Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Write your name and study number at the top right-hand of your work.

Exam CT4150 Plastic Analysis of Structures
Tuesday 21 August 2007, 14:00 – 17:00 hours

Problem 1

A frame consists of two columns, one beam and two brackets for a travelling crane (Fig. 1). The yield strengths of all members and joints are M_p. The columns are fixed in the foundation. The frame is loaded by a distributed load q and two vertical forces $F = 10qa$. The following relation exists between the plastic moment M_p and the plastic normal force N_p (Fig. 2).

$$N_p = \beta \frac{M_p}{a}$$

The influence of shear on the yield contour is neglected. Buckling and second order effects are not considered.

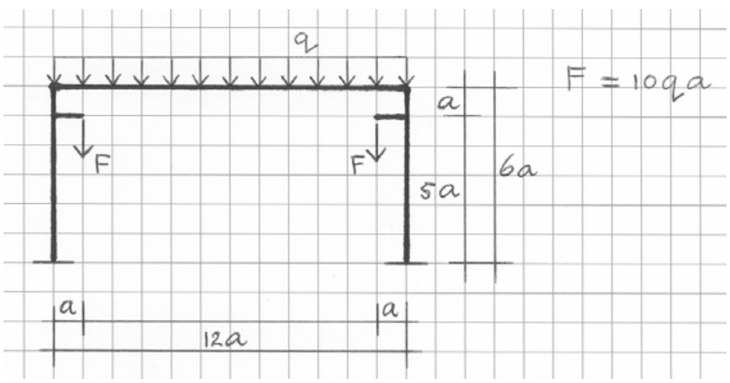


Figure 1. Frame

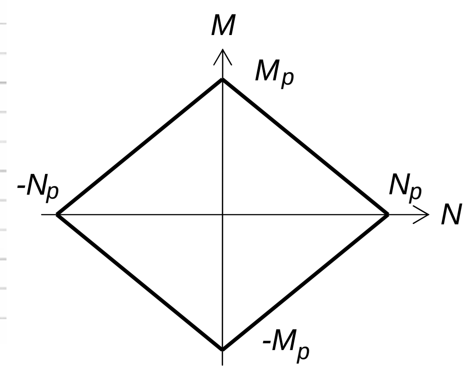


Figure 2. Yield contour

a Assume $\beta \to \infty$. Determine the collapse load q for all important mechanisms. Write the collapse loads as functions of M_p and a. What is the decisive collapse load? (1.5 points)

b Assume $\beta \to \infty$. Draw the bending moment diagram and normal force diagram for the structure at the moment of collapse. (1 point)

c Assume $\beta = 64$. Choose one of the following problems (You need not do both).

– Determine the largest lower-bound for F.

– Determine the smallest upper-bound for F.

If you choose the upper-bound you only need to write down the equations and not solve the equations. (2 points)

Problem 2

A plate has hinged edges and free edges (Fig. 3). The plate carries a perpendicular evenly distributed load q [kN/m²]. The plate is homogeneous. There is only reinforcement near the bottom face of the plate in the x and y directions. $m_{px} = m_{py} = m_p$ and $m'_{px} = m'_{py} = 0$.

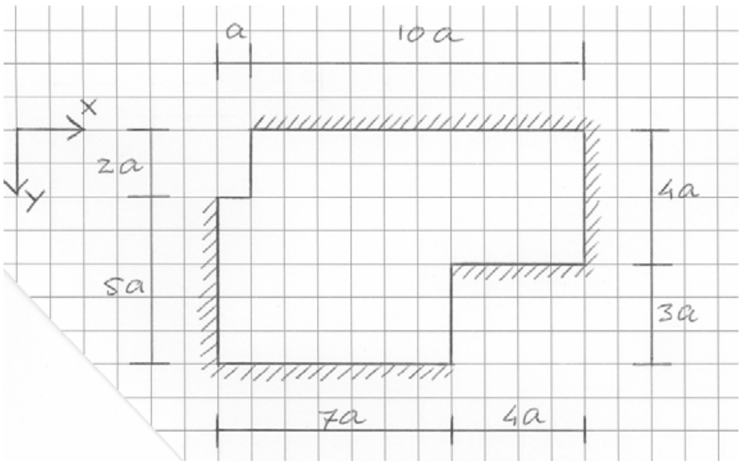


Figure 3. Plate with dimensions

a Consider the yield line patterns of Figure 4. Which of these patterns give kinematically possible mechanisms. (1 point)

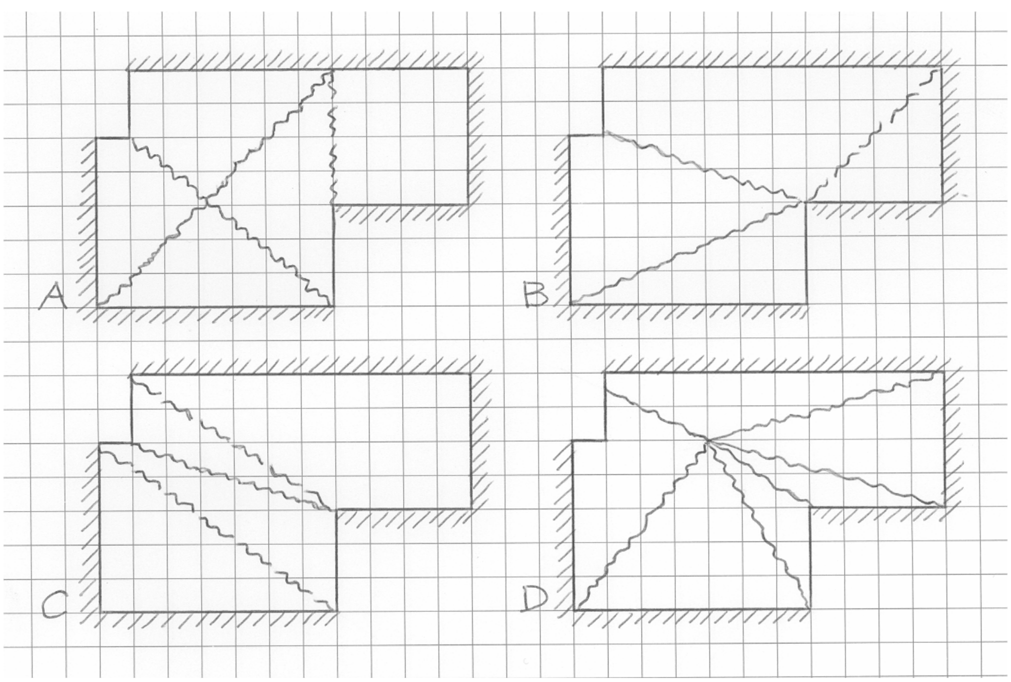


Figure 4. Proposed yield line patterns

b Consider the yield line pattern of Figure 5. Determine an <u>upper bound</u> for q expressed in m_p and a. (1.5 points)

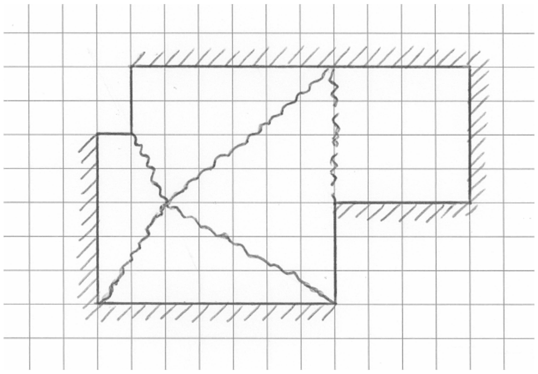


Figure 5. Actual yield line pattern

c Determine the largest <u>lower-bound</u> for q using torsion free beams ($m_{xy} = 0$) in the x direction and y direction. You only need to write down all equations and not solve the equations. (1.5 points)

Problem 3

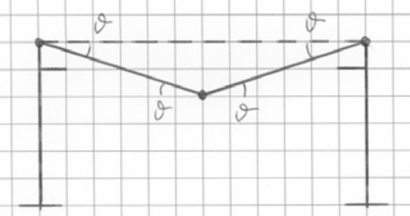
a When we do not choose the right mechanism in an upper-bound analysis we will notice that when plotting the moments. In what way? Choose A, B, C or D. (0.5 point)

A – The moment distribution will not be in equilibrium.
B – Somewhere a moment will be larger than the moment capacity.
C – There will not be sufficient plastic hinges to plot all of the moment distribution.
D – The largest moment does not occur where the shear force is zero.

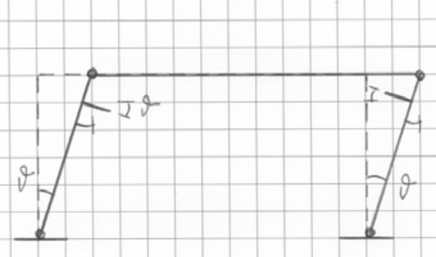
b A ductile structure has a convex yield contour. This is important because … Choose A, B, C or D. (0.5 point)
A – it proves that the solution of plastic problems is independent of the load path.
B – structures need to be able to resist unexpected accidents and terrorist attacks.
C – it determines the proportions of the plastic deformation in finite element analyses.
D – therefore we need to check just a few load combinations.

c In engineering practice, elastic analysis is used much more than plastic analysis (upper-bound, lower-bound) What is the reason for this? Choose A, B, C or D. (0.5 point)
A – There is little software available for plastic analysis.
B – Codes of practice do not allow plastic deformations.
C – Plastic analysis gives information on serviceability limit states only.
D – Elastic analysis is faster than plastic analysis.

Exam CT 4150, 21 August 2007
Answer to Problem 1a

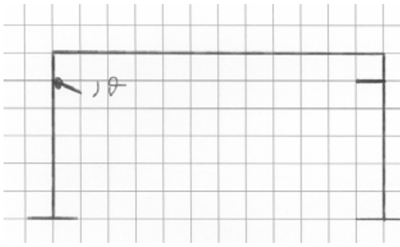


$$E = M_p\vartheta + M_p(\vartheta+\vartheta) + M_p\vartheta = 4M_p\vartheta$$
$$A = q\,6a\,\vartheta\,3a = 18qa^2\vartheta$$
$$E = A \Rightarrow q = \frac{2}{9}\frac{M_p}{a^2}$$

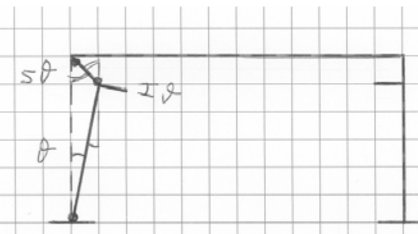


$$E = M_p\vartheta + M_p\vartheta + M_p\vartheta + M_p\vartheta = 4M_p\vartheta$$
$$A = F\vartheta a - F\vartheta a = 0q$$
$$E = A \Rightarrow q = \infty$$

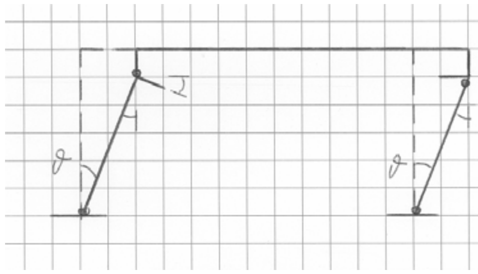
$$E = M_p\vartheta + M_p\vartheta + M_p\tfrac{6}{5}\vartheta + M_p\tfrac{6}{5}\vartheta = \frac{22}{5}M_p\vartheta$$
$$A = F\vartheta a = 10qa^2\vartheta$$
$$E = A \Rightarrow q = \frac{22}{50}\frac{M_p}{a^2}$$



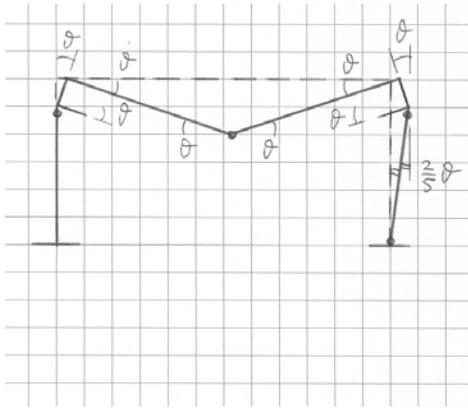
$$E = M_p\vartheta$$
$$A = F\vartheta a = 10qa^2\vartheta$$
$$E = A \Rightarrow q = \frac{1}{10}\frac{M_p}{a^2}$$



$$E = M_p\vartheta + M_p 5\vartheta + M_p(\vartheta + 5\vartheta) = 12M_p\vartheta$$
$$A = F\vartheta a = 10qa^2\vartheta$$
$$E = A \Rightarrow q = \frac{12}{10}\frac{M_p}{a^2}$$

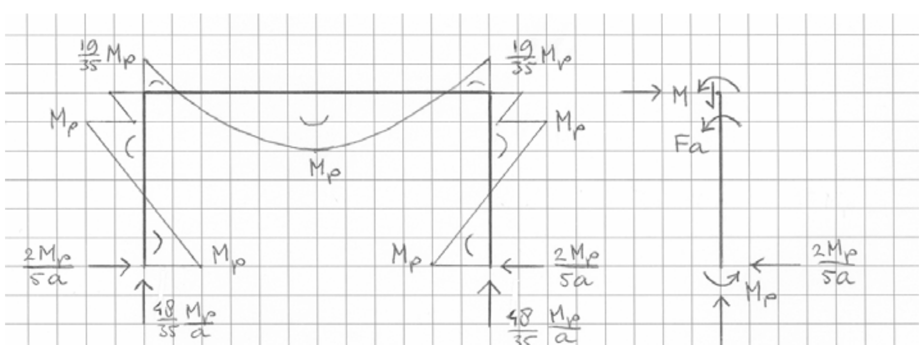


$$E = M_p\vartheta + M_p\vartheta + M_p\vartheta + M_p\vartheta$$
$$= 4M_p\vartheta$$
$$A = F\vartheta a = 10qa^2\vartheta$$
$$E = A \Rightarrow q = \frac{4}{10}\frac{M_p}{a^2}$$

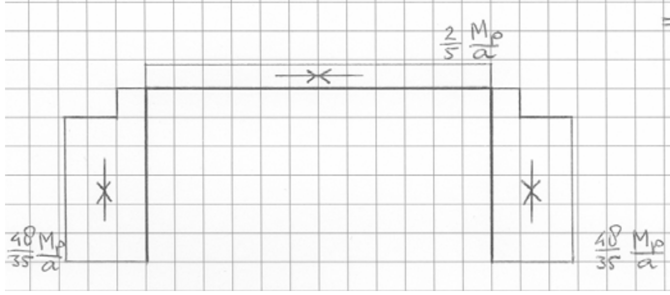


$$E = M_p\vartheta + M_p(\vartheta+\vartheta) + M_p\left(\vartheta + \tfrac{2}{5}\vartheta\right)$$
$$+ M_p\tfrac{2}{5}\vartheta = \frac{24}{5}M_p\vartheta$$
$$A = 2F\vartheta a + 2q\,6a\,\vartheta\,3a$$
$$= 56qa^2$$
$$E = A \Rightarrow \boxed{q = \frac{3}{35}\frac{M_p}{a^2}} \text{ decisive}$$
$$0{,}0857$$

Answer to Problem 1b



$$M = \frac{2M_p}{5a}6a - M_p - Fa$$
$$= \frac{19}{35}M_p$$



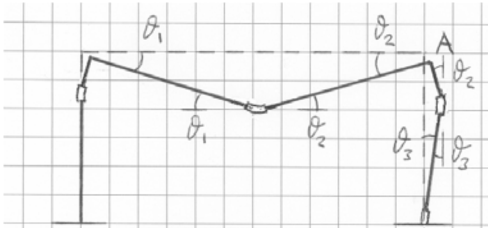
Answer to problem 1c Lower-bound

$$N = \alpha \frac{48}{35}\frac{M_p}{a} = (1-\alpha) N_p = (1-\alpha)\beta \frac{M_p}{a}$$

$$\alpha \frac{48}{35} = (1-\alpha)\beta \Rightarrow \alpha = \frac{\beta}{\frac{48}{35}+\beta} = \frac{140}{143}$$

$$q = \alpha \frac{3}{35}\frac{M_p}{a^2} = \frac{140}{143}\frac{3}{35}\frac{M_p}{a^2} = \frac{12}{143}\frac{M_p}{a^2} = 0{,}0839\,\frac{M_p}{a^2}$$

Answer to Problem 1c Upper-bound



$u = \vartheta \frac{a}{\beta} \quad \beta = 6a$

displacement of point A $\rightarrow$

$$\vartheta_1 a - (\vartheta_1+\vartheta_2)\frac{a}{\beta} = \vartheta_3 5a - \vartheta_2 a$$

$F = 10 q a$

displacement of point A $\downarrow$

$$\vartheta_1 \frac{a}{\beta} + \vartheta_1 6a - \vartheta_2 6a = (\vartheta_2+\vartheta_3)\frac{a}{\beta} + \vartheta_3 \frac{a}{\beta}$$

$$E = M_p \vartheta_1 + M_p(\vartheta_1+\vartheta_2) + M_p(\vartheta_2+\vartheta_3) + M_p \vartheta_3$$

$$A = F(\vartheta_1 \frac{a}{\beta} + \vartheta_1 a) + q\,6a\,(\vartheta_1 \frac{a}{\beta} + \vartheta_1 3a) +$$
$$+ q\,6a\,(\vartheta_1 \frac{a}{\beta} + \vartheta_1 6a - \vartheta_2 3a) + F(\vartheta_3 \frac{a}{\beta} + (\vartheta_2+\vartheta_3)\frac{a}{\beta} + \vartheta_2 a)$$

$$E = A$$

the solution of the equations is

$$\vartheta_1 = \frac{8809}{8791}\vartheta_2 \qquad q = \frac{1532}{18143}\frac{M_p}{a^2} = 0{,}0844\frac{M_p}{a^2}$$

$$\vartheta_3 = \frac{3465}{8791}\vartheta_2$$

Answer to Problem 2a

Kinematically possible is pattern D only.

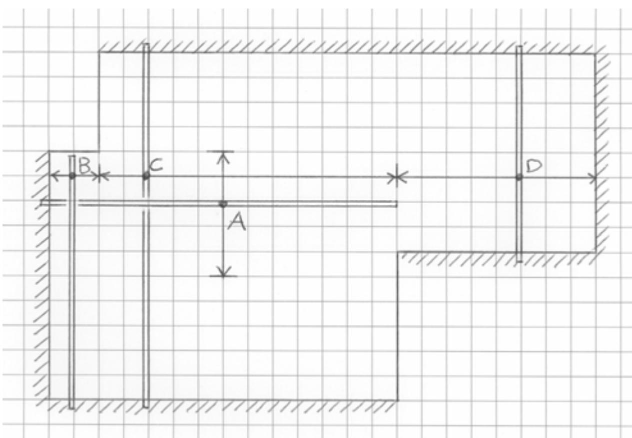
Answer to Problem 2b

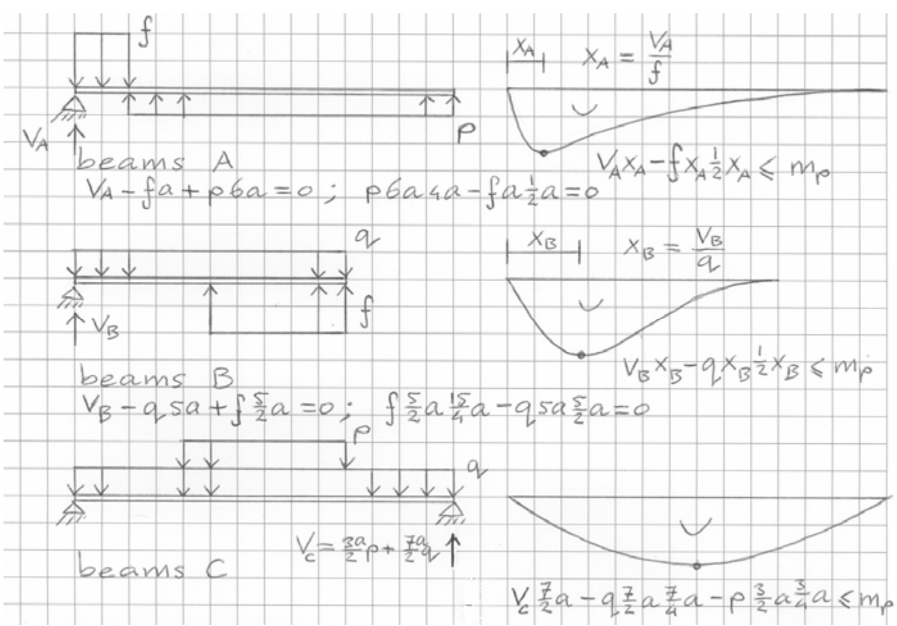
$$E = m_p \frac{w}{2a}3a + m_p \frac{w}{3a}2a + m_p \frac{w}{5a}3a + m_p \frac{w}{3a}5a +$$
$$+ m_p \frac{w}{5a}4a + m_p \frac{w}{4a}5a + m_p \frac{w}{2a}2a + m_p \frac{w}{4a}a$$
$$= m_p \frac{116}{15} w$$

$$A = q \tfrac{1}{2} 7a\, 3a\, \tfrac{1}{3} w + q \tfrac{1}{2} 7a\, 5a\, \tfrac{1}{3} w + q \tfrac{1}{2} 7a\, 4a\, \tfrac{1}{3} w -$$
$$- q \tfrac{1}{2} 1a\, 2a\, \tfrac{1}{6} w + q \tfrac{1}{2} 7a\, 2a\, \tfrac{1}{3} w - q \tfrac{1}{2} 2a\, a\, \tfrac{1}{6} w$$
$$= q a^2 16 w$$

$$E = A \Rightarrow \quad q = \frac{29}{60} \frac{m_p}{a^2}$$

Answer to Problem 2c





beams A

$$V_A - f a + p 6a = 0; \quad p 6a\, 4a - f a \tfrac{1}{2} a = 0$$

$$x_A = \frac{V_A}{f}$$

$$V_A x_A - f x_A \tfrac{1}{2} x_A \leq m_p$$

beams B

$$V_B - q 5a + f \tfrac{5}{2} a = 0; \quad f \tfrac{5}{2} a \tfrac{15}{4} a - q 5a \tfrac{5}{2} a = 0$$

$$x_B = \frac{V_B}{q}$$

$$V_B x_B - q x_B \tfrac{1}{2} x_B \leq m_p$$

beams C

$$V_c = \tfrac{3a}{2} p + \tfrac{7a}{2} q$$

$$V_c \tfrac{7}{2} a - q \tfrac{7}{2} a \tfrac{7}{4} a - p \tfrac{3}{2} a \tfrac{3}{4} a \leq m_p$$

beams D

$$\frac{1}{8} q (4a)^2 \leq m_p$$

The solution of the equations is $q = \frac{96}{599} \frac{m_p}{a^2} = 0{,}160 \frac{m_p}{a^2}$

Beam C is decisive

Answer to Problem 3a

B is correct.

Answer to Problem 3b

D is correct.

Answer to Problem 3c

A is correct.