Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Write your <u>name</u> and <u>study number</u> at the top right-hand of your work.

Exam CT4150 Plasticity Theory
Thursday 19 August 2004, 14:00 – 17:00 hours

Problem 1

A simply supported frame consists of three members that are rigidly connected (Figure 1). The frame is loaded by two forces $9\lambda F$ and λF. The members have different yield contours (Figure 2). The top and right-hand member have a yield moment $2M_p$. The bottom member has a yield moment M_p. The following relation exists between the plastic moment M_p and the plastic normal force N_p.

$$N_p = \beta \frac{M_p}{a}$$

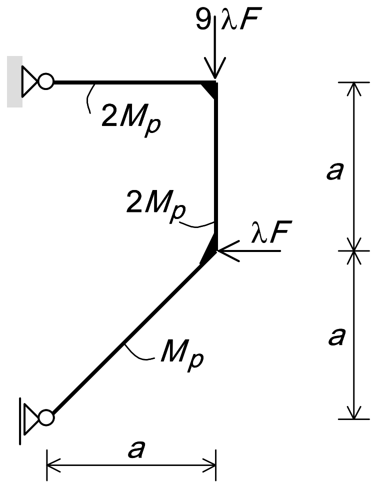


Figure 1. Simply supported frame

The influence of shear on the yield contour is neglected. Buckling and second order effects are not considered.

a Assume $\beta \to \infty$. Determine the collapse load λF for every possible mechanism. Write the collapse loads as functions of M_p and a. Which mechanism is decisive? What is the corresponding collapse load? (1 point)

b Assume $\beta \to \infty$. Draw the bending moment and normal force diagram for the whole structure at the moment of collapse (1 point).

c Assume $\beta = 30\sqrt{2}$. Choose one of the following problems (You need not do both). Use the decisive mechanism of problem **1a** (3 points).

– Determine the largest <u>lower-bound</u> for λF.

– Determine the smallest <u>upper-bound</u> for λF.

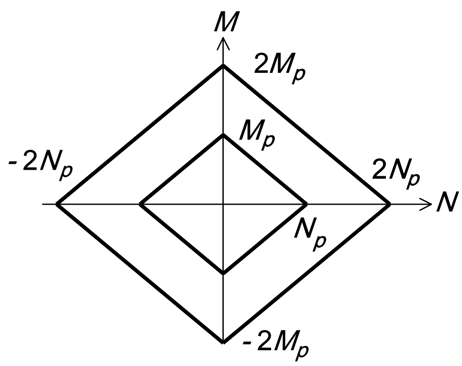


Figure 2. Yield contour

Problem 2

A square plate is simply supported at parts of the edges (Figure 3). One edge of the plate carries a evenly distributed load λf [kN/m]. The plate is homogeneous. The yield moment in the y direction is m_p. The yield moment in the x direction is $3\,m_p$ [kNm/m].

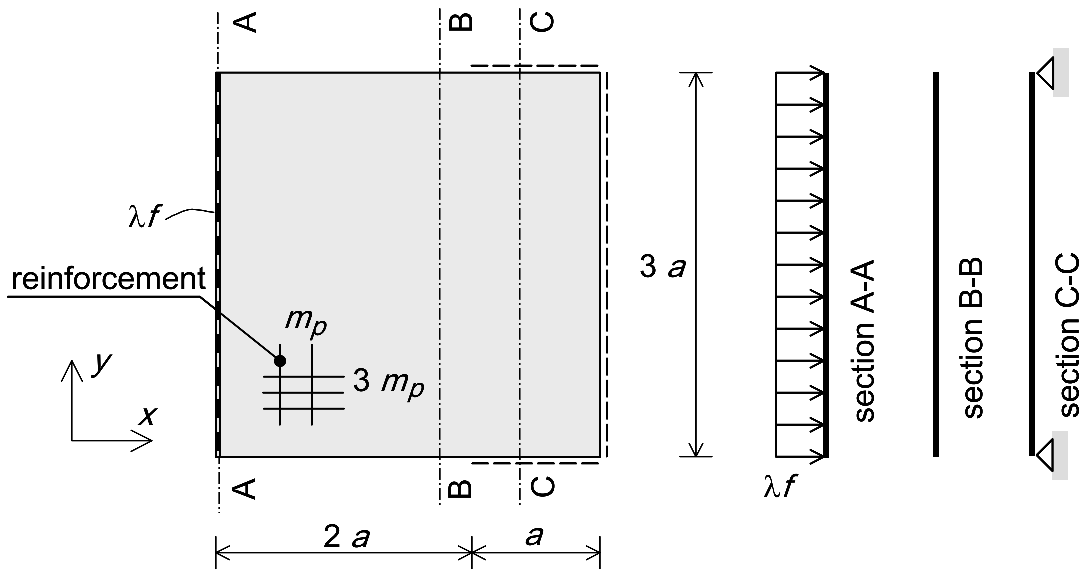


Figure 3. Simply supported square plate

a We consider the yield line patterns of Figure 4 and Figure 5. Which of these patterns give kinematically possible mechanisms (2 points).

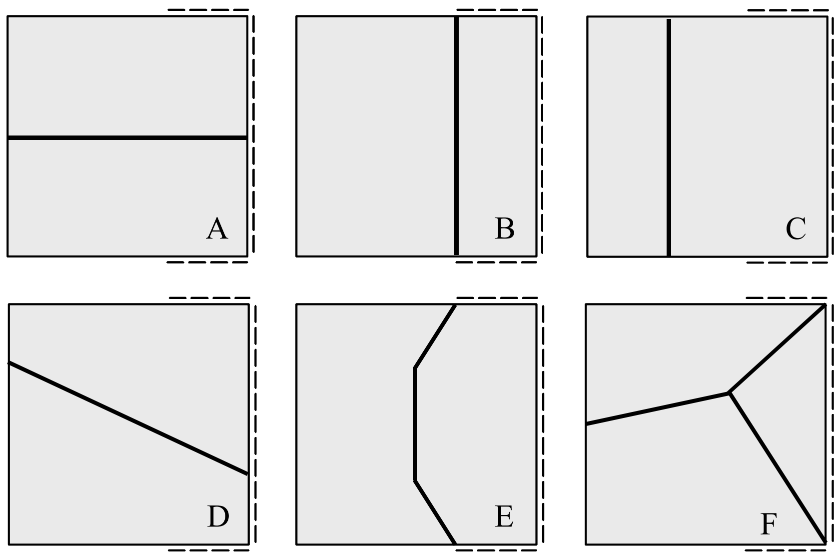


Figure 4. Yield line patterns of problem 2a (see also Figure 5)

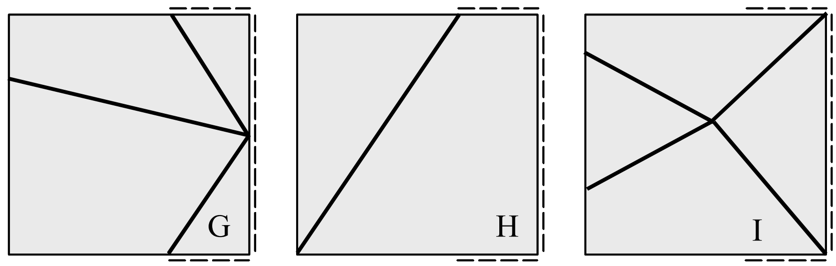


Figure 5. Yield line patterns of problem 2a

b We consider the yield line pattern of Figure 6. Determine an upper bound for λf expressed in m_p and a (1 point).

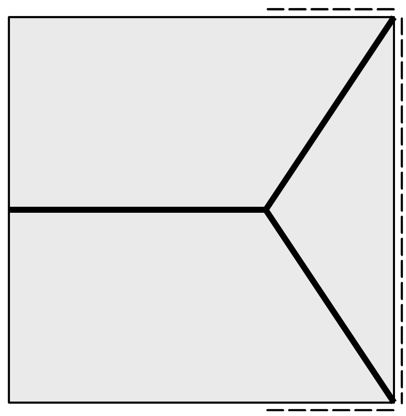
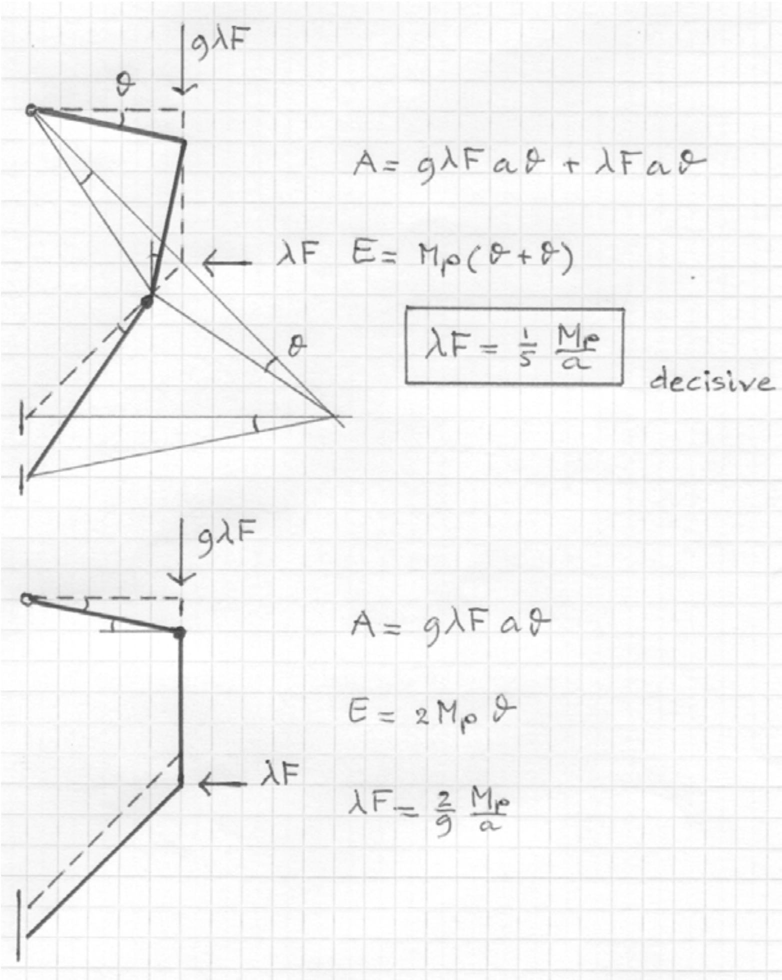


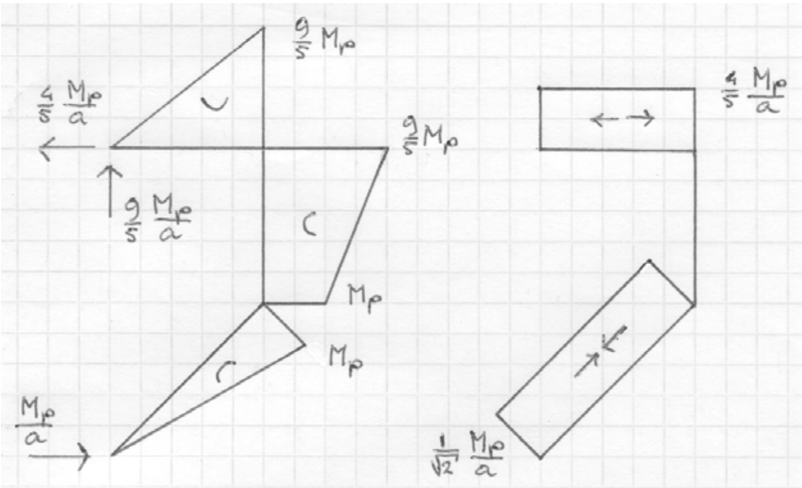
Figure 6. Yield line pattern of problem 2b

c Determine the largest lower-bound for λf using torsion free beams ($m_{xy} = 0$) in the x direction and y direction (2 points).

Answer to Problem 1a



Answer to Problem 1b



Answer to problem 1c Lower-bound

We reduce the loading with a factor α. Also the moments and the normal forces are reduced with α. In a section with a moment αM_p we can allow a normal force $(1-\alpha)N_p$. Therefore, in the plastic hinge

$$N = \alpha \frac{1}{\sqrt{2}} \frac{M_p}{a} = (1-\alpha) N_p = (1-\alpha)\beta \frac{M_p}{a}.$$

Consequently,

$$\alpha \frac{1}{\sqrt{2}} = (1-\alpha)\beta$$

and

$$\alpha = \frac{\beta}{\frac{1}{\sqrt{2}} + \beta}.$$

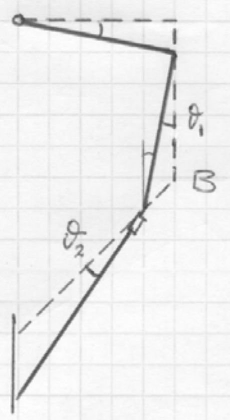
Since $\beta = 30\sqrt{2}$ we find

$$\alpha = \frac{30\sqrt{2}}{\frac{1}{\sqrt{2}} + 30\sqrt{2}} = \frac{60}{61}.$$

The collapse load is

$$\lambda F = \alpha \frac{1}{5} \frac{M_p}{a} = \frac{12}{61} \frac{M_p}{a}.$$

Answer to Problem 1c Upper-bound



$$u = \vartheta \frac{a}{30\sqrt{2}}$$

horizontal displacement of B

$$\vartheta_1 a = \vartheta_2 a + (\vartheta_1 + \vartheta_2) \frac{a}{30\sqrt{2}} \frac{1}{\sqrt{2}}$$

$$\vartheta_1 a - \vartheta_1 \frac{a}{60} = \vartheta_2 a + \vartheta_2 \frac{a}{60}$$

$$\vartheta_1 \left(1 - \frac{1}{60}\right) = \vartheta_2 \left(1 + \frac{1}{60}\right)$$

$$\vartheta_1 = \vartheta_2 \frac{\frac{61}{60}}{\frac{59}{60}} = \vartheta_2 \frac{61}{59}$$

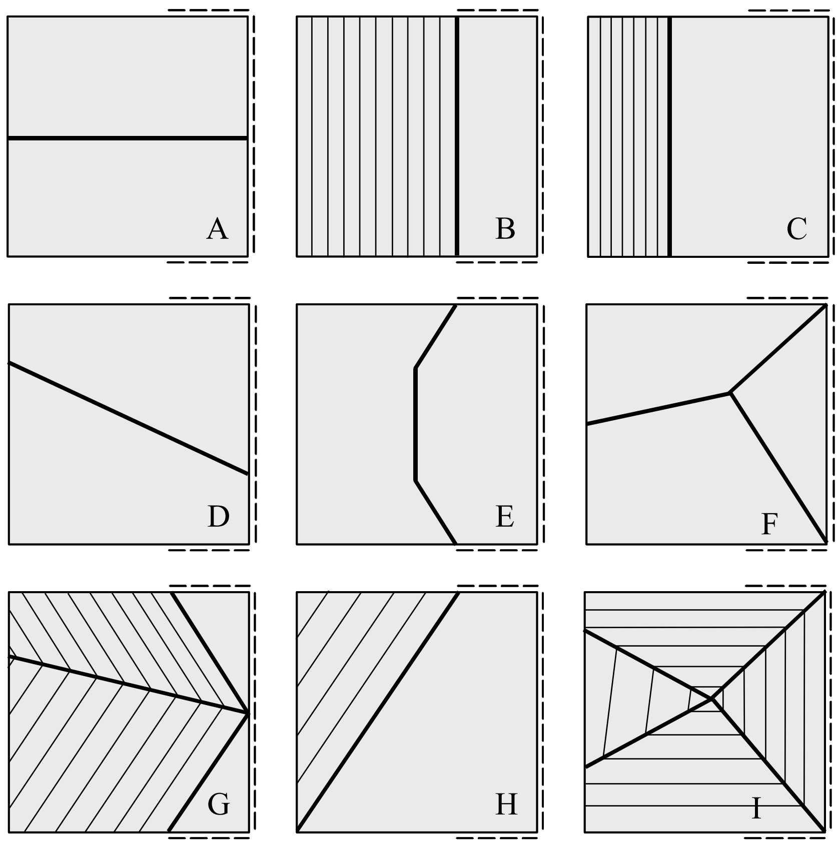
$$A = 9\lambda F a \vartheta_1 + \lambda F a \vartheta_1 = 10\lambda F a \vartheta_1 = \lambda F a \vartheta_2 \frac{610}{59}$$

$$E = (\vartheta_1 + \vartheta_2) M_p = \left(\frac{61}{59} + 1\right) \vartheta_2 M_p = \frac{120}{59} \vartheta_2 M_p$$

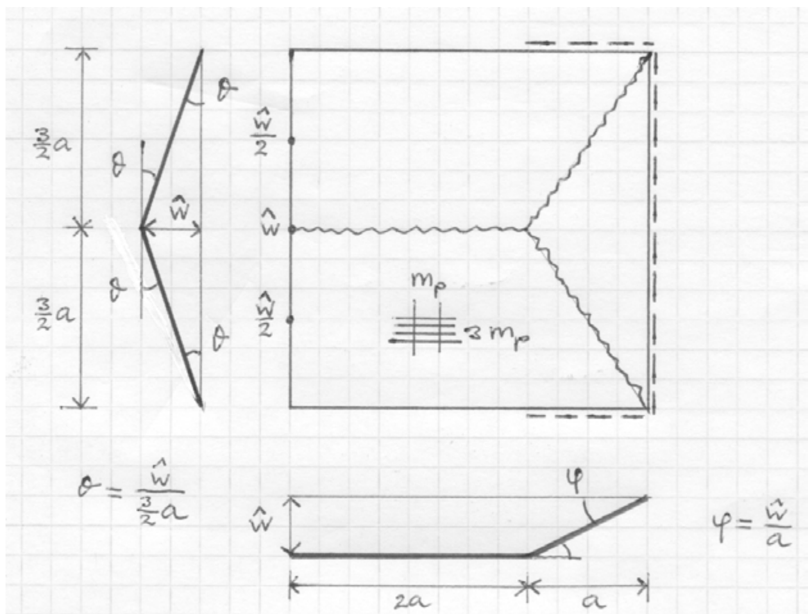
$$\lambda F = \frac{120}{610} \frac{M_p}{a} = \frac{12}{61} \frac{M_p}{a}$$

Answer to Problem 2a

Kinematically possible are patterns B, C, G, H and **I**. The figure below shows the altitude lines of the deformed mechanisms.



Answer to Problem 2b

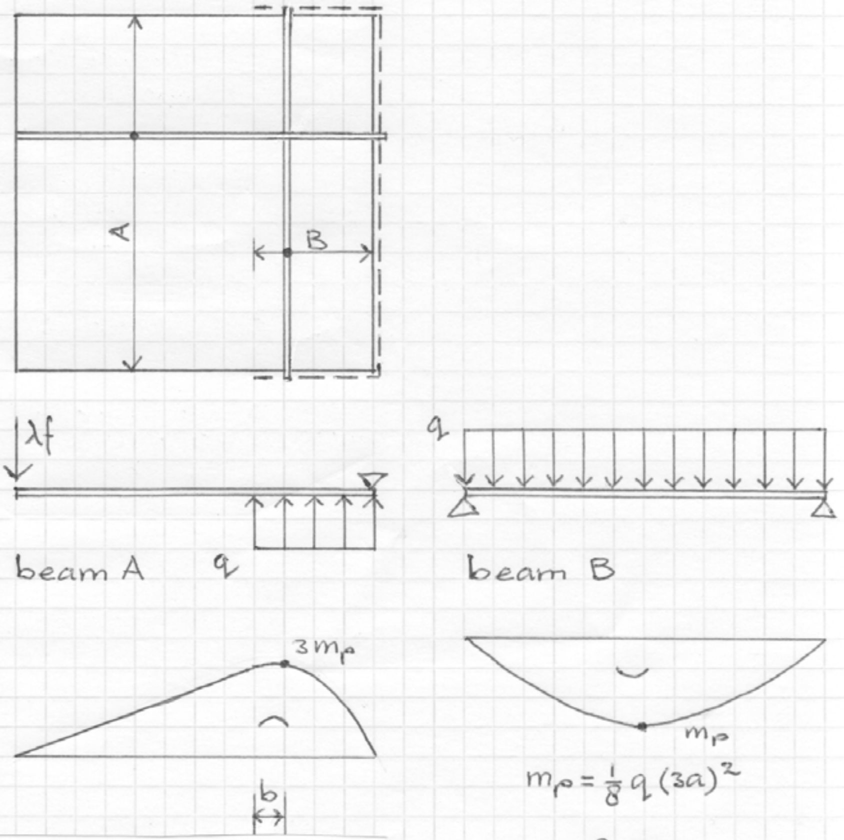


$$E = m_p\, 2a\, 2\vartheta + 2\left[m_p\, a\, \vartheta + 3m_p\, \tfrac{3}{2}a\, \varphi \right] = 13\, m_p\, \hat{w}$$

$$A = 2\left[\lambda f\, \tfrac{3}{2}a\, \tfrac{\hat{w}}{2} \right] = \tfrac{3}{2}\lambda f a\, \hat{w}$$

$$\lambda f = \frac{26}{3}\,\frac{m_p}{a}$$

Answer to Problem 2c



$$m_p = \tfrac{1}{8}\, q\, (3a)^2$$

$$q = \frac{8}{9}\,\frac{m_p}{a^2}$$

$$\lambda f\, 3a = q a \tfrac{1}{2} a \;\Rightarrow\; \lambda f = \tfrac{1}{6} q a$$

$$\lambda f = \frac{4}{27}\,\frac{m_p}{a}$$

Check max moment in beam A

$$\lambda f = bq \;\Rightarrow\; b = \frac{\lambda f}{q} = \frac{1}{6}a$$

$$m_{max} = \lambda f\,(2a + b) - q b \tfrac{1}{2} b$$

$$= \frac{4}{27}\,\frac{m_p}{a}\left(2a + \tfrac{1}{6}a\right) - \frac{8}{9}\,\frac{m_p}{a^2}\left(\tfrac{1}{6}a\right)^2 \frac{1}{2}$$

$$= \frac{25}{81}\, m_p \;<\; 3\, m_p \quad \text{okay}$$