Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Write your name and study number at the top right-hand of your work.

Exam CT4150 Plastic Analysis of Structures
Thursday 8 April 2010, 14:00 – 17:00 hours

Problem 1

A frame consists of two columns and an arch (Fig. 1). The arch is twice as strong as the columns. All joints between the members and with the foundation are fixed connections. The structure is loaded by a vertical load F and a horizontal evenly distributed load $\frac{1}{4}F/a$. The following relation exists between the plastic moment M_p and the plastic normal force N_p (Fig. 2).

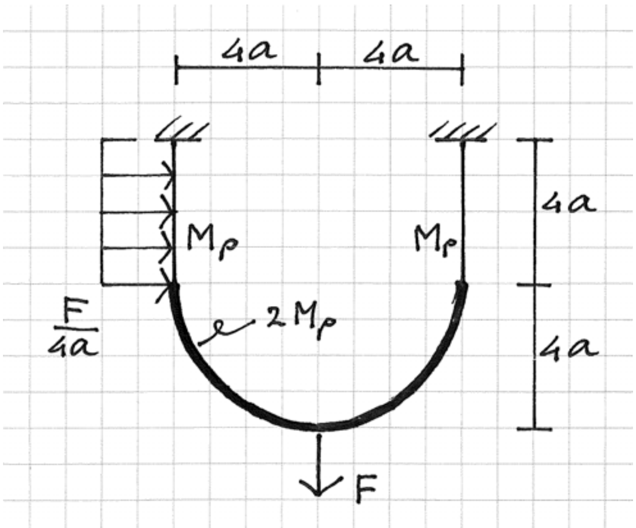


Figure 1. Frame structure

$$N_p = \beta \frac{M_p}{a}$$

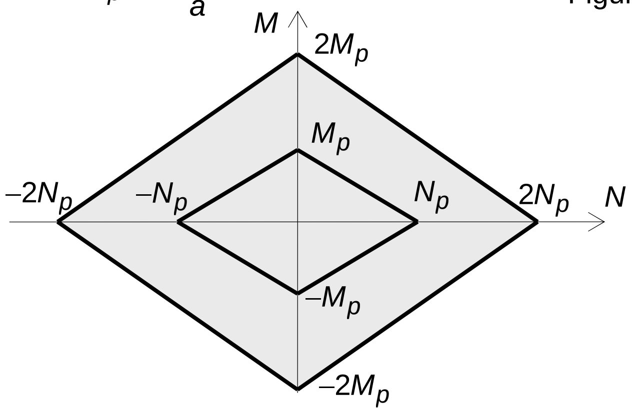


Figure 2. Yield contours

The influence of shear on the yield contour is neglected. Buckling and second order effects are not considered.

a Assume $\beta \to \infty$. Determine the collapse load F for all possible mechanisms. Write the collapse loads as functions of M_p and a. What is the decisive collapse load? (1.5 point)

b Assume $\beta \to \infty$. Draw the bending moment diagram and normal force diagram for the structure at the moment of collapse. (1.5 points)

c Assume β = 5. Choose one of the following problems (You need not do both).
– Use Fig. 3 to determine the largest lower-bound for F.
– Determine the smallest upper-bound for F.
You only need to write down the equations and not solve the equations (1.5 points).

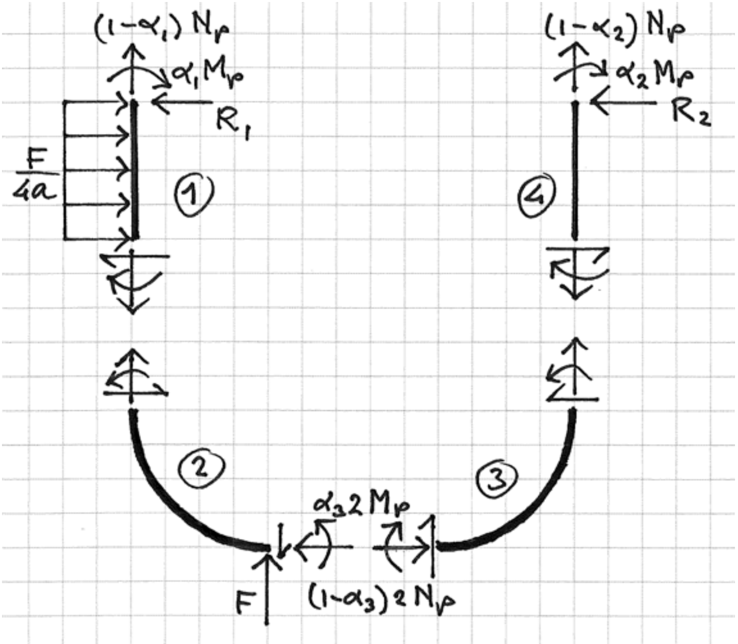


Figure 3. Equilibrium system for including *M-N* interaction

Problem 2

A reinforced concrete plate has simply supported edges and free edges (Fig. 4). It carries an evenly distributed load q. The plate is homogeneous and orthotropic.

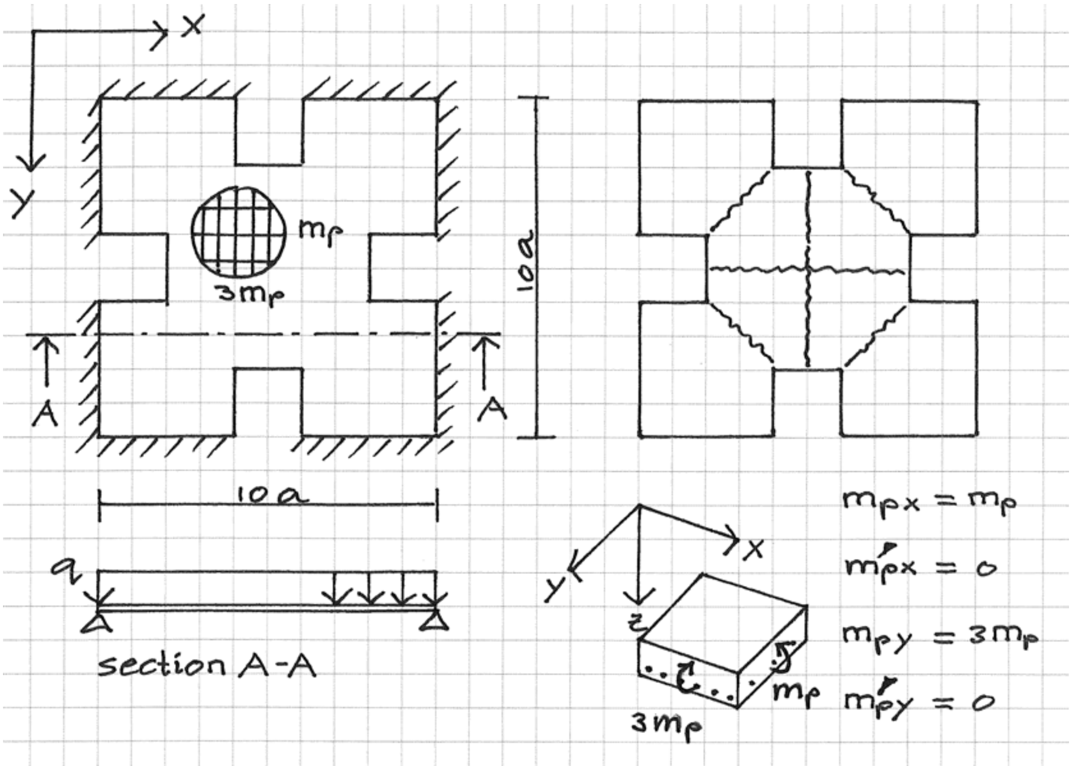


Figure 4. Plate dimensions, supports, reinforcement and yield line pattern

a Consider the yield line patterns of Figure 5. Which of these patterns give kinematically possible mechanisms. (1 point)

b Consider the yield line pattern of Figure 4. Determine an <u>upper-bound</u> for q expressed in m_p and a (1.5 point).

c Determine the largest <u>lower-bound</u> for q using torsion free beams ($m_{xy} = 0$) (1.5 point).

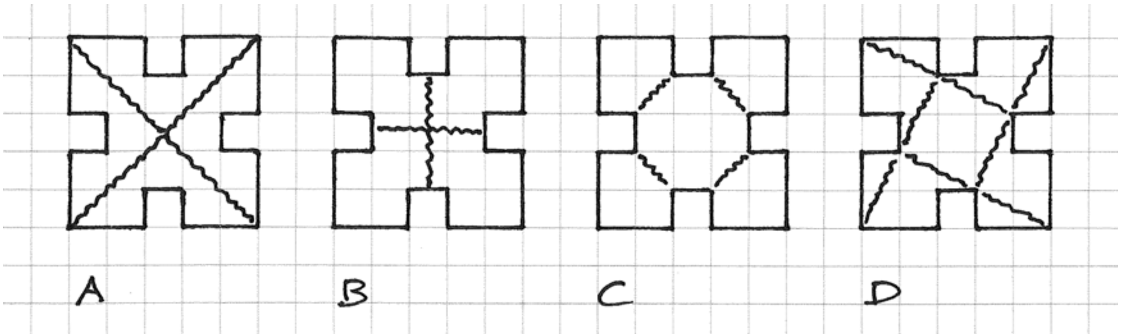


Figure 5. Yield line patterns of problem 2a

Problem 3

a Which criterion is very suitable for modelling yielding of metals? Choose A, B, C, or D (0.5 point).

A Tresca's criterion,
B Rankine's criterion,
C De Saint Venant's criterion,
D Von Mises' criterion.

b The yield contour of a plastic structure is always convex. The yield contour of a structure made of brittle materials can be nonconvex. What is a consequence of a nonconvex yield contour? Choose A, B, C, or D (0.5 point).

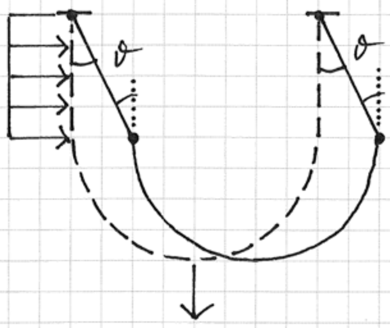
A The normality rule for computing plastic deformation increments is not valid.
B Many load combinations need to be analysed to make sure that the structure will not fail.
C The validity of the virtual work equation is restricted to special situations.
D No important consequences

c Suppose that you have studied several structural analysis methods for modelling a particular structure. Now you want to select one method for application in design practice. Which of the following make a good set of criteria? Choose from A to H. More than one can be selected (0.5 point).

A Accurate results,
B Agrees with relevant codes of practice,
C Easy to understand and to report,
D Includes the latest scientific research,
E Quickly to perform,
F Suitable for computer implementation,
G Free of patents,
H Safe results.

Exam CT4150, 8 April 2010

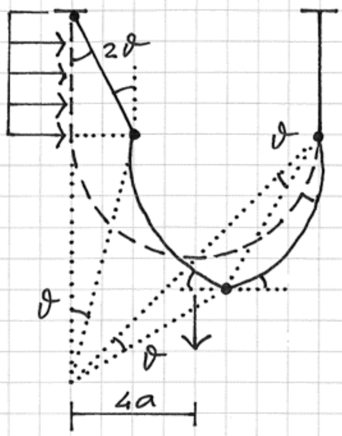
Answer to Problem 1a



$$E = M_p\vartheta + M_p\vartheta + M_p\vartheta + M_p\vartheta$$

$$A = \frac{F}{4a}\,4a\,2a\,\vartheta$$

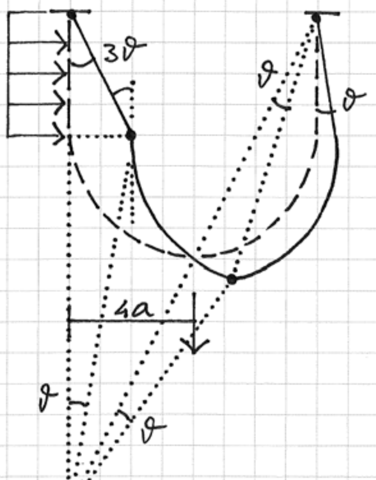
$$E = A \;\Rightarrow\; F = 2\,\frac{M_p}{a}$$



$$E = M_p\,2\vartheta + M_p(2\vartheta+\vartheta) + 2M_p(\vartheta+\vartheta) + M_p\vartheta$$

$$A = \frac{F}{4a}\,4a\,2a\,2\vartheta + F\,\vartheta\,4a$$

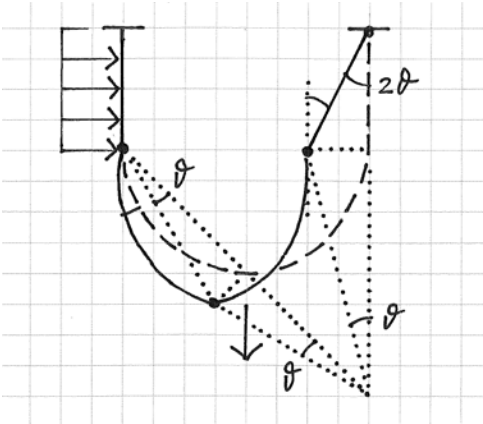
$$E = A \;\Rightarrow\; F = \frac{5}{4}\,\frac{M_p}{a}$$



$$E = M_p\,3\vartheta + M_p(3\vartheta+\vartheta) + 2M_p(\vartheta+\vartheta) + M_p\vartheta$$

$$A = \frac{F}{4a}\,4a\,2a\,3\vartheta + F\,\vartheta\,4a$$

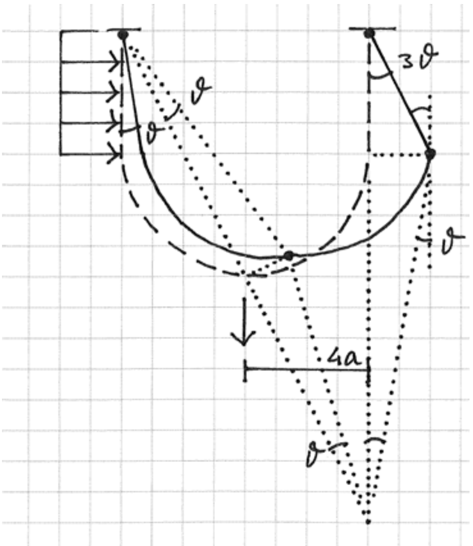
$$E = A \;\Rightarrow\; \boxed{F = \frac{6}{5}\,\frac{M_p}{a}}$$ decisive



$$E = M_p\,\vartheta + 2M_p(\vartheta+\vartheta) + M_p(\vartheta+2\vartheta) + M_p\,2\vartheta$$

$$A = F\,\vartheta\,4a$$

$$E = A \Rightarrow F = \frac{5}{2}\frac{M_p}{a}$$



$$E = M_p\,\vartheta + 2M_p(\vartheta+\vartheta) + M_p(\vartheta+3\vartheta) + M_p\,3\vartheta$$

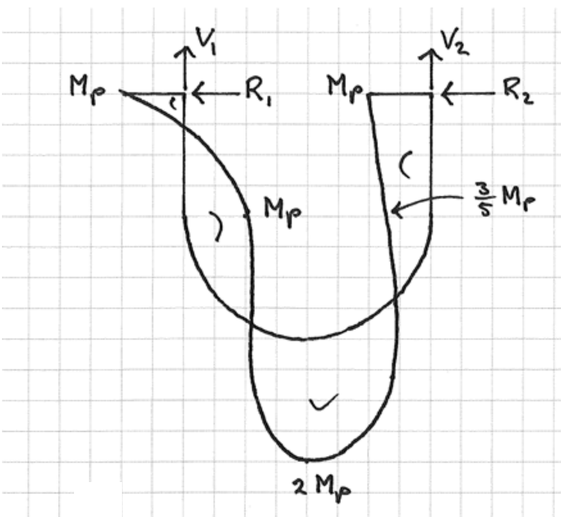
$$A = \frac{F}{4a}\,4a\,2a\,\vartheta - F\,\vartheta\,4a$$

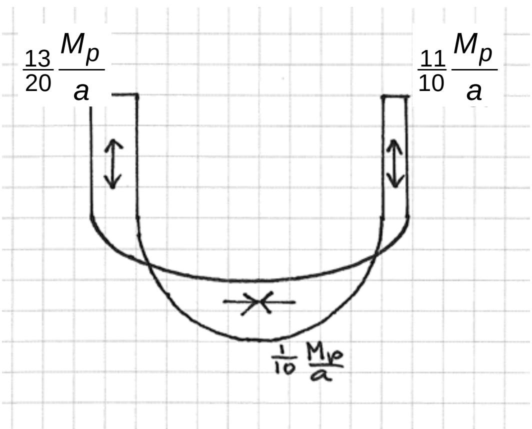
$$E = A \Rightarrow F = -6\frac{M_p}{a}$$

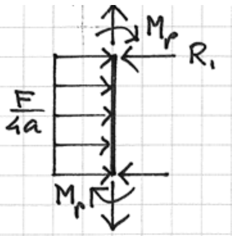
wrong direction of ϑ

$$F = 6\frac{M_p}{a}$$

Answer to Problem 1b

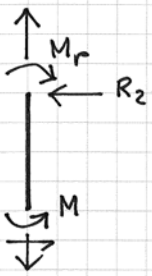






$$M_p = R_1 4a - \frac{F}{4a}4a\,2a - M_p \Rightarrow R_1 = \frac{11}{10}\frac{M_p}{a}$$

$$\frac{F}{4a}4a = R_1 + R_2 \Rightarrow R_2 = \frac{1}{10}\frac{M_p}{a}$$

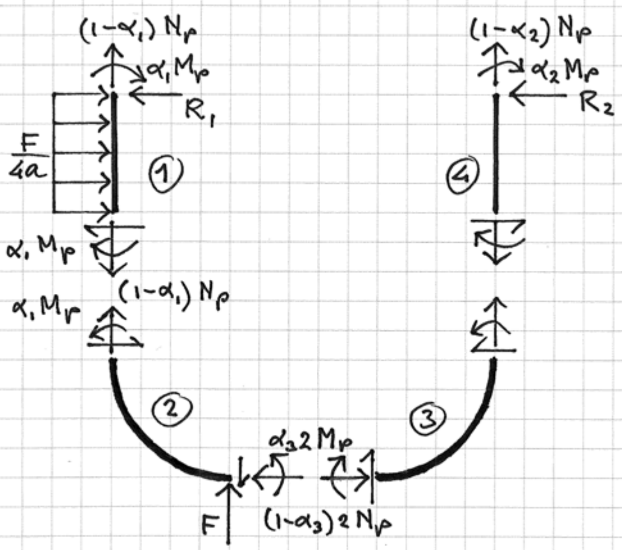


$$M = M_p - R_2 4a = \frac{3}{5}M_p \quad \text{check} \le M_p \quad \text{OK}$$

$$M_p = \frac{F}{4a}4a\,2a - F4a + V_2\,8a - M_p \Rightarrow V_2 = \frac{11}{20}\frac{M_p}{a}$$

$$F = V_1 + V_2 \Rightarrow V_1 = \frac{13}{20}\frac{M_p}{a}$$

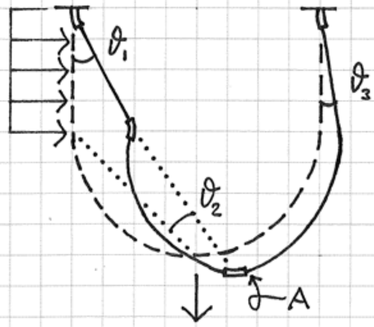
Answer to problem 1c Lower-bound




```
> restart:
> b:=5:
> Np:=b*Mp/a:
> eq1:= F/(4*a)*4*a = R1 +R2:
> eq2:= F = (1-a1)*Np +(1-a2)*Np:
> eq3:= a1*Mp = F/(4*a)*4*a*2*a -F*4*a +(1-a2)*Np*8*a -a2*Mp:
> eq4:= a1*Mp = F*4*a -R2*4*a - (1-a2)*Np*8*a + a2*Mp:
> eq5:= a3*2*Mp = R2*8*a +(1-a2)*Np*4*a -a2*Mp:
> eq6:= (1-a3)*2*Np =R2:
> solve({eq1,eq2,eq3,eq4,eq5,eq6},{F,a1,a2,a3,R1,R2});
```

$$\left\{F = \frac{4170\,Mp}{3787\,a},\ R1 = \frac{3750\,Mp}{3787\,a},\ R2 = \frac{60\,Mp}{541\,a},\ a1 = \frac{3330}{3787},\ a2 = \frac{3410}{3787},\ a3 = \frac{535}{541}\right\}$$

Answer to Problem 1c Upper-bound



$$u = \vartheta \frac{a}{\beta}$$

horizontal displacement of A →

$$\vartheta_1 4a - \vartheta_2 4a - (\vartheta_2+\vartheta_3)\frac{a}{\beta} = \vartheta_3 8a$$

vertical displacement of A ↓

$$\vartheta_1 \frac{a}{\beta} + (\vartheta_1+\vartheta_2)\frac{a}{\beta} + \vartheta_2 4a = \vartheta_3 \frac{a}{\beta} + \vartheta_3 4a$$

$$E = M_p \vartheta_1 + M_p(\vartheta_1+\vartheta_2) + 2M_p(\vartheta_2+\vartheta_3) + M_p \vartheta_3$$

$$A = \frac{F}{4a} 4a\, 2a\, \vartheta_1 + F\left(\vartheta_3 \frac{a}{\beta} + \vartheta_3 4a\right)$$

$$E = A$$

```
> restart:
> b:=5:
> eq1:= t1*4*a -t2*4*a -(t2+t3)*a/b = t3*8*a:
> eq2:= t1*a/b +(t1+t2)*a/b +t2*4*a = t3*a/b +t3*4*a:
> E:= Mp*t1 +Mp*(t1+t2) +2*Mp*(t2+t3) +Mp*t3:
> A:= F/(4*a)*4*a*2*a*t1 +F*(t3*a/b+t3*4*a):
> eq3:= E = A:
> solve({eq1,eq2,eq3},{t3,t2,F});
```

$$\left\{F = \frac{4170\,Mp}{3787\,a},\ t2 = \frac{169\,t1}{651},\ t3 = \frac{11\,t1}{31}\right\}$$

Answer to Problem 2a

Kinematically possible is pattern A.

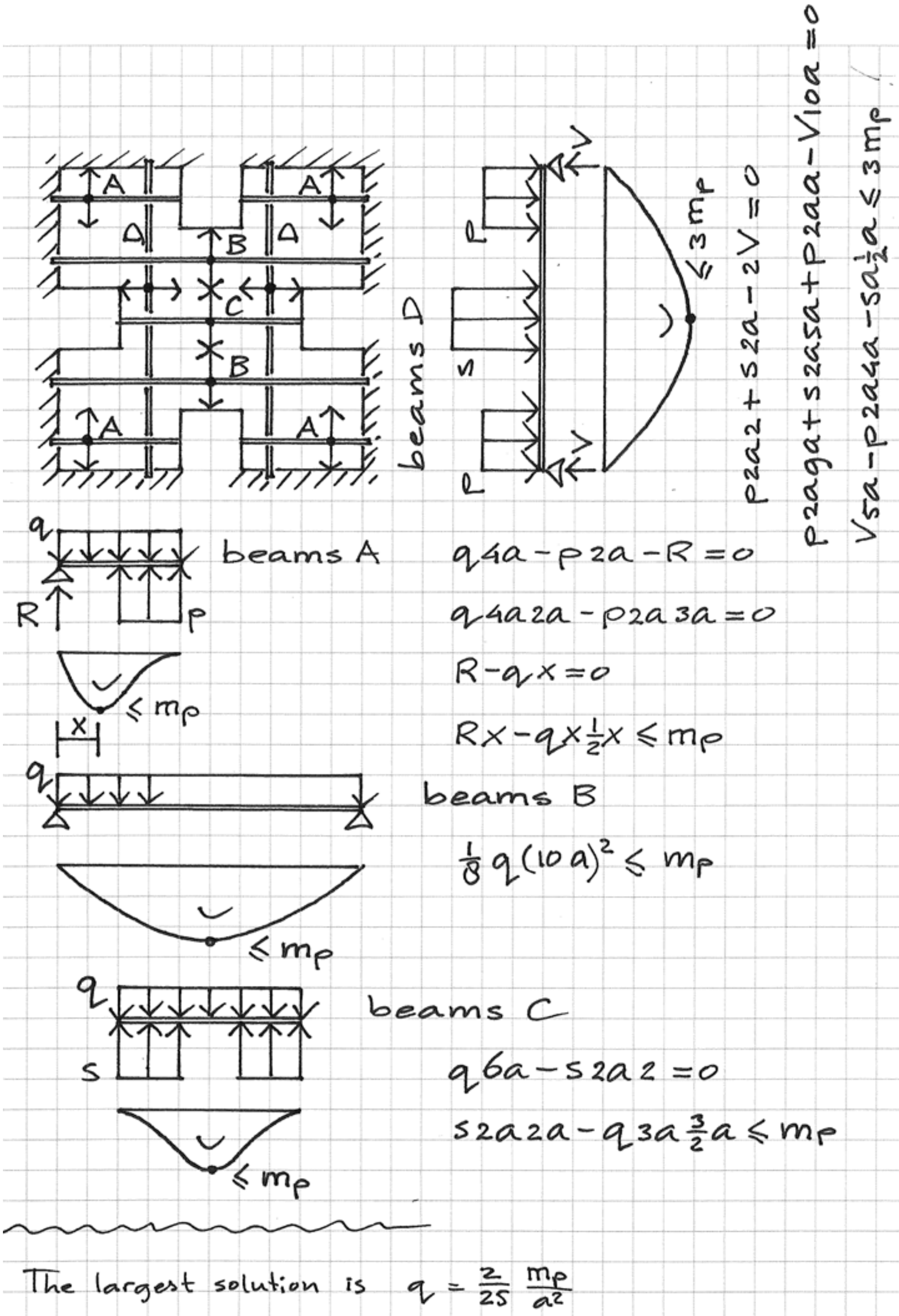
Answer to Problem 2b

$$E = m_p 6a\, 2\frac{w}{4a} + 3 m_p 6a\, 2\frac{w}{4a} = 12 m_p w$$

$$A = 4\left\{q \frac{4a\,4a}{2}\frac{w}{3} - 2q\frac{a\,a}{2}\frac{1}{4} w \frac{1}{3}\right\} = \frac{31}{3} q a^2 w$$

$$E = A \Rightarrow q = \frac{36}{31}\frac{m_p}{a^2}$$

Answer to Problem 2c



Answer to Problem 3a
D

Answer to Problem 3b
B

Answer to Problem 3c
A, B, C, E and H.