Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Write your name and study number at the top right-hand of your work.

Exam CT4150 Plasticity Theory
Wednesday 11 June 2003, 9:00 – 12:00 hours

Problem 1

A frame consists of two columns and a roof beam which are rigidly connected (Figure 1). The frame is simply supported in point A and point B. The frame is loaded by three forces.
The columns and beam have the yield contour drawn in Figure 2. The following relation exists between the plastic moment M_p and the plastic normal force N_p.

$$N_p = \beta \frac{M_p}{a}$$

The influence of shear on the yield contour is neglected and buckling is not considered.

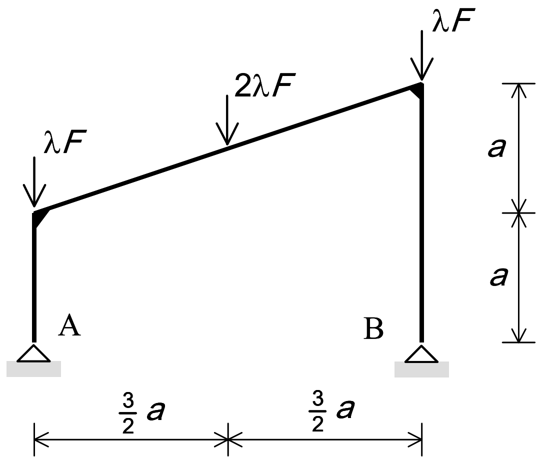


Figure 1. Simply supported frame

a Assume $\beta \to \infty$. Determine the collapse load λF for every possible mechanism. Write the collapse loads as functions of M_p and a. Which mechanism is decisive? What is the corresponding collapse load? (1.5 points)

b Assume $\beta \to \infty$. Draw the bending moment and normal force diagram for the whole structure at the moment of collapse (1 point).

c Assume $\beta = 2$ for the columns and $\beta \to \infty$ for the roof beam. Choose one of the following problems (You need not do both) Use the decisive mechanism of problem **1a** (2.5 points).

– Determine the largest lower-bound for λF.

– Determine the smallest upper-bound for λF.

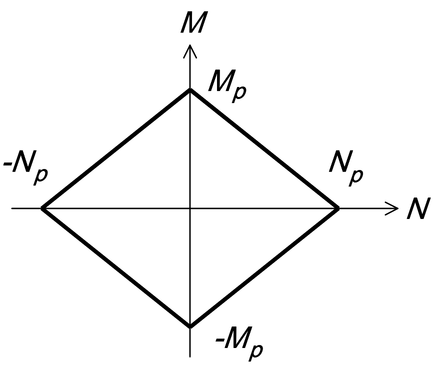


Figure 2. Yield contour

Problem 2

A square plate with an opening is simply supported at the edges (Figure 3). The plate carries an equally distributed load λf [kN/m²]. The plate is homogeneous and isotropic. The yield moment is m_p [kNm/m].

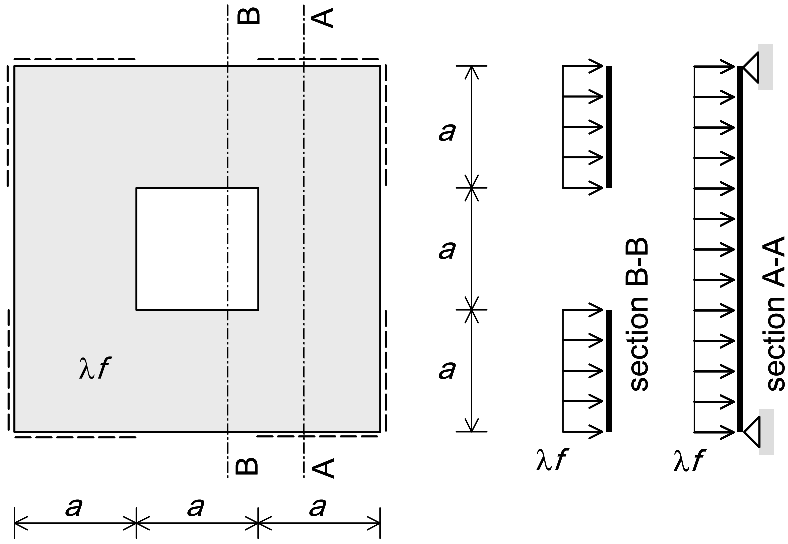


Figure 3. Simply supported square plate

a We consider the yield line pattern of Figure 4. Determine an <u>upper-bound</u> for λf expressed in m_p and a (1 point).

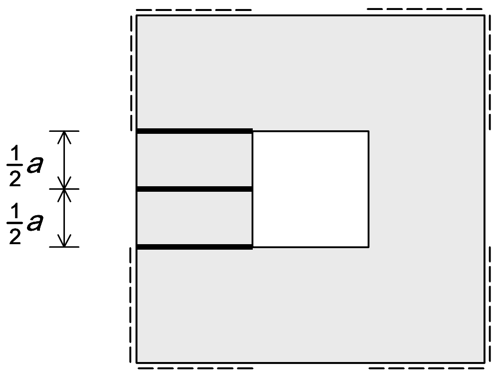


Figure 4. Yield line pattern of problem 2a

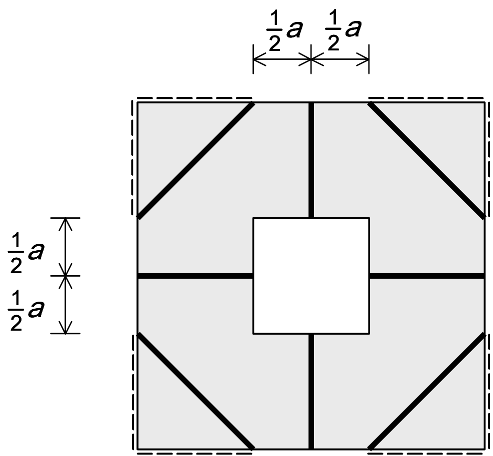
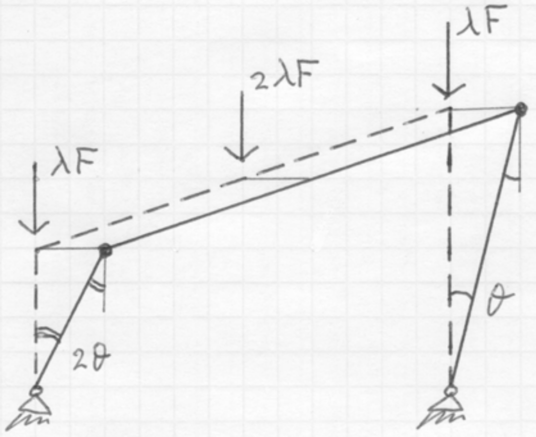


Figure 5. Yield line pattern of problem 2b

b We consider the yield line pattern of Figure 5. Determine an <u>upper bound</u> for λf expressed in m_p and a (2 points).

c Determine the largest <u>lower-bound</u> for λf using torsion free beams ($m_{xy} = 0$) in the x direction and y direction (2 points).

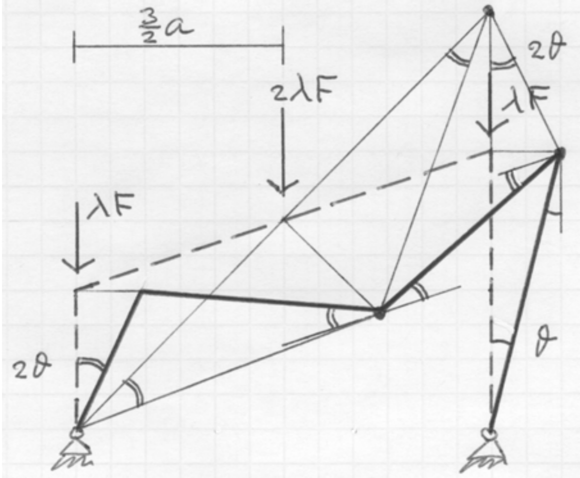
Answer to Problem 1a



$A = 0$

$E = M_p(2\vartheta + \vartheta)$

λf = indetermined

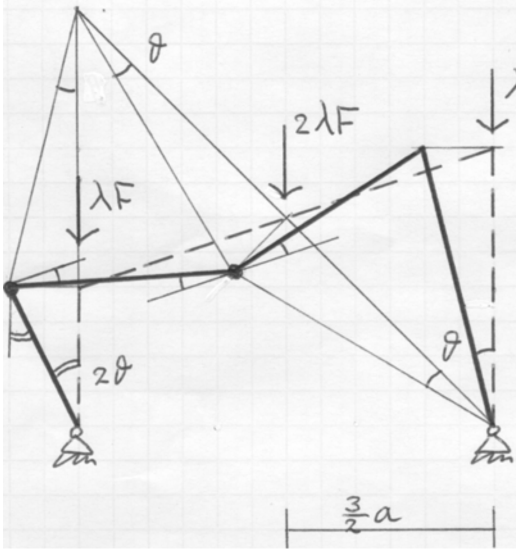


$A = 2\vartheta \cdot \frac{3}{2}a \cdot 2\lambda F = 6\vartheta a \lambda F$

$E = M_p(4\vartheta + 3\vartheta) = 7\vartheta M_p$

$\boxed{\lambda F = \frac{7}{6}\frac{M_p}{a}} \approx 1{,}17 \frac{M_p}{a}$

decisive

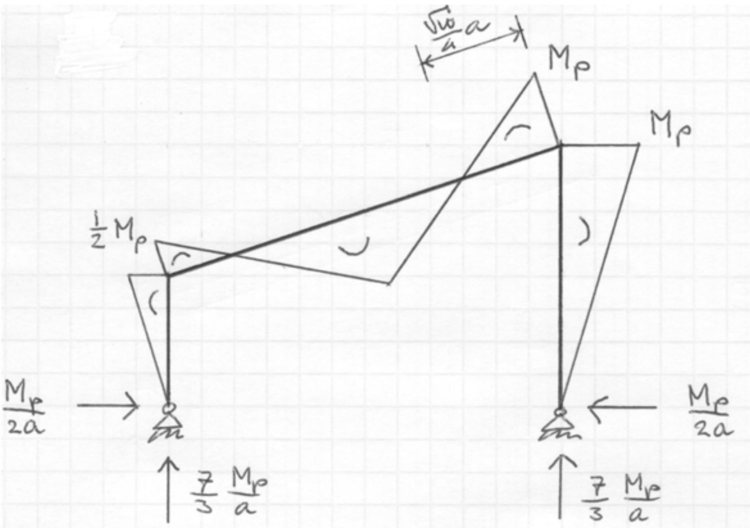


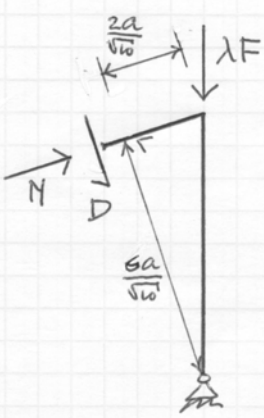
$A = \vartheta \cdot \frac{3}{2}a \cdot 2\lambda F = 3\vartheta a \lambda F$

$E = M_p(3\vartheta + 2\vartheta) = 5\vartheta M_p$

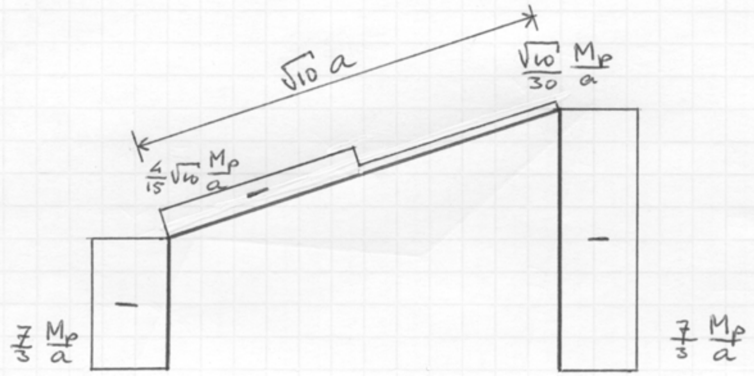
$\lambda F = \frac{5}{3}\frac{M_p}{a}$

Answer to Problem 1b





$$D = \frac{M_p}{\frac{\sqrt{10}}{4}a}$$



$$N\,\frac{6a}{\sqrt{10}} = D\left(\frac{\sqrt{10}}{4}a - \frac{2a}{\sqrt{10}}\right)$$

$$N\,\frac{6a}{\sqrt{10}} = \frac{M_p}{a}\,\frac{4}{\sqrt{10}}\left(\frac{\sqrt{10}}{4} - \frac{2}{\sqrt{10}}\right)a$$

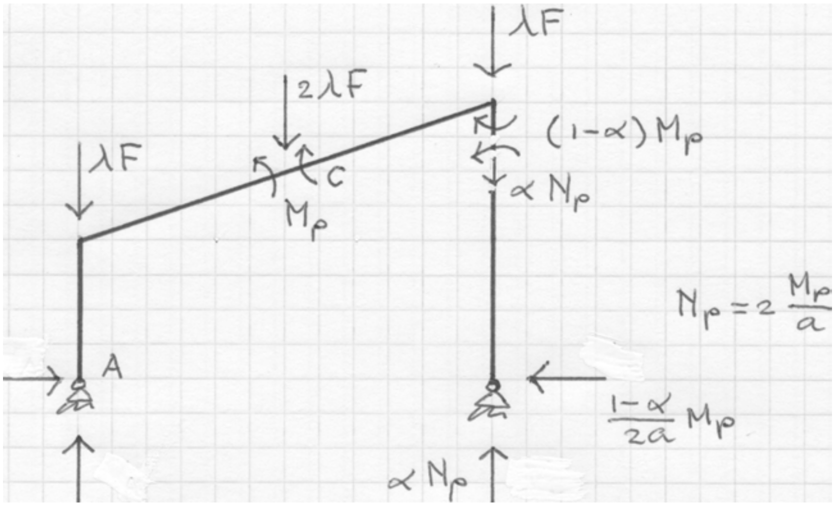
$$= M_p\left(1 - \frac{8}{10}\right)$$

$$= M_p\,\frac{2}{10}$$

$$N = \frac{M_p}{a}\,\frac{2}{10}\,\frac{\sqrt{10}}{6}$$

$$= \frac{\sqrt{10}}{30}\,\frac{M_p}{a}$$

Answer to problem 1c Lower-bound



Σ mom A

$$0 = 2\lambda F \tfrac{3}{2}a + \lambda F\, 3a - \alpha N_p\, 3a$$

$$= 6a\lambda F - \alpha\, 2\frac{M_p}{a}\, 3a$$

$$= 6a\lambda F - \alpha 6 M_p \quad \Rightarrow \quad \lambda F = \alpha \frac{M_p}{a} \qquad (1)$$

Σ mom C right-hand side

$$M_p = \alpha N_p \tfrac{3}{2}a - \frac{1-\alpha}{2a} M_p \tfrac{3}{2}a - \lambda F \tfrac{3}{2}a$$

$$\frac{M_p}{\frac{3}{2}a} = \alpha\, 2\frac{M_p}{a} - \frac{M_p}{2a} + \alpha \frac{M_p}{2a} - \lambda F$$

$$\left(\tfrac{2}{3} + \tfrac{1}{2}\right)\frac{M_p}{a} = \tfrac{5}{2}\alpha \frac{M_p}{a} - \lambda F \quad \Rightarrow \quad \lambda F = -\frac{M_p}{a}\left(\tfrac{7}{6} - \tfrac{5}{2}\alpha\right) \qquad (2)$$

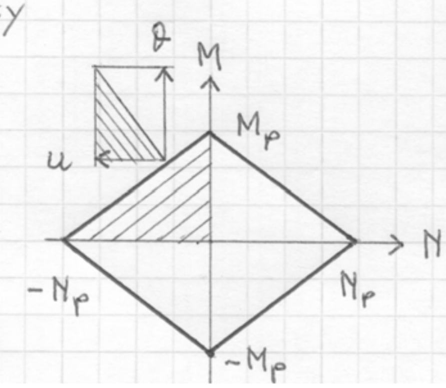
(1) and (2)

$$\alpha = -\tfrac{7}{6} + \tfrac{5}{2}\alpha \quad \Rightarrow \quad -\tfrac{3}{2}\alpha = -\tfrac{7}{6} \quad \Rightarrow \quad \alpha = \tfrac{7}{9}$$

$$\lambda F = \tfrac{7}{9}\frac{M_p}{a}$$

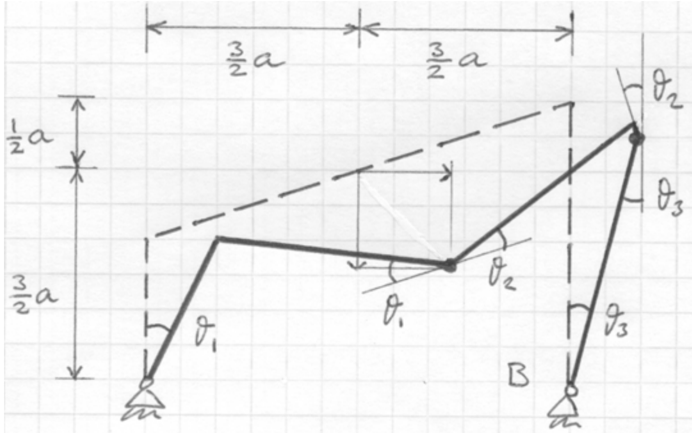
Answer to Problem 1c Upper-bound

normality



$$\frac{u}{\vartheta} = \frac{M_p}{N_p} = \frac{a}{\beta}$$

$$u = \vartheta \frac{a}{\beta} = \frac{1}{2}\vartheta a$$



horizontal displacement of point B

$$\vartheta_1 \tfrac{3}{2}a - \vartheta_2 \tfrac{1}{2}a - \vartheta_3\, 2a = 0$$

vertical displacement of point B

$$\vartheta_1 \tfrac{3}{2}a - \vartheta_2 \tfrac{3}{2}a - \tfrac{1}{2}a(\vartheta_2+\vartheta_3) = 0$$

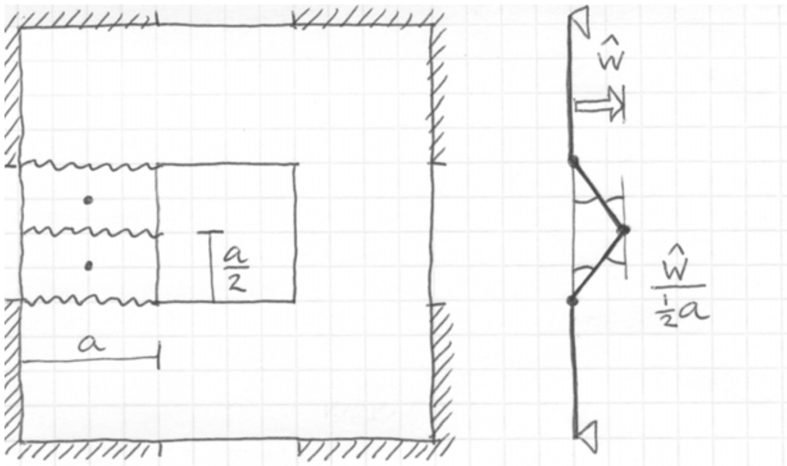
$$\begin{cases} \vartheta_1 = \tfrac{5}{3}\vartheta_3 \\ \vartheta_2 = \vartheta_3 \end{cases}$$

$$A = 2\lambda F\, \vartheta_1 \tfrac{3}{2}a + \lambda F \tfrac{1}{2}a(\vartheta_2+\vartheta_3) = 6\lambda F \vartheta_3 a$$

$$E = M_p(\vartheta_1+\vartheta_2) + M_p(\vartheta_2+\vartheta_3) = \tfrac{14}{3} M_p \vartheta_3$$

$$A = E \Rightarrow \lambda F = \frac{7}{9}\frac{M_p}{a} \approx 0{,}78\frac{M_p}{a}$$

Answer to Problem 2a



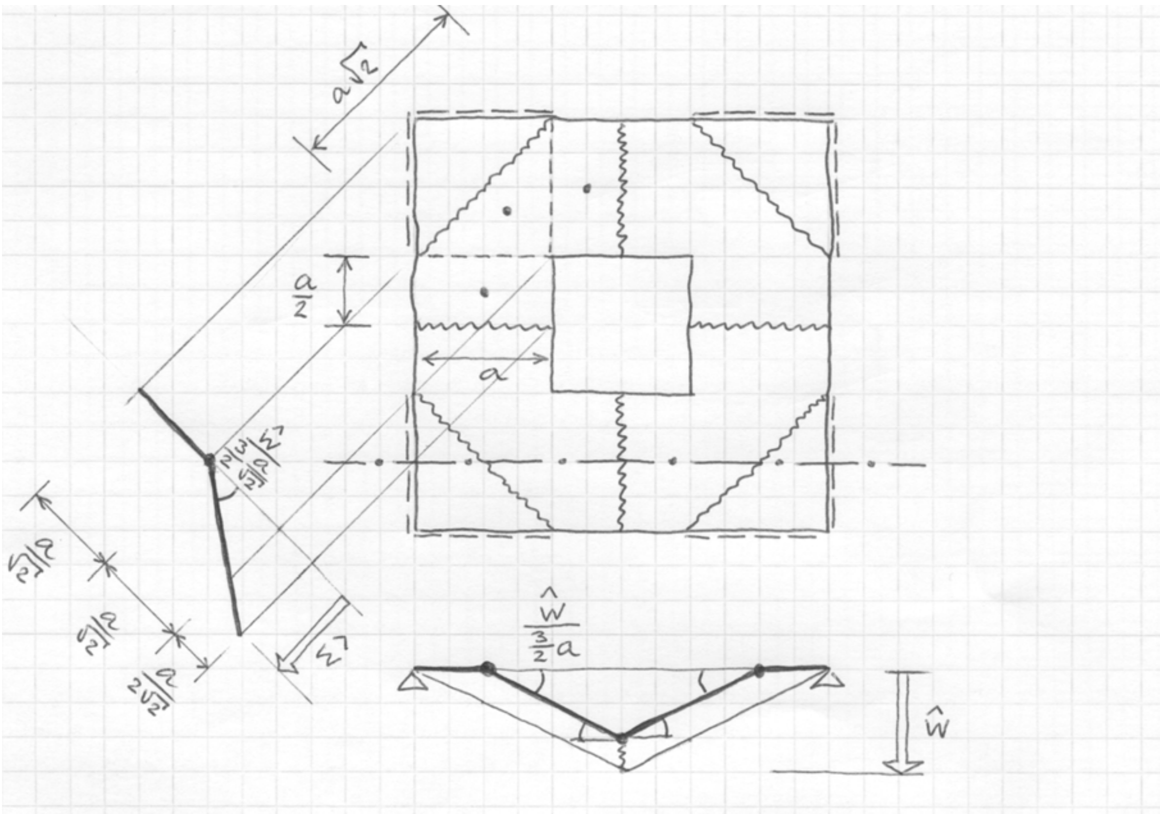
$$A = 2\left(\frac{\hat{w}}{2}\lambda f a \frac{a}{2}\right) = \frac{1}{2}a^2 \hat{w} \lambda f$$

$$E = \frac{\hat{w}}{\frac{1}{2}a} m_p a + 2\frac{\hat{w}}{\frac{1}{2}a} m_p a + \frac{\hat{w}}{\frac{1}{2}a} m_p a$$

$$= 8 \hat{w} m_p$$

$$A = E \Rightarrow \lambda f = 16 \frac{m_p}{a^2}$$

Answer to Problem 2b

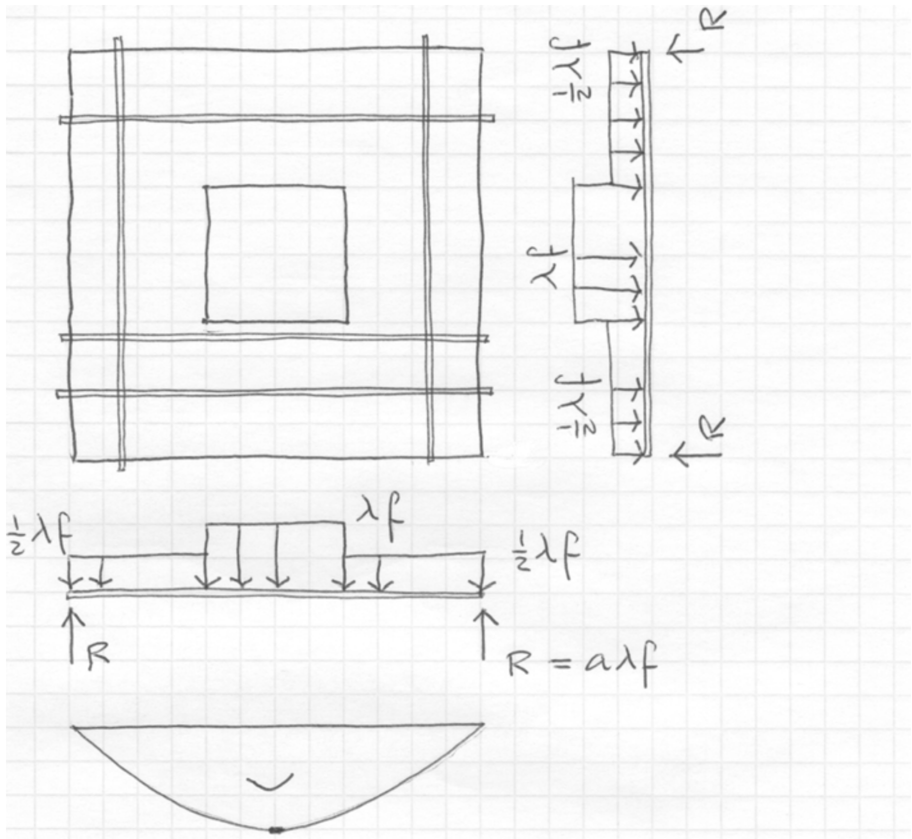


$$A = 4\left[\frac{\hat{w}}{2}\lambda f\, a\frac{a}{2} + \frac{\hat{w}}{2}\lambda f\, a\frac{a}{2} + \frac{2}{9}\hat{w}\,\lambda f\,\frac{a\,a}{2}\right] = \frac{22}{9}a^2\hat{w}\,\lambda f$$

$$E = 4\left[\frac{\hat{w}}{\frac{3}{2}\frac{a}{\sqrt{2}}}\,a\sqrt{2}\,m_p\right] + 4\left[2\,\frac{\hat{w}}{\frac{3}{2}a}\,a\,m_p\right] = \frac{32}{3}\hat{w}\,m_p$$

$$\lambda f = \frac{48}{11}\,\frac{m_p}{a^2} \approx 4{,}36\,\frac{m_p}{a^2}$$

Answer to Problem 2c



$$m_p = \frac{3}{2}a\, a\lambda f - \frac{1}{2}\lambda f\, a a - \lambda f\, \frac{1}{2}a\, \frac{1}{4}a$$

$$= \frac{7}{8}\, a^2 \lambda f \;\Rightarrow$$

$$\lambda f = \frac{8}{7}\,\frac{m_p}{a^2}$$