Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Write your name and study number at the top right-hand of your work.

Exam CT4143 Shell Analysis
Monday 1 July 2013, 14:00 – 17:00 hours

Problem 1 (36 minutes) 2 points

Explain shortly the meaning of the following eight words in relation to structural shells.

Zebra analysis
Hypar
Lamé parameters
Intrinsic surface property
Stringer
Particle spring method
Sanders-Koiter equations
Geodesic sphere

Problem 2 (36 minutes) 2 points

a Figure 1 shows a shell part with forces and moments in the positive directions. Add to this figure the variables for the directions, forces and moments (for example x, n_{xx} , ... Make clear which variable belongs to which arrow.) (You can hand in this page with your answers.)

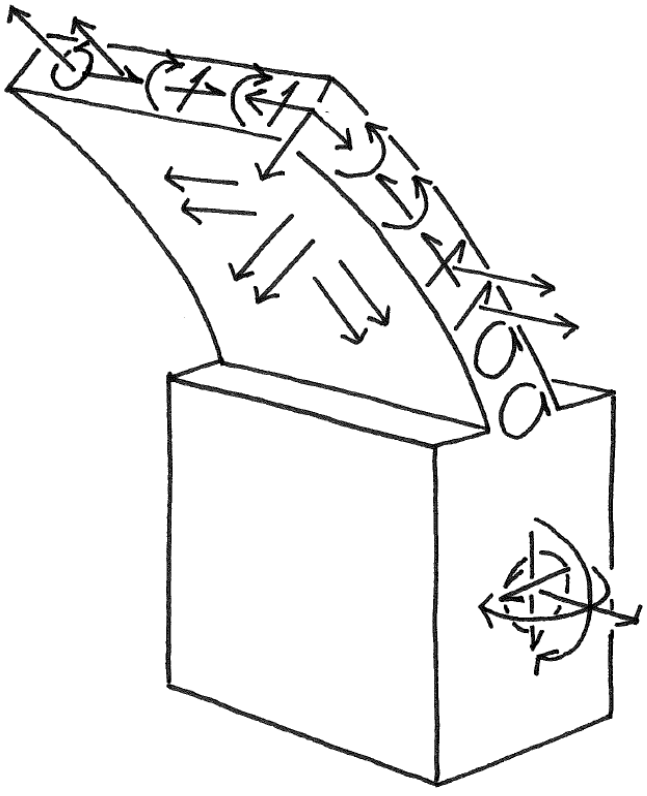


Figure 1. Shell part with foundation beam

b Figure 2 shows a structural model of the shell part. The shell elements are not square but rectangular. What is the reason for this element shape?

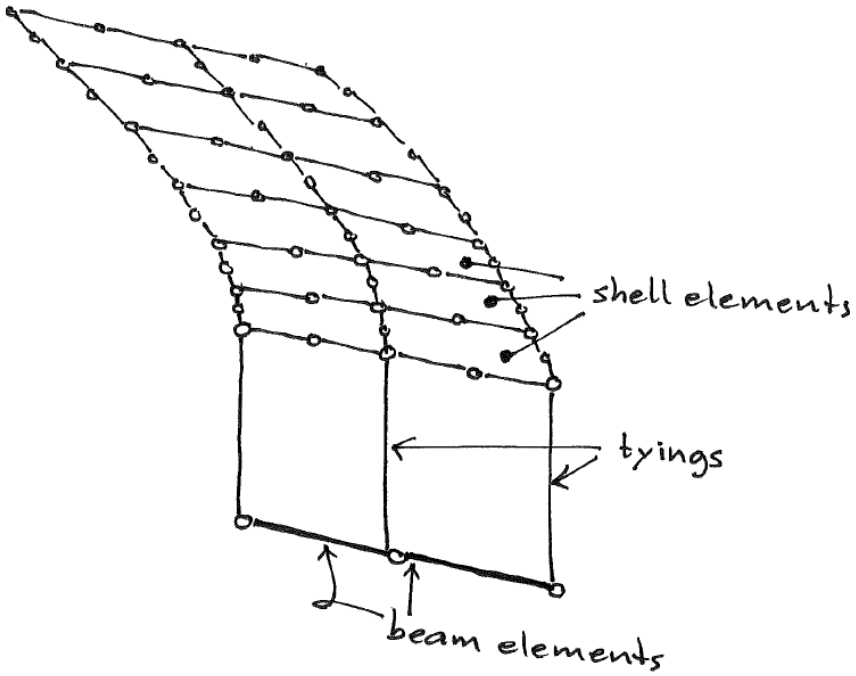


Figure 2. Element distribution of the shell part of Figure 1.

Problem 3 (36 minutes) 2 points

a Hypars are not sensitive to imperfections. Why is this? Choose A, B, C or D.

A The imperfections are usually very small due to the simple construction method.
B The edge and interior beams give sufficient support.
C Tensile membrane forces remove the imperfections.
D Often cables are necessary to prevent buckling of the parts.

b In 1928 it was discovered that thin shell structures buckle at loads much smaller than predicted by the shell differential equation. The correct explanation was found 12 years later in 1940. What is the correct explanation? Choose A, B, C or D.

A The differential equation had not been solved properly for cylindrical shells.
B There is an essential error in the shell differential equation related to the strain energy.
C Shells are sensitive to shape imperfections that are invisible to the naked eye.
D If load is applied quickly the reflecting shock wave significantly decreases the strength of cylindrical test specimen.

c In 1955 a Japanese professor stayed at the Californian Institute of Technology where he wrote about his work on buckling of thin shell structures. His report (NASA 1390) had such un impact that today we still use the name of this professor when we refer to the shape in which thin shells buckle. What is the name of this buckling shape? Choose A, B, C or D.

A Yoshimura pattern

B Fukushima pattern
C Yahama pattern
D Origami pattern

d The in-plane shear forces [kN/m] include a quantity n_z. Does this n_z exist in reality of is it virtual?

A It is virtual because it is not computed by any finite element program.
B It is virtual because moment equilibrium shows that $n_{xy} = n_{yx}$.
C It is real because it follows from the derivations of both Sanders and Koiter.
D It is real because the shell edges impose it onto the structure.

e The moments due to a point load on a shell structure often do not depend on the boundary conditions (hinged or fixed edges). What causes this?

A The strain energy due to the point load is small compared to the strain energy of the boundary conditions.
B In-extensional deformation carries the moments in a shell.
C The influence length is often smaller than the distance from the point load to the boundaries.
D This statement is not true. The boundary conditions do have a large influence.

Problem 4 (36 minutes) 2 points

Consider a reinforced concrete hypar which will be cast on a timber framework. The hypar radius of curvature is a = 140 m. The formwork consists of straight beams in parallel to the hypar edges and multiplex plates. The plates will be twisted. Clearly, we do not want them to buckle. The factory dimensions of the plates are 2440 x 1220 x 18 mm. Do the plates need to be cut to a smaller size? Explain your answer.

Problem 5 (36 minutes) 2 points

Consider an assembly of four identical reinforced concrete hypars (Fig. 3). The shell thickness is t = 120 mm. The distributed load including self-weight is 5 kN/m². (safety factors are included in all numbers.)

a Does this design need cables? If so, add these.

b Calculate the membrane force and related stress.

c Calculate the largest bending moment and related stress. (Assume that the material is linear-elastic.)

d Calculate the forces in the beams and columns and cables if present.

e Check whether the shell will buckle. Use Young's modulus is 15 x 10^6 kN/m². (This is the tangent stiffness of cracked reinforced concrete at small stresses.)

f Calculate the reinforcement required to carry the membrane forces. Use a yield stress of 430 N/mm². The bar directions are parallel to the shell edges.

Common rebar diameters in the Netherlands are ø8, ø10, ø12, ø16, ø20, ø25, ø32, ø40.

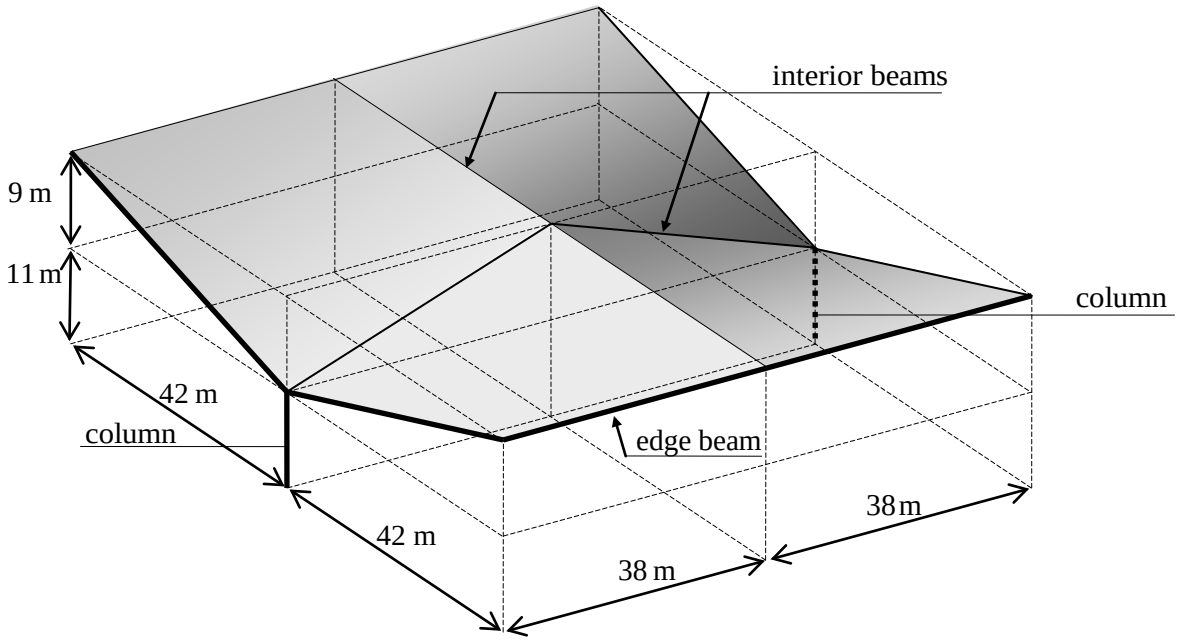


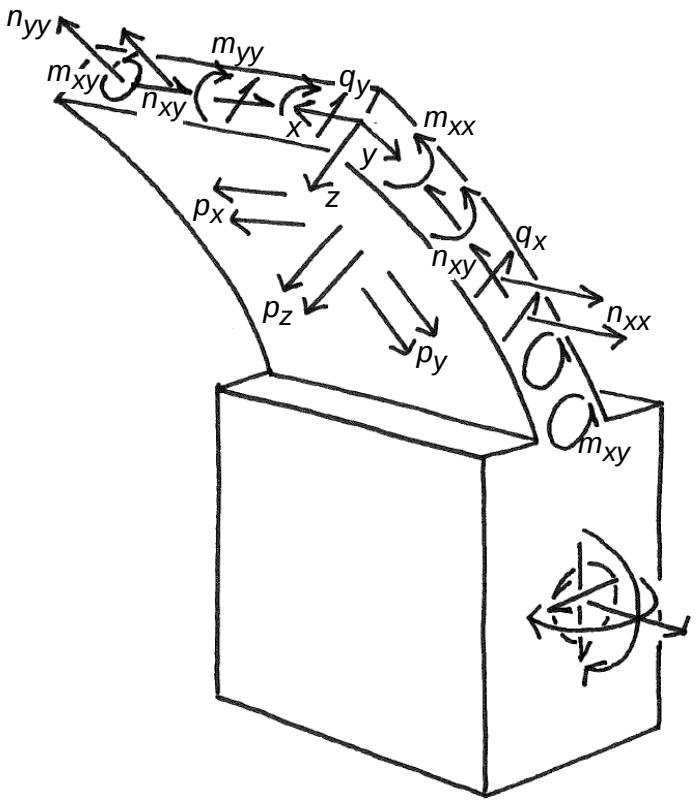
Figure 3. Four hypar roof

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Solution to Problem 1

Zebra analysis …….	Inspection method for the smoothness of a shell surface
Hypar ……...	Hyperbolical paraboloid
Lamé parameters ……..	Distances between the gridlines of a curved coordinate system
Intrinsic surface property …..	Property that depends on measurements within a surface only
Stringer ……..	Stiffener in thin shell structure; for example an aeroplane
Particle spring method ……..	Virtual points connected by springs for formfinding the grid of a grid shell
Sanders-Koiter equations ….	21 Equations that describe the behaviour of thin shells
Geodesic sphere …….	An icosahedron which is further divided in triangles of which the nodes are projected onto a sphere

Solution to Problem 2

a Variables



b Element shape

There is an edge disturbance in this part of the shell. To calculate the stresses accurately the elements need to be small in the direction perpendicular to the shell edge. In the direction of the shell edge there is no need for small elements. It is better to have large element dimensions in this direction in order to have not too many dofs in the shell model.

Solution to Problem 3

a C

b C

c A

d C

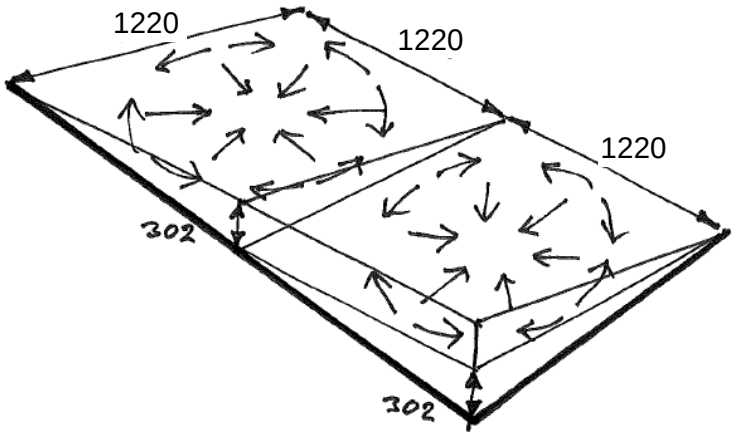
e D

Solution to Problem 4

The plates need to be twisted until they match the shell radius of curvature.

$$a = \frac{bc}{h} \Rightarrow 140000 = \frac{2440 \times 1220}{h} \Rightarrow h = 21\text{mm}$$

A square plate can be twisted 16.8 times the plate thickness before it buckles (Staak's formula). If the plate is cut in halves the parts will be square and buckle at a corner displacements of 16.8 x 18 = 302 mm.
The whole plate will buckle at a corner displacement of 2 x 302 = 604 mm (see figure below).



Since 21 < 604 the plate will not buckle.

Solution to Problem 5

a Cables
Yes; See figure below (This follows from problem 5d.)

b Membrane force

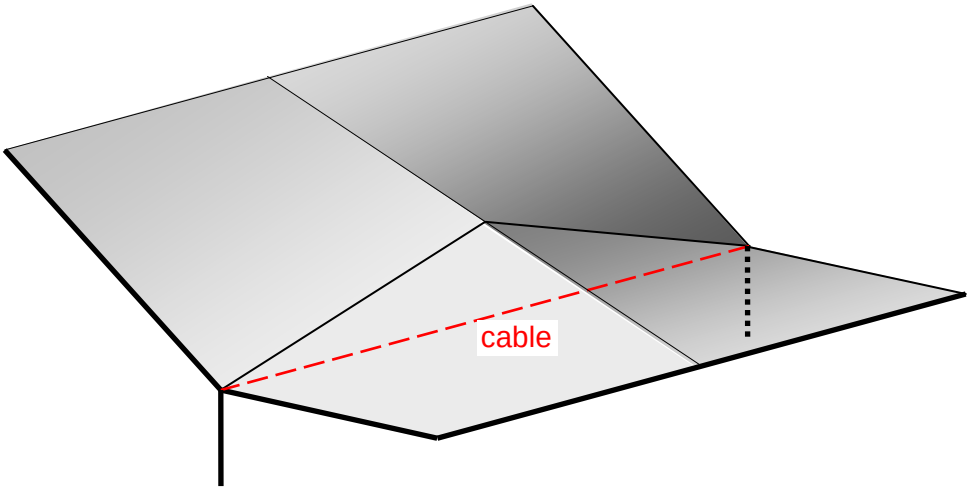
Radius of curvature $a = \frac{38 \times 42}{9} = 177$ m

The direction of the dead load is approximately perpendicular to the shell surface.
Therefore, the distributed membrane force is

$$n_{xy} = \tfrac{1}{2} a\, p_z = \tfrac{1}{2} 177 \times 5 = 443\,\text{kN/m}.$$

The shear stress is

$$\sigma_{xy} = \frac{n_{xy}}{t} = \frac{443\,\text{N/mm}}{120\,\text{mm}} = 3.6\,\text{N/mm}^2\,.$$



c Moment

The largest moment in the shell occurs at the inclined interior beams. (Here, a rotation will not occur due to symmetry.)

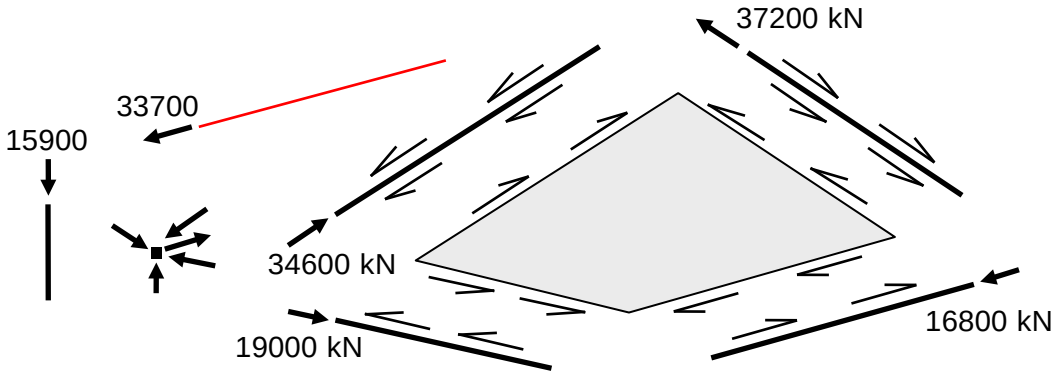
$$l_i = \sqrt{38^2 + 9^2} = 39\,\text{m}$$

The moment is (Loof's formula)

$$m_{xx} = 0.511\frac{p_z}{l_i^2}\sqrt[3]{(a\,t\,l_i)^4} = 0.511\frac{5}{39^2}(177\times 0.120\times 39)^{\frac{4}{3}} = 13.1\ \text{kNm/m}.$$

The bending stress is

$$\sigma_{xx} = \frac{m_{xx}}{\frac{1}{6}t^2} = \frac{1310\,\text{Nmm/mm}}{\frac{1}{6}(120\,\text{mm})^2} = 5.5\,\text{N/mm}^2\,.$$



d Force flow

Inclined edge beam length $l_e = \sqrt{42^2 + 9^2} = 43$ m

Forces in the beams $2\times 443\times 42 = 37200\,\text{kN}$

$443\times 43 = 19000\,\text{kN}$

$443\times 38 = 16800\,\text{kN}$

$$2 \times 443 \times 39 = 34600 \text{ kN}$$

Cable force $H = 34600 \dfrac{38}{39} = 33700 \text{ kN}$

Column force $V = 34600 \dfrac{9}{39} + 2 \times 19000 \dfrac{9}{43} = 15900 \text{ kN}$

Check: column force $\dfrac{84 \times 76 \times 5}{2} = 16000 \text{ kN}$ OK.

e Buckling

The critical membrane force is

$$n_{xy,cr} = 0.6 \frac{E t^2}{a} = 0.6 \frac{15 \times 10^6 \times 0.120^2}{177} = 731 \text{ kN/m}$$

(A knockdown factor is not needed because hypars are not sensitive to imperfections.)
The actual membrane force of 443 kN/m is smaller, therefore, it will not buckle.

f Reinforcement

$n_{sx} = n_{xx} + |n_{xy}| = 0 + 443 = 443 \text{ kN/m}$

$n_{sy} = n_{yy} + |n_{xy}| = 0 + 443 = 443 \text{ kN/m}.$

Proposed ∅ 16 – 190 in two directions

$A_s = \frac{1}{4} \pi 16^2 \times 1000 / 190 = 1060 \text{ mm}^2\text{/m}$

$n_{max} = 1060 \times 430 = 455000 \text{ N/m} = 455 \text{ kN/m} > 443 \text{ kN/m}$ OK