Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CIE4143 Shell Analysis
Tuesday 21 June 2022, 13:30 – 16:30 hours

Choose A, B, C or D.

1. A dome has a foundation diameter of 40 m and a sagitta of 10 m.
   What is its radius of curvature?

   A 20 m
   B 25 m
   C 30 m
   D 40 m

2. What is a funicular?

   A The path of a cannon ball
   B A type of minimal surface
   C A train that obtains electricity from curved power lines
   D The shape of a hanging chain

3. A shell has the shape $\bar{z} = \frac{1}{2}\frac{1}{20\ \mathrm{m}}\bar{x}^2 + \frac{1}{10\ \mathrm{m}}\bar{x}\,\bar{y} + \frac{1}{2}\frac{-1}{30\ \mathrm{m}}\bar{y}^2$.
   What are the principal curvatures in the origin?

   A $k_2 = \frac{-1}{30\ \mathrm{m}},\ k_1 = \frac{1}{20\ \mathrm{m}}$
   B $k_2 = \frac{-6}{60\ \mathrm{m}},\ k_1 = \frac{7}{60\ \mathrm{m}}$
   C $k_2 = \frac{2}{30\ \mathrm{m}},\ k_1 = \frac{3}{20\ \mathrm{m}}$
   D $k_2 = \frac{-1}{60\ \mathrm{m}},\ k_1 = \frac{1}{40\ \mathrm{m}}$

4. A planetarium dome has a foundation diameter of 9340 mm, a sagitta of 2700 mm and a thickness of 57 mm. Is it optimal? Is it thin?

   A yes, yes
   B yes, no
   C no, yes
   D no, no

5. Which shape is this?

$$\bar{x} = u\cos v$$
$$\bar{y} = u\sin v$$
$$\bar{z} = u$$

A Cylinder
B Sphere
C Cone
D Ellipse

6. A dome is shaped like half a sphere and loaded by gravity. The radius is 50 m. The thickness is 80 mm. The mass density of the material is 2400 kg/m^3. How large is the stress due to the hoop force at the bottom?

A 1.20 N/mm^2
B 0.12 N/mm^2
C -1.20 N/mm^2
D -12.00 N/mm^2

7 The shell membrane equations can describe …

A … moments in the thickness.
B … buckling.
C … edge disturbances.
D … in-extensional deformation.

8. The shell membrane equations have boundary conditions. How many on each edge?

A 2
B 3
C 4
D 5

9. What is a monocoque?

A An old type of airplane
B A word sometimes used for a shell structure
C The fuselage of an airplane, for example the Airbus A380
D A shell structure that is in fact a truss structure

10. Are the Sanders-Koiter equations valid for thin shells?
Are the Sanders-Koiter equations valid for large displacements?

A yes, yes
B no, yes
C yes, no
D no, no

11. Which approximation is made in deriving the shallow shell differential equation?

A Curvatures k_{xx}, k_{xy} and k_{yy} are zero.
B Gaussian curvature k_G is small in absolute value but not exactly zero.
C In plane loads p_x and p_y are zero.
D Rigid translations are small.

12. In the shell membrane equations $n_{xy} = n_{yx}$. How can we proof this?

A Finite element programs display n_{xy} only, so n_{yx} must be the same.
B This follows from the thick shell definitions of n_{xy} and n_{yx} (Appendix 7).
C $n_{xy} = n_{yx}$ is written in the shell reader (p. 36), teachers know these things.
D Start with the Sanders-Koiter equations (p. 52) and make all moments zero.

13. Construction workers have made the reinforcement for a plate. They are about to start casting concrete, however, several bars in the plate edges are missing. These bars are u-shaped and carry the top bars. What do you decide?

A We cast and during casting we do not step on the edge bars.
B We cast and at the end of casting we pull up the edge bars to the right cover.
C We stop and support the edge bars by quickly hammering large nails into the formwork.
D We stop, find the missing u-shaped bars and put them in place before casting.

14. Often, finite element membrane forces and moments do not exactly agree with the shell boundary conditions. What causes this?

A Finite elements are based on a different theory than the Sanders-Koiter equations.
B Smaller elements would give better approximations of membrane forces and moments.
C The shell boundary conditions are approximations of reality and not perfect.
D On edges, special edge elements should be used instead of finite elements.

15. The accuracy of a finite element model depends on model accuracy and arithmetic accuracy. Suppose that we have a model and we increase the number of dofs. What will happen?

A The model accuracy increases and the arithmetic accuracy decreases.
B The model accuracy increases and the arithmetic accuracy increases.
C The model accuracy decreases and the arithmetic accuracy decreases.
D The model accuracy decreases and the arithmetic accuracy increases.

16. A shell has a surface area of 4 m^2 and a thickness of 10 mm. The radius of curvature is 5 m. The Gaussian curvature is positive. The finite element model uses 4 node elements which are all about the same size. How many dofs are in this model?

    A 300
    B 3000
    C 30 000
    D 300 000

17. In a shell part, the membrane force is n_{xx} = 500 kN/m. The bending moment is m_{xx} = –5 kNm/m. The shell has a thickness t = 50 mm. What is the largest normal stress?

    A –2 N/mm^2
    B 10 N/mm^2
    C 12 N/mm^2
    D 22 N/mm^2

18. Somebody designed a shell structure. It has solid supports and edge beams where necessary. However, finite element analysis shows that the deflection is extremely large. This could be caused by …

    A … imperfection sensitivity.
    B … the middle third rule.
    C … in-extensional deformation.
    D … edge disturbances.

19. Can the *theorema egregium* be proved from the Sanders-Koiter equations?

    A No, the Sanders-Koiter equations are valid for small displacements only.
    B No, Gauß did not consider elastic deformation.
    C Yes, the proof starts with the compatibility equation.
    D Yes, shells can behave like a plate.

20. Consider a linear elastic finite element analysis of a shell structure. The elements have an error $O(h^2)$. In an important shell part the stress is 54.20 N/mm^2. A second analysis is made with smaller elements and the same load. The smaller elements have half the lengths and widths of the first elements. The second stress in the part is 55.81 N/mm^2. What is the best approximation of the stress?

    A 54.20 N/mm^2
    B 55.81 N/mm^2
    C 56.35 N/mm^2
    D 57.42 N/mm^2

21. An architect is designing a timber grid shell. The thickness of the laths is 30 mm. The allowable stress is 10 N/mm^2 during construction (also considering relaxation and subsequent loading). Young's modulus of timber is 10 000 N/mm^2.
First a shape study is made with NURBSes. What curvatures are possible?

A principal curvatures smaller than 1/(3.7 m)
B principal curvatures smaller than 1/(7.5 m)
C principal curvatures smaller than 1/(15 m)
D principal curvatures smaller than 1/(30 m)

22. Consider two shells; an elliptical shell and a hyperbolical shell. The shells have the same thickness, curvature, and Young's modulus. Each shell carries a perpendicular point load of 1 kN. The point loads are not close to the shell edges. The shell edges are supported properly. Which shell deflects most?

A The elliptical shell
B The hyperbolical shell
C They have the same deflection.
D This depends on the Gaussian curvature.

23. Why do shell natural frequencies need to be larger than 1 Hz?

A A load factor smaller than 1 Hz would lead to buckling.
B Cross-sections would fail (unity check).
C Many shell structures have a natural tendency to resonate with frequencies up to 1 Hz.
D Wind gusts with a frequency larger than 1 Hz do not have much energy.

24. For computing natural frequencies in SCIA we need to specify mass groups and mass combinations. Why?

A The program needs to know which loads have a mass too.
B Natural frequencies depend on the mass; not on the load.
C If we would not attach mass to the roof, it will slide of during the vibrations.
D Some loads have a large mass and a small weight.

25. What is the buckling load of a long beam supported by springs?

A 0
B $2\sqrt{kEI}$
C $\pi\sqrt[4]{\frac{EI}{k}}$
D $\frac{\pi^2 EI}{l^2}$

26. Consider a hemisphere and a sphere of similar material, size and thickness. They have no supports. The sound of the hemisphere is much lower than the sound of the sphere. What causes this?

    A The hemisphere has a smaller Gaussian curvature, therefore it is more flexible.
    B The hemisphere is open and soundwaves can project easily.
    C The hemisphere deforms in-extensionally, therefore it has no membrane forces.
    D The hemisphere has half the amount of material, therefore less mass.

27. What is influence length?

    A The width of an edge disturbance
    B The buckling length of a beam on distributed springs
    C The width of shell that participates in carrying a point load
    D The mean length of a shape imperfection that resembles a buckling shape

28. Part of a shell has the following membrane forces
    n_{xx} = 5 kN/m, n_{xy} = 5 kN/m, n_{yx} = 5 kN/m, n_{yy} = 5 kN/m.
    The shear forces v_x and v_y are zero.
    The curvatures are
    k_{xx} = 1/(5 m), k_{xy} = -1/(3 m), k_{yy} = -1/(8 m)
    The shell thickness t = 0.152 m.
    How large is the load p_z ?

    A 0.00296 kN/m^2
    B 0.0296 kN/m^2
    C 0.296 kN/m^2
    D 2.96 kN/m^2

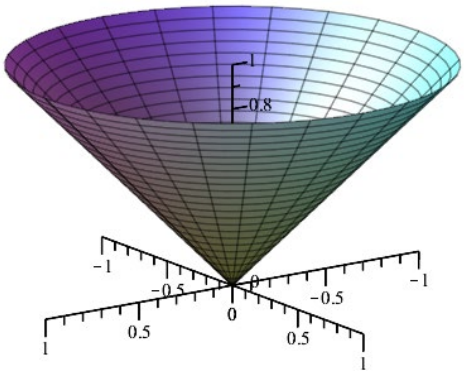
29. A square panel with a small curvature is simply supported (diaphragm boundary condition).
    l = 0.5 m, t = 4 mm, E = 2.1 10^5 N/mm^2, ν = 0.35, ρ = 7000 kg/m^3
    k_{xx} = 1/(5 m), k_{yy} = -1/(8 m), k_{xy} = -1/(3 m),
    The panel is not loaded. What is its smallest natural frequency?

    A 64 Hz
    B 85 Hz
    C 292 Hz
    D 304 Hz

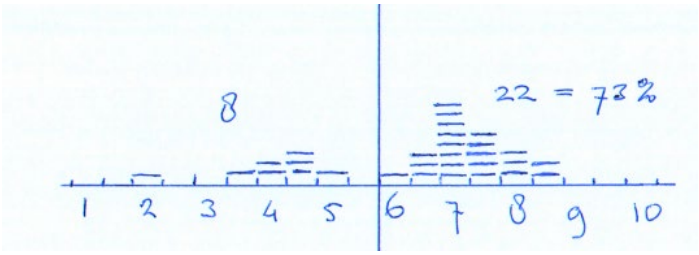
30. Write a good shell multiple choice question for next year's exam. The question needs to be not too easy and not too difficult. The question needs to be not just copied and not copied-and-modified-a-bit. (Unless the latter modification is substantial and original.)

Answers

1. B 93% of the answers were correct.
2. D 97%
3. B 83%
4. A 90%
5. C 70% … x:=u*cos(v); y:=u*sin(v); z:=u; plot3d([x,y,z],u=0..1,v=0..2*Pi);
6. A 90%
7. D 80%
8. A 20% … the same as plates loaded in plane. (B 33%, C 47%, D 0%)



9. B 73%
10. C 87%
11. C 43% … (B 43% is wrong, for example a cylinder has zero Gaussian curvature.)
12. D 80%
13. D 97%
14. B 20% … (A 37%, C 17%, D 23%) Often, we use normal size elements and the difference between computation and dynamic boundary conditions is due to the element size. Occasionally, we use very small elements and we still see small differences between computation and boundary conditions. It is not clear wat causes these differences.
15. A 93%
16. B 87%
17. D 63% … (A 33% is wrong; the largest stress is not in the shell top but in the bottom.)
18. C 57% … (A 23%, B 3%, D 17%)
19. A 20% … (C 67% is wrong because the derivation in the reader is not valid for large displacements, while *theorema egregium* is.)
20. C 70%
21. C 90%
22. B 43% … (See equations on page 116 and 117. D 40% is wrong because they have the same curvature. The Gaussian curvatures are only different in sign.)
23. D 63% … (C 37% is just nonsense.)
24. A 67% … (B 33% is wrong because both mass and load are important.)
25. B 83%
26. C 97%
27. A 80%
28. D 93%
29. D 73%
30. 80%



The grade is $\frac{40s-290}{91}$ rounded to the closest halve, where s is the number of correct answers.

The grade formula above is derived as follows. A student with grade g knows $3g$ answers. This student guesses $29 - \frac{29}{10}g$ answers. The expected number of correct answers is $s = 3g + (29 - \frac{29}{10}g)/4$, from which g can be solved.