Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CIE4143 Shell Analysis
Tuesday 12 July 2022, 13:30 – 16:30 hours

Choose A, B, C or D.

1. Consider an arch that carries a bridge deck.
   The bridge deck is very heavy compared to the arch.
   What is the shape of the pressure line?

   A semi-circle
   B parabola
   C funicular
   D hyperbolic cosine

2. Consider the figure below.
   What happens to the pressure line when the pole is moved to the right? [1]

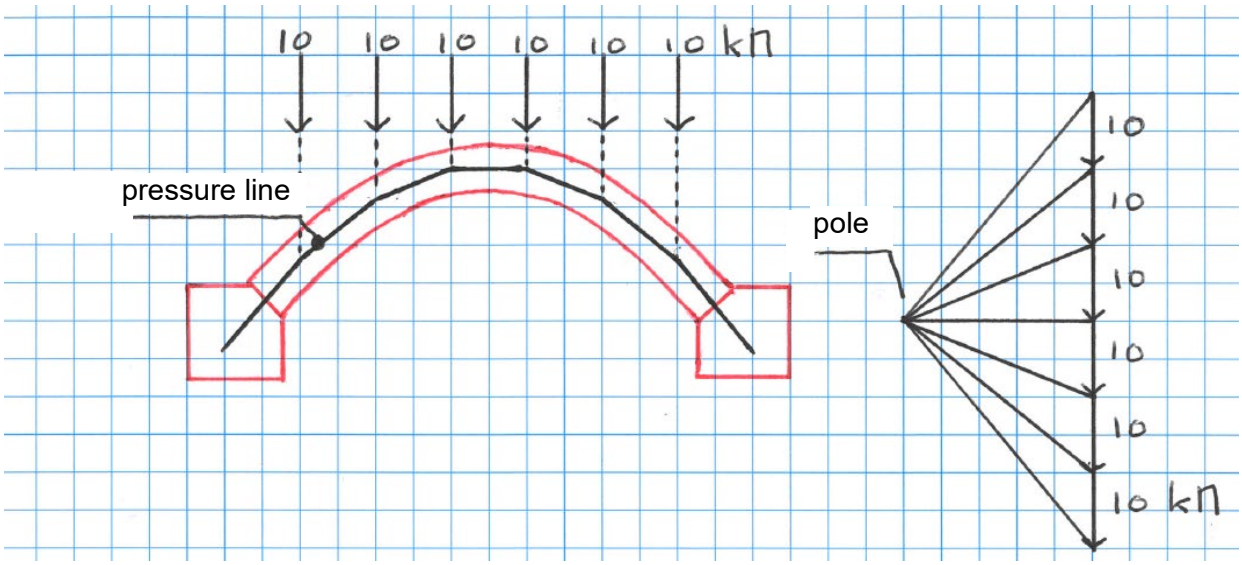


Figure: Line of action plan *Figure: magnitude plan*

   A It becomes higher.
   B It becomes flatter.
   C It slopes to the left.
   D It slopes to the right.

3. Consider a spherical shell that contains a nuclear reactor. There is an accident and the inside pressure increases. At what over pressure does the shell fail?
   diameter = 31 m; thickness = 16 mm; Young's modulus = 2.1 10^5 N/mm^2
   yield strength = 400 N/mm^2

   A 0.83 N/mm^2
   B 0.41 N/mm^2
   C 0.21 N/mm^2
   D 0.10 N/mm^2

4. A dome has a foundation diameter of 13 m, a sagitta of 2 m and a thickness of 100 mm. How do we classify this shell?

   A membrane
   B thin
   C thick
   D very thick

5. Consider Palazzetto dello sport in Rome. What is its Gaussian curvature?

   A 0.1 / m^2
   B 0.01 / m^2
   C 0.001 / m^2
   D 0.0001 / m^2

6. Consider the parametrisation $\bar{x} = u\cos v$
   $\bar{y} = u\sin v$
   $\bar{z} = u$

   Is it orthogonal? What are the Lame parameters?

   A yes, $\alpha_x = |u|$, $\alpha_y = \sqrt{2}$
   B yes, $\alpha_x = \sqrt{2}$, $\alpha_y = |u|$
   C no, $\alpha_x = |u|$, $\alpha_y = \sqrt{2}$
   D no, $\alpha_x = \sqrt{2}$, $\alpha_y = |u|$

7. What is the difference between k_{xx}, k_{yy}, k_{xy} and k_x, k_y?

   A k_{xx}, k_{yy}, k_{xy} describe the parameter lines and k_x, k_y describe the principal directions.
   B k_{xx}, k_{yy}, k_{xy} describe the principal curvatures and k_x, k_y describe the shell shape.
   C k_{xx}, k_{yy}, k_{xy} describe the shell shape and k_x, k_y describe the parameter lines.
   D k_{xx}, k_{yy}, k_{xy} describe the curvature directions and k_x, k_y describe the shell principals.

8. The shell membrane equations can describe …

   A … imperfection sensitivity.
   B … edge disturbances.
   C … in-extensional deformation.
   D … Brazier's effect.

9. A tube carries a shear force V = 23.1 kN.
The tube diameter is 700 mm.
The wall thickness is 12 mm.
What is the largest shear stress in the cross-section?

A 0.98 N/mm^2
B 1.41 N/mm^2
C 1.53 N/mm^2
D 1.78 N/mm^2

10. A shell part has curvatures k_{xx} = 1/(200 m), k_{yy} = 1/(150 m), k_{xy} = –1/(180 m)
and moments m_{xx} = 5 kNm/m , m_{yy} = 8 kNm/m , m_{xy} = 5 kNm/m.
What is $n_{xy} - n_{yx}$? [2]

A 0 N/m
B 64 N/m
C –25 N/m
D 8 N/m

11. How did construction workers build Palazzetto dello sport?

A Almost all cassettes were cut by a CNC milling machine using a virtual representation.
B The cassette forms were copied from a live size dome part made of timber and plaster.
C The formworks of the cassettes were enlarged from a wood scale model.
D One formwork was used for all cassettes, for which a drawing was available.

12. A long beam EI = 700 kNm^2 is supported by springs k = 50 kN/m^2.
On the beam middle is a perpendicular point load F = 12 kN.
What is moment under the point load? What is the deflection under the point load?

A M = 1.0 kNm, w_0 = 6 mm
B M = 2.0 kNm, w_0 = 11 mm
C M = 4.1 kNm, w_0 = 22 mm
D M = 8.2 kNm, w_0 = 44 mm

13. For plates the recommended finite element size is two times the thickness.
Is the recommended finite element size for thin shells smaller or larger?

A smaller because shells are thinner than plates.
B smaller, especially at edges, to compute edge disturbances accurately.
C larger because $\frac{1}{6} 2.4 \sqrt{30 t \times t} = 2.2 t$.
D larger; there would be too many $2t$ elements to fit in normal computer memory.

14. Pressure vessels have stress concentrations, for which equations are available. How were these equations derived?

    A For pressure vessels the Sanders-Koiter equations can be solved analytically.
    B Many finite element analyses were performed. The data was curve fitted.
    C The equations were derived from design graphs which are based on experiments.
    D Some engineers have extensive experience and they wrote them down.

15. The semi-loof element is derived from ... [3]

    A … the Sanders-Koiter equations.
    B … solid elements.
    C … the shell membrane equations.
    D … flat plate elements.

16. A finite element program in essence consists of many commands that operate on a database. The commands can be divided in three types. In the design process we use these types in a specific order (often cyclically). Which order is this?

    A Analysis – Modelling – Studying the results
    B Modelling – Analysis – Studying the results
    C Analysis – Studying the results – Modelling
    D Studying the results – Modelling – Analysis

17. We want computation results with an arithmetic error smaller than 1%. How many significant digits does the computation results need to have?

    A no digit
    B 1 digit
    C 3 digits
    D 4 digits

18. What is a simple way of checking for arithmetic error in finite element analyses?

    A Compare the computation result with the result of a hand calculation.
    B Compute the sum of loads and support reactions. It needs to be zero.
    C Repeat the computation. The difference between the results should be small.
    D Repeat the computation on another computer and compare the results.

19. Which can be plotted by Mohr's circle?

    A The Von Mises stress in a shell
    B The curvature of a shell
    C The Sanders-Koiter equilibrium equations
    D An elliptical paraboloid

20. A shell structure consists of curved steel plates that are welded together.
How do we obtain small stresses in the welds?

A By designing the same curvature on either side of the welds
B By consistent quality control of all welds and weld edges directly after production
C By removing stress concentrations
D By preventing the occurrence of edge disturbances

21. Load on a shell structure is carried by membrane forces and by bending moments.
Curvature makes these work together.
How is the exception called?

A In-extensional deformation
B A flat plate
C *Theorema Egregium*
D Imperfection sensitivity

22. When do we need in-extensional deformation?

A Never; extensional deformation is always the more efficient way of carrying load.
B For thick and very thick shell structures
C For snow load
D In case of foundation settlements

23. In SCIA Engineer we can program what to plot.
It may be interesting to plot n_{xy} and n_{yx} separately.
However, there is a problem. What is the problem?

A The finite element coordinate systems need to be aligned to obtain proper plots.
SCIA does not have functionality for this. It would have to be programmed too.
B Finite element derivations assume that $n_{xy} = n_{yx}$.
So, of course we will find that in SCIA n_{yx} is exactly the same as n_{xy}.
C SCIA elements are flat; they do not have information on k_{xx}, k_{yy} or k_{xy}.
So, we cannot fill in Sanders-Koiter 6 or 18 to compute $n_{xy} - n_{yx}$ for each element.
D Stresses in shells are calculated using $\frac{1}{2}(n_{xy} + n_{yx})$, which is displayed by SCIA.
No equation uses n_{xy} or n_{xy} separately. So, there is no point in plotting n_{yx} too.

24. A shell linear buckling analysis produces a buckling load factor of 9.09.
Subsequently, the load is increased by a factor 9.09 and a nonlinear analysis is performed.
The load is applied in 90 steps. The analysis diverges in step 48.
What is the knockdown factor?

A 0.11
B 0.15
C 0.17
D 0.52

25. Before a nonlinear analysis we add shape imperfections to a shell finite element model. Often, we choose a buckling mode imperfection with a positive amplitude and a negative amplitude. Why both positive and negative?

    A This is an extra check. The ultimate loads should be the same.
    B If one computation diverges, we still have the other for determining the ultimate load.
    C The buckling mode is randomly inward or outward. Analysing both makes it not random.
    D We do not know the governing shape imperfection. We need to try as many as possible.

26. Consider two reinforced concrete shells roof designs.
    The shells have the same surface area.
    We made finite element models.
    Shell A has a buckling length of 2 m. Shell B has a buckling length of 3 m.
    We need to add shape imperfections to the models.
    How much larger is the imperfection amplitude of shell B?

    A 18%
    B A factor 1.5
    C 3.2 mm
    D Undetermined: we do not know the imperfection lengths.

27. A cylinder is axially loaded in compression.
    The edges are simply supported.
    The cylinder almost buckles.
    What is its smallest natural frequency? [4]

    A $f_n = \infty$
    B $f_n > 0$
    C $f_n \approx 0$
    D $f_n < 0$

28. How many rigid body modes does a properly supported shell structure have? [5]

    A 0
    B 3
    C 6
    D ∞

29. A structure is exited in a normal mode. The load is removed and the stress amplitude is measured. It is 50.3 N/mm^2. Twenty peaks later the stress amplitude is just 9.12 N/mm^2. What is the damping ratio of this mode? [6]

    A 0.12 %
    B 1.30 %
    C 8.53 %
    D 85.3 %

See page 7.

30. Write a good shell multiple choice question for next year's exam. The question needs to be not too easy and not too difficult. The question needs to be not just copied and not copied-and-modified-a-bit. (Unless the latter modification is substantial and original.)

Answers

1 B 100% of the answers were correct.
2 A 82%
3 A 73% … $n = \frac{1}{2} pa, \quad n = f\,t, \quad p = \ldots$
4 B 91% … thin because 30 < *a* / *t* < 4000
5 C 100%
6 B 91%
7 C 55% … (A 7%, B 18%, D 18%)
8 C 91%
9 D 91%
10 C 73%
11 B 82%
12 D 36% … (A 0%, B 27%, C 36%) …
13 C 82%
14 A 55% … (B 9%, C 27%, D 9%)

> $V := -\frac{1}{2} \cdot F$:

> $eq1 := \frac{V}{\mathrm{sqrt}(2) \cdot w0 \cdot k^{\left(\frac{3}{4}\right)} \cdot EI^{\left(\frac{1}{4}\right)}} = -1$:

> $eq2 := \frac{M}{w0 \cdot \mathrm{sqrt}(k \cdot EI)} = 1$:

> $opl := solve(\{eq1, eq2\}, \{M, w0\})$; $assign(opl)$;

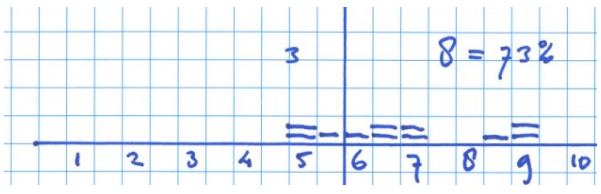
$$opl := \left\{ M = \frac{F\sqrt{2}\sqrt{k\,EI}}{4\,k^{3/4}\,EI^{1/4}},\ w0 = \frac{F\sqrt{2}}{4\,k^{3/4}\,EI^{1/4}} \right\}$$

> $EI := 700 \cdot kN \cdot m^2$: $k := 50. \frac{kN}{m^2}$: $F := 12 \cdot kN$: $assume(kN > 0)$: $assume(m > 0)$:

> $w0 := simplify(w0)$; $M := simplify(M)$;

$$w0 := 0.04386662672\ m\sim$$
$$M := 8.206694391\ kN\sim m\sim$$

15 A 100%
16 B 91%
17 C 100%
18 B 100%
19 B 100%
20 A 45% … C 18% and D 36% are wrong because these are just rearrangements of the question with different words.
21 A 36% … B 18% is wrong because a shell can be curved and have no interaction between membrane forces and moments,
C 36% is related but if you would call it *theorema egregium*, nobody would understand you, not even a mathematician,
D 9% is just wrong.
22 D 100%
23 C 18% … D 64% is wrong because reinforcement is calculated with $n_{xy} n_{yx}$.
24 D 36% … C 45% is wrong because 0.17 = 1/6 is used if nothing better is available.
25 C 45% … A 18% is wrong because the results will not be the same at all.
B 9% is wrong because both computations will diverge at ultimate load.
D 27% is wrong: probably there exists an imperfection worse than the buckling shape but the chance that this imperfection will occur is very small.
26 A 64% … D 36% is wrong because the imperfection length is the buckling length.
27 C 45% … B 55% is wrong from an engineering point of view because it is almost zero. Apparently, many shell students aim for a career in mathematics, therefore, answer B was accepted as well.
28 A 91%
29 A 100%
30 59%



The grade is $\frac{40s-290}{91}$ rounded to the closest halve, where s is the number of correct answers.

The grade formula above is derived as follows. A student with grade g knows $3g$ answers. This student guesses $29 - \frac{29}{10}g$ answers. The expected number of correct answers is $s = 3g + (29 - \frac{29}{10}g)/4$, from which g can be solved.

Endnotes

[1] Question by Laslo Bleker, August 2019
[2] Question by Willem Wolswijk, June 2022
[3] Question by Hemin Sindy, June 2022
[4] Question by Brahim Bougarchouh, June 2022
[5] Question by Daan Smolders, June 2022
[6] Question by Nora Kovacs, June 2022