The Strength of Tubular Struts.

By Andrew Robertson, D.Sc., University of Bristol.

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[Plates 10–14.]

Introduction.

The problem of the strength of a circular tubular strut differs from that of a solid strut of the same material in that there may be a condition of instability in the wall of the tube apart altogether from that type of instability considered in the Eulerian analysis which involves only the bending of the centre line of the strut. It is found that when a thin tube is tested in direct compression up to collapse, it fails by the formation of characterisic folds in the wall, examples of which are given in the photographs, figs. 7, 10 and 11 (Plate 11). The folds are of two types depending upon the ratio of the thickness t to the radius R of the tube. Above a certain value of t/R the fold consists of a uniform bulge in which all cross sections remain circular; below this value of t/R the walls "cave in" in several places producing a series of lobes. The phenomenon has been called wrinkling, and also secondary flexure, to distinguish it from the flexure produced by the bending of the centre line of the strut.

The problem has been investigated mathematically and experimentally. On the mathematical side the best-known investigations are those of Lilly, Southwell and Dean; on the experimental side those of Lilly, Mason, Barling and Webb, and Popplewell and Carrington.* The present investigation is experimental and the greater part of it was carried out in 1915, at which date the only investigations with which the author was acquainted were those of Lilly and Mason. A paper describing the work was presented in 1915 to a Sub-Committee of the War Committee of the Royal Society and was also communicated to the Air Departments of the Army and Navy. General publication, however, was withheld until the end of the war. A short summary of the conclusions reached was given to the British Association Meeting in 1919. Since the major part of the work was completed and reported upon, experimental investigations have been published by Messrs. Barling and Webb and Messrs.

* Lilly, 'Inst. Civ. Eng. Irel.,' March, 1906; Southwell, 'Phil. Trans.,' A, vol. 213, p. 187 (1914); Dean, 'Roy. Soc. Proc.,' A, vol. 107, p. 734 (1925); Mason, 'Proc. Inst. Mech. Eng.,' parts 3, 4 (1909); Barling and Webb, 'J. R. Aeron. Soc.,' Oct. (1918); Popplewell and Carrington, 'Proc. Inst. Civ. Eng.,' vol. 203, p. 381 (1917).

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Popplewell and Carrington. As these results differ notably from the author's, some additional experiments were carried out on large tubes made from high tensile steel strip and the whole of the work is now presented in this paper. It will be convenient to depart from the strict chronological order in which the various experiments were made and examine the whole problem in the light of all the existing work known to the author.

Other Investigations.

Theoretical.—On the theoretical side Lilly obtained formulæ for the collapsing load of a tube and for the length of the characteristic wave-like fold produced when failure occurred by the formation of a uniform bulge. The formulæ are

$$p = \frac{\mathrm{E}}{\sqrt{3}}\frac{t}{\mathrm{R}}, \qquad \lambda = \frac{\pi}{4\sqrt{12}}\sqrt{\mathrm{R}t},$$

where p = collapsing load per square inch, E = Young's modulus, t = thickness, R = radius, λ = wave-length.

Southwell has investigated the same problem in his paper on the general theory of elastic stability and arrived at a formula identical with that of Lilly if the term $\sqrt{m^2}/(m^2 - 1)$ be added, *i.e.*,

$$p = \frac{\mathrm{E}}{\sqrt{3}}\frac{t}{\mathrm{R}}\sqrt{\frac{m^2}{m^2 - 1}}, \qquad \frac{1}{m} = \text{Poisson's ratio.}$$

Southwell has also investigated (using the theory of thin shells) the conditions under which the lobed form of failure is produced and has given the formula

$$p = \frac{\mathrm{E}}{\sqrt{3}}\frac{t}{\mathrm{R}}\sqrt{\frac{m^2}{m^2 - 1}}\left(\frac{\mathrm{K}^2 - 1}{\mathrm{K}^2 + 1}\right),$$

where K is the number of lobes formed. He has also given a curve representing the way in which the collapsing load depends on the length of the strut.

Dean has also obtained these formulæ by other methods of attack.

In a later portion of this paper detailed consideration is given to these formulæ. It is suggested that an oversight in the interpretation by Southwell of his general analytical result has led him to omit consideration of the conditions which are most likely to occur in practice and that in consequence his formula for the lobed type of failure is incorrect. Dean appears to have fallen into the same error.

It appears that, correctly interpreted, Southwell's general equation yields

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the same value for the minimum collapsing load in the lobed as in the symmetrical type of failure.

From the theoretical investigations it is clear that only very thin tubes may be expected to fail by elastic instability. Taking mild steel, for example, for which the proportional elastic limit in compression may be 20 tons per square inch, the ratio of t/R which would be required to produce failure by elastic instability under this load per square inch would be about 1/400, *i.e.*, an 8-inch tube 1/100-inch thick. Such tubes are not met with in ordinary engineering work, so that the mathematical investigations would suggest that in tubes of ordinary proportions the proportional elastic limit of the material would be reached before any question of failure by elastic instability could arise. For stresses greater than the proportional elastic limit the formulæ cease to apply, since they are, of course, based upon the assumption of linearity between stress and strain. In some materials, however, the deviation from linearity of the stress-strain curve between the proportional elastic limit and the yield is so small that if the yield stress is used the formulæ will probably give a reasonable estimate of the value of t/R above which this type of deformation would not be expected to occur as a primary cause of failure.

It should be noted that these formulæ give a value for the load which will produce a condition of instability in the walls. Strictly interpreted, this is not necessarily a collapsing load, just as in the case of a solid strut the Euler load is not really a collapsing load. If the conditions could be properly chosen it would be possible to apply the Southwell load and note the formation of the bulge or lobes, and, on releasing the load, find the tube uninjured, *i.e.*, exactly as one may test a very long strut under its Euler load. Unfortunately, the difficulty of realising the conditions is so great that if any deflection does occur the material is probably strained beyond its proportional elastic limit and complete collapse generally occurs.

Experimental Investigations.

Lilly.—The most extensive series of experiments known to the author are those of Lilly, who used tubes of mild steel varying from $\frac{3}{8}$-inch diameter 22 gauge to 1-inch diameter 18 gauge. From his results he deduced that the collapsing load was given by the formulæ

$$f = \mathrm{F} \Big/ 1 + \mathrm{K}\left(\frac{\rho}{t}\right)^2,$$

where f = load per square inch at collapse, F = strength in compression,

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K = constant = 1/60 for mild steel, ρ = radius of gyration, t = thickness. In another paper he modified this to

$$f = F \Big/ 1 + K\left(\frac{\rho}{t}\right) \qquad (K = 1/8 \text{ for mild steel}).$$

It is difficult to discuss these experiments since no stress-strain diagrams of the material in tension or compression are given and all the tubes are treated as if they were of the same material, despite the fact that the tensile strength varied from 62,000 lbs. per sq. inch to 80,000 per sq. inch.

The present author has discussed elsewhere* the importance of stress-strain diagrams in dealing with strut tests and the inadvisability of using specimens from different batches of material which have been subjected to different amounts of cold work when the object of the experiment is to ascertain how a material behaves under particular stress distribution.

Mason.—Probably the most carefully conducted tests bearing on the problem are those described by Mason in his paper on mild steel under combined stress. In the course of this research Mason carried out simple compression tests on tubes $2\frac{3}{4}$-inch bore 10 gauge (*i.e.*, $t/R = 0{\cdot}089$) and tubes 3-inch bore 14 gauge (*i.e.*, $t/R = 0{\cdot}052$). The yield in compression as obtained by an extensometer was designated Y_1 and as obtained by compasses designated Y_2. In some of the tests on the thinner tubes ($t/R = 0{\cdot}052$) apparent yielding (attributed by the author to flexure, presumably the formation of a wrinkle) was observed at a stress below that at which the yield could be detected with a pair of compasses. In several other tests, however, on specimens cut from the same tube, the difference between Y_1 and Y_2 was "only about the same difference as between similar yield stresses of the tube in tension."

It may also be noted that in one series of tests on specimens from the same tube, Y_2 was 20·5 tons per square inch, Y_1 for a tube 19 inches long was 19·25 tons per square inch and for tubes 4 inches long 14·7 and 16·1 tons per square inch respectively. In another series also the longest tube gave a value of Y_1 much greater than the shorter tubes and greater than several of the values of Y_2 on other tubes. The length of the tube in every case was many times the wave-length of a wrinkle, so that the present author considers that the anomalies given above are probably due to unknown inequalities in the stress distribution which would be greater with a short specimen than with a long one, and that the actual wrinkle (if produced) was due to larger stresses than the average load per square inch recorded.

* 'Inst. Civ. Eng.,' Selected Paper 28.

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For the other series of tubes $t/R = 0{\cdot}087$, in no case did any sign of flexure occur and the collapsing load was greater than the yield.

Barling and Webb.—These authors describe tests carried out at the Royal Aircraft Factory on annealed mild steel tubes. They defined the wrinkling stress as the stress at which the relation between the stress and the strain departs appreciably from the straight-line law, and to determine this point they used apparatus which gave autographic records of the relation between the load and the strain. They state that for $t/R =$ or $> 0{\cdot}06$ the wrinkling stress is constant at 30,000 lbs. per square inch, and for $t/R < 0{\cdot}06$ the wrinkling stress is directly proportional to t/R.

Popplewell and Carrington.—These authors carried out an investigation on high tensile steel tubes at the request of the Superintendent of the Royal Aircraft Factory. They adopted the definition of Messrs. Barling and Webb for the wrinkling stress and followed them also in using autographic diagrams, though they checked some of them with an extensometer. They tested the material in the hardened and annealed conditions.

For the hardened tubes the value of the wrinkling stress as defined above was uniform at about 50 tons per square inch between $t/R = 0{\cdot}049$ and $t/R = 0{\cdot}1$, and for values of $t/R > 0{\cdot}1$ the stress diminished by a linear law. For the annealed tubes the wrinkling stress was directly proportional to t/R up to $t/R = 0{\cdot}1$, and above this value was approximately uniform at 30 tons per square inch.

These experiments, and also those of Messrs. Barling and Webb, are open to the same criticism as Lilly's in that tubes of different sizes $\frac{3}{4}$-inch 22 gauge to 2-inch 20 gauge are treated as though they were identical material in spite of the fact that tensile tests showed that a hardened $1\frac{1}{4}$-inch 20 gauge tube had an ultimate stress of 104 tons per square inch, whilst a hardened $1\frac{1}{4}$-inch 12 gauge tube had an ultimate stress of only 75 tons per square inch. The inadvisability of this is also very clearly shown by the result Messrs. Popplewell and Carrington give of a test on a thin tube in the hardened condition which was turned down from a thick one. This thin tube ($t/R = 0{\cdot}055$) gave nearly the same value as the thick one ($t/R = 0{\cdot}17$), *i.e.*, about 36 tons per square inch as against about 39, whilst the other tubes of the same t/R as this thin one gave values of about 50 tons per square inch. It is thus clear that if all the tests had been made on tubes of the same material (*i.e.*, by turning down the thick ones) the results would have been the same over this range $t/R = 0{\cdot}055$ to $0{\cdot}17$, and there would have been no discontinuity at $t/R = 0{\cdot}1$. Moreover, the fact that over such a large range of t/R the value of what they call the

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wrinkling stress is constant is probably an indication of a stress failure rather than failure by elastic instability, since this must surely depend upon t/R.

From this review of the available experimental work it is clear that there is a very great diversity of opinion as to the value of the collapsing load for a tubular strut.

Author's Experiments.

The author's experiments have been made on a variety of different tubes, but the principal results can be stated by reference to four sets of experiments, *i.e.*,

A. Experiments on nickel-chrome steel tubes.
B. ,, ,, mild steel tubes.
C. ,, ,, large tubes made from thin steel strips.
D. ,, ,, tubes of silver spruce.

Experimental Methods.—In testing specimens in compression it is important to take precautions to ensure that the stress in the portion of the specimen under investigation is as uniform as possible. The ordinary compression plates of a testing machine are not necessarily parallel to one another at the beginning of a test, nor will they remain parallel under increased loading. In order to avoid inequalities of stress from this cause, the apparatus illustrated in fig. 1

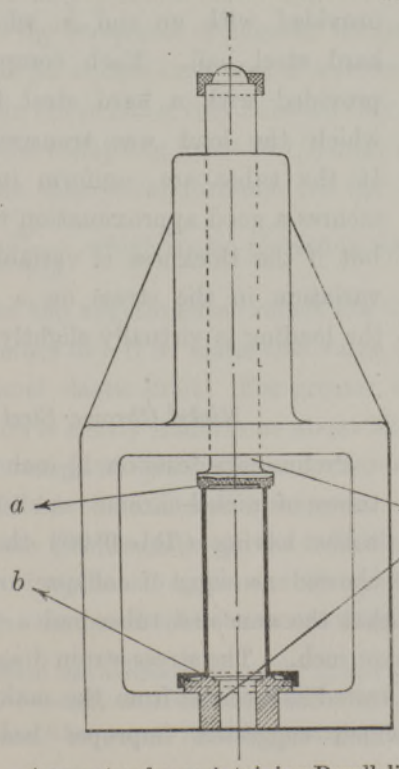

FIG. 1.—Apparatus for maintaining Parallelism of Ends ($\times \frac{1}{4}$).

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was made. The construction is clear from the diagram. Before testing, each specimen was ground against the faces a and b, so as to secure good initial bedding. A special extensometer was constructed for a gauge length of 1 inch. The construction is obvious from the diagrammatic sketch in fig. 2

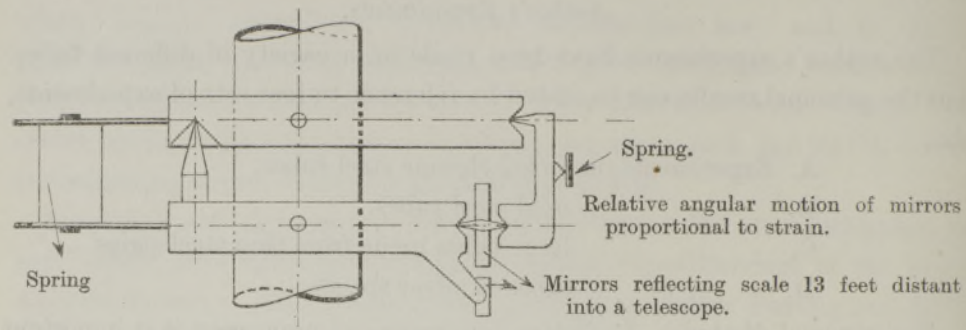

Fig. 2.—Diagrammatic Sketch of Extensometer.

and the photographs in figs. 3 and 4 (Plate 10). This apparatus was used on all the tubes about $1\frac{1}{4}$-inch diameter. For larger specimens no similar apparatus was available, so each specimen was provided with an end in which was centred a hard steel ball. Each compression plate was provided with a hard steel flat plate through which the load was transmitted (see fig. 5). If the tubes are uniform in themselves this secures a good approximation to uniform loading, but if the thickness is variable there will be a variation in the stress on a cross section, *i.e.*, the loading is virtually slightly eccentric.

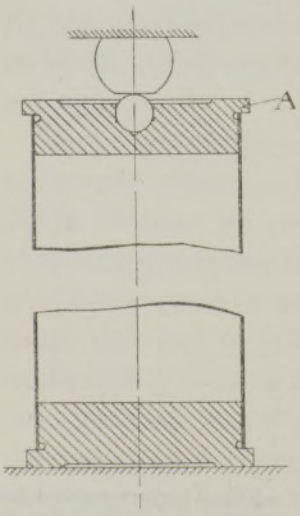

Fig. 5.

Nickel-Chrome Steel Tubes.

Preliminary tests on $1\frac{1}{4}$-inch diameter 18-gauge tubes of nickel-chrome steel indicated that for tubes having $t/R = 0{\cdot}083$ the hardened tubes showed no signs of collapse under a stress of 72 tons per square inch, and that the annealed tubes had a yield which varied from 28 to 39 tons per square inch. The stress-strain diagrams of specimens from different tubes when tested as received from the makers indicated very considerable differences which suggested improper heat treatment. An

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investigation was therefore made into the heat treatment of this material. The investigation showed the importance of three conditions, *i.e.*,

(*a*) Annealed.*
(*b*) Hardened from 800° C.
(*c*) Hardened from 800° C. and then tempered at 400° C.

Three series of tests were, therefore, carried out on tubes with varying values of t/R. In every case the specimens in each series were from the same tube and the various ratios of t/R were obtained by machining or grinding the tube.

The stress-strain diagrams and the collapsing loads plotted against t/R are given in fig. 6 and photograph of typical collapsed specimens in fig. 7 (Plate 11).

In the case of the thinnest tubes, which were only 0·005 inch thick, the collapsing load only was obtained.

For the annealed specimens the proportional elastic limit varied from 35 to 40 tons per square inch, and the yield about 45 tons per square inch. For the hardened specimens the proportional elastic limit is low, *i.e.*, from 28 to 43 tons, but there is no yield, the stress-strain curve resembling in shape that of cast iron. For the tempered specimens the proportional elastic limit is high, 58 and 78 tons per square inch, and at a stress of 80 to 90 tons per square inch the stress-strain curve has a very considerably increased slope.

The relation of the collapsing loads to t/R may most easily be summarised by reference to the theoretical formulæ for the collapse by wrinkling, *i.e.*, $p = \frac{E}{\sqrt{3}} \frac{t}{R} \sqrt{\frac{m^2}{m^2-1}}$, which is also plotted in fig. 6. For the tempered and annealed specimens the experimental values are about 0·4 of the theoretical ones up to such values of t/R as make this value (*i.e.*, 0·4 p) slightly greater than the proportional elastic limit. For greater values of t/R the collapsing load per square inch is nearly uniform at about 92 tons per square inch up to $t/R = 0·08$ for the tempered specimens, and rises slightly with t/R up to 50 tons at $t/R = 0·08$ for the annealed specimens.

This generalisation would also cover the mild steel tubes, as is shown by the results of a series of machined specimens also plotted in this diagram.

The results on the hardened tubes are more irregular, probably the low result

* The actual treatment was hardening from 800° C. and subsequent tempering at 650° C., and would be more correctly described as "softened." The term annealed has been retained as it is used in other papers for this condition.

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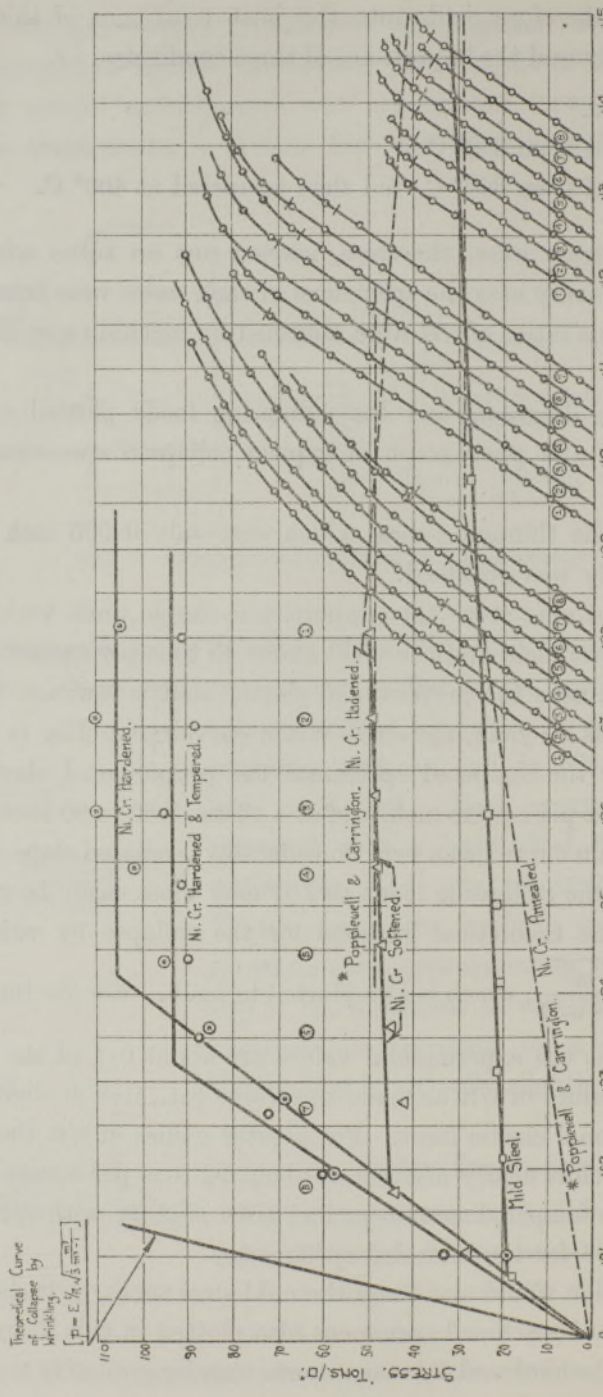

FIG. 6.—Compression Tests on Nickel-Chrome Steel Tubes.*

* These tests are for the first departure from linearity and should be compared with the stress-strain diagrams.

for $t/R = 0{\cdot}01$ is due to unequal stress distribution. Apart from this one, however, the results are not greatly different from those of the tempered speci-

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mens up to $t/R = 0{\cdot}04$. For larger values of t/R the collapsing load is about 105 tons per square inch. In considering the results it must be remembered that owing to the enlarged ends there is an unknown additional stress produced which is not included in the theoretical analysis.

These results may be compared with those of Messrs. Popplewell and Carrington, which were made on tubes to the same specification, and supplied, as were the author's tubes, by the Superintendent of the Royal Aircraft Factory. The results of Messrs. Popplewell and Carrington are shown by dotted lines in fig. 6. Their results refer to the first deviation from the straight line.

If these results be compared with the proportional elastic limits as found by the author, it will be seen that there is considerable divergence. The author considers that Messrs. Popplewell and Carrington's results are too low for the annealed tube and too high for the hardened tube, and that this is due to their method of test not securing a good distribution of stress and to their autographic diagram and extensometer being less sensitive than the author's. Moreover, they used tubes of different diameters and thicknesses and assumed that they were dealing with the same material. It has previously been pointed out that their published results show that if they had machined the specimens from the same tube they would have had no marked break at $t/R = 0{\cdot}1$ in the case of the hardened tubes. The author, therefore, dissents entirely from the conclusions which Messrs. Popplewell and Carrington drew from their own experiments.

For almost identical reasons the experiments of Messrs. Barling and Webb, which were carried out on similar lines to those of Messrs. Popplewell and Carrington, and which Messrs. Popplewell and Carrington claim are verified by their investigations, are also misleading.

Mild Steel Tubes, $2\frac{3}{4}$-inch Diameter.

For these experiments the method of loading was that shown in fig. 5. This method avoids serious inequalities of strain at the ends, and though the strain is not uniform unless the tube is of uniform thickness, an estimate of the variation can be made if the variation in the thickness is known.

For all these tests autographic records of load and strain were obtained. The actual decrease in length of the tube under test was multiplied by a system of levers so as to give on the diagram a magnification of about 8.

In taking the diagram one of the levers used to magnify the motion was kept in contact with the underside of the flange A. On account of the variation

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in thickness the tube rarely collapsed uniformly with the ends parallel, so the motion of the end of the lever does not give a correct measure of the mean longitudinal motion after yield has occurred. In a few cases the lever was either pushed away by the formation of a fold or the end tilted so much in collapsing that the diagram was reduced to the single line traced during the loading period.

The specimens were about 6 inches long and several of them were machined on the outside to give various ratios of t/R, the lowest being 0·0057. The unannealed specimens IV and V were machined over their entire length.

Compression Tests.

Unannealed Mild Steel Tubes 2·689 Int. Dia. Table I

1 Tube	2 No.	3 Area □"	4 Thickness Mean	5 Thickness Least	6 $\frac{t}{R}$	7 Collapsing Load Tons □" Actual	8 Collapsing Load Tons □" Calculated	9 Max. Stress.	10 Manner of Loading
A	I	·253	·0297	·0279	·0217	20·4			Plates
"	II	·253	·0296	·0278	·0217	20·4	21·6	21·7	Ball
"	III	·253	·0297	·0288	·0217	20·6	22·3	21·0	Ball
"	IV	·159	·0187	·0182	·0138	21·2	22·6	21·8	"
"	V	·108	·0127	·0120	·0094	18·9	21·8	20·0	"
"	II r	·107	·0126	·0101	·0093	15·9	19·1	19·1	"
"	III r*	·125	·0147	·0146	·0109	21·2	23	21·2	"
"	IV r	·0865	·0102	·0100	·0076	16·6	22·6	16·9	"
B	I	·253	·0297	·0285	·0217	22·1	22·6	23·0	"
"	I r	·206	·0242	·0	·0179	20·7	19·8	24·8	"
"	II	·253	·0297	·0283	·0217	20·8			Plates.

Annealed Mild Steel Tubes. Table II.

B	I	·252	·0295	·0290	·0216	19·5	19·5	19·8	Ball
"	II	·166	·0195	·0185	·0144	18·8	18·8	19·8	"
"	III	·129	·0153	·0137	·0114	19·3	17·9	21·3	"
"	IV	·064	·0076	·0060	·0057	14·7	16·3	17·8	"

Tension Tests.

Strips cut From Tube. Table IV

Tube.	No.	Area	Stress. Yield.	Stress. Max.	% Elongation. 2"	% Elongation. 6"	% Elongation. at Max.	
A	I	·0249	22·9	27·3			6·0	Broke outside gauge length.
A	II	·0190	24·0	27·7	19	11·9	8·25	
A	III	·0177	22·2	25·9	"		6·0	"
B	I	·0168	23·6	28·0	17	on 5" 11·9	9·7	
B Annealed	I	·0177	19·5	26·7	32	on 5" 23·6	18	
"	II	·0176	20·1	26·4			12·8	"

FIG. 8.—Compression and Tension Tests—Tables.

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After the first test the collapsed portions (which were at the ends) of Nos. II III, IV were cut off, and the remainder machined down over the middle 4 inches. These tests were denoted by II_r, III_r, IV_r. When necessary, the tubes were cut in order to determine the variation of thickness in the region close to that which had collapsed.

The results of these tests are given in fig. 8, some of the autographic diagrams in fig. 9, and photographs of some of the collapsed specimens in

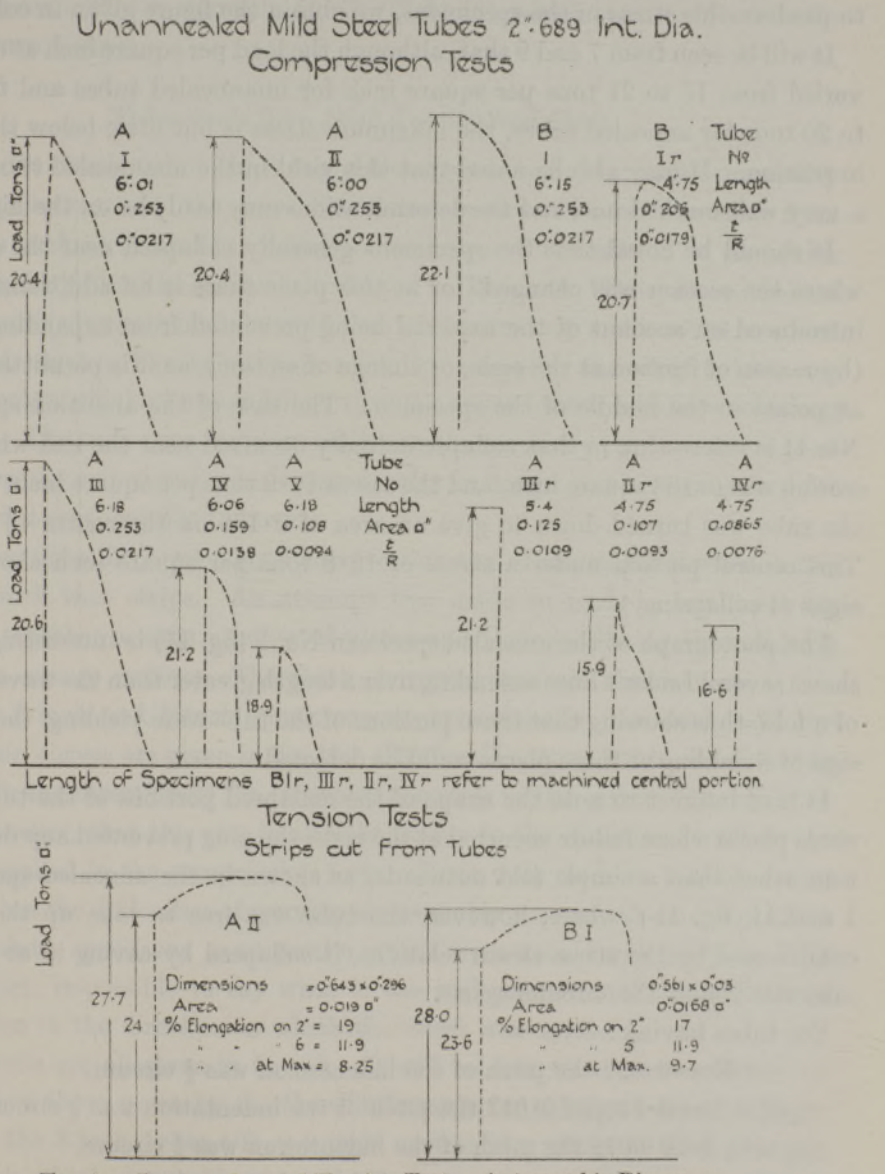

FIG. 9.—Compression and Tension Tests—Autographic Diagrams.

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figs. 10 and 11 (Plate 11). The results of some tension tests made on a strip cut from the tube are also given in fig. 9.

In column 7 is given the actual collapsing load per square inch, and in column 9 the calculated maximum stress, allowing for the eccentricity of loading produced by the variation of thickness. The tubes were too thin to obtain the real yield in compression, but if we assume it to be the same as that observed in the tension test and then calculate the load per square inch required to produce this stress in the specimens, we obtain the figure given in column 8.

It will be seen from 7 and 9 that, although the load per square inch at collapse varied from 16 to 21 tons per square inch for unannealed tubes and from 15 to 20 tons for annealed tubes, the maximum stress is but little below the yield in tension. It may also be noted that this yield in the unannealed tube is not a very well-marked one, and the determinations may easily be on the high side.

It should be noted that the specimens generally collapsed near the ends or where the section was changed, for at this place there is an additional stress introduced on account of the material being prevented from expanding freely (by reason of friction at the ends, or change of section), as it is permitted to do at points in the middle of the specimen. The case of the annealed specimen No. II is interesting in that collapse actually occurred near the end where the section was 0·188 square inch, and the stress 16·6 tons per square inch, though the tube was turned down to give an area of 0·166 on the centre $4\frac{1}{2}$ inches. This central portion under a stress of 18·8 tons per square inch showed no signs of collapsing.

The photograph of the annealed specimen No. I (fig. 11) is interesting, for it shows several Luder's lines extending over a length greater than the wave-length of a fold, thus showing that these portions of the tube were yielding, though no sign of wrinkling at these places could be detected.

It is of interest to note the shape of the collapsed portions of the tube. In those places where failure occurred at the ends, the plug prevented any deformation other than a simple fold outwards, as shown in the annealed specimens I and II, fig. 11; where, however, the tube was free to take up the shape conditioned by the stress-strain relations, it collapsed by caving in at regular intervals round the circumference.

For tubes having :—

$t/R = 0{\cdot}022$ the pitch of the indentation was $\frac{1}{4}$ circum.

$= 0{\cdot}01$ and $0{\cdot}017$ the pitch of the indentation was $\frac{1}{6}$ circum.

$= 0{\cdot}0076$ the pitch of the indentation was $\frac{1}{8}$ circum.

$= 0{\cdot}0057$ the pitch of the indentation was $\frac{1}{9}$ circum.

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It may be noted that thick tubes of mild steel give an autographic diagram very similar to an autographic diagram in tension. This is shown by the diagram in fig. 12 (Plate 12). After yield has been passed it can generally be noticed that the walls of the tube have bulged, but complete collapse does not occur till the load is much greater than the yield. Generally a specimen which gives a diagram showing a horizontal line at yield collapses with a characteristic uniform bulge, whilst one giving only a peak will cave in in several places. This is shown in fig. 14 (Plate 12), which represents a remarkably uniform failure.

Tubes made from High Tensile Steel Strip.

In order to obtain tubes which might be expected to fail by elastic instability rather than by reason of the stress distribution, a series of tubes was made from steel strip such as is used for metal construction of aircraft. The particular steel used, for which the author is indebted to Major H. Wylie, has a limit of proportionality in tension of about 40 tons per square inch and is rolled to about 0·015 inch thick. The tubes were made by wrapping the strip (which was about 5 inches wide) on a mandril so that the edges butted and then soldering the joint. The ends were faced up whilst on the mandril so as to secure a good bedding on the end-plates, which were similar in design to those employed for the $2\frac{3}{4}$-inch tubes described above.

It is very difficult to get a good compression test of the material when in the form of such thin strips. An attempt was made to make a test piece by soldering together a large number of pieces and then machining the resulting block to a square section. The result, however, was not satisfactory.

On the 3-inch and 5-inch specimens an extensometer was used, and the stress-strain curves are given in fig. 15. The tension stress-strain curve for the material in the direction of the length of the strip is also given. It should be noted, however, that this test is at right angles to the direction in which the stress is applied in the compression tests. Tests were made in tension across the strip, but the difficulties of securing a reasonably accurate stress distribution on such short specimens gave results which were considered fictitious. It is, therefore, impossible to say what is the real proportional elastic limit in compression in the direction in which the stress was applied.

The results are all given in fig. 15. It will be seen that the results are all about 0·6 of those given by Southwell's formula for a bulge type of collapse and that the 3-inch tubes $t/R = 0{\cdot}0093$ sustained a stress of 43·5 tons per square inch, which is probably above the limit of proportionality of the material.

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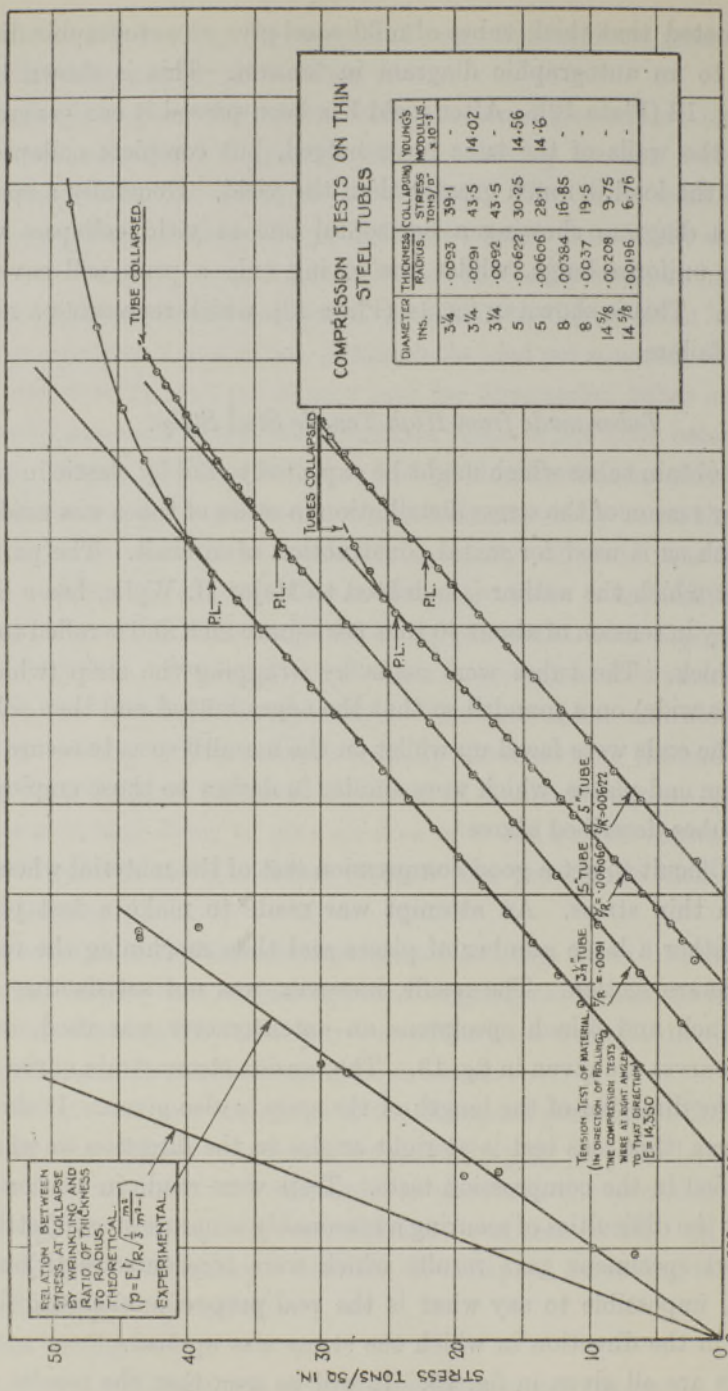

FIG. 15.

It has to be remembered that the strip is not exactly straight, and that the tubes cannot be exactly circular. These two defects would, of course, lead to values less than the theoretical ones.

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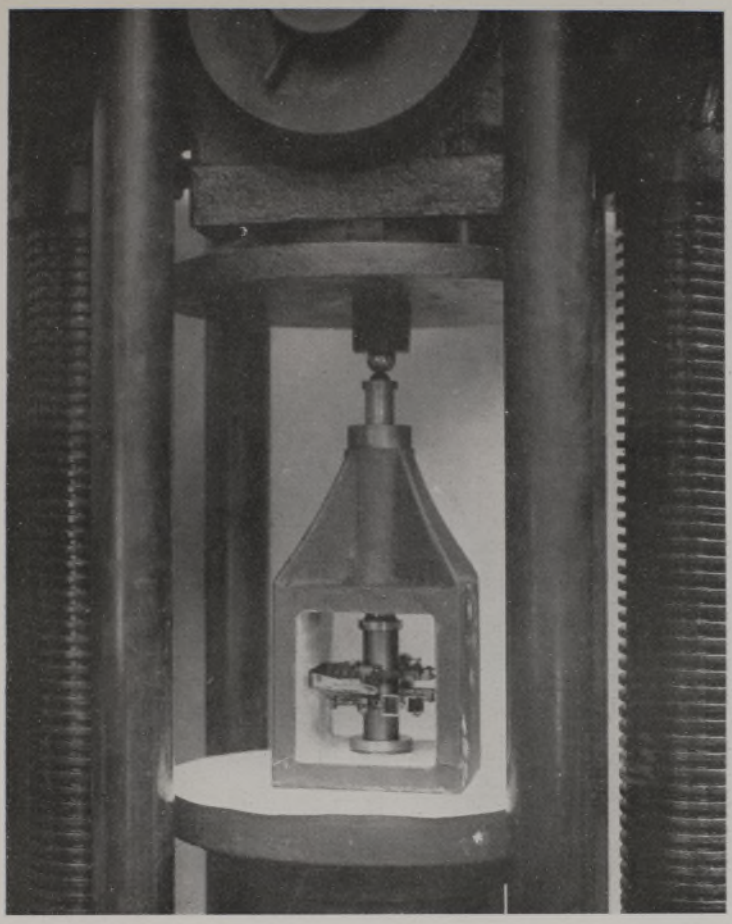

FIG. 3

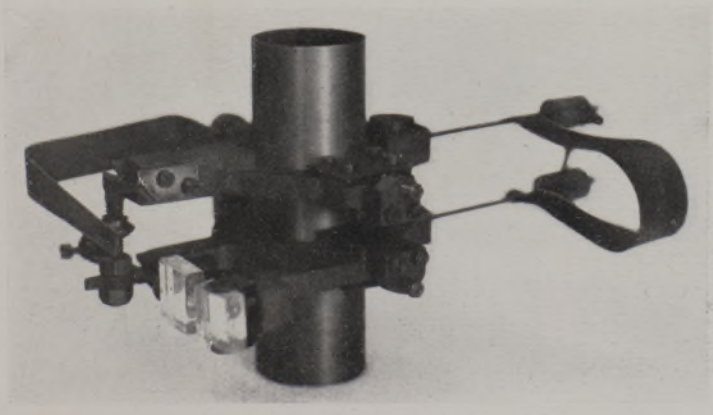

FIG. 4.

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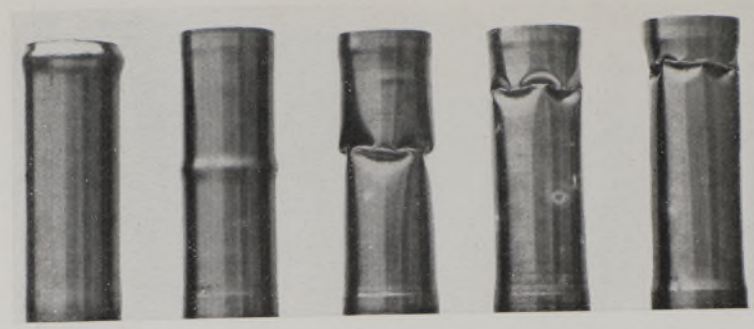

0·0801 0·0627 0·0279 0·0174 0·0104

FIG.—7.—Values of Thickness/Radius. Ni-Cr Steel, C annealed.



FIG. 10.—Unannealed Mild Steel Tubes.



FIG. 11.—Annealed Mild Steel Tubes.

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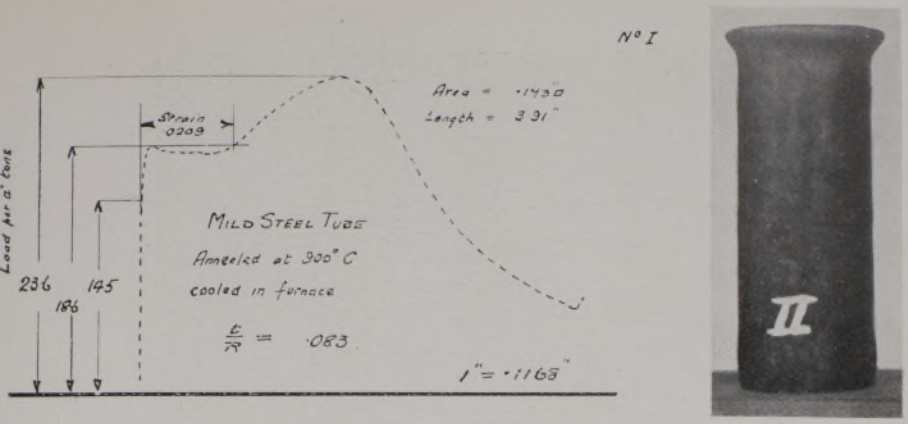

FIG. 12.

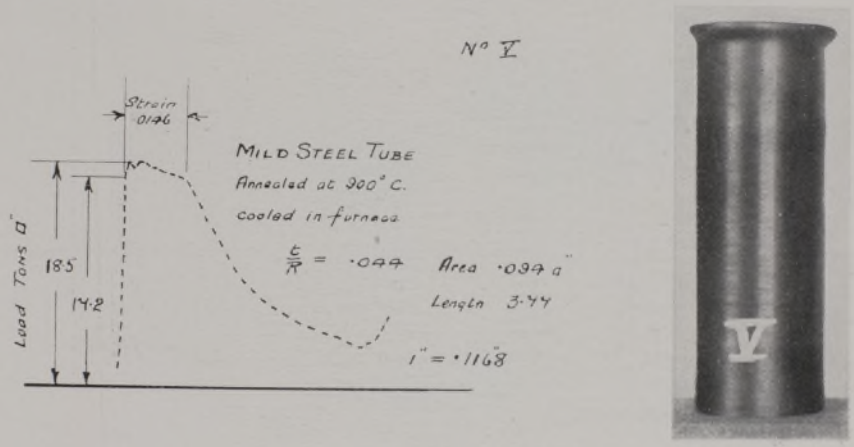

FIG. 13.

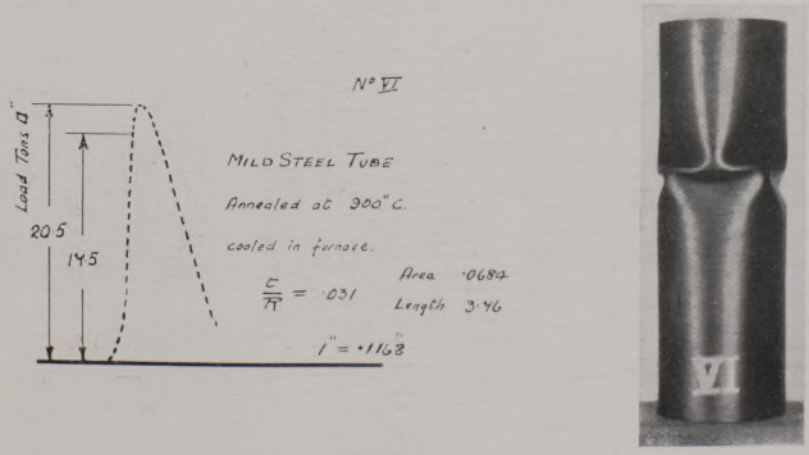

FIG. 14.

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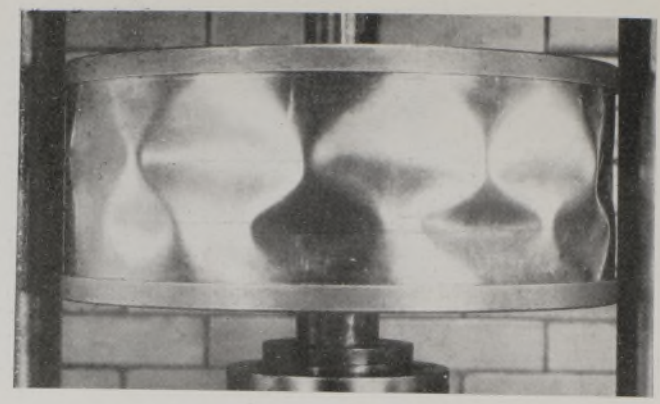

FIG. 16.—Photograph of Large Tube, before removing Collapsing Load.

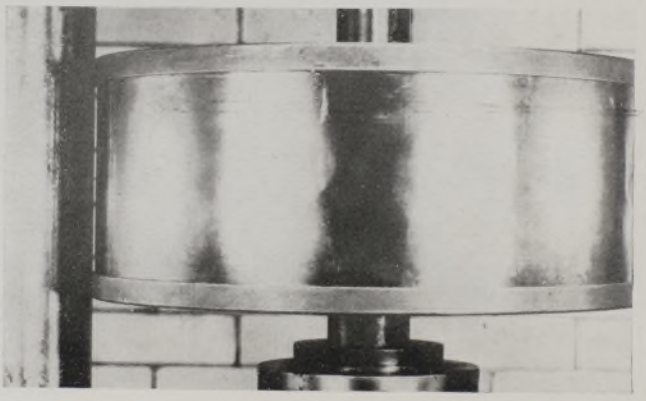

FIG. 17.—Photograph of Large Tube, after removing Collapsing Load.



0·187 0·082 0·052

FIG. 21.—Values of Thickness/Radius. Tubular Struts—Spruce.

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The largest tubes were tested in a special machine in series with a specimen of tool steel, to which an extensometer was attached. This enabled the deformation at failure to be restricted to a smaller value than it would have had if tested in the usual testing machine. A photograph, fig. 16, was taken of the tube just after failure, and another, fig. 17, when the load had been relieved (Plate 13). These show that a considerable amount of the deformation was elastic, since the characteristic lobes have nearly disappeared on the removal of the load.

Comparison of Experimental Results with the Theoretical Formula.

This series of tests is the only one that really comes within the scope of the available theoretical analysis, and in particular of Southwell's theory of the formation of lobes obtained by using the theory of thin shells. The author has recently had occasion to examine Southwell's work in the light of the experimental results just described, and has found reasons for considering that an oversight in the interpretation of a long equation (expressing the results of the analytical investigations) has rendered Southwell's formula* (105) and diagram, fig. 6 (p. 235), inapplicable to the problems here considered. It will be necessary to consider this point at some length.

Southwell obtained the equations which follow, and these are supported by an independent investigation recently published by W. R. Dean.

$$\psi = -\frac{m^2-1}{m^2}\frac{p}{E} = -\frac{m^2-1}{m^2}\frac{\mathfrak{S}}{4\pi a t_1 E}, \tag{96}$$

$$\begin{aligned}&\psi^2[\tfrac{1}{2}(m+1)\,K^2q^4/m]+\psi[\tfrac{1}{2}(m-1)\,q^2\{(K^2+q^2)^2+K^2\}/m]\\&+\tfrac{1}{2}(m^2-1)\,(m-1)\,q^4/m^3+\tfrac{1}{3}t_1^2/a^2\{\psi q^2[\tfrac{1}{2}(m+1)\,K^2\{(K^2+q^2)^2-K^2\}/m\\&+\tfrac{1}{2}(m^2-7m+4)\,K^2q^2/m^2+(m-1)\,q^4/m]\\&+\tfrac{1}{2}(m-1)/m[(K^2+q^2)^4-q^6/m-K^2\{2K^4+7K^2q^2+(7m^2+m-2)\,q^4/m^2\}\\&+K^4+3K^2q^2+(2m^2-1)q^4/m^2]\}=0,\end{aligned} \tag{98}$$

where p = load per square inch, E = Young's modulus, $1/m$ = Poisson's ratio, K = number of lobes, $q = 2\pi a/\lambda$, λ = wave-length of a lobe, a = radius = R, $t_1 = \frac{1}{2}$ thickness of tube, t = thickness of tube.

The cases to be considered are (1) $K = 0$, (2) $K = 1, 2, 3$, etc.

* The references in this section of the paper are to Southwell's original paper, 'Phil. Trans.,' A, vol. 213, and his nomenclature has been adopted except that t_1 is used for the half thickness of the tube instead of t.

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When $K = 0$ the collapsing load is the same as that obtained by his more rigorous analysis in the early part of his paper, *i.e.*,

$$p = E\frac{2t_1}{a}\sqrt{\frac{1}{3}\frac{m^2}{m^2-1}} = E\frac{t}{R}\sqrt{\frac{1}{3}\frac{m^2}{m^2-1}}.$$

This formula is obtained from equation 98 in virtue of simplifications which can be introduced on account of the fact that p/E must be very small in any case of practical importance. When $K = 0$, q must be great in order that this condition may be satisfied, and only the highest powers of q need be retained. When $K \neq 0$ Southwell concluded (p. 234) that q must be small (*i.e.*, the wave-length must be great), and his formula (105) and fig. 6 (p. 235) were obtained on this assumption. But although a small value of q (when t_1/a is very small) may correspond with a possible value of p/E, it is not a necessary condition for this result. A small value of p/E can also occur with a large value of q (small wave-length), and this is, in fact, the dominating condition in the problem. We therefore require to examine the consequence of (98) when the conditions are not limited in the way Southwell considered.

Examination of the relative value of the terms in any practicable case shows that the equation may to a close approximation be reduced to

$$\frac{p}{E} = \left(\frac{q}{K^2+q^2}\right)^2 + \frac{1}{3}\left(\frac{t_1}{a}\right)^2 \frac{m^2}{m^2-1}\left(\frac{K^2+q^2}{q}\right)^2 \tag{1}$$

$$= \frac{1}{\alpha^2} + A\alpha^2, \tag{2}$$

where $\alpha = (K^2+q^2)/q$, $A = \frac{1}{3}t_1^2 m^2/a^2(m^2-1)$.

We desire to know the minimum value of p/E since this will be the load at which instability will occur, and the corresponding values of K and q. For a minimum

$$\frac{d(p/E)}{dq} = \left(2A\alpha - \frac{2}{\alpha^3}\right)\left(1 - \frac{K^2}{q^2}\right) = 0, \tag{3}$$

and

$$\frac{d(p/E)}{dK} = \left(2A\alpha - \frac{2}{\alpha^3}\right)\left(\frac{2K}{q}\right) = 0. \tag{4}$$

To satisfy both (3) and (4), $\alpha^4 = 1/A$. (5)

Substituting in (2)

$$\frac{p}{E} = \sqrt{A} + \frac{A}{\sqrt{A}} = 2\sqrt{A} = 2\frac{t_1}{a}\sqrt{\frac{1}{3}\frac{m^2}{m^2-1}},$$

i.e., exactly the same equation as that for case 1 when $K = 0$.

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From (5) we can find the value of q (for any given value of K) when the collapsing stress is a minimum.

$$(K^2 + q^2)/q = \sqrt[4]{(1/A)}. \tag{6}$$

$$q = \{ \sqrt[4]{(1/A)} \pm \sqrt{(1/A)^{\frac{1}{2}} - 4K^2} \}/2. \tag{7}$$

For real values of q

$$K^2 < 1/(4\sqrt{A}). \tag{8}$$

From (6), (7) and (8) it is evident that the tube can fail in any number of lobes less than the maximum given by (8), and that for any given value of K there are two possible values of q.

This is clearly shown in fig. 18, in which the collapsing stress for a particular tube is plotted against $1/q$. The maximum value of K, to give a minimum

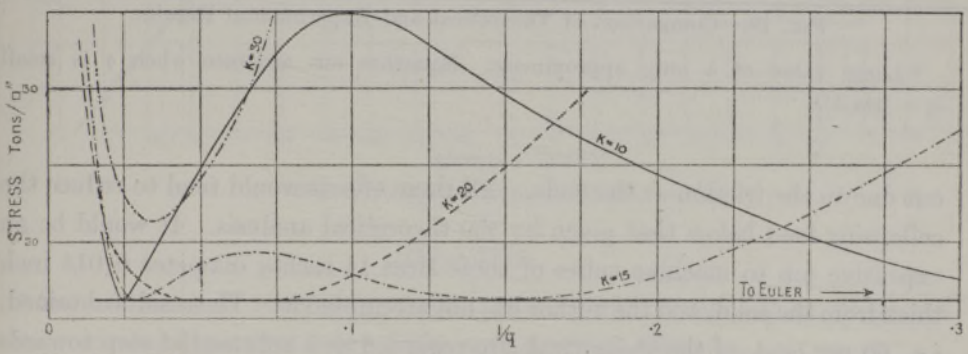

Fig. 18.—Collapsing Stress of Thin-Walled Steel Tube. $2t/a = 0{\cdot}002$, $E = 3 \times 10^7$, $m = 10/3$.

collapsing stress, for this particular tube, is 20, so that it could fail by the formation of 20, or any smaller number of lobes. Curves for $K = 20$, $K = 15$, and $K = 10$ are shown plotted, and it will be seen that each curve has two minima for two different values of q, and that all these minima occur at the same collapsing stress. The two minimum values for $K = 20$ are practically coincident, and the second value for $K = 10$ is to the right of the diagram. The curve for $K = 30$ is also shown, and it will be seen that its minimum is considerably higher than the minima of all curves in which K is less than 20.

A comparison of the experimental results with the theoretical ones is set out in fig. 19. It will be seen that the actual number of lobes into which the tube collapsed was always less than their minimum, and that the wave-length as measured after collapse had occurred was always greater than the theoretical value. The actual collapsing load was about 0·6 the theoretical. It must, however, be remembered that these tubes deviate from the assumption in two

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important particulars, for they cannot be considered as either truly circular or quite straight. Moreover, there is also the complication previously pointed

TUBE	a	t/a	k		λ		λ_e/λ_t	p		p_e/p_t
			NO. OF LOBES.		WAVE LENGTH.			COLLAPSING STRESS.		
			EXPERIMENTAL	THEORETICAL	EXPERIMENTAL	THEORETICAL.				
DIAMETER.	MEAN RADIUS.	HALF THICKNESS / MEAN RADIUS. $\times 10^{-3}$		Any number not exceeding:-		Corresponding to number of lobes in which tube actually failed.	RATIO OF EXPERIMENTAL TO THEORETICAL WAVE LENGTH.	EXPERIMENTAL Tons/in^2	THEORETICAL. Tons/in^2	RATIO OF EXPERIMENTAL TO THEORETICAL COLLAPSING STRESS
3¼"	1·63"	4·55	7	10	·8"	·64" or 3·4" *	1·25	42	70·6	·595
5"	2·50"	3·05	8	12	1·1"	·77" or 5·0" *	1·43	29	48·0	·602
8"	4·00"	1·90	8	15	1·8"	·93" or 10·7" *	1·94	18	29·8	·606
14⅝"	7·32"	1·00	12	20	1·95"	1·25" or 11·6" *	1·56	9	15·9	·569

FIG. 19.—Comparison of Theoretical and Experimental Results.

* Large value of λ only approximate. Equation not accurate when q is small ($q = 2\pi a/\lambda$).

out due to the friction at the ends. All these effects would tend to reduce the collapsing load below that given by the theoretical analysis. It would be an expensive job to machine tubes of these sizes 14 inches diameter 0·014 inch thick from the solid, and the author has not attempted it. The results obtained, *i.e.*, 60 per cent. of the theoretical, are regarded as a substantial step towards the verification of the Southwell formula.

Timber.—One series of tests on tubes made from spruce was carried out. Thin strips from the same plank were carefully planed to a fairly uniform thickness, about 0·025 inch. The various tubes were made by wrapping the necessary number of strips round a mandril having first spread a fine coating of glue on all the faces that were to come together. To minimise the risk of failure at the ends an extra strip, ½ inch wide, was glued on. Care was taken that the grain of the timber was parallel to the axis of the mandril.

On a solid specimen from the same plank a compression test was carried out. The results are given in fig. 20 and a photograph of some of the collapsed specimens in fig. 21 (Plate 13).

It will be seen that the collapsing stress is uniform and practically that of the solid specimen for all values of $t/R > 0{\cdot}08$. It should be noted that in the case of timber, which has such different elastic properties in different directions, there is as yet no theory giving the collapsing load in terms of the elastic constants. In view, however, of the extensive use of timber in aeronautical

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work, it was considered advisable to carry out this series so as to determine the probable limit of t/R for hollow struts.

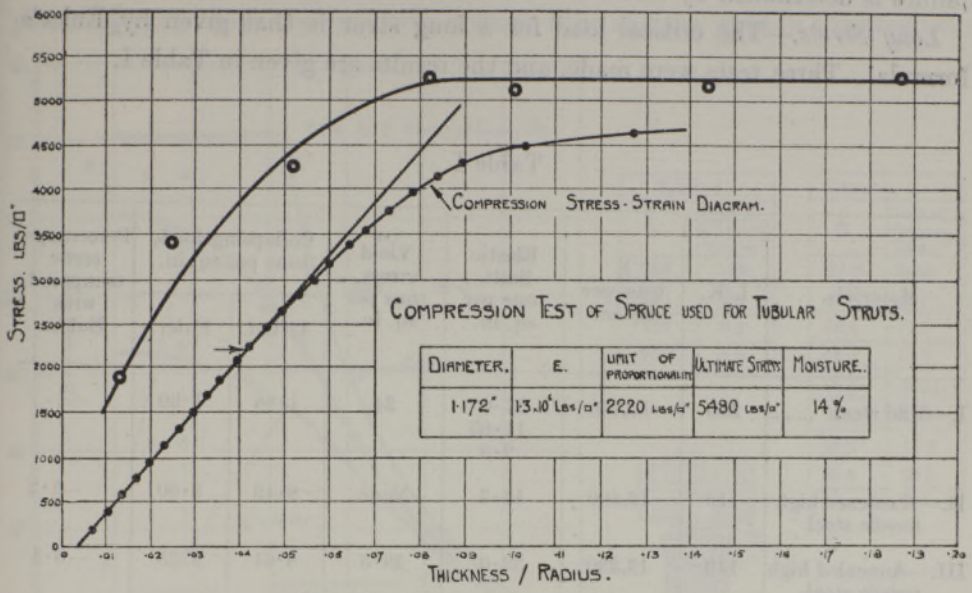

Fig. 20.—Spruce Tubular Struts.

Strut Tests.

Several series of tests of free-ended struts were carried out on mild steel and nickel-chrome steel tubes. For each specimen the tubes were cut into lengths, the ends faced up and provided with a plug, in which a steel ball was centred. During the tests the load on the strut was transmitted through hardened flat plates (usually flats ground on 1-inch balls), as shown in fig. 22. The length of the strut was taken as the distance between the centres of the balls. The load was therefore applied at the end of the strut in the centre line of the bore. Since the tubes were not uniformly thick, this introduced a small eccentricity.

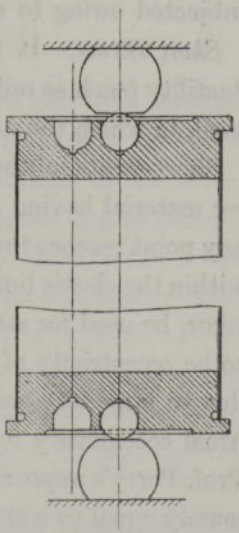

Fig. 22.

It will be convenient to treat long and short struts separately, using the term "long" to apply to struts of such a length that the Eulerian value is much less than the elastic limit of the material. This distinction is convenient since the factors producing failure are different in the two cases. In a long strut failure is chiefly determined by the

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elastic instability which occurs at the Euler load, whilst in the short strut failure is determined by stress considerations.

Long Struts.—The critical load for a long strut is that given by Euler's formula. Three tests were made, and the results are given in Table I.

Table I.

Materials.	L/K.	E tons per sq. in.	Elastic limit, tons per sq. in.	Yield stress, tons per sq. in.	Collapsing load, tons per sq. in.		Percentage error compared with Euler.
					Actual.	Euler.	
I.—Mild steel	256	12,760	17·6 11·65 9·5	24	1·88	1·90	−1
II.—Hardened high tensile steel	119	12,400	16·3	None	8·49	8·60	−1·3
III.—Annealed high tensile steel	119	13,330	33·0	38·5	8·61	9·25	−6·8

It will be seen that the experimental values agree very closely with the theoretical despite the inevitable eccentricity of loading to which the tubes are subjected owing to unequal wall thickness.

Short Struts.—It is convenient to consider materials having considerable ductility (such as mild steel) separately from those like hardened nickel-chrome steel, in which the ductility is relatively very small.

Short struts fail by reason of stress conditions. The author has shown* that for material having a drop of stress at yield, failure occurs when the stress at any point reaches the yield. The usual formulæ are only applicable for stresses within the elastic limit, but, in the case of mild steel, they may, without serious error, be used for stresses up to the yield. In all tests of tubes there is likely to be eccentricity of loading due to the unequal wall thickness as well as that due to imperfect end fittings. To represent the results, therefore, either the usual eccentricity formula may be employed or the curvature formula, using Prof. Perry's approximation that a strut having an eccentricity of e is approximately equal to a strut bent initially into a sine curve and having a deflection at the centre as measured from the line joining the ends of $C_0 = \frac{6}{5}e$.

* 'Inst. Civ. Eng.,' Selected Paper 28.

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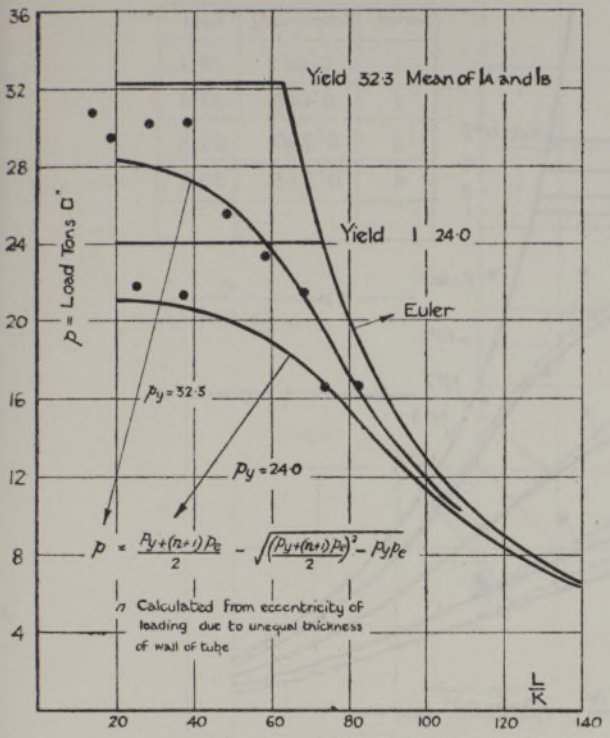
Ductile Materials. Tubes 1¼ inch, 18 Gauge.

Mild Steel.—The results of two series of tests are given in fig. 23. The test pieces for each series were cut from a single tube and the yield in compression



Tube I	
L/K	Load at collapse Tons □"
256	1·88
74·1	16·6
37·1	21·3
25·2	21·8

Tubes I_A I_B	
L/K	Load at Collapse Tons □"
82·9	16·65
68·6	21·4
58·4	23·3
48·3	25·5
38·2	30·2
28·2	30·2
18·3	29·4
13·5	30·8

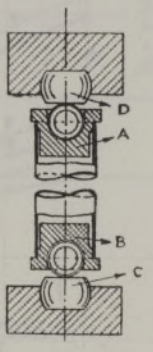


FIG. 23.—Mild Steel Tubes, 1¼ inch 0.0.18 gauge.

was determined by an extensometer test. If the specimens had been straight and axially loaded the results would have followed fairly closely the yield line and the Euler curve. Owing to the variation in thickness of the walls, however, the test pieces are loaded eccentrically by an amount approximately 0·027 inch. Assuming that failure will occur when the stress reaches the yield stress in compression of the material the curves for this eccentricity have been plotted and the results are seen to agree fairly closely with these curves.

Annealed High Tensile Steel.—The results of a series of tests are given in fig. 24, series 1. It will be seen that, allowing for the inevitable eccentricity due to unequal wall thickness, the results agree very well with the eccentricity formula in which the stress term is the yield stress.

On the same diagram are given the results of several series under eccentric loading of various amounts. The experimental results are all above those given by the formula, very appreciably in cases of the short struts and slightly

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in cases of longer ones. This arises from the fact that when loaded with considerable eccentricity a large portion of the stress is due to bending, and

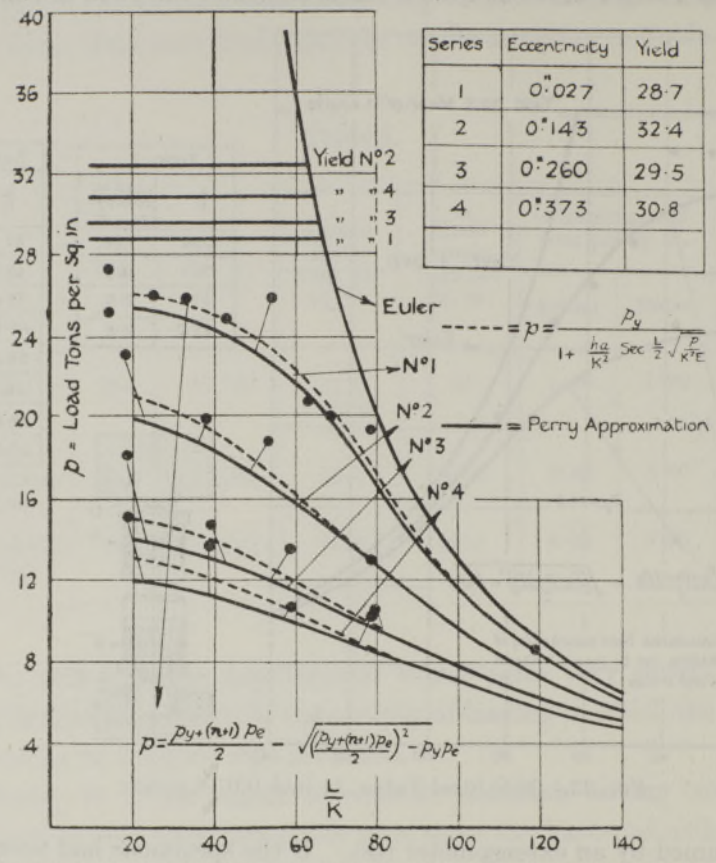

FIG. 24.—Eccentrically Loaded Struts—Annealed NiCr Steel, tubes $1\frac{1}{4}$ inches out. diam., 18 gauge.

hence with increasing eccentricity the behaviour of the strut tends to resemble that of a beam. Moreover, there is little drop of stress at yield in this material, and, therefore, the collapsing load is appreciably greater than that calculated on a basis of complete collapse when the yield stress is attained at any point. This point has been discussed by the author in his paper on struts.

Strut Tests on Mild Steel Tubes, $2\frac{3}{4}$ inches O.D., 22 Gauge, $t/R = 0{\cdot}022$.

The struts were provided with the same type of end as was used in the other series, by which the load was applied through the centre of the bore. The wall thickness was measured and the eccentricity of loading calculated.

The results are given in Tables II, III. and photographs of the portions of

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FIG. 25.

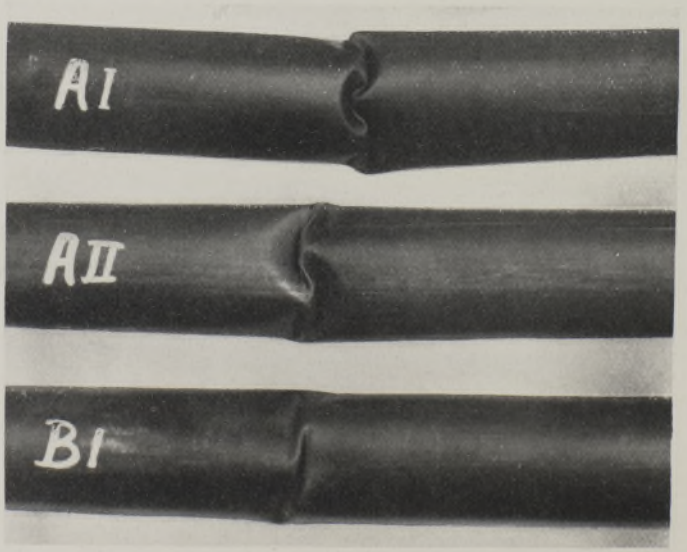

FIG. 26.

(*Facing p.* 581.)

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the struts affected by the collapse are given in figs. 25 and 26 (Plate 14); one view being taken in the plane of bending, the other at right angles to it.

Table II.—Strut Tests.

Tube.	No.	Area.	Thickness.		Eccentricity.	L/K.	Collapsing load.		Max. stress at collapse.†
			Mean.	Least.			Actual. (8)	Perry.* (9)	(10)
A	I	0·253	0·0296	0·0280	0·037	33·8	20·7	21·2	22·2
A	II	0·253	0·0296	0·0284	0·625	33·8	10·9	10·3	22·4
B	I	0·253	0·0296	0·0277	0·044	94·2	12·7	13·0	21·5

* On assumption that the yield in tension = the yield in compression.

† Max. stress = Load per square inch $\times \left\{1 + \frac{ah}{K^2} \text{Sec.} \frac{\pi}{2} \sqrt{\left(\frac{p}{p_e}\right)}\right\}$.

p = Load per sq. in. p_e = Eulerian value. h = Eccentricity. a = Radius.

Table III.—Tension Tests. (Strips cut from Tube.)

Tube.	No.	Area.	Stress.		Percentage elongation.			—
			Yield.	Max.	2″	6″	At max.	
A	I	0·0249	22·9	27·3			6·0	Broke outside gauge length.
A	II	0·0190	24·0	27·7	19	11·9	8·25	
A	III	0·0177	22·2	25·9			6·0	,, ,,
B	I	0·0168	23·6	28·0	17	On 5″ 11·9	9·7	
B	I	0·0177	19·5	26·7	32	On 5″ 23·6	18	
Annealed ,,	II	0·0176	20·1	26·4			12·8	,, ,,

The probable collapsing load has been calculated from the Perry formula, the yield in compression being taken as equal to the yield in tension. From the actual collapsing load the maximum stress produced on the concave side has been calculated.

It will be seen from an inspection of columns 8, 9 and 10 of Table II that in every case (*i.e.*, L/K = 33·8, 33·8, 94·2, and eccentricities 0·037 inch, 0·625 inch and 0·044 inch, respectively) the maximum stress is practically the yield stress, and that the Perry formula gives a good approximation of the collapsing load even with an eccentricity as great as 0·625 inch.

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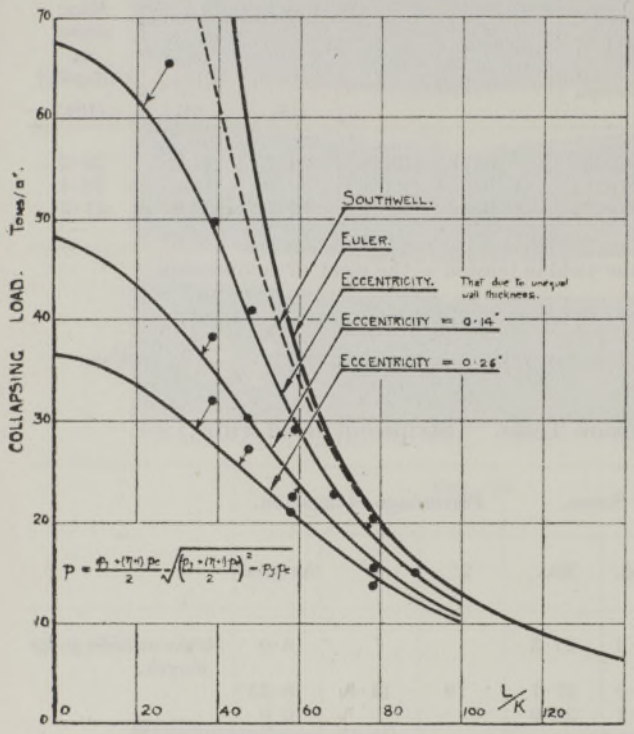
Hardened High Tensile Steel.

The results of the test are given in fig. 27. It will be seen that for the series in which the eccentricity is only that due to unequal wall thickness the results follow the Southwell curve for axial loading, but are decidedly below it.



Eccentricity.	L/K.	Collapsing load, tons/sq. in.
That due to unequal wall thickness	88·6	15·0
	78·2	20·5
	68·4	22·8
	59·0	29·2
	48·6	41·0
	39·6	49·6
	27·7	65·3
0·14"	78·0	15·6
	58·5	22·6
	47·4	30·3
	38·9	38·4
0·26"	78·1	13·7
	58·1	21·0
	47·8	27·3
	38·6	32·1

FIG. 27.—Hardened High Tensile Steel Tubes. $1\frac{1}{4}$ inches out. diam., 18 gauge.

There is no theoretical analysis available to enable the effect of eccentricity to be calculated for a strut made of a material of this type. For materials with very considerably greater ductility than that of this material in this condition, the author has obtained a fair agreement with experimental results by taking the stress term in the curvature formula as that at which the tangent to the stress-strain curve is one-third the value it has in the elastic portion of the curve. The method is artificial, but even in this case gives an approximate representation of the probable collapsing load, as is shown by the curves given in the diagram.

In the absence of a mathematical analysis in a material of this type it is not considered worth while to obtain an empirical formula to fit the results.

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Conclusions.

From the work described in this paper the following conclusions may be drawn :—

I. For tubular struts of mild steel having a yield in compression of 22 tons per square inch the strength depends on the yield stress and not on the wrinkling stress provided $t/R \geqslant 0{\cdot}022$.

II. For short specimens of thin tubes tested in compression failure will occur either at the yield or at a stress which is some fraction of the Southwell value (p_s) for the load required to produce elastic instability in the walls. For the tubes tested the fraction varied from 0·4 for solid-drawn small tubes of mild steel and nickel-chrome steel to 0·6 for tubes made from the steel strip used in metal construction of aircraft. The value of this fraction doubtless depends upon the accuracy of the tubes, and even with perfect tubes would be less than unity on account of the additional stresses introduced in applying the load. In all cases of thin tubes the wave-length of a deformation is small, being considerably less than the radius of the tube.

III. For tubes having t/R greater than the value as determined by II above, the proportional elastic limit and also yield, where this is well defined, precedes wrinkling, and the particular type of deformation produced depends upon the ratio t/R.

From these conclusions the author considers that a reasonable estimate of the strength of a steel tubular strut can be made by using the eccentricity formula (Prof. Smith) or the modification by Prof. Perry and inserting as the stress term the lower of two values, *i.e.*, the yield or an appropriate fraction of the Southwell value for collapse by elastic instability. In order, therefore, to avoid elastic instability as a primary cause of failure the value of t/R for steel tubes should be greater than $0{\cdot}4\ p_s$, *i.e.*, 0·006 for mild steel (20 tons yield). It is hardly likely that tubes of such a character will be used in ordinary engineering work, but this investigation has an important bearing upon the design of metal construction for aircraft. In this work one of the important elements of design is to avoid failure by elastic instability when compression stresses have to be taken. In order to do this flat surfaces are avoided and the flanges of girders, etc., are made curved or indented.

These conclusions derive considerable support from certain experiments carried out by Fairbairn* in 1846 in connectionwith the Conway tubular bridge.

* W. Fairbairn, 'Conway and Menai Tubular Bridges,' London (1849); E. Clark, Britannia and Conway Tubular Bridges,' London (1850).

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He tested under a central load tubes built up from iron plates. The tubes varied from 12 inches diameter 0·048 inch thick to 24 inches diameter 0·095 inch thick, and the length between the supports was varied between 15 feet 6 inches and 31 feet. In Table IV are given the results of these tests, the experiments being arranged in order of the value of t/R. As the tubes were of wrought iron with an elastic limit of probably 14 tons per square inch, the tubes would require to have a t/R of 0·0018 (calculated from Southwell's theoretical formula) in order to fail from elastic instability. One would, therefore, on the basis of the author's work, expect them to stand up to a stress equal to the proportional elastic limit before failure by wrinkling. In only two cases did failure occur by wrinkling on the compression side of the beam, and these at stresses of about 15 tons per square inch, *i.e.*, well above the proportional elastic limit. In all the other cases failure occurred on the tension side.

Table IV.

Fairbairn. No. of Experiment.	t/R.	Maxmium stress at failure.	Type of failure.
2	0·00623	14·92	Compression.
6	0·00636	15·1	Tension.
1	0·00673	14·92	Compression.
4	0·00715	14·49	Tension.
7	0·00733	10·04	,,
9	0·00747	11·2	,,
8	0·0112	10·1	,,
5	0·0132	13·4	,,
3	0·0214	15·8	,,

In this connection the tubes of the Chepstow Bridge, built by Brunel* about 1850, are also very instructive. The compression member of the bridge is a tube 9 feet diameter and $\frac{5}{8}$ inch thick, *i.e.*, $t/R = 0·0116$. Diaphragms are provided every 26 feet to preserve the shape. The distance, however, is so great compared with the wave-length of a lobe that they have no strengthening effect. The whole truss with transverse girder and roadway weighs 460 tons, and this was assembled on the river bank and tested with 770 tons before being built in position. Under this loading the stress must have been of the order of $4\frac{3}{4}$ tons per square inch. Brunel had, of course, the experiments of Fairbairn on which to base his design. It is significant that Lilly's formula would give 2·45 tons per square inch as the collapsing load of a tube of this

* I. Brunel, 'Life of Brunel' (1870).

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value of t/R, and that Messrs. Barling and Webb's formula would give 2·6 tons as the value of the stress at which wrinkling occurs. Further comment as to the insufficiency of these formulæ is unnecessary.

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On the Stability of Running of Locomotives.

By F. W. Carter, M.A., Sc.D., M.Inst.C.E., M.I.E.E.

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1. *Introduction.*

In a paper entitled "The Electric Locomotive," read before the Institution of Civil Engineers in 1916,* the author gave the beginnings of a rational discussion of the riding qualities of electric locomotives, as affected by their fundamental type. It was shown that, aside from all questions of unevenness of track or of the nature of the springing and equalizing arrangements, there might exist in a locomotive an inherent tendency to deviate from a motion of pure progression, which, although perhaps within the control of the wheel flanges and therefore deemed unobjectionable on good track, might give reason for anxiety if poor track were encountered or other external circumstances conspired with inherent tendencies to bring about a condition involving danger of derailment. It is the intention, in the present paper, to discuss the matter further with the view of determining to what extent inherent riding qualities of a design can be brought within the realm of rational investigation.

* 'Proc. Inst. C.E.,' vol. 201, p. 221.

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