in Shells
Monday 29 June 2009, 14:00 – 17:00 hours

Problem 1 (2 point, 36 minutes)

Explain shortly the meaning of the following eight words in relation to structural shells.

Yoshimura pattern
In-extensional deformation
Koiter's half power law
Normal mode
Diaphragm boundary condition
NURBS
C2 continuity
Fully stressed design

Problem 2 (2 points, 36 minutes)

a Figure 1 shows a shell part with forces and moments in the positive directions. Add to this figure the variables for the directions, forces and moments (for example x, n_{xx} , ... Make clear which variable belongs to which arrow.)

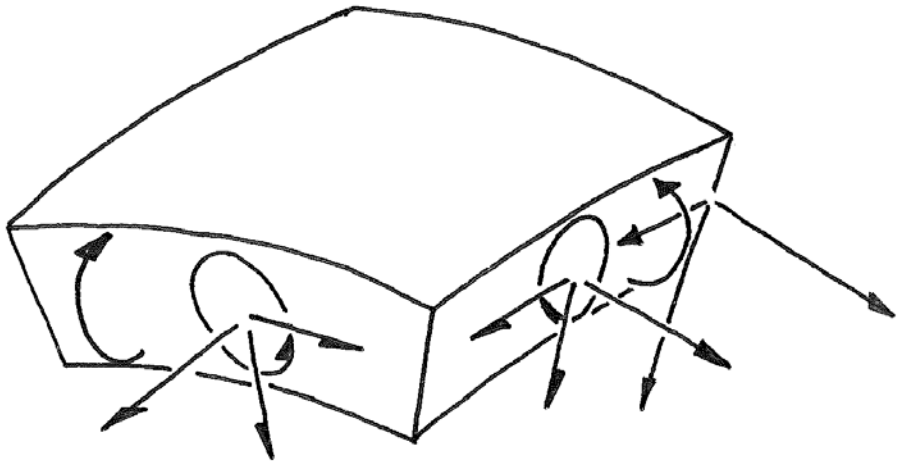


Figure 1. Representative shell part

b In a well designed shell structure the membrane forces are caused by the loading. What causes the moments in a well designed shell structure?

c All these membrane forces, moments and shear forces make it difficult to understand the force flow in a shell structure. Therefore, simple concepts are introduced like pressure lines and force paths. Unfortunately, computers cannot plot pressure lines or force paths for shell structures. What is the reason for this? What can a computer plot that comes close to a pressure line or force path?

d In case of a reinforced concrete shell, what are the best directions of the reinforcing bars?

e How can crushing failure of concrete be checked?

Problem 3 (2 points, 36 minutes)

a In one of the lectures we assembled a small hypar shell that consisted of 25 plate parts, 4 edge beams and a tension rod (rope). What can be learned of this exercise? Select one of the following answers.

A All parts have almost the same shape so they do not need to be numbered.
B A shell needs to be assembled in the right shape otherwise the parts do not fit.
C Shell construction is very simple and can be done quickly without preparation.
D The tension rod is necessary to maintain the right shape already before external loading is applied.

b In one of the lectures we loaded an assembled hypar shell. What can be learned of this exercise? Select one of the following answers.

A After failure the shell still possessed considerable load carrying capacity.
B The edge beams are not really contributing to the strength of the structure.
C The stiffness of the shell is small (due to the joints between the parts).
D The shell can easily carry the design load.

c What is the greatest challenge in realising a shell structure? Select one of the following answers.

A Architectural design; Can we come up with a shape that is functional and beautiful?
B Structural analysis; Do we have analysis methods and software available?
C Construction; How can we build this within budget?
D Legal issues; Who is responsible if it goes wrong?

Problem 4 (2 points, 36 minutes)

In-extensional deformation is an important concept for shell structures.

a What is the relation between in-extensional deformation and dead load?

b What is the relation between in-extensional deformation and differential support settlements?

c What is the relation between in-extensional deformation and Gaussian curvature?

Problem 5 (2 points, 36 minutes)

An assembly of four identical hypar shells is supported by one column in the middle (Fig. 4). The shell thickness is t = 75 mm. The dead weight of the shell including roofing and live load is p = 4 kN/m².

a Does this design need cables or tension rods? If so, add these to the design.

b Calculate the membrane forces and the largest bending moment in the shell.

c Calculate the forces in the edge beams, interior beams, cables and column.

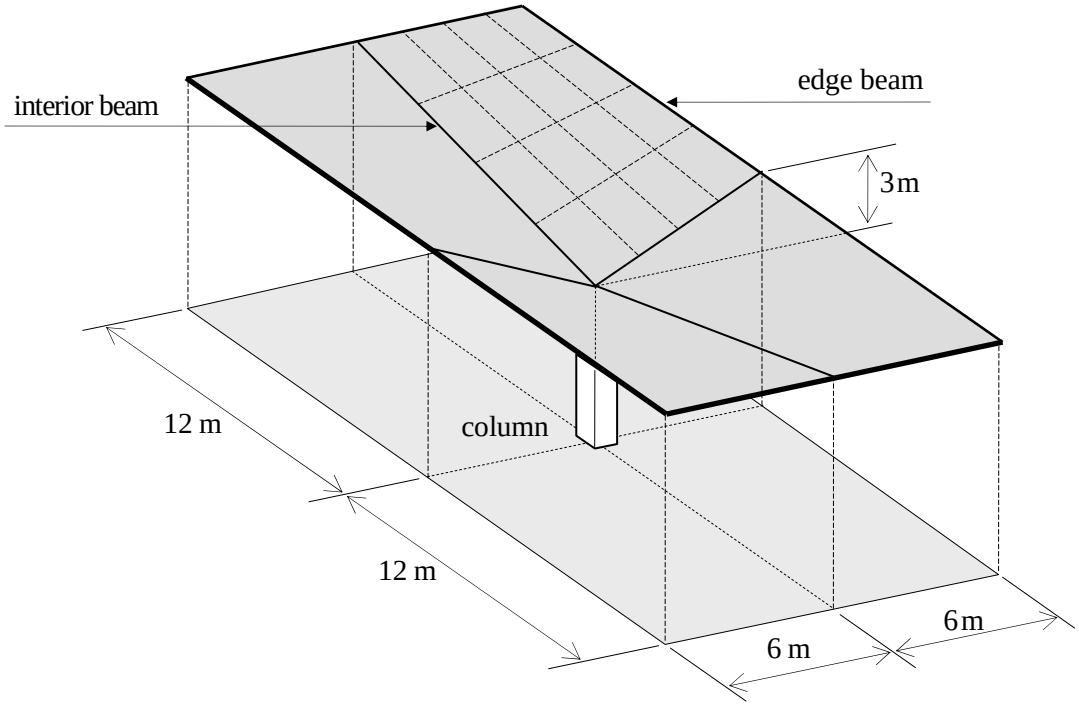


Figure 4. Four hypar shell

d Check whether the shell will buckle. Use Young's modulus is 15 x 10^6 kN/m² (which is the tangent stiffness of cracked reinforced concrete at small stresses).

e Is the following reinforcement sufficient for strength in the middle of each hypar?

2Ø10 – 150

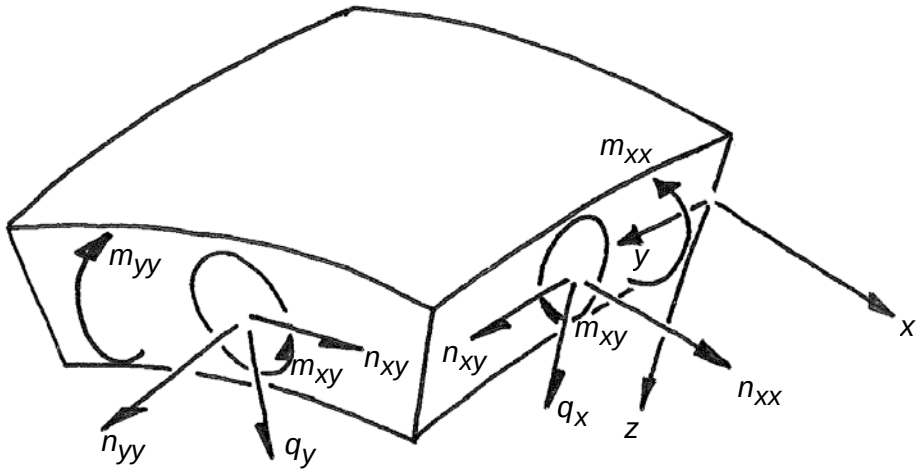
(Reinforcing net of bars with a diameter of 10 mm spaced at 150 mm centre-to-centre in two perpendicular directions). Use a bar yield stress of 450 N/mm². Partial safety factors can be ignored.

Exam CT5143, 29 June 2009
Solution to Problem 1

Yoshimura pattern …….. Buckling shape of a thin shell
In-extensional deformation …… Deformation with no extension of the middle surface and only bending
Koiter's half power law …….. $\lambda_{\max} = \lambda_{cr}\left(1 - 2(w_o \rho c_1)^{\frac{1}{2}}\right)$
Normal mode …….. Deformation in which a structure can vibrate
Diaphragm boundary condition …… Mathematical formulation of the stresses and displacements imposed by a thin sheet that is very stiff in extension, very flexible in bending and does not buckle
NURBS …… Curved surface element in CAD programs
C2 continuity …… Curvatures of the surfaces at the connection line are the same
Fully stressed design …….... Optimal shape for which everywhere the stress is equal to the yield stress or crushing stress

Solution to Problem 2

a Variables



b Moments
The loaded shell deforms. The boundary does not deform (or deforms differently). Closing the gap between the shell and the boundary gives the moments (compatibility). The same cause can give moments at connection lines between shell parts.

c Pressure line and force path
Pressure lines and such simplify the true force flow too much. Moreover, they are not clearly defined for shells. Therefore, they cannot be computed. The principal directions of the membrane forces come closest to a pressure line.

d Reinforcement directions
If 2 layers of bars are used, the principal directions of the membrane forces.
If 4 layers of bars are used, the principal directions of the stresses at the surfaces.

e Crushing

Mohr-Coulomb criterion

Solution to Problem 3

a Assembling B

b Loading C

c Challenge C

Solution to Problem 4

a Dead load
For dead load in-extensional deformation should not occur.

b Support settlements
For support settlements in-extensional deformation needs to occur.

c Gaussian curvature
When the deformation is in-extensional the Gaussian curvature does not change.

Solution to Problem 5

a Cables
Not needed. (Equilibrium is possible without cables.)

b Membrane force and moment
The radius of the hypar is

$$a = \frac{bc}{h} = \frac{6 \times 12}{3} = 24 \text{ m.}$$

The direction of the dead load is approximately perpendicular to the shell surface. Therefore the distributed membrane force is

$$n_{xy} = \tfrac{1}{2} a p_z = \tfrac{1}{2} 24 \times 4 = 48 \text{ kN/m.}$$

The length of the long interior beams is $l = \sqrt{12^2 + 3^2} = 12.37$ m.

The length of the short interior beams is $l = \sqrt{6^2 + 3^2} = 6.71$ m.

The largest moment in the shell occurs at the short interior beam. Rotation will not occur due to symmetry. The moment is

$$m_{xx} = 0.511 \frac{p}{l^2} \sqrt[3]{(a t l)^4} = 0.511 \frac{4}{6.71^2} \sqrt[3]{(24 \times 0.075 \times 6.71)^4} = 1.258 \text{ kNm/m.}$$

c Force flow
The normal force in the long interior beams is $2 \times 12.37 \times 48 = 1188$ kN compression.
The normal force in the short interior beams is $2 \times 6.71 \times 48 = 644$ kN compression.

The horizontal and vertical component the long interior beam force are

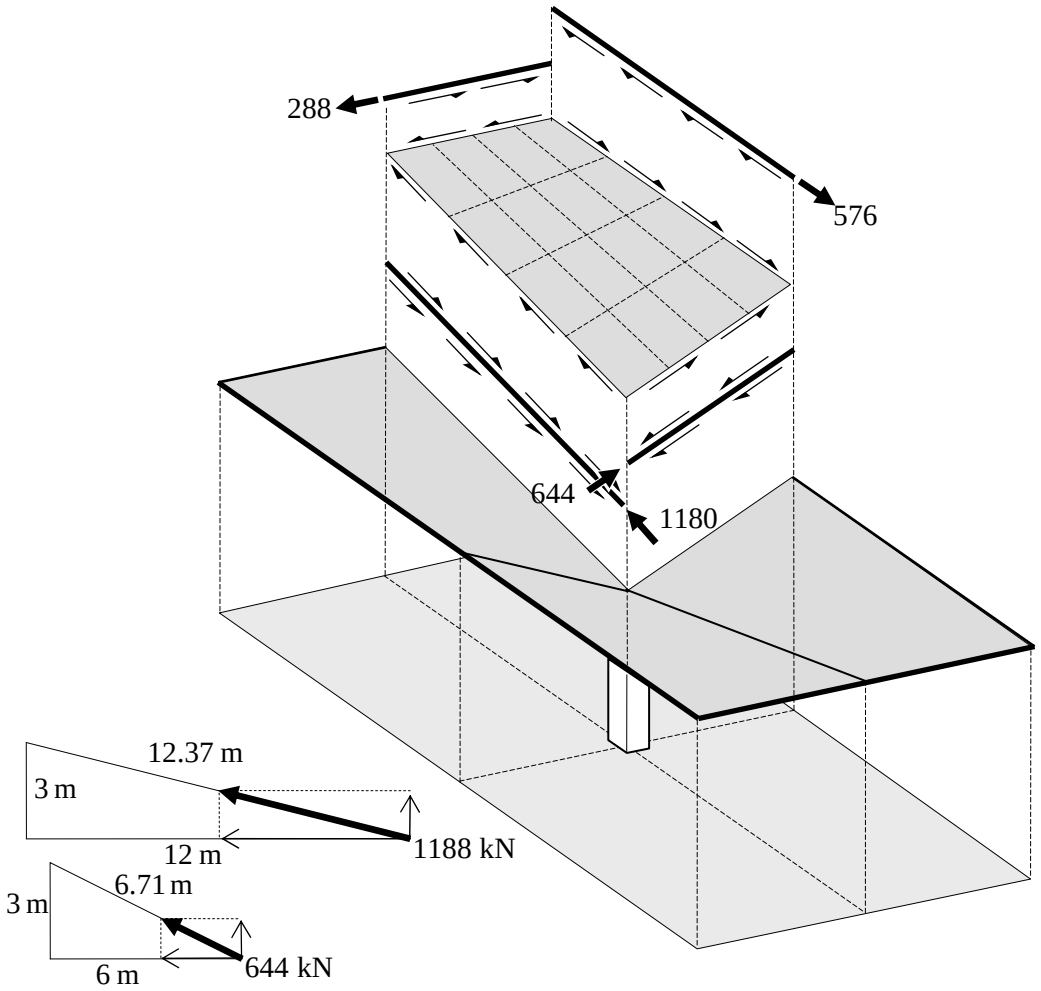
$$N_h = 1188\frac{12}{12.37} = 1152\,\text{kN}. \quad N_v = 1188\frac{3}{12.37} = 288\,\text{kN}.$$

The horizontal and vertical component the short interior beam force are

$$N_h = 644\frac{6}{6.71} = 576\,\text{kN}. \qquad N_v = 644\frac{3}{6.71} = 288\,\text{kN}.$$

The force in the column is $R = 2\times 288 + 2\times 288 = 1152$ kN.

As a check we calculate the total load $G = 48\times 12\times 2 = 1152$ kN which is equal to the column force.



d Buckling

The critical membrane force is

$$n_{xy,cr} = 0.6\frac{E t^2}{a} = 0.6\frac{15\times 10^6 \times 0.075^2}{24} = 2109 \text{ kN/m}$$

(A knockdown factor is not needed because hypars are not sensitive to imperfections.)
The actual membrane force (48 kN/m) is much smaller, therefore, it will not buckle.

e Reinforcement

Required reinforcement membrane force

$n_{sx} = n_{xx} + |n_{xy}| = 0 + 48 = 48$ kN/m

$n_{sy} = n_{yy} + |n_{xy}| = 0 + 48 = 48$ kN/m.

Available reinforcement membrane force

$A_s = \frac{1}{4}\pi 10^2 \times 1000/150 = 524$ mm²/m

$n_{max} = 524 \times 450 = 235800$ N/m = 236 kN/m.

Available is much more than required therefore the reinforcement is OK.