Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Write your <u>name</u> and <u>study number</u> at the top right-hand of your work.

Exam CT4150 Plastic Analysis of Structures
Thursday 21 January 2010, 14:00 – 17:00 hours

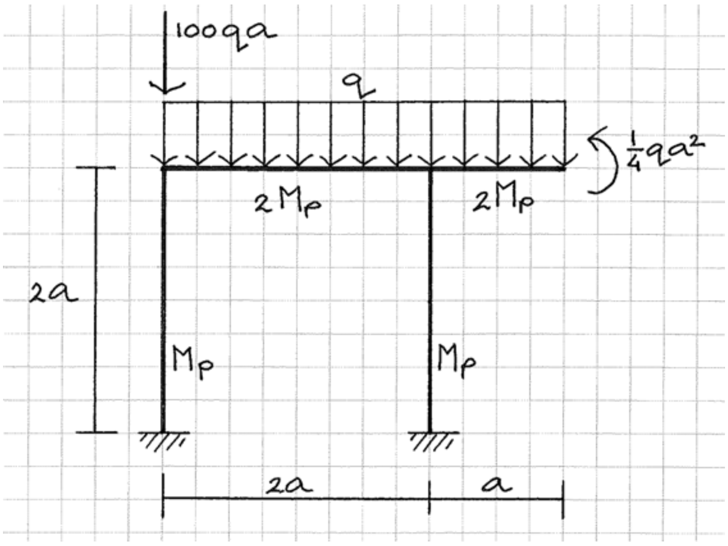

Figure 1. Frame structure

Problem 1

A frame consists of two columns and a beam (Fig. 1). The beam is twice as strong as the columns. All joints between the members and with the foundation are fixed connections. The structure is loaded by a vertical evenly distributed load q, a vertical force $100qa$ and a moment ¼ qa^2. The following relation exists between the plastic moment M_p and the plastic normal force N_p (Fig. 2).

$$N_p = \beta \frac{M_p}{a}$$

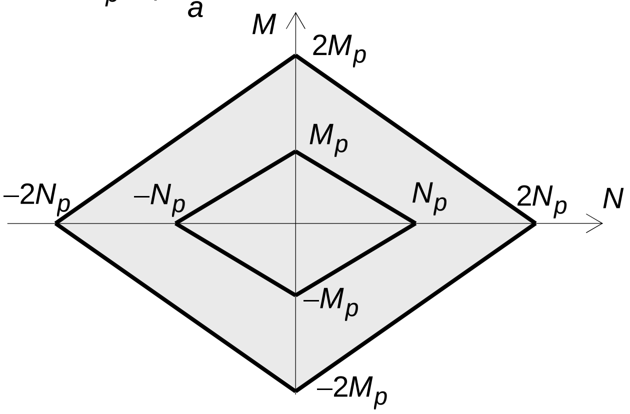

Figure 2. Yield contour

The influence of shear on the yield contour is neglected. Buckling and second order effects are not considered.

a Assume $\beta \to \infty$. Determine the collapse load *q* for all possible mechanisms. Write the collapse loads as functions of M_p and *a*. What is the decisive collapse load? (1.5 point)

b Assume $\beta \to \infty$. Draw the bending moment diagram and normal force diagram for the structure at the moment of collapse. (1.5 points)

c Assume β = 82. Choose one of the following problems (You need not do both).
– Use Fig. 3 to determine the largest <u>lower-bound</u> for *q*.
– Use Fig. 4 to determine the smallest <u>upper-bound</u> for *q*.
You only need to write down the equations and not solve the equations (1.5 points).

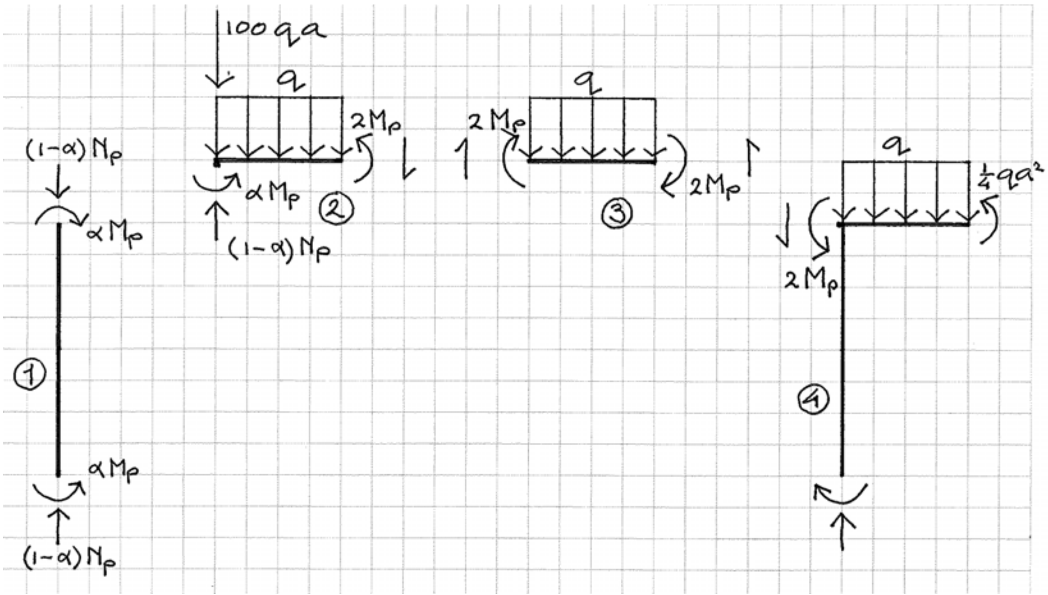

Figure 3. Equilibrium system for including M-N interaction

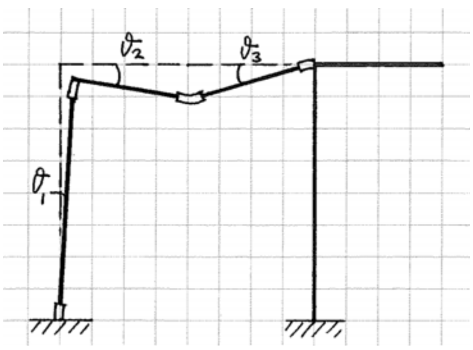

Figure 4. Mechanism for including M-N interaction

Problem 2

A reinforced concrete plate has fixed and simply supported edges (Fig. 5). It carries an evenly distributed load q. The plate is homogeneous and orthotropic.

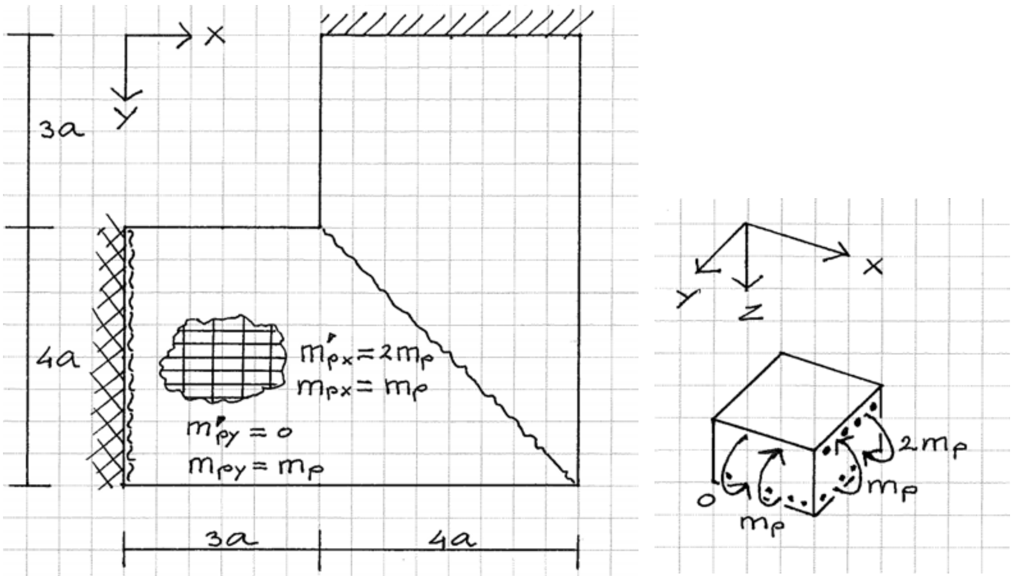

Figure 5. Plate dimensions, reinforcement and yield line pattern

a Consider the yield line patterns of Figure 6. Which of these patterns give kinematically possible mechanisms. (1 point)

b Consider the yield line pattern of Figure 5. Determine an <u>upper bound</u> for q expressed in m_p and a (1.5 point).

c Determine the largest <u>lower-bound</u> for q using torsion free beams ($m_{xy} = 0$) (1.5 point).

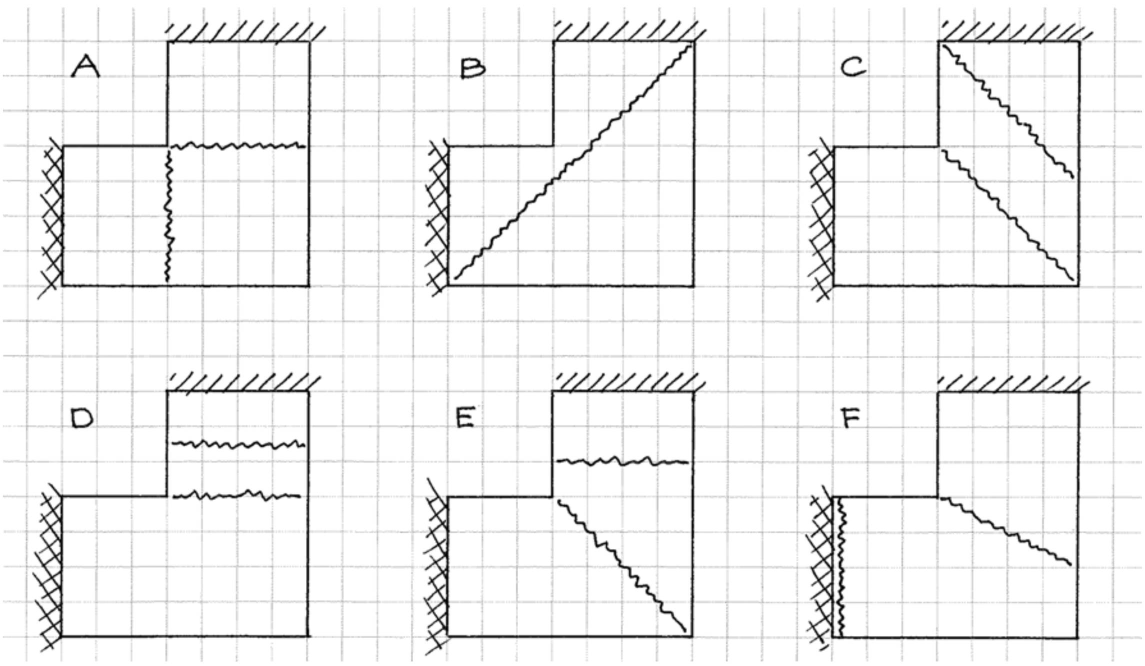

Figure 6. Yield line patterns of problem 2a

Problem 3

a The hardening property of metals makes it possible to increase the tensile yield strength by imposing a cold deformation. However, at the same time the compressive yield strength is reduced. How is this called? Choose A, B, C, or D (0.5 point).

A Hoogenboom's law,
B Bauschinger effect,
C Turkstra's rule,
D Stüssi-Kollbrunner paradox.

b In nonlinear finite element analysis of reinforced concrete floors we often observe that many of the cracks have an angle of 45º with the reinforcement. Why is this?
Choose A, B, C, or D (0.5 point).

A In general, the crack direction of the best mechanism has an angle of 45º with the floor edges. Reinforcement is often designed parallel to the edges,
B Apparently, the reinforcement has been designed using $m_{px} = m_{xx} + |m_{xy}|$ etc.,
C Moment trajectories often have a 45º angle with the edges and the reinforcement,
D The shortest distance for the shear force to go to the edges is in a 45º angle with the edges and the reinforcement.

c In structural design often the force flow is analysed elastically while the local capacity is determined plastically. This is a remarkable contradiction in methods. We could use plastic analysis for determining the force flow too. Better still, we could use accurate nonlinear analysis (incremental analysis) for all analyses. In the future - with powerful computers, excellent software and well educated engineers - what will be a reason for still using elastic analysis in design? Choose A, B, C, D or E (0.5 point).

A Computers are not fast enough for routine application of nonlinear analysis.
B Most engineers are not qualified to do nonlinear analysis. For this they would need to take extra courses.
C Redistributions of the force flow due to yielding or cracking are usually small, especially when the design is based on a linear analysis.
D Nonlinear analyses show how a structure would collapse, but they do not show how to improve the design.
E Nonlinear analyses require extra input data which is not available or takes much time to find.

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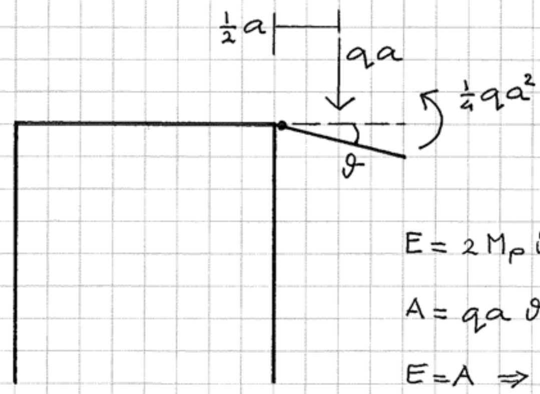
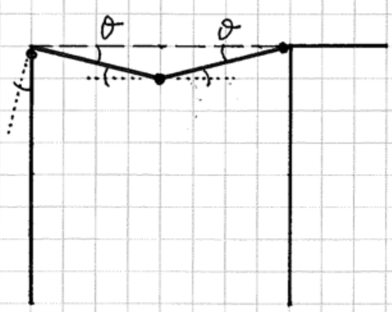
Answer to Problem 1a



$$E = 2 M_p \vartheta$$

$$A = qa\,\vartheta\,\tfrac{1}{2}a - \tfrac{1}{4}qa^2\,\vartheta$$

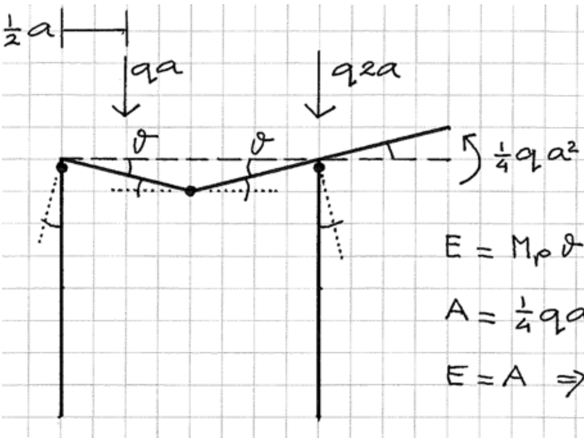
$$E = A \Rightarrow q = 8\frac{M_p}{a^2}$$



$$E = M_p\vartheta + 2M_p(\vartheta+\vartheta) + 2M_p\vartheta$$

$$A = qa\,\vartheta\,\tfrac{1}{2}a + qa\,\vartheta\,\tfrac{1}{2}a$$

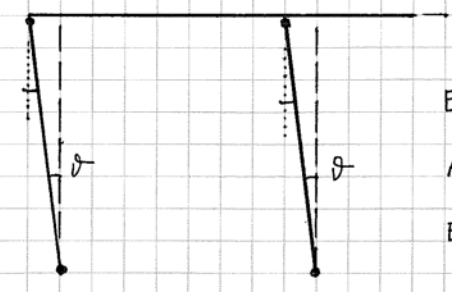
$$E = A \Rightarrow \boxed{q = 7\frac{M_p}{a^2}} \quad \text{decisive}$$



$$E = M_p\vartheta + 2M_p(\vartheta+\vartheta) + M_p\vartheta$$

$$A = \tfrac{1}{4}qa^2\,\vartheta + qa\,\vartheta\,\tfrac{1}{2}a$$

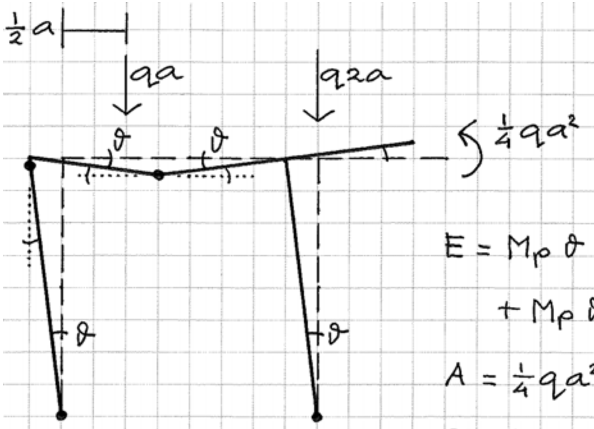
$$E = A \Rightarrow q = 8\frac{M_p}{a^2}$$



$$E = M_p\vartheta + M_p\vartheta + M_p\vartheta + M_p\vartheta$$

$$A = q0$$

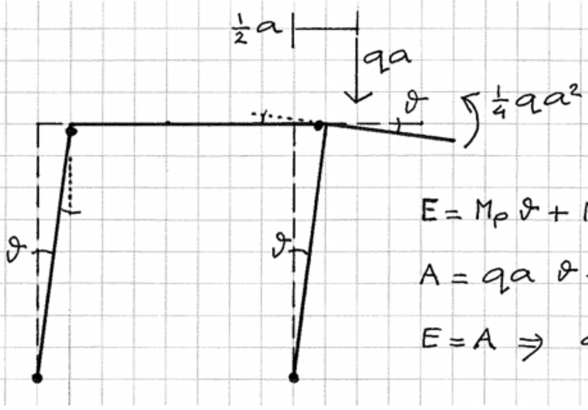
$$E = A \Rightarrow q = \infty$$



$$E = M_p\vartheta + M_p(\vartheta+\vartheta) + 2M_p(\vartheta+\vartheta) + M_p\vartheta$$

$$A = \frac{1}{4}qa^2\vartheta + qa\,\vartheta\,\frac{1}{2}a$$

$$E = A \Rightarrow q = \frac{32}{3}\frac{M_p}{a^2}$$

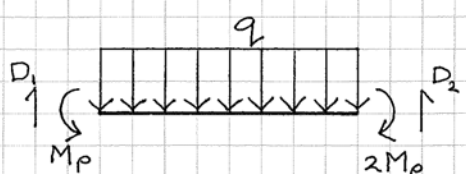


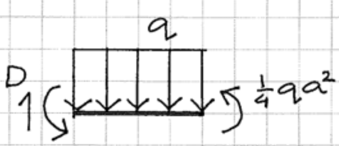
$$E = M_p\vartheta + M_p\vartheta + 2M_p\vartheta + M_p\vartheta$$

$$A = qa\,\vartheta\,\frac{1}{2}a - \frac{1}{4}qa^2\vartheta$$

$$E = A \Rightarrow q = 20\frac{M_p}{a^2}$$

Answer to Problem 1b





$$0 \overset{\curvearrowright +}{=} -M_p + q2a\,a + 2M_p - D_2\,2a$$

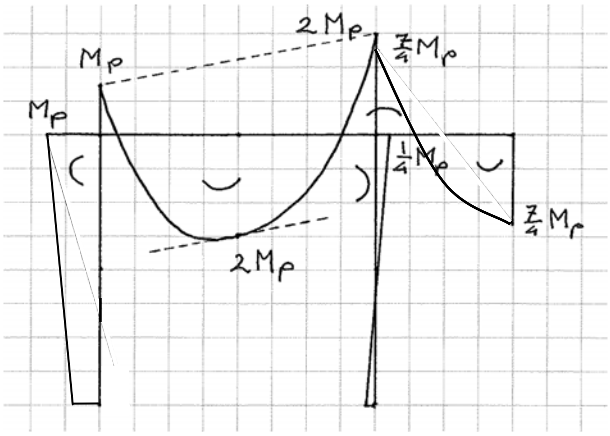
$$\Rightarrow D_2 = \frac{15}{2}\frac{M_p}{a}$$

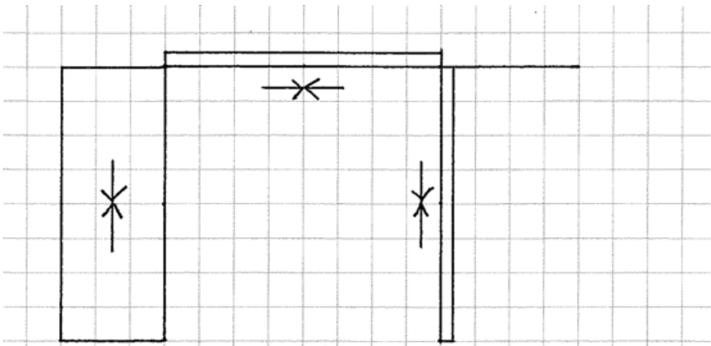
$$0 \overset{\downarrow +}{=} q2a - D_1 - D_2$$

$$\Rightarrow D_1 = \frac{13}{2}\frac{M_p}{a}$$

$$M = qa\frac{1}{2}a - \frac{1}{4}qa^2 = \frac{1}{4}qa^2 = \frac{7}{4}M_p$$

$$D = qa = 7\frac{M_p}{a}$$





$$100qa + D_1 = \frac{1413}{2}\frac{M_p}{a} \qquad D + D_2 = \frac{29}{2}\frac{M_p}{a}$$

Answer to problem 1c Lower-bound

$$N_p = \beta \frac{M_p}{a}$$

moment equilibrium of part ②

$$0 = 100qa\,a + qa\tfrac{1}{2}a + \alpha M_p - (1-\alpha) N_p a + 2M_p$$

moment equilibrium of part ② and ③ connected

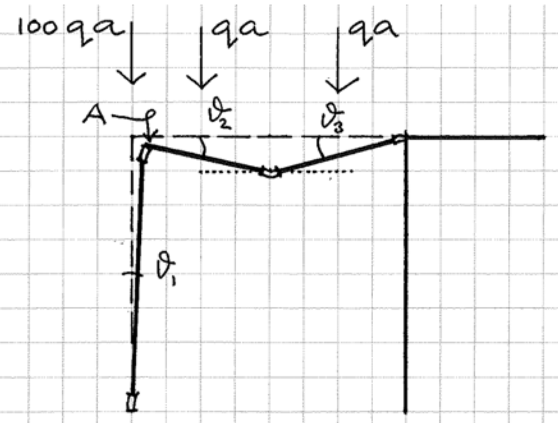
$$0 = 100qa\,2a + q\,2a\,a + \alpha M_p - (1-\alpha) N_p\, 2a - 2M_p$$

The solution to the equations is

$$\alpha = -\frac{1046}{367}, \quad q = \frac{1156}{367}\frac{M_p}{a^2}$$

Note that α is smaller than 0. Clearly, α should have a value between 0 and 1. This is caused by a mistake in the exam problem. The value of β is unrealistically small. In this problem it should have been 605 or larger. For β = 605 the solution is α = 0 and q = 6 M_p / a^2

Answer to Problem 1c Upper-bound



$$u = \vartheta \frac{a}{\beta}$$

horizontal displacement of A →

$$\vartheta_1 2a = (\vartheta_2 + \vartheta_3)\frac{a}{\beta} + \vartheta_3 \frac{a}{\beta}$$

vertical displacement of A ↓

$$\vartheta_1 \frac{a}{\beta} + (\vartheta_2 - \vartheta_1)\frac{a}{\beta} = \vartheta_3 a - \vartheta_2 a$$

$$E = M_p \vartheta_1 + M_p(\vartheta_2 - \vartheta_1) + 2M_p(\vartheta_2 + \vartheta_3) + 2M_p \vartheta_3$$

$$A = qa\,\vartheta_3 \tfrac{1}{2}a + qa(\vartheta_3 a - \vartheta_2 \tfrac{1}{2}a) + 100qa(\vartheta_3 a - \vartheta_2 a)$$

$$E = A$$

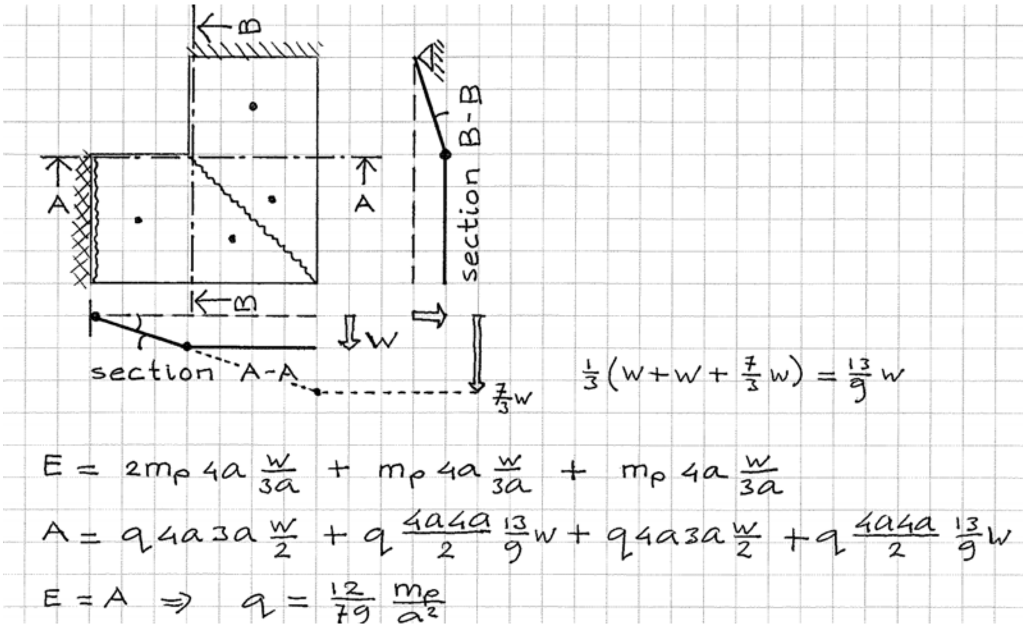
The solution to the equations is

$$\vartheta_2 = \frac{1681}{31}\vartheta_1, \quad \vartheta_3 = \frac{3403}{62}\vartheta_1, \quad q = \frac{1156}{367}\frac{M_p}{a^2} \approx 3.15\frac{M_p}{a^2}$$

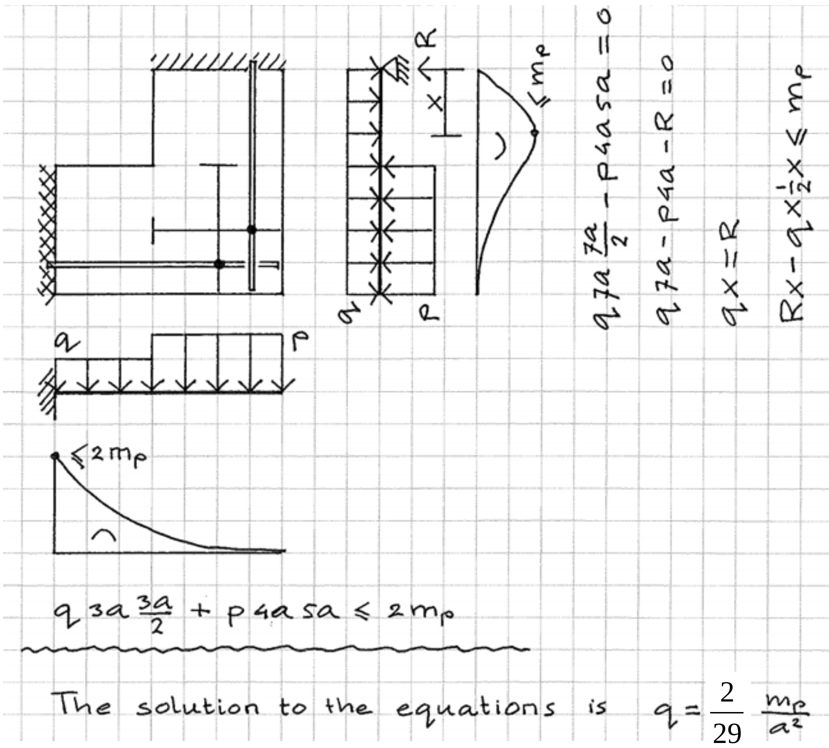
Answer to Problem 2a

Kinematically possible are pattern B and D.

Answer to Problem 2b



Answer to Problem 2c



Answer to Problem 3a
B (see lecture book 1 page 6)

Answer to Problem 3b
B (see handout 8 slide 15)

Answer to Problem 3c
C or D (see handout 8 last slide)