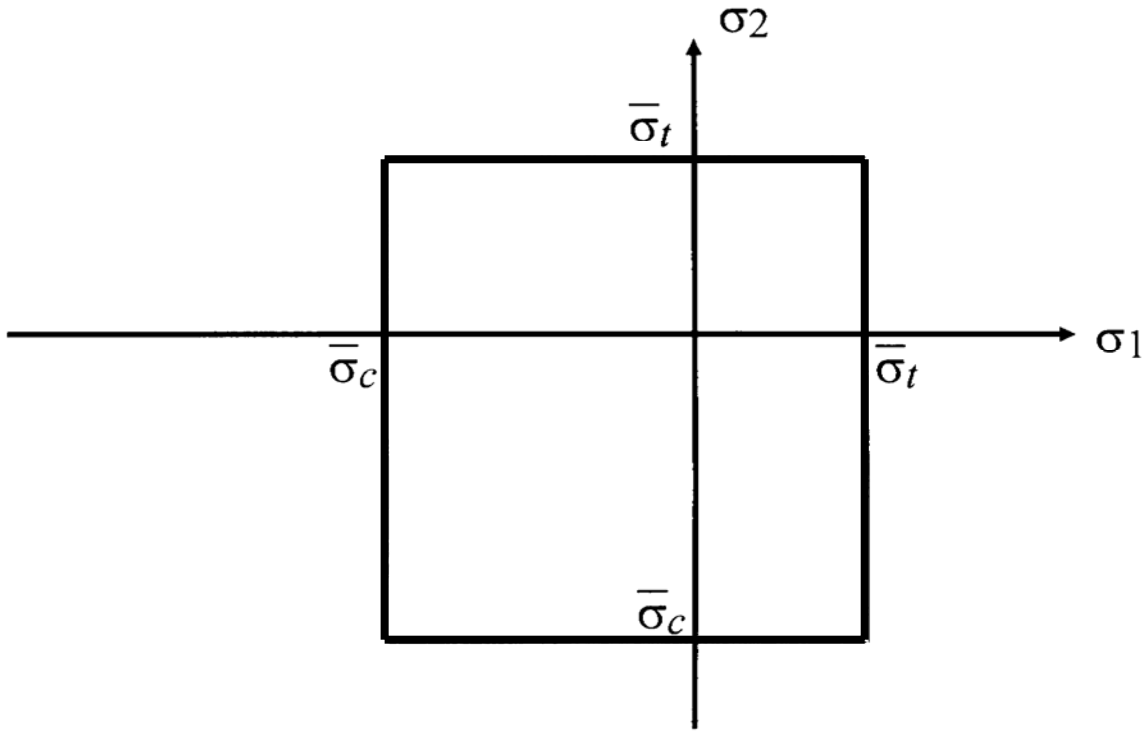
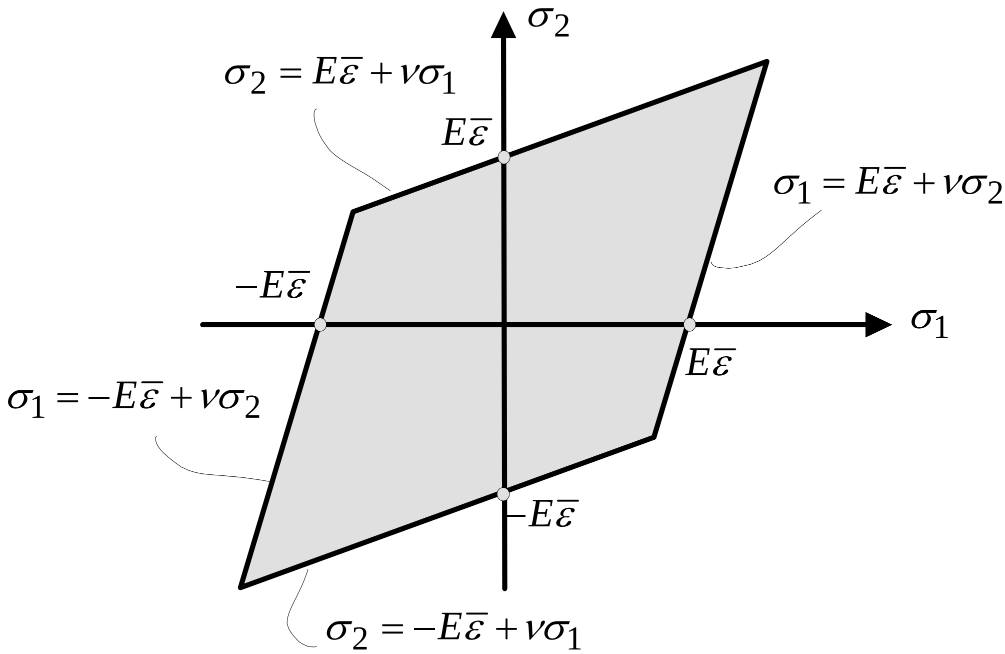
RANKINE



Limits to the principal stresses

$$\bar{\sigma}_c \leq \sigma_1, \sigma_2, \sigma_3 \leq \bar{\sigma}_t$$

De SAINT VENANT



Limit to the principal strains

$$-\bar{\varepsilon} \leq \varepsilon_1, \varepsilon_2, \varepsilon_3 \leq \bar{\varepsilon}$$

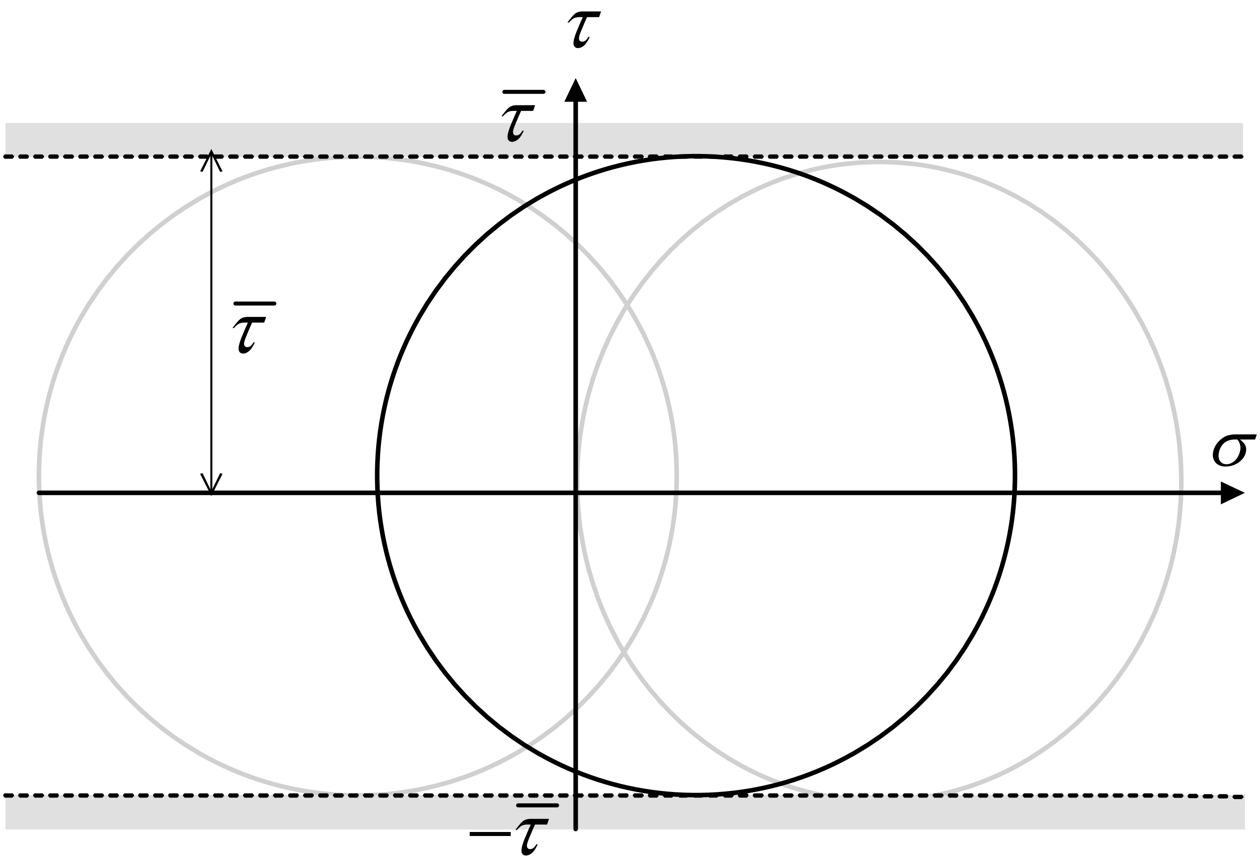
$$\bar{\varepsilon} = \varepsilon_1 = \frac{1}{E}[\sigma_1 - \nu(\sigma_2 + \sigma_3)]$$

$$\bar{\varepsilon} = \frac{1}{E}[\sigma_1 - \nu\sigma_2]$$

$$\sigma_1 = \bar{\varepsilon}E + \nu\sigma_2$$

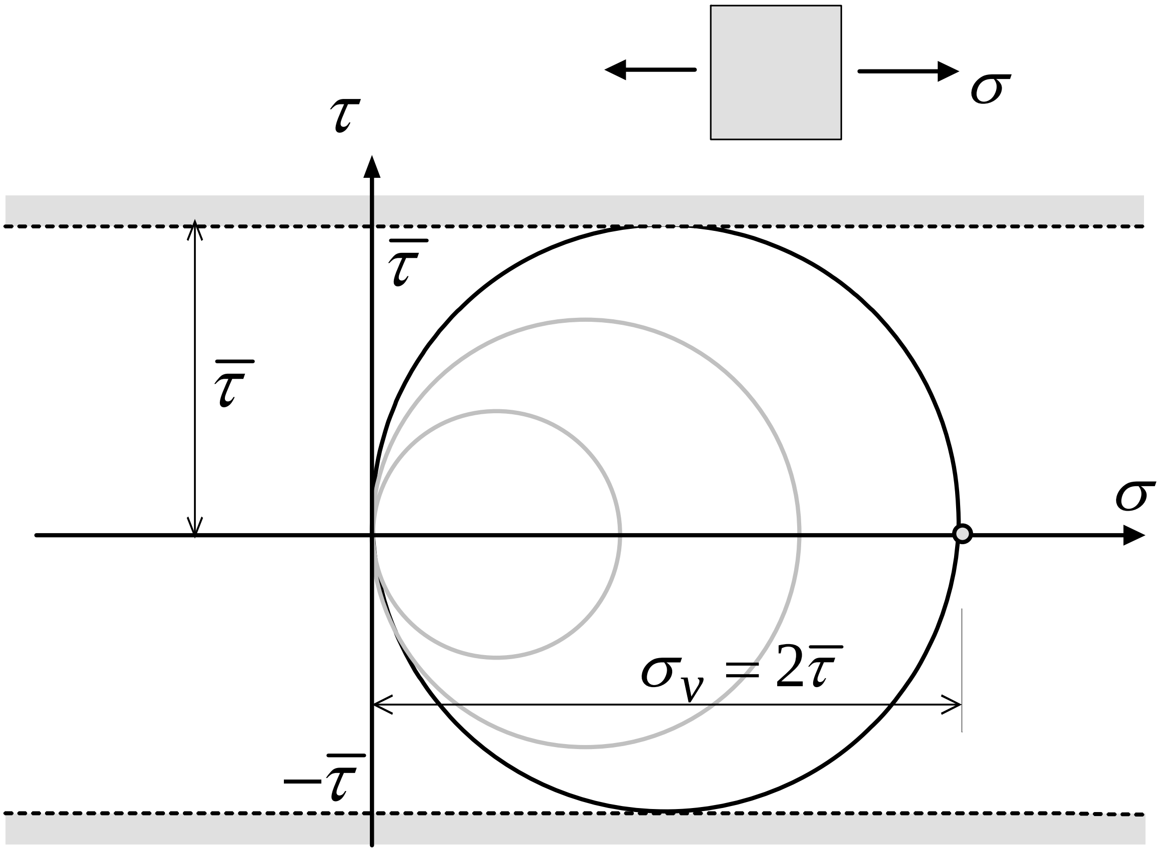
TRESCA

Limit to the largest shear stress on every possible section



$$\tau \leq \bar{\tau}$$

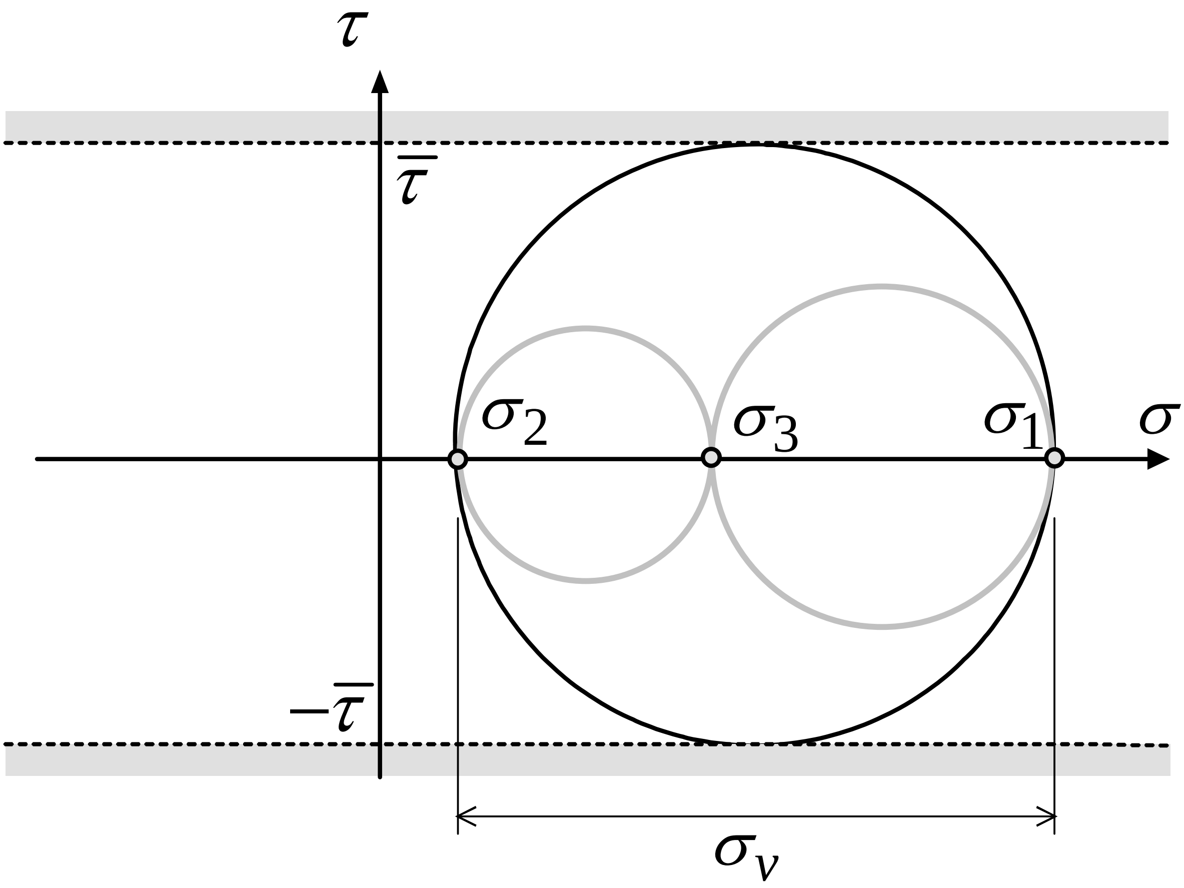
TRESCA



So, we can write

$$2\tau \leq \sigma_v$$

TRESCA



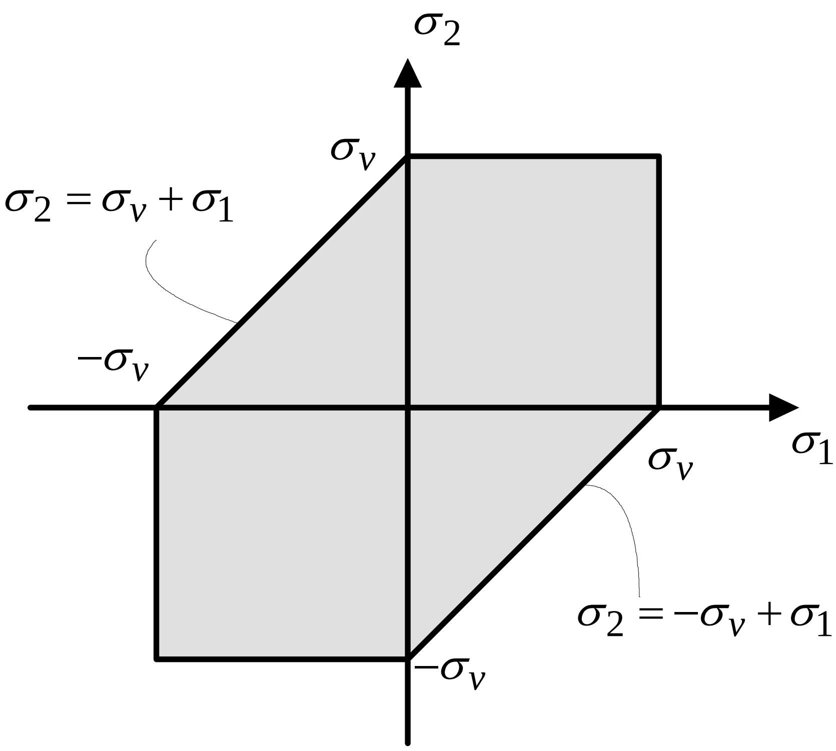
So, we can also write

$$|\sigma_1 - \sigma_2| \leq \sigma_v$$

$$|\sigma_2 - \sigma_3| \leq \sigma_v$$

$$|\sigma_3 - \sigma_1| \leq \sigma_v$$

TRESCA


σ_2
σ_v
$\sigma_2 = \sigma_v + \sigma_1$
$-\sigma_v$
σ_1
σ_v
$\sigma_2 = -\sigma_v + \sigma_1$
$-\sigma_v$

BELTRANI

Limit to the strain energy

$$W = \tfrac{1}{2}\left(\sigma_1\varepsilon_1 + \sigma_2\varepsilon_2 + \sigma_3\varepsilon_3\right)$$

$$met:$$

$$\varepsilon_1 = \frac{1}{E}\left(\sigma_1 - \nu\left(\sigma_2 + \sigma_3\right)\right)$$

$$\varepsilon_2 = \frac{1}{E}\left(\sigma_2 - \nu\left(\sigma_3 + \sigma_1\right)\right)$$

$$\varepsilon_3 = \frac{1}{E}\left(\sigma_3 - \nu\left(\sigma_1 + \sigma_2\right)\right)$$

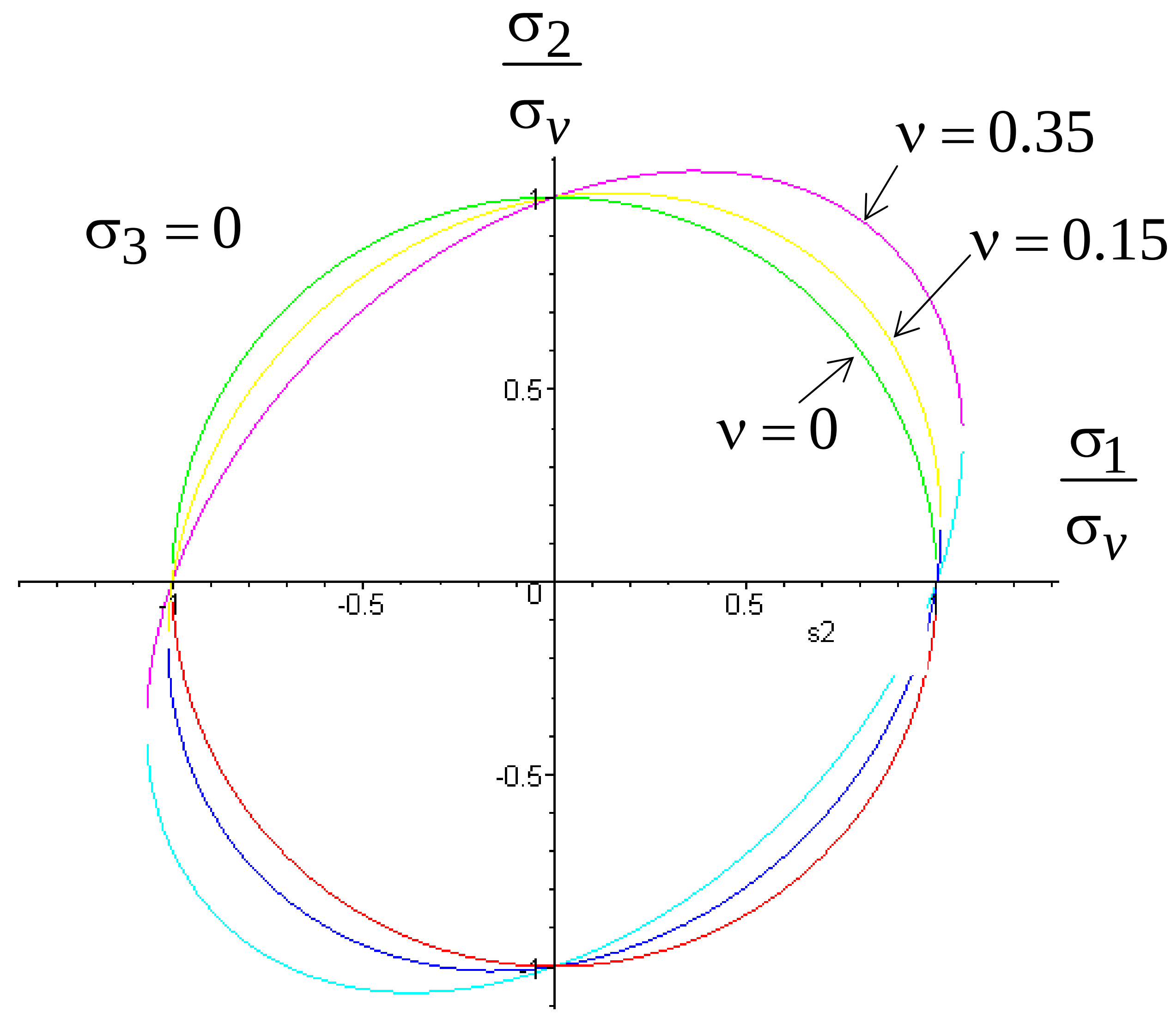
$$dus$$

$$W = \frac{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu\left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1\right)}{2E} \leq \bar{W}$$

$$\bar{W} = \frac{\sigma_v^2}{2E}$$

$$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu\left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1\right) \leq \sigma_v^2$$

BELTRANI



$\frac{\sigma_2}{\sigma_v}$
$\nu = 0.35$
$\nu = 0.15$
$\sigma_3 = 0$
$\nu = 0$
$\frac{\sigma_1}{\sigma_v}$
1
0.5
-0.5
-1
0
s2

VON MISES

Limit to the distorsional part of the strain energy

$$\overline{W} = \frac{1}{2} \sum_{i=x,y,z} \sum_{j=x,y,z} s_{ij} \varepsilon_{ij}$$

$$s_{xx} = \sigma_{xx} - \sigma_o$$

$$s_{xy} = \sigma_{xy} \qquad etc.$$

$$\sigma_o = \frac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})$$

Result

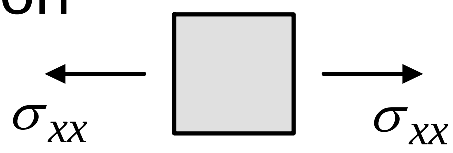
$$\sqrt{\frac{1}{2}[(\sigma_{xx} - \sigma_{yy})^2 + (\sigma_{yy} - \sigma_{zz})^2 + (\sigma_{zz} - \sigma_{xx})^2] + 3(\sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{zx}^2)} \leq \sigma_v$$

Or

$$\sqrt{\frac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]} \leq \sigma_v$$

VON MISES

Uni axial tension

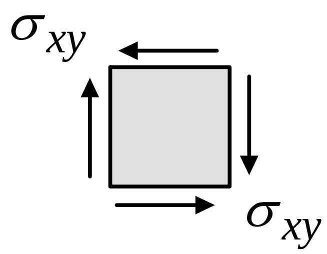


$$0 = \sigma_{yy} = \sigma_{zz} = \sigma_{xy} = \sigma_{yz} = \sigma_{xz}$$

So

$$\sigma_{xx} \leq \sigma_v$$

Shear



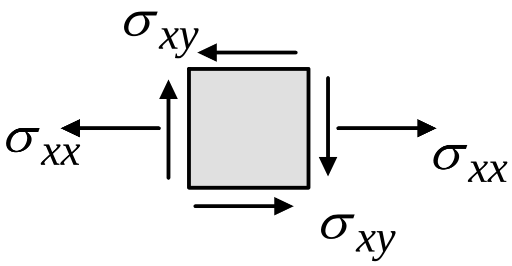
$$0 = \sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \sigma_{yz} = \sigma_{xz}$$

So

$$\sqrt{3}\sigma_{xy} = \sigma_v \Rightarrow \boxed{\sigma_{xy} \leq 0,58\sigma_v}$$

VON MISES

Tension + shear



$$0 = \sigma_{yy} = \sigma_{zz} = \sigma_{yz} = \sigma_{xz}$$

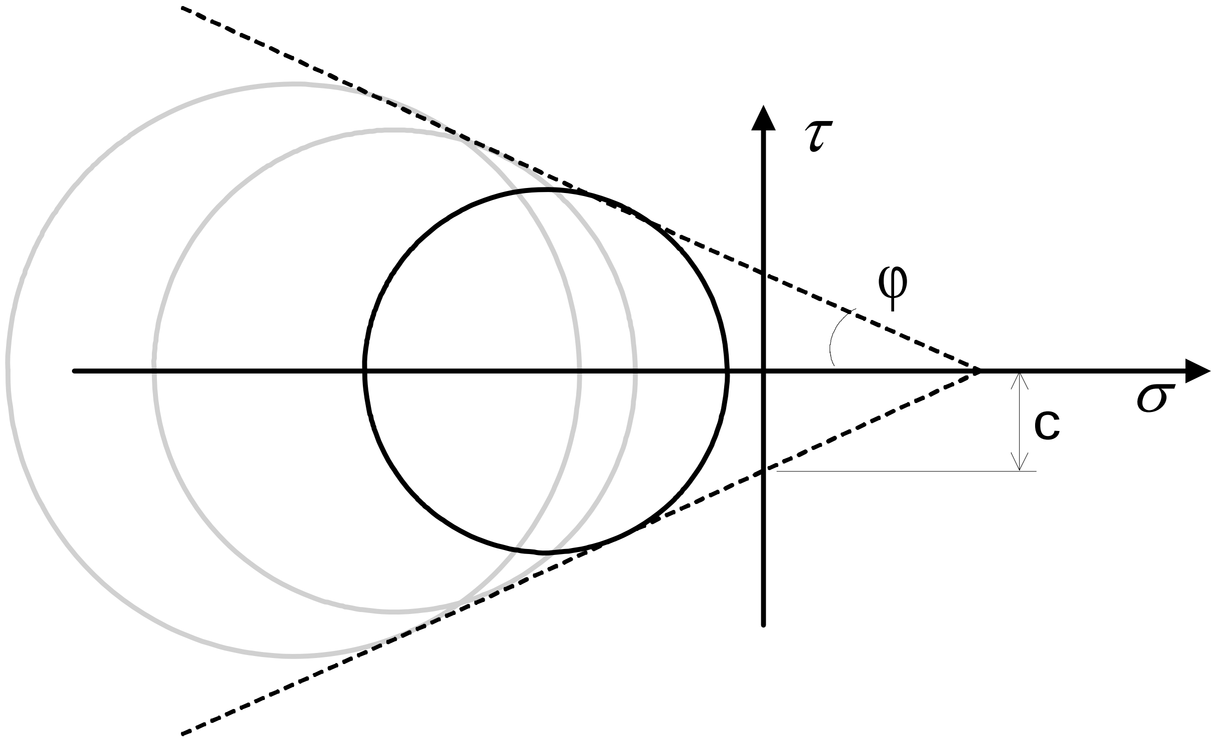
So

$$\sqrt{\sigma_{xx}^{2} + 3\sigma_{xy}^{2}} \leq \sigma_{v}$$

(Huber Hencky)

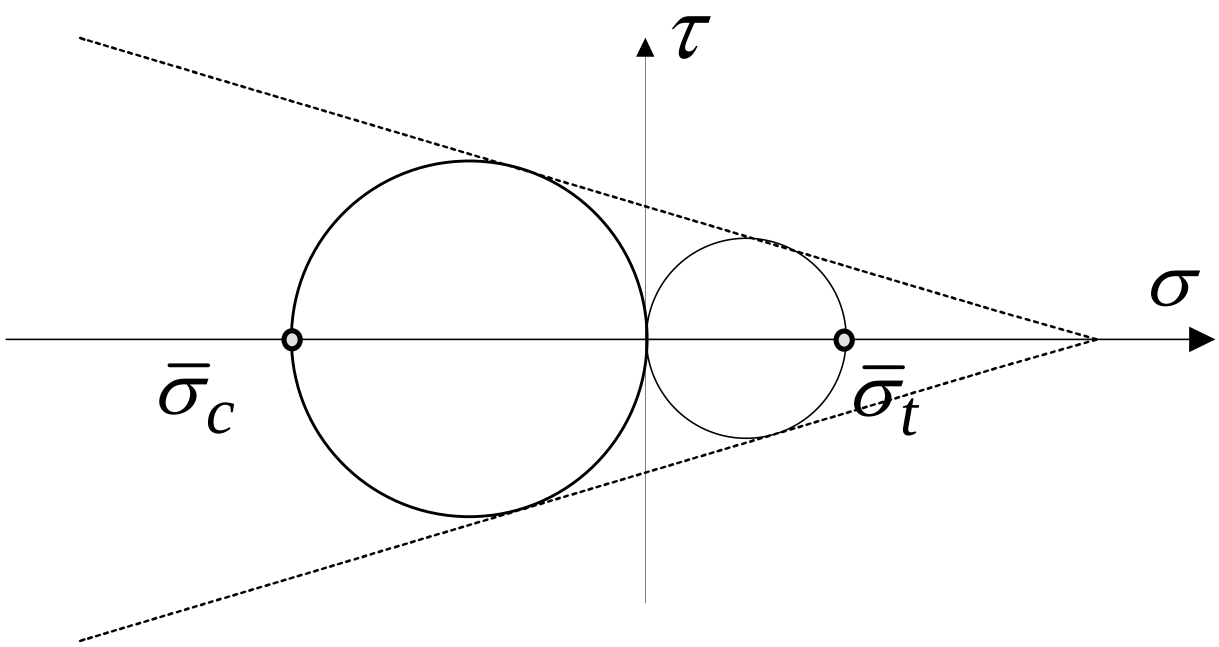
MOHR-COULOMB

Limit to the combination of τ and σ at every possible section



$$\tau \leq c - \sigma \tan \varphi$$

MOHR-COULOMB



Compression

$$\bar{\sigma}_c = \frac{2c\cos\varphi}{\sin\varphi - 1} \qquad \bar{\sigma}_t = \frac{2c\cos\varphi}{1+\sin\varphi}$$

In general

$$\frac{\max(\sigma_1,\sigma_2,\sigma_3)}{\bar{\sigma}_t} + \frac{\min(\sigma_1,\sigma_2,\sigma_3)}{\bar{\sigma}_c} \le 1$$

MOHR-COULOMB Example

Soil c = 3,0 MPa

φ = 0,2 rad

Stress to carry

$$\sigma_1 = -0,86 \quad \sigma_2 = -4,16 \quad \sigma_3 = 3,88$$

Will it fail?

$$\bar{\sigma}_t = \frac{2c\cos\varphi}{1+\sin\varphi} = \frac{2*3,0*\cos 0,2}{1+\sin 0,2} = 4,91$$

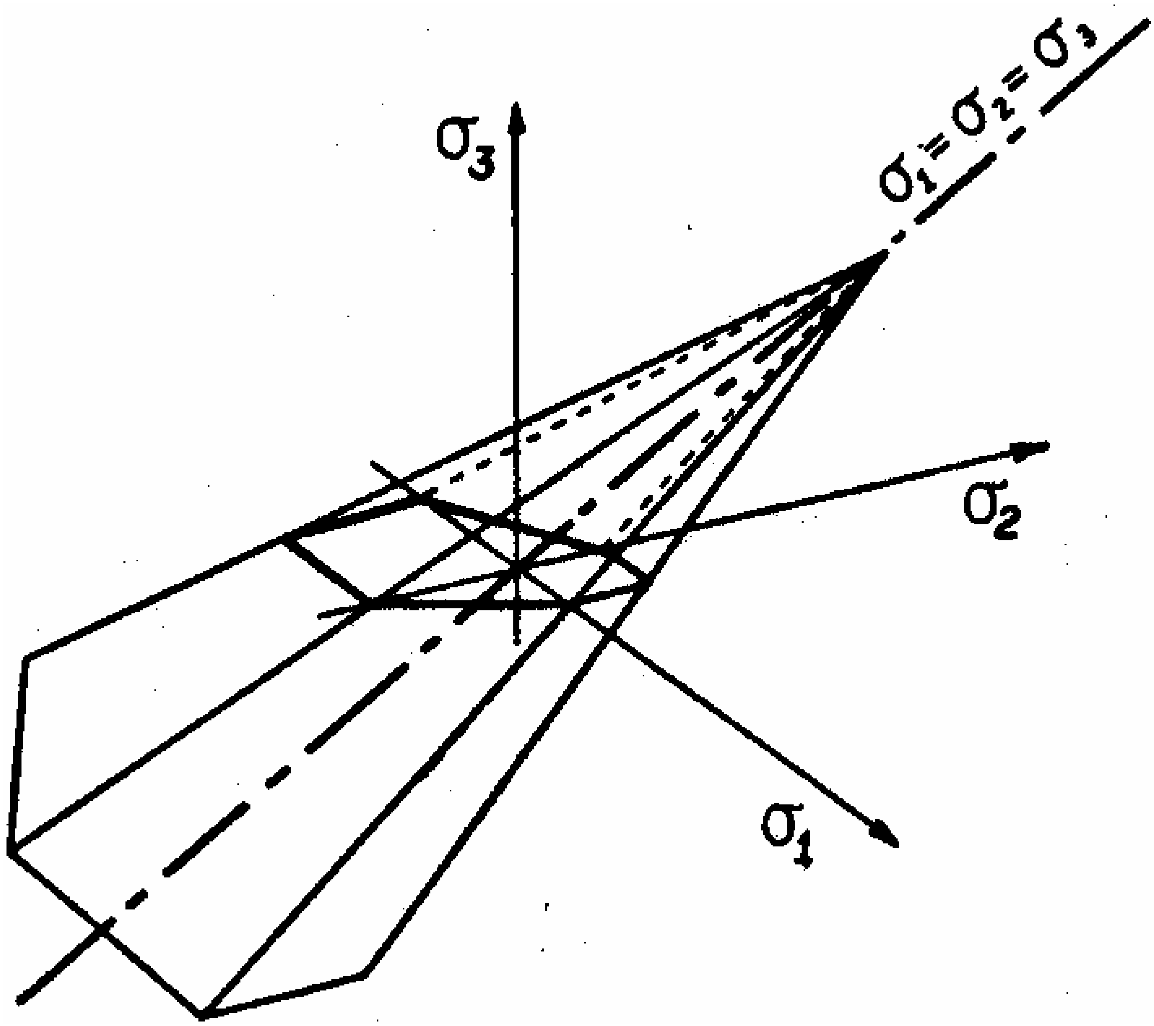
$$\bar{\sigma}_c = \frac{2c\cos\varphi}{\sin\varphi - 1} = \frac{2*3,0*\cos 0,2}{\sin 0,2 - 1} = -7,34$$

$$\frac{\max(\sigma_1,\sigma_2,\sigma_3)}{\bar{\sigma}_t} + \frac{\min(\sigma_1,\sigma_2,\sigma_3)}{\bar{\sigma}_c} =$$

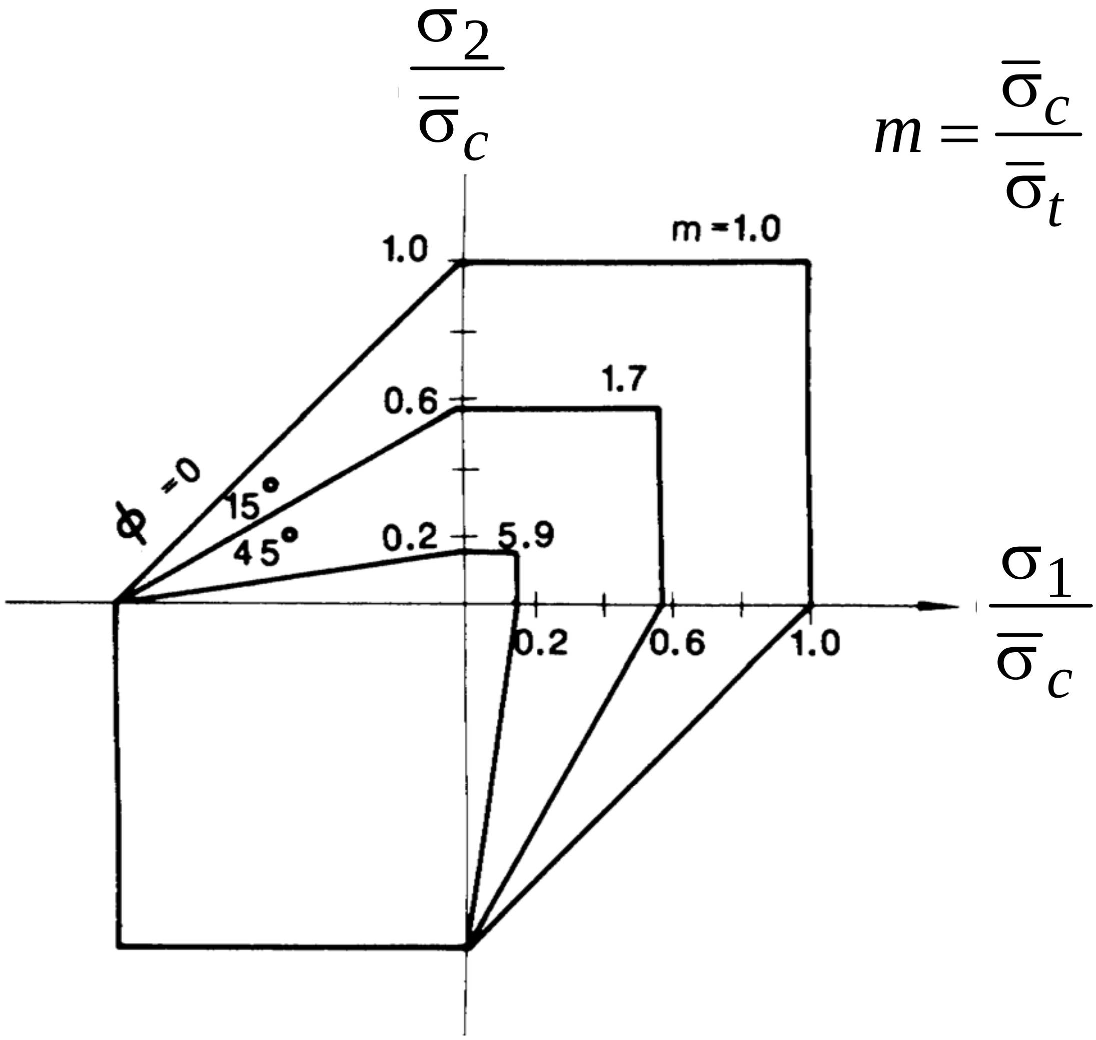
$$\frac{3,88}{4,91} + \frac{-4,16}{-7,34} = 1,36$$

1,36 is larger than 1, so, it will fail.

MOHR–COULOMB

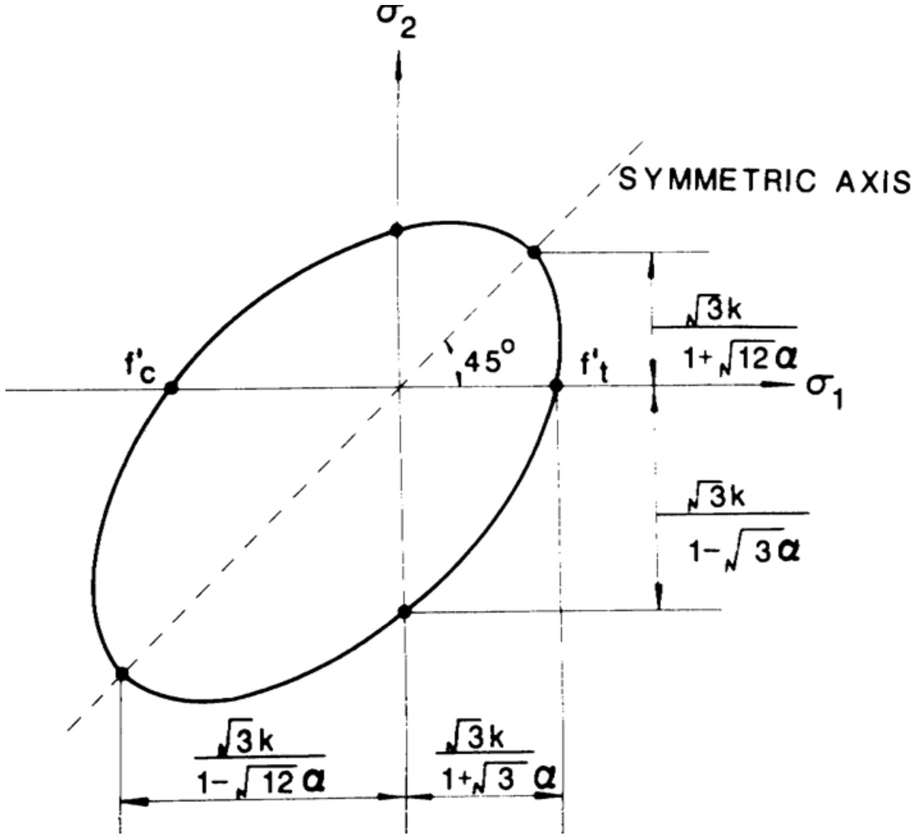


MOHR–COULOMB



$$m = \frac{\bar{\sigma}_c}{\bar{\sigma}_t}$$

DRUCKER-PRAGER



MOHR-COULOMB
versus
DRUCKER-PRAGER