Model answers

Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CIEM5301 Shell Structures
Thursday 5 November 2026, 9:00 – 12:00 hours

Find and correct the errors in this calculation.
You do not need to improve the design.
However, correct suggestions for improvement will yield points too.

A reinforced concrete hypar roof will be built in Arnhem (Fig. 1).
The roof is supported by heavy edge beams, which are supported by columns.
The roof will cover a swimming pool.

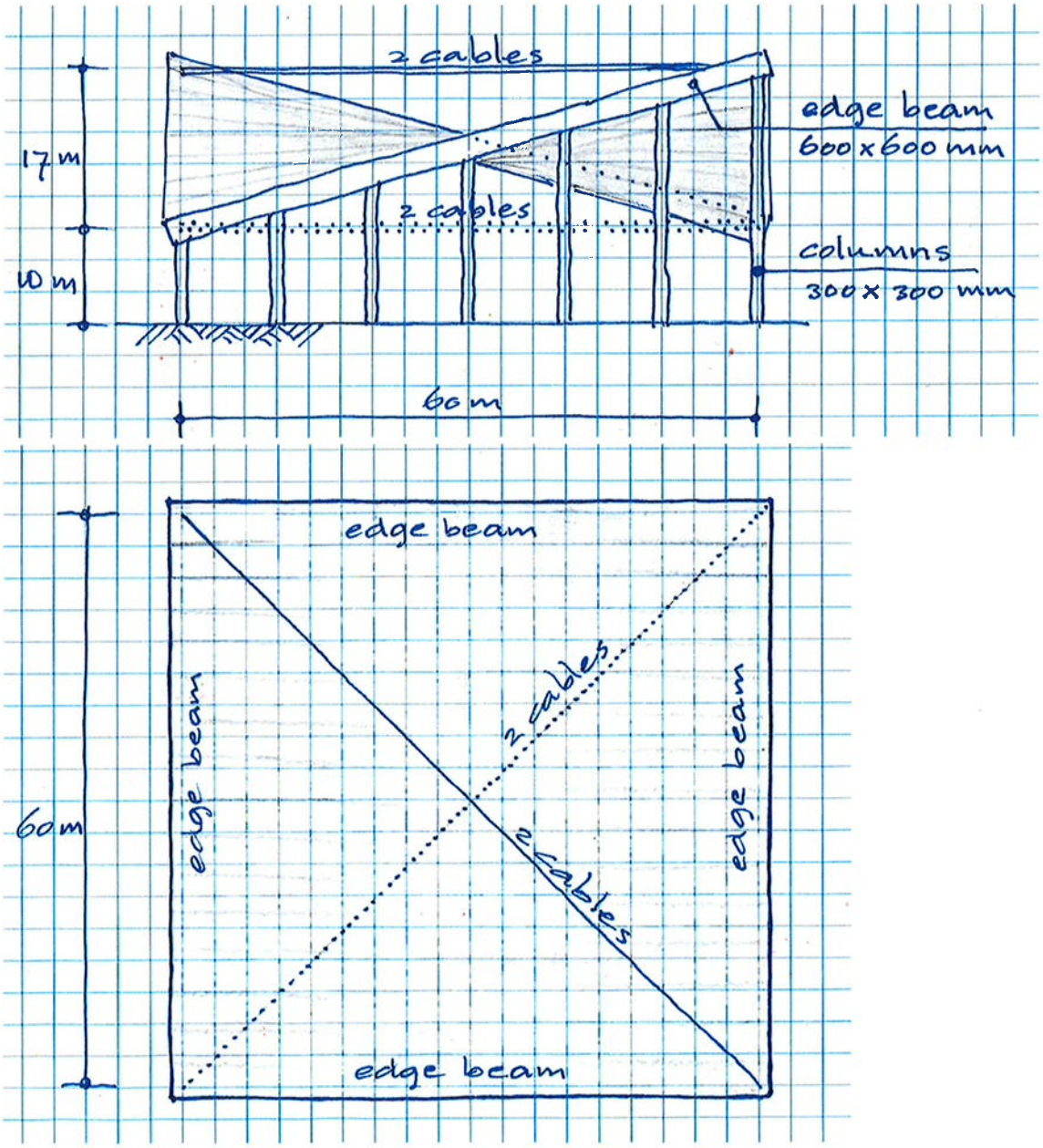


Figure 1. Hypar roof

span ..... l = 60 m

sagitta ..... s = 17 m

hypar thickness ..... t = 150 mm

Young's modulus ..... E = 10 000 N/mm² ✓ (cracked reinforced concrete)

Poison's ratio ..... ν = 0 ✓ (concrete lateral expansion is absorbed by cracks)

concrete strength ..... fc = 35 / 1.5 = 23.3 N/mm² ✓

reinforcing bars ..... 12 mm diameter, 150 mm c.t.c. spacing, 4 layers (Fig. 2)

steel strength ..... fs = 500 / 1.15 = ~~478~~ 435 N/mm²

edge beam ..... 600 x 600 mm

cable diameter ..... 60 mm

Load ½ point

self-weight	2400 kg/m³ x 0.150 m x 10 N/kg = ...	~~2.88~~ 3,60 kN/m²
insulation	~~10~~ 40 kg/m³ x 0.100 m x 10 N/kg = ...	~~0.01~~ 0.04 kN/m²
asphalt water proofing	~~2000 kg/m³ x 0.012 m~~ 10 kg/m² x 10 N/kg = ...	~~0.24~~ 0.10 kN/m²
steel fasteners	0.050 kg x 4 / m² x 10 N/kg = ...	0 kN/m² ✓
inside decoration		0.50 kN/m²
		---------- +
		~~3.05~~ 4.24 kN/m²
wind load	at 20 m in Arnhem	0.75 kN/m²

↕ not used

Membrane and moment capacity [4] 1 point

tension ..... nxx = ¼ π (12 mm)² x fs / (150 mm) = ~~332~~ 656 N/mm = ~~332~~ 656 kN/m (2 layers; 435)

bending ..... mxx = $\frac{nxx}{2}$ (t – 30 mm – ~~d / 2~~ 0,9) = 30 ~~N~~ 000 = 30 kNm/m (-12-6) ✓

compression nxx = –fc t = $\frac{35.0}{1,5}$ N/mm² x 150 mm = ~~–5.25~~ 3500 N/mm = ~~–5250~~ 3500 kN/m

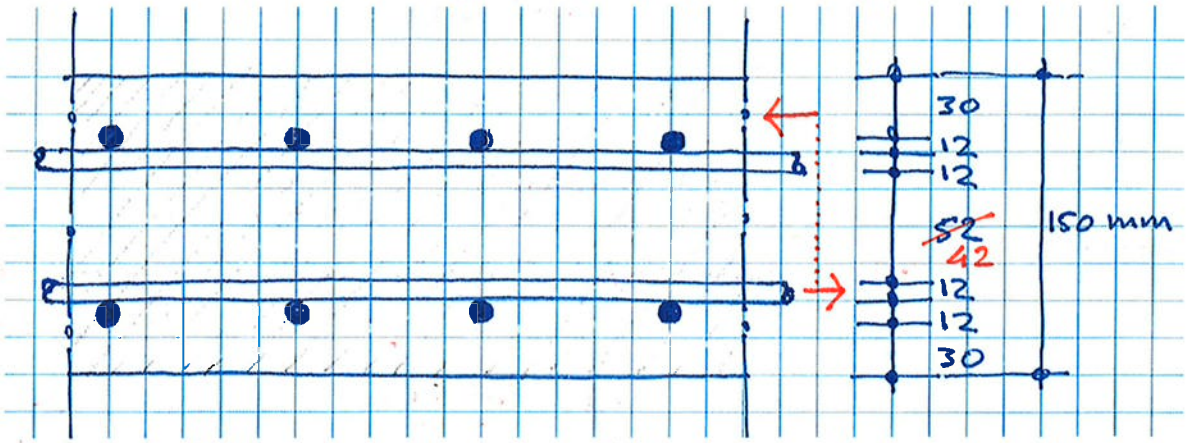


Figure 2. Reinforcing bars

Hypar buckling capacity 1 point $\frac{a}{t} = \frac{106}{0.150} = 706 \approx 500$ · [1, p. 1] .... OK

hypar radius of curvature ..... $a = \frac{(\frac{1}{2}l)^2}{\frac{1}{2}s} = \frac{30^2}{8.5} = 106$ m ..... [1, p. 101] ✓

local critical buckling load .... $n_{cr} = -0.6\frac{Et^2}{a} = -0.6\frac{10000 \times 150^2}{100\,600} = -8\,946$ kN/m ... [1, p. 139] ✓

(provided there is no inextensional deformation)

buckling load including imperfections ... $n_{ult} = \frac{1}{6}n_{cr} = -\frac{1}{6} \times 8\,946 = -1\,491$ kN/m . [1, p. 145] ✓

Edge beam capacity and cable capacity

edge beam strength ..... - (600 mm)² x *fc* = - 8400 000 N = - 8400 kN ✓

cable strength ..... ¼ π (60 mm)² x *fs* = 2488 000 N = 2488 kN

Load combinations [3] ½ point

SLS ... permanent load

... permanent load & wind load ..... not governing ← additional deformation

ULS ... 1.35 permanent load

... 1.35 permanent load & 1.50 wind load ..... not governing because wind counteracts self-weight

... 1.35 perm. load & 1.50 snow load & wind load

Deflection (SLS)

There are no formulas for the deflection of hypars.
It is expected that the deflection is very small
The finite element analysis confirms this (see p. 5). ..... OK ✓

$n_2 = \frac{n_{xx}+n_{yy}}{2} - \sqrt{\left(\frac{n_{xx}-n_{yy}}{2}\right)^2 + n_{xy}^2} = -303 \overset{?}{\not<} 213$ kN/m .... NOT OK

[1, p.97] buckling

Forces and moments (ULS) 3 points

membrane forces .... $n_{xx} = 0$ ..... [1, p. 102] ✓

$n_{yy} = 0$

$n_{xy} = -\frac{1}{2}pa = -\frac{1}{2} \times 1.35 \times \overset{4.24}{\cancel{3.05}}$ kN/m^2 $\times 106$ m $= \overset{-303}{\cancel{310}}$ kN/m ~~< 332 .... OK~~

[1, p.104] $n_{sx} = n_{xx} + |n_{xy}| = 0 + 303 = 303 < 656$ .... OK

tension

hypar edge length ... $l_{edge} = \sqrt{l^2 + s^2} = \sqrt{60^2 + 17^2} = 62$ m ✓

edge moment ..... $m_{xx} = -0.511\frac{p}{l^2}(a\,t\,l)^{\cancel{\frac{3}{4}}\,\frac{4}{3}} = -0.511\frac{\overset{1.35\times4.24}{\cancel{3.05}}}{62^2}(106 \times 0.15 \times 62)^{\cancel{\frac{3}{4}}\,\frac{4}{3}} =$ [1, p. ~~119~~ 127]

$= -\overset{7.4}{\cancel{5.6}}$ kNm/m < 30 kNm/m ... OK

edge beam force ..... $N = n_{xy}\,\cancel{\frac{1}{2}}\,l_{edge} = -\overset{303}{\cancel{1.35 \times 3.05}}$ kN/m $\times \cancel{\frac{1}{2}} \times 62$ m $= -\overset{18897}{\cancel{4836}}$ kN/m

900 x 900 would be OK.

~~<~~ 4800 ..... NOT OK

cable force ..... $T = -N\sqrt{2} = \overset{18897}{\cancel{4836}}$ kN $\sqrt{2} = \overset{26725}{\cancel{3839}}$ kN < 2 x $\overset{1229}{\cancel{2488}}$ ..... NOT OK

22 cables would be OK

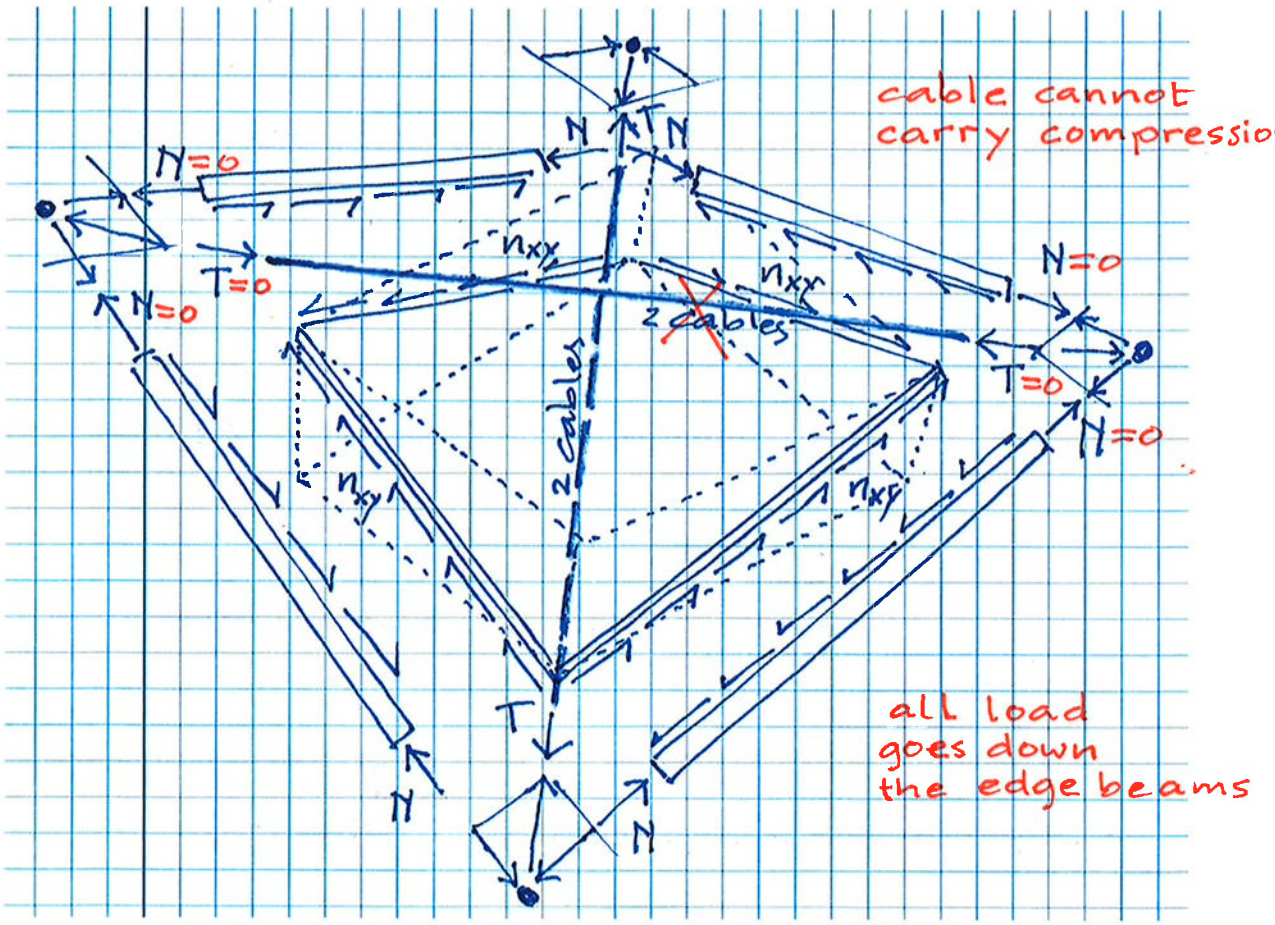


Figure 3. Exploded view of the hypar, edge beams and cables

Finite element model 2 points

buckling

~~influence~~ length..... $l_f = 2.4\sqrt{at} = 2.4\sqrt{62\text{ m} \times 0.14\text{ m}} = 7.1\text{ m}$ ..... [1, p. 74]

recommended element size..... $h = \frac{1}{6} l_f = \frac{1}{6} \times 7.1\text{ m} = 1.2\text{ m}$ ..... [1, p. 75]

applied element size ..... $h = 1$ m

theory ..... Kirchhoff (~~includes~~ no out of plane shear deformation)
Young's modulus ..... $E = 10\,000$ N/mm²
Poison's ratio ..... $\nu = 0$

The hypar edges are fixed because the large edge beams fix the thin hypar at its edges.

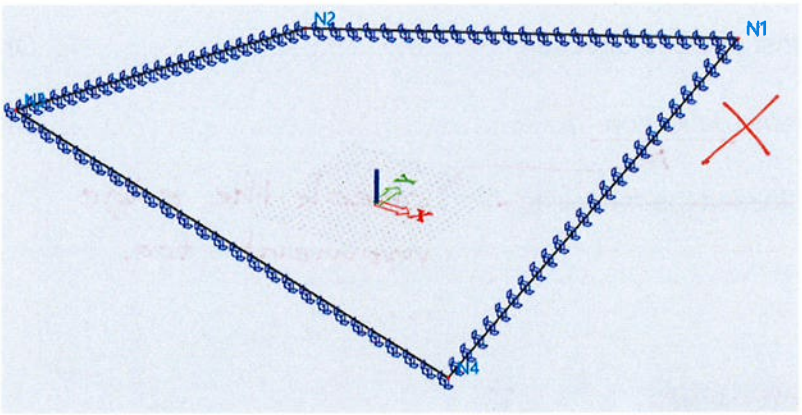


Figure 3. Boundary condition

The edge beams are not stiff! They need to be included in the model. The bottom 11 cables need to be included too. Their deformation changes the force flow.

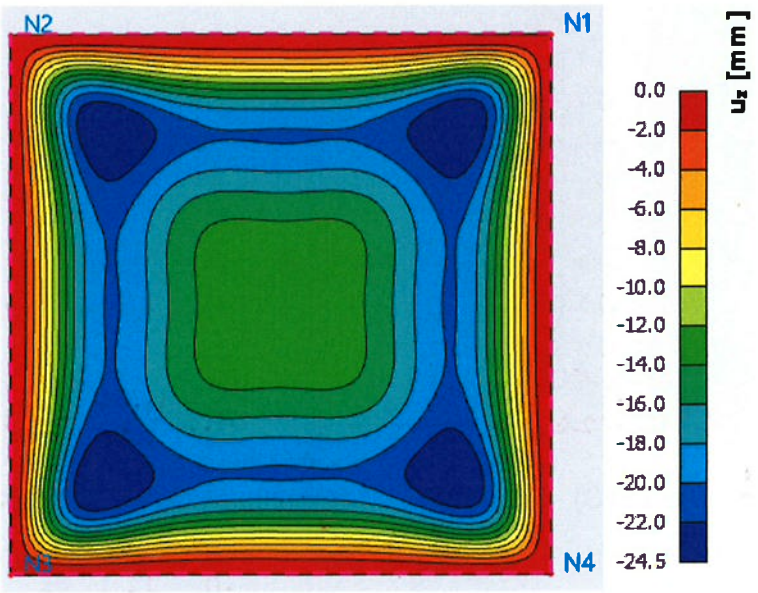


Figure 4. Deformation due to self-weight

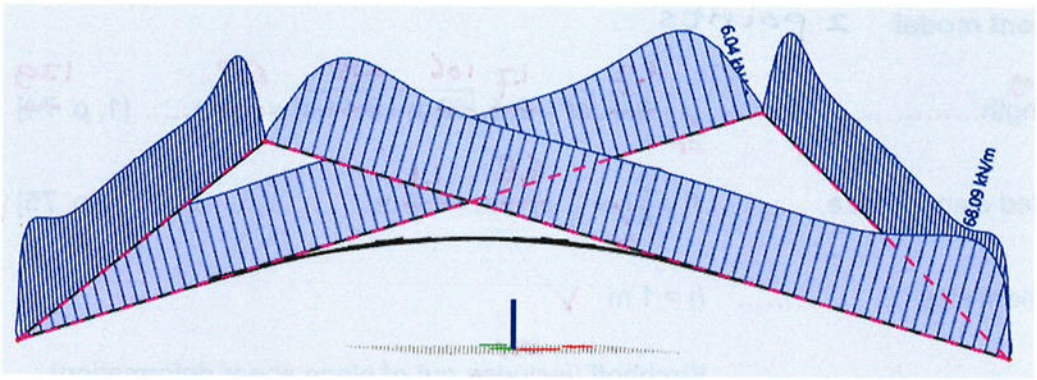


Figure 5. Support reactions due to self-weight

The displacement is in the load direction. ..... OK ✓

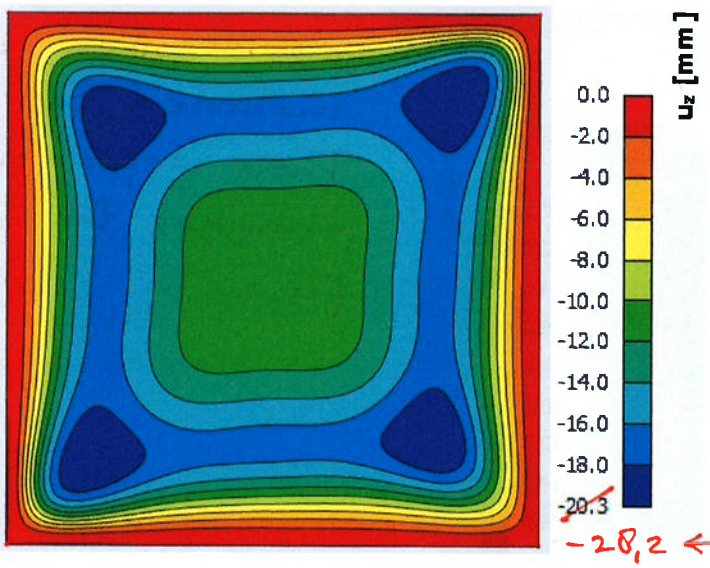
The edge displacement is between -2 mm and 0. ..... OK ✓

The support reactions are against the load direction ..... OK ✓

0.5 m element size gives the same deflection. ..... OK ✗

~~This shows that there is no mistake in the modelling.~~ check the edge moment too.

Response 2 points



$-28{,}2 \leftarrow -20.3 \frac{4.24}{3.05} = -28.2$

Figure 6. Deformation due to ~~-3.05~~ 4.24 kN/m² (SLS) ..... ~~-20~~ 28 mm is small ..... OK ✓

creep will double this in 10 years

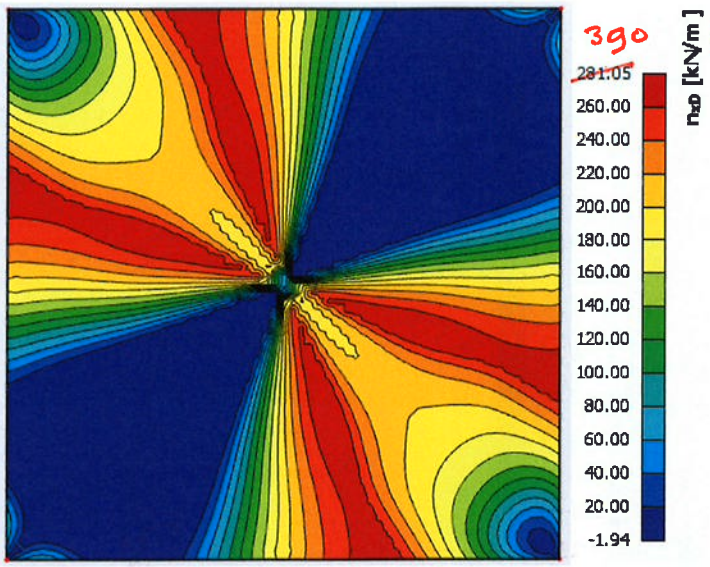


Figure 7. Membrane force *nxx* + |*nxy*| due to 1.35 x ~~-3.05~~ 4.24 = ~~-4.12~~ 5.72 kN/m² (ULS) ... [1, p. 103]

~~281~~ 390 < ~~332~~ 656 kN/m² ..... OK ✓

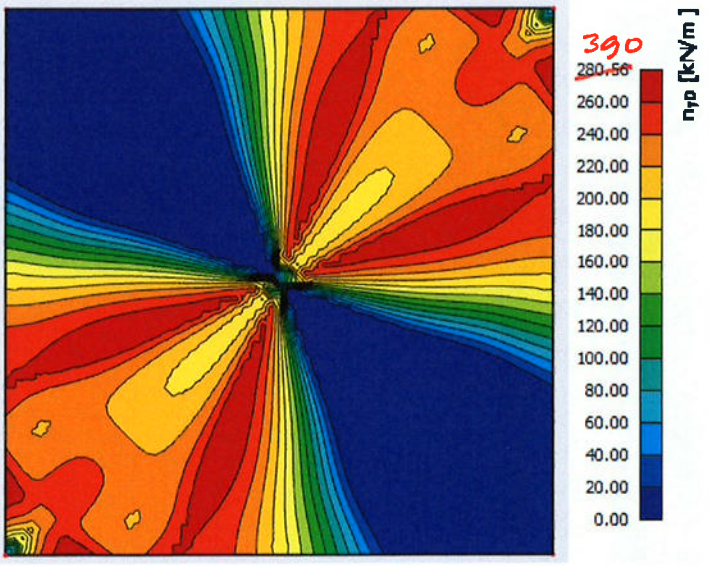
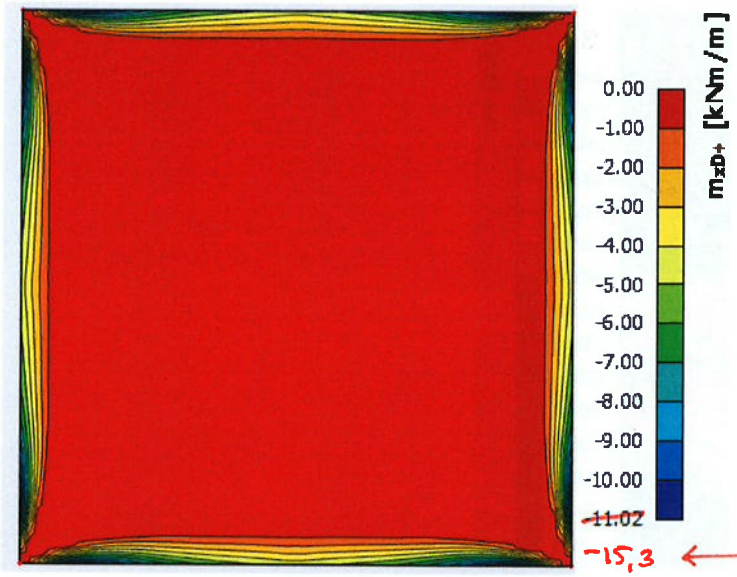


Figure 8. Membrane force *nyy* + |*nxy*| due to 1.35 x ~~-3.05~~ 4.24 = ~~-4.12~~ 5.72 kN/m² (ULS)

~~281~~ 390 < ~~332~~ 656 kN/m² ..... OK ✓



-15,3 ← 11.02 5.72/4.12

Figure 9. Moment *mxx* + |*mxy*| to 1.35 x ~~-3.05~~ 4.24 = ~~-4.12~~ 5.72 kN/m² (ULS)

~~11~~ 15 < 30 kNm/m ..... OK ✓

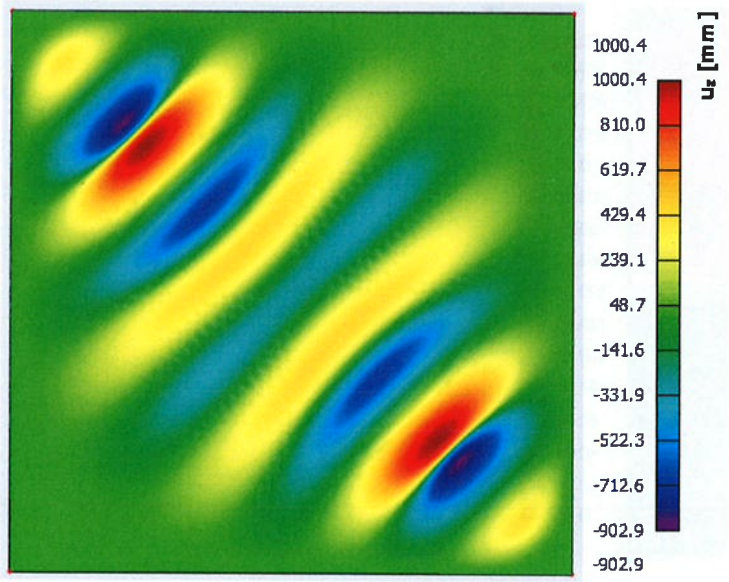


Figure 9. First buckling shape due to ~~-3.05~~ 5.72 kN/m² (~~SLS~~ ULS)

λ = 7.61, 7.61, 7.71, 7.71, 8.00, 8.00, 8.34, 8.34 ~~>~~ / 6 ..... NOT OK

7.61 3.05/5.72 = 4.06, 4.06, ...

hypar edge length ..... $l_{edge} = \sqrt{l^2 + s^2} = \sqrt{60^2 + 17^2} = 62$ m ✓

hypar area ..... $A \approx l_{edge}^2 = 62^2 = 3\,889$ m² ✓

imperfection amplitude ... $d = \frac{1}{\sim\!\!109\ 108} A^{0.3} l^{0.2} = \frac{1}{109}(3889\ \text{m}^2)^{0.3}(\sim\!\!62\ \text{m})^{0.2} = \sim\!\!0.353$ m . [1, p. 148] ✓

0.4 ↑ l_{cr} ; 6.8 ; 0.4 ; 0.33 ; 1.4

first mode imperfections of 0.35 m or -0.35 m have been added to the model shape.

✓ ✓

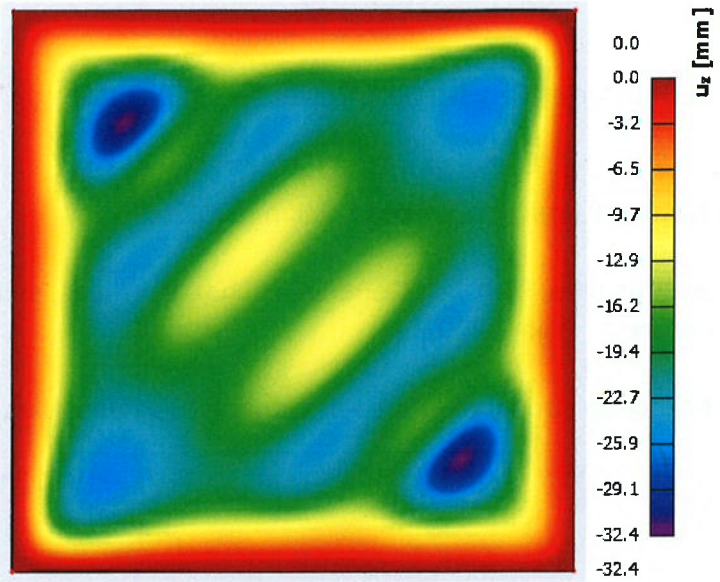


Figure 10. Geometrical non-linear deformation due to 1.35 x ~~-3.05~~ 4.24 = ~~-4.12~~ 5.72 kN/m² (~~GLS~~ ULS)

~~-32~~ ? mm is small ..... OK ✓

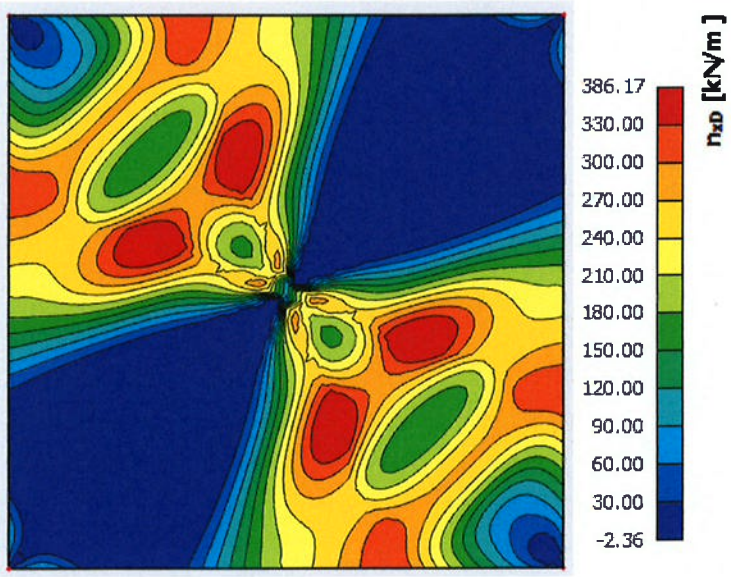


Figure 11. GNL membrane force *nxx* + |*nxy*| due to 1.35 x ~~-3.05~~ 4.24 = ~~-4.12~~ 5.72 kN/m² (ULS)

~~386~~ ? < ~~332~~ 656 kN/m² ..... OK ✓

Conclusions

The deflection due to the permanent load is acceptable. ✓

The extra deformation in a storm is acceptable. ← Js not checked.

The hypar moments and membrane forces can be carried.

The edge beams and cables can carry the forces.

The roof will not buckle.

This roof can be built.

- thicker shell
- no top cables
- larger edge beams: 900 x 900 mm
- 22 bottom cables
- better FEM model
- check snow
- check storm

Literature

1. P.C.J. Hoogenboom, Notes on Shell Structures, Reader Delft University of Technology, 2024, 215 pages

2. Anonymous, Eurocode 0, Basis of structural design, European Union, 2002

3. Anonymous, Eurocode 1, Actions on structures, Part 1-1: General actions - Densities, self-weight, imposed loads for buildings, European Union, 2002

4. Anonymous, Eurocode 2 Design of concrete structures, Part 1-1: General rules and rules for buildings, European Union, 2004