Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Write your <u>name</u> and <u>study number</u> at the top right-hand of your work.

Exam CT4150 Plastic Analysis of Structures
Thursday 15 January 2009, 9:00 – 12:00 hours

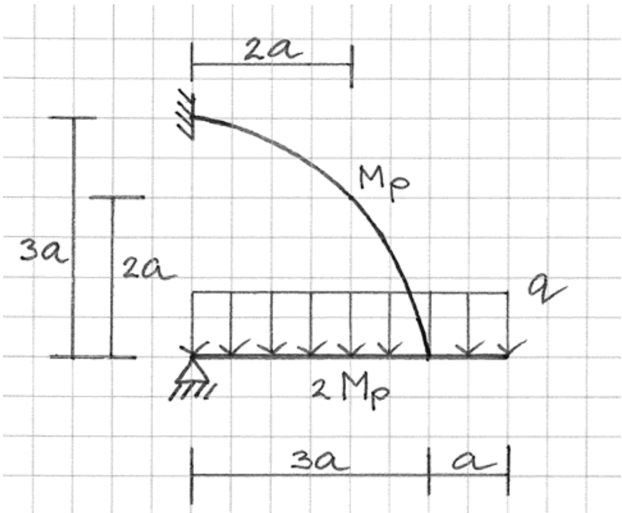

Figure 1. Frame

Problem 1

A frame consists of a straight and a curved beam with strengths $2M_p$ and M_p (Fig. 1). The beams are fixed to each other. The straight beam is loaded by an evenly distributed load q. The following relation exists between the plastic moment M_p and the plastic normal force N_p (Fig. 2).

$$N_p = \beta \frac{M_p}{a}$$

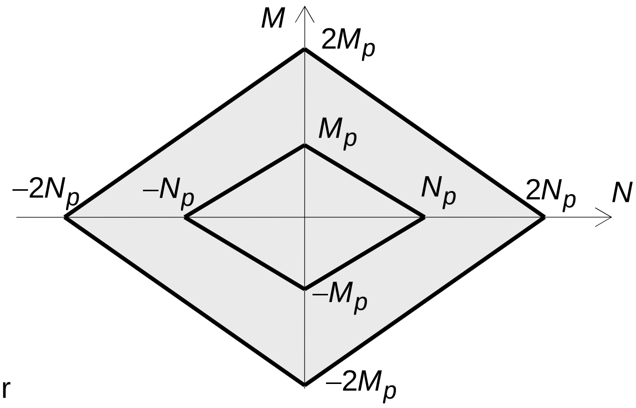

Figure 2. Yield contour

The influence of shear on the yield contour is neglected. Buckling and second order effects are not considered.

a Assume $\beta \to \infty$. Determine the collapse load q for all possible mechanisms. Write the collapse loads as functions of M_p and a. What is the decisive collapse load? (1.5 point)

b Assume $\beta \to \infty$. Draw the bending moment diagram and normal force diagram for the structure at the moment of collapse. (1 point)

c Assume β = 70. Choose one of the following problems (You need not do both).
– Determine the largest <u>lower-bound</u> for *q*.
– Determine the smallest <u>upper-bound</u> for *q*.
If you choose the upper-bound you only need to write down the equations and not solve the equations (2 points).

Problem 2

A plate has simply supported edges (Fig. 3). It carries an evenly distributed load *q*. The plate is homogeneous and orthotropic $m_{px} = m_p$, $m'_{px} = 0$, $m_{py} = m_p$, $m'_{py} = 0$.

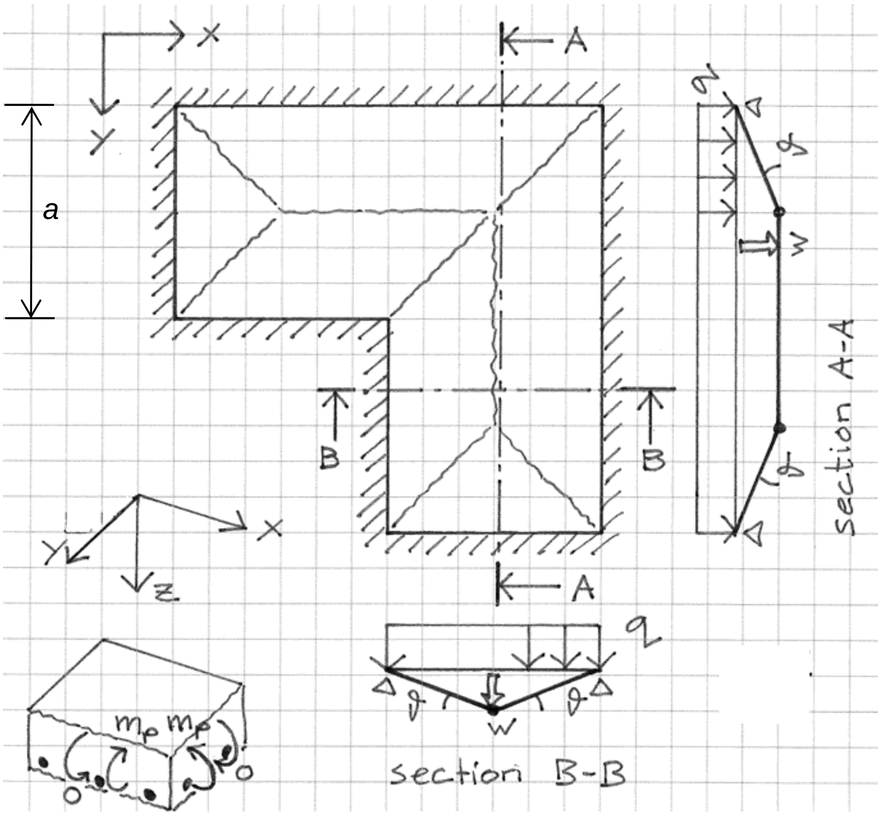

Figure 3. Plate dimensions and loading

a Consider the yield line patterns of Figure 4. Which of these patterns give kinematically possible mechanisms. (1 point)

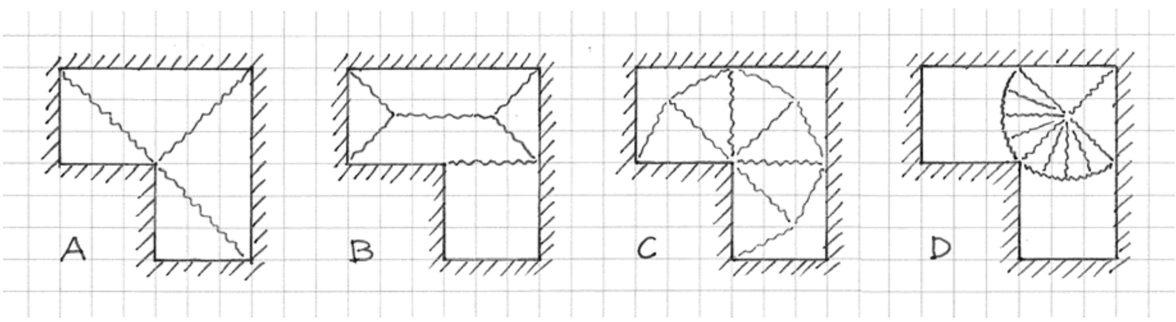

Figure 4. Yield line patterns of problem 2a

b Consider the yield line pattern of Figure 3. Determine an <u>upper bound</u> for q expressed in m_p and a (1.5 point).

c Determine the largest <u>lower-bound</u> for q using torsion free beams ($m_{xy} = 0$) (1.5 point).

Problem 3

a In problem 1c the interaction of moments and normal forces is taken into account. Shear forces are not considered? What is the reason for this? Choose A, B, C, or D (0.5 point).

A Shear forces should be included but this makes the problem too difficult for solving by hand.
B The influence of shear forces on the strength of plastic hinges is very small for steel structures.
C Shear forces are automatically included as the derivative of the moments.
D The shear force in a plastic hinge is zero because all material strength is already used to carry the moment and normal force.

b In Figure 5a a steel plate, with a thickness t, is loaded in the middle. In Figure 5b a large number of steel beams, width depths t, are loaded in the middle. The steel plate can carry 15% more load than the beams. Explain this (0.5 point).

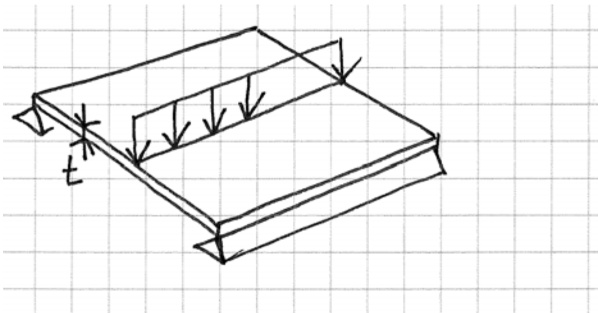

Figure 5a. Simply supported steel plate

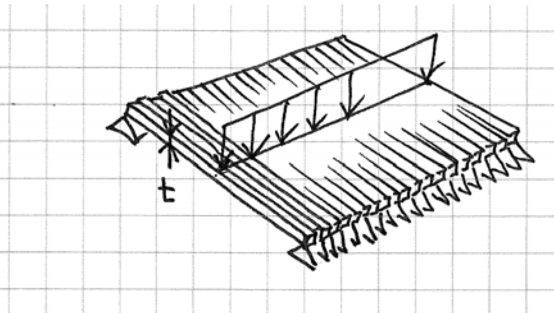

Figure 5b. Simply supported steel beams

c Codes of practice do not require that foundation settlements are considered in structural analysis for the ultimate limit state. Many engineers feel uncomfortable about this because foundation settlements occur often and give very large linear elastic stresses. Do we need to feel uncomfortable about ignoring foundation settlements? Explain your answer (0.5 point).

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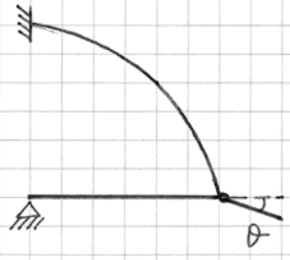
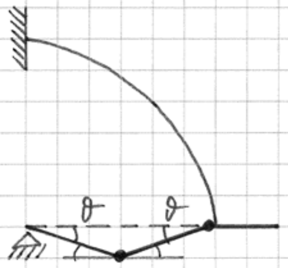
Answer to Problem 1a



$E = 2M_p\vartheta$

$A = q a \vartheta \frac{a}{2}$

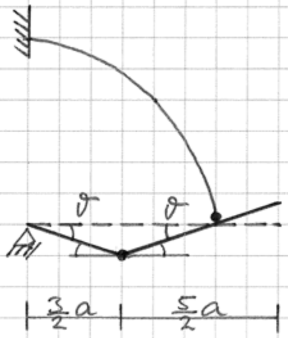
$E = A \Rightarrow q = 4\frac{M_p}{a^2}$



$E = 2M_p(\vartheta+\vartheta) + 2M_p\vartheta = 6M_p\vartheta$

$A = 2\left[q\frac{3}{2}a\vartheta\frac{3}{4}a\right] = \frac{9}{4}qa^2\vartheta$

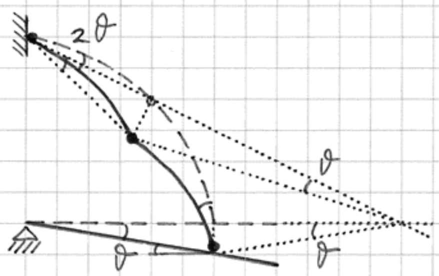
$E = A \Rightarrow q = \frac{8}{3}\frac{M_p}{a^2}$



$E = 2M_p(\vartheta+\vartheta) + M_p\vartheta = 5M_p\vartheta$

$A = q\frac{3}{2}a\vartheta\frac{3}{4}a + q\frac{5}{2}a\vartheta\frac{1}{4}a = \frac{7}{4}qa^2\vartheta$

$E = A \Rightarrow q = \frac{20}{7}\frac{M_p}{a^2}$

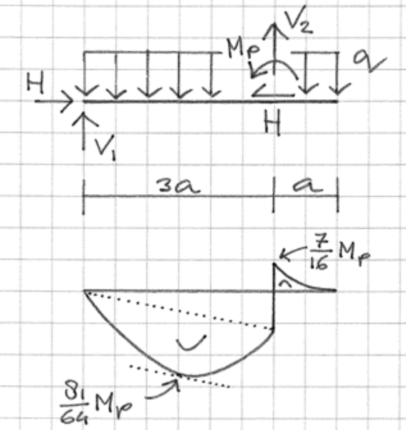


$E = M_p 2\vartheta + M_p(2\vartheta+\vartheta) + M_p(\vartheta+\vartheta) = 7M_p\vartheta$

$A = q 4a\vartheta 2a = 8qa^2\vartheta$

$E = A \Rightarrow \boxed{q = \frac{7}{8}\frac{M_p}{a^2}}$ decisive

Answer to Problem 1b

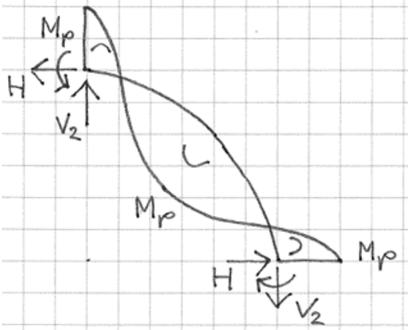


$$V_2 3a + M_p - q4a2a = 0 \Rightarrow V_2 = 2\frac{M_p}{a}$$

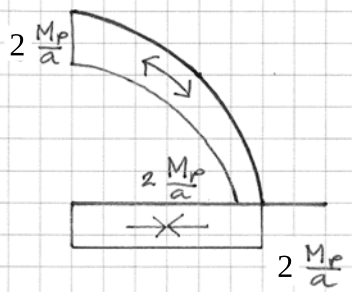
$$V_1 + V_2 = q4a \Rightarrow V_1 = \frac{3}{2}\frac{M_p}{a} \approx 1{,}5\frac{M_p}{a}$$

$$M = qa\frac{1}{2}a = \frac{7}{16}M_p \approx 0{,}44\,M_p$$

$$M = V_1\frac{3}{2}a - q\frac{3}{2}a\frac{3}{4}a = \frac{81}{64}M_p \approx 1{,}3\,M_p$$



$$M_p = M_p + V_2 3a - H3a \Rightarrow H = 2\frac{M_p}{a}$$



Answer to problem 1c Lower-bound

$$N = \alpha\frac{7}{3}\frac{M_p}{a} = (1-\alpha)N_p = (1-\alpha)70\frac{M_p}{a}$$

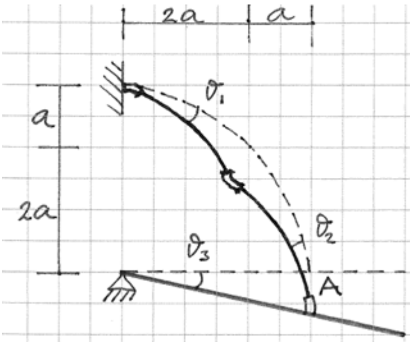
$$\alpha\frac{7}{3} = (1-\alpha)70$$

$$\alpha\frac{7}{3} + \alpha 70 = 70$$

$$\alpha = \frac{70}{\frac{7}{3}+70} = \frac{30}{31}$$

$$q = \alpha\frac{7}{8}\frac{M_p}{a^2} = \frac{105}{124}\frac{M_p}{a^2} \approx 0.847\,\frac{M_p}{a^2}$$

Answer to Problem 1c Upper-bound



$$u = \vartheta \frac{a}{\beta}$$

horizontal displacement of A →

$$\vartheta_1 \frac{a}{\beta} - \vartheta_1 a + (\vartheta_1 + \vartheta_2)\frac{a}{\beta}\frac{1}{\sqrt{2}} + \vartheta_2 2a = 0$$

vertical displacement of A ↓

$$\vartheta_1 2a + (\vartheta_1 + \vartheta_2)\frac{a}{\beta}\frac{1}{\sqrt{2}} - \vartheta_2 a + (\vartheta_2 + \vartheta_3)\frac{a}{\beta} = \vartheta_3 3a$$

$$E = M_p \vartheta_1 + M_p(\vartheta_1 + \vartheta_2) + M_p(\vartheta_2 + \vartheta_3)$$

$$A = q\, 4a\, \vartheta_3\, 2a$$

$$E = A$$

The solution to the equations is

$$\vartheta_1 = 2.06\,\vartheta_2\,, \quad \vartheta_3 = 1.06\,\vartheta_2\,, \quad q = 0.847\frac{M_p}{a^2}\,.$$

Answer to Problem 2a

Kinematically possible are pattern B and D.

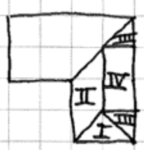
Answer to Problem 2b

$$E = 6\left[m_p \vartheta \frac{a}{2} + m_p \vartheta \frac{a}{2}\right] + 2\left[m_p 2\vartheta a\right] = 10\, m_p a \vartheta$$

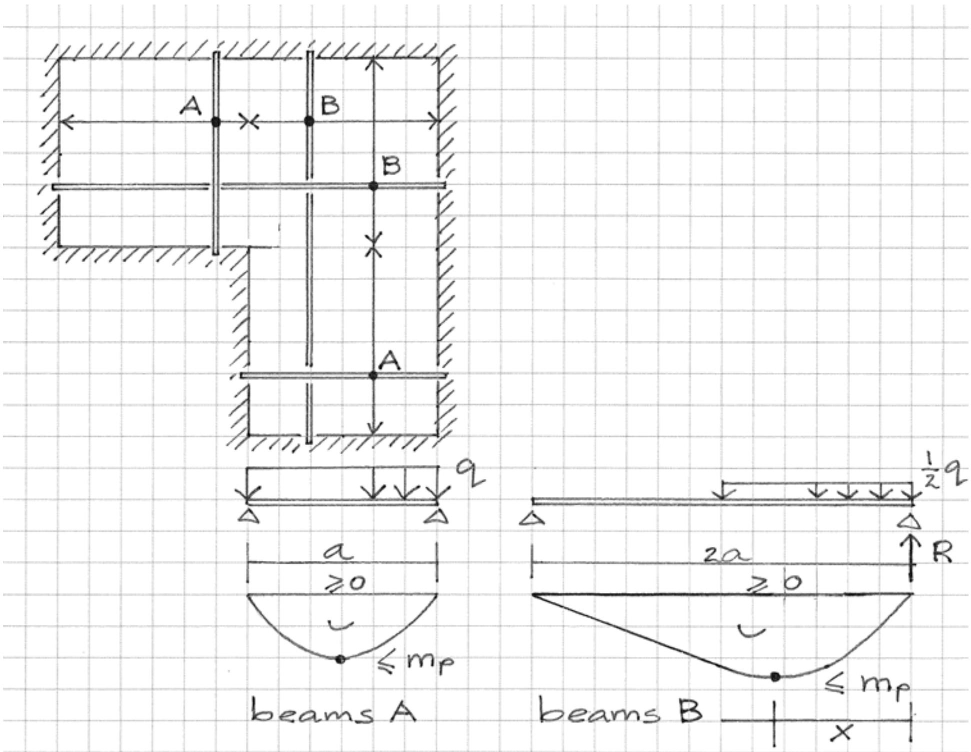
$$A = 2\Big[q \underbrace{\frac{1}{2} a \frac{a}{2}}_{\mathrm{I}} \vartheta \frac{1}{3}\frac{a}{2} + q \underbrace{a \frac{a}{2}}_{\mathrm{II}} \vartheta \frac{1}{2}\frac{a}{2} + 2\Big(q \underbrace{\frac{1}{2}\frac{a}{2}\frac{a}{2}}_{\mathrm{III}} \vartheta \frac{1}{3}\frac{a}{2}\Big) + q\, a \underbrace{\frac{a}{2}}_{\mathrm{IV}} \vartheta \frac{1}{2}\frac{a}{2}\Big]$$

$$= \frac{2}{3} q a^3 \vartheta$$

$$E = A \Rightarrow q = 15\frac{m_p}{a^2}$$



Answer to Problem 2c



$\frac{1}{8} q a^2 \leq m_p$

$0 = \frac{1}{2} q a \frac{3}{2} a - R 2a$

$\frac{1}{2} q x = R$

$-\frac{1}{2} q x \frac{1}{2} x + R x \leq m_p$

The solution to the equations is

$q = \frac{64}{9} \frac{m_p}{a^2} \approx 7.11 \frac{m_p}{a^2}$

Answer to Problem 3a

Answer B is correct (See beam lecture book Chap. 8).

Answer to Problem 3b

In a fold line the plastic rotation is in the first principal direction $\theta_1 = \theta$. The nature of folding prevents a rotation in the other direction $\theta_2 = 0$. Steel behaves according to the Von Mises yield contour (Fig. 6). Normality requires that the plastic rotation vector is perpendicular to the yield contour. Therefore, $m_1 = 1.15\ m_p$ and $m_2 = 0.58\ m_p$.

In the beams the plastic rotation θ_2 is free and $m_2 = 0$. Therefore, $m_1 = m_p$.

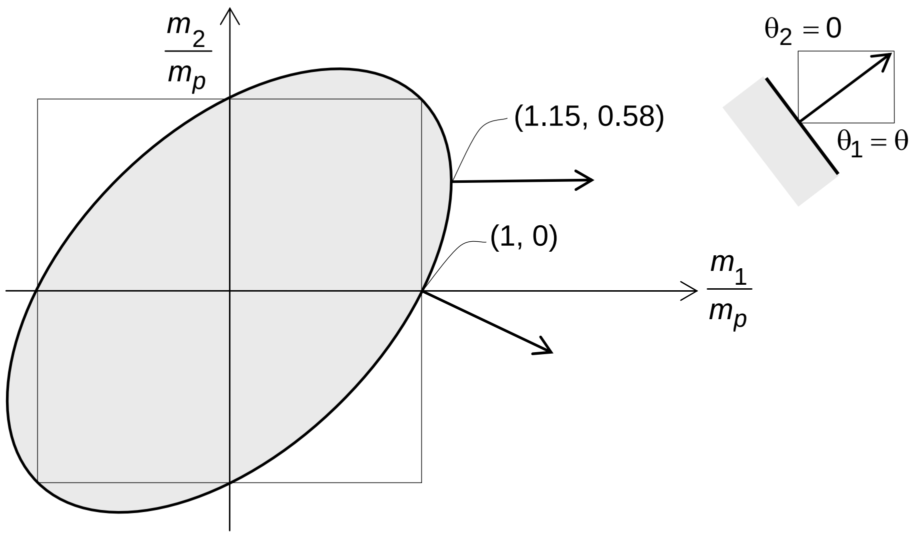


Figure 6. Yield contour for steel plates

Answer to Problem 3c
No. Foundation settlements do not influence the ultimate load of ductile structures.