Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Write your <u>name</u> and <u>study number</u> at the top right-hand of your work.

Exam CT4150 Plastic Analysis of Structures
Thursday 17 January 2008, 9:00 – 12:00 hours

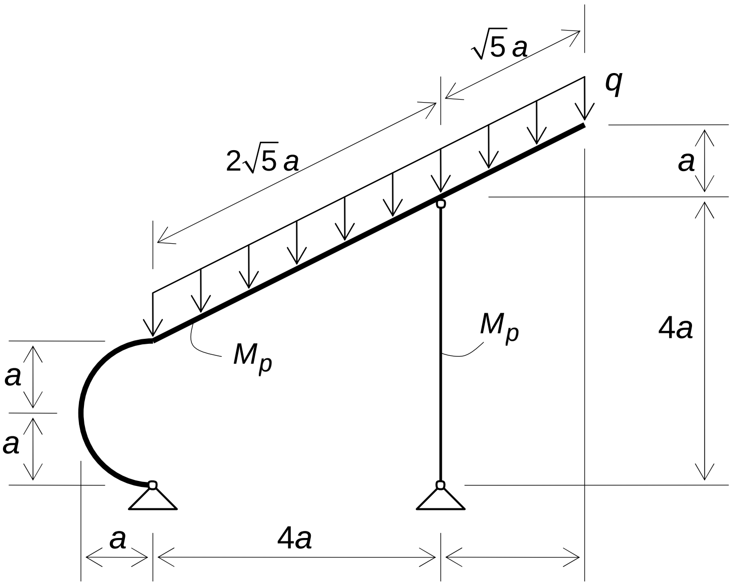

Figure 1. Curved frame

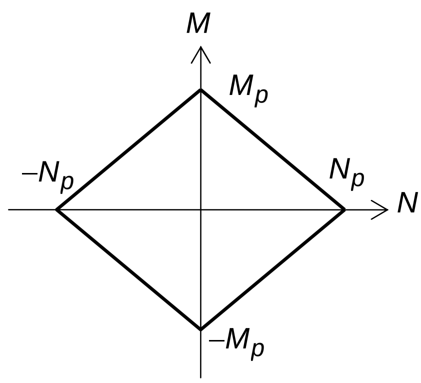

Figure 2. Yield contour

Problem 1

A frame consists of a curved beam and a column with strengths M_p (Fig. 1). The beam is connected with hinges to the foundation and to the column. The beam is loaded by an evenly distributed force *q*. The following relation exists between the plastic moment M_p and the plastic normal force N_p (Fig. 2).

$$N_p = \beta \frac{M_p}{a}$$

The influence of shear on the yield contour is neglected. Buckling and second order effects are not considered.

a Assume $\beta \to \infty$. Determine the collapse load *q* for all possible mechanisms. Write the collapse loads as functions of M_p and *a*. What is the decisive collapse load? (1.5 points)

b Assume $\beta \to \infty$. Draw the bending moment diagram and normal force diagram for the structure at the moment of collapse. (1 point)

c Assume $\beta = 30\sqrt{5}$. Choose one of the following problems (You need not do both).
– Determine the largest <u>lower-bound</u> for *q*.
– Determine the smallest <u>upper-bound</u> for *q*.
If you choose the upper-bound you only need to write down the equations and not solve the equations (2 points).

Problem 2

A square plate is supported on all edges except for a length of $a/2$ (Fig. 3). The support is special in that it resists only compression and no tension. Neither it resists moments (Fig. 4). The plate carries a point load F in the middle. The plate is isotropic and homogeneous, therefore $m_{px} = m_{py} = m'_{px} = m'_{py} = m_p$.

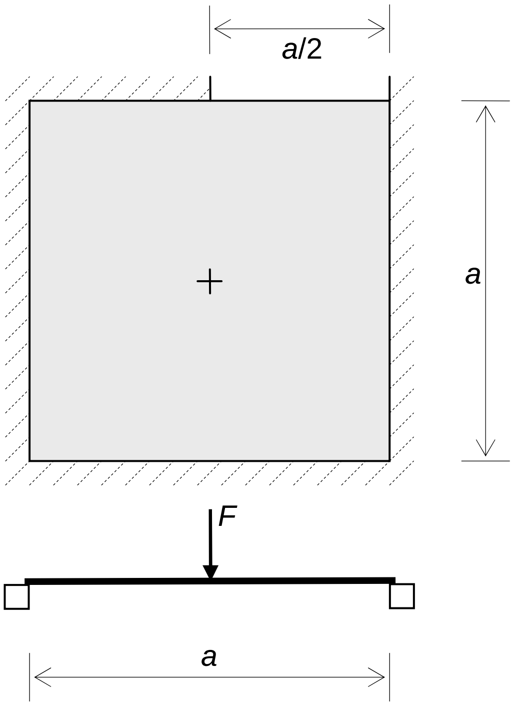

Figure 3. Plate dimensions and loading

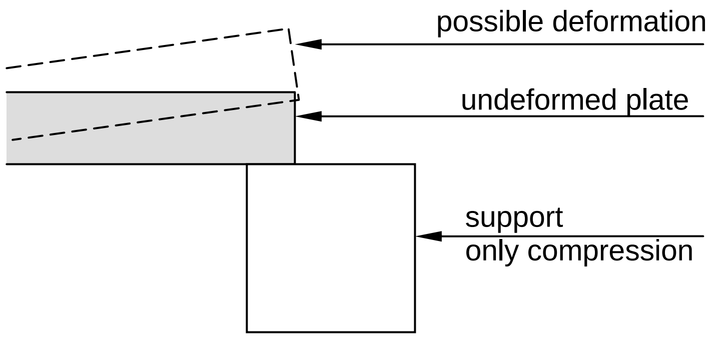

Figure 4. Plate support

a Consider the yield line patterns of Figure 5. Which of these patterns give kinematically possible mechanisms. (1 point)

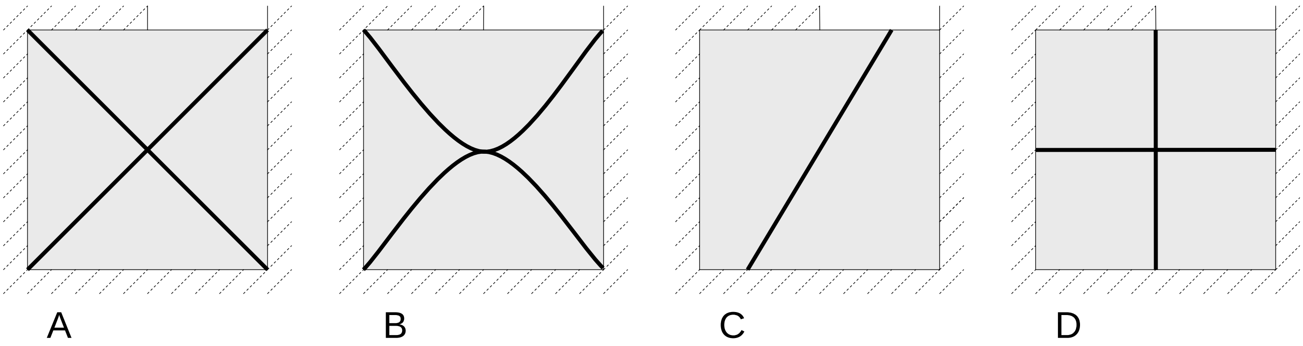

Figure 5. Yield line patterns of problem 2a

b Consider the yield line pattern of Figure 6. Determine an upper bound for F expressed in m_p (1.5 points).

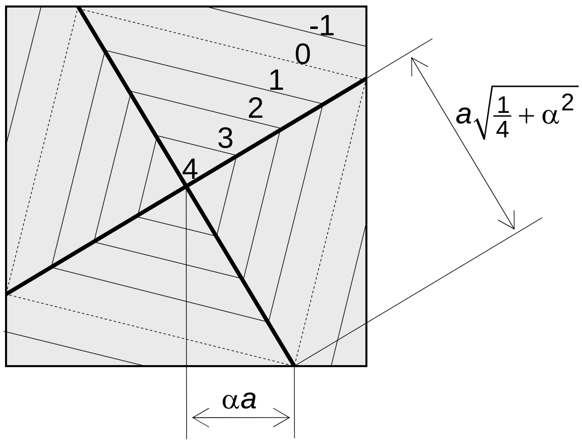

Figure 6. Yield line pattern of problem 2b

c Determine the largest <u>lower-bound</u> for F using torsion free beams ($m_{xy} = 0$) in the x direction and y direction. Assume that the point load is distributed over an area with a width a/10. (1.5 points)

Problem 3

a When we design reinforcement according to

$$m_{px} = m_{xx} + |m_{xy}|$$
$$m_{py} = m_{yy} + |m_{xy}|$$
$$m'_{px} = -m_{xx} + |m_{xy}|$$
$$m'_{py} = -m_{yy} + |m_{xy}|$$

what will be the crack direction (angle α)? (0.5 point)

b In elastic analysis a point load F on a plate gives unrealistic large moments directly under the point load. Designers cut these moments at a value F/3. Is this safe for a mid load, a freed edge load and a corner load? Assume that $m_{px} = m_{py} = m'_{px} = m'_{py} = m_p$. (0.5 point)

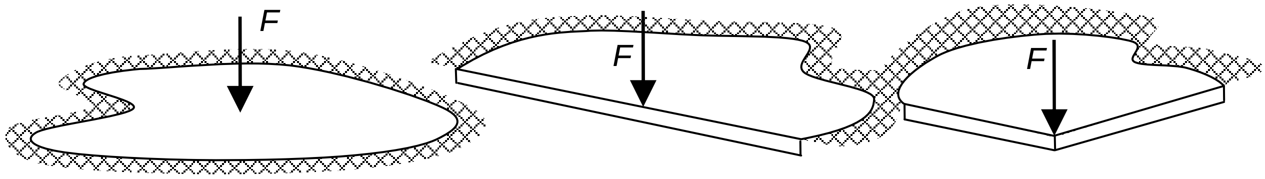

Figure 7. Point loads at three locations

c Which of the following reinforcement detailing of a column bracket will provide the largest strength? (0.5 point)

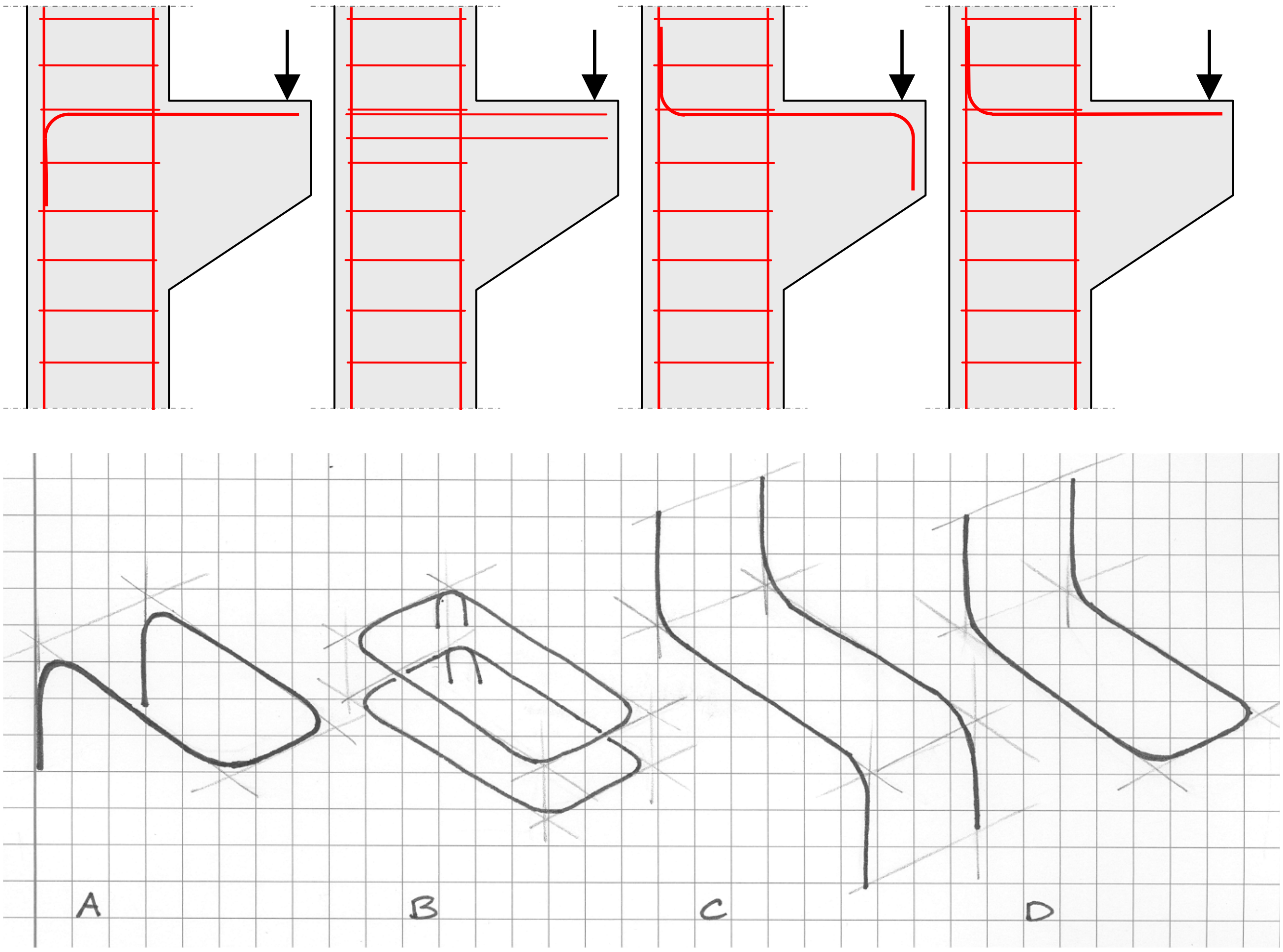

Figure 8. Reinforcement detailing

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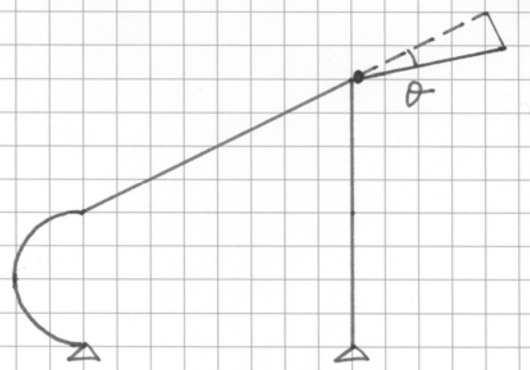
Answer to Problem 1a

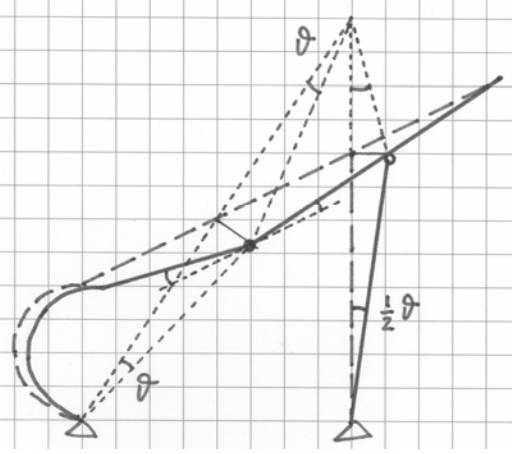


$E = M_p \vartheta$

$A = q\sqrt{5}a\ \vartheta a$

$E = A \Rightarrow \boxed{q = \frac{1}{\sqrt{5}} \frac{M_p}{a^2}} \approx 0.447 \frac{M_p}{a^2}$

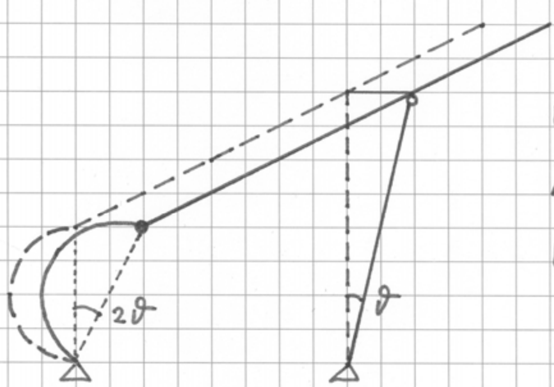
decisive



$E = M_p\, 2\vartheta$

$A = q\sqrt{5}a\ \vartheta\ a + q\, 2\sqrt{5}a\, 0$

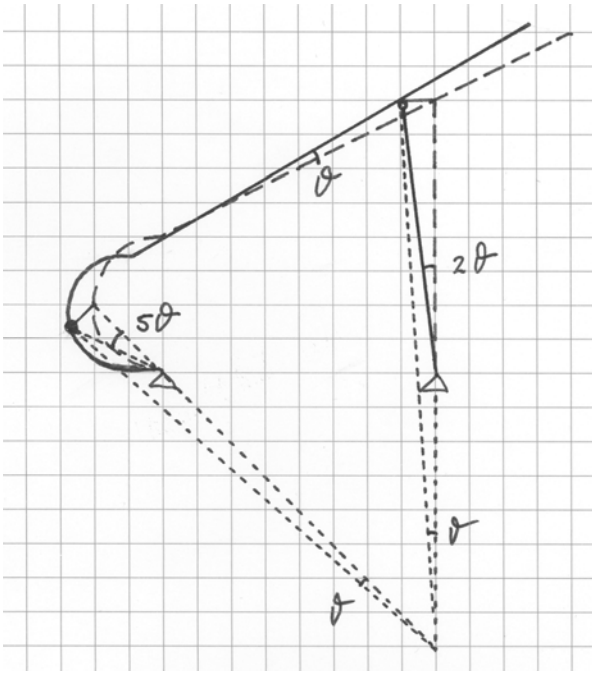
$E = A \Rightarrow \quad q = \frac{2}{\sqrt{5}} \frac{M_p}{a^2}$



$E = M_p\, 2\vartheta$

$A = q\, 0$

$E = A \Rightarrow \quad q = \infty$

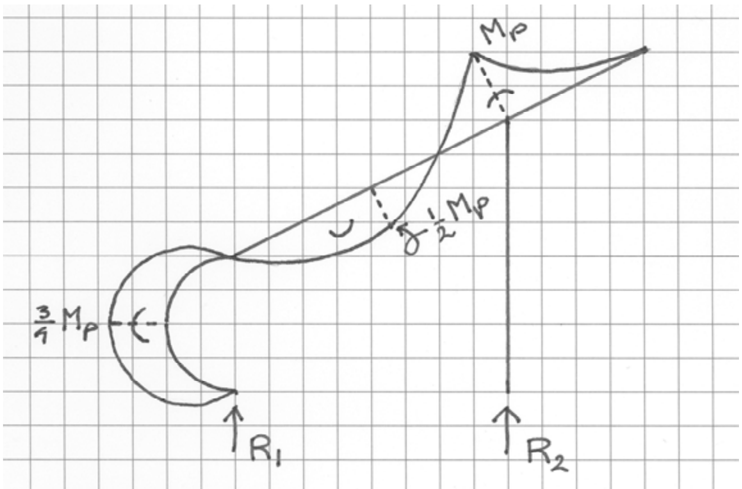


$$E = M_p(5\vartheta - \vartheta)$$

$$A = q\,3\sqrt{5}a\;a\vartheta$$

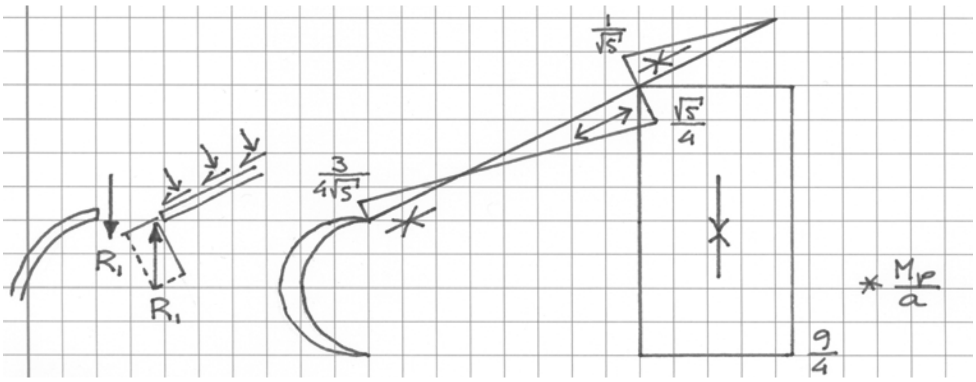
$$E = A \Rightarrow q = \frac{4}{3\sqrt{5}}\frac{M_p}{a^2}$$

Answer to Problem 1b



$$q\,3\sqrt{5}a\,a - R_1 4a = 0 \Rightarrow R_1 = \frac{3}{4}\sqrt{5}qa$$

$$q\,3\sqrt{5}a - R_1 - R_2 = 0 \Rightarrow R_2 = \frac{9}{4}\sqrt{5}qa$$



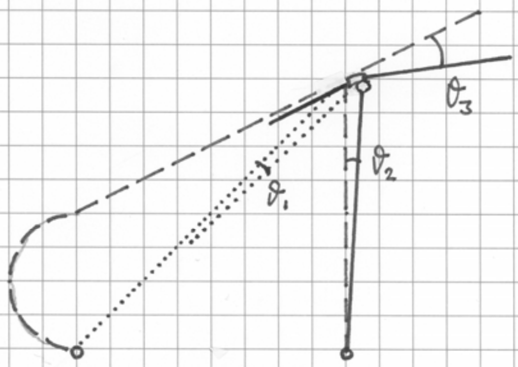
Answer to problem 1c Lower-bound

$$N = \frac{\sqrt{5}}{4}\frac{M_p}{a}\alpha = (1-\alpha)N_p = (1-\alpha)\beta\frac{M_p}{a}$$

$$\alpha\frac{\sqrt{5}}{4} = (1-\alpha)\beta \quad \Rightarrow \quad \alpha = \frac{\beta}{\beta+\frac{\sqrt{5}}{4}} = \frac{120}{121}$$

$$q = \alpha\frac{1}{\sqrt{5}}\frac{M_p}{a^2} = \frac{120}{121}\frac{1}{\sqrt{5}}\frac{M_p}{a^2} \approx 0{,}444\frac{M_p}{a^2}$$

Answer to Problem 1c Upper-bound



$$u = \vartheta\frac{a}{\beta}$$

$$\beta = 30\sqrt{5}$$

hor. displ. $\vartheta_1 4a + (\vartheta_3 - \vartheta_1)\frac{a}{\beta}\frac{2}{\sqrt{5}} = \vartheta_2 4a$

vert. displ. $\vartheta_1 4a - (\vartheta_3 - \vartheta_1)\frac{a}{\beta}\frac{1}{\sqrt{5}} = 0$

$E = M_p(\vartheta_3 - \vartheta_1)$

$A = q\sqrt{5}a\,\vartheta_3 a + q 2\sqrt{5}a\,\vartheta_1 2a$

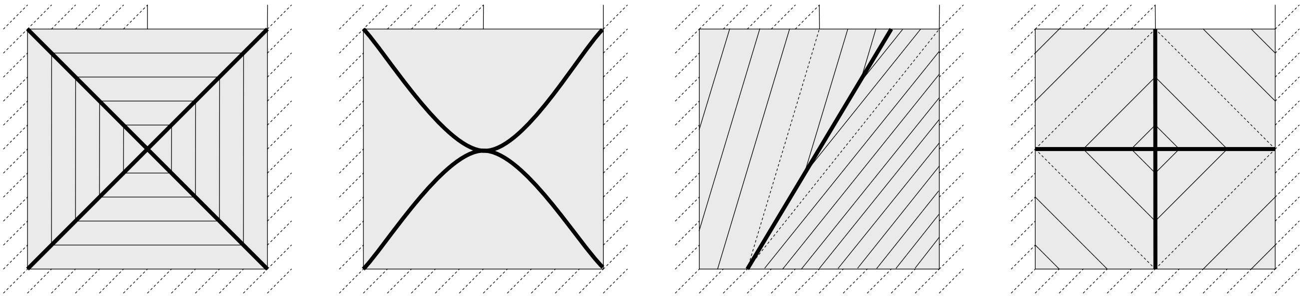
$E = A$

The solution to the equations is

$$\vartheta_1 = \frac{1}{601}\vartheta_3 \quad \vartheta_2 = \frac{3}{601}\vartheta_3 \quad q = \frac{24}{121}\sqrt{5}\frac{M_p}{a^2} \approx 0.444\frac{M_p}{a^2}$$

Answer to Problem 2a

Kinematically possible are pattern A, C, D. The figure below shows the altitude lines of the deformed mechanisms.



A B C D

Answer to Problem 2b

$$E = 4\left[\varphi m_p l\right] = 8\,w\,m_p$$

$$l = a\sqrt{\tfrac{1}{4} + \alpha^2}$$

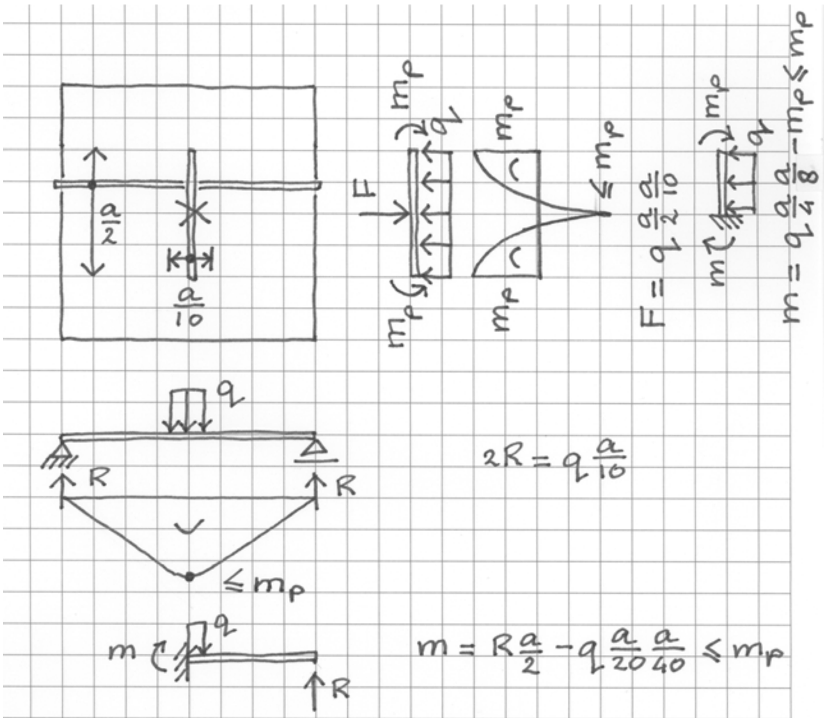
$$\varphi = 2\frac{w}{l}$$

$$A = Fw$$

$$E = A \Rightarrow \boxed{F = 8 m_p}$$

Note that the result does not depend on α.

Answer to Problem 2c



The solution of the equations is

$F = \frac{40}{19} m_p \approx 2.11\, m_p$

Answer to Problem 3a

$$tan\,\alpha = \frac{m_{px} - m_{xx}}{m_{xy}} \qquad (12.10)$$

$$= \frac{\left(m_{xx} + \left|m_{xy}\right|\right) - m_{xx}}{m_{xy}} = \frac{\left|m_{xy}\right|}{m_{xy}} = \pm 1$$

α = 45º when $m_{xy} \geq 0$

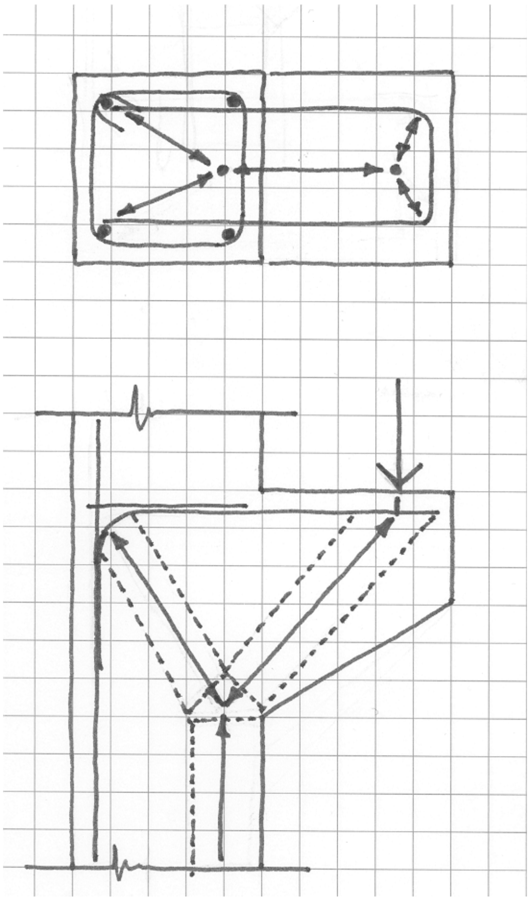
α = -45º when $m_{xy} \leq 0$

Answer to Problem 3b

Mid plate (p. 47)	$F = 4\pi m_p \Rightarrow m_p = \dfrac{F}{12.6}$	safe
Plate edge (p. 50)	$F = (2+\pi) m_p \Rightarrow m_p = \dfrac{F}{5.14}$	safe
Plate corner (p. 50)	$F = 2 m_p \Rightarrow m_p = \dfrac{F}{2}$	not safe

Answer to Problem 3c

A or B are correct (see figure)
C will split at the load point
D does not connect properly to the left inclined strut.



Strut-and-tie model for detailing A