Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Write your name and study number at the top right-hand of your work.

Exam CT4150 Plastic Analysis of Structures
Thursday 18 June 2008, 9:00 – 12:00 hours

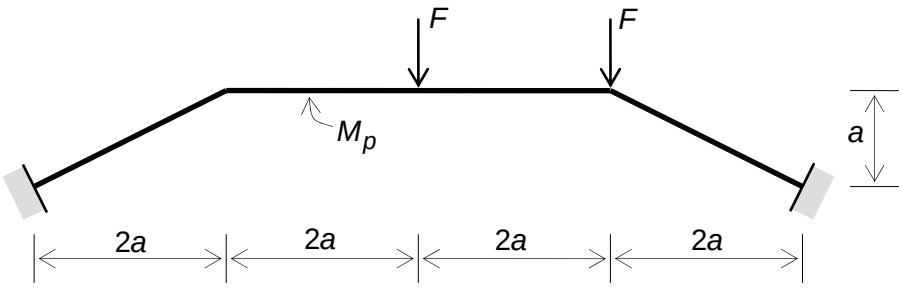

Figure 1. Frame

Problem 1

A frame consists of a three beams with strengths M_p (Fig. 1). The beams are fixed to each other and to the foundation. The frame is loaded by two forces *F*. The following relation exists between the plastic moment M_p and the plastic normal force N_p (Fig. 2).

$$N_p = \beta \frac{M_p}{a}$$

The influence of shear on the yield contour is neglected. Buckling and second order effects are not considered.

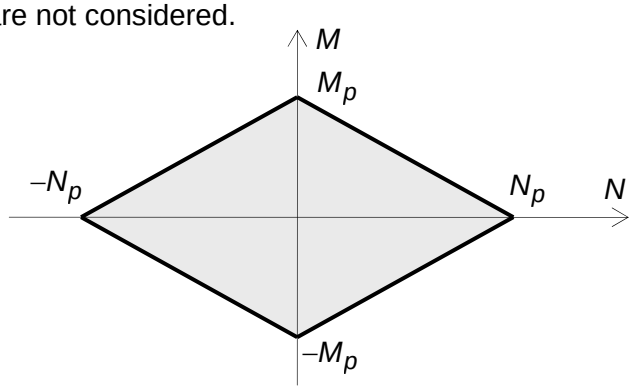

Figure 2. Yield contour

a Assume $\beta \to \infty$. Determine the collapse load *F* for all possible mechanisms. Write the collapse loads as functions of M_p and *a*. What is the decisive collapse load? (1.5 point)

b Assume $\beta \to \infty$. Draw the bending moment diagram and normal force diagram for the structure at the moment of collapse. (1 point)

c Assume $\beta = 20$. Choose one of the following problems (You need not do both).
– Determine the largest lower-bound for *F*.
– Determine the smallest upper-bound for *F*.
If you choose the upper-bound you only need to write down the equations and not solve the equations (2 points).

Problem 2

A plate has round and straight edges (Fig. 3). It is fixed at the straight edges. The plate carries an evenly distributed load q. The plate is homogeneous and orthotropic $m_{px} = 0$, $m'_{px} = 4m_p$, $m_{py} = m_p$, $m'_{py} = m_p$.

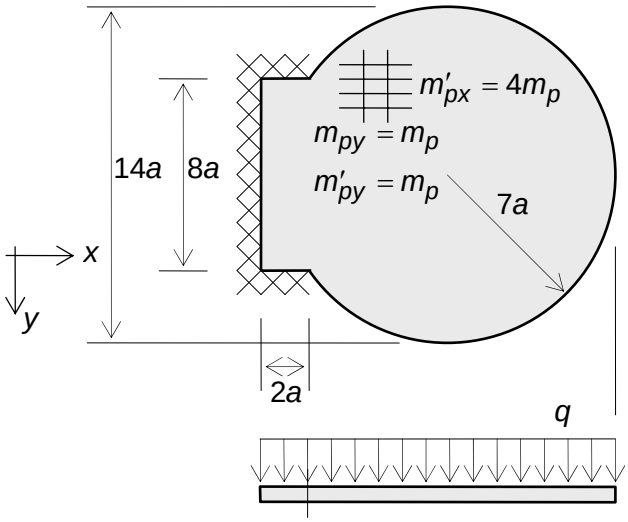

Figure 3. Plate dimensions and loading

a Consider the yield line patterns of Figure 4. Which of these patterns give kinematically possible mechanisms. (1 point)

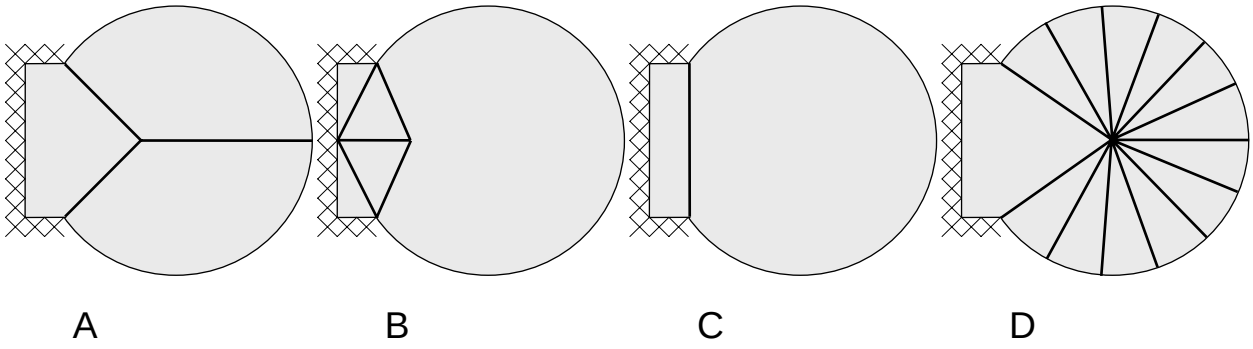

Figure 4. Yield line patterns of problem 2a

b Consider the yield line pattern of Figure 5. Determine an upper bound for q expressed in m_p and a (1.5 point).

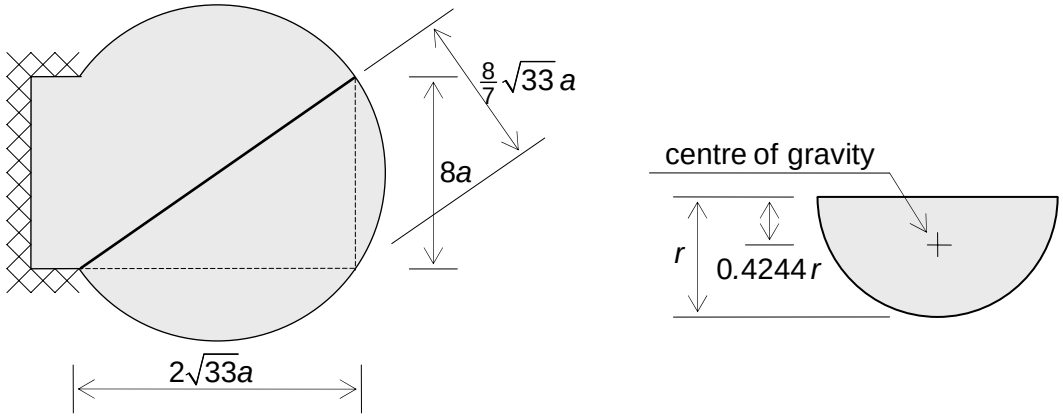

Figure 5. Yield line pattern of problem 2b

c Determine the largest lower-bound for q using torsion free beams ($m_{xy} = 0$) (1.5 point).

Problem 3

a Prestressing does not change the ultimate load of a structure made of plastic materials. Is this correct? Choose A, B, C, or D (0.5 point).

A Yes, because initial stress does not influence upperbound calculations or lowerbound calculations.
B No, because prestressing tendons carry part of the load.
C No, because if the construction workers apply too much prestress the structure will collapse.
D No, otherwise concrete bridges would not be prestressed.

b The evenly distributed load q at which a simply supported circular plate collapses is

$$q = 24\frac{m_p}{a^2}.$$

Does this solution need top reinforcement? Explain your answer (0.5 point).

c Make a drawing of the idealised yield contour of reinforced concrete plates in case only bottom reinforcement is applied (0.5 point).

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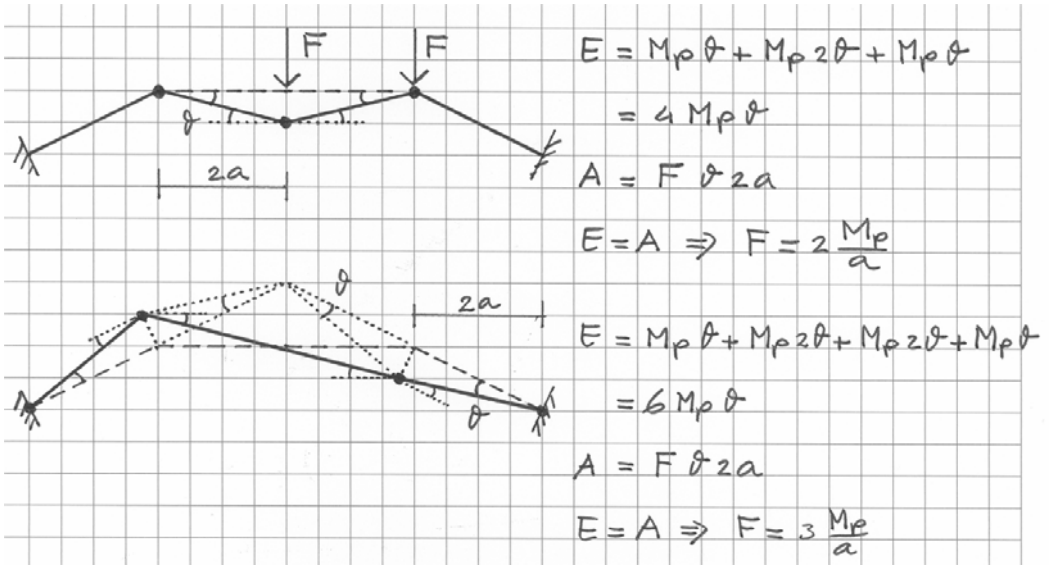
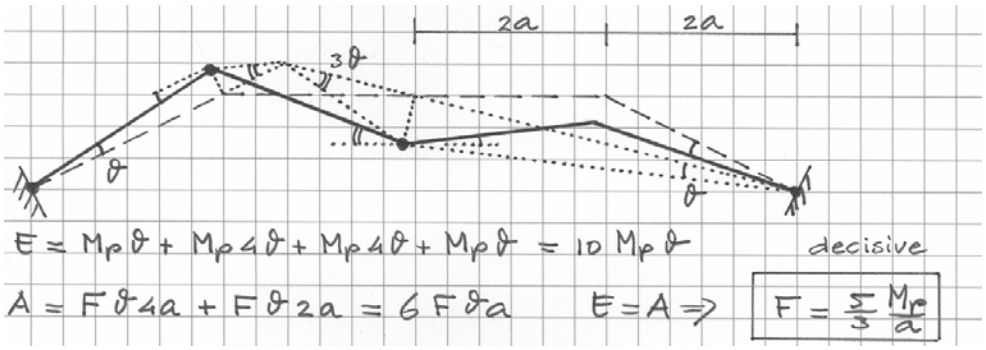
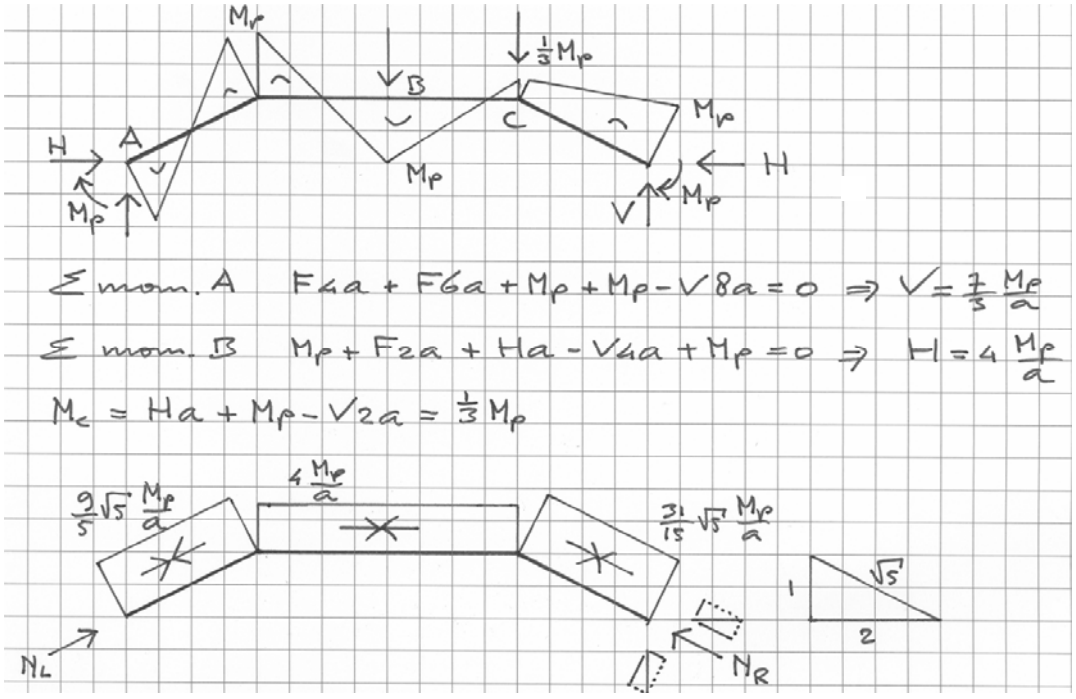
Answer to Problem 1a





Answer to Problem 1b



$$N_R = \frac{2}{\sqrt{5}} H + \frac{1}{\sqrt{5}} V = \frac{31}{15}\sqrt{5}\,\frac{M_p}{a} \approx 4.62\,\frac{M_p}{a}$$

$$N_L = \frac{2}{\sqrt{5}} H + \frac{1}{\sqrt{5}} (2F - V) = \frac{9}{5}\sqrt{5}\,\frac{M_p}{a} \approx 4.02\,\frac{M_p}{a}$$

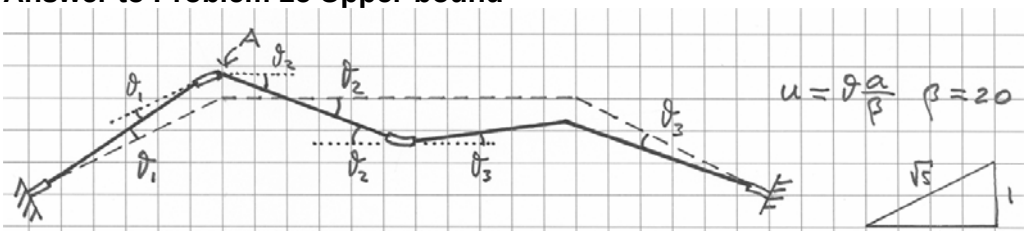
Answer to problem 1c Lower-bound

$$N = \alpha\,\frac{31}{15}\sqrt{5}\,\frac{M_p}{a} = (1-\alpha)\,N_p = (1-\alpha)\,\beta\,\frac{M_p}{a}$$

$$\alpha\,\frac{31}{15}\sqrt{5} = (1-\alpha)\,\beta \Rightarrow \alpha = \frac{\beta}{\frac{31}{15}\sqrt{5}+\beta} = \frac{20}{\frac{31}{15}\sqrt{5}+20} = 0.812$$

$$F = \alpha\,\frac{5}{3}\,\frac{M_p}{a} = 1.35\,\frac{M_p}{a}$$

Answer to Problem 1c Upper-bound



$u = \vartheta\frac{a}{\beta}$ $\beta = 20$

displacement of point A ←

$$\vartheta_1\frac{a}{\beta}\frac{2}{\sqrt{5}} + \vartheta_1 a + (\vartheta_1+\vartheta_2)\frac{a}{\beta}\frac{2}{\sqrt{5}} = -\vartheta_3\frac{a}{\beta}\frac{2}{\sqrt{5}} + \vartheta_3 a - (\vartheta_2+\vartheta_3)\frac{a}{\beta}$$

displacement of point A ↑

$$-\vartheta_1\frac{a}{\beta}\frac{1}{\sqrt{5}} + \vartheta_1 2a - (\vartheta_1+\vartheta_2)\frac{a}{\beta}\frac{1}{\sqrt{5}} = -\vartheta_3\frac{a}{\beta}\frac{1}{\sqrt{5}} - \vartheta_3 4a + \vartheta_2 2a$$

$$E = M_p\vartheta_1 + M_p(\vartheta_1+\vartheta_2) + M_p(\vartheta_2+\vartheta_3) + M_p\vartheta_3$$

$$A = F\left(\vartheta_3\frac{a}{\beta}\frac{1}{\sqrt{5}} + \vartheta_3 2a\right) + F\left(\vartheta_3\frac{a}{\beta}\frac{1}{\sqrt{5}} + \vartheta_3 4a\right)$$

$E = A$

the solution to the equations is

$\vartheta_1 = 0.236\,\vartheta_2$

$\vartheta_3 = 0.388\,\vartheta_2$

$$F = 1.38\,\frac{M_p}{a}$$

Answer to Problem 2a

Kinematically possible are pattern A, C and D.

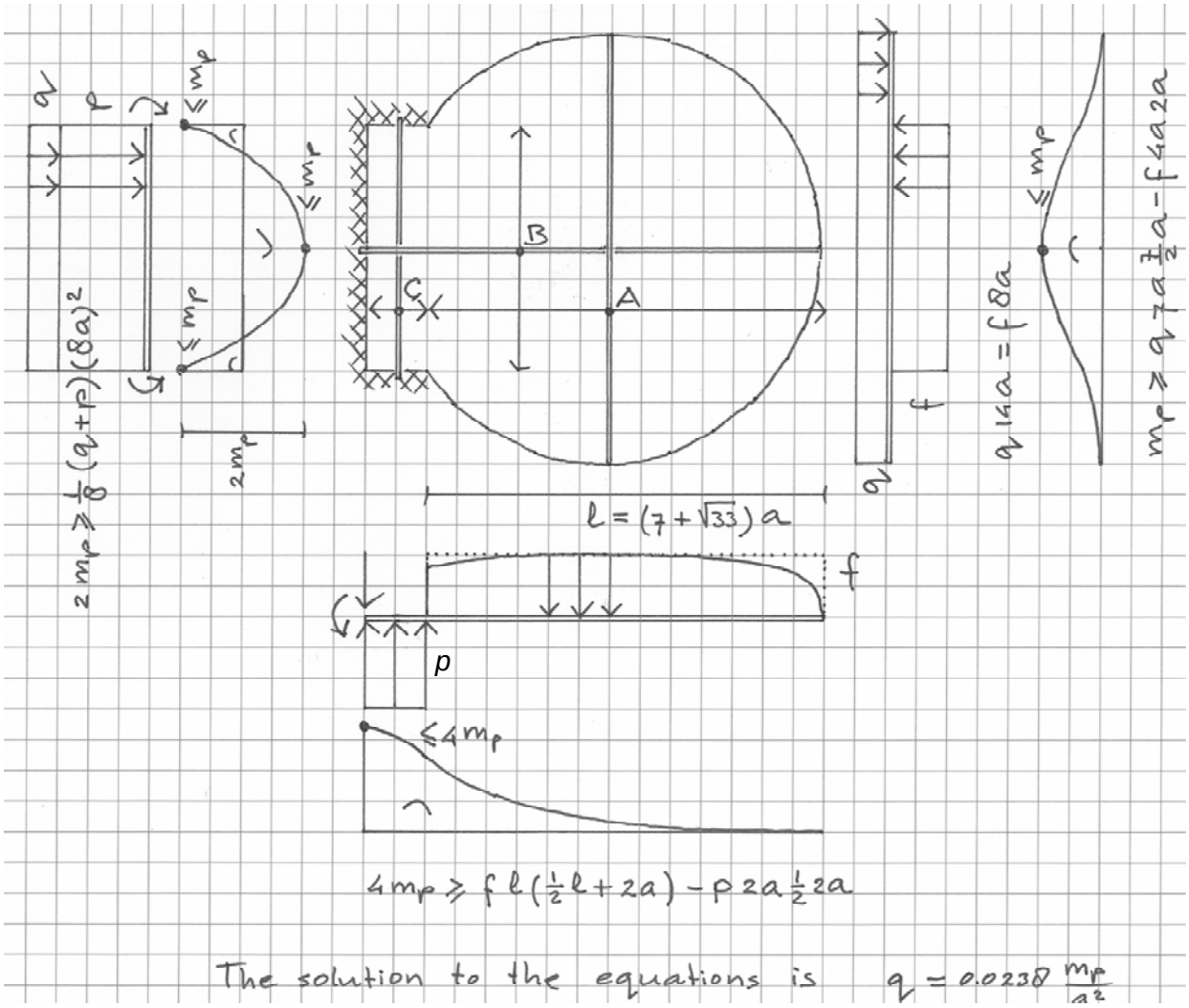
Answer to Problem 2b

$$E = m'_{px}\,l_y\,\varphi_x + m'_{py}\,l_x\,\varphi_y = 4m_p \times 8a \times \frac{w}{2\sqrt{33}\,a} + m_p \times 2\sqrt{33}\,a \times \frac{w}{8a} = 4.221 m_p\,w$$

$$A = q\,Au = q \times \tfrac{1}{2}\pi(7a)^2 \times w\,\frac{0.4244 \times 7a}{\frac{8}{7}\sqrt{33}\,a} = 34.83\,q\,a^2 w$$

$$E = A \Rightarrow \quad q = 0.121\frac{m_p}{a^2}$$

Answer to Problem 2c



Answer to Problem 3a

Answer A is correct.
(B The presence of tendons is not the question.)
(C Most of the time they do it right.)
(D Concrete bridges are prestressed for crack control and smaller deflections.)

Answer to Problem 3b

No. The moment distribution is (plate lecture book, p. 41).

$$m_{rr} = m_p \left[1 - \left(\frac{r}{R} \right)^2 \right]$$

$$m_{tt} = m_p$$

These are positive values only. Therefore, only bottom reinforcement is needed. (Assuming that the *z*-axis is pointing down. Assuming that the bars are placed in radial and tangential direction.)

Answer to Problem 3c