Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CIE4143 Shell Analysis
Tuesday 27 June 2023, 13:30 – 16:30 hours

Choose A, B, C or D.

1. What is a spline?

   A a parabolic pressure line
   B a flexible rod
   C a funicular
   D a polynomial of the fourth order

2. The main structure of a small submarine is a cylinder of carbon fibre. Its diameter is 1500 mm. The wall thickness is 20 mm. What is the largest material stress at 4000 m depth?

   A 40 N/mm^2
   B 200 N/mm^2
   C 750 N/mm^2
   D 1500 N/mm^2

3. The stresses in supported shell edges are often large. How do we deal with this?

   A We build the shell on roller supports.
   B We increase the shell thickness.
   C We accept plastic deformation.
   D We strengthen the shell supports.

4. Consider Palazzetto dello sport in Rome. What is its mean curvature?

   A 0.32 / m
   B 0.032 / m
   C 0.0032 / m
   D 0

5. The geometry of a shell is described by k_{xx}, k_{yy}, k_{xy}, α_x and α_y. One could think that we can create a shape by freely choosing five functions of u and v for these quantities. However, this is not true. Which relation needs to be fulfilled?

   A The shell membrane equations and boundary conditions
   B The chain rule of differentiation
   C The equation of Gauß and the equations of Codazzi
   D The compatibility equation

6. A dome is loaded by self-weight only. Radius a = 50 m, thickness t = 15 mm, mass density ρ = 7000 kg/m^3. What is the stress in the top?

   A -0.18 N/mm^2
   B -1.75 N/mm^2
   C -3.50 N/mm^2
   D -26.25 N/mm^2

7. Consider a shell with supports and loading. We need to check whether the stresses are too large. The result depends on the ...

   A ... global coordinate system $\bar{x}$, $\bar{y}$, $\bar{z}$.
   B ... local coordinate system x, y.
   C ... curved coordinate system u, v.
   D ... material properties.

8. The shell membrane equations can describe ...

   A ... the deflection of a dome.
   B ... the influence length of the boundary conditions.
   C ... moments in the shell wall.
   D ... buckling of the shell wall.

9. We perform a small-scale experiment of a simply supported tube loaded by self-weight. All dimensions are 1/10 of the real tube. The model material is the same as the real material.

   What is the ratio of the real *deflection* and that of the model?
   What is the ratio of the real *strength* and that of the model?
   What is the ratio of the real *elastic edge disturbance stress* and that of the model?

   A 10; 10; 1
   B 10; 10; 10
   C 10; 1; 1
   D 100; 0.1; 10

10. A shell part has curvatures k_{xx} = 1/(200 m), k_{yy} = 1/(150 m), k_{xy} = -1/(180 m) and membrane forces n_{xx} = 99 kN/m, n_{yy} = -100 kN/m, $n_{xy} = n_{yx}$ = 55 kN/m. The shear forces v_x and v_y can be neglected. The shell thickness is 80 mm. What load does this shell carry?

    A p_z = 0.78 kN/m^2
    B p_z = -0.25 kN/m^2
    C p_z = -0.81 kN/m^2
    D p_z = -1.00 kN/m^2

11. The Sanders-Koiter equations can be written as one differential equation, provided we make several approximations. If we do not make any approximation to the Sanders-Koiter equations, substitutions lead to more than one differential equation? How many?

    A 2
    B 3
    C 14
    D 21

12. For most shell structures, the Sanders-Koiter equations cannot be solved analytically. What causes this?

A We do not know the exact boundary conditions.
B A general solution has not been found; it probably does not exist.
C The Sanders-Koiter equations are incomplete for many shells.
D The mathematics takes too much time.

13. What is approximately the stiffness of a ping pong ball?

A $p_z =$ 1.0 N/mm^3 × u_z
B $p_z =$ 1.3 N/mm^3 × u_z
C $p_z =$ 1.5 N/mm^3 × u_z
D $p_z = 1.3 \times 1.5$ N/mm^3 × u_z

14. In a plate edge, m_{xx} = 1 kNm/m, m_{yy} = 2 kNm/m, m_{xy} = 3 kNm/m. What is the concentrated shear force?

A 0.24 kN
B 2.64 kN
C 3.00 kN
D -1.75 kN

15. Consider a shell edge with the local *y* axis pointing outwards perpendicular to the edge. v_x = 15 kN/m, v_y = 3 kN/m, m_{xy} = -30 kNm/m and t = 180 mm.
What is the shear stress in the edge?

A -4.26 N/mm²
B 0.13 N/mm²
C 4.52 N/mm²
D 13.32 N/mm²

16. A shell edge is in the local *x* direction. The *y* direction points into the shell. We impose an edge displacement u_x = 3 mm. What is the corresponding support reaction?

A n_{yx}
B n_{xy}
C $k_{xx} m_{xy} - n_{yx}$
D v_x

17. A tube edge is fixed. The tube is loaded by a radial load p_z. An edge disturbance occurs. What is the peak bending stress?

A p_z
B $\sqrt{3}\,\dfrac{a\,p_z}{t(1-\nu)}$
C $\dfrac{p_z}{\frac{1}{6}t^2}$
D ∞

18. A hypar has straight edge beams with lengths of 10 m. The shell thickness is 6 cm. Three corners are in a plane and one corner has a distance of 2 m to this plane. What is the influence length?

A 0.4 m
B 4 m
C 7 m
D 14 m

19. A hypar has a length of 13 m and a width of 10 m. The shell thickness is 10 cm. Three corners are in a plane and one corner has a distance of 2 m to this plane. How do we classify this shell?

A membrane
B thin
C thick
D very thick

20. A shell is modelled with flat quadrilateral finite elements. However, some elements are not flat but twisted. What is the right thing to do?

A Ignore warnings generated by the program.
B Replace twisted quadrilaterals by two triangles.
C Generate a new mesh in the principal curvature directions.
D Use Irons-Loof elements.

21. We want computation results with a finite element error smaller than 10%. Which elements should we choose?

A Elements with an error of O(1)
B Elements with an error of O(h)
C Elements with an error of $O(h^2)$
D Elements with an error of $O(h^3)$

22. Consider a weld between a thin shell part and a thinner shell part. So, there is a jump in the thickness of the shell. The weld is in the *x* direction. A finite element program can remove jumps by averaging the values around a node. For which result should we *not* do this?

A n_{xx} in weld nodes
B n_{yy} in weld nodes
C n_{xy} in weld nodes
D n_{yx} in weld nodes

23. Which can be plotted by Mohr's circle?

A The Mohr-Coulomb unity check
B The shell membrane equations
C The Gaussian curvature
D The membrane force tensor

24. Which of the following depends on the direction of the local coordinate system?

A Sanders-Koiter equations
B Von Mises stress check
C Required reinforcement
D Smallest natural frequency

25. Somebody designed reinforcement for a shell part n_{sx} = 290, n_{sy} = 340 kN/m (material factor included). The membrane forces are n_{xx} = 233, n_{yy} = 59, n_{xy} = -123, n_{yx} = -132 kN/m (load factor included). Is the reinforcement sufficient? Is the reinforcement close to optimal?

A Yes, Yes
B Yes, No
C No, Yes
D No, No

26. The laths of a timber grid shells point in the principal curvature directions. Why?

A Laths can be bend k_{xx} but not much twisted k_{xy} without breaking.
B The Gaussian curvature k_G needs to be uniform for an even stiffness to point loads.
C The particle spring method does not include torsion deformation k_{xy} .
D The giants may be not available for spring back analysis.

27. The differential equation for shell buckling is derived from the Sanders-Koiter equations. To this end, Sanders-Koiter equation 1 is improved for large displacements. The 20 other equations are not improved. Why?

A Equilibrium of the deformed shell contributes most to buckling.
B The other equations are already accurate for larger displacements.
C We do the same in the derivation of the Euler column differential equation.
D We cannot solve a more complicated partial differential equation.

28. Consider a cylinder. We hit it with a hammer and measure a natural frequency of 9 Hz. Subsequently, the cylinder is loaded by an axial force. We increase the load until the cylinder buckles at 22 kN. We now take a similar cylinder, load it up to 10 kN in the same way as before and hit it with a hammer in the same way as before. What is the natural frequency?

A 6.6 Hz
B 8.3 Hz
C 9.0 Hz
D 10.9 Hz

29. A structure is exited in a normal mode. The load is removed and the stress amplitude is measured. It is 23.7 N/mm^2. Ten peaks later the stress amplitude is 1.91 N/mm^2. What is the damping ratio of this mode? [1]

A 0.21 %
B 1.00 %
C 6.51 %
D 99.50 %

30. Write a good shell multiple choice question for next year's exam. The question needs to be not too easy and not too difficult. The question needs to be not just copied and not copied-and-modified-a-bit. (Unless the latter modification is substantial and original.)

Answers

1 B 80% of the answers were correct. Answer D 20% is wrong because a spline is piecewise third order polynomial.

2 D 40% … A 30% is the water pressure,
C 30% is the pressure in the axial direction,
D is the pressure in the hoop direction calculated with Barlow's formula.

3 C 90%

4 B 70%

5 C 10% … A 30%, B 20%, D 40%

6 B 60%

7 D 60%

8 A 100%

9 D 20% … C 80% ……

$$> lm := \frac{l}{10} : am := \frac{a}{10} : tm := \frac{t}{10} : pm := \frac{p}{10} :$$

$$> w := \frac{5}{192} \cdot \frac{p \cdot l^4}{a^2 \cdot E \cdot t} : wm := \frac{5}{192} \cdot \frac{pm \cdot lm^4}{am^2 \cdot E \cdot tm} :$$

$$> \mathrm{lambda} := \frac{f \cdot 4 \cdot a \cdot t}{p \cdot l^2} : lambdam := \frac{f \cdot 4 \cdot am \cdot tm}{pm \cdot lm^2} :$$

$$> m := 1.5 \cdot p \cdot a \cdot t : mm := 1.5 \cdot pm \cdot am \cdot tm :$$

$$> \mathrm{sigma} := \frac{m}{\frac{1}{6} t^2} : sigmam := \frac{mm}{\frac{1}{6} tm^2} :$$

$$> \frac{w}{wm} ; \frac{\mathrm{lambda}}{lambdam} ; \frac{\mathrm{sigma}}{sigmam}$$

$$100$$

$$\frac{1}{10}$$

$$10.00000000$$

10 A 80%

11 B 10% … A 30%, C 30%, D 30%, We can substitute all S-K equations in the equilibrium equations for *px*, *py* and *pz*.

12 B 30% … A 40% is wrong because we do know the shell boundary conditions.

13 D 20% … C 60% is wrong because the graph shows a factor of approximately 1.3 between the peak displacement and the peak force.

14 B 100%

15 C 80%

16 C 90%

17 B 90%

18 C 80%

19 B 100%

20 B 70% … C 30% is wrong because this functionality is not available in finite element programs.

21 D 0% … C 80% is wrong because we choose the most accurate element available unless just testing.

22 A or C 30% … B 50% is wrong because nyy_left_of_the_joint = nyy_right_of_the_joint.

23 D 50% … C 50% is wrong because the Gaussian curvature is not proportional to anything in Mohr's circle.

24 C 70%

25 D 60% … C 40% is wrong …

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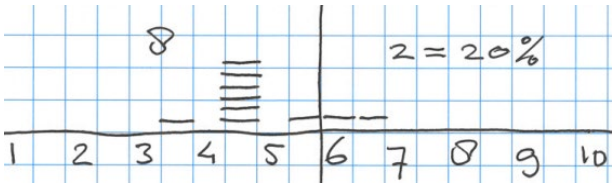
> nsx := 290·kN/m : nsy := 340·kN/m :
> nxx := 233·kN/m : nyy := 59·kN/m : nxy := -123.·kN/m : nyx := -132·kN/m :
> nxx < nsx; # OK
                              233 kN~/m~ < 290 kN~/m~
> nyy < nsy; # OK
                               59 kN~/m~ < 340 kN~/m~
> nxy·nyx < (nsx - nxx)·(nsy - nyy); # not OK
                          16236. kN~^2/m~^2 < 16017 kN~^2/m~^2
> nsx := nxx + sqrt(nxy·nyx); # Selected should be 24 % larger
                                nsx := 360.4205635 kN~/m~
> nsy := nyy + sqrt(nxy·nyx); # Selected should be 45 % smaller
                                nsy := 186.4205635 kN~/m~
```

26 A 90%
27 A 80%
28 A 70%
29 B 100%
30 100%



The grade is $\frac{40s-290}{91}$ rounded to the closest halve, where *s* is the number of correct answers.

The grade formula above is derived as follows. A student with grade *g* knows 3*g* answers. This student guesses $29 - \frac{29}{10}g$ answers. The expected number of correct answers is $s = 3g + (29 - \frac{29}{10}g)/4$, from which *g* can be solved.

Endnotes

[1] Question by Nora Kovacs, June 2022