Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Write your name and study number at the top right-hand of your work.

Exam CT4143 Shell Analysis
28 August 2012, 14:00 – 17:00 hours

Problem 1 (36 minutes) 2 points

Explain shortly the meaning of the following eight words in relation to structural shells.

Elephant foot
Festoon
Flame
Isler
Fourier series
Funicular
Order h
Lamé parameters

Problem 2 (36 minutes) 2 points

a Figure 1 shows a shell part with forces and moments in the positive directions. Add to this figure the variables for the directions, forces and moments (for example x, n_{xx} , ... Make clear which variable belongs to which arrow.) (You can hand in this page with your answers.)

b What is the correct interpretation of the shear force q_x on the free shell edge? (Fig. 1)

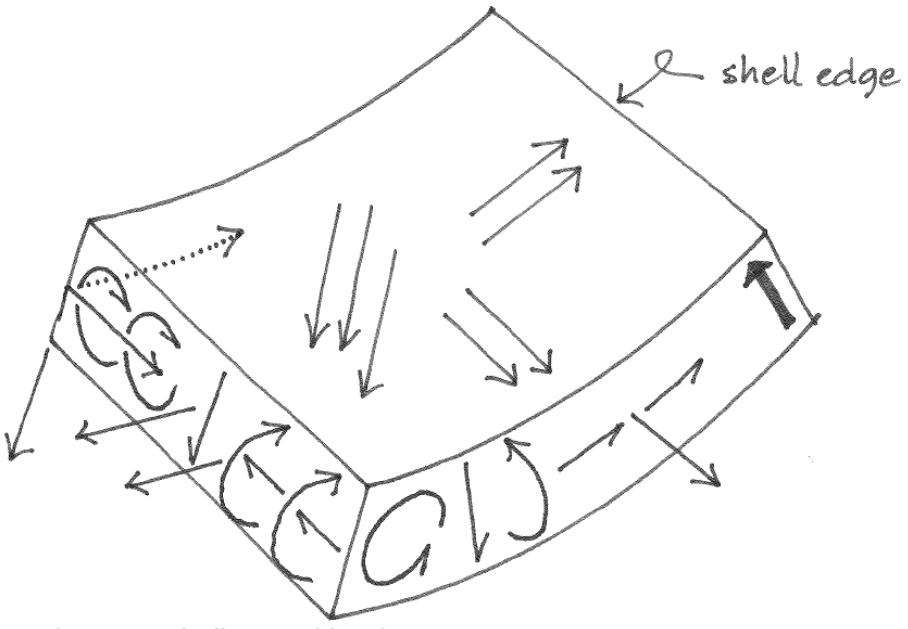

Figure 1. Shell part with edge

c Is this shell part thin or thick (Fig. 1). Explain your answer. You can measure dimensions in Figure 1.

d Suppose this shell part is modelled with finite elements for computing the stresses (Fig. 1). In which direction (*x* or *y*) need the elements be smallest? Explain your answer.

e Are semi-Loof elements suitable for modelling the shell part (Fig. 1)? Explain your answer.

Problem 3 (36 minutes) 2 points

a How many unbilics occur in the curvature trajectories of a sphere?

A 0
B 1
C 2
D infinite

b What is a whispering gallery?

A A floor level under the dome of Sint Paul's cathedral
B A shell shape that magnifies sound to a disturbing level
C A BLOB or free form object intended for sharing gossip
D A situation in which sound waves are guided by a convex hard surface

c In a gridshell the lath joints can slide during construction. Why is this?

A To make adjustments possible to the shell surface shape
B To reduce stresses in the laths
C To obtain full contact in the joints
D To save labour costs in fastening and removing screws

d Who was the first to calculate the fundamental frequency of a thin wall sphere?

A W. de Viking
B Dr. H. Lamb
C Dr. E. Reissner
D Dr. H. Reissner

e Why are shells sensitive to imperfections? Choose from A, B, C or D.

A The stiffness due to perpendicular loading is highly non-linear.
B An imperfection reduces the curvature which is further reduced during loading.
C Interaction of buckling modes.
D This cannot be explained in one line.

Problem 4 (36 minutes) 2 points

Consider an optimal glass dome covering a city.
city diameter 3 km
glass Young's modulus 80000 N/mm^2
glass mass 2500 kg/m^3

Earth gravitational acceleration 9.81 m/s^2
Moon gravitational acceleration 1.62 m/s^2
Earth atmospheric pressure 0.101 N/mm^2
Moon atmospheric pressure 0 N/mm^2
partial safety factors are included in the above numbers

a Calculate the sagitta.

b Calculate the thickness.

c Calculate the thickness if the dome were built on the Moon.

d Calculate the stress in the top of the Moon dome.

e Calculate the stress in the Moon dome due to Earth's atmospheric pressure inside.

Problem 5 (36 minutes) 2 points

Consider an assembly of 8 identical reinforced concrete hypars (Fig. 2). The columns are fixed in the foundation and can resist large moments. The shell thickness is t = 70 mm. The distributed load including self-weight is 4 kN/m². (safety factors are included in all numbers.)

a Does this design need cables or extra beams? If so, add these.
(Note that the columns can resist moments.)

b Calculate the membrane force and related stress.

c Calculate the largest bending moment and related stress. (Assume that the material is linear-elastic.)

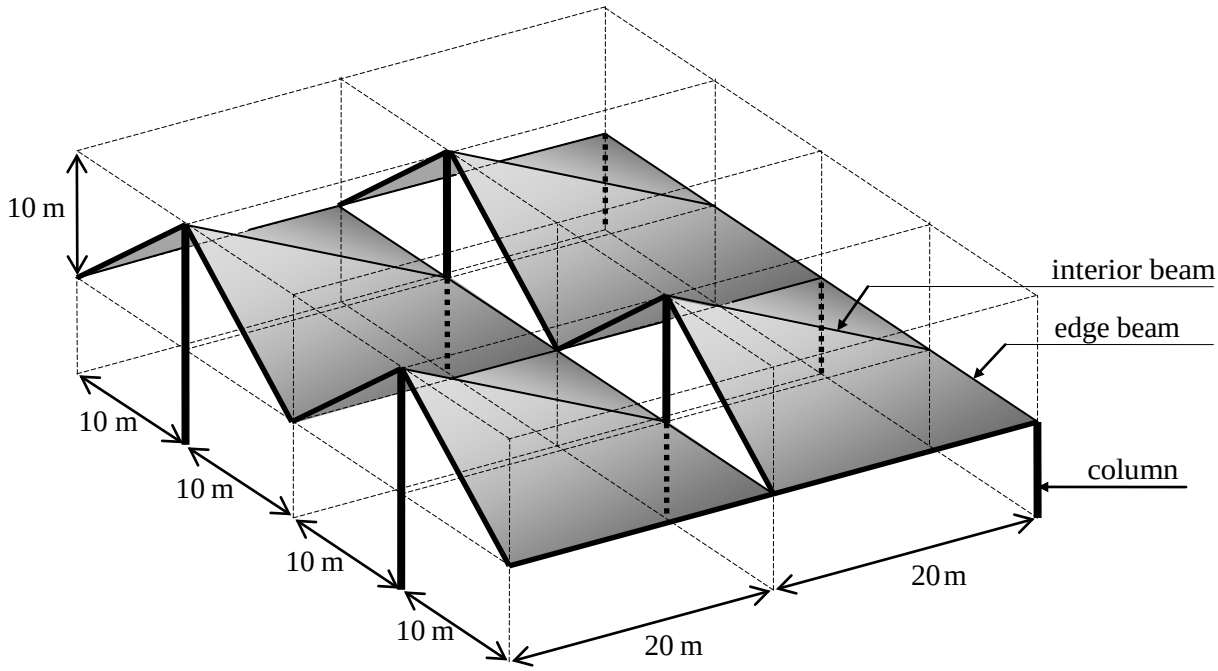

Figure 2. Eight hypar roof

d Calculate the forces in the beams and on the column heads.

e Check whether the shell will buckle. Use Young's modulus is 15 x 10^6 kN/m². (This is the tangent stiffness of cracked reinforced concrete at small stresses.)

f Calculate the reinforcement required to carry the membrane forces. Use a yield stress of 400 N/mm². The bar directions are parallel to the shell edges.

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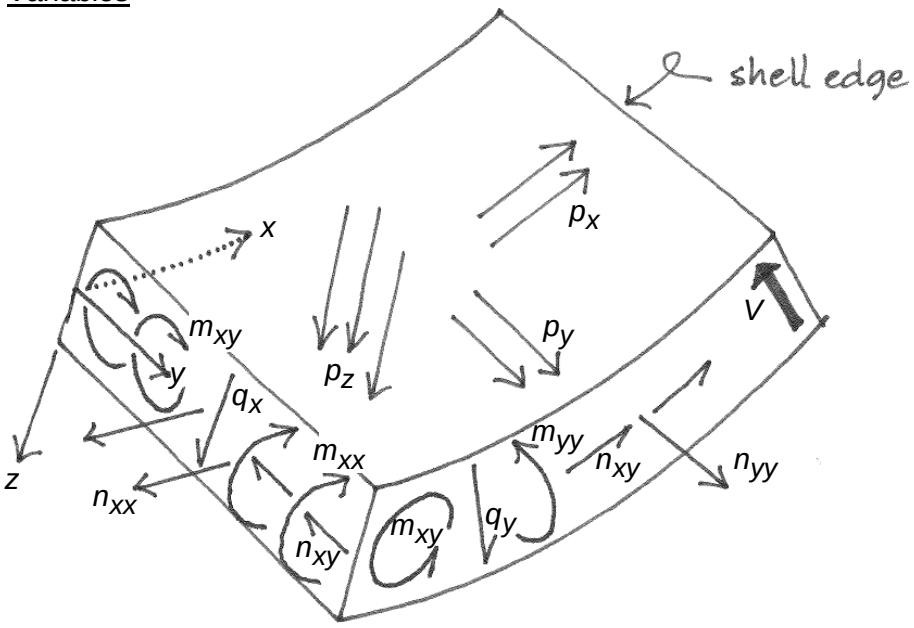
Solution to Problem 1

Elephant foot ..…….... Ring pattern buckle at the bottom of cylindrical tank
Festoon ……. Envelope of curved graphs
Flame …….. Umbilic where two ridges cross
Isler …….. Designer of over 1200 reinforced concrete shell structures
Fourier series ……. Approximation of function by a summation of sine and cosine
Funicular ……. Shape of a hanging chain
Order h …… $C_1h + C_2h^2 + C_3h^3 + \ldots$
Lamé parameters ……..... $\alpha_1 = \frac{\partial x}{\partial \tilde{x}}$, $\alpha_2 = \frac{\partial y}{\partial \tilde{y}}$

Solution to Problem 2

a Variables



b Shear
A shear force on a free edge is in reality an increase of the concentrated shear force *V* along the edge.

c Thin or thick
The radius of curvature in the *x* direction is approximately 5.5 times the thickness. This is less than 30. Therefore, this shell part is thick.

d Smallest direction
In the *y* direction there is no curvature. Therefore, stresses will have a small gradient in this direction. Therefore, the elements can be large in this direction.
In the *x* direction there is an edge disturbance. At least 6 elements are needed in the influence length. Therefore, the elements need to be small in this direction.

e Semi-Loof
Semi-Loof elements can be suitable. However, they are based on the Sanders-Koiter equations, which do not include shear deformation. In a thick shell shear deformation can be important. Then shell elements derived from a solid element need be used. These do include shear deformation.

Solution to Problem 3

a D (k1 = k2 everywhere. We see only 2 umbilics. The others have 2 orthogonal ridges.)

b D (A is half okay)

c C

d B

e Each answer is okay. Thanks for your opinion.
(A is Von Karman's, B is Calladine's, C is Koiter's)

Solution to Problem 4

```
> restart:
> l:=3000  *m:
> E:=80000 *N/mm^2:
> rho:=2500*kg/m^3:
> gE:=9.81 *m/s^2:
> gM:=1.62 *m/s^2:
> p:=0.101 *N/mm^2:
>
> m:=1000*mm:
> kg:=N/(m/s^2):
>
> # a Sagitta
> s:=sqrt(3.)/6*l;
```

$$s := 866025.4040\ mm$$

```
> # b Earth shell thickness
> t:=2/9*10*rho*gE*l^2/E;
```

$$t := 6131.250002\ mm$$

```
> # c Moon shell thickness
> t:=2/9*10*rho*gM*l^2/E;
```

$$t := 1012.500000\ mm$$

```
> # d stress
> a:=1/2*s +1/8*l^2/s:
> sigma:=0.1*E*t/a; # compression
```

$$\sigma := \frac{4.676537182\ N}{mm^2}$$

```
> # d stress alternative
> ps:=rho*gM*t:
> sigma:=1/2*ps*a /t; # compression
```

$$\sigma := \frac{3.507402884\ N}{mm^2}$$

```
> # e stress (Barlow's formula)
> sigma:=1/2*p*a /t; # tension
```

$$\sigma := \frac{86.38870690\ N}{mm^2}$$

Solution to Problem 5

a Cables and extra beams
No

b Membrane force

Radius of curvature $a = \dfrac{20 \times 10}{10} = 20$ m

The direction of the dead load is approximately perpendicular to the shell surface. Therefore, the distributed membrane force is

$$n_{xy} = \tfrac{1}{2} a\, p_z = \tfrac{1}{2} 20 \times 4 = 40 \text{ kN/m}.$$

The shear stress is

$$\sigma_{xy} = \frac{n_{xy}}{t} = \frac{40 \text{ N/mm}}{70 \text{ mm}} = 0.571 \text{ N/mm}^2.$$

c Moment

The largest moment in the shell occurs at the horizontal interior beams. (Here, a rotation will not occur due to symmetry.) The moment is (Loof's formula)

$$m_{xx} = 0.511 \frac{p_z}{l^2} \sqrt[3]{(a\,t\,l)^4} = 0.511 \frac{4}{20^2} (40 \times 0.070 \times 20)^{\frac{4}{3}} = 0.434 \text{ kNm/m}.$$

The bending stress is

$$\sigma_{xx} = \frac{m_{xx}}{\frac{1}{6} t^2} = \frac{434 \text{ Nmm/mm}}{\frac{1}{6} (70 \text{ mm})^2} = 0.532 \text{ N/mm}^2.$$

d Force flow

Edge beam length $\sqrt{10^2 + 10^2} = 14.1$ m

Interior beam length $\sqrt{10^2 + 20^2} = 22.4$ m

Force in the beams
$40 \times 20 = 800$ kN
$40 \times 10 = 400$ kN
$40 \times 14.1 = 566$ kN
$2 \times 40 \times 22.4 = 1789$ kN

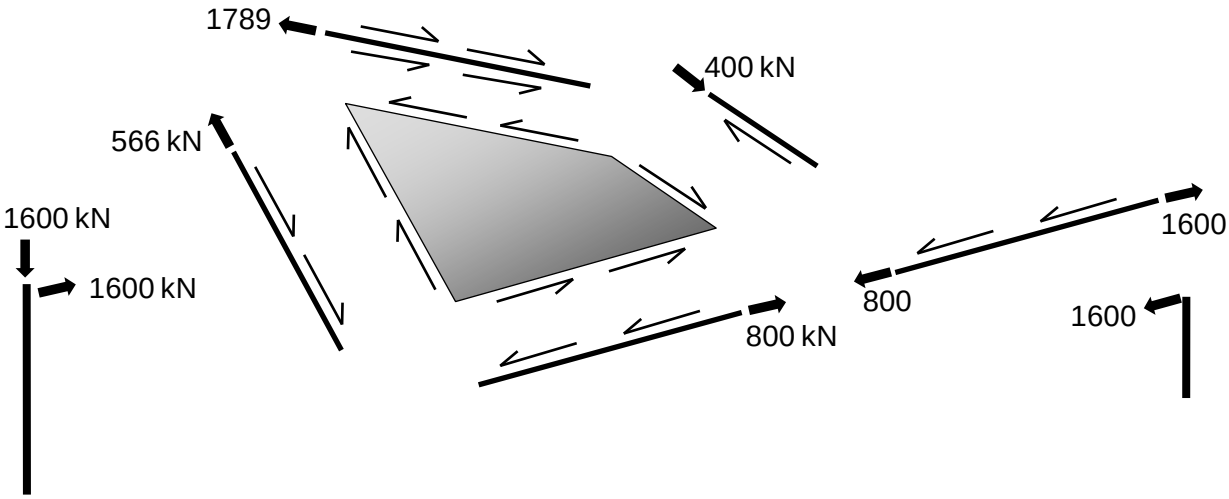
Vertical component of the interior beam force $1789 \dfrac{10}{\sqrt{10^2 + 20^2}} = 800$ kN

Horizontal component of the interior beam force $1789 \dfrac{20}{\sqrt{10^2 + 20^2}} = 1600$ kN

Vertical component of the exterior beam forces $566\sqrt{2} = 800$ kN

Column force $800 + 800 = 1600$ kN

Check: column force $\dfrac{20 \times 20 \times 4}{4} = 1600$ kN OK.



e Buckling

The critical membrane force is

$$n_{xy,cr} = 0.6\frac{E\,t^2}{a} = 0.6\frac{15\times10^6\times0.070^2}{20} = 2205\ \text{kN/m}$$

(A knockdown factor is not needed because hypars are not sensitive to imperfections.)
The actual membrane force of 40 kN/m is much smaller, therefore, it will not buckle.

f Reinforcement

$$n_{sx} = n_{xx} + \left|n_{xy}\right| = 0 + 40 = 40\ \text{kN/m}$$

$$n_{sy} = n_{yy} + \left|n_{xy}\right| = 0 + 40 = 40\ \text{kN/m}.$$

Proposed ∅ 6 – 200 in two directions

$$A_s = \tfrac{1}{4}\pi 6^2 \times 1000 / 200 = 141\ \text{mm}^2\text{/m}$$

$$n_{\max} = 141\times 400 = 56400\ \text{N/m} = 56.4\ \text{kN/m} > 40\ \text{kN/m OK}$$