Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Write your name and study number at the top right-hand of your work.

Exam CT4143 Shell Analysis
Tuesday 13 August 2013, 14:00 – 17:00 hours

Problem 1 (27 minutes) 1.5 points

Explain shortly the meaning of the following eight words in relation to structural shells.

Abutment force
Spline
Thick shell
Fully stressed design
C1 continuity
Eric Reissner
Benchmark
Consistent shell boundary conditions

Problem 2 (27 minutes) 1.5 points

a Figure 1 shows a shell part with forces and moments in the positive directions. Add to this figure the variables for the directions, forces and moments (for example x, n_{xx} , ... Make clear which variable belongs to which arrow.) (You can hand in this page with your answers.)

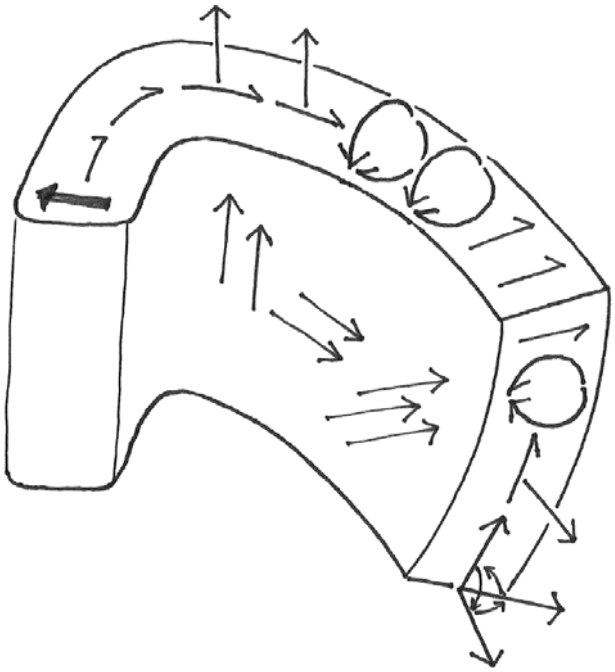


Figure 1. Shell part with curved edge

b Is the shell part of Figure 1 thick? Explain your answer.

c Figure 2 shows a finite element model of the shell part. It consists entirely out of 8-node shell elements. Can this model be used to accurately compute the stresses in the shell edge? Explain your answer.

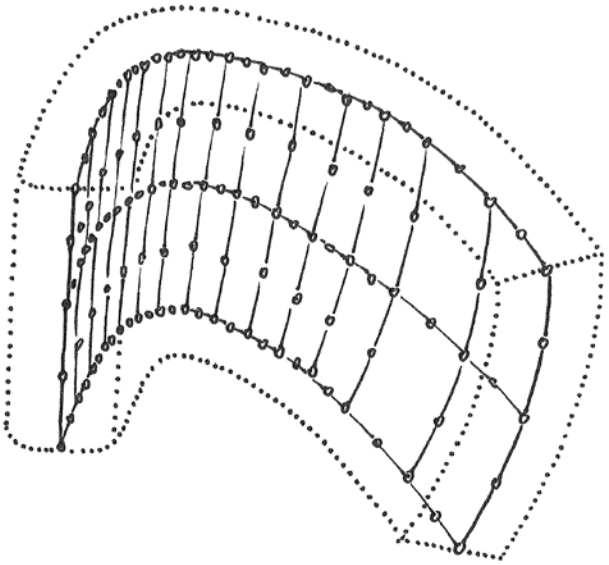


Figure 2. Element distribution of the shell part of Figure 1.

Problem 3 (27 minutes) 1.5 points

a What is Koiter's law? Choose A, B, C or D.

A Shape imperfections of a shell are usually very small and can be safely ignored.
B The real buckling load (ultimate load) is inversely proportional to the imperfection amplitude.
C All you need is Love.
D The critical load reduction is proportional to the square root of the imperfection amplitude.

b The three most important shell phenomena are …. Choose A, B, C or D.

A … natural frequency, edge disturbance and in-extentional deformation.
B … natural frequency, edge disturbance and imperfection sensitivity.
C … natural frequency, in-extensional deformation and imperfection sensitivity.
D … edge disturbance, in-extensional deformation and imperfection sensitivity.

c The force flow in a square hypar shell is $n_{xx} = 0$, $n_{yy} = 0$ and $n_{xy} = n$. (The x and y axis are along the straight edges.) The reinforcement can be directed 1) along the hypar edges or 2) along the hypar diagonals. Which direction gives the smallest reinforcement? Choose A, B, C or D.

A the diagonal directions; only half as much is needed
B the diagonal directions; only 2/3 as much is needed
C the edge directions; only half as much is needed
D the edge directions; only 2/3 as much is needed

d There is a remarkable difference between the smallest natural frequency of a free spherical shell and that of a free hemispherical shell. What causes this? Choose A, B, C or D.

A One deforms extensionally the other in-extensionally.
B They might have different diameters.
C One has more mass than the other.
D One is closed the other is open.

e Prof. Metrikine does not agree with the course notes on shell dynamics. Why is this? Choose A, B, C, D.

A Because they contain many mistakes.
B Because all derivations have been omitted.
C Because some notes recommend using the finite element method.
D Because no proper reference is made to any of his publications.

Problem 4 (27 minutes) 1.5 points

a In-extensional deformation can be seen as an *disease*. ("A shell design suffers from in-extensional deformation.") What are the *symptoms* and what are possible *cures*?

b On the other hand, in-extensional deformation can be necessary. In what situation is it necessary?

c What causes in-extensional deformation?

Problem 5 (27 minutes) 1.5 points

The smallest natural frequency of a curved square panel is

$$f_n = \sqrt{\frac{\pi^2}{12(1-\nu^2)}\frac{Et^2}{\rho l^4} + (k_x + k_y)^2 \frac{E}{16\pi^2\rho} + \frac{n_{xx} + n_{yy}}{4\rho t l^2}} \,.$$

Derive from this formula the buckling load formula of a spherical shell.

Problem 6 (27 minutes) 1.5 points

Consider an assembly of four identical reinforced concrete hypars (Fig. 3). The shell thickness is t = 70 mm. The distributed load including self-weight is 3 kN/m². (safety factors are included in all numbers.)

a Does this design need cables? If so, add these.

b Calculate the membrane force and related stress.

c Calculate the largest bending moment and related stress. (Assume that the material is linear-elastic.)

d Calculate the forces in the beams and columns and cables if present.

e Check whether the shell will buckle. Use Young's modulus is 15 x 10^6 kN/m². (This is the tangent stiffness of cracked reinforced concrete at small stresses.)

f Calculate the reinforcement required to carry the membrane forces. Use a yield stress of 430 N/mm². The bar directions are parallel to the shell edges.

Common rebar diameters in the Netherlands are ø8, ø10, ø12, ø16, ø20, ø25, ø32, ø40.

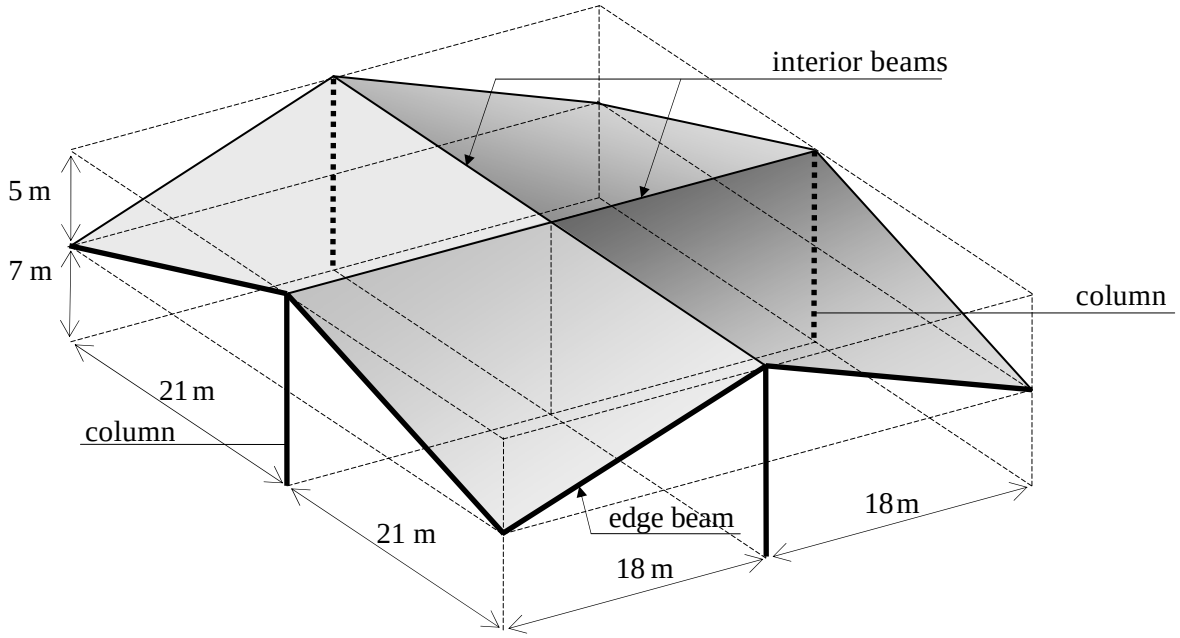


Figure 3. Four hypar roof

Problem 7 (18 minutes) 1 point

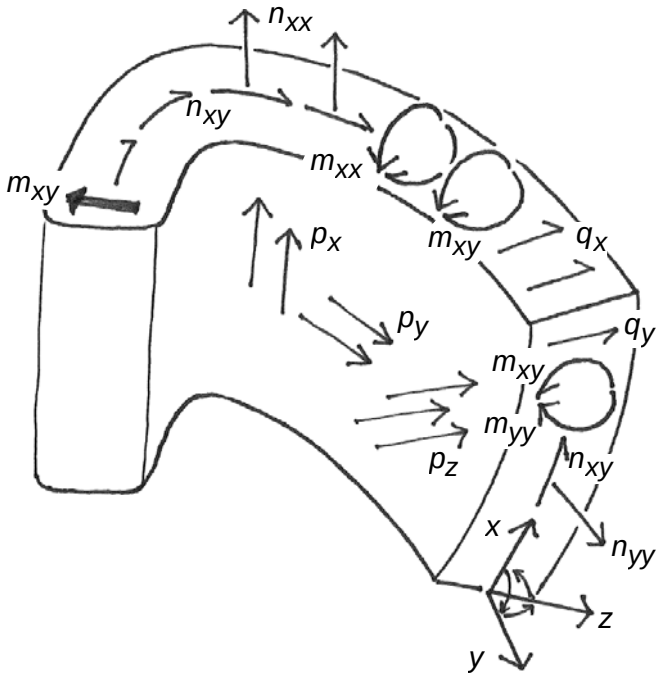
Design an assembly of hypar shells to be analysed in the next Shell exam (July 2014).
It needs to be
1) different than in this exam and different than in the previous exams,
2) stable,
3) not extremely complicated.

Exam CT4143, 1 July 2013
Solution to Problem 1

Abutment force	Horizontal component of the support reaction of an arch or a shell
Spline	Flexible strip of metal, wood or plastic used for drawing curved lines
Thick shell	Shell part with a ratio a / t < 30
Fully stressed design	Design for which in every location the material is loaded to its ultimate stress
C1 continuity	Connection between two surfaces in which the tangents are the same
Eric Reissner	USA scientist who solved several shell problems
Benchmark	Test model for checking finite elements
Consistent shell boundary conditions	Boudary conditions that perfectly agree with the Sanders-Koiter equations (derived by Koiter in 1959)

Solution to Problem 2

a Variables



b Thickness

This shell part is very thick. The reason is that in the curve of the edge a / t ≈ 0.7 (measured from the drawing) which is much smaller than 5.

c Accuracy

Stresses calculated with the model of figure 2 will not be accurate. The reason is that the shell part is very thick. (Shell elements are derived from Bernoulli's hypothesis. However, in a very thick shell, plane sections do not remain plane.) If the stresses in the curved edge are important than it needs to be modelled by volume elements.

Solution to Problem 3

a D

b D

c A

d A

e B

Solution to Problem 4

a Symptoms

- large deformation due to imposed forces
- large bending stresses due to imposed forces
- loading carried by moments instead of normal forces
- very small natural frequencies
- mode of the smallest natural frequency has small wave numbers
- very small buckling load factors
- very large buckling lengths

Possible cures

- add edge beams
- increase curvature
- add support points
- close the shell

b Necessary
When displacements are imposed, for example foundation settlement or temperature expansion.

c Cause
The deformation is such that the Gaussian curvature does not change

Solution to Problem 5

$$f_n = \sqrt{\frac{\pi^2}{12(1-\nu^2)}\frac{Et^2}{\rho l^4} + (k_x + k_y)^2 \frac{E}{16\pi^2\rho} + \frac{n_{xx} + n_{yy}}{4\rho t l^2}}$$

$$0 = \sqrt{\frac{\pi^2}{12(1-\nu^2)}\frac{Et^2}{\rho l^4} + (\frac{1}{a} + \frac{1}{a})^2 \frac{E}{16\pi^2\rho} + \frac{n_{cr} + n_{cr}}{4\rho t l^2}}$$

$$0 = \frac{\pi^2}{12(1-\nu^2)}\frac{Et^2}{l^4} + \frac{4}{a^2}\frac{E}{16\pi^2} + \frac{n_{cr}}{2tl^2}$$

$$n_{cr} = -2tl^2\left(\frac{\pi^2}{12(1-\nu^2)}\frac{Et^2}{l^4}+\frac{4}{a^2}\frac{E}{16\pi^2}\right)$$

$$n_{cr} = \frac{\pi^2 Et^3}{6(1-\nu^2)}\frac{-1}{l^2}-\frac{Et}{2\pi^2 a^2}l^2$$

$$\frac{\partial n_{cr}}{\partial l} = \frac{\pi^2 Et^3}{6(1-\nu^2)}\frac{2}{l^3}-\frac{Et}{2\pi^2 a^2}2l = 0$$

$$\frac{\pi^2 t^2}{3(1-\nu^2)} = \frac{1}{\pi^2 a^2}l^4$$

$$l^4 = \frac{\pi^4 t^2 a^2}{3(1-\nu^2)}$$

$$l^2 = \frac{ta\pi^2}{\sqrt{3(1-\nu^2)}} \qquad \text{note that } l \approx 2.4\sqrt{at}$$

$$n_{cr} = \frac{\pi^2 Et^3}{6(1-\nu^2)}\frac{-\sqrt{3(1-\nu^2)}}{ta\pi^2}-\frac{Et}{2\pi^2 a^2}\frac{ta\pi^2}{\sqrt{3(1-\nu^2)}}$$

$$n_{cr} = \frac{Et^2}{2a}\frac{-1}{\sqrt{3(1-\nu^2)}}-\frac{Et^2}{2a}\frac{1}{\sqrt{3(1-\nu^2)}}$$

$$\boxed{n_{cr} = -\frac{1}{\sqrt{3(1-\nu^2)}}\frac{Et^2}{a}} \quad \text{Q. E. D.}$$

Solution to Problem 6

a Cables

No (This follows from problem 5d.)

b Membrane force

Radius of curvature $a = \frac{18\times 21}{5} = 75.6\,\text{m}$

The direction of the dead load is approximately perpendicular to the shell surface. Therefore, the distributed membrane force is

$$n_{xy} = \tfrac{1}{2}a\,p_z = \tfrac{1}{2}75.6\times 3 = 113\,\text{kN/m}.$$

The shear stress is

$$\sigma_{xy} = \frac{n_{xy}}{t} = \frac{113\ \text{N/mm}}{70\ \text{mm}} = 1.62\ \text{N/mm}^2 .$$

c Moment

The largest moment in the shell occurs at the short interiour beams. (Here, a rotation will not occur due to symmetry.)

$$l_i = \sqrt{38^2 + 9^2} = 39\ \text{m}$$

The moment is (Loof's formula)

$$m_{xx} = 0.511 \frac{p_z}{l_i^2} \sqrt[3]{(a\, t\, l_i)^4} = 0.511 \frac{3}{18^2} (75.6 \times 0.070 \times 18)^{\frac{4}{3}} = 2.06\ \text{kNm/m}.$$

The bending stress is

$$\sigma_{xx} = \frac{m_{xx}}{\frac{1}{6} t^2} = \frac{2060\ \text{Nmm/mm}}{\frac{1}{6}(70\ \text{mm})^2} = 2.5\ \text{N/mm}^2 .$$

d Force flow (see figure)

Inclined edge beam lengths $l_s = \sqrt{18^2 + 5^2} = 18.7$ m

$l_l = \sqrt{21^2 + 5^2} = 21.6$ m

Forces in the beams $2 \times 113 \times 18 = 4082$ kN

$113 \times 18.7 = 2118$ kN

$113 \times 21.6 = 2448$ kN

$2 \times 113 \times 21 = 4763$ kN

Column forces $2 \times 2118 \frac{5}{18.7} = 1134$ kN

$2 \times 2448 \frac{5}{21.6} = 1134$ kN

Check: total load $2 \times 1134 + 2 \times 1134 = 4536$ kN

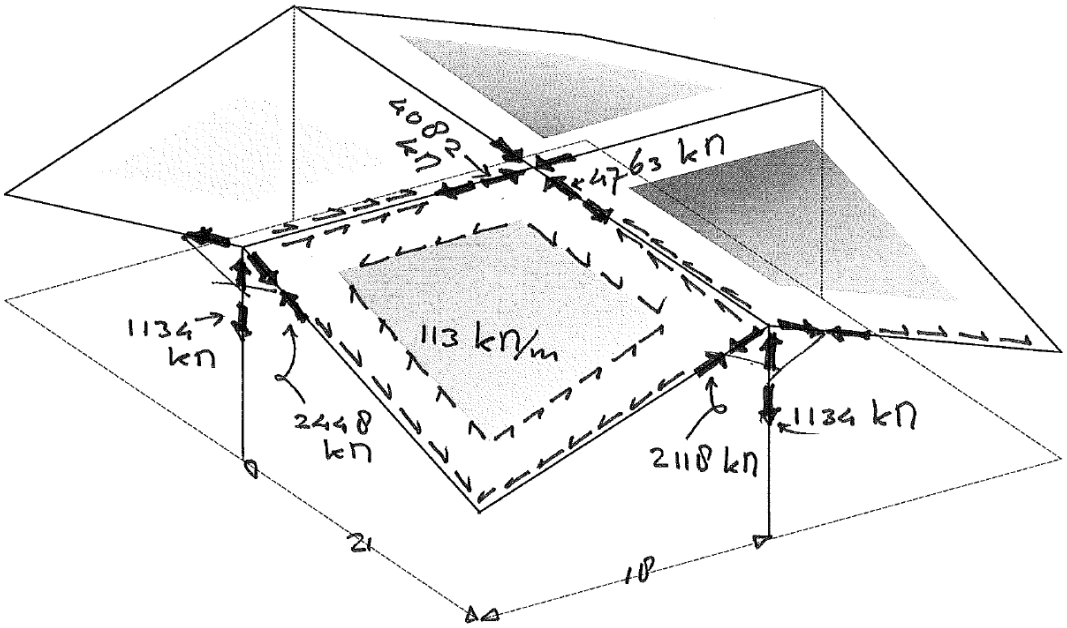
$3(2 \times 18 \times 2 \times 21) = 4536$ kN OK.

e Buckling

The critical membrane force is

$$n_{xy,cr} = 0.6 \frac{E\, t^2}{a} = 0.6 \frac{15 \times 10^6 \times 0.070^2}{75.6} = 583\ \text{kN/m}$$

(A knockdown factor is not needed because hypars are not sensitive to imperfections.)
The actual membrane force of 113 kN/m is smaller, therefore, it will not buckle.



f Reinforcement

$n_{sx} = n_{xx} + |n_{xy}| = 0 + 113 = 113$ kN/m

$n_{sy} = n_{yy} + |n_{xy}| = 0 + 113 = 113$ kN/m.

Proposed ∅ 8 – 180 in two directions

$A_s = \frac{1}{4}\pi 8^2 \times 1000 / 180 = 279$ mm²/m

$n_{max} = 279 \times 430 = 120000$ N/m = 120 kN/m > 113 kN/m OK

Solution to problem 7

See next exam.