Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Write your name and study number at the top right-hand of your work.

Exam CT4150 Plastic Analysis of Structures
Thursday 17 June 2009, 9:00 – 12:00 hours

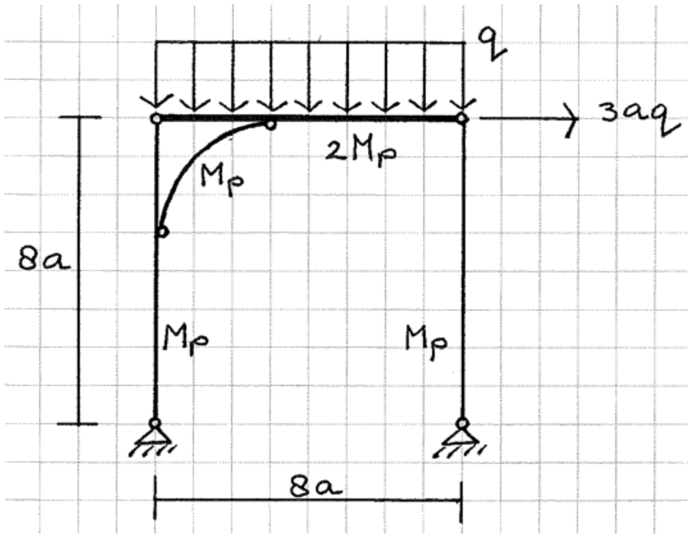

Figure 1. Frame

Problem 1

A frame consists of two columns, a beam and a curved bracket (Fig. 1). All joints between the members and with the foundation are pinned connections. The structure is loaded by a vertical evenly distributed load q and a horizontal load $3aq$. The following relation exists between the plastic moment M_p and the plastic normal force N_p (Fig. 2).

$$N_p = \beta \frac{M_p}{a}$$

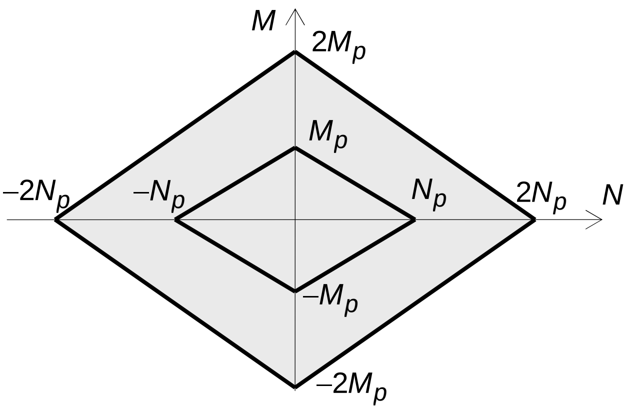

Figure 2. Yield contour

The influence of shear on the yield contour is neglected. Buckling and second order effects are not considered.

a Assume $\beta \to \infty$. Determine the collapse load q for all possible mechanisms. Write the collapse loads as functions of M_p and a. What is the decisive collapse load? (1.5 point)

b Assume $\beta \to \infty$. Draw the bending moment diagram and normal force diagram for the structure at the moment of collapse. (1.5 points)

c Assume β = 90. Choose one of the following problems (You need not do both).
– Determine the largest <u>lower-bound</u> for q.
– Determine the smallest <u>upper-bound</u> for q.
You only need to write down the equations and not solve the equations (1.5 points).

Problem 2

A reinforced concrete plate has fixed and simply supported edges (Fig. 3). It carries an evenly distributed load q. The plate is homogeneous and orthotropic.

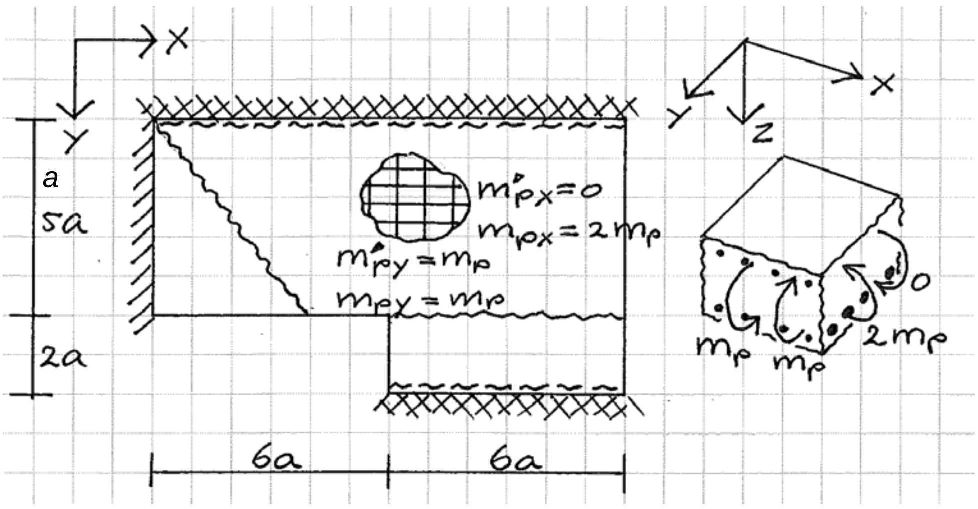

Figure 3. Plate dimensions, reinforcement and yield line pattern

a Consider the yield line patterns of Figure 4. Which of these patterns give kinematically possible mechanisms. (1 point)

b Consider the yield line pattern of Figure 3. Determine an <u>upper bound</u> for q expressed in m_p and a (1.5 point).

c Determine the largest <u>lower-bound</u> for q using torsion free beams ($m_{xy} = 0$) (1.5 point).

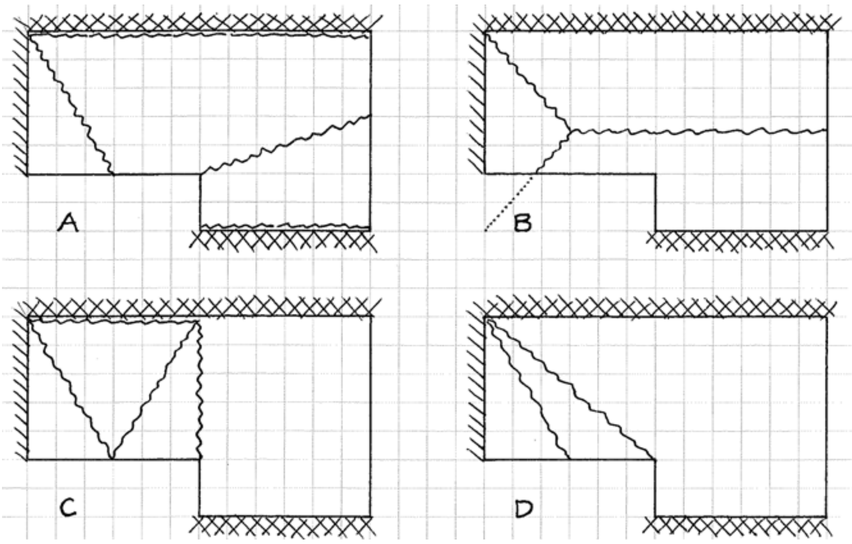

Figure 4. Yield line patterns of problem 2a

Problem 3

a Eurocode 2 uses strut-and-tie models for designing reinforced concrete structures. How are strut-and-tie models related to plastic analysis? Choose A, B, C, or D (0.5 point).

A Strut-and-tie models are applications of the lower bound theorem.
B Both the strut-and-tie method and plastic analysis require sufficient ductility of the material.
C Nodes in strut-and-tie models are similar to plastic hinges in frames and yield lines in plates.
D The strut directions need to be in the direction of the elastic stress trajectories. Therefore, there is no relation.

b Consider an elementary plate part with reinforcing moments
$m_{px} = 17, m_{py} = 0,\ m'_{px} = 0,\ m'_{py} = 10$ kNm/m.
The plate part is loaded by the section moments
$m_{xx} = 13,\ m_{yy} = -8$ and $m_{xy} = 5$ kNm/m.
Will this plate part yield? Explain your answer.

c In plastic analysis of plates the obtained smallest upper bound often is not equal to the exact plastic collapse load. What causes this? Choose A, B, C, or D (0.5 point).

A The true mechanism is to complicated for analysis.
B The torsion moments have been neglected.
C The virtual work equation gives an approximation of equilibrium (weak formulation).
D The round-off errors made in the hand calculations.

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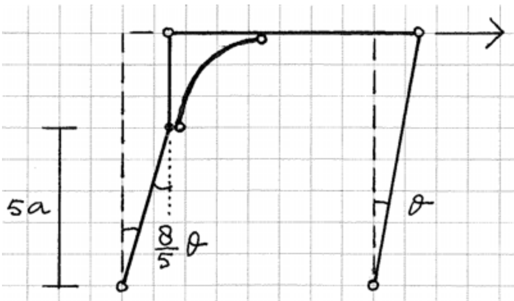
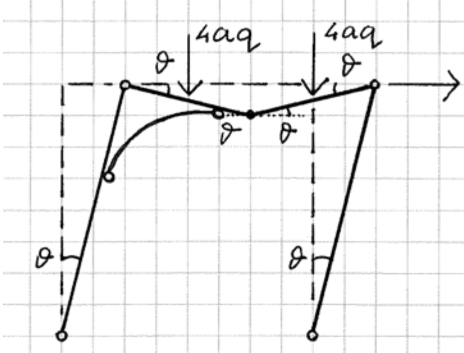
Answer to Problem 1a



$$E = M_p \tfrac{8}{5}\vartheta$$

$$A = 3aq\,\vartheta\,8a$$

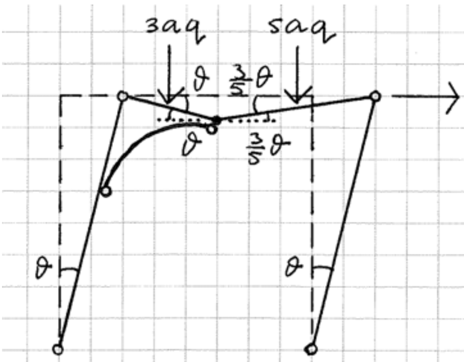
$$E = A \Rightarrow \boxed{q = \tfrac{1}{15}\tfrac{M_p}{a^2}} \text{ decisive}$$



$$E = 2M_p(\vartheta + \vartheta) = 4M_p\vartheta$$

$$A = 4aq\,\vartheta\,2a + 4aq\,\vartheta\,2a + 3aq\,\vartheta\,8a = 40qa^2\vartheta$$

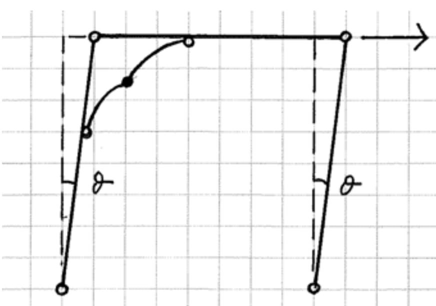
$$E = A \Rightarrow q = \tfrac{1}{10}\tfrac{M_p}{a^2}$$



$$E = 2M_p(\vartheta + \tfrac{3}{5}\vartheta) = \tfrac{16}{5}M_p\vartheta$$

$$A = 3aq\,\vartheta\,\tfrac{3}{2}a + 5aq\,\tfrac{3}{5}\vartheta\,\tfrac{5}{2}a + 3aq\,\vartheta\,8a = 36qa^2\vartheta$$

$$E = A \Rightarrow q = \tfrac{4}{45}\tfrac{M_p}{a^2}$$

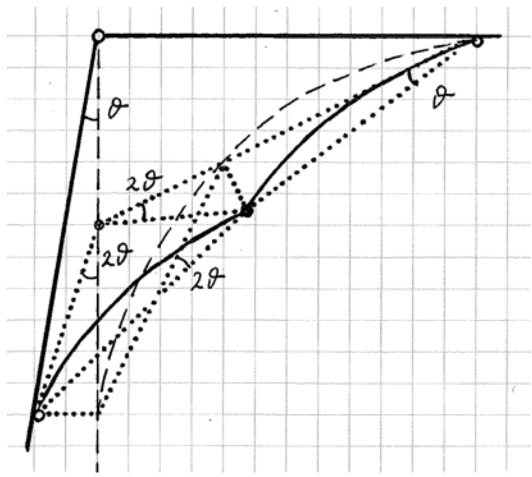


$$E = M_p(2\vartheta + \vartheta)$$

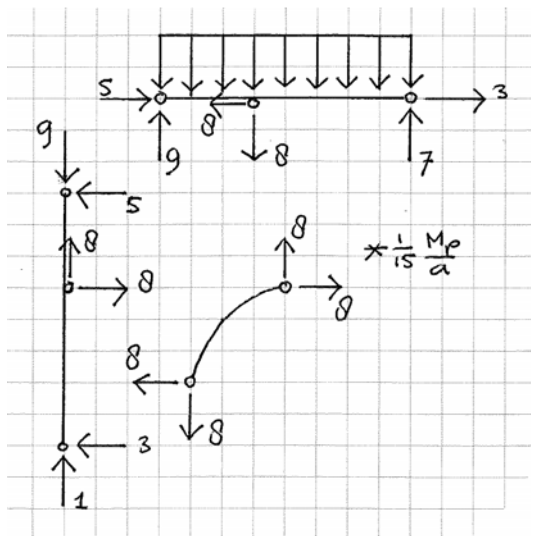
$$A = 3aq\,\vartheta\,8a$$

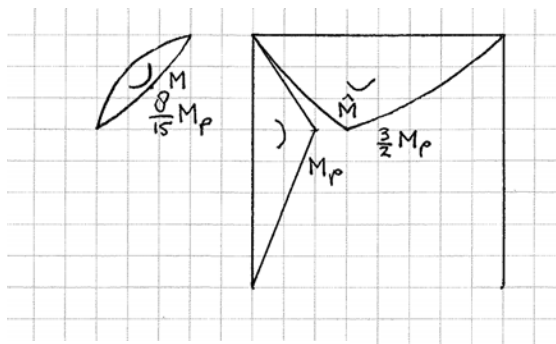
$$E = A \Rightarrow q = \tfrac{1}{8}\tfrac{M_p}{a^2}$$

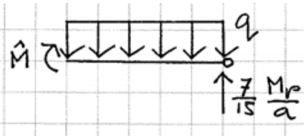
(See bracket detail at page 5.)



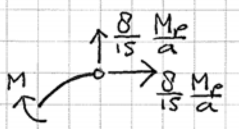
Answer to Problem 1b



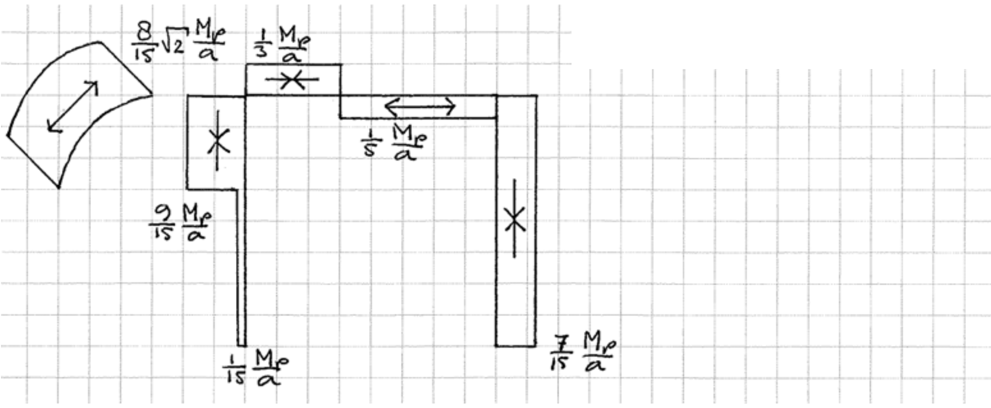




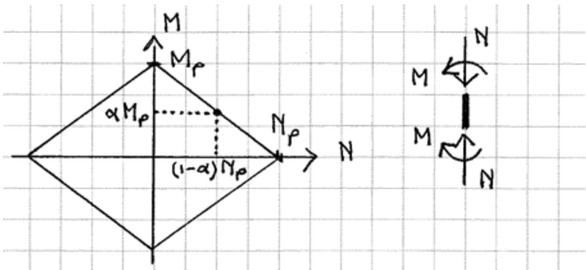
$$\hat{M} = \frac{7}{15}\frac{M_p}{a}5a - q\,5a\,\frac{5a}{2} = \frac{3}{2}M_p$$



$$M = \frac{8}{15}\frac{M_p}{a}2a - \frac{8}{15}\frac{M_p}{a}a = \frac{8}{15}M_p$$



Answer to problem 1c Lower-bound



$$N = \frac{1}{15}\frac{\alpha M_p}{a} = (1-\alpha)N_p = (1-\alpha)\,\beta\,\frac{M_p}{a} \Rightarrow$$

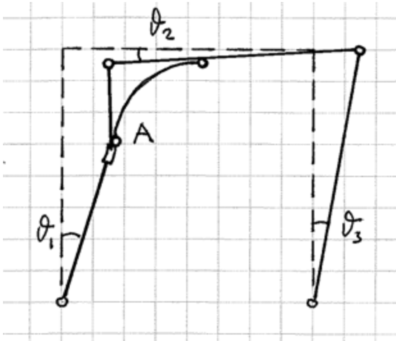
$$\frac{1}{15}\alpha = (1-\alpha)\beta \Rightarrow$$

$$\frac{1}{15}\alpha + \beta\alpha = \beta \Rightarrow$$

$$\alpha = \frac{\beta}{\frac{1}{15}+\beta} = \frac{90}{\frac{1}{15}+90} = \frac{1350}{1351}$$

$$q = \alpha\,\frac{1}{15}\,\frac{M_p}{a^2} = \frac{90}{1351}\,\frac{M_p}{a^2}$$

Answer to Problem 1c Upper-bound



$$u = \vartheta\,\frac{a}{\beta}$$

horizontal displacement of A →

$$\vartheta_1\, 5a = \vartheta_3\, 8a + \vartheta_2\, 3a$$

vertical displacement of A ↓

$$(\vartheta_1 + \vartheta_2)\frac{a}{\beta} = \vartheta_2\, 8a$$

$$E = M_p \, (\vartheta_1 + \vartheta_2)$$

$$A = q \, 8a \, \vartheta_2 \, 4a + 3aq \, \vartheta_3 \, 8a$$

$$E = A$$

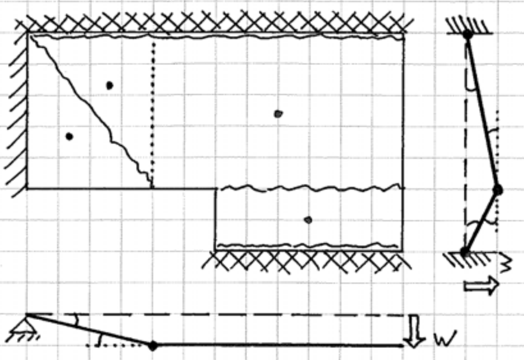
The solution to the equations is

$$\vartheta_1 = 719 \, \vartheta_2 \;, \quad \vartheta_3 = 449 \, \vartheta_2 \;, \quad q = \frac{90}{1351} \frac{M_p}{a^2}$$

Answer to Problem 2a

Kinematically possible are pattern C and D.

Answer to Problem 2b



$$E = m_p \, 6a \left(\frac{w}{2a} + \frac{w}{5a}\right) + 2m_p \, 5a \, \frac{w}{4a} + m_p \, 4a \, \frac{w}{5a} +$$

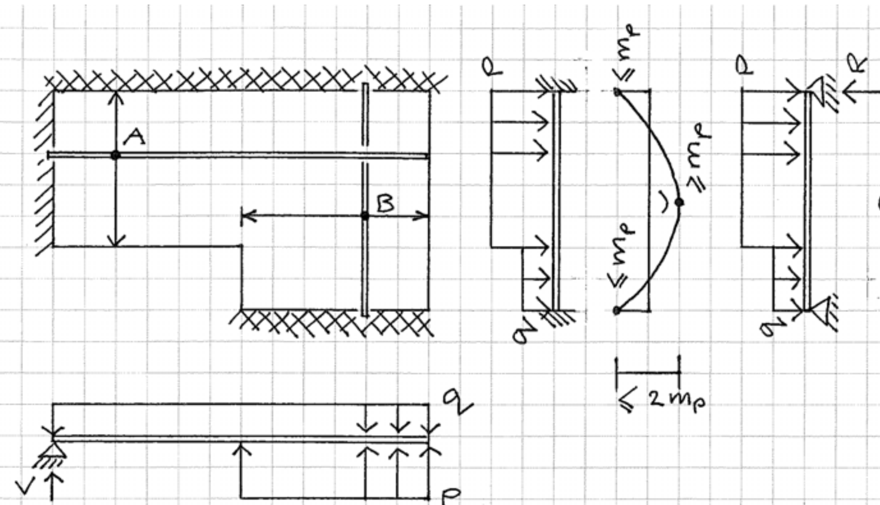
$$+ m_p \, 6a \, \frac{w}{2a} + m_p \, 12a \, \frac{w}{5a} = \frac{129}{10} m_p \, w$$

$$A = q \, 2a \, 6a \, \frac{w}{2} + q \, 8a \, 5a \, \frac{w}{2} + q \, \frac{1}{2} \, 4a \, 5a \, \frac{1}{3} w +$$

$$+ q \, \frac{1}{2} \, 4a \, 5a \, \frac{1}{3} w = \frac{98}{3} q a^2 w$$

$$E = A \;\Rightarrow\; q = \frac{387}{980} \frac{m_p}{a^2} \approx 0.39 \frac{m_p}{a^2}$$

Answer to Problem 2c



$q\,2a\,a + p\,5a\,4.5a - R\,7a = 0$

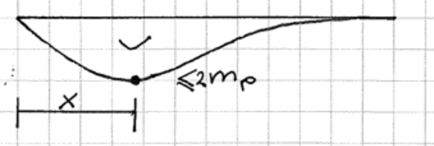
$R - px = 0$

$Rx - px\tfrac{1}{2}x \leq 2m_p$

beams A

$q\,12a\,6a - p\,6a\,9a = 0$

$q\,12a - V - p\,6a = 0$



$V - qx = 0$

$Vx - qx\tfrac{1}{2}x \leq 2m_p$

The solution to the equations is $q = \frac{1}{4}\frac{m_p}{a^2}$

Answer to Problem 3a

Answer A is correct.

Answer to Problem 3b

No yielding when

$$m_{xy}^2 < \min\left[(m_{px} - m_{xx})(m_{py} - m_{yy}),\,(m'_{px} + m_{xx})(m'_{py} + m_{yy})\right] \qquad (12.13)$$

$$5^2 < \min\left[(17-13)(0+8),\,(0+13)(10-8)\right]$$

$$25 < \min\left[(32, 26)\right]$$

Therefore, no yielding.

Answer to Problem 3c

Answer A is correct.