Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Write your name and study number at the top right-hand of your work.

Exam CT4150 Plasticity Theory
Thursday 21 August 2003, 14:00 – 17:00 hours

Problem 1

A frame consists of two columns and a roof beam which are rigidly connected (Figure 1). The frame is simply supported in point A and point B. The frame is loaded by three forces. The columns and beam have the yield contour drawn in Figure 2. The following relation exists between the plastic moment M_p and the plastic normal force N_p.

$$N_p = \beta \frac{M_p}{a}$$

The influence of shear on the yield contour is neglected and buckling is not considered.

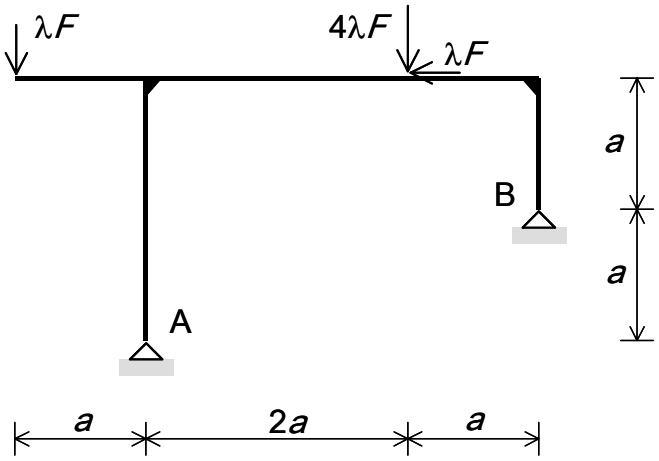


Figure 1. Simply supported frame

a Assume $\beta \to \infty$. Determine the collapse load λF for every possible mechanism. Write the collapse loads as functions of M_p and a. Which mechanism is decisive? What is the corresponding collapse load? (1.5 points)

b Assume $\beta \to \infty$. Draw the bending moment and normal force diagram for the whole structure at the moment of collapse (1 point).

c Assume $\beta = 1$. Choose one of the following problems (You need not do both) Use the decisive mechanism of problem **1a** (2.5 points).

– Determine the largest lower-bound for λF.

– Determine the smallest upper-bound for λF.

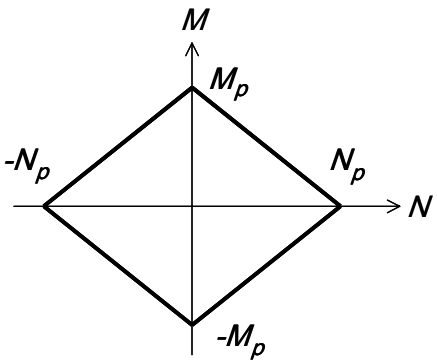


Figure 2. Yield contour

Problem 2

A square plate is simply supported at the middle part of each edge (Figure 3). The plate carries an equally distributed load λf [kN/m²]. The plate is homogeneous and isotropic. The yield moment is m_p [kNm/m].

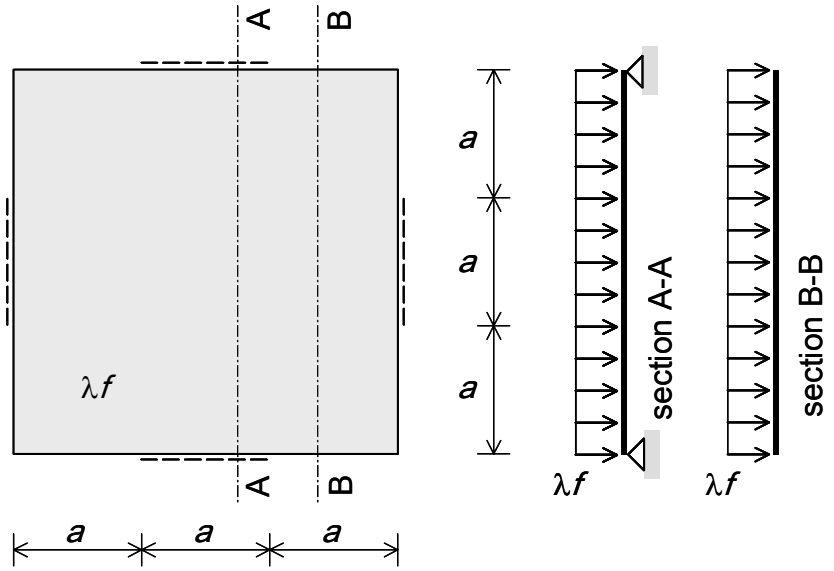


Figure 3. Simply supported square plate

a We consider the yield line patterns of Figure 4. Which of these patterns give kinematically possible mechanisms (2 points).

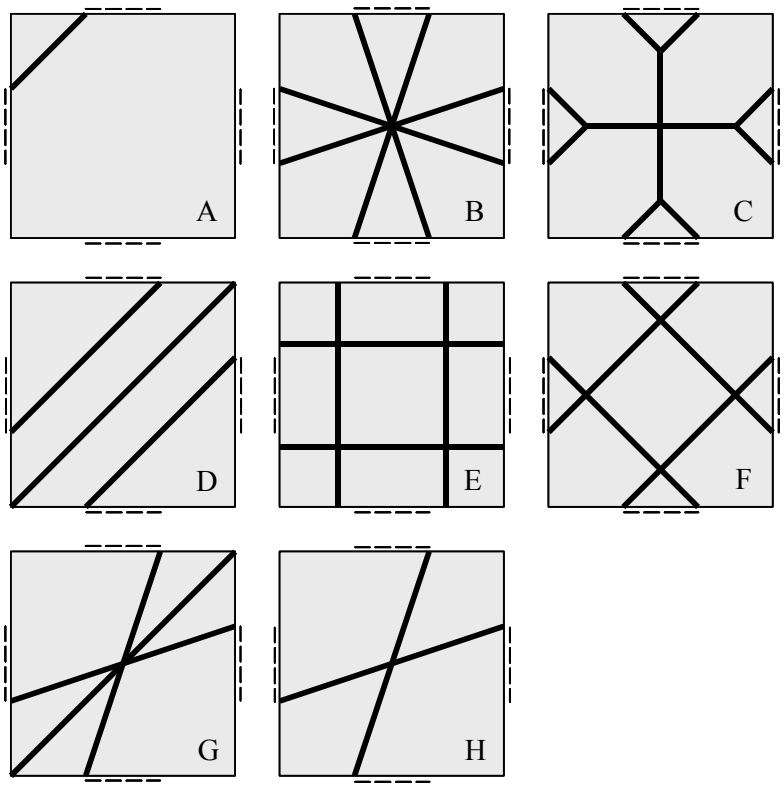


Figure 4. Yield line patterns of problem 2a

b We consider the yield line pattern of Figure 5. Determine an <u>upper bound</u> for λf expressed in m_p and a (1 point).

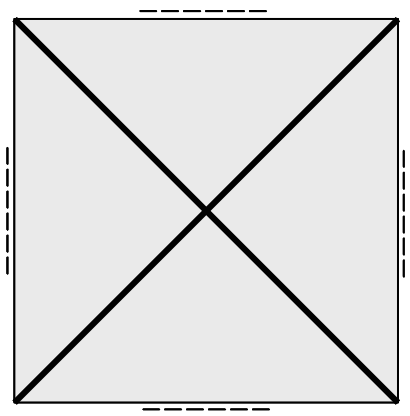
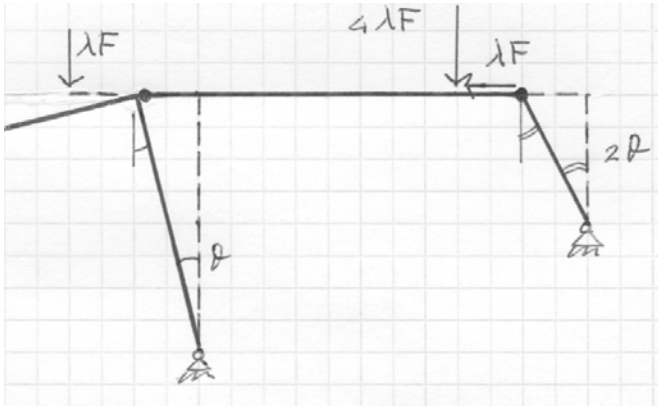


Figure 5. Yield line pattern of problem 2b

c Determine the largest <u>lower-bound</u> for λf using torsion free beams ($m_{xy} = 0$) in the x direction and y direction (2 points).

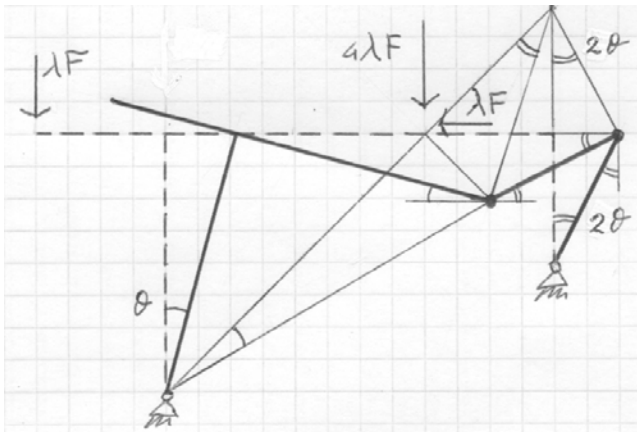
Answer to Problem 1a



$$A = \lambda F\, 2\vartheta a + \lambda F \vartheta a$$

$$E = M_p(\vartheta + 2\vartheta)$$

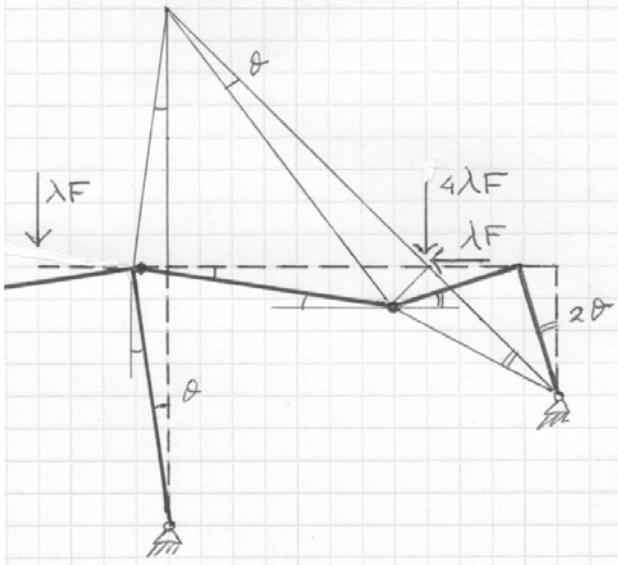
$$\lambda F = \frac{M_p}{a}$$



$$A = 4\lambda F\, \vartheta 2a - \lambda F\, \vartheta 2a - \lambda F \vartheta a$$
$$= 5\lambda F \vartheta a$$

$$E = M_p(3\vartheta + 4\vartheta)$$
$$= 7 M_p \vartheta$$

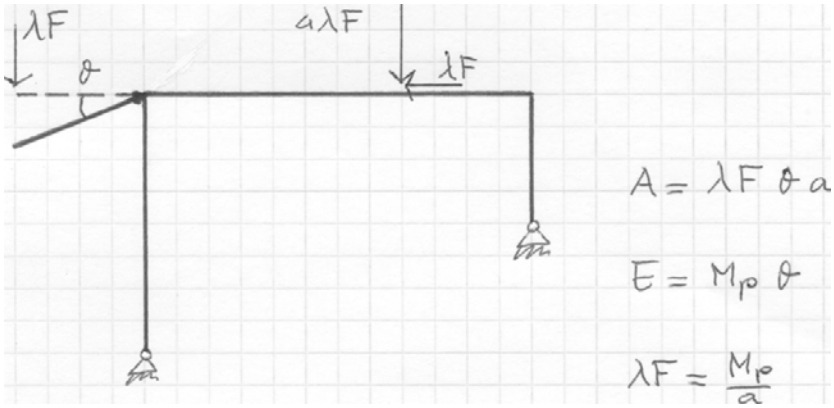
$$\lambda F = \frac{7}{5}\frac{M_p}{a}$$



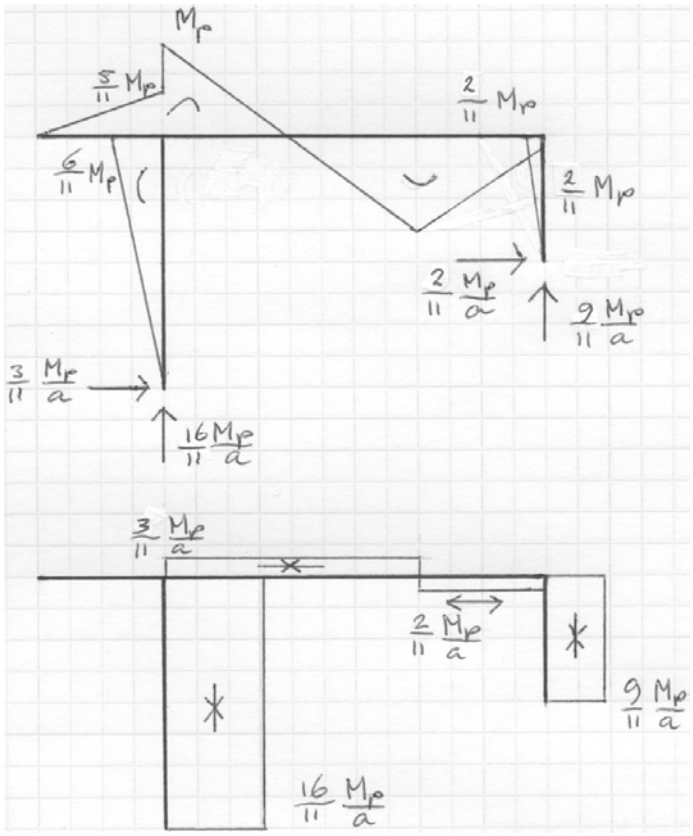
$$A = 4\lambda F\, 2\vartheta a + \lambda F\, 2\vartheta a + \lambda F \vartheta a$$
$$= 11\lambda F \vartheta a$$

$$E = M_p(2\vartheta + 3\vartheta)$$
$$= 5 M_p \vartheta$$

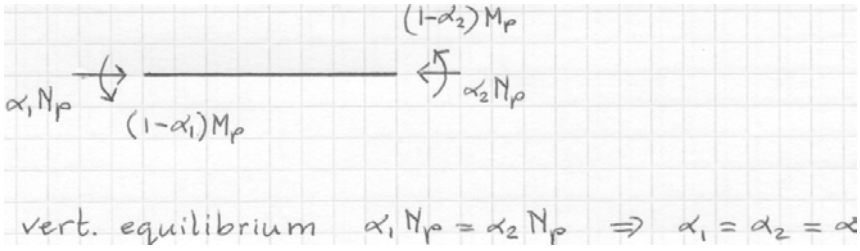
decisive $\longrightarrow$ $\boxed{\lambda F = \frac{5}{11}\frac{M_p}{a}}$

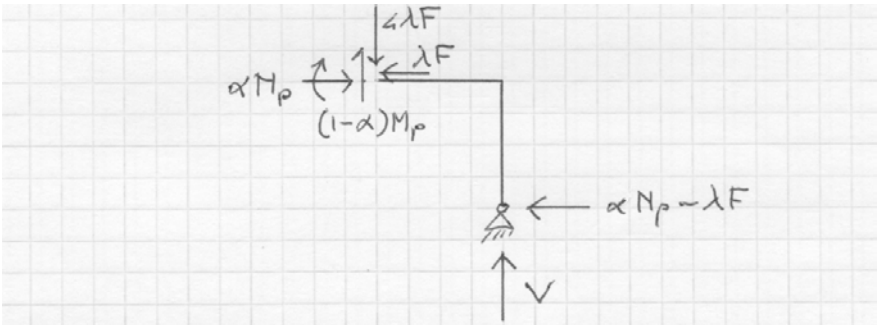


Answer to Problem 1b



Answer to problem 1c Lower-bound



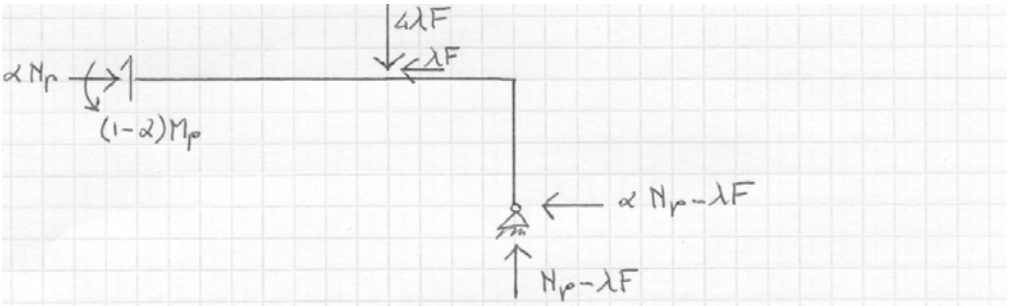


mom. equilibrium $(1-\alpha)M_p = Va - (\alpha N_p - \lambda F)a \Rightarrow$

$$V = (1-\alpha)\frac{M_p}{a} + \alpha N_p - \lambda F$$

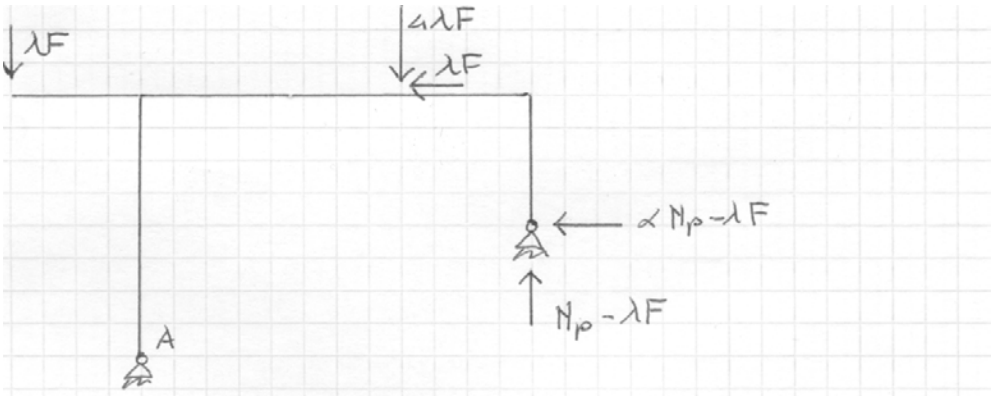
$$= (1-\alpha) N_p + \alpha N_p - \lambda F$$

$$= N_p - \lambda F$$



mom. equilibrium $(1-\alpha)M_p = 4\lambda F\, 2a - (N_p - \lambda F)\, 3a + (\alpha N_p - \lambda F)a$

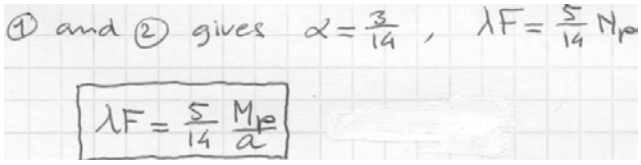
$$(2-\alpha) N_p = 5\lambda F \qquad (1)$$



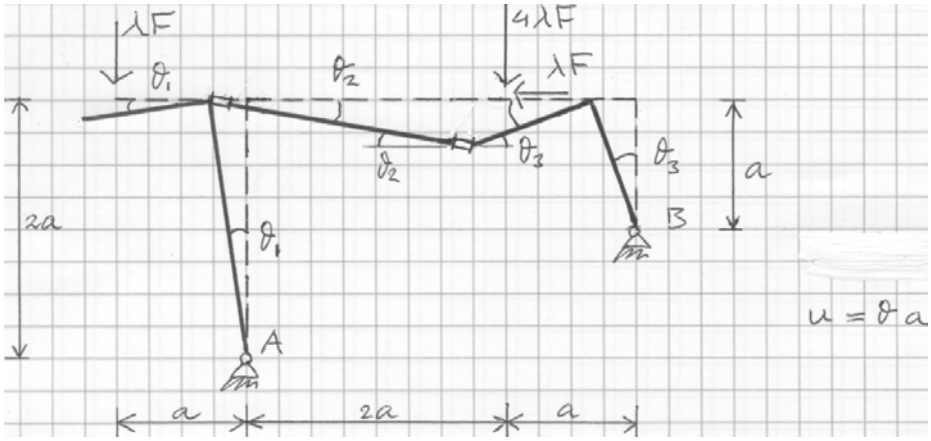
mom. equilibrium of A

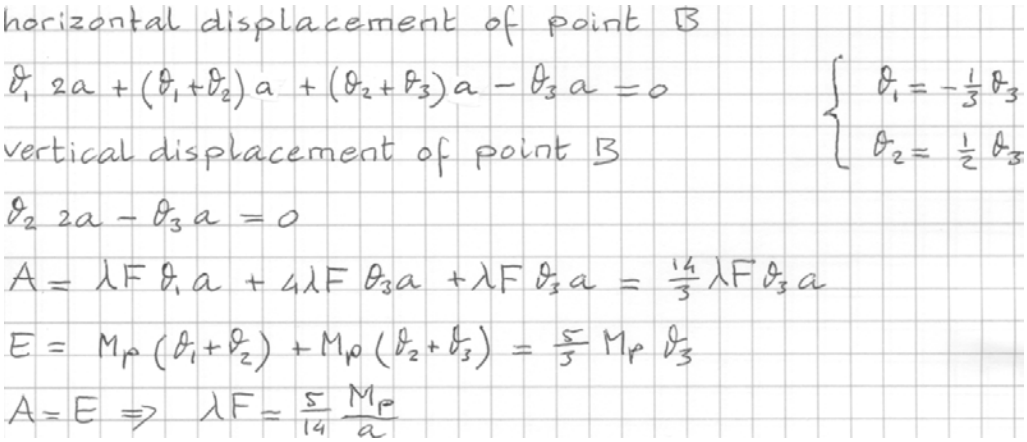
$$0 = -\lambda F a + 4\lambda F\, 2a - \lambda F\, 2a - (N_p - \lambda F)\, 3a - (\alpha N_p - \lambda F)a$$

$$0 = 9\lambda F - (3+\alpha) N_p \qquad (2)$$



Answer to Problem 1c Upper-bound

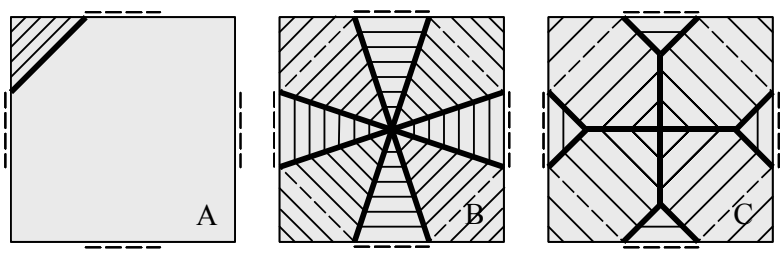


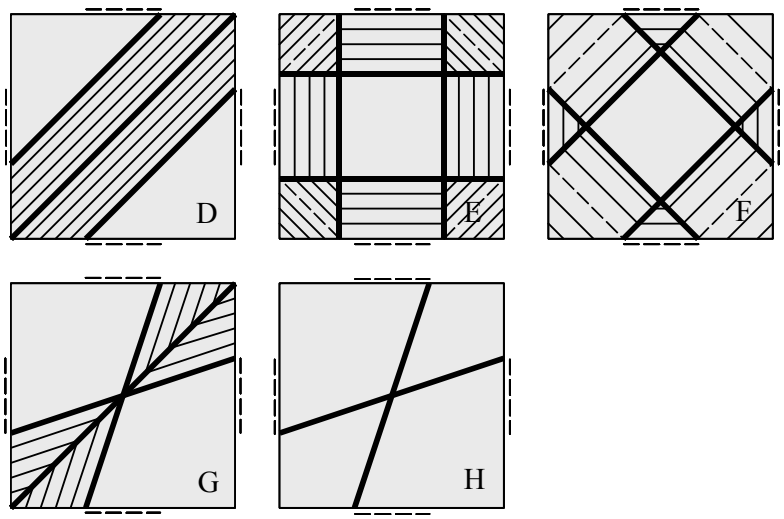


Note that θ_1 is negative. Apparently the roof beam shortens substantially during plastic deformation. This is due to the small value of β.

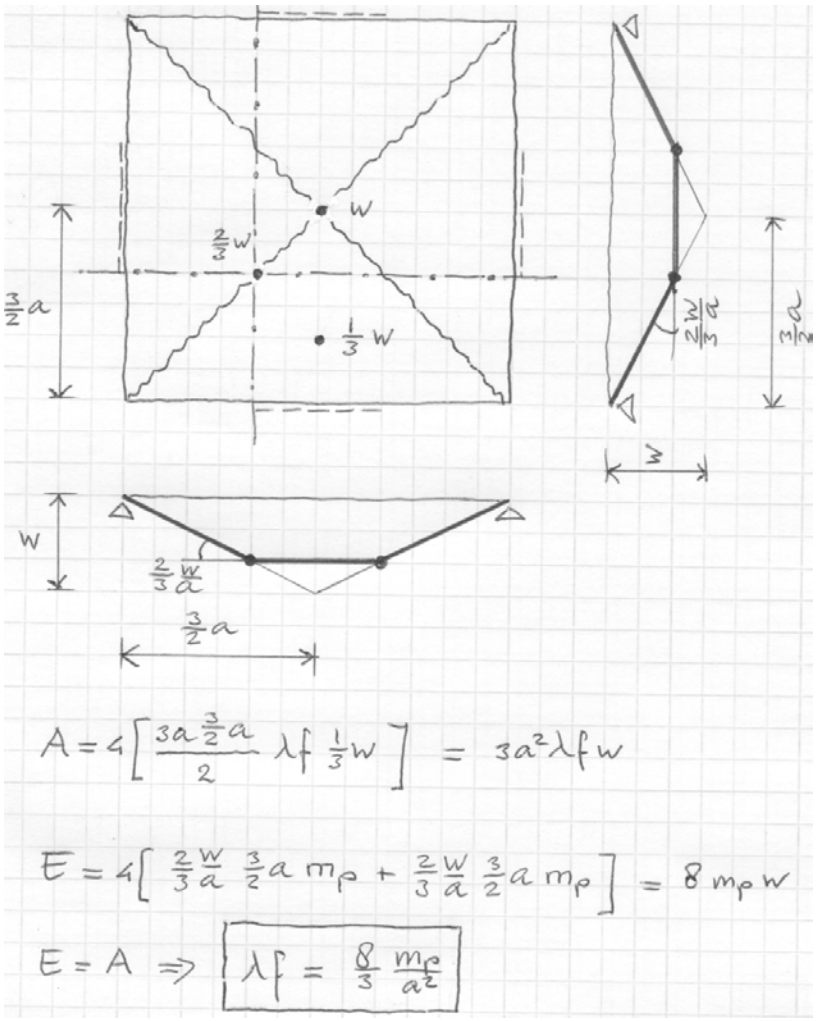
Answer to Problem 2a

All yield line patterns are possible mechanisms except for pattern H. The figure below shows the altitude lines of the deformed mechanisms.





Answer to Problem 2b



Answer to Problem 2c