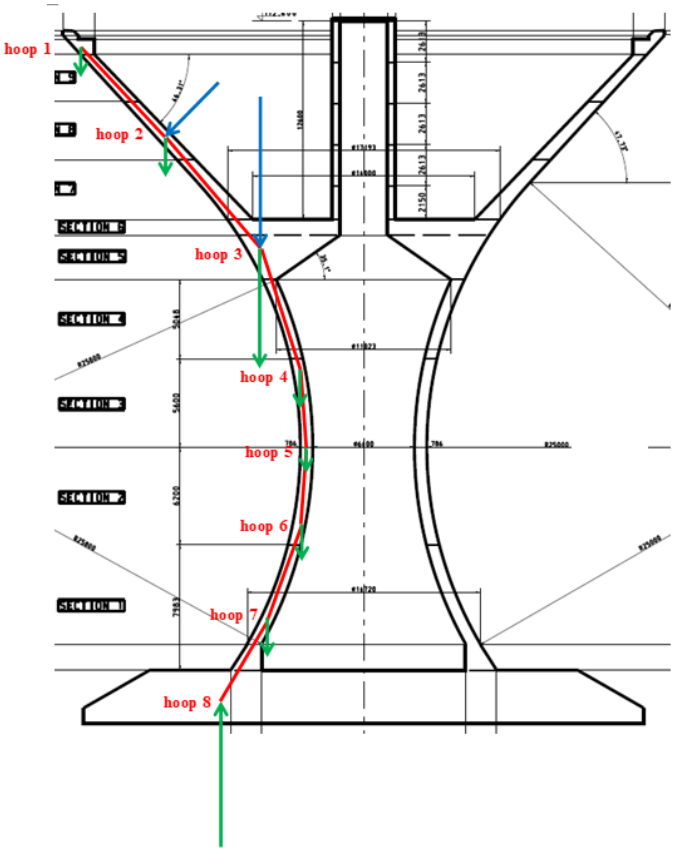
Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CIE4143 Shell Analysis
Tuesday 29 June 2020, 13:30 – 16:30 hours

Choose A, B, C or D.



1. Consider Sillogue water tower. The figure to the right shows the pressure line (red), gravity acting on the concrete (green) and water pressure (blue). The lengths of the arrows are proportional to the magnitudes of the forces. Hoop forces (not shown) push and pull the pressure line into the middle third. The hoops are numbered 1 to 8. How many of the hoops are tensioned?

   A 0
   B 2
   C 4
   D 6

2. An egg has a smallest diameter of 48 mm and a shell thickness of 0.35 mm. How is it classified?

   A membrane
   B thin shell
   C thick shell
   D very thick shell

3. A shell has a mean curvature $k_m = \frac{1}{11\,\text{m}}$ and a Gaussian curvature $k_G = \frac{1}{100\,\text{m}^2}$.
   What are the principal curvatures of this shell?

   A $k_1 = \frac{1}{11\,\text{m}}$, $k_2 = \frac{1}{11\,\text{m}}$
   B $k_1 = \frac{1}{10\,\text{m}}$, $k_2 = \frac{1}{10\,\text{m}}$
   C This shell does not exist.
   D Other (not A, B or C)

4. Curvature is a tensor. How do we know?

   A We can write it as a small square matrix.
   B If we choose a different coordinate system, it changes according to specific equations.
   C Most physical quantities are tensors.
   D The questions says it. The teacher wrote the question. Teachers know these things.

5. In shell geometry; what are Lamé parameters?

   A Transition factors for length between *x*, *y*, *z* space and *u*, *v* space.
   B Parameters for calculating in plane curvature in two directions.
   C Distances between the curvature trajectories.
   D Strains of the middle surface.

6. Consider a torus (handout 3, page 30). What are the in plane curvatures k_x and k_y?

   A $k_x = 0$ $\quad k_y = \infty$
   B $k_x = \infty$ $\quad k_y = \infty$
   C $k_x = \frac{1}{b} + \frac{1}{a \sin \frac{v}{a}}$ $\quad k_y = 0$
   D $k_x = \frac{\cos \frac{v}{a}}{b + a \sin \frac{v}{a}}$ $\quad k_y = 0$

7. What are the shell membrane equations used for?

   A tent structures
   B moderately thick shells, thick shells and very thick shells
   C approximations and when moments are small
   D shells with simple shapes, like spheres, cones and cylinders

8. The Von Mises stress in the middle bottom of a simply supported tube is

$$\sigma_{VM\max} = \frac{pl^2}{4at}\sqrt{1 - 4\frac{a^2}{l^2} + 16\frac{a^4}{l^4}}\,.$$

   What is the stress in the top?

   A the same
   B the same but negative
   C smaller because the top can buckle locally
   D $\sigma_{VM\max} = -\frac{pl^2}{4at}\sqrt{1 + 10\frac{a^4}{l^2}}$

9. It is often said that in deep beams, shear deformation cannot be neglected.
We check this for a rather deep simply supported tube.
Suppose that the length $l = 10a$ and Poisson's ratio ν = 0.35
What is the contribution of shear deformation to the stress in the middle bottom?
What is the contribution of shear deformation to the deflection of the middle bottom?

A 2%, 17%
B 17%, 2%
C 12%, 9%
D 9%, 2%

10. The shallow shell differential equation is rather short. This is due to the notation with operators ∇^2 and Γ. Suppose we expand the equation. In other words, we write the equation without operators. In some terms something will be differentiated 2 times, in others perhaps 3 times. In some of the terms something will be differentiated the largest number of times. How many times?

A 3
B 4
C 6
D 8

11. To solve the Sanders-Koiter equations, boundary conditions are specified for shell edges.
How many boundary conditions are specified on one edge?

Boundary conditions are used to model types of support, such as fixed and pinned.
How many types of support are theoretically possible?

A 4, 4
B 4, 16
C 5, 5
D 5, 32

12. We know that $n_{xy} \neq n_{yx}$ because of Sanders-Koiter equation 6. Which is a sound argument?

A If n_{xy} were equal to n_{yx}, Koiter would have simplified equation 6.
Since he did not, it is not.

B We can build any shell, so k_{xx}, k_{yy} and k_{xy} can certainly have any small values.
We can impose any deformation, so m_{xx}, m_{yy} and m_{xy} can certainly have any small values.
To always fulfil equation 6, $n_{xy} - n_{yx}$ must often have a nonzero value.

C Koiter could have chosen $m_{xy} \neq m_{yx}$ and $n_{xy} = n_{yx}$.
This would have also fulfilled moment equilibrium around the *z* axis.

D Suppose that $k_{xx} = k_{xy} = 0$ and $k_{yy} = \frac{1}{30t}$ (almost thick) and $m_{xy} = \frac{1}{6}t^2 f$ (almost yielding)
then equation 6 reduces to $n_{xy} = n_{yx} - \frac{1}{180}tf$. The yield strength of n_{yx} is tf but it will often be much smaller, for example $n_{yx} = \frac{1}{20}tf$. In this situation $-\frac{1}{180}tf$ is not negligible.

13. A shell can be sometimes interpreted as a beam supported by uniformly distributed springs. Which shell part is the beam and which shell part is the springs?

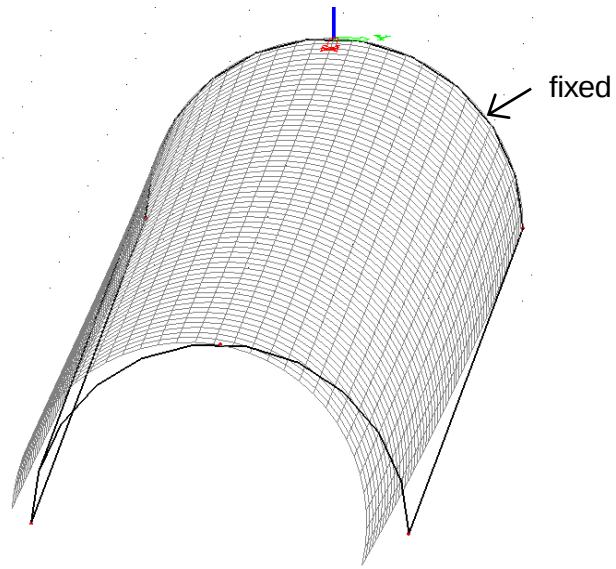
    A The shell edge is the beam and the support is the springs.
    B A hoop is the beam and the adjacent support is the springs.
    C A strip of shell is the beam and perpendicular hoops are the springs.
    D A cylindrical shell is the beam and the supports of the cylinder are the springs.

14. Had Saturn V a monocoque structure?

    A Yes; its skin carried the load.
    B Yes; it went very fast.
    C No; most shell parts had stiffeners.
    D No; it was a rocket not an airplane.

15. Half a cylinder is fixed at one edge and free at the other edges (see figure). The radius a = 2 m, the length l = 12 m and the thickness t = 60 mm. The number of elements in the length is 60. The number of elements in the width is 36. Should the elements be smaller?

    A No, this figure is in the reader too so it should be all right.
    B No, the elements are 200 mm and the recommend size is 1155 mm.
    C Yes, the elements are 200 mm and the recommend size is 139 mm.
    D Yes, the elements are 196 mm and the recommend size is 139 mm.



16. What is an hourglass mode?

    A shell deformation in which the middle surface does not stretch
    B spurious energy shell omission for hyper extensional catenoids
    C strong sensitivity to invisible shape imperfections
    D unrealistic deformation of the mesh of a finite element model

17. In SCIA Engineer, a hypar has a local coordinate system *x*, *y*, *z*. The directions of the axis depend on the order in which the hypar vertices are entered. If the vertices are entered clockwise, the *z* axis points outwards. If the vertices are entered counter clockwise, the *z* axis points inwards.
Suppose that we build a shell model out of several hypars. Some of the hypars are entered clockwise and others are entered counter clockwise. All *x* axes happen to be aligned.
Then, we display bending moment m_{xx} . What do we see?

A what is the top surface moment in some hypars, is the bottom surface moment in others
B what is a positive moment in some hypars, is a negative moment in the others
C what is m_{xx} in some hypars, is m_{yy} in the others
D beautiful colours in vibrant patterns

18. Suppose we make a finite element model of a shell loaded by gravity only.
We do a linear buckling analysis. A buckling mode can move up or down.
What makes the difference?

A computational inaccuracies
B the direction of gravity as selected in the software
C the distribution of the compression membrane forces
D the local coordinate systems of the finite elements

19. In a shell at some location
t = 3 mm,
m_{xx} = 256 N, m_{yy} = 132 N, m_{xy} = -97 N,
n_{xx} = 354 N/mm, n_{yy} = -354 N/mm, n_{xy} = 146 N/mm, n_{yx} = 146 N/mm.
q_x = 97 N/mm, q_y = -31 N/mm
What is σ_{xx} in the middle surface?

A 4.74 N/mm^2
B 43.5 N/mm^2
C 48.7 N/mm^2
D 118 N/mm^2

20. What word do we use when a shell behaves like a flat plate?

A middle third rule
B theorema egregium
C Gaussian curvature
D in-extensional deformation

21. A shell is loaded by a perpendicular point load of 25 kN.
k_G = 1/(100 m^2), E = 2.1 10^5 N/mm^2, ν = 0.35, t = 5 mm.
This causes compression in the middle surface.
What is the membrane force under the point load?

A -1014 N/mm
B -203 N/mm
C -203 N/mm and +203 N/mm in the perpendicular direction
D infinite

22. People in Switserland use the words *Kammweg* and *Talweg*.
What do these words mean in geometry?

A direction of k_1, direction of k_2
B lemon umbilic, star umbillic
C line over the peak, line through the valley
D positive and negative curvature trajectories

23. SCIA Engineer generates finite element meshes that consist of quadrilaterals and triangles.
Triangles are less accurate than quadrilaterals. Why does the program choose triangles?

A This is a mistake in the software.
B Quadrilaterals are randomly replaced by triangles. This speeds up the computation.
C To force cracks to travel in the correct direction in nonlinear analysis.
D SCIA uses flat elements. Twisted quadrilaterals have to be replaced by flat triangles.

24. Every finite element program consists essentially of three parts. Which are these?

A functionality setup – drawing the model, supports and loads – design improvements
B modelling – computation – displaying the results
C installing – learning – using
D linear elastic analysis – linear buckling analysis – nonlinear analysis

25. A cylinder is loaded axially by a uniformly distributed edge load.
E = 2.1 10^5 N/mm^2, ν = 0.35, l = 800 mm, a = 200 mm, t = 1 mm
What is the critical buckling load?

A 192 kN
B 630 kN
C 792 kN
D 991 kN

26. A linear buckling analysis gives us critical loads.
How do we account for imperfection sensitivity?

A select the highest consequence class
B specify appropriate partial load factors and partial material factors
C apply a knockdown factor
D add shape imperfections

27. Suppose that you are analysing a reinforced concrete shell roof. The shell surface area is 360 m^2. You made a finite element model, performed a linear buckling analyses and observed that the buckling length is about 1.2 m. Now, you want to do a nonlinear analysis. So, you need to add a shape imperfection to the model. What imperfection amplitude do you apply?

A 58 mm
B 58 mm and -58 mm
C 82 mm
D 114 mm and -114 mm

28. Consider a hemisphere and a sphere of similar material, size and thickness. The sound of the sphere is much higher than the sound of the hemisphere. What causes this?

A The sphere deforms extensionally, the hemisphere deforms in-extensionally.
B The sphere encloses air, the hemisphere does not.
C The sphere is symmetrical, the hemisphere is not.
D The sphere has twice the mass of the hemisphere.

29. Some engineers think that a modern course on shell structures should focus on performing finite element analyses and leave out most of the theory. What is an advantage of this idea?

A Students would be better at recognising shell phenomena in computation results.
B Students could easily understand how the design equations were derived.
C Students would be better at checking finite element programs.
D Students would be able to solve unforeseen future problems.

30. Write a good shell multiple choice question for next year's exam. The question needs to be not too easy and not too difficult. The question needs to be not just copied and not copied-and-modified-a-bit. (Unless the latter modification is substantial and original.)

Answers

1. C 78% of the answers were correct …. Hoop 1, 2, 3 and 8 are tensioned.

2. B 98% ……

> $d := 48.\cdot mm : \; t := 0.35\cdot mm : \; a := \frac{d}{2} : \; \frac{a}{t};$

$$68.57142857$$

3. C 68% ……

> $km := \frac{1}{11\cdot m} : kG := \frac{1}{100\cdot m^2} :$

> $eq1 := km = \frac{k1 + k2}{2} : eq2 := kG = k1\cdot k2 :$

> $opl := solve(\{eq1, eq2\}, \{k1, k2\}) :$

> $allvalues(opl[1]); allvalues(opl[2]);$

$$k1 = -\frac{-10 + I\sqrt{21}}{110\, m}, \; k1 = -\frac{-10 - I\sqrt{21}}{110\, m}$$

$$k2 = \frac{\frac{10}{11} + \frac{I\sqrt{21}}{11}}{10\, m}, \; k2 = \frac{\frac{10}{11} - \frac{I\sqrt{21}}{11}}{10\, m}$$

4. B 98% complex curvatures …. not physical
5. A 83%
6. D 71% ……

> $alphax := 1 + \frac{a}{b}\cdot \sin\left(\frac{v}{a}\right) : \qquad alphay := 1 :$

> $kx := simplify\left(\frac{1}{alphax}\cdot diff(alphax, v)\cdot \frac{1}{alphay}\right);$

$$kx := \frac{\cos\left(\frac{v}{a}\right)}{a\sin\left(\frac{v}{a}\right) + b}$$

> $ky := simplify\left(\frac{1}{alphay}\cdot diff(alphay, u)\cdot \frac{1}{alphax}\right);$

$$ky := 0$$

7. C 80%
8. A 49% …… This follows from evaluating the analytical solution in the reader. (B 27% Von Mises stress in not negative, C 12% buckling gives bending moments which gives more stress not less, D 12%)
9. A 68% ……

> $l := 10.\cdot a : \mathrm{nu} := 0.35 :$

> $stress_met := \frac{p\cdot l^2}{4\cdot a\cdot t}\cdot \mathrm{sqrt}\left(1 - 4\cdot\frac{a^2}{l^2} + 16\cdot\frac{a^4}{l^4}\right) :$

> $stress_zonder := \frac{p\cdot l^2}{4\cdot a\cdot t}\cdot \mathrm{sqrt}\left(1 + 16\cdot\frac{a^4}{l^4}\right) :$

> $verschil := \frac{stress_met - stress_zonder}{stress_met}\cdot 100; \#\,\%$

$$verschil := -2.058676143$$

> $deflection_met := \frac{p\cdot l^4}{a^2\cdot E\cdot t}\left(\frac{5}{192} + \frac{\mathrm{nu} + 4}{8}\cdot\frac{a^2}{l^2} + \frac{a^4}{l^4}\right) :$

> $deflection_zonder := \frac{p\cdot l^4}{a^2\cdot E\cdot t}\left(\frac{5}{192} + \frac{a^4}{l^4}\right) :$

> $verschil := \frac{deflection_met - deflection_zonder}{deflection_met}\cdot 100; \#\,\%$

$$verschil := 17.21863042$$

10. D 90%
11. B 61% ..... Choose kinematic or dynamic, four times; 2^4=16 (A 32%, C 2%, D 5%)
12. B 44% ..... (A 2% appeal to authority,
C 20% builds on argument B,
D 34% example that builds on B)
13. C 98%
14. C 24% ..... (A 71% All parts except the domes had stiffeners of some sort.
Stiffeners form a frame. A frame is not monocoque, B 5%, D 0%)
15. C 78% .....

$$> a := 2.\cdot m : l := 12.\cdot m : t := 60\cdot mm : nw := 32 : nl := 60 : m := 1000\cdot mm :$$

$$> li := 2.4\cdot \mathrm{sqrt}(a\cdot t) : hr := \frac{li}{6}; h := \frac{l}{nl};$$

$$hr := 138.5640646\sqrt{mm^2}$$

$$h := 200.0000000\ mm$$

(A 0% appeal to authority, B 17% The edge disturbance is in the supported curved edge; not in the unsupported straight edges,
D 5% The element size in the length is important; not in the width.)

16. D 98%
17. B 61% ..... (A 22% Stresses are different in top surface and bottom surface; moments are not. C 10% The *x* directions are the same,
D 7% You should become an architect … or a lawyer.)
18. A 71% ..... Shell buckling is as column buckling: The direction is random.
19. D 95% .....

$$> t := 3\cdot mm : nxx := 354\cdot\frac{N}{mm} : sigma_xx_m := \frac{nxx}{t};$$

$$sigma_xx_m := \frac{118\ N}{mm^2}$$

20. D 98%
21. A 71% .....

$$> P := 25\cdot kN:\ E := 2.1e5\cdot\frac{N}{mm^2}:\ \mathrm{nu} := 0.35:\ t := 5\cdot mm:\ kN := 1000\cdot N:$$

$$> n := -\frac{\mathrm{sqrt}(3.)}{8}\cdot\frac{P}{t}\cdot\mathrm{sqrt}\left(1-\nu^2\right);$$

$$n := -\frac{1014.061296\ N}{mm}$$

(D 20% The singularity is in the bending moments; not in the membrane forces.)

22. C 44% ..... Cannot be defined mathematically, but intuitively easy:
We go to the next town following the road on the mountain ridges or following the road through the valley. These are the black lines on the monkey saddle figure in the reader. (A 7%, B 15%, D 34%)
23. D 83%
24. B 95%
25. C 41% .....

$$> E := 2.1e5\cdot\frac{N}{mm^2}:\ \mathrm{nu} := 0.35 : l := 800\cdot mm : a := 200\cdot mm : t := 1\cdot mm : N := \frac{kN}{1000}$$

$$> ncr := -0.6\cdot\frac{E\cdot t^2}{a} : P := 2\cdot\mathrm{Pi}\cdot a\cdot ncr;$$

$$P := -791.6813488\ kN$$

(B 59% Calculated n_{cr} in kN/m but forgot *P* in kN)

26. C 95%
27. C 12% .....

$$> A := 360\cdot m^2 : l := 1.2\cdot m : m := 1000\cdot mm :$$

$$> d := simplify\left(\frac{1}{108}\cdot A^{0.3}\cdot l^{0.4}\right)\cdot 1.4;$$

$$d := 81.52063031\ mm\sim$$

An even better answer is 82 mm and -82 mm.
(A 59% Buckling is ULS, so we better apply safety factors.)

28. A 98%
29. A 63% …… (C 32% For checking we need alternative ways to produce accurate answers.)
30. 88%

The grade is $\frac{40s - 290}{91}$ rounded to the closest half, where *s* is the number of correct answers.

The grade formula above is derived as follows. A student with grade *g* knows 30/10 *g* answers. This student guesses 29 - 29/10 *g* answers. The expected number of correct answers is *s* = 30/10 *g* + (29 - 29/10 *g*)/4, from which *g* was solved.

