Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CIE4143 Shell Analysis
Tuesday 18 July 2023, 13:30 – 16:30 hours

Choose A, B, C or D.

1 What is an oculus?

A opening in a dome
B window in the top of the Pantheon
C pair of glasses in Latin
D stress concentration at a window in an aeroplane fuselage

2 The stresses in supported shell edges are often large. How do we call this?

A imperfection sensitivity
B middle-third rule
C in-extensional deformation
D edge disturbance

3 Do pressure lines in shells always go through the middle third?

A Yes, but not in arches and only if it does not go outside the middle third.
B Yes, hoop forces pull and push the pressure line into the middle third.
C No, not in the top of a dome.
D No, not in edge disturbances.

4 Consider a sphere at the bottom of the ocean. We want to replace the sphere by a cylinder of the same diameter and wall thickness. What will be the largest stress in the cylinder wall?

A 1/2 the stress in the sphere
B the same as in the sphere
C 2 times the stress in the sphere
D 4 times the stress in the sphere

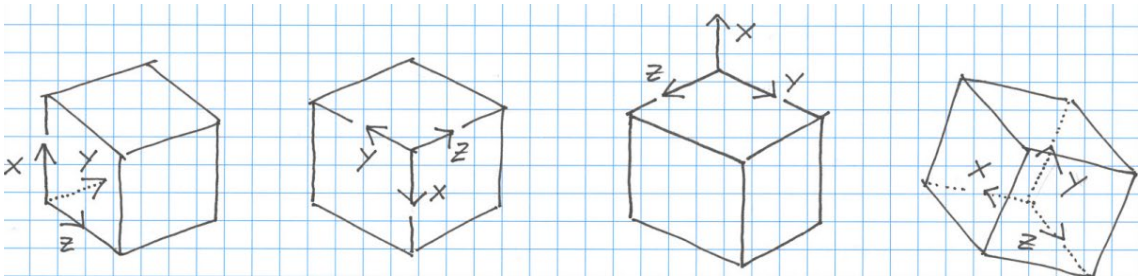
5 What is a fully stressed dome?

A stress unity check of 1, everywhere
B optimal shape and thickness for carrying a particular load
C dome with the minimal thickness for carrying self-weight
D dome that is stressed to the full extent

6 What is the optimal proportion of a spherical dome of constant thickness?

A thickness / sagitta = 30%
B span / radius = 2
C sagitta / span = $\sqrt{3}$ / 6
D radius / thickness = 500

7 Which is a positive Cartesian coordinate system?



A B C D

8 We perform a small-scale experiment on a shell model. All dimensions are 1/10 of the real structure. The model material is the same as the real material. The distributed load (kN/m^2) is the same as the real load.

What is the ratio of the real *deflection* and that of the model?
What is the ratio of the real *stress* and that of the model?
What is the ratio of the real *buckling load factor* and that of the model?

A 10; 10; 1
B 10; 10; 10
C 10; 1; 1
D 100; 0.1; 10

9 Consider a shell with supports and loading. We need to check the buckling load. The result depends on the ...

A ... global coordinate system $\bar{x}$, $\bar{y}$, $\bar{z}$.
B ... local coordinate system *x*, *y*.
C ... curved coordinate system *u*, *v*.
D ... material properties.

10 What is the difference between κ_{xx} and k_{xx}?

A κ_{xx} is in-plane curvature, k_{xx} is out-of-plane curvature.
B κ_{xx} is in the *x*-*y* coordinate system, k_{xx} is in the *u*-*v* coordinate system.
C κ_{xx} is the increase of k_{xx} due to loading.
D κ_{xx} is the curvature of the deformation, k_{xx} is the curvature of the shape.

11 Which of the following quantities depends on the choice of the local *x*-*y* coordinate system?

A $k_{xx}n_{xx} + k_{xy}(n_{xy} + n_{yx}) + k_{yy}n_{yy}$
B $k_{xy}(m_{xx} - m_{yy}) - (k_{xx} - k_{yy})m_{xy}$
C $u_x^2 + u_y^2$
D $\kappa_{xx}\kappa_{yy} - \rho_{xy}^2$

12 What is the compatibility equation used for?

A To connect strains to stresses.
B To proof that rigid translations are described accurately.
C To derive the second shell differential equation.
D To build the formwork of Palazzetto dello sport.

13 The shell differential equations are derived from the Sanders-Koiter equations. In the process several terms are removed. Why?

A Otherwise the differential equations produce invalid solutions (complex numbers).
B These terms are not just removed but cancel each other out.
C This is a generalisation which leads to the correct differential equations.
D These terms are small and their removal simplifies the differential equations.

14 A shell edge is fixed. The edge disturbance is modelled as a beam on elastic foundation. What is the peak moment?

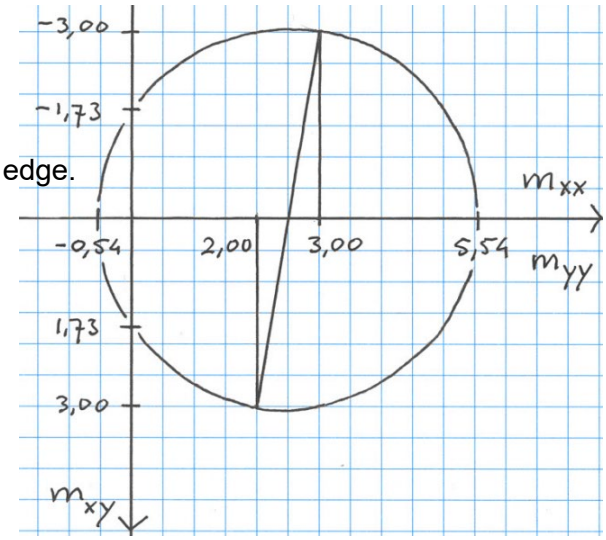
A $m = -0.30\, p_z a t$

B $m = -0.30 \dfrac{p_z a}{t}$

C $m = -0.30 \dfrac{p_z a t^2}{E}$

D $m = -0.30 \dfrac{a t}{p_z}$

15 The figure shows More's circle of the moments in a shell edge. What is the concentrated shear force?



A -0.54 kN
B 1.73 kN
C 3.00 kN
D 5.54 kN

16 A shell edge can be free or hinged or fixed et cetera. How many types of support are possible in theory?

A 3
B 4
C 2^4
D 4!

17 A sphere with supports and load is analysed with a finite element program. The diameter is 13 m. The sphere wall thickness is 100 mm. What element size is recommended?

A 10 mm
B 200 mm
C 300 mm
D 2000 mm

18 Shell finite elements have a limit to the ratio length over width. Why?

A Long thin elements are not accurate.
B Matrix solution algorithms are not accurate for elongated elements.
C This happens when the stress field varies strongly.
D This is a remnant from the past. It can be ignored.

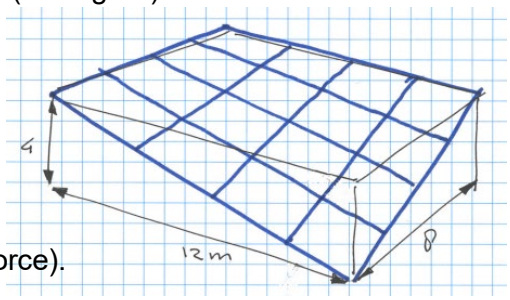
19 A finite element analysis shows that the deflection of a shell is 39 mm. We repeat the analyses with half element size. The new deflection is 42 mm. We do not know the computation accuracy. What is the best prediction of the shell deflection?

A 39 mm
B 42 mm
C 43 mm
D 45 mm

20 We study a singularity. Half the finite element size gives an increase in stress with a constant value. Which function describes this?

A y = C x
B y = C ln(x)
C y = C / x
D y = C exp(-x)

21 A hypar has a length of 12 m, a width of 8 m and a height of 4 m (see figure). The shell thickness is 1 cm. How do we classify this shell?

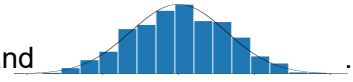


A membrane
B thin
C thick
D very thick

22 In arches we define eccentricity as M/N (moment over normal force). In shells we can define eccentricity as ...

A ... $m_{xx}/n_{xx} + m_{yy}/n_{yy}$.
B ... $(m_{xx} + m_{yy})/(n_{xx} + n_{yy})$.
C ... largest of m_{xx}/n_{xx} and m_{yy}/n_{yy} .
D ... $\begin{bmatrix} e_{xx} & e_{xy} \\ e_{yx} & e_{yy} \end{bmatrix}\begin{bmatrix} n_{xx} & n_{xy} \\ n_{yx} & n_{yy} \end{bmatrix} = \begin{bmatrix} m_{xx} & m_{xy} \\ m_{yx} & m_{yy} \end{bmatrix}$.

23 We make a 1/10 scale model of a timber grid shell design. This goes well and just 2 of the 1000 laths broke. How many laths will break building the real grid shell?

A Also 2 because $\sigma_{xx} = \frac{1}{2} E\, 10t \frac{k_{xx}}{10}$.
B More than 2 because scale model laths have more knots than real laths.
C At least 10 because $\sigma_{xx} = \frac{1}{2} E\, 10t\, k_{xx}$.
D Probably all laths will break because $\sigma_{xx} = \frac{1}{2} E\, 10t\, k_{xx}$ and  .

24 In-extensional deformation is a special case of …

A … extensional deformation.
B … the middle surface is not stretched or shrunk in any direction.
C … there-is-no-change-in-Gaussian-curvature.
D … a plate behaving like a shell.

25 A monstar is …

A … a particular trajectory pattern.
B … a point in a shell that needs extra stress checks.
C … a point in which principal values are the same.
D … an umbilic with two ridges.

26 Shells with a negative Gaussian curvature are often **not** sensitive to imperfections. What causes this?

A They can be built without imperfections.
B Tension dominates compression.
C They already have minimal surfaces.
D They buckle like flat plates.

27 Often we use a knockdown factor of 1/6. What is this based on?

A Nonlinear finite element analysis of various shells with realistic imperfections
B 172 Aluminium cylinder test
C The solution to the shell differential equation for buckling
D Analyses by Von Karman and Qian

28 Consider the course handout on shell dynamics, table 25, page 161. Equation j is different from equations f, g, h, i. What causes it to be different?

A In-extensional deformation
B The cylinder boundary conditions vary.
C The j cylinder vibrates in a different normal mode.
D A mistake must have been made because the cylinder length is not in the j equation.

29 Consider the course handout on shell dynamics. The shell of figure 103 is also present in figure 107. Which curve?

A curve h
B curve f
C curve g
D curve i

30 Write a good shell multiple choice question for next year's exam. The question needs to be not too easy and not too difficult. The question needs to be not just copied and not copied-and-modified-a-bit. (Unless the latter modification is substantial and original.)

Answers

1 A 100%
2 D 100%
3 D 50% B 50% is wrong. For example figure 22c .....

```
> m := 1.5·p·a·t :
> G := p · 4·Pi·a^2 : # selfweight
> q := G / (2·Pi·a/2) : # vertical support reation
> n := q/2 : # menbrane force
> m/n; # excentricity
                                0.7500000000 t
> t/6.; # middle third
                                0.1666666667 t
```

4 C 50% B 38% is wrong because of Barlow's formula.
5 A 75%
6 C 100%
7 B 88%
8 C 38% D 50% is wrong because ...

```
> lm := l/10 : am := a/10 : tm := t/10 :
> w := 5/192 · p·l^4/(a^2·E·t) :   wm := 5/192 · p·lm^4/(am^2·E·tm) :
> sigma := p·l^2/(4·a·t) :   sigmam := p·lm^2/(4·am·tm) :
> n := 1/2·p·a :   nm := 1/2·p·am :
> lambda := -0.6·E·t^2/(n·a) : lambdam := -0.6·E·tm^2/(nm·am) :
> w/wm; sigma/sigmam; lambda/lambdam;
                                10
                                1
                                1.000000000
```

9 D 88%
10 D 100%
11 A or D 25% A 25% is variant because exchanging *x* and *y* will change the sign.
B 50% is invariant.
C 25% is invariant.
D 0% is variant because $\kappa_{xx}\kappa_{yy} - (\frac{1}{2}\rho_{xy})^2$ is invariant.
12 C 0% A 50% is done by the constitutive equations.
B 50% is done by the kinematic equations.
13 D 88%
14 A 75% kNm/m = kN/m² · m · m
15 B 50% C 50% is wrong because the concentrated shear force is m_{xy} in the direction of the edge and on a free or hinged edge the bending moment is zero. (This is not a fixed edge; in a fixed edge the principal moments have the same sign.)
16 C 63% B 25% is wrong. We choose 4 boundary conditions and each has 2 options.
17 C 75% B 25% is wrong. ...

```
> a := 13.·m/2 : t := 100·mm :
> m := 1000·mm : assume(mm > 0) :
> li := 2.4·sqrt(a·t);
                        li := 1934.941860 mm
> h := li/6;
                        h := 322.4903100 mm
```

18 D 50% B 50% is wrong because it really is not important anymore.

19 C or D 100% C 25% is right because the most used element is O(h^2), so add 1/3 of the difference.
D 75% is right because assuming O(h) elements, so add the difference, is safe.

20 B 0% …
```
> y1 := -1·ln(0.20) :
> y2 := -1·ln(0.10) :
> y3 := -1·ln(0.05) :
> y2 - y1;
                    0.693
> y3 - y2;
                    0.693
```

21 B 100% …
```
> a := 12·8/4;
                    a := 24
> a/0.010;
                  2400.000000
```

22 D 63% A 13% is wrong because this quantity does not need to be within the middle third.
B 0% is invariant but adding forces in different directions has no physical meaning.
C 25% is wrong because it depends on the direction of the *x* and *y* axis.
D 63% is right because a tensor has a core that does not depend on the direction of the *x* and *y* axis.

23 A 38% The equation shows that the stress does not change when scaling up.
B 0% is wrong because model laths have no knots at all. A model lath with a knot part would break without load.
C 0% is wrong because the real curvature is 1/10 of the model curvature.
D 63% is wrong. If two laths have broken and the stress is increased by a factor 10, then it is likely that all laths break. However, the stress is not increased by a factor 10.

24 C 75% B 25% is wrong because this is the definition of in-extensional deformation.

25 A 100%

26 D 38% B 63% is wrong because it also occurs when compression dominates tension.

27 B 88%

28 A 50% B 25% or C 25% are wrong because these are not specific to situation j.

29 B 75% diaphragm boundary conditions

30 100%

The grade is (40*s* – 290)/91 s rounded to the closest halve, where *s* is the number of correct answers.

