Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Write your name and study number at the top right-hand of your work.

Exam CIE4143 Shell Analysis
Monday 2 July 2018, 13:30 – 16:30 hours

Problem 1

Explain shortly the meaning of the following eight words in relation to shell structures. (1.6 point)

Edge disturbance
Qian
Grid shell
NURBS
Tensor
Fully stressed
Middle surface
Buckling load factor

Problem 2

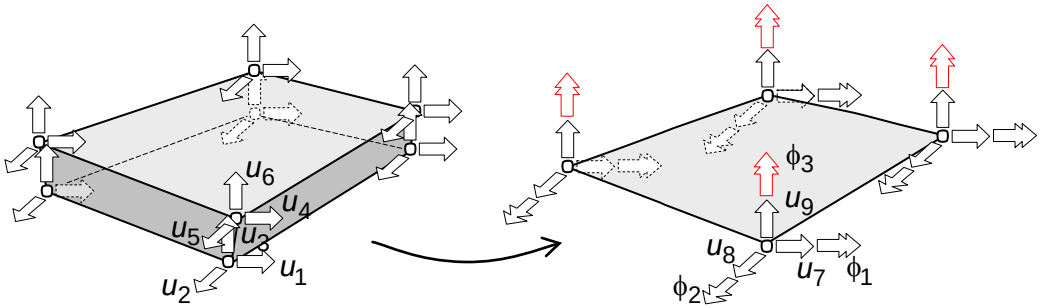
a In predicting the experimental strength of shell objectives we found that some parameters are 1) difficult to determine and 2) have a large influence on the strength. Give an example of such a parameter. Explain your answer. (0.4 point)

b There is disagreement amongst scientists on what are the correct equations describing shell structures, who formulated them first and in what year. Please give a short answer to these questions. Make a reference to literature. Do not make a reference to the handouts of the shell course. (0.4 point)

c Shell structures tend to deform in a wave that does not move (undulation). Give the formula of the halve wave length in a sphere. (Distance from zero displacement to zero displacement.) How is this length called? (0.4 point)

d A vibrating load can make a shell resonate. To avoid this we can design the shell to have natural frequencies that are well above and well below the load frequency. Which design parameters have an influence on the shell natural frequencies? (0.4 point)

Problem 3
An 8 node solid finite element is changed into a 4 node shell finite element (see figure below). The following assumption is used: Straight pin remains straight during deformation. The shell thickness is *t*. (0.3 point)



a What is the number of degrees of freedom of the solid element?
What is the number of degrees of freedom of the shell element? (0.3 point)

b Give the equations that express $u_7, u_8, u_9, \phi_1, \phi_2$ in $u_1, u_2, u_3, u_4, u_5, u_6$. (0.3 point)

c What causes ϕ_3 to be an exception? (0.3 point)

d The *z* axis is perpendicular to the shell surface for both the solid and the shell element.
Is this essential? Explain your answer. (0.3 point)

e Can this shell element be used to model a shell part that is twisted? Explain your answer.
(0.3 point)

Problem 4

a Few experiments have been done to test the shell membrane equations or the Sanders-Koiter equations. Why is this? Choose A, B, C or D. (0.2 point)

A In science it is not common to do experiments for testing a hypothesis.
B The equations have been derived from Hooks law in 3 dimensions.
C Koiter was brilliant and the work of brilliant scientists does not need to be tested.
D It is not necessary that the equations describe reality. They only need to provide a consistent legal framework adopted by the current standards.

b A cylinder shell can be interpreted as a beam supported by distributed springs.
The springs represent … Choose A, B, C or D. (0.2 point)

A … the radial component of the loading
B … imaginary rings or hoops
C … the diaphragms that close the cylinder
D … a strip of material in the axial direction of the cylinder.

c Which are the important shell phenomena? Choose from A to H. (0.2 point)

A thickness
B edge disturbance
C spurious vibrations
D middle third rule
E in-extensional deformation
F Lamé parameters
G Intrinsic property
H imperfection sensitivity

d A beautifully curved thin shell can behave as thick flat plate. How is this called? Choose A, B, C or D. (0.2 point)

A in-extensional deformation
B middle third rule
C imperfection sensitivity
D Intrinsic property

e In the shell theory the displacement u_z … Choose A, B, C or D. (0.2 point)

A … is always in the *z* direction of the global coordinate system.
B … is directed away from the origin of the global coordinate system.
C … follows the direction of one of the curvilinear coordinates.
D … is in a direction perpendicular to the shell middle surface.

f An axially loaded cylinder has not only membrane forces in the axial direction but also large membrane forces in the hoop direction. What causes this? Choose A, B, C or D. (0.2 point)

A edge disturbances
B Poisson's ratio
C invisible shape imperfections
D local element coordinate system

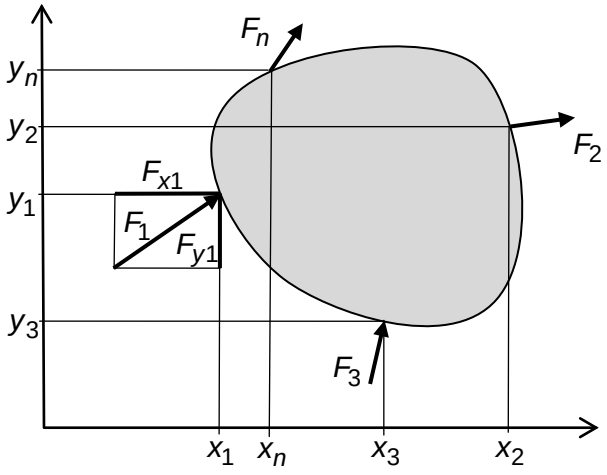
g We do not include shape imperfections in linear buckling analyses. Why? Choose A, B, C or D. (0.2 point)

A The software does not allow this as yet.
B The shape imperfections in well-made shells are very small and can be neglected.
C The imperfections increase when the loading increases.
This is not described by linear buckling analyses.
D If shape imperfections were included, the knockdown factor would not be valid.

Problem 5 (1 point)
What is the level of continuity of the shape of a drafting spline? Explain your answer.

Problem 6 (1.3 point)

Consider a volume of material.



Forces *F* act on the volume. The number of forces is *n*. The forces are in equilibrium. Each force *F* has an *x* component F_x and a *y* component F_y. A quantity *g* is defined.

$$g_{xx} = \sum_{i=1}^{n} x_i F_{xi} \qquad g_{yx} = \sum_{i=1}^{n} y_i F_{xi}$$

$$g_{xy} = \sum_{i=1}^{n} x_i F_{yi} \qquad g_{yy} = \sum_{i=1}^{n} y_i F_{yi}$$

Proof that *g* is a tensor.

Problem 7

A shell has a shape imperfection with magnitude *d*, length *l* and width *l* (see figure).
We assume that *d*, *s* << *l* .

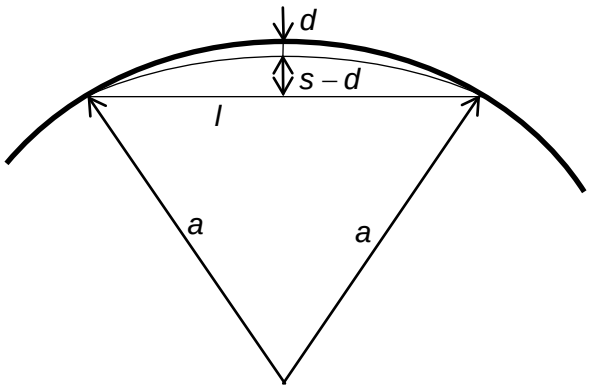
a Derive the following relations between the perfect and imperfect (') curvatures. (0.4 point)

$$k_1' = k_1 - 8\frac{d}{l^2}$$

$$k_2' = k_2 - 8\frac{d}{l^2}$$

$$k_m' = k_m - 8\frac{d}{l^2}$$

$$k_G' = k_G - 16\frac{d}{l^2}\frac{k_m + k_m'}{2}$$



Suppose that the buckling shape is the same as the imperfection shape. Consequently, when some load is applied the imperfection amplitude *d* increases.

b For which shell shape does the Gaussian curvature change most?
For which shell shape does the Gaussian curvature change least? (0.4 point)

c Why is the change in the Gaussian curvature important? (0.4 point)

d Calculate the extra membrane forces in the middle of the imperfection shape due to the change in the Gaussian curvature. (0.4 point)

Answers to problem 1

Edge disturbance …. large stresses at edges – or at internal edges – of a shell structure
Qian ……. scientist who discovered imperfection sensitivity
Grid shell ……. curved laths connected to form a shell
NURBS …….. mathematical way of defining free form surfaces
Tensor ……. physical quantity that transforms by particular equations
Fully stressed ..... loaded to the failure stress everywhere
Middle surface …….. imaginary surface half way between the inner and outer surfaces
Buckling load factor .. factor with which the load is multiplied to obtain buckling

Answers to problem 2

a - Thickness is difficult to measure because you need to demolish the object for it. Also it varies from place to place.
The buckling load is proportional to the thickness squared.
- Imperfections are difficult to measure because they are invisibly small.
The knockdown factor is $1-C\sqrt{d}$, where d is the imperfection amplitude and C is a constant.
- The strength of glass is difficult to determine because it is depends on invisible scratches in the surface which are random by nature.
The crushing load is just linear in the material strength.

b Sanders-Koiter equations, formulated by Sanders and Koiter independently. Their papers were published in a conference in Delft in 1959.
W.T. Koiter, A consistent first approximation in the general theory of thin elastic shells, Part 1, Foundations and linear theory, report, Delft University of Technology, 1959, 47 pp.

c $l_i \approx 2\sqrt{at}$ **,** influence length

d This statement is wrong: Shell natural frequencies have very little space in between them. It is not possible to design a shell such that the load frequency is in between the shell natural frequencies.
The following parameters influence the natural frequencies:
- E, ν, ρ
- length, width, thickness
- supports, edge beams
- curvature
- membrane forces

Answers to problem 3

a 24, 24

b $u_7 = \frac{u_1+u_4}{2}$ $u_8 = \frac{u_2+u_5}{2}$ $u_9 = \frac{u_3+u_6}{2}$ $\phi_1 = \frac{u_5-u_2}{t}$ $\phi_2 = \frac{u_1-u_4}{t}$

c It does not correspond to any dof of the solid. It exists to fit the shell element in a general purpose finite element program. We give it some stiffness otherwise the stiffness matrix is singular but it does not contribute to the accuracy of the shell element.

d Yes, the equations of answer **3b** would not be correct for a rotated coordinate system.

e Yes, the solid element can be twisted so the shell element can be twisted.

Answers to problem 4

a B
b B
c B,D,E,H
d A
e D

f C

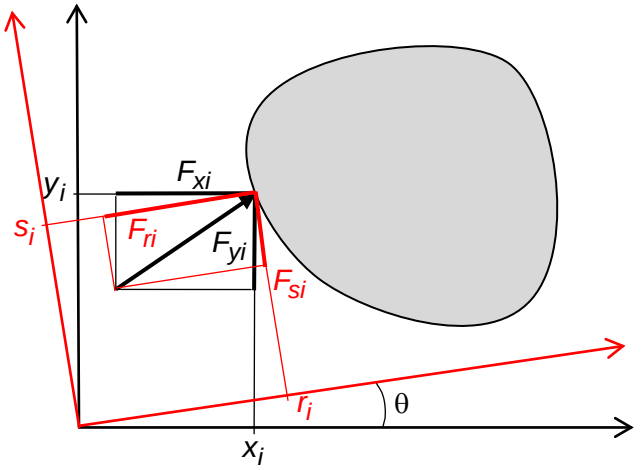
g C

Answer to problem 5

C_2 continuity.

The hooks at a drafting spline are point loads. The shear force jumps at the location of the point loads. The moment distribution is continuous. Moment is proportional to the curvature which is the second derivative of the displacement. The second derivative is continuous so it has C_2 continuity.

Answer to problem 6



$$r_i = x_i \cos\theta + y_i \sin\theta \qquad F_{ri} = F_{xi} \cos\theta + F_{yi} \sin\theta$$

$$g_{rr} = \sum_{i=1}^{n} r_i F_{ri}$$

$$g_{rr} = \sum_{i=1}^{n} (x_i \cos\theta + y_i \sin\theta)(F_{xi} \cos\theta + F_{yi} \sin\theta)$$

$$= \sum_{i=1}^{n} x_i F_{xi} \cos^2\theta + y_i F_{xi} \sin\theta\cos\theta + x_i F_{yi} \cos\theta\sin\theta + y_i F_{yi} \sin^2\theta$$

$$= \cos^2\theta \sum_{i=1}^{n} x_i F_{xi} + \sin\theta\cos\theta \sum_{i=1}^{n} y_i F_{xi} + \sin\theta\cos\theta \sum_{i=1}^{n} x_i F_{yi} + \sin^2\theta \sum_{i=1}^{n} y_i F_{yi}$$

$$= g_{xx} \cos^2\theta + g_{yx} \sin\theta\cos\theta + g_{xy} \sin\theta\cos\theta + g_{yy} \sin^2\theta$$

$$= g_{xx} \cos^2\theta + (g_{xy} + g_{yx}) \sin\theta\cos\theta + g_{yy} \sin^2\theta$$

$$M = \sum_{i=1}^{n} x_i F_{yi} - \sum_{i=1}^{n} y_i F_{xi} = g_{xy} - g_{yx} = 0 \quad \Rightarrow \quad g_{xy} = g_{yx}$$

$$g_{rr} = g_{xx} \cos^2\theta + 2g_{xy} \sin\theta\cos\theta + g_{yy} \sin^2\theta$$

The latter is the rotation equation of the first component of a tensor (see reader Appendix 3) For g_{ss} and g_{rs} a similar result can be obtained. Therefore, g is a tensor.

(In fact, the quotient of g and the volume is the average stress in the volume. The above equations are useful if we have analysed a grid shell and want to transform the results to those of a normal shell. This transformation is called homogenisation.)

Answers to problem 7

a

$$k_1 = \frac{1}{a} = \frac{1}{\frac{1}{2}s_1 + \frac{1}{8}\frac{l^2}{s_1}} \approx \frac{1}{\frac{1}{8}\frac{l^2}{s_1}} = 8\frac{s_1}{l^2} \qquad k_1' \approx 8\frac{s_1 - d}{l^2} = 8\frac{s_1}{l^2} - 8\frac{d}{l^2} = k_1 - 8\frac{d}{l^2}$$

$k_2' = \ldots$ *ditto*

$$k_m = \frac{k_1 + k_2}{2} \qquad k_m' = \frac{k_1' + k_2'}{2} = \frac{k_1 - 8\frac{d}{l^2} + k_2 - 8\frac{d}{l^2}}{2} = k_m - 8\frac{d}{l^2}$$

$$k_G = k_1 k_2 \qquad k_G' = k_1' k_2' = (k_1 - 8\frac{d}{l^2})(k_2 - 8\frac{d}{l^2}) = k_1 k_2 - 8\frac{d}{l^2}k_2 - k_1 8\frac{d}{l^2} + 8\frac{d}{l^2}8\frac{d}{l^2} =$$

$$= k_G - 8\frac{d}{l^2}(k_2 + k_1 - 8\frac{d}{l^2}) = k_G - 8\frac{d}{l^2}(k_m + k_m - 8\frac{d}{l^2})$$

$$= k_G - 8\frac{d}{l^2}(k_m + k_m') = k_G - 16\frac{d}{l^2}\frac{k_m + k_m'}{2}$$

b When the mean curvature is largest, for example a sphere.
When the mean curvature is zero, for example a cooling tower.

c It produces membrane forces.

d membrane forces

$$k_G' - k_G = -16\frac{\Delta d}{l^2}\frac{k_m + k_m'}{2}$$

$n_1 = n_2 = \frac{1}{64}Et\,l^2(k_G' - k_G)$ (plate twisting)

$$n_1 = n_2 = -\frac{1}{4}Et\,\Delta d\,\frac{k_m + k_m'}{2}$$