Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CIE4143 Shell Analysis
Monday 1 July 2019, 13:30 – 16:30 hours

Choose A, B, C or D.

1. What is a kissing circle?

   A A circle that touches a curve and has the same curvature in the point where it touches.
   B A circle that is drawn on the middle surface of a shell and represents its curvature.
   C A graphical representation of the curvature tensor.
   D A mathematical concept related to curved surfaces.

2. What is an example of a star umbilic?

   A Membrane forces in a wall around a square opening.
   B Tensor field of a lemon.
   C Curvature of a monkey saddle.
   D Edge moments in a sphere.

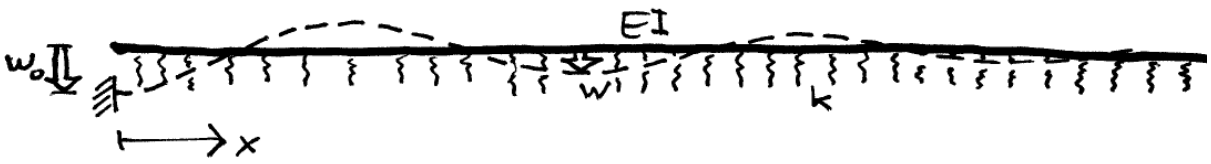
3. On a fixed shell edge $\varphi_z = 0$. How do we know?

   A This follows from Sanders-Koiter equation 18.
   B This is a boundary condition on the fixed edge.
   C It follows from finite element analyses.
   D None of the above because $\varphi_z \neq 0$.

4. Settlement of shell supports should be followed by in-extensional deformation. What happens if it is not?

   A large stresses
   B inactive membrane forces
   C no change in Gaussian curvature
   D large displacements

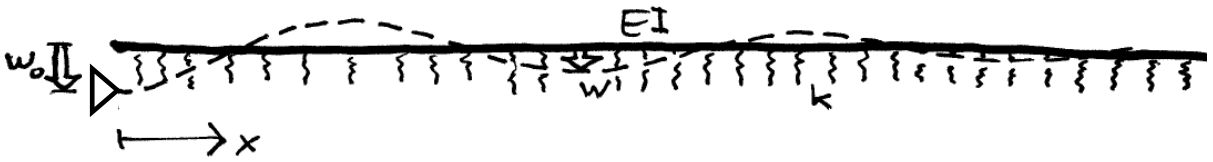
5. Consider a long beam supported by uniformly distributed springs. The right-hand edge is far away. The left edge is fixed and translated by w_o (see figure).



   What is the largest moment in the beam?

A 0
B $w_o\sqrt{k\,EI}$
C $\exp\frac{-\pi x}{l_i}$
D $\sqrt{2}\pi\sqrt[4]{\frac{EI}{k}}$

6. Consider again a long beam supported by uniformly distributed springs. The right-hand edge is far away again. The left edge is <u>pinned</u> and translated by w_o (see figure).



What is the ratio of the largest moment in this beam to the largest moment in the previous question?

A -0.32
B -0.68
C -1.00
D -1.41

7 A cylinder can sometimes be interpreted as a beam supported by uniformly distributed springs. Which cylinder part is the beam and which cylinder part are the springs?

A The beam represents the cylinder.
The springs represent the edge disturbance.
B The beam represents the cylinder.
The springs represent the boundary conditions.
C The beam represents a strip of material in the axial direction.
The springs represent imaginary hoops of the cylinder.
D The beam represents a strip of material in the hoop direction.
The springs represent boundary conditions.

8. The shell reader gives two parameterisations of a Bohemian dome (p. 79). Neither of these parameterisations are in the principal curvature directions. How do we know this?

A Principal curvatures are perpendicular and kxy = 0.
B Principal curvatures are the largest and smallest of the curvatures in all directions.
C In a Bohemian dome each direction is a principal curvature direction.
D Principal curvatures are perpendicular to the edges of a shell.

9. What is the level of continuity of the shape of a drafting spline?

A C0
B C1
C C2
D C3

10. A catenoid is a minimal surface. So a soap film between two rings assumes a catenoid shape. However, a cylinder shape between the rings has an even smaller surface area. Explain this inconsistency.

    A There are several equilibrium shapes of the soap film possible.
    B The word “minimal surface” is not chosen well.
    C This is due to approximations. The real soap film shape is somewhere between the catenoid and the cylinder.
    D In the catenoid shape the air pressure is less than around it.

11. What causes an edge disturbance in a torus?

    A Discontinuity in the edge moment
    B Discontinuity in the slope of the middle surface
    C Change in sign of the Gaussian curvature
    D Discontinuity in the material stiffness

12. Membrane forces and moments on shell edges computed by the finite element method do not agree exactly with the shell boundary conditions. What causes this?

    A Element size; smaller elements would compute the correct values.
    B Approximations made in deriving the shell elements.
    C Approximations made in deriving the Sanders-Koiter equations.
    D Approximations made in deriving the boundary conditions.

13. For plates the recommended element size is 2*t*. Suppose that a shell needs elements this size. What is the *a* / *t* ratio of this shell?

    A 5.67
    B 25
    C 100
    D >500

14. In a point of a shell n_{xx} = -500 kN/m, n_{yy} = 0, n_{xy} = 0, k_{xx} = 0.002 /m, k_{yy} = 0.040 /m, k_{xy} = 0.010 /m, E = 2.1 10^5 N/mm^2, t = 0.007 m. What is the load factor for the critical load?

    A 0.02
    B 0.08
    C 0.49
    D 2.02

15. A shell has a surface area of 144 m^2 and a thickness of 20 mm. The radius of curvature is 15 m. The Gaussian curvature is positive. The finite element model uses 4 node elements which are all about the same size. How many dofs are in this model?

    A 18 000
    B 180 000
    C 1 800 000
    D 18 000 000

16. The bending moment in a shell part is m_{xx} = 5 kNm/m and the membrane force is n_{xx} = 300 kN/m. The shell has a thickness t = 60 mm. What is the largest normal stress?

    A 0.45 N/mm^2
    B 1.36 N/mm^2
    C 2.37 N/mm^2
    D 13.33 N/mm^2

17. How many boundary conditions does a thin shell edge have?

    A 2, 3 or 4
    B 4
    C 4 or 5
    D 5

18. The accuracy of a finite element model depends on model accuracy and arithmetic accuracy. Suppose that we have a model and we increase the number of dofs. What will happen?

    A The model accuracy increases and the arithmetic accuracy increases.
    B The model accuracy increases and the arithmetic accuracy decreases.
    C The model accuracy decreases and the arithmetic accuracy increases.
    D The model accuracy decreases and the arithmetic accuracy decreases.

19. A shell structure can be too flexible. A possible explanation is …

    A … imperfection sensitivity.
    B … the middle third rule.
    C … in-extensional deformation.
    D … edge disturbance.

20. Buckling modes computed in a linear buckling analysis can move inwards or outwards. What is the difference?

    A Inward buckles have smaller buckling load factors than outward buckles.
    B If the load is outward the buckle is outwards, if the load is inward the buckle is inwards.
    C It depends on the direction of the applied invisible shape imperfections.
    D There is no physical difference. This is caused by numerical round off errors.

21. Suppose that the smallest buckling mode of a shell structure is in-extensional. Should we apply a knockdown factor?

    A Yes, we always need to apply a knockdown factor.
    B No, the smallest buckling mode is not sensitive to imperfections.
    C No, in-extensional deformation is not sensitive to imperfections.
    D No, in-extensional deformation has no imperfections.

22. What is a fully stressed dome?

    A Everywhere in this dome the material is loaded to its capacity.
    B A dome that carries the load with the minimum amount of material.
    C The optimally shaped dome.
    D The largest dome possible.

23. Which shell buckling shapes are observed in axial compression tests of cylinders?

    A Ring patterns and chess board patterns
    B Ring patterns and elephant feet
    C Elephant feet and Yoshimura patterns
    D Yoshimura patterns and ring patterns

24. Thin shell structures are sensitive to invisible shape imperfections. Who discovered this?

    A Koiter
    B Von Karman, Dunn and Qian
    C Timoshenko
    D Robertson

25. A shell roof is loaded by earthquakes. Which rule of thumb applies for design?

    A The shell natural frequencies need to be within 1 and 10 Hz.
    B All shell natural frequencies need to be smaller than 1 Hz.
    C All shell natural frequencies need to be larger than 1 Hz.
    D All shell natural frequencies need to be larger than 10 Hz.

26. Consider a hemisphere and a sphere of similar material, size and thickness. They have no supports. The sound of the sphere is much higher than the sound of the hemisphere. What causes this?

    A The sphere deforms extensionally; the hemisphere deforms in-extensionally.
    B The sphere is open; the hemisphere is closed.
    C The Gaussian curvature of the sphere is fixed; the Gaussian curvature of the hemisphere changes.
    D The sphere has twice as much material as the hemisphere.

27. What is an influence length?

    A Half the wave length of a beam on distributed springs
    B The distance from an edge disturbance at which its effects are small
    C The mean length of a shape imperfection that resembles a buckling shape
    D The width of shell that participates in carrying a point load

28. What is the buckling length of a paper cylinder? The sheet of paper has a width of 210 mm (circumference) and a length of 297 mm. A pile of 50 sheets has a thickness of 6 mm.

    A 3.4 mm
    B 4.8 mm
    C 42 mm
    D 149 mm

29. k_{xx} = 1/(5 m), k_{yy} = -1/(8 m), k_{xy} = -1/(3 m). What the Gaussian curvature?

    A -1/(40 m^2)
    B -0.136 / m^2
    C 0.075 / m^2
    D 0.086 / m^2

30. We can accurately determine deformation, stresses and buckling load of any shell structure by finite element analyses. Why do we also do hand calculations of shell structures?

    A Every finite element result should be checked by a hand calculation.
    B Hand calculations are used in exams for testing student knowledge.
    C Hand calculations give quick estimates for designing and checking.
    D Hand calculations are necessary when computers are not available.

Answers

1. A 98%
2. C 90%
3. D 21% … This follows from substituting the boundary conditions in S-K equation 18.
4. A 98%
5. B 100%
6. A 46% …

> $li := \mathrm{sqrt}(2)\cdot \mathrm{Pi}\cdot\left(\frac{EI}{k}\right)^{\left(\frac{1}{4}\right)}$:

> $w := wo\cdot\cos\left(\frac{\mathrm{Pi}\cdot x}{li}\right)\cdot\exp\left(-\frac{\mathrm{Pi}\cdot x}{li}\right)$:

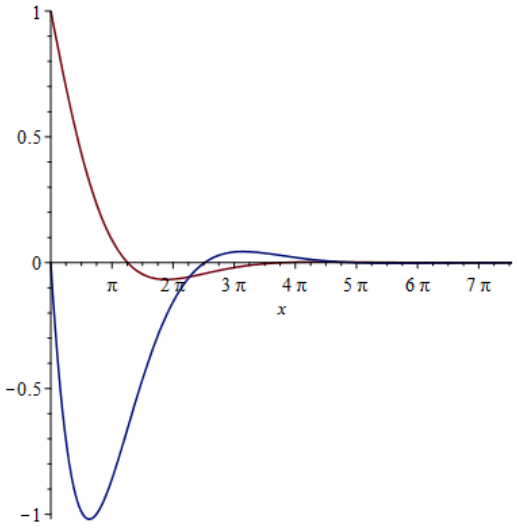
> $simplify(EI\cdot diff(w, x, x, x, x) + k\cdot w)$;

$$0$$

> $M := -EI\cdot diff(w, x, x)$:

> $V := diff(M, x)$:

> $EI := 10 : k := 1 : wo := 1 : plot(\{w, M\}, x = 0..3\cdot li)$; $EI := 'EI' : k := 'k' : wo := 'wo'$:



> $x := solve(V = 0, x)$;

$$x := \frac{1}{4}\sqrt{2}\,\pi\left(\frac{EI}{k}\right)^{1/4}$$

> $assume(k > 0) : assume(EI > 0) : evalf\left(simplify\left(\frac{M}{wo\cdot\mathrm{sqrt}(k\cdot EI)}\right)\right)$;

$$-0.3223969418$$

7. C 94%
8. A 77% … The first parameterisation doesn't have the first property (see figure) and the second parameterisation doesn't have the second property (see equations).
9. C 94%
10. B 71% … (A 6%, C 10%, D 13%)
11. C 67%
12. B 29% … (38% chose A, which is incorrect because it does not converge to the correct values. Fortunately, the difference between the computed edge values and the correct edge values is small.)
13. B 77% …

> $eq1 := h = 2\cdot t$:

> $eq2 := h = \frac{li}{6}$:

> $eq3 := li = 2.4\cdot\mathrm{sqrt}(a\cdot t)$:

> $sol := solve(\{eq1, eq2, eq3\}, \{a, t, li\})$; $assign(sol)$;

$$sol := \{a = 12.50000000\ h,\ li = 6.\ h,\ t = 0.5000000000\ h\}$$

> $\frac{a}{t}$;

$$25.00000000$$

14. C 40% …

$> nxx := -500 \cdot \frac{kN}{m} : nyy := 0 : nxy := 0 : kxx := -0.002 \cdot \frac{1}{m} : kyy := 0.040 \cdot \frac{1}{m} : kxy := 0.010 \frac{1}{m} :$

$> E := 2.1e5 \cdot \frac{N}{mm^2} : t := 0.007 \cdot m :$

$> m := 1000 \cdot mm : kN := 1000 \cdot N :$

$> a := \frac{1}{kyy} : ncr := -0.6 \cdot \frac{E \cdot t^2}{a} :$

$> \text{lambda} := \frac{ncr}{nxx};$

$$\lambda := 0.4939200000$$

15. A 56% …

$> A := 144 \cdot m^2 : t := 20. \cdot mm : a := 15 \cdot m :$

$> m := 1000 \cdot mm :$

$> li := 2.4 \cdot \text{sqrt}(a \cdot t) : h := \frac{li}{6} :$

$> nr_elements := \frac{A}{h^2} :$

$> nr_nodes := nr_elements :$

$> nr_dofs := 6 \cdot nr_nodes;$

$$nr_dofs := 18000$$

16. D 92% …

$> mxx := 5 \cdot \frac{kN \cdot m}{m} : nxx := 300 \cdot \frac{kN}{m} : t := 60. \cdot mm :$

$> kN := 1000 \cdot N : m := 1000 \cdot mm :$

$> s := \frac{mxx}{\frac{1}{6} t^2} + \frac{nxx}{t};$

17. B 83%
18. B 92%

$$s := \frac{13.33333334\, N}{mm^2}$$

19. C 98%
20. D 50% ... See handout conoid.
21. C 79%
22. A 100%
23. C 38% … (58% chose A, which is wrong because we don't see these modes in tests.)
24. B 71% … (27% chose D. Indeed Robertson discovered that something was wrong but he had no idea what it was.)
25. D 92%
26. A 88%
27. B 85%
28. A 69% …

$> t := \frac{6 \cdot mm}{50} :$

$> w := 210 \cdot mm :$

$> a := solve(w = 2 \cdot 3.1415 \cdot a, a) :$

$> lcr := 1.7 \cdot \text{sqrt}(a \cdot t);$

$$lcr := 3.404596606\, mm$$

29. B 100% …

$> kxx := \frac{1}{5 \cdot m} : kyy := -\frac{1}{8. \cdot m} : kxy := -\frac{1}{3. \cdot m} : kG := kxx \cdot kyy - kxy^2;$

$$kG := -\frac{0.1361111111}{m^2}$$

30. C 85% ... (A 13%, B 0%, D 2%)
A is wrong because it is not possible to check every finite element result by hand. The reason for doing finite element analysis is that we cannot do it by hand.

The grade is $\frac{4}{9}s - \frac{10}{3}$ rounded to the closest halve, where *s* is the number of correct answers.

The grade formula above is derived as follows. A student with grade *g* knows 3*g* answers. This student guesses 30-3*g* answers. The expected number of correct answers is *s* = 3*g* + (30-3*g*)/4, from which *g* can be solved.