Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Write your name and study number at the top right-hand of your work.

Exam CT4150 Plasticity Theory
Wednesday 15 June 2005, 9:00 – 12:00 hours

Problem 1

A frame consists of two columns and a roof beam (Figure 1). The left hand column is rigidly connected to the roof beam and the foundation. The right hand column is connected with hinges to the roof beam and the foundation. The frame is loaded by forces F and $2F$. The columns and beam have different yield contours (Figure 2). The following relation exists between the plastic moment M_p and the plastic normal force N_p.

$$N_p = \beta \frac{M_p}{a}$$

The influence of shear on the yield contour is neglected. Buckling and second order effects are not considered.

a Assume $\beta \to \infty$. Determine the collapse load F for possible mechanisms. Write the collapse loads as functions of M_p and a. What is the decisive collapse load? (1.5 points)

b Assume $\beta \to \infty$. Draw the bending moment diagram for the whole structure at the moment of collapse. (The normal force diagram is plotted in Figure 3.) (1 point)

c Assume $\beta = 50$. Choose one of the following problems (You need not do both). Use the decisive mechanism of problem **1a** (2 points).

– Determine the largest lower-bound for F.

– Determine the smallest upper-bound for F.

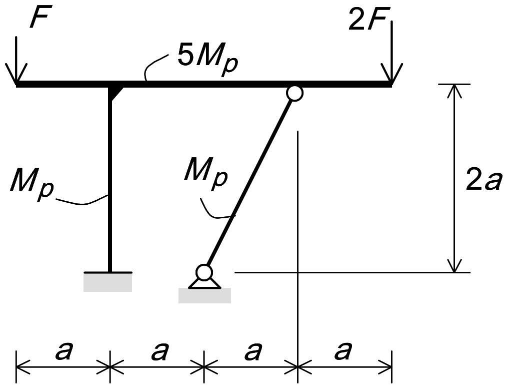


Figure 1. Frame

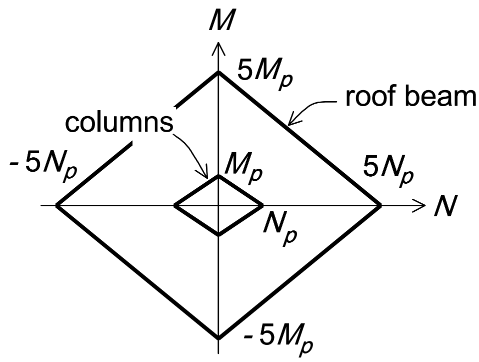


Figure 2. Yield contour

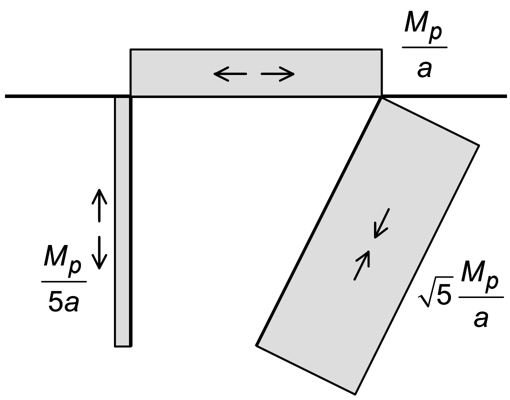


Figure 3. Normal force distribution

Problem 2

A square plate is simply supported in its interior (Figure 4). All plate edges are free. The plate carries an evenly distributed load q [kN/m²]. The plate is homogeneous. The yield moment in the x direction is m_p. The yield moment in the y direction is $3\,m_p$ [kNm/m].

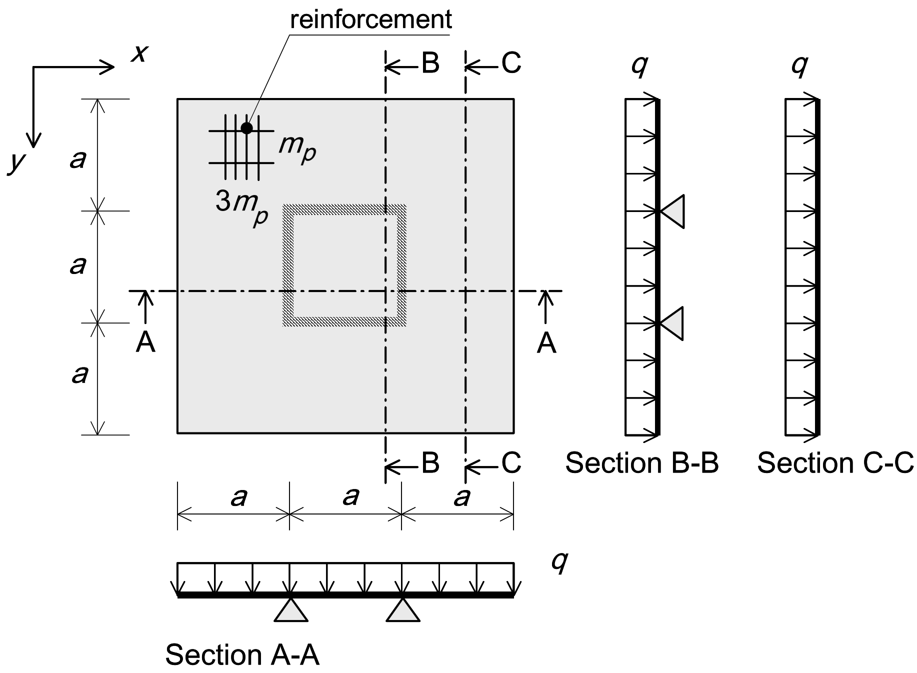


Figure 4. Simply supported square plate

a We consider the yield line patterns of Figure 5. Which of these patterns give kinematically possible mechanisms (1 point).

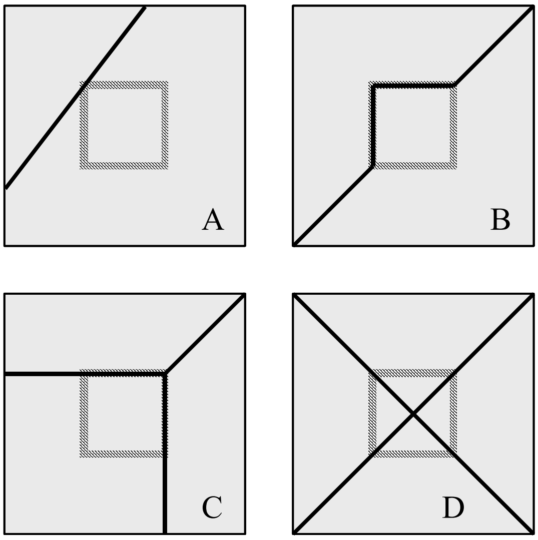


Figure 5. Yield line patterns of problem 2a

b We consider the yield line pattern of Figure 6. Determine an <u>upper bound</u> for q expressed in m_p and a (1.5 points).

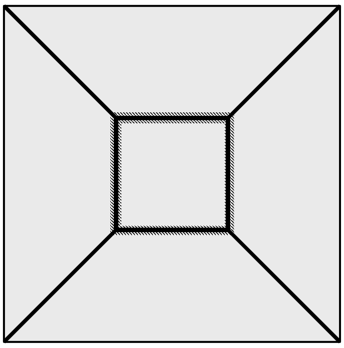


Figure 6. Yield line pattern of problem 2b

c Determine the largest <u>lower-bound</u> for q using torsion free beams ($m_{xy} = 0$) in the x direction and y direction (1.5 points).

Problem 3

a We assume that a structure is designed with a linear elastic frame program and that ductile components and joints are used. Many load cases have not been considered in design, such as temperature loading, foundation settlements and mis fits. Will this structure be safe despite ignoring these load cases? Choose one of the following answers (0.5 point).

A Yes, the forces and moments caused by these load cases are small and can be neglected.
B Yes, according to the lower bound theorem the mentioned load cases do not cause any stresses.
C Yes, the mentioned load cases do not change the plastic collapse load.
D No, a linear elastic frame program is not suitable for calculating the plastic ultimate load.

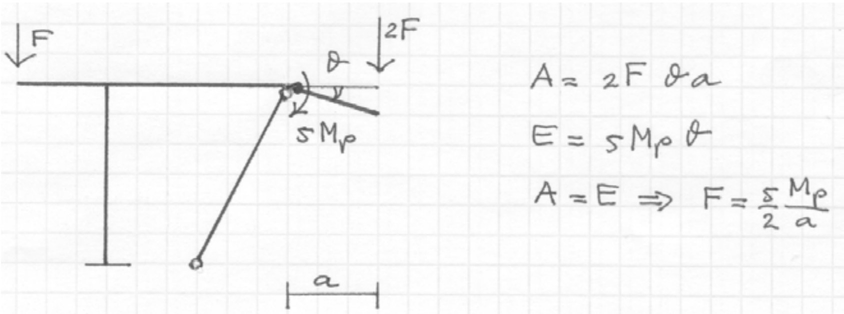
b Rolling stresses in steel beams influence the ... Complete this sentence with one of the following answers (0.5 point).
A plastic bending moment
B bending stiffness
C interaction diagram
D ductility

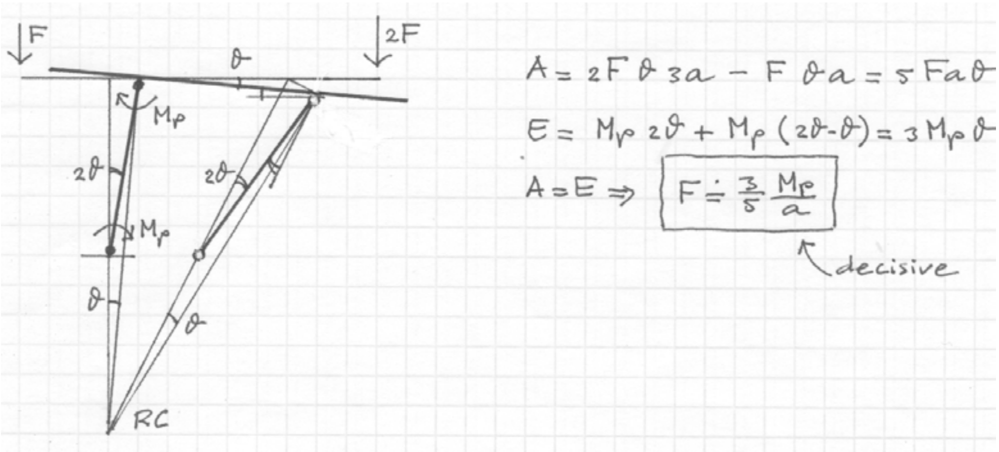
c Many finite element programs can make contour plots of the following plate quantities (0.5 point).

$$m_{xx} + |m_{xy}|$$
$$-m_{xx} + |m_{xy}|$$
$$m_{yy} + |m_{xy}|$$
$$-m_{yy} + |m_{xy}|$$

What are these used for? Do they provide a lower bound or an upper bound approximation of the structural strength? Explain the latter answer.

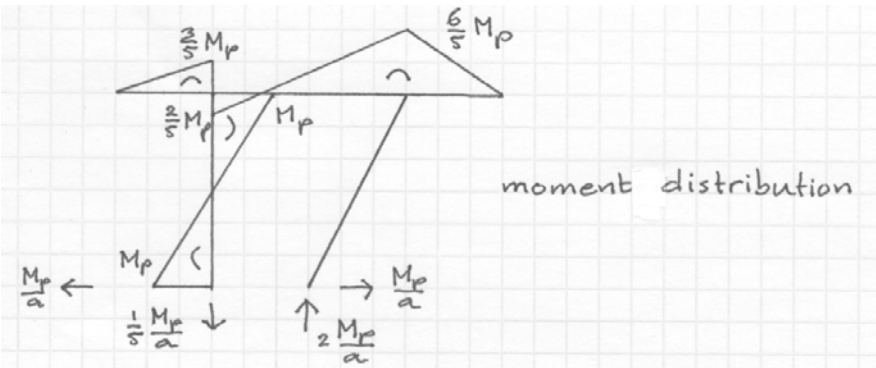
Answer to Problem 1a





More mechanisms are possible but they do not seem to be decisive. Plotting the moment distribution will show whether we found the correct mechanism.

Answer to Problem 1b



Answer to problem 1c Lower-bound

We reduce the loading with a factor α. Also the moments and the normal forces are reduced with α. In a section with a moment αM_p we can allow a normal force $(1-\alpha)N_p$. Therefore, at the bottom and top of the left hand column we have

$$N = \alpha \frac{1}{5}\frac{M_p}{a} = (1-\alpha)N_p = (1-\alpha)\beta\frac{M_p}{a}.$$

Consequently,

$$\alpha \tfrac{1}{5} = (1-\alpha)\beta$$

and

$$\alpha = \frac{\beta}{\tfrac{1}{5}+\beta}\,.$$

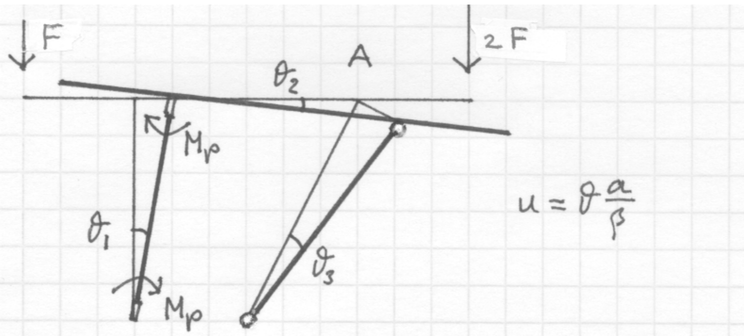
Since β = 50 we find

$$\alpha = \tfrac{250}{251}\,.$$

The collapse load is

$$F = \alpha \tfrac{3}{5}\frac{M_p}{a} = \tfrac{250}{251}\tfrac{3}{5}\frac{M_p}{a} = \tfrac{150}{251}\frac{M_p}{a}$$

Answer to Problem 1c Upper-bound



horizontal displacement of point A →

$$\vartheta_1 2a = \vartheta_3 2a \Rightarrow \vartheta_1 = \vartheta_3$$

vertical displacement of point A ↑

$$\vartheta_1 \frac{a}{\beta} + (\vartheta_1 - \vartheta_2)\frac{a}{\beta} - \vartheta_2 2a = -\vartheta_3 a \Rightarrow$$

$$\vartheta_1\left(\frac{a}{\beta}+\frac{a}{\beta}\right) - \vartheta_2\left(\frac{a}{\beta}+2a\right) = -\vartheta_3 a$$

$$\vartheta_3\left(\frac{a}{\beta}+\frac{a}{\beta}+a\right) = \vartheta_2\left(\frac{a}{\beta}+2a\right)$$

$$\vartheta_2 = \frac{2\frac{a}{\beta}+a}{\frac{a}{\beta}+2a}\vartheta_3 = \frac{2+\beta}{1+2\beta}\vartheta_3 = \frac{52}{101}\vartheta_3$$

$$A = 2F(\vartheta_3 a + \vartheta_2 a) + F(\vartheta_3 a - \vartheta_2 3a)$$

$$= Fa(3\vartheta_3 - \vartheta_2)$$

$$= Fa\left(3 - \frac{52}{101}\right)\vartheta_3$$

$$E = M_p \vartheta_1 + M_p(\vartheta_1 - \vartheta_2)$$

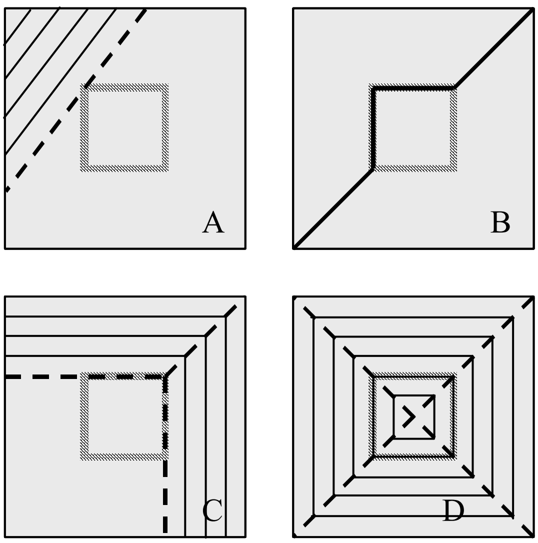
$$= M_p(2\vartheta_1 - \vartheta_2)$$

$$= M_p\left(2 - \frac{52}{101}\right)\vartheta_3$$

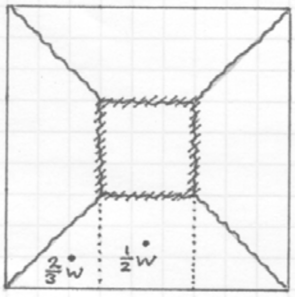
$$A = E \Rightarrow F = \frac{2 - \frac{52}{101}}{3 - \frac{52}{101}}\frac{M_p}{a} \Rightarrow \boxed{F = \frac{150}{251}\frac{M_p}{a}}$$

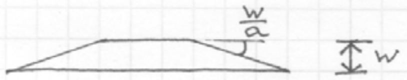
Answer to Problem 2a

Kinematically possible are patterns A, C, D. The figure below shows the altitude lines of the deformed mechanisms.



Answer to Problem 2b





$$A = 4\left[q a^2 \tfrac{1}{2} w\right] + 8\left[q \tfrac{1}{2} a^2 \tfrac{2}{3} w\right]$$
$$= q a^2 w \left[2 + \tfrac{8}{3}\right]$$
$$= \tfrac{14}{3} q a^2 w$$

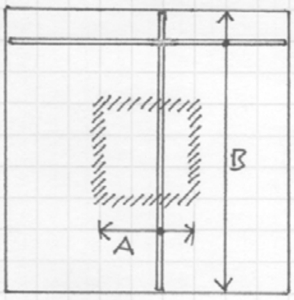
$$E = 4\left[3 m_p a \tfrac{w}{a} + m_p a \tfrac{w}{a}\right] + 2\left[m_p a \tfrac{w}{a}\right] +$$
$$+ 2\left[3 m_p a \tfrac{w}{a}\right]$$

$$E = m_p w [16 + 2 + 6] = 24\, m_p w$$

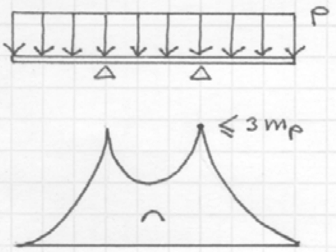
$$A = E \Rightarrow \quad q = \tfrac{3}{14} 24 \frac{m_p}{a^2}$$

$$\boxed{q = \frac{36}{7} \frac{m_p}{a^2}}$$

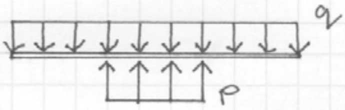
Answer to Problem 2c

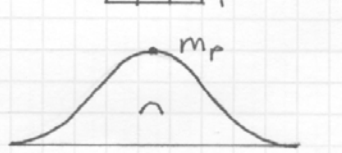


beam A



beam B





mom. equilibrium

$$3m_p \overset{?}{\geq} p\, a \tfrac{1}{2} a$$

$$\geq \tfrac{3}{2} q a^2$$

$$\geq \tfrac{3}{2}\tfrac{4}{3} m_p$$

$$\geq 2 m_p$$

OK.

vert. equilibrium

$$q \cdot 3a = p \cdot a \Rightarrow p = 3q$$

mom. equilibrium

$$m_p = q \tfrac{3}{2}a \tfrac{3}{4}a - p \tfrac{1}{2}a \tfrac{1}{4}a$$

$$= q a^2 \left(\tfrac{3}{2}\tfrac{3}{4} - 3\tfrac{1}{2}\tfrac{1}{4}\right) = \tfrac{3}{4} q a^2$$

$$\boxed{q = \tfrac{4}{3}\tfrac{m_p}{a^2}}$$

Answer to Problem 3a

C Yes, the mentioned load cases do not change the plastic collapse load.

Answer to Problem 3b

B Rolling stresses in steel beams influence the bending stiffness.

Answer to Problem 3c

These plate quantities are used to determine the required reinforcement. This provides a lower bound of the strength because the moments are in equilibrium with the load and boundary conditions, and nowhere the yield condition is violated.