Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Write your name and study number at the top right-hand of your work.

Exam CIE4143 Shell Analysis
Tuesday 14 August 2018, 13:30 – 16:30 hours

Choose A, B, C or D.

1. What is sagitta?

   A The height of an arch or a shell structure.
   B The largest span of a shell structure.
   C The radius of curvature of an arch or a shell structure.
   D The slenderness of a shell structure.

2. Consider the shell that carries its self-weight with the least amount of material. What is the ratio sagitta over span?

   A $\frac{1}{6}\sqrt{3}$
   B at most 500
   C approximately 0.3
   D larger than 30

3. Suppose that $k_1' = k_1 + dk$ and $k_2' = k_2 + dk$. Which equation is correct?

   A $k_G' - k_G = k_m$
   B $k_G' - k_G = 2dk^2$
   C $k_G' - k_G = -2k_m dk$
   D $k_G' - k_G = k_m'^2 - k_m^2$

4. In an orthogonal parameterisation the grid lines are perpendicular to each other. Each space between the grid lines is a little square. Since they are all squares, how can a shell have shear deformation?

   A The little squares are only approximately square.
   B The grid of the deformed shell is not orthogonal.
   C The shear deformation is very small and often invisible.
   D The Gaussian curvature changes from location to location.

5. Which are correct equations for computing deformation curvature?

   A $\kappa_{xx} = k'_{xx} - k_{xx}$ $\quad \kappa_{yy} = k'_{yy} - k_{yy}$ $\quad \rho_{xy} = k'_{xy} - k_{xy}$
   B $\kappa_{xx} = k_{xx} - k'_{xx}$ $\quad \kappa_{yy} = k_{yy} - k'_{yy}$ $\quad \rho_{xy} = 2(k_{xy} - k'_{xy})$
   C $\kappa_{xx} = k'_{xx} + k_{xx}$ $\quad \kappa_{yy} = k'_{yy} + k_{yy}$ $\quad \rho_{xy} = k'_{xy} + k_{xy}$

D $\kappa_{xx} = \frac{k'_{xx} - k_{xx}}{2}$ $\kappa_{yy} = \frac{k'_{yy} - k_{yy}}{2}$ $\rho_{xy} = \frac{k'_{xy} - k_{xy}}{2}$

where k_{xx} is the curvature in the local x direction in the undeformed state and k'_{xx} is the same curvature in the deformed state, et cetera.

6. What is the ratio *a/t* of a thin shell?

   A $a/t < 5$
   B $5 < a/t < 30$
   C $30 < a/t < 4000$
   D $4000 < a/t$

7. Which statement is true about Gaussian curvature?

   A When Gaussian curvature changes, membrane forces develop.
   B When Gaussian curvature changes, the deformation is in-extensional.
   C When Gaussian curvature changes, the shell shape is uniform.
   D When Gaussian curvature changes, the shell is flexible.

8. Are the following statements true?
   The matrix sum of two tensors is a tensor.
   The matrix product of two tensors is a tensor.

   A Yes, Yes
   B Yes, No
   C No, Yes
   D No, No

9. What is an edge disturbance?

   A A strongly curved part at the edge of a shell. It prevents the need for an edge beam
   B An unsupported shell edge sensitive to in-extensional deformation.
   C A deformation wave that occurs at fixed supports of spherical caps.
   D Large stresses at edges or internal edges of shells.

10. What causes edge disturbances close to fixed supports?

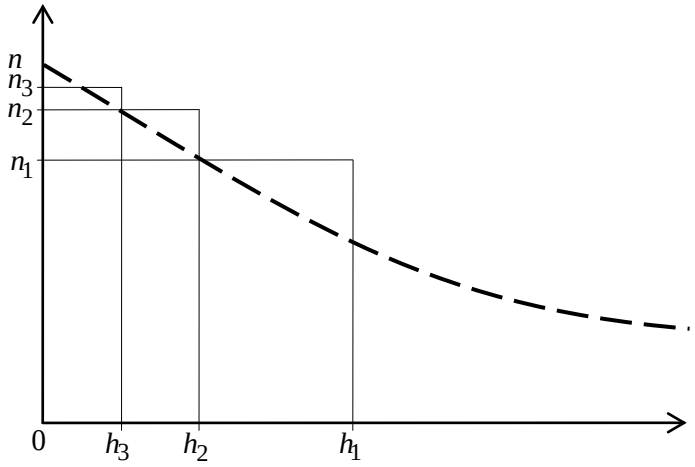
    A Lateral contraction as described by Poisson's ratio.
    B Invisible shape imperfections.
    C The middle third rule.
    D The shell deforms but close to the support it bends back to meet the boundary condition.

11. A curved shell can behave as a flat plate. What is the name of this phenomena?

    A Van der Neut effect
    B imperfection sensitivity

C middle third rule
D in-extensional deformation

12. A shell finite element type approximates the membrane force as uniform over the element surface (see figure). We want to compute the membrane force in a shell edge. What is the order of the error?



A O(h)
B $O(h^2)$
C $O(h^3)$
D ∞

13. Do the Sanders-Koiter equations include out of plane shear deformation?

A Yes, these equations are similar to the Kirchhoff equations for plates.
B Yes, these equations are similar to the Mindlin-Reissner equations for plates.
C No, because these equations are for thin shells only.
D No, because these equations are for small displacements only.

14. Which simple model can be used to understand edge disturbances?

A column buckling
B cantilever
C beam on elastic foundation
D mass spring system

15. Wat is a monocoque?

A an historic airplane
B the shell structure of cars, airplanes, etc.
C a construction method for vehicles that do not have a frame
D the carbon fibre technology for velocity enhancing aspects of advertising mono objectives

16. In a free shell edge the out of plane shear force q_x = 41 kN/m, the bending moment m_{xx} = 3 kNm/m and the torsion moment m_{xy} = 4 kNm/m. The shell has a thickness t = 60 mm. The y axis points outwards. What is the largest shear stress in the edge?

A 0
B 1 N/mm^2
C 6 N/mm^2
D -4 N/mm^2

17. Does the correct answer to question 16 depend on the direction of the y axis?

A Yes, otherwise the direction would not have been mentioned in the question.
B Yes, otherwise the direction would not have been mentioned in the reader.
C Yes, when the y axis point inwards, the formula for the shear stress is different.
D No, when the y axis points inwards, the formula for the shear stress is the same.

18. What is a hoop force in a shell of revolution?

A The membrane force in the direction of the rotation.
B The opposite of the meridional force.
C Loading perpendicular to the shell surface.
D Confinement by the support.

19. Consider linear analysis and geometrically nonlinear analysis. For many shell structures these two analyses produce very different results. What causes this difference?

A imperfection sensitivity
B middle third rule
C in-extensional deformation
D edge disturbance

20. When are curvature trajectories in the principal directions?

A in hypars
B always
C $k_{xy} = 0$
D in an umbilic

21. Sometimes moments are neglected in the analysis of a shell structure. How are the applied equations called?

A Sanders-Koiter equations
B first approximation theory
C shell membrane equations
D moment-free theory

22. When do we add shape imperfections to shell finite element models?

A Before performing natural frequency analyses.
B Before performing linear buckling analyses.
C Before performing nonlinear analyses.
D After the linear analysis and before post processing the stresses.

23. Which shape imperfection is often added to shell finite element models?

A A cosine wave in two directions with an amplitude of 2*t*.
B Normally distributed node shifts with a standard deviation of 2*t*.
C The first buckling mode with an amplitude of 2*t*.
D A random field with a spectral density of 2*t*.

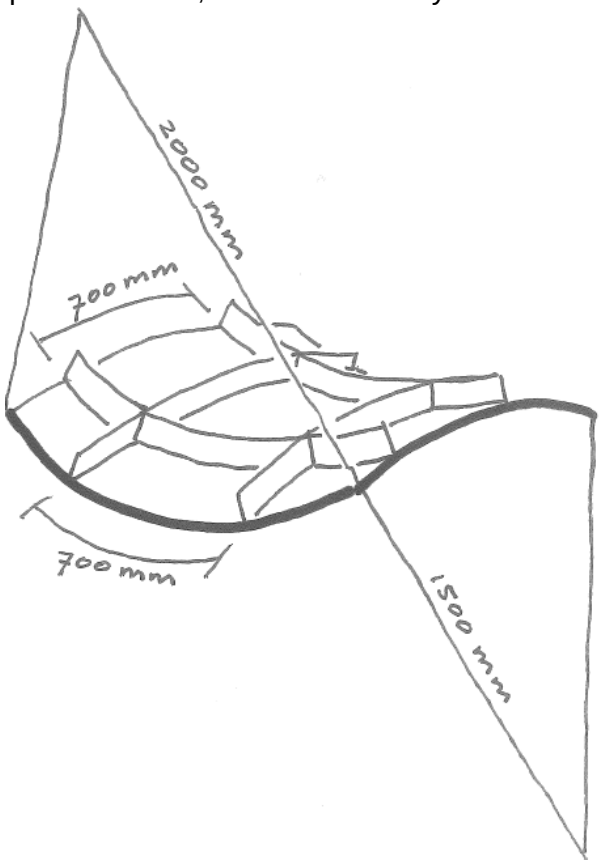
24. What is a zebra analysis?

A Computer supported method for inspecting the continuity of shells.
B Algorithm for projecting neon light tubes on car bodies.
C Curvature enhancing method inspired by the camouflage of African zebras.
D Plotting finite element displacements on a shell surface in bands of black and white.

25. Most airplanes and ships have shell structures with stiffeners. Most concrete shell roofs, silos and flower pots have shell structures without stiffeners. What causes the difference?

A In airplanes and ships buckling is the governing failure mechanism, in roofs, silos and flower pots yielding or crushing is governing.
B Stiffeners would lead to lighter roofs, silos and flower pots but stiffeners can be ugly, expensive or impractical for example for cleaning.
C Airplanes and ships are subjected to a few well-known load combinations for which they are optimised while roofs, silos and flower pots are subjected to a much wider range of loads.
D Stiffeners provide extra safety which is not required for roofs, silos and certainly not for flower pots.

26. Consider a curved steel plate with stiffeners. The figure shows a part of this plate, which continues in both directions with the same grid of stiffeners. What is the smallest natural frequency?



E = 2.1 10^5 N/mm^2
ν = 0.35
ρ = 7800 kg/m^3
t = 3 mm
k_{xy} = 0
n_{xx} = 0
n_{yy} = -10 kN/m
n_{xy} = 0

A 23 Hz
B 74 Hz
C 77 Hz
D 482 Hz

27. In question 26 the gravitational acceleration g = 9.8 m/s^2 is not provided.
Does this curved steel plate have the same natural frequencies on Mars?

A No, on Mars the ρ of steel is different
B No, the higher frequencies depend on g
C No, the inertia and the gravitation compensate each other
D Yes

28. Consider the theoretical situation of a perfect shell structure that is loaded such that it does not deform before buckling. This structure buckles at a particular load magnitude. How is this magnitude called?

A Ultimate load
B Critical load or bifurcation load
C Post buckling load
D Knockdown load

29. What is a knockdown factor?

A A partial factor for uncertainties in load, for example snow load magnitude
B A partial factor for uncertainties in strength, for example steel yield strength
C A reduction factor for loads on structures with a design live smaller than 50 years
D A model factor for uncertainties in the exact geometry

30. In science, when is something true?

A When almost all well-educated people think it is true.
B When an important person with demonstrated knowledge on the subject says it is true.
C When it can be logically derived from laws that are confirmed by experiments.
D When it is falsifiable, testable and reproducible.

Answers

1. A 100%
2. C 50% …… (50% chose A, which is incorrect because the exact optimum shell shape has not been found as yet.)
3. D 50% ……

$k1p := k1 + dk : k2p := k2 + dk :$

$km := \frac{1}{2} \cdot (k1 + k2) : kmp := \frac{1}{2} \cdot (k1p + k2p) :$

$kG := k1 \cdot k2 : kGp := k1p \cdot k2p :$

$simplify(kGp - kG - km);$

$$dk^2 + \frac{1}{2}\,(2\,k1 + 2\,k2)\,dk - \frac{1}{2}\,k1 - \frac{1}{2}\,k2$$

$simplify(kGp - kg - 2 \cdot dk^2);$

$$-dk^2 + (k1 + k2)\,dk + k1\,k2 - kg$$

$simplify(kGp - kG + 2 \cdot km \cdot dk);$

$$dk\,(dk + 2\,k1 + 2\,k2)$$

$simplify(kGp - kG - kmp^2 + km^2);$

$$0$$

4. B 38% …….. (A 25%, C 6%, D 31%)
5. B 69% …….. $z \approx \frac{1}{2}k_{xx}x^2 + k_{xy}xy + \frac{1}{2}k_{yy}y^2$ …….. page 21

$z' \approx c_1 + c_2x + c_3y + \frac{1}{2}k'_{xx}x^2 + k'_{xy}xy + \frac{1}{2}k'_{yy}y^2$

$u_z \approx z' - z = c_1 + c_2x + c_3y + \frac{1}{2}(k'_{xx} - k_{xx})x^2 + (k'_{xy} - k_{xy})xy + \frac{1}{2}(k'_{yy} - k_{yy})y^2$

$\kappa_{xx} \approx -\frac{\partial^2 u_z}{\partial x^2} = k_{xx} - k'_{xx}$ …… Sanders-Koiter equations 19, 20, 21

$\kappa_{yy} \approx -\frac{\partial^2 u_z}{\partial y^2} = k_{yy} - k'_{yy}$

$\rho_{xy} \approx -2\frac{\partial^2 u_z}{\partial x \partial y} = 2(k_{xy} - k'_{xy})$

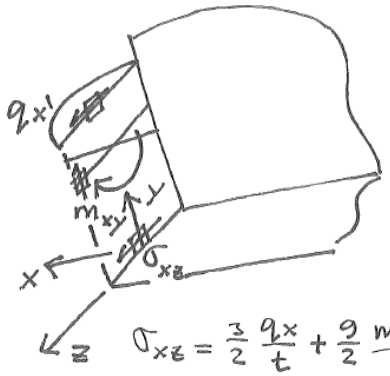
6. C 100%
7. A 100%
8. A 63% …….

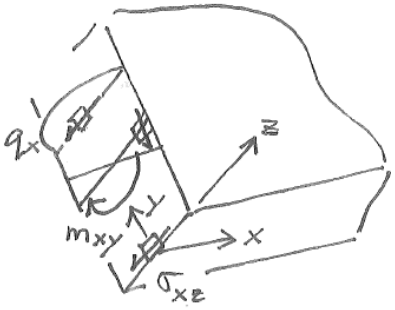
$B' + D' = RBR^{\mathsf{T}} + RDR^{\mathsf{T}} = R(B + D)R^{\mathsf{T}}$

$B'D' = RBR^{\mathsf{T}}RDR^{\mathsf{T}} = RBR^{-1}RDR^{\mathsf{T}} = RBIDR^{\mathsf{T}} = RBDR^{\mathsf{T}}$

9. D 100%
10. D 100%
11. D 100%
12. A 44% ……. (B 43%, C 0%, D 13%)
13. C 63% ……. (A 12%, B 6%, D 19%)
14. C 31% ……. (A 19%, B 44%, D 6%)
15. B 31% ……. (19% chose A. A was the correct answer 100 years ago, however the meaning has expanded. 44% chose C. C is incorrect because a monocoque is a structure and not a construction method. Moreover, we do not define something by what it is not.
Blackadder: “Your definition of dog is?” Baldrick: “Not a cat.”)
16. D 75% ……. $\sigma_{xz} = \frac{3}{2}\frac{q_x}{t} - \frac{9}{2}\frac{m_{xy}}{t^2} = -3.98\,\text{N/mm}^2$

17. C 75 % ……





$$\sigma_{xz} = \frac{3}{2}\frac{q_x}{t} + \frac{9}{2}\frac{m_{xy}}{t^2}$$

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18. A 81%
19. A 94%
20. B 38% ……

$$\sigma_{xz} = \frac{3}{2}\frac{q_x}{t} + \frac{9}{2}\frac{m_{xy}}{t^2}$$

$$\sigma_{xz} = \frac{3}{2}\frac{q_x}{t} + \frac{9}{2}\frac{m_{xy}}{t^2}$$ es?)

21. C 100%
22. C 94%
23. C 94%
24. A 100%
25. B 13% …….. (75% chose A. If A were true it could be an explanation of the difference, however, many roofs, silos and flower pots fail in buckling too. The course has many examples of this.)
26. B 19% …….. The mode shape is a chess board pattern. The nodes are at the stiffeners.

$$E := 2.1e5\cdot\frac{N}{mm^2} : \text{nu} := 0.35 : \text{rho} := 7800\cdot\frac{kg}{m^3} : t := 3\cdot mm : nyy := -10\cdot\frac{kN}{m} : l := 700\cdot mm :$$

$$kN := 1000\cdot N : N := kg\cdot\frac{m}{s^2} : m := 1000\cdot mm : assume(s > 0) :$$

$$km := \frac{1}{2}\cdot\left(\frac{1}{1500\cdot mm} + \frac{-1}{2000\cdot mm}\right) : kG := \frac{1}{1500\cdot mm}\cdot\frac{-1}{2000\cdot mm} :$$

$$t1 := \frac{\pi^2}{12\cdot\left(1-\nu^2\right)}\cdot\frac{E\cdot t^2}{\text{rho}\cdot l^4} : t2 := \left(km^2 + \frac{kG^2\cdot t^2}{1100}\right)\cdot\frac{E}{4\cdot\pi^2\cdot\text{rho}} : t3 := \frac{nyy}{4\cdot\text{rho}\cdot t\cdot l^2} :$$

$$fn := \text{sqrt}(t1 + t2 + t3);$$

$$fn := \frac{74.14526315}{s\sim}$$

27. D 56% ……. (A 25%, B 13%, C 6%)
28. B 100%
29. D 100%
30. C 44% ……. (56% chose D. D is the answer to another question: When is something scientific?)

The grade is $s\frac{4}{9} - \frac{10}{3}$ rounded to the closest halve, where *s* is the number of correct answers.

The grade formula above is derived as follows. A student with grade *g* knows 3*g* answers. This student guesses 30-3*g* answers. The number of correct answers is *s* = 3*g* + (30-3*g*)/4, from which *g* can be solved.

A lucky student will guess more answers correctly. For example, *s* = 3*g'* + (30-3*g'*)/2 . Suppose that this student is content with the grade *g* = 5.75 which is rounded to 6. The number of correct answers needs to be *s* = 3x5.75 + (30-3x5.75)/4 = 20.4. The grade *g*' this student needs to study for is: 21 = 3*g'* + (30-3*g'*)/2 from which can be solved *g*' = 4. This lucky student guesses 9 answers right or more out of 30-3x4 = 18 answers. The probability that this happens is [3^9 x 18!/(9!9!) + 3^8 x 18!/(10!8!) + … + 3x18 + 1] / 4^18 = 0.02 .